Financial Regulation, Interest Rate Responses, and Distributive Effects[1]

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Abstract

This paper examines financial regulation and corrective taxes in a heterogeneous-agents economy with pecuniary externalities induced by a collateral constraint. A loan-to-value reduction benefits only few unconstrained borrowers and reduces social welfare. A Pigouvian-type debt-tax/savings-subsidy raises collateral prices and lowers interest rates, which stimulates borrowing and generates welfare gains for almost all income groups. A Pigouvian-type asset subsidy induces a wealth appreciation, while an asset tax particularly benefits low-wealth borrowers and enhances social welfare. Overall, collateral effects are of minor importance and interest rate rather than asset price responses are decisive for welfare effects of corrective policies.

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1 Introduction

The recent financial crisis has steered attention toward the interaction between de-leveraging and asset prices. Given that the scope to borrow against collateral crucially depends on the price of (pledgeable) assets, borrowers tend to de-leverage in states where asset prices fall, giving rise to a financial amplification mechanism (see e.g. Kiyotaki and Moore, 1997). This can even be more pronounced when adverse effects of de-leveraging induce a further decline in the price of collateral.[3] Given that (borrowing) agents do not internalize the impact of their behavior on prices, they might tend to overborrow. This pecuniary externality with regard to the collateral price provides a straightforward rationale for macroprudential financial regulation, as for example shown by Lorenzoni (2008), Bianchi (2011), Jeanne and Korinek (2017, 2019), or Bianchi and Mendoza (2018).[4] At the heart of this mechanism are borrowing limits that positively depend on the current price of collateral. As a novel contribution, we examine financial regulation and corrective taxes in a prototype heterogeneous agents framework with collateralized loans,[5] providing an empirically relevant specification of household debt.[6] While previous studies focused on asset prices and collateral effects,[7] our analysis reveals that the welfare consequences of policy interventions in credit and asset markets mainly depend on induced interest rate responses and distributive effects.

For the analysis of financial regulation based on the above mentioned mechanism, existing studies consider the case where borrowing constraints bind only in extreme states (e.g. financial crises). When agents tend to overborrow, policy interventions can be beneficial if they induce agents to borrow less before borrowing constraints become binding. Ideally, policy should not only intervene ex ante, but also ex post to mitigate adverse effects of the financial amplification mechanism (see Benigno et al., 2016, or Jeanne and Korinek, 2017). So far, these analyses have been conducted in a framework with a single endogenous price. They usually employ infinite-horizon small open economy models (based on Mendoza, 2002, or 2010), where a representative domestic agent borrows from abroad, or three-period closed economy models (e.g. Lorenzoni, 2008, or Jeanne and Korinek, 2017) with distinct types of agents, who either borrow or lend. In these studies, agents can only borrow against the current market value of collateral. Given that the interest rate is exogenously fixed, changes in the terms of borrowing are mainly induced by variations of the price of collateral. The above cited studies then typically find that the pecuniary externality regarding the collateral price can be corrected by an ex-ante policy that constrains or dis-incentivizes borrowing, such as a reduction in the loan-to-value ratio or a Pigouvian tax

[3]This mechanism can give rise to "fire sales", when assets are sold at dislocated prices (see Davila and Korinek, 2018).

[4]Based this mechanism, policy interventions with several types of instruments can be justified (Fornaro, 2015, Benigno et al., 2016, Schmitt-Grohé and Uribe, 2017, Korinek, 2018).

[5]Gottardi and Kubler (2015) examine a complete markets model with a collateral constraint and find that tighter restrictions on borrowing can enhance (constrained) efficiency.

[6]Since the 1980s, household debt secured by durable consumption goods (like vehicles or especially residential real estate) has accounted for more that 90% of US household debt in the United States (see Hintermaier and Koeniger, 2016), which we will calibrate our model to. Similar to Diaz and Luengo-Prado (2010), our model can replicate several distributional statistics observed in the data (see Section 3.1). This framework has been shown by Aaronson et al. (2012) to be consistent with individual household consumption behavior (see also Parker et al., 2013). A related model is used by Guerrieri and Lorenzoni (2017) to quantitatively analyze a debt-deleveraging crisis.

[7]An exception is Davila and Korinek (2018) who restrict their study to an analytically tractable (rather than quantitative) framework. We relate our analysis to theirs in Section 2.1.

on borrowing.[8]

In this paper, we assess corrective policies when the fundamental element of this amplification mechanism, namely, a borrowing limit that depends on the current value of collateral, is integrated into an incomplete markets framework (see Huggett, 1993). Individual agents cannot fully ensure against idiosyncratic income risk and might face a binding collateral constraint depending on their stochastic income and their endogenously determined wealth. In this framework, the interest rate is not invariant and the borrowing constraint occasionally binds for individual agents, while it is regularly binding for a non-zero fraction of the population. The model is calibrated to match several aggregate and distributional targets based on US data. In contrast to the above-mentioned studies on financial regulation, we abstract from aggregate risk and aim at addressing the following questions:

1. Is financial regulation in form of a loan-to-value ratio reduction recommendable under an empirically plausible distribution of secured household debt?

2. What are the distributional consequences of financial regulation and corrective taxes in the markets for debt and assets?

3. How important are changes in the interest rate compared to asset prices when borrowing is constrained by the value of assets?

To address these questions, we start with a simplified model version with two types of agents and a time horizon of three periods. This choice is made for comparability with the study of Davila and Korinek (2018), who provide an analysis of pecuniary externalities under financial frictions in a closely related framework. The main differences to their model are that agents face a borrowing constraint that depends on the current value of collateral not only in the second but also in the first period, and that there is no superior borrowers' use for assets (besides serving as collateral). As in Davila and Korinek (2018), the constrained-efficient allocation can be implemented by type-specific Pigouvian taxes on debt and on assets, which are both compensated by a set of type-dependent lump-sum transfers. Davila and Korinek (2018) show that the effect of pecuniary externalities on the collateral constraint "generally entails overborrowing" under "natural conditions", i.e. when asset prices increase with agents' net worth. In contrast, we find that "collateral effects", i.e. effects of pecuniary externalities on the collateral constraint (see Davila and Korinek, 2018), are not unambiguous, since debt market interventions affect borrowers not only ex ante (before they are constrained) but also ex post. The analysis further shows that the design of corrective policies in markets for debt and assets depends on how changes in the asset price as well as in the interest rate exert "distributive effects" via agents' intertemporal choices.

We then examine corrective policies in a calibrated heterogeneous agents model, which is essentially a Huggett (1993) model with (utility-providing) durable goods and a borrowing limit, which is based on limited commitment and depends on the current market value of end-of-period durable goods. To

[8]An exception is Schmitt-Grohé and Uribe (2018), who demonstrate the existence of underborrowing in a small-open economy model with equilibrium multiplicity.

isolate the main effects, the model is kept deliberately simple, while it nevertheless features elements that allow for an empirically relevant specification of household (secured) debt (see Diaz and Luengo-Prado, 2010, Aaronson et al. 2012, or Guerrieri and Lorenzoni, 2017). We calibrate the model to match several aggregate and distributional targets following Diaz and Luengo-Prado (2010). In contrast to their analysis, we endogenize the durables price, which serves as a main object of our analysis. We then use the model to assess price effects and the ability of different types of corrective policies to enhance social welfare.[9] Given that our calibrated (discrete-time) model features both, an endogenous wealth distribution and a borrowing constraint that depends on the market price of collateral, numerical computation of constrained-efficient policies is – to our knowledge – not feasible.[10] For the purpose of our analysis (see questions 1-3 above), we focus on a loan-to-value ratio reduction as a typical measure of financial regulation and anonymous (rather than type-specific) corrective taxes, i.e. Pigouvian-type tax interventions in the markets for debt and durables that equally apply to all agents, and that are intended to manipulate market prices in beneficial ways. Our main findings can be summarized as follows.

First, an unforeseen permanent reduction in the loan-to-value ratio (LTV) has direct and indirect effects. It directly limits the borrowing capacity of constrained agents and thereby leads to a decline in aggregate credit volume, which further causes indirect price effects. The reduction in credit demand leads to a lower equilibrium interest rate. Savers respond by raising their demand for durables as a store of wealth, such that the price of durables increases. The LTV reduction tends to reduce welfare of agents in all income groups. For only few borrowers with relatively high income the indirect price effects dominate the direct effect such that they experience a welfare improvement. These unconstrained agents reduce their borrowing, which contributes to the lower interest rate. Hence, these agents would have been better off under laissez faire if they were able to internalize the interest rate effects of borrowing decision. While the LTV reduction can in principle address this pecuniary externality, social welfare falls.[11] We further find that the effects of the LTV reduction on prices and welfare are slightly more pronounced in an artifical case of a price-inelastic borrowing constraint case where the borrowing limit depends on the value of collateral at a fixed price (at the laissez faire level). Thus, we do not find support for a substantial role of collateral effects in this experiment.

Second, we examine corrective taxes in the debt market. Specifically, we (unexpectedly and permanently) introduce an anonymous tax on debt, implying a subsidy on savings, which is aimed at manipulating market prices by affecting – in contrast to the LTV reduction – both sides of the credit market. Effects on agents' available resources are neutralized by type-specific lump-sum transfers/taxes (as in Davila and Korinek, 2018). Due to this Pigouvian-type tax, lenders tend to save more and borrowers

[9]Following several normative studies in the incomplete markets literature (see Conesa et al., 2009, Krueger et al., 2016, or Nuño and Moll, 2018), we measure social welfare as ex-ante expected lifetime utility, which is identical to utilitarian welfare.

[10]Davila et al (2012) derive constrained-efficient policies for a heterogeneous-agent economy à la Aiyagari (1994), which has an endogenous distribution of wealth but price-inelastic borrowing constraints. Nuño and Moll (2018) propose a numerical strategy for computing constrained-efficient allocations in continuous-time heterogeneous-agent models. They also do not consider environments where market prices enter borrowing constraints.

[11]Gottardi and Kubler (2015) find that tighter restrictions on borrowing can enhance (constrained) efficiency in a model with state-contingent debt and an endogenous collateral constraint.

tend to dis-save less, such that the interest rate declines and the price of durables increases. Compared to a LTV reduction, interest rate responses are relatively more pronounced than collateral price responses. In contrast to the LTV reduction, the debt-tax/saving-subsidy raises rather than lowers the aggregate credit volume, and also induces constrained borrowers to issue more debt. Overall, we find that aggregate welfare in all (except the highest) income states increases after this intervention and that it enhances social welfare. Specifically, borrowers tend to gain and lenders tend to lose from the decline in the interest rate. Thus, this policy intervention induces price changes that serve as partial insurance for borrowers from an ex-ante perspective, which is not internalized by individual agents in the laissez-faire economy. The analysis further shows that interest rate responses are more relevant than collateral price responses for the overall welfare results under the debt-tax/saving-subsidy.[12]

Third, we (unexpectedly and permanently) introduce a Pigouvian-type tax/subsidy on end-of-period holdings of durables. As a direct effect, a tax on durables dis-incentivizes purchases of durables and lowers their price. Agents substitute investment in durables in favor of investment in bonds, such that the interest rate decreases. Thus, low-wealth borrowers tend to benefit from the intervention, whereas high-wealth savers tend to lose. Like the tax on debt, the durables tax induces price changes that partially insure borrowers from an ex-ante perspective. Overall, the durables tax induces an increase in social welfare over the transition phase as well as in the long run. Notably, the welfare gain is higher under a price-inelastic borrowing constraint (since the borrowing constraint is not tightened by the lower durables price), indicating non-negligible collateral effects. Yet, the associated increase in borrowing reflects the decisive role of interest rate responses. By contrast to the durables tax, the social welfare effects of a Pigouvian-type subsidy on durables differ between the short run and the long run. Due to the increase in the durables price and in the interest rate low-wealth agents tend to lose and high-wealth agents tend to gain in the long run.[13] In the long run, the adverse effects on the former group dominate, such that the subsidy leads to a social welfare loss. Immediately after the introduction of the subsidy, when the distribution of bonds and durables is not yet adjusted, all income groups tend to gain from the wealth increase induced by the durables price appreciation. Given that a higher durables price further benefits low income agents who sell durables, social welfare including the transition phase increases under a durables subsidy. These results indicate that the distributive effects of corrective policy interventions are decisive and can qualitatively differ between the short run and the long run.

The Pigouvian-type tax interventions in the markets for debt and durables were scaled to induce equally-sized effects on the long-run price of collateral. We then observe that the simultaneous change in the interest rate is much more pronounced under the tax/subsidy on debt. More specifically, whereas a tax on debt of 5% lowers the long-run real interest rate by 5.9 percentage points, a subsidy on durables of 0.6%, which results in the same long-run price of durables, yields an interest rate increase of 0.7 percentage

[12]Consistently, we find that price and welfare effects are almost identical under a price-inelastic borrowing constraint, indicating a negligible role of collateral effects.

[13]The latter do not internalize that raising their holdings of durables contributes to a beneficial increase in the durables price, which can be addressed by the durables subsidy.

points. We further find that the overall welfare effects of the former tax is about 20-times larger than under the tax/subsidy on durables. Hence, the impact of corrective policies on the price of collateral is much less relevant than the impact on the interest rate in an empirically relevant model of household (collateralized) debt. This finding suggests that the role of collateral price effects is overestimated in studies on financial regulation where credit supply and interest rates changes are disregarded.

The remainder is structured as follows. Section 2 develops the simplified model, examines the constrained-efficient allocation, and describes issues of its implementation. Section 3 describes the Huggett (1993)-type model and its calibration, and presents results for our policy experiments. Section 4 concludes.

2 A model with limited commitment and incomplete markets

In this section, we develop a basic framework with financial frictions and assess effects of policy intervention with corrective instruments in an analytical way. As main ingredients for our analysis, the model features heterogeneous agents and a financial constraint that can induce inefficiencies due to pecuniary externalities. We examine government interventions in markets for debt and assets (here, durable goods). We start with a two-agent and three-period framework, which facilitates the derivation of analytical results and direct comparisons with Davila and Korinek (2018). In Section 3, we extend the analysis to an infinite-horizon model, based on Huggett (1993), where the status of each agents is endogenous, and examine the effects of policy interventions numerically.

We assume that financial markets are incomplete and that the only financial asset is a non-state-contingent one-period bond. A bond issued in period t trades at price $1/r_t$ and promises the payment of one unit of a non-durable good, which serves as the numeraire in the model, in period $t+1$. We further assume that there exists a financial friction, which gives rise to a borrowing constraint that can induce pecuniary externalities. Specifically, we assume that borrowers cannot commit to repay debt and that debt can be renegotiated after issuance in the same period. We allow borrowers to make a take-it-or-leave-it offer to reduce the value of debt. If the lender rejects the offer, he can seize a fraction γ of the borrower's assets (durable goods), which he can sell at the competitive market price q_t. Offers are therefore accepted when the repayment value of debt at least equals the current value of seizable assets. Without loss of generality, we assume that default and renegotiation never happen in equilibrium. Hence, when debt is issued, an individual borrower i has to take into account that the amount of debt $-b_{i,t}$ is constrained by

$$-b_{i,t} \leq \gamma q_t d_{i,t}, \tag{1}$$

where $d_{i,t}$ denotes the amount of the asset (durable good) held during the debt contract.

Given that renegotiation of debt issued in period t takes place in period t rather than in the subsequent period $t+1$, the borrowing constraint (1) features the price of the asset for the period of issuance q_t. This type of borrowing constraint is shared by many recent studies on macroprudential regulation (see, e.g., Stein, 2012, Jeanne and Korinek, 2017, or Bianchi and Mendoza, 2018), it is also common in quantitative studies with collateralized debt (see e.g. Favilukis et al., 2017, Lorenzoni and Guerrieri, 2017, or Berger et al., 2018), and it is consistent with empirical evidence (see Cloyne et al., 2019).

The borrowing constraint (1) can generate a feedback from sales of durables and price declines to a reduction of the debt limit, which induces borrowers to de-leverage. Given that the effects of individual behavior on the price of durables are not internalized, pecuniary externalities can be relevant for the allocation of resources. If, for example, the price of durables increases with borrowers' net wealth, agents tend to *overborrow* (see Davila and Korinek, 2018). In this case, borrowers do not internalize price effects of a higher debt burden, such that efficiency can principally be enhanced by corrective ex-ante policies that limit the build-up of debt.

2.1 A finite-horizon model with two types of agents

We develop a model that is structured to facilitate comparisons with the analysis of Davila and Korinek (2018). Our model essentially differs from theirs by *i.*) considering price dependent borrowing limits not only in the second, but also in the first period and *ii.*) by neglecting borrowers' prior use of assets (beyond their ability to serve as collateral). We consider three periods $t = 1, 2, 3$ and two mass-one groups $\{b, l\}$ with infinitely many households each. In each period t, a household $i \in \{b, l\}$ derives utility from consumption of a non-durable good, $c_{i,t}$, and a durable good, $d_{i,t}$, as given by the function

$$u_{i,t} = u(c_{i,t}, d_{i,t}), \tag{2}$$

which is increasing and concave with respect to both arguments. The budget constraint of a household i for period t is given by

$$c_{i,t} + q_t(d_{i,t} - d_{i,t-1}) + b_{i,t}/r_t = b_{i,t-1} + y_{i,t}, \tag{3}$$

where $y_{i,t}$ denotes the household's exogenous endowment of non-durable goods. Households of group b have initial assets $b_{b,0}$ and $d_{b,0}$ and an initial endowment $y_{b,1}$, and households of group l initial assets $b_{l,0}$ and $d_{l,0}$ and an initial endowment $y_{l,1}$. Households of the former (latter) group will be called borrowers (lenders). In period 2, the state of nature is random $\omega \in \{u, e\}$. With probability p, state u realizes and households face an unequal distribution of endowment (u), where borrowers receive $y_{b,2}$ and lenders $y_{l,2}$, such that they do not change roles. In state e, which realizes with probability $1 - p$, both types of households receive an equal endowment y. In period 3, endowment is identical for all households and given by y.

As discussed above, limited commitment implies that borrowing (in periods 1 and 2) is restricted by the collateral constraint (1). A household i aims at maximizing expected lifetime utility, given by $E_1[\sum_{t=1}^{T} \beta^{t-1} u(c_{i,t}, d_{i,t})]$, where $T = 3$ and E_1 denotes an expectation operator, subject to (1) and (3) for a given initial endowment $b_{i,0} = 0\ \forall i$ and $d_{i,0} > 0\ \forall i$. The first-order conditions for consumption, durables, and debt for $i \in \{b, l\}$ can be summarized as

$$u'_c(c_{i,1}, d_{i,1})q_1 = u'_d(c_{i,1}, d_{i,1}) + \beta \left(pq_2^u u'_c(c_{i,2}^u, d_{i,1}^u) + (1-p)q_2^e u'_c(c_{i,2}^e, d_{i,1}^e)\right) + \mu_{i,1}\gamma q_1, \quad (4)$$

$$u'_c(c_{i,1}, d_{i,1})/r_1 = \beta \left(pu'_c(c_{i,2}^u, d_{i,2}^u) + (1-p)\, u'_c(c_{i,2}^e, d_{i,2}^e)\right) + \mu_{i,1}, \quad (5)$$

$$u'_c(c_{i,2}^\omega, d_{i,2}^\omega)q_2^\omega = u'_c(c_{i,2}^\omega, d_{i,2}^\omega) + \beta q_3^\omega u'_c(c_{i,3}^\omega, d_{i,3}^\omega) + \mu_{i,2}^\omega \gamma q_2^\omega, \quad (6)$$

$$u'_c(c_{i,2}^\omega, d_{i,2}^\omega)/r_2^\omega = \beta u'_c(c_{i,3}^\omega, d_{i,3}^\omega) + \mu_{i,2}^\omega, \quad (7)$$

$$u'_c(c_{i,3}^\omega, d_{i,3}^\omega)q_3^\omega = u'_d(c_{i,3}^\omega, d_{i,3}^\omega), \quad (8)$$

where $\mu_{i,t}^\omega \geq 0$ denotes the multiplier on the collateral constraint (1), which satisfies $\mu_{l,1} = \mu_{l,2}^\omega = \mu_{b,2}^e = 0$ as well as $\mu_{b,1}(b_{b,1} + \gamma q_1^e d_{b,1}) = 0$ and $\mu_{b,2}^u(b_{b,2}^u + \gamma q_2^u d_{b,2}) = 0$. For lenders, the multipliers $\mu_{l,1}$ and $\mu_{l,2}$ equal zero as well as the multiplier for the borrower in state $\omega = e$, $\mu_{b,2}^e = 0$.

To close the model, we assume that the supply of durables in each period is fixed and equal to $\bar{d}$. A competitive equilibrium can then be defined as follows: A competitive equilibrium is given by an allocation of durables, non-durables, and debt $\{c_{i,1}, d_{i,1}, b_{i,1}, c_{i,2}^\omega, d_{i,2}^\omega, b_{i,2}^\omega, c_{i,3}^\omega, d_{i,3}^\omega\}\ \forall i$, a set of prices $\{r_1, r_2^\omega, q_1, q_2^\omega, q_3^\omega\}$ and multipliers $\{\mu_{b,1}, \mu_{b,2}^u\}$, satisfying the budget constraints (3) $\forall t$, (4)-(8), the market clearing conditions $d_{b,t} + d_{l,t} = \bar{d}\ \forall t$, and $b_{b,t} + b_{l,t} = 0$ for $t \in \{1, 2\}$, and the collateral constraints (1) for $i = b$ and $t \in \{1, 2\}$ with $\mu_{b,1}(b_{b,1} + \gamma q_1 d_{b,1}) = 0$ and $\mu_{b,2}^\omega(b_{b,2}^\omega + \gamma q_2^\omega d_{b,2}) = 0$, given an initial distribution of assets and sequences of endowments.[14]

2.2 Pecuniary externalities and corrective policies

To assess the potential for welfare improvements, we consider the problem of a social planner who faces the households' constraints and optimal price taking behavior in a competitive equilibrium. We restrict our attention to the case, where the social planner is not able to implement first best, but a constrained-efficient allocation. Thus, we aim at identifying how changes in household behavior with regard to borrowing/lending and purchases of durables, which are induced by a specific set of instruments, can improve upon the equilibrium allocation under laissez faire. To facilitate comparisons, we thereby closely follow the set-up of the policy problem of Davila and Korinek (2018).

We consider two types of instruments. The social planner has access to a tax (subsidy) on borrowing $\tau_b^b > 0$ ($\tau_b^b < 0$) and a tax (subsidy) on borrowers' purchases of durables $\tau_b^d > 0$ ($\tau_b^d < 0$). Both instruments are only applied in period 1. We disregard further instruments, e.g. taxes on lenders, which would render implementation of first best possible.[15] We assume that these taxes/subsides are introduced at constant rates and are fully compensated by type-specific transfers/taxes, such that the instruments are purely corrective and exclusively affect the perceived after-tax prices for borrowers. The tax on borrowing

[14] Appendix A describes the equilibrium solution.

[15] A subsidy on durables can for example stimulate the collateral value such that collateral constraint never binds. Davila and Korinek (2018) allow for taxes on borrowers and lenders. The policy problem is nonetheless non-trivial, since the borrowing constraint in period 1 does not depend on prices.

represents regulatory instruments on borrowing, which has been the focus of studies on macroprudential regulation under pecuniary externalities (see e.g. Bianchi, 2011, Benigno et al., 2016, or Bianchi and Mendoza, 2018). While the latter studies feature just one relevant endogenous price (namely, the price of collateral), we introduce a second instrument to be able to address pecuniary externalities related to the two endogenous prices in our model, q and r.[16]

Given this set of instruments, the social planner can choose the period 1 allocation $\bar{c}_{b,1}, \bar{c}_{l,1}, \bar{d}_{b,1}, \bar{d}_{l,1}, \bar{b}_{b,1}$, and $\bar{b}_{l,1}$ subject to the budget constraints of all agents, market clearing conditions, and the lenders' first-order conditions for the period 1, while accounting for the optimizing behavior of private agents for periods 2 and 3, summarized by $V_2^{i,\omega}(\cdot)$ (see Appendix A). The social planner problem then reads

$$\max_{\{\bar{c}_{i,1}, \bar{d}_{i,1}, \bar{b}_{i,1}\}_{i\in\{b,l\}}} \sum_i \theta_i \left\{ u_1(\bar{c}_{i,1}, \bar{d}_{i,1}) + \beta E_1 \left[V_2^{i,\omega}(\bar{d}_{i,1}, \bar{b}_{i,1}; \bar{d}_1, \bar{b}_1) \right] \right\} \tag{9}$$

$$\begin{aligned} \text{s.t. } & y_{b,1} = \bar{c}_{b,1} + q_1(\bar{d}_{b,1} - \bar{d}_{b,0}) + \bar{b}_{b,1} r_1^{-1}, \; y_{l,1} = \bar{c}_{l,1} + q_1(\bar{d}_{l,1} - \bar{d}_{l,0}) + \bar{b}_{l,1} r_1^{-1}, \\ & -\bar{b}_{b,1} \leq \gamma q_1 \bar{d}_{b,1}, \; \bar{d}_{b,1} + \bar{d}_{l,1} = \bar{d}, \; \bar{b}_{b,1} + \bar{b}_{l,1} = 0, \\ & u_c'(\bar{c}_{l,1}, \bar{d}_{l,1}) q_1 = u_d'(\bar{c}_{l,1}, \bar{d}_{l,1}) + \beta E_1 [q_2^\omega u_c'(\bar{c}_{l,2}^\omega, \bar{d}_{l,2}^\omega)], & \text{(L1)} \\ & u_c'(\bar{c}_{l,1}, \bar{d}_{l,1}) r_1^{-1} = \beta E_1 \left[u_c'(\bar{c}_{l,2}^\omega, \bar{d}_{l,2}^\omega) \right], & \text{(L2)} \end{aligned}$$

where θ_i denotes the Pareto weight assigned to household type i by the social planner. For the quantitative analysis in Section 3, we consider ex-ante expected utility, i.e. Pareto weights that reflect a household's ex-ante probability of being a specific type. Since the planner knows that all agents of type $i \in \{b, l\}$ act identically, the individual states, $d_{i,1}$ and $b_{i,1}$, can as arguments of the continuation value $V_2^{i,\omega}(\cdot)$ be replaced by the type-specific aggregate states, $\bar{d}_{i,1}$ and $\bar{b}_{i,1}$ in the policy problem (9). The distinction between individual and aggregate states is relevant to understand why the social planner solution might differ from the competitive one. The problem for the social planner can be written as

$$\begin{aligned} \mathcal{L} \equiv & \sum_i \theta_i \left\{ u_1(\bar{c}_{i,1}, \bar{d}_{i,1}) + \beta E_1 \left[V_2^{i,\omega}(\bar{d}_{i,1}, \bar{b}_{i,1}; \bar{d}_1, \bar{b}_1) \right] \right\} & (10) \\ & + \theta_b \lambda_{b,1}^{bud} \left[y_{b,1} - \bar{c}_{b,1} - q_1(\bar{d}_{b,1} - \bar{d}_{b,0}) - \bar{b}_{b,1} r_1^{-1} \right] \\ & + \theta_l \lambda_{l,1}^{bud} \left[y_{l,1} - \bar{c}_{l,1} - q_1(\bar{d}_{l,1} - \bar{d}_{l,0}) - \bar{b}_{l,1} r_1^{-1} \right] \\ & + \theta_b \mu_{b,1} \left[\bar{b}_{b,1} + \gamma q_1 \bar{d}_{b,1} \right], \end{aligned}$$

where we use the market clearing conditions for debt, $\bar{b}_{l,1} = -\bar{b}_{b,1}$, and durables, $\bar{d} = \bar{d}_{l,1} + \bar{d}_{b,1}$, implying for the aggregate state variables $\bar{d}_1 = \left(\bar{d}_{b,1}, \bar{d} - \bar{d}_{b,1}\right)$ and $\bar{b}_1 = \left(\bar{b}_{b,1}, -\bar{b}_{b,1}\right)$. Furthermore, the planner takes into account that prices are functions of the allocation and implicitly given by (L1) and (L2). In contrast to the first period, the allocation in the second period is determined by the laissez-faire equilibrium solution, given the beginning-of-period-2 states, that the planner can control in the first

[16] Notably, instrument *ii.)* can alternatively be specified as a single tax/subsidy on purchases of durables of both agents (borrowers and lender), where the borrowing constraint depends on the after tax price of collateral.

period. This dependence is captured by $V_2^{i,\omega}(\bar{d}_{i,1}, \bar{b}_{i,1}; \bar{d}_1, \bar{b}_1)$. Notably, period-1-prices r_1 and q_1 not only depend on lenders' non-durables $\bar{c}_{l,1}$ and durables $\bar{d}_{l,1}$, but also on the aggregate states $\bar{d}_1$ and $\bar{b}_1$ via their impact on next period's equilibrium objects. Moreover, the end-of-period 1 aggregate states alter next period prices q_2^ω and r_2^ω via their impact through the continuation values $V_2^{l,\omega}(\cdot)$ and $V_2^{b,\omega}(\cdot)$. These price effects, i.e. $\frac{\partial q_1}{\partial \bar{x}_{j,1}}$, $\frac{\partial r_1^{-1}}{\partial \bar{x}_{j,1}}$, $\frac{\partial q_2^\omega}{\partial \bar{x}_{j,1}}$, and $\frac{\partial (r_2^\omega)^{-1}}{\partial \bar{x}_{j,1}}$ for $\bar{x} \in \{\bar{d}, \bar{b}\}$ and $j \in \{b, l\}$, are not internalized by any individual agent, but by the social planer, who accounts for their impact via the agents' budget constraints and the borrowing constraint. These effects need to be distinguished from those effects that are internalized by the households, which correspond to the derivatives of $V_2^{i,\omega}(\cdot)$ with respect to the individual state $x_{i,1}$, which however coincides with $\bar{x}_{i,1}$ in equilibrium.

The social planner's first-order conditions for $\bar{d}_{b,1}$ and $\bar{b}_{b,1}$ can be written as (see Appendix B)

$$u_c'(\bar{c}_{b,1}, \bar{d}_{b,1})q_1 = u_d'(\bar{c}_{b,1}, \bar{d}_{b,1}) + \beta E_1 \left[u_c'(\bar{c}_{b,2}^\omega, \bar{d}_{b,2}^\omega)q_2^\omega\right] + \mu_{b,1}\gamma q_1 + \Delta_1^d + \beta E_1 \left[\Delta_2^{d,\omega}\right], \tag{11}$$

$$u_c'(\bar{c}_{b,1}, \bar{d}_{b,1})r_1^{-1} = \beta E_1 \left[u_c'(\bar{c}_{b,2}^\omega, \bar{d}_{b,2}^\omega)\right] + \mu_{b,1} + \Delta_1^b + \beta E_1 \left[\Delta_2^{b,\omega}\right]. \tag{12}$$

These first-order conditions solely deviate from those of the competitive equilibrium by the wedges Δ_1^x and $\Delta_2^{x,\omega}$. They capture the social value that changes of $\bar{x}_{b,1}$ and – due to market clearing – of $\bar{x}_{l,1}$ have because of their impact on first and second period prices $(q_1, 1/r_1)$ and $(q_2^\omega, 1/r_2^\omega)$. The wedges $\Delta_2^{x,\omega}$ account for the marginal effect that an increase in $\bar{x}_1$ has on prices in the second period, conditional on the realization of ω. Since these prices only respond to aggregate quantities, the wedges depend on the derivatives of $V_2^{b,\omega}(\cdot)$ and $V_2^{l,\omega}(\cdot)$ with respect to the aggregate states, $\Delta_2^{x,\omega} = \frac{\partial V^{b,\omega}}{\partial \bar{x}_1}\frac{\partial \bar{x}_1}{\partial \bar{x}_{b,1}} - \frac{\partial V^{b,\omega}}{\partial \bar{x}_1}\frac{\partial \bar{x}_1}{\partial \bar{x}_{l,1}} + \frac{\theta_l}{\theta_b}\frac{\partial V^{l,\omega}}{\partial \bar{x}_1}\frac{\partial \bar{x}_1}{\partial \bar{x}_{b,1}} - \frac{\theta_l}{\theta_b}\frac{\partial V^{l,\omega}}{\partial \bar{x}_1}\frac{\partial \bar{x}_1}{\partial \bar{x}_{l,1}}$. The wedges for the first period Δ_1^x account for the marginal effects of changes in $\bar{x}_1$ on first period prices via agents' choices in the first period (L1) and (L2). Specifically, the wedges Δ_1^x and $\Delta_2^{x,\omega}$ are given by

$$\Delta_1^x = \left(u_c'(\bar{c}_{b,1}, \bar{d}_{b,1}) - \frac{\theta_l}{\theta_b}u_c'(\bar{c}_{l,1}, \bar{d}_{l,1}) - \mu_{b,1}C_{c_{l,1}}^b\right)\frac{D_{x_{b,1}}^b - D_{x_{l,1}}^b}{1 + D_{c_{l,1}}^b} + \mu_{b,1}\left(C_{x_{b,1}}^b - C_{x_{l,1}}^b\right), \tag{13}$$

$$\Delta_2^{x,\omega} = \left(u_c'(\bar{c}_{b,2}^\omega, \bar{d}_{b,2}^\omega) - \frac{\theta_l}{\theta_b}u_c'(\bar{c}_{l,2}^\omega, \bar{d}_{l,2}^\omega)\right)\left(D_{x_{b,2}}^{b,\omega} - D_{x_{l,2}}^{b,\omega}\right) + \mu_{b,2}^\omega\left(C_{x_{b,1}}^{b,\omega} - C_{x_{l,1}}^{b,\omega}\right), \tag{14}$$

for $x \in \{b, d\}$. Following Davila and Korinek (2018), effects of un-internalized price changes are separated in *distributive effects* (D), which affect the budget sets of agents, and *collateral effects* (C), which affect the borrowing constraint:

$$D_{c_{l,1}}^b = -\frac{\partial q_1}{\partial \bar{c}_{l,1}}(\bar{d}_{b,1} - \bar{d}_{b,0}) - \frac{\partial r_1^{-1}}{\partial \bar{c}_{l,1}}\bar{b}_{b,1},$$

$$D_{x_{j,1}}^b = -\frac{\partial q_1}{\partial \bar{x}_{j,1}}(\bar{d}_{b,1} - \bar{d}_{b,0}) - \frac{\partial r_1^{-1}}{\partial \bar{x}_{j,1}}\bar{b}_{b,1},$$

$$D_{x_{j,2}}^{b,\omega} = -\frac{\partial q_2^\omega}{\partial \bar{x}_{j,1}}(\bar{d}_{b,2}^\omega - \bar{d}_{b,1}) - \frac{\partial \left(r_2^\omega\right)^{-1}}{\partial \bar{x}_{j,1}}\bar{b}_{b,2}^\omega,$$

$$C_{c_{l,1}}^b = \gamma\frac{\partial q_1}{\partial \bar{c}_{l,1}}\bar{d}_{b,1}, \ C_{x_{j,1}}^b = \gamma\frac{\partial q_1}{\partial \bar{x}_{j,1}}\bar{d}_{b,1}, \ C_{x_{j,1}}^{b,\omega} = \gamma\frac{\partial q_2^\omega}{\partial \bar{x}_{j,1}}\bar{d}_{b,2}^\omega.$$

The effects of pecuniary externalities in this model differ from those in Davila and Korinek (2018), since prices here are also relevant in period 1 for the social planer, whereas only second period prices matter in their analysis (which corresponds to $\Delta_1^x = 0$) As a consequence, we have to account additionally for the un-internalized effects of changes in $\bar{c}_{l,1}$, $\bar{d}_{l,1}$, $\bar{d}_{b,1}$, $\bar{b}_{b,1}$ and $\bar{b}_{l,1}$ on period 1 prices. This for example implies that the shadow price of the lender's budget constraint in the first period in (10), $\lambda_{l,1}^{bud}$, deviates from $u_c'(\bar{c}_{l,1})$. Given that we only consider taxes/subsidies on borrowers, the relative effects $D_{x_{b,1}}^{b,\omega} - D_{x_{l,1}}^{b,\omega}$ and $C_{x_{b,1}}^b - C_{x_{l,1}}^b$ show up in the externality terms of the social planner's first-order conditions for $x_{b,1}$, since raising $x_{b,1}$, the planner also directly lowers $x_{l,1}$ because of fixed aggregate supply and market clearing.[17] Apparently, the collateral effects in periods 1 and 2 only apply to household types $i = b$ since $\mu_{l,1} = \mu_{l,2}^{\omega} = 0$ holds by construction. Further note that the distributive effects on the budget sets of borrowers and lenders are – as in Davila and Korinek (2018) – symmetric, such that $\sum_{i=b,l} D_{x_{j,1}}^i = \sum_{i=b,l} D_{x_{j,2}}^{i,\omega} = 0$ where $D_{x_{j,1}}^i = -\frac{\partial q_1}{\partial \bar{x}_{j,1}}(\bar{d}_{i,1} - \bar{d}_{i,0}) - \frac{\partial r_1^{-1}}{\partial \bar{x}_{j,1}}\bar{b}_{i,1}$ and $D_{x_{j,2}}^{i,\omega} = -\frac{\partial q_2^{\omega}}{\partial \bar{x}_{j,1}}(\bar{d}_{i,2}^{\omega} - \bar{d}_{i,1}) - \frac{\partial (r_2^{\omega})^{-1}}{\partial \bar{x}_{j,1}}\bar{b}_{i,2}^{\omega}$ for $i, j \in \{b, l\}$. Given that the social planer has access to borrower-specific Pigouvian tax instruments, the borrowers' first-order conditions are given by

$$
\begin{aligned}
(1 + \tau_b^d) u_c'(\bar{c}_{b,1}, \bar{d}_{b,1}) q_1 &= u_d'(\bar{c}_{b,1}, \bar{d}_{b,1}) + \beta E_1 \left[u_c'(\bar{c}_{b,2}^{\omega}, \bar{d}_{b,2}^{\omega}) q_2^{\omega} \right] + \mu_{b,1} \gamma q_1, \\
(1 - \tau_b^b) u_c'(\bar{c}_{b,1}, \bar{d}_{b,1}) r_1^{-1} &= \beta E_1 \left[u_c'(\bar{c}_{b,2}^{\omega}, \bar{d}_{b,2}^{\omega}) \right] + \mu_{b,1}.
\end{aligned}
$$

Comparing these two conditions with the first-order conditions of the social planer (11) and (12) shows that the constrained-efficient allocation can be implemented by tax rates satisfying

$$
\tau_b^b = \frac{\Delta_1^b + \beta E_1 \left[\Delta_2^{b,\omega} \right]}{u_c'(\bar{c}_{b,1}) r_1^{-1}} \quad \text{and} \quad \tau_b^d = -\frac{\Delta_1^d + \beta E_1 \left[\Delta_2^{d,\omega} \right]}{q_1 u_c'(\bar{c}_{b,1})}, \tag{15}
$$

and by associated lump-sum taxes/transfers, which neutralize the effects of τ_b^d and τ_b^b on the borrowers' budget set.

Collateral effects The signs of the wedges Δ_1^x and $\Delta_2^{x,\omega}$ depend on a variety of potentially opposed price effects as well as on differences in the valuation of funds between agents and their asset positions (see 13 and 14). For example, an increase in durables held by lenders at the end of period 1 tends to alter the current price q_1 and the future price q_2^{ω} in opposite ways (see L2). Apparently, the collateral effects in period 1 are only relevant if the borrowing constraint is binding in period 1, $\mu_{b,1} > 0$. These externalities are evidently absent when the borrowing constraint does not depend on the price of collateral in period 1 (as in Davila and Korinek, 2018), such that $C_{c_{l,1}}^b = C_{x_{j,1}}^b = C_{x_{j,1}}^{b,\omega} = 0$. Here, the sign of the collateral effects depends on sign of the derivatives $\partial q_1/\partial \bar{c}_{l,1}$, $\partial q_1/\partial \bar{x}_{b,1}$, $\partial q_1/\partial \bar{x}_{l,1}$, $\partial q_2^{\omega}/\partial \bar{x}_{b,1}$, and $\partial q_2^{\omega}/\partial \bar{x}_{l,1}$. To assess the impact of the collateral effects on the tax/subsidy on borrowing, suppose that the following

[17] If, as in Davila and Korinek (2018), taxes/subsidies were also imposed on lenders, the solution to his problem would involve separate first order conditions for $x_{b,1}$ and $x_{l,1}$.

inequality holds (which relates to Condition 1 in Davila and Korinek, 2018)

$$\gamma\mu^{\omega}_{b,2}\left(\frac{\partial q^{\omega}_2}{\partial\bar{b}_{b,1}}-\frac{\partial q^{\omega}_2}{\partial\bar{b}_{l,1}}\right)\bar{d}^{\omega}_{b,2}>0. \tag{16}$$

Then, the collateral externality included in $\Delta_2^{b,\omega}$ (see 14) calls for a tax on borrowing $\tau_b^b > 0$ (see 15) as implied by studies on macroprudential regulation where agents tend to overborrow (see e.g. Bianchi and Mendoza, 2018). The inequality (16) requires that the increasing impact of a lower debt position at the beginning of period 2 $\bar{b}_{b,1}$ (or higher bond holdings) on the collateral price q_2^{ω} in period 2 is larger than the simultaneous decreasing impact of the reduction in lenders' bond holdings $\bar{b}_{l,1}$. Then, the borrowers' increased willingness to spend on durables dominates the response of lenders. Now consider the effects of a change in the borrowers' (lenders') bond position $\bar{b}_{b,1}$ ($\bar{b}_{l,1}$) at the end of period 1 on prices in period 1. Focusing on the impact on available resources in a particular period, a change in the end-of-period bond holdings tends to lead to price effects that are opposed to the price effects of equal changes in the beginning-of-period bond holdings. Thus, the term representing the collateral effects in the wedge Δ_1^b (see 13), i.e.

$$\gamma\mu_{b,1}\left(\frac{\partial q_1}{\partial\bar{b}_{b,1}}-\frac{\partial q_1}{\partial\bar{b}_{l,1}}\right)\bar{d}_{b,1}, \tag{17}$$

tends to be negative under (16). This tendency is further strengthened by the fact that an increase in $\bar{b}_{b,1}$ (decline in end-of-period debt) tends to reduce borrowers' marginal valuation of durables as collateral. Hence, even if agents tend to overborrow, as they do not internalize the adverse impact of de-leveraging on the collateral price in period 2, this externality is not sufficient to imply a tax on borrowing, if borrowers are already constrained in period 1, $\mu_{b,1} > 0$. Apparently, the size of the effects given in (16) and (17) also depend on the tightness of the borrowing constraints and the stock of durables held by borrowers in both periods, such that the total impact of the collateral effects on the tax rate τ_b^b (see 15) is ambiguous. In our numerical analysis (see Section 3.1), we actually find that distributive effects rather than collateral effects are decisive for the welfare effects of interventions in the credit market.

Distributive effects Now suppose that the borrowing limit were price-inelastic, i.e. $-\gamma\overline{q}d_{i,t}$, with a fixed price $\overline{q} > 0$ rather than an endogenous price q_t given by the RHS of (1). This assumption will also be used as a reference case in the subsequent quantitative analysis. Based on the quantitative results, which will be presented in Section 3, we will conclude that the effects of policy interventions under the price-inelastic borrowing constraint are very similar to those under the price-elastic collateral constraint (1). Under a price-inelastic borrowing constraint, the wedges Δ_1^b and $\Delta_2^{b,\omega}$, which are relevant for the borrowing tax τ^b (see 15), would solely depend on the distributive effects, i.e.

$$\Delta_1^b = \frac{u_c'(\bar{c}_{b,1}, \bar{d}_{b,1}) - \frac{\theta_l}{\theta_b} u_c'(\bar{c}_{l,1}, \bar{d}_{l,1})}{1 - \frac{\partial q_1}{\partial \bar{c}_{l,1}}(\bar{d}_{b,1} - \bar{d}_{b,0}) - \frac{\partial r_1^{-1}}{\partial \bar{c}_{l,1}} \bar{b}_{b,1}} \times \left[\left(\frac{\partial q_1}{\partial \bar{b}_{l,1}} - \frac{\partial q_1}{\partial \bar{b}_{b,1}} \right) \{\bar{d}_{b,1} - \bar{d}_{b,0}\} + \left(\frac{\partial r_1^{-1}}{\partial \bar{b}_{l,1}} - \frac{\partial r_1^{-1}}{\partial \bar{b}_{b,1}} \right) \bar{b}_{b,1} \right], \tag{18}$$

$$\Delta_2^{b,\omega} = \left(u_c'(\bar{c}_{b,2}^\omega, \bar{d}_{b,2}^\omega) - \frac{\theta_l}{\theta_b} u_c'(\bar{c}_{l,2}^\omega, \bar{d}_{l,2}^\omega) \right) \times \left[\left(\frac{\partial q_2^\omega}{\partial \bar{b}_{l,1}} - \frac{\partial q_2^\omega}{\partial \bar{b}_{b,1}} \right) \{\bar{d}_{b,2}^\omega - \bar{d}_{b,1})\} + \left(\frac{\partial \left(r_2^\omega\right)^{-1}}{\partial \bar{b}_{l,1}} - \frac{\partial \left(r_2^\omega\right)^{-1}}{\partial \bar{b}_{b,1}} \right) \bar{b}_{b,2}^\omega \right]. \tag{19}$$

The wedges Δ_1^b and $\Delta_2^{b,\omega}$, which are decisive for the size and sign of the borrowing tax, depend not only on un-internalized changes in the interest rates $\{r_1,\ r_2^\omega\}$ that are induced by changes in debt/savings $\{\bar{b}_{b,1},\ \bar{b}_{l,1}\}$, but also on their un-internalized impact on the durables prices $\{q_1, q_2^\omega\}$ (see the terms in the square brackets in 18 and 19). The subsequent quantitative analysis will reveal that interventions in the credit market alter the interest rate to a larger extent than the durables price, and that social welfare effects will be dominated by the former price effects. Consider, for example, the case where the first factor on the RHS of (18) is positive.[18] Given that the price of debt r_1^{-1} tends to increase with savings and to decrease with borrowing, such that $(\partial r_1^{-1}/\partial \bar{b}_{l,1}) - (\partial r_1^{-1}/\partial \bar{b}_{b,1}) > 0$ holds, and that borrowing implies $\bar{b}_{b,1} < 0$, the wedge Δ_1^b would then call for a subsidy, $\tau^b < 0$, rather than a tax on debt.

Likewise, the wedges that are relevant for the sign of the tax on durables τ^d, also depend on un-internalized changes of the price of durables and the interest rate as well as the allocation of bonds and durables,

$$\Delta_1^d = \frac{u_c'(\bar{c}_{b,1}, \bar{d}_{b,1}) - \frac{\theta_l}{\theta_b} u_c'(\bar{c}_{l,1}, \bar{d}_{l,1})}{1 - \frac{\partial q_1}{\partial \bar{c}_{l,1}}(\bar{d}_{b,1} - \bar{d}_{b,0}) - \frac{\partial r_1^{-1}}{\partial \bar{c}_{l,1}} \bar{b}_{b,1}} \times \left[\left(\frac{\partial q_1}{\partial \bar{d}_{l,1}} - \frac{\partial q_1}{\partial \bar{d}_{b,1}} \right) \{\bar{d}_{b,1} - \bar{d}_{b,0}\} + \left(\frac{\partial r_1^{-1}}{\partial \bar{d}_{l,1}} - \frac{\partial r_1^{-1}}{\partial \bar{d}_{b,1}} \right) \bar{b}_{b,1} \right], \tag{20}$$

$$\Delta_2^{d,\omega} = \left(u_c'(\bar{c}_{b,2}^\omega, \bar{d}_{b,2}^\omega) - \frac{\theta_l}{\theta_b} u_c'(\bar{c}_{l,2}^\omega, \bar{d}_{l,2}^\omega) \right) \times \left[\left(\frac{\partial q_2^\omega}{\partial \bar{d}_{l,1}} - \frac{\partial q_2^\omega}{\partial \bar{d}_{b,1}} \right) \{\bar{d}_{b,2}^\omega - \bar{d}_{b,1})\} + \left(\frac{\partial \left(r_2^\omega\right)^{-1}}{\partial \bar{d}_{l,1}} - \frac{\partial \left(r_2^\omega\right)^{-1}}{\partial \bar{d}_{b,1}} \right) \bar{b}_{b,2}^\omega \right], \tag{21}$$

where collateral effects are again not present. In contrast to the uninternalized effects on the interest rate, the impact of un-internalized durables price changes that are induced by interventions in the durable market are not scaled with a stock variable, i.e. the stock of debt, but with a flow variable, i.e. the change in holdings of durables (see the terms in the curly brackets in 20 and 21). Hence, the impact of durables

[18] This holds if the weighted marginal utility of non-durable consumption of the borrower exceeds the weighted marginal utility of non-durable consumption of the lender, $u_c'(\bar{c}_{b,1}, \bar{d}_{b,1}) - \frac{\theta_l}{\theta_b} u_c'(\bar{c}_{l,1}, \bar{d}_{l,1}) > 0$, and if $D^b_{c_{l,1}} \geq -1$. We find that the former holds in our quantitative model evaluation, i.e. lenders tend to have a lower marginal utility of non-durable consumption than borrowers.

price changes increases with the adjustments of durables. In general, these adjustments are likely to be larger in the short run, i.e. on impact or during the transition after the policy intervention, than in the long run, i.e. in a new stationary equilibrium when adjustments in quantities are completed. In Section 3.3.2, we will consistently observe that durable price effects are quantitatively more relevant in the short run, whereas interest rate effects (that are scaled with the debt level) tend to dominate welfare results in the long run.

The expressions in (18)-(21) apparently indicate that without further information on preferences and the distributions of bonds and durables, the sign of these wedges are in general unclear, so that the implementation of the constrained-efficient allocation might either require taxes or subsides on debt and durables.

3 Quantitative analysis

The analysis in the previous section has shown that the identification of welfare-enhancing corrective policies requires further information on agents' preferences and the allocation of bonds and durables. To restrict our attention to an empirically relevant specification of (secured) household debt (see Diaz and Luengo-Prado, 2010, Aaronson et al. 2012, Guerrieri and Lorenzoni, 2017), we use a model version with an infinite horizon and potentially varying income and wealth state of agents, which can be calibrated to reasonably match the data. We start by examining the consequences of changing the loan-to-value ratio γ, which directly affects households via the collateral constraint but also indirectly via general equilibrium price effects. Then, we look at taxes on debt and end-of-period durables, similar to the ones considered in Section 2.1. Given that the distribution of wealth and the market price of collateral both are endogenous in the model, the implementation of a constrained-efficient allocation would require a set of individual tax rates that depend on income and wealth states, which cannot be computed in a straightforward way.[19] For the purpose of the paper, it however suffices to locally examine anonymous Pigouvian-type taxes on debt and durables.[20] For all policy experiments, we start with the laissez-faire economy. We then unexpectedly (and permanently) change the policy tool of interest, look at the equilibrium effects along the transition path to the new long-run equilibrium, and analyze the welfare implications of these policy experiments.

[19] Davila et al. (2012) calculate optimal corrective taxes for an Aiyagari (1994)-type economy by first directly solving for the constrained-efficient allocation. Doing so is not feasible in our model due to the presence of a borrowing constraint that depends on the endogenous collateral price. See Nuño and Moll (2018) for an approach similar to Davila et al. (2012) in continuous time.

[20] While it would be interesting to impose such a tax on borrowers only, i.e. asymmetrically, doing so makes the household problem non-convex, such that first-order conditions are no longer sufficient to find the optimal decision rules. A solution approach like this is however necessary to solve a model with Pigouvian-type taxes.

Parameter	Value	Target
α	0.9480	$q\bar{d}/c = 1.4$
β	0.8811	Real interest rate 4%
γ	0.8000	Empirical LTV ratio
δ	0.4500	Literature
θ	2.0000	Standard value
ρ	0.9895	Diaz and Luengo-Prado (2010)
σ	0.1257	Diaz and Luengo-Prado (2010)
$\pi_{R,S} \times 100$	0.0125	Gini coefficient income
$\pi_{S,R} \times 100$	0.2063	Gini coefficient wealth
$\bar{d}$	0.0724	Relative durable distribution

Table 1: Model parameters

3.1 An infinite-horizon model with an endogenous wealth distribution

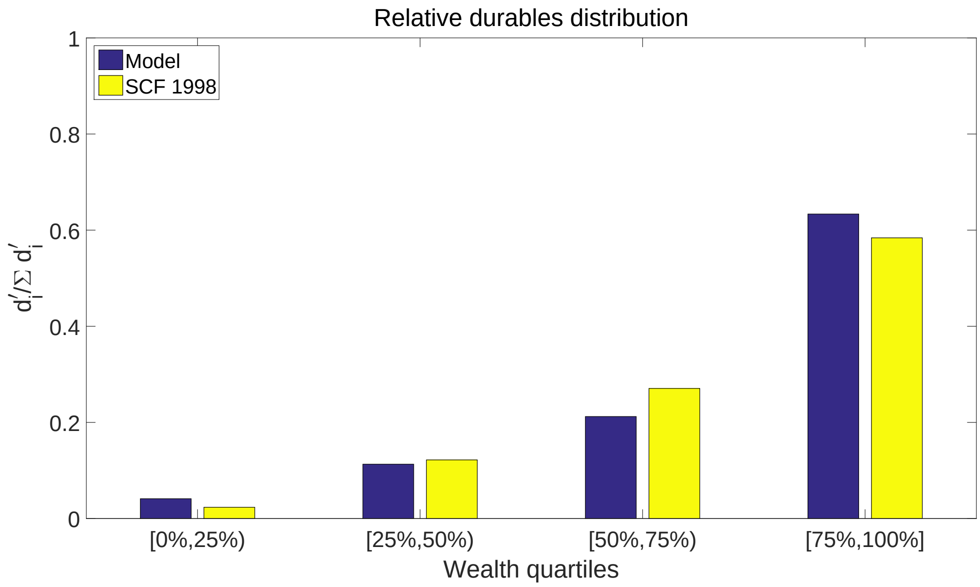
In this section, we set up a version of the model that can be calibrated reasonably well and be used to quantitatively assess the effects of corrective policies. The model is an incomplete-markets economy à la Huggett (1993) that is extended to allow for durable goods and a collateral constraint (see 1). In contrast to the 3-period model studied so far, households are infinitely-lived ($T = \infty$), i.e. $E_1 \sum_{t=1}^{\infty} \beta^{t-1} u(c_{i,t}, d_{i,t})$, and face a random, idiosyncratic income $y_{i,t} \in Y \equiv \{y_1, y_2, ..., y_N\}$, with $y_1 < y_2 < ... < y_N$, which follows a first-order Markov process with conditional transition probabilities $\pi(y_{i,t+1}|y_{i,t})$. The presence of uninsurable idiosyncratic risk results in an endogenous and non-degenerate wealth distribution. Importantly, whether a household is a borrower or a saver is not fixed over time (as in the model of Section 2.1), but an endogenous outcome that depends on an individual household's history of shocks.[21] We consider unanticipated permanent policy changes and study transition dynamics. For the quantitative analysis, we check (ex post) that, for the considered policy experiments, net wealth is always positive (and default never occurs).

Functional forms and parameters Since the model is solved numerically, functional forms and parameters have to be specified.[22] We calibrate the model by choosing suited parameter values from related studies and by targeting selected statistics of the income, wealth, and durables distribution observed for the United States, similar to Diaz and Luengo-Prado (2010), based on data from the Survey of Consumer Finances (SCF) for 1998. The parameter values are summarized in Table 1. In contrast to the latter study we define the empirical counterpart of durable consumption not only as residential housing but add vehicles as well, given that these two categories account for the majority of collateral used for household credit. For the household utility function, we use the specification

$$u(c, d) = \frac{\left[\alpha c^{\delta} + (1-\alpha) d^{\delta}\right]^{\frac{1-\theta}{\delta}}}{1-\theta},$$

[21]The equilibrium for the infinite-horizon model is defined in Section C.
[22]Details about the numerical solution procedure can be found in Appendix D.

Figure 1: Relative durable holdings for different wealth quartiles (data vs model)



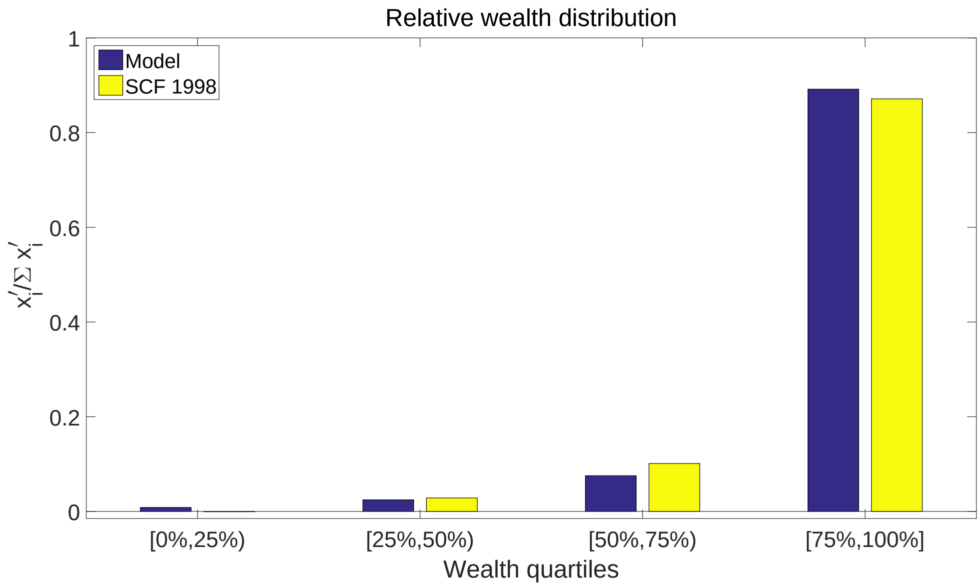
where $0 < \theta \neq 1$ is the inverse of the intertemporal elasticity of substitution with respect to a constant-elasticity-of-substitution (CES) consumption aggregate that consists of non-durable and durable consumption, c and d, with $\delta > 0$ controlling the degree of substitution between the two types of consumption goods. For θ, we choose a standard value of two, whereas δ is set to 0.45, which is the average of values used by Benhabib et al. (1991), McGrattan et al. (1997), and Piazzesi et al. (2007), who use the same functional form for the utility function. The values for the utility function parameter α (0.948) and the discount factor β (0.8811) are set to match two empirical targets, namely, the ratio of aggregate durable-to-non-durable-consumption of 1.4 and a real interest rate of 4%. The fraction of seizable collateral γ is set at 0.8, implying an empirically plausible loan-to-value ratio of 80% (see Diaz and Luengo-Prado, 2010).

The income support Y and the associated income transition probabilities π are chosen to match the Gini coefficients for income $y_{i,t}$ (0.43) and (net-)wealth $x_{i,t} \equiv b_{i,t} + q_t d_{i,t}$ (0.8). A household's wealth position serves as the single endogenous individual state variable, which can take on finitely many values (see Appendix D for details on the numerical computation). As is well known in the literature (see e.g. Di Nardi et al., 2015), without additional assumptions, a standard Bewley-Aiyagari-Huggett-type incomplete-markets model fails to match important features of the wealth distribution, the concentration of wealth at the top in particular. To address this shortcoming, we follow Diaz and Luengo-Prado (2010) and assume that individual income follows a log-normal AR(1) process,

$$\ln y_{i,t} = \rho \ln y_{i,t-1} + \sigma \varepsilon_{i,t},$$

with autocorrelation $\rho = 0.9895$ and standard deviation $\sigma = 0.1257$, and that, additionally, there is also a

Figure 2: Relative net-wealth for different wealth quartiles (data vs model)



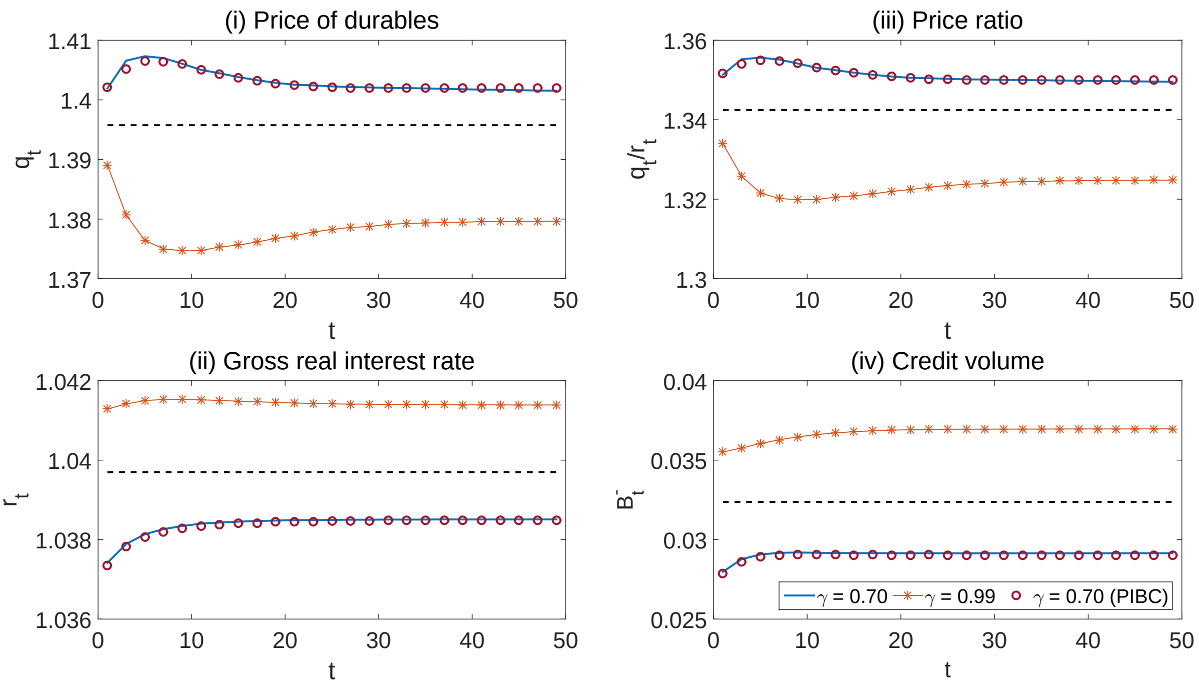
small probability $\pi_{R,S}$ of transitioning to a "superstar" income state, which is left with probability $\pi_{S,R}$. While the AR(1) process provides a good fit for most of the population, it cannot suitably account for the top 1% of the income distribution. The "regular" income states y_1 to y_6 are obtained by discretizing the AR(1) process via the method proposed by Tauchen and Hussey (1991), while the superstar income value y_7 is set to match the empirical ratio $y_7/y_6 = 6$ and the transition probabilities are $\pi_{R,S} = 0.000125$ and $\pi_{S,R} = 0.002063$. Combining these values with the transition probabilities for the regular income states, obtained by discretizing the AR(1) process, yields the transition probabilities $\pi(y_{i,t+1}|y_{i,t})$, which are given in Appendix D.3. Lastly, the aggregate supply of the durable good $\bar{d} = 0.0724$ is chosen to provide a reasonable fit for the durable distribution, as given by Figure 1. Figure 2 shows the distribution of net-wealth for the model and the data.

3.2 Loan-to-value ratio

In this section, we discuss the effects of permanently and unexpectedly changing the loan-to-value ratio in the economy. For convenience, we abstain from introducing an additional policy instrument that enters the collateral constraint and assume that the policy maker directly reduces γ.[23] Although, as illustrated in Section 2.1, type-dependent Pigouvian taxes on borrowing and durables can induce (welfare-improving) corrections of prices, they are unlikely going to be implemented in practice. The loan-to-value ratio by contrast is a policy instrument that is typically considered as an useful instrument to regulate borrowing. In the model, changes in the loan-to-value ratio have two types of effects. First, they directly affect households' decisions to borrow and – as a result – to buy durable goods. Second, the resulting reactions

[23]Equivalently, we could introduce a policy instrument Γ, such that the borrowing constraint (1) changes to $-b_{i,t} \leq \Gamma\gamma q_t d_{i,t}$.

Figure 3: Transition paths for prices and credit after an unexpected change of loan-to-value ratio γ



Notes: The panels also show the transition paths for a price-inelastic constraint (PIBC). The dashed black lines denote the respective laissez-faire steady-state values.

affect equilibrium prices of debt and durables, which in turn lead households to (re-)adjust their behavior. In the remainder, we will refer to effects of the first kind as "direct effects" and those of the second kind as "indirect effects". The direct effects work mechanically and as expected, such that it will be particularly important to understand the indirect effects. To do so, we will look at the responses of equilibrium prices as well as of the aggregate credit volume in the short run and in the long run following an unexpected change of γ. Figure 3 visualizes the results by plotting the transition path for prices, their ratio and the credit volume, which we denote as B_t^-.[24]

Consider a reduction of the loan-to-value ratio (from $\gamma = 0.8$) to $\gamma = 0.7$, which tightens the collateral constraint for all households. Ceteris paribus, this change directly reduces the borrowing limit of constrained households and low-income/low-wealth borrowers who have previously been unconstrained become constrained as well. As a direct effect, these types of households respond by reducing their debt as well as their holdings of durables since debt-financing becomes more restricted. These direct effects suggest that the credit volume, the interest rate, and the price of durables fall. Yet, we find that the durables price q_t instead increases, as shown by the solid blue line in Panel (i) of Figure 3 (which almost coincides with the magenta-colored circles displaying a reference case, see below). The effect on the debt market is in line with the intuition suggested by the direct effects. Credit volume declines and the interest rate falls and settles at a higher value that is nevertheless lower compared to the old steady state. Furthermore, the price ratio q_t/r_t increases when γ declines (see Panel (iii) of Figure 3).

[24]Credit volume is calculated by aggregating all negative end-of-period bond positions across agents.

Why does the price of durables increase? The overall response of q_t particularly depends on how savers and richer unconstrained borrowers respond. These types of households are not directly affected by the tightening of the collateral constraint and also unlikely going to be in the near future. Their response will therefore mainly reflect how the prices of durables and bonds change when borrowers de-leverage. The lower real interest rate makes investing in bonds less attractive for these households who increase their holdings of durables. These responses create upward pressure on both prices, q_t and $1/r_t$, with the price of durables ultimately experiencing an increase. Although a higher value for q_t counteracts the lower loan-to-value ratio, it is not sufficient for γq_t to go up and to relax household borrowing constraints. For demonstrative purposes, we also consider a higher loan-to-value ratio $\gamma = 0.99$. Reverting the direction of the change in γ (marked orange line) leaves the propagation mechanism unchanged and switches the sign of the effects.

We further examine an artificial reference case, which allows abstracting from the collateral effects of pecuniary externalities. Specifically, we consider a price-inelastic borrowing constraint (PIBC), for which we hold the durables price in the collateral constraint fixed at the laissez-faire equilibrium level. The magenta-colored circles in Figure 3 show that interest rate effects are almost identical and the impact on the durables price during the transition phase is slightly less pronounced under the price-inelastic borrowing constraint. Panel (iv) in Figure 3 further shows that there is only a tiny difference between both cases with respect to the credit volume, indicating that the results are hardly affected by price-induced changes in the collateral constraint.

Welfare implications What do the effects of the policy experiments imply for welfare? To assess the welfare effects, taking into account the transition to the new steady state, we consider three measures. The first one is the welfare-equivalent consumption bundle variation $CEV_i \equiv CEV(x_i, y_i)$ with

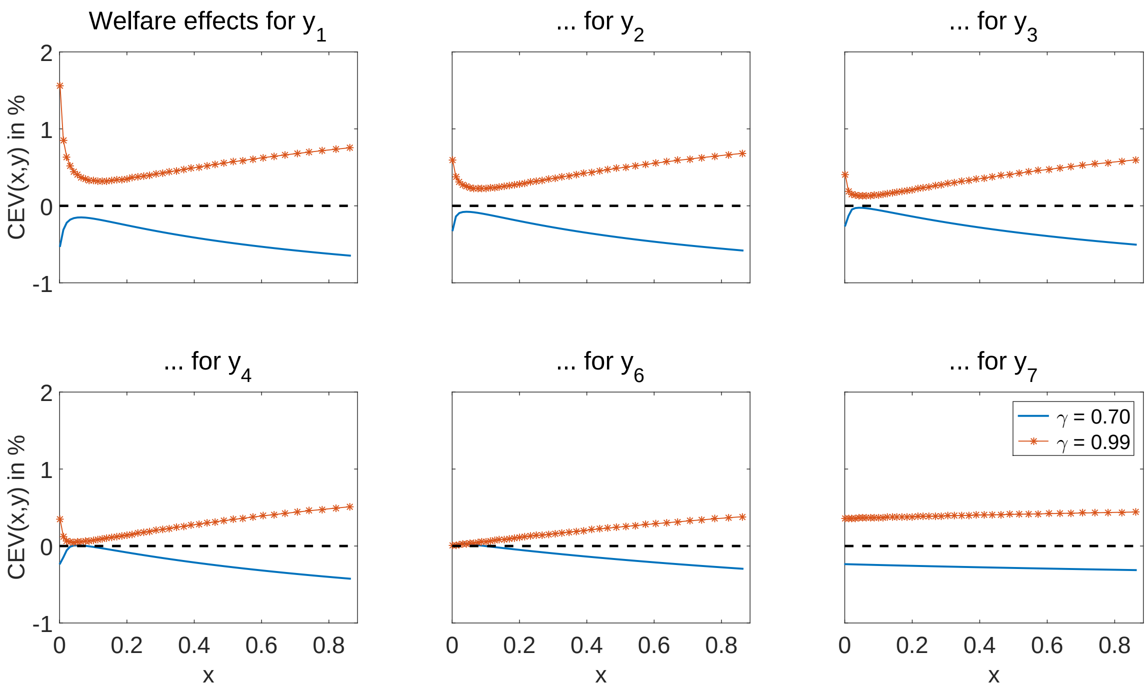
$$CEV_i = \left[\frac{\tilde{V}_1(x_{i,t}, y_{i,t})}{V(x_{i,t}, y_{i,t})}\right]^{\frac{1}{1-\theta}} - 1,$$

where $V(\cdot)$ denotes the value of a household with beginning-of-period wealth $x_{i,t} = b_{i,t-1} + q_t d_{i,t-1}$ and income $y_{i,t}$ in the laissez-faire economy, which satisfies

$$V(x_{i,t}, y_{i,t}) = \max_{b_{i,t}, c_{i,t}, d_{i,t}} \left\{ u(c_{i,t}, d_{i,t}) + \beta \sum_{y_{i,t+1} \in Y} \pi\left(y_{i,t+1} | y_{i,t}\right) V(b_{i,t} + q d_{i,t}, y_{i,t+1}) \right\} \quad \text{s.t. (1) and (3)},$$

and $\tilde{V}_1(\cdot)$ denotes the corresponding value of a household in the impact period $t = 1$ of an economy that is experiencing a policy change. The value $\tilde{V}_t(\cdot)$ carries a time index because the prices q_t and r_t change during the transition period and are therefore a function of time in this case. By contrast, prices in the stationary laissez-faire economy are constant in all periods and the associated individual household values only changes over time via the individual states. The welfare measure CEV_i allows assessing how welfare

Figure 4: Welfare effects conditional on income and wealth type (change in LTV)



of individual types of households changes after the policy intervention, where a positive (negative) value for CEV_i means that a household is better (worse) off.

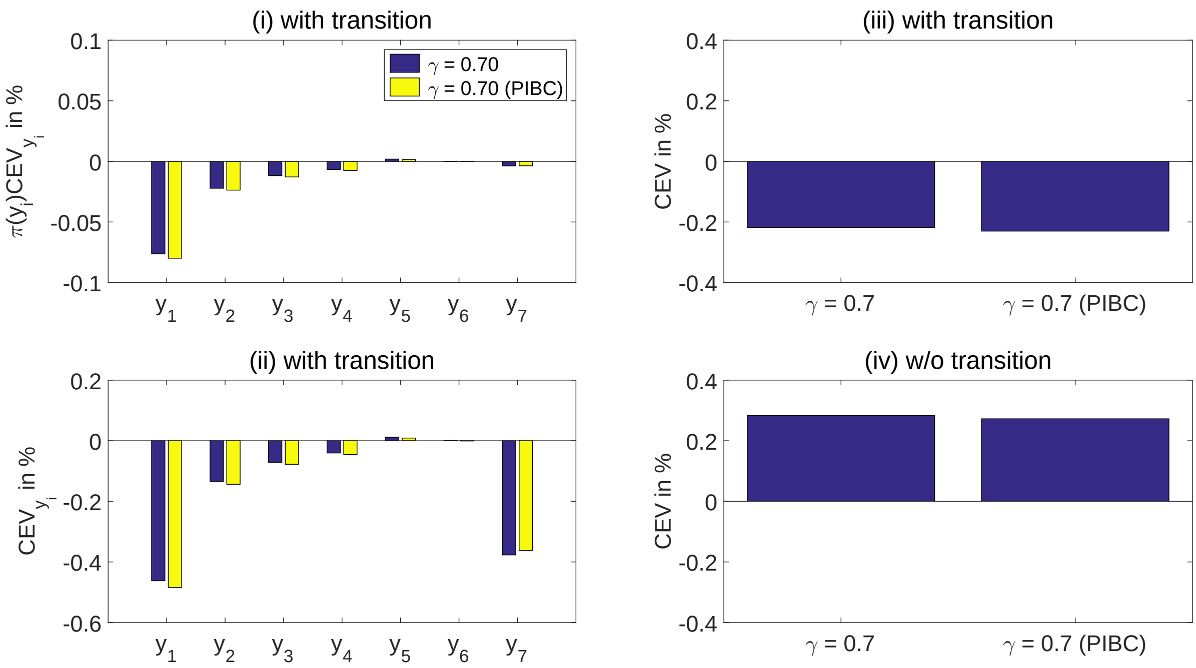
We further want to examine policy effects on social welfare. Pareto improvements are hardly possible under dispersed agents' endowments of wealth and income (see Davila et al., 2012). Thus, we consider ex-ante expected lifetime utility (see e.g. Conesa et al., 2009, Krueger et al., 2016, or Nuño and Moll, 2018) and compute variations of this measure expressed in equivalent consumption bundle units

$$CEV = \left[\frac{\sum_{x_{i,t},y_{i,t}} \tilde{\lambda}(x_{i,t},y_{i,t})\tilde{V}_1(x_{i,t},y_{i,t})}{\sum_{x_{i,t},y_{i,t}} \lambda(x_{i,t},y_{i,t})V(x_{i,t},y_{i,t})}\right]^{\frac{1}{1-\theta}} - 1,$$

where $\lambda(x_{i,t},y_{i,t})$ denotes the (unconditional) probability of individual state $(x_{i,t},y_{i,t})$ in the laissez-faire economy and $\tilde{\lambda}(x_{i,t},y_{i,t})$ the corresponding probability in the period of the policy change.[25] This welfare criterion, which is identical to a utilitarian welfare measure, can be interpreted as measuring whether an unborn household, who is randomly assigned to an idiosyncratic state, would prefer to be born into the laissez-faire economy or into an economy that experiences a sudden policy change. In contrast to the state-dependent and household-specific measure CEV_i, CEV can be used to assess the overall welfare implications for the economy.

[25] While individual holdings of bonds and durables are predetermined, wealth $x_{i,t} = b_{i,t} + q_t d_{i,t}$ also depends on the price q_t, which can shift the wealth distribution on impact.

Figure 5: Transition paths for prices and credit after an unexpected change of loan-to-value ratio γ



Notes: Panels (i) and (ii) display welfare conditional on the income state. In Panel (i) the welfare effects are weighted with the probability mass of the respective income states. Panels (iii) and (iv) display aggregate welfare with and without taking into account the transition periods. The case of a price-inelastic collateral constraint is denoted as "PIBC".

The final welfare measure that we use is the income-specific measure CEV_{y_i} which is defined as

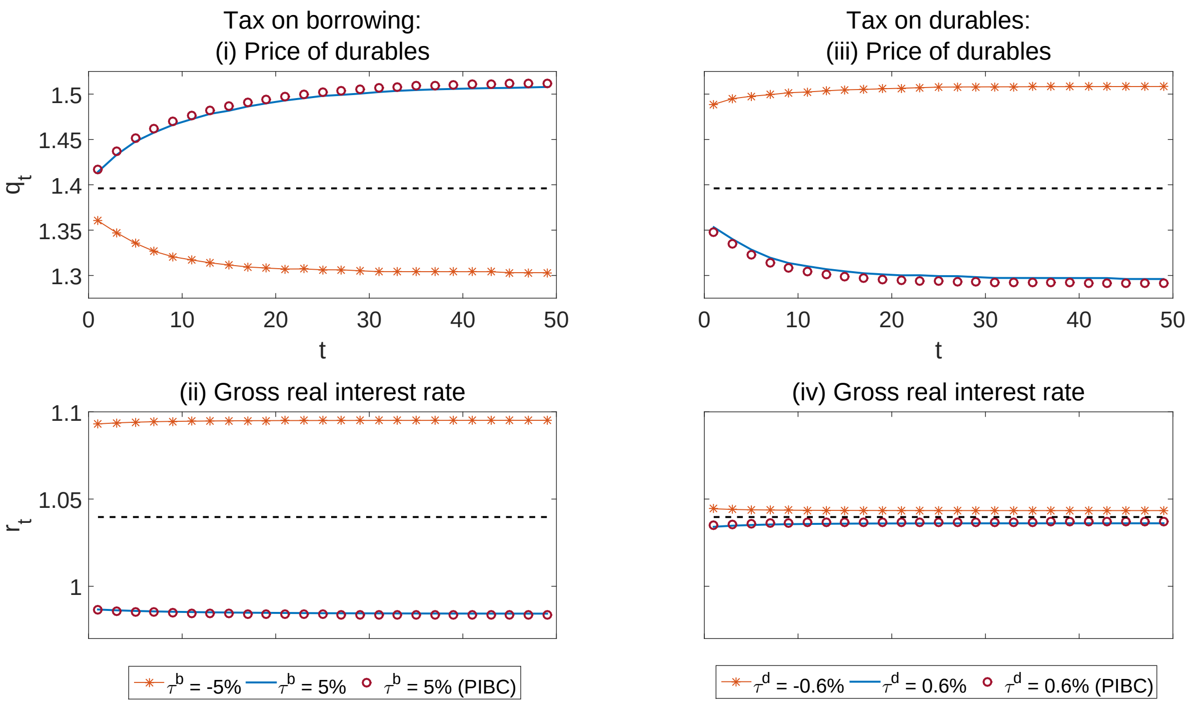
$$CEV_{y_i} = \left[\frac{\sum_{x_{i,t}} \tilde{\lambda}_{y_i}(x_{i,t}) \tilde{V}_1(x_{i,t}, y_i)}{\sum_{x_{i,t}} \lambda_{y_i}(x_{i,t}) V(x_{i,t}, y_i)} \right]^{\frac{1}{1-\theta}} - 1,$$

where $\lambda_{y_i}(x_{i,t})$ and $\tilde{\lambda}_{y_i}(x_{i,t})$ denote the probability of state $x_{i,t}$ for the laissez-faire economy and the economy subject to a policy change, respectively, given income y_i. This welfare measure is a refinement of CEV that takes into account the distribution of wealth conditional on income y_i, which will help to shed light on the source of aggregate social welfare changes (see Krueger et al., 2016).

Figure 4, which displays CEV_i for different income and wealth levels, shows that almost all types of households are worse off after a decrease in γ (see solid blue lines). Constrained borrowers can borrow less, whereas savers suffer from lower interest rates. The only types of households who gain under a lower γ-value are income-rich unconstrained borrowers who are not directly affected by the tighter collateral constraint but benefit from the lower interest rate. These unconstrained households reduce their borrowing (see Figure 14 in Appendix E), which contributes to the lower interest rate. Hence, these agents would have benefited under laissez faire if they internalized the price effects of their behavior. These types of households, however, constitute a tiny fraction of households in the economy.

Panel (i) of Figure 5 depicts welfare conditional on specific income states and weighted with the respective probability, i.e. $\pi(y_i)CEV_{y_i}$. The unweighted welfare measure CEV_{y_i} is displayed in Panel

Figure 6: Price transition paths after an unexpected tax change



Notes: The panels also show the transition paths for a price-inelastic constraint (PIBC). The dashed black lines denote the respective laissez-faire steady-state values.

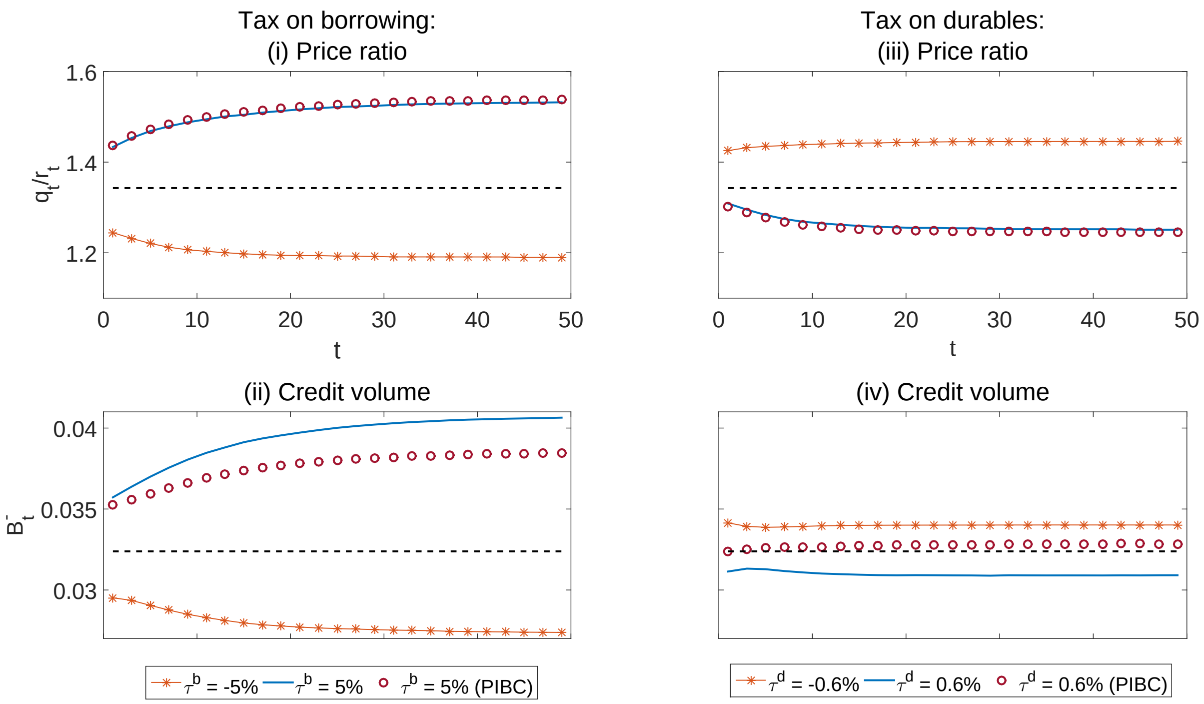
(ii). These figures reveal that the welfare losses clearly dominate. Consistently, social welfare effects as measured by CEV are negative for $\gamma = 0.7$ (see Panel (iii)). When the transition is however not taken into account, these results are reversed, since the increase (decrease) in the price of durables due to the LTV reduction (increase) leads to a general upward shift in wealth (see Panel (iv)).[26] Overall, Figure 5 shows that the welfare losses of a LTV reduction under a price-inelastic borrowing constraint (PIBC) are slightly larger for low income groups and that social welfare effects hardly differ under a price-inelastic borrowing constraint. According to this experiment, we do not find evidence for substantial collateral effects.

3.3 Corrective taxes

This section examines the effects of corrective tax policies. These anonymous taxes affect prices by altering the marginal valuation of goods and assets, while payments or receipts of funds are individually compensated in a lump-sum way. Thereby, these Pigouvian-type tax policies do not directly redistribute resources across households and – like the loan-to-value ratio – affect the economy via price effects that can address pecuniary externalities. We first consider a symmetric tax on debt $-b$ at the rate τ_b, which implies a tax on borrowing and a subsidy on savings. We focus on local effects in the neighborhood of the

[26] In this case, the computed welfare measure is $CEV = \left[\sum_{s_{i,t}} \bar{\lambda}(s_{i,t})\bar{V}(s_{i,t}) / \sum_{s_{i,t}} \lambda(s_{i,t})V(s_{i,t})\right]^{\frac{1}{1-\theta}}$, with $s_{i,t} \equiv (x_{i,t}, y_{i,t})$. The bars denote that the probability measure and the value function are associated with the new long-run equilibrium.

Figure 7: Transition path for price ratio and credit volume after an unexpected tax change



Notes: The panels also show the transition paths for a price-inelastic constraint (PIBC). The dashed black lines denote the respective laissez-faire steady-state values.

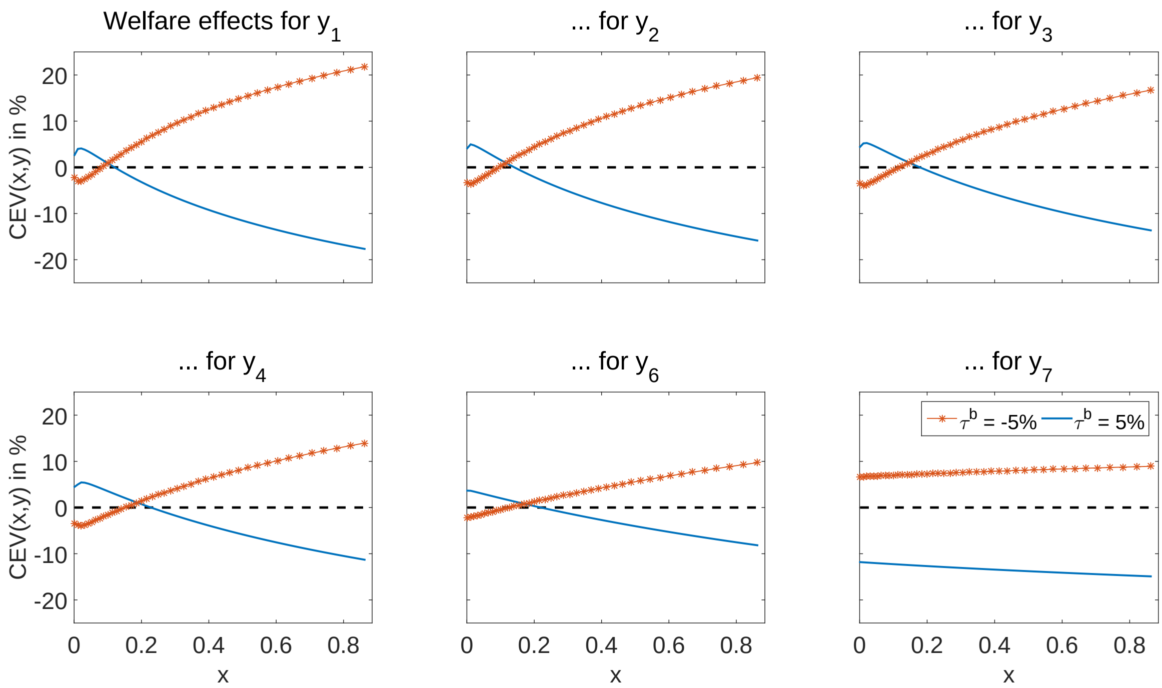
laissez faire equilibrium induced by $\tau^b > 0$ and $\tau^b < 0$; the latter implying a subsidy on borrowing and a savings tax. As a second Pigouvian-type tax policy, we introduce an anonymous tax on end-of-period holdings of durables τ^d. To facilitate comparisons between both Pigouvian-type tax policies, the values for τ^d are chosen to yield equally-sized changes in the long-run durable price q_t.

3.3.1 Debt-tax/Saving-subsidy

First, suppose that the tax on durables is kept at zero ($\tau^d = 0$) and a tax on debt, which implies a subsidy on savings, is unexpectedly and permanently imposed ($\tau^b = 0.05$). To illustrate the policy effects, we examine time paths of prices and of the credit volume in the left hand columns of Figures 6 and 7. Relative to the laissez-faire steady-state values (dashed black lines), the durables price q_t and the interest rate r_t, given by the solid blue lines in the left hand column of Figure 6, move into opposite directions. The price of durables q_t jumps up and then gradually moves up to a higher new steady-state value, while the interest rate r_t immediately drops and further declines until it arrives at the new (lower) long-run value.

What drives these responses? Ceteris paribus, since it is imposed symmetrically, the tax τ^b induces all types households, i.e. borrowers and savers, to save more and to dis-save less, i.e. to choose higher values of $b_{i,t}$. These direct effects in turn create downward pressure on the interest rate r_t to ensure market clearing for debt and simultaneously also upward pressure on the price q_t (see solid blue line in Panels

Figure 8: Welfare effects conditional on income and wealth type of tax/subsidy on borrowing

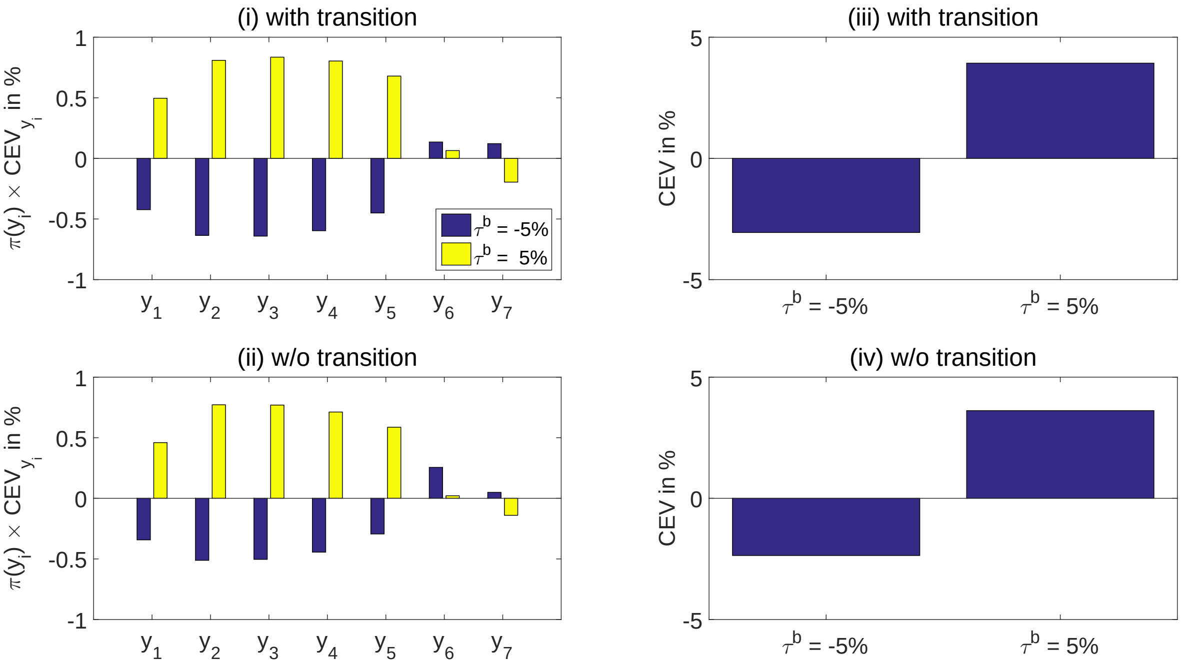


(i) and (ii) of Figure 6). The reason for the latter is that durables are untaxed and now hence provide a relatively higher return, which stimulates the demand for durables. These price responses tend to ease borrowing conditions, while the increase in the price ratio q/r raises the amount of funds borrowed per unit of collateral. Consistently, we observe a gradual increase in the aggregate credit volume (see solid blue line in Panel (ii) of Figure 7), reflecting that income- and wealth-poor households tend to increase their borrowing. It should be noted that even constrained borrowers issue more debt, even though the policy (tax on debt) tends to make it less attractive for households to borrow. At the same time, savers are however incentivized to save more, which induces the interest rate to decline and the credit volume to increase. Under a subsidy on borrowing ($\tau^b = -0.05$), given by the marked orange lines, prices and the credit volume move into the opposite direction, showing that the mechanism just discussed operates in a symmetric way.

As in Section 3.2, we further consider a price-inelastic borrowing constraint (PIBC) where we fix the durables price in the collateral constraint at the steady-state laissez-faire value. In this case, we hardly observe any difference with regard to the responses of q_t and r_t to the introduction of the borrowing tax compared to the benchmark case of a price-elastic borrowing constraint (see magenta circles in Figure 6).[27] In contrast to the LTV-reduction, the credit volume response is however substantially affected by keeping the durables price fixed in the collateral constraint: The increase in the collateral price raises the borrowing limit only in the benchmark case, such that the increase in the credit volume is apparently less pronounced under a price-inelastic borrowing constraint (see Panel (ii) of Figure 7).

[27] The debt-subsidy/saving-tax has the same qualitative implications with the responses having the opposite sign.

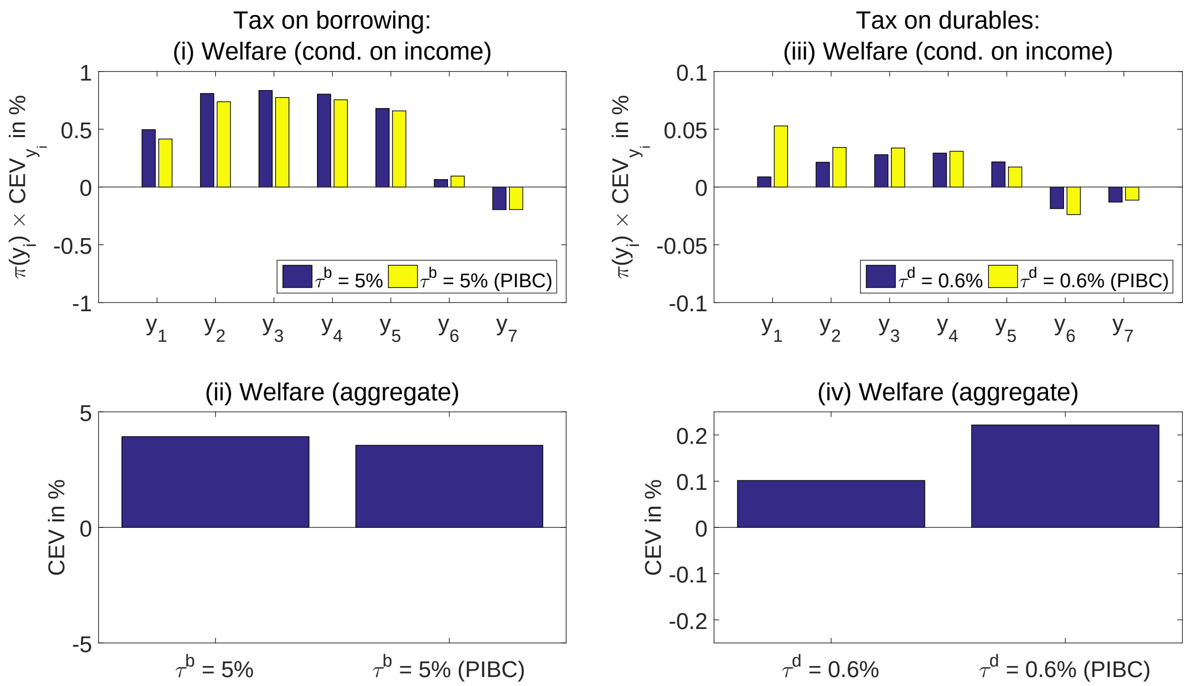
Figure 9: Welfare effects of a tax/subsidy on borrowing



Notes: Panels (i) and (ii) display welfare conditional on the income state, with and without taking into account the transition periods Panels (iii) and (iv) display aggregate welfare with and without transition periods.

Welfare implications The welfare effects of a tax on debt are visualized in Figures 8 and 9. As shown by the solid blue lines in Figure 8, wealth-poor households in all income groups (except the highest) gain from a debt-tax/saving-subsidy ($\tau^b > 0$). Both price effects of the debt-tax/saving-subsidy, i.e. a higher price of durables and lower interest rates, are beneficial for borrowers. The tax on debt thus induces price responses that serve as partial insurance for borrowers from an ex-ante perspective. While the price effects are qualitatively identical to the effects of a LTV reduction, they here are associated with a higher credit volume induced by agents' increased willingness to lend. Notably, wealth-rich agents tend to lose, as they earn a lower interest rate on their savings. To understand the welfare effects displayed by the yellow bars in Panels (i) and (ii) of Figure 9 (compared to Figure 8), one has to further take into account that endogenous shifts in the durables price q_t lead to a change in agents wealth $x_{i,t} = b_{i,t-1} + q_t d_{i,t-1}$ and does so the more a household owns durables $d_{i,t-1}$. When aggregating within income groups, which takes changes of the wealth distribution into account, one can see that all income groups except the highest benefit from a tax on debt, which reflects that this group mostly consists of lenders. Panels (iii) and (iv) of Figure 9, which display the social welfare gains/losses with and without transition, reveal that the welfare gains from the debt-tax/saving-subsidy ($\tau^b = 0.05$) are due to positive short-run and long-run effects. Furthermore, the welfare effects hardly change under a price-inelastic borrowing constraint, indicating a negligible role of collateral effects for welfare, both within income groups as well as the aggregate (see Panels (i) and (ii) of Figure 10).

Figure 10: Welfare effects of a Pigouvian-type tax with and without price-inelastic borrowing constraint

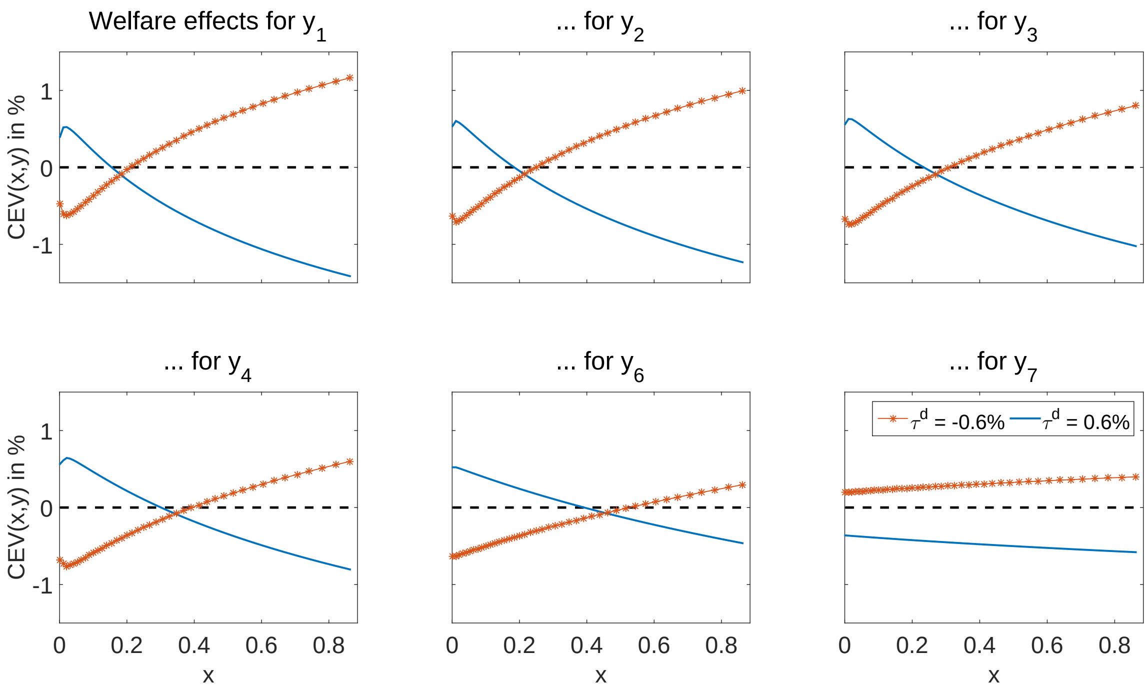


3.3.2 Durables tax

Now consider an unexpected permanent increase in the tax (subsidy) on end-of-period holdings of durables by $\tau^d = 0.006$, which is compensated by lump-sum transfers (taxes). The tax on debt is set at zero ($\tau^b = 0$). The size of the Pigouvian-type policy intervention in the market for durables is chosen to yield a change in the long-run price of durables that is of the same (long-run) magnitude as under debt market interventions, $-\tau^b \neq 0$. The associated price and credit responses are given by the solid blue lines in the Panels (iii) and (iv) of Figures 6 and 7. Ceteris paribus, the taxation of durables causes households to substitute durable goods in favor of non-durable goods. Furthermore, agents who are willing to transfer wealth intertemporally tend to substitute durables in favor of bonds, such that credit supply increases. These direct effects imply that q_t and r_t fall to clear markets, which is shown in the Panels (iii) and (iv) of Figure 6. While the taxes on durables and debt both lower the real interest rate, the responses of the aggregate credit volume substantially differ. A main difference is that the price ratio q/r decreases under the durables tax, which reduces the maximum amount of funds that can borrowed against collateral, whereas the price ratio increases under the debt tax (see solid blue lines in Panels (i) and (iii) of Figure 7). Correspondingly, the credit volume increases under the debt tax and decreases under the durables tax, as shown in the Panels (ii) and (iv) of Figure 7.

Like under the debt-tax/savings-subsidy, the responses of the durables price q_t and the interest rate r_t to a durables tax introduction hardly change when we consider a price-inelastic borrowing constraint (see magenta circles in Panels (i) and (ii) of Figure 6). In contrast, the credit volume responses show

Figure 11: Welfare effects conditional on income and wealth type of tax/subsidy on durables

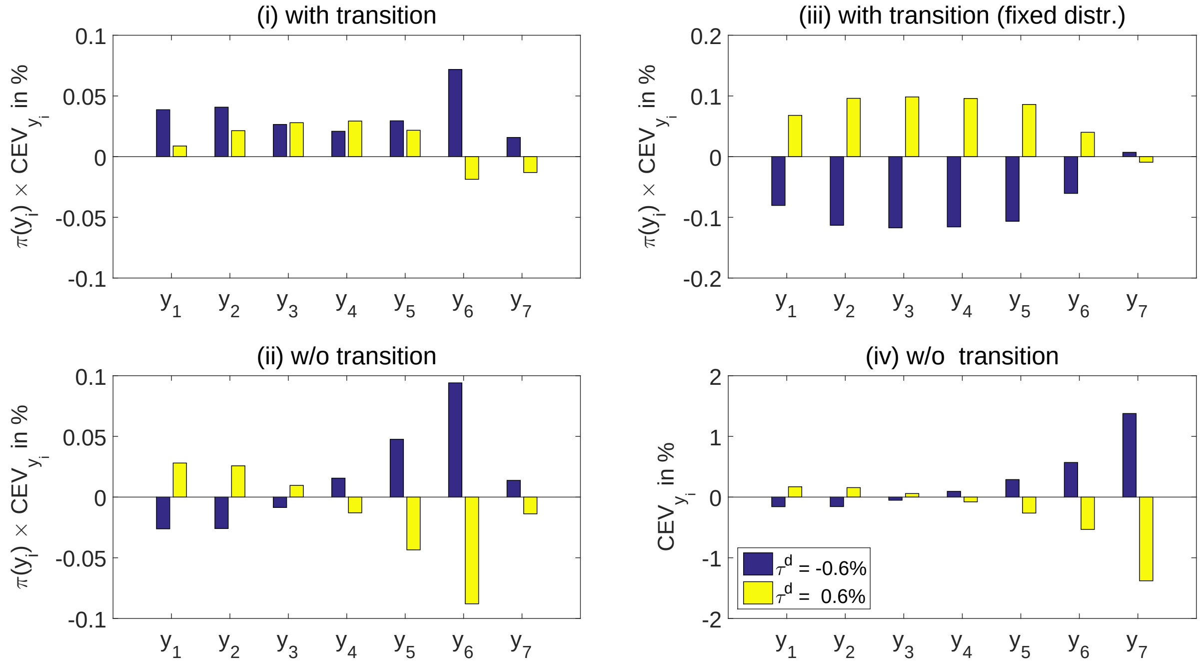


remarkable differences: The durables tax leads to a decline in the interest rate, which tends to stimulate borrowing, and to a decline in the durables price, which tends to reduce the borrowing limit under the benchmark (price-elastic) collateral constraint. Under a price-inelastic borrowing constraint, however, a decline in the durables price does not have a direct impact on the borrowing constraint. As a result, the credit volume increases relative to the laissez-faire case due to the decline in the interest rate, whereas it decreases under the price-elastic borrowing constraint (see Panel (iv) of Figure 7).

Since we considered values for the durables tax τ^d that induce long-run changes in q_t that are of the same magnitude as those associated a debt tax $-\tau^b$, one can compare the response of the real interest rate under the two tax instruments. Figure 6 shows that the interest rate adjustment is much more pronounced under the borrowing tax τ^b, which directly affects the market for debt. More specifically, a tax on debt of 5% lowers the long-run real interest rate by 5.9 percentage points, while a subsidy on durables of 0.6%, which results in the same long-run price of durables, yields an interest rate increase of 0.7 percentage points.

Welfare implications Who benefits from a tax on durables? As shown by the solid blue lines in Figure 11, low-wealth households, which are typically borrowers, benefit from the lower interest rate in almost all income groups. Even constrained households gain despite the drop in the price of durables, which tends to tighten their borrowing constraints. Like tax on debt, the durables tax leads to responses of the interest rate that partially insure borrowers from a social perspective. Welfare declines for the highest income groups (see yellow bars in the Panels (i) and (ii) of Figure 12), while social welfare increases for

Figure 12: Welfare effects of a tax/subsidy on durables conditional on income state



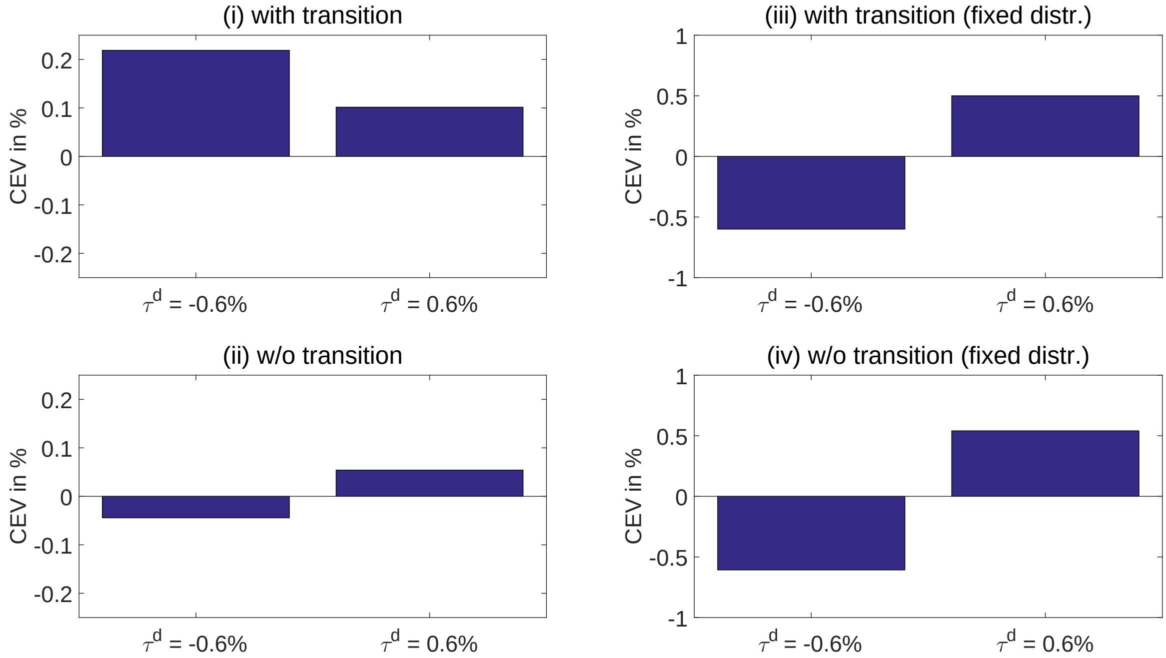
Notes: Panels (i) to (ii) display welfare conditional on the income state. Panel (iii) shows the measure when the transition period is taken into account but the wealth distribution in the impact period is kept fixed. Panel (iv) displays the measure in Panel (i), weighted by the probability mass of the income states.

$\tau^d > 0$ (see Panels (i) and (ii) of Figure 13). It should further be noted that the decline in the durables price reduces wealth of agents in terms of non-durables, which shifts the wealth distribution such that the mass of agents who gain from lower interest rates increases. Under a price-inelastic borrowing constraint, there are visibly larger social welfare gains compared to the benchmark case (see Panels (iii) and (iv) of Figure 10), which correspond to the qualitative difference in the credit volume response (see Figure 7). While this indicates the quantitative relevance of collateral effects, it should be noted that it is the decline in the interest rate which is ultimately responsible for the increase in the credit volume and in social welfare.

A subsidy on durables, $\tau^d < 0$, reverses the qualitative responses of prices and the credit volume (see marked orange lines in the Panels (iii) and (iv) Figures 6 and 7). The long-run welfare effects, i.e. those not accounting for the transition dynamics, of a durables subsidy ($\tau^d = -0.006$) are negative and mirror those of the tax on durables with the opposite sign, as shown in the Panel (ii) of Figures 12 and 13. Specifically, savers tend to gain from the increase in the interest rate and the price of durables compared to the laissez-faire case (see marked orange line in Figure 11), where they do not internalize that raising their holdings of durables contributes to the increase in the durables price.[28] However, the overall aggregate welfare effect, which includes the transition phase, is positive (see Panels (i) of Figures

[28]This can be seen from the policy functions shown in Figure 15 in Appendix E.

Figure 13: Aggregate welfare effects of a tax/subsidy on durables



Note: Panels (iii) and (iv) use the wealth distribution prior to the policy shock for the welfare calculation.

12 and 13).[29] The durable subsidy leads to an increase in the durables price q_t, leading to a higher wealth level of all agents. This effect due to a higher durables price is particularly more pronounced in the short run, where adjustments of the quantities are not yet made. Computing the welfare measures by fixing the wealth distribution prior to the policy change (see Panel (iii) of Figures 12 and 13), shows that welfare then declines in (almost) all income groups, such that aggregate welfare CEV declines as well.

It should be noted that the welfare effects of the durables price increase are non-trivial, since it is beneficial for agents who are net-sellers of durables ($d_{i,t} < d_{i,t-1}$), whereas net-buyers ($d_{i,t} > d_{i,t-1}$) suffer from the price increase. In the experiment, a durables subsidy thereby tends to benefit poor households more than it hurts rich ones. These effects are particularly larger in the transition phase where agents adjust their holdings of durables more than in a stationary equilibrium. Note that these effects relate to the distributive effects included in the wedges Δ_1^d and $\Delta_2^{d,\omega}$ that are relevant for the corrective durables tax identified in Section 2.2 (see 18-21). The beneficial effects of higher durables prices even compensate agents that would otherwise lose under the durables subsidy (see marked orange lines in Figure 11). As a result, all income groups are better off and social welfare increases for the durables subsidy $\tau^d < 0$ (see Panel (i) of Figures 12 and 13).[30]

Finally, recall that the subsidy on durables and the tax on debt have been scaled to lead to equally-sized long-run changes in the durable prices. Figure 13 however shows that the implied aggregate welfare

[29] Note that CEV does not equal $\sum_{y_i} \pi(y_i) CEV_{y_i}$ by construction.
[30] Within income groups, there might however be some losers, such that there is no strict Pareto improvement.

effects are more than 20-times larger under the debt market intervention. As revealed in Figure 6, this difference can mainly be attributed to the change in the interest rate, which is more pronounced under the debt tax. This observation thus implies that interest rate responses are even more relevant for the overall welfare effects of corrective policies than changes in the collateral price.

4 Conclusion

Pecuniary externalities with regard to the price of collateral can justify financial regulation. This paper examines financial regulation and corrective taxes in a prototype incomplete markets economy with limited commitment featuring collateral constraints. We find that a loan-to-value reduction positively affects welfare of only income-rich unconstrained borrowers, whereas social welfare decreases. By contrast, a Pigouvian-type debt-tax/savings-subsidy generates welfare gains for (almost) all income groups due to a higher collateral price and a lower interest rate. Borrowers tend to gain and income-rich lenders tend to lose from the decline in the interest rate, providing a partial insurance for borrowers from an ex-ante perspective. Interventions in the market for durables (collateral) exert ambiguous welfare effects due to price-induced shifts in the wealth distribution. The resulting short-run welfare effects can qualitatively overturn the long-run welfare implications, which tend to be positive (negative) for households with a low (high) income in the case of a tax (subsidy) on durables. Overall, the analysis reveals that interest rate responses and distributive effects rather than changes in the durables price and collateral effects are decisive for the total welfare effect of corrective policies. While the previous literature has found that financial regulation can be beneficial under pecuniary externalities due to financial frictions, our analysis indicates that this principle cannot be generalized to a macroeconomic structure with an endogenous interest rate and a non-trivial distribution of income and wealth.

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Appendix

A Competitive equilibrium of the three-period model

The competitive equilibrium can be characterized as follows. Consider the terminal period 3. Conditional upon previous choices for debt $b^{\omega}_{i,2}$ and durables $d^{\omega}_{i,2}$ as well as on the exogenous period-2-state ω and the fixed endowment y, agents $i \in \{b, l\}$ allocate their available resources between durables $d^{\omega}_{i,3}$ and non-durables $c^{\omega}_{i,3}$ according to the first-order conditions (8) and the budget constraint in period 3, while the relative price of durables q^{ω}_3 ensures that $d^{\omega}_{b,3} + d^{\omega}_{l,3} = \bar{d}$ holds.

In period 2, agents choose to borrow/lend $b^{\omega}_{i,2}$, to buy/sell durables $d^{\omega}_{i,2} - d_{i,1}$, and to buy non-durables $c^{\omega}_{i,2}$ contingent upon the predetermined states $\{d_{i,1}, b_{i,1}\}$ and the realization of the random variable $\omega \in \{u, e\}$. Like in Davila and Korinek (2018), we define $\bar{d}_1 \equiv \left(\bar{d}_{l,1}, \bar{d}_{b,1}\right)$ and $\bar{b}_1 \equiv \left(\bar{b}_{l,1}, \bar{b}_{b,1}\right)$ to distinguish the states variables of an individual household of type i from the state variables aggregated across all i-type households. The combined problem for periods 2 and 3 of household $i \in \{b, l\}$ can be summarized as

$$V^{i,\omega}_2(d_{i,1}, b_{i,1}; \bar{d}_1, \bar{b}_1) = \max_{c^{\omega}_{i,2}, d^{\omega}_{i,2}, b^{\omega}_{i,2}, c^{\omega}_{i,3}, d^{\omega}_{i,3}} u(c^{\omega}_{i,2}, d^{\omega}_{i,2}) + \beta u(c^{\omega}_{i,3}, d^{\omega}_{i,3}) \tag{22}$$

$$\text{s.t. } b^{\omega}_{i,1} + y^{\omega}_{i,2} = c^{\omega}_{i,2} + q^{\omega}_2(d^{\omega}_{i,2} - d_{i,1}) + b^{\omega}_{i,2}/r^{\omega}_2, \tag{23}$$

$$b^{\omega}_{i,2} + y = c^{\omega}_{i,3} + q^{\omega}_3(d^{\omega}_{i,3} - d^{\omega}_{i,2}), \tag{24}$$

$$-b^{\omega}_{i,2} \leq \gamma q^{\omega}_2 d^{\omega}_{i,2}, \tag{25}$$

where prices are functions of $\bar{d}_1$ and $\bar{b}_1$, leading to the first-order conditions (6)-(8). The latter as well as (23) and (24) all holding for $i \in \{b, l\}$ and $\omega \in \{e, u\}$, $d^{\omega}_{b,t} + d^{\omega}_{l,t} = d$ for $\omega \in \{e, u\}$ and $t \in \{2, 3\}$, $b^{\omega}_{b,2} + b^{\omega}_{l,2} = 0$ for $\omega \in \{e, u\}$, $\mu^{\omega}_{l,2} = 0$ for $\omega \in \{e, u\}$, $\mu^{e}_{b,2} = 0$ and $-b^{u}_{b,2} = \gamma q^{u}_2 d^{u}_{b,2}$ determine the equilibrium solution for the household choices $\{c^{\omega}_{i,2}, d^{\omega}_{i,2}, b^{\omega}_{i,2}, c^{\omega}_{i,3}, d^{\omega}_{i,3}\}$ for $i \in \{b, l\}$ and $\omega \in \{e, u\}$, prices $\{q^{\omega}_2, r^{\omega}_2, q^{\omega}_3\}$ for $\omega \in \{e, u\}$, and the multipliers $\{\mu^{\omega}_{b,2}, \mu^{\omega}_{l,2}\}$ for $\omega \in \{e, u\}$, given $\{d_{i,1}, b_{i,1}\}$ for $i \in \{b, l\}$.

Then, a household i's problem in period 1 can be summarized as

$$\max{}_{c_{i,1}, d_{i,1}, b_{i,1}} u(c_{i,1}, d_{i,1}) + \beta E_1[V^{i,\omega}_2(d_{i,1}, b_{i,1}; \bar{d}_1, \bar{b}_1)]$$
$$\text{s.t. } c_{i,1} + q_1(d_{i,1} - d_{i,0}) + b_{i,1}/r_1 = b_{i,0} + y_{i,1},$$
$$- b_{i,1} \leq \gamma q_1 d_{i,1},$$

leading to the first-order conditions (4) and (5). The latter for $i \in \{b, l\}$, the budget constraint (3) for $t = 1$, for $i \in \{b, l\}$, $-b_{b,1} \leq \gamma q_1 d_{b,1}$ and $\mu_{b,1}(b_{b,1} + \gamma q^{\varepsilon}_1 d_{b,1}) = 0$, $\mu_{l,1} = 0$, $d_{b,t} + d_{l,t} = \bar{d}$ and $b_{b,1} + b_{l,1} = 0$, then determine the equilibrium solution for the allocation $\{c_{i,1}, d_{i,1}, b_{i,1}\}$ for $i \in \{b, l\}$, prices $\{q_1, r_1\}$ and multipliers $\{\mu_{b,1}, \mu_{l,1}\}$, given initial values $b_{i,0} = 0$ and $d_{i,0} > 0$.

B Social planner problem in the three-period model

The social planner problem is given as

$$
\begin{aligned}
\mathcal{L} \equiv \sum_i \theta_i & \left\{ u_1(\bar{c}_{i,1}, \bar{d}_{i,1}) + \beta E_1 \left[V_2^{i,\omega}(\bar{d}_{i,1}, \bar{b}_{i,1}; \bar{d}_1, \bar{b}_1) \right] \right\} \\
& + \theta_b \lambda_{b,1}^{bud} \left[y_{b,1} - \bar{c}_{b,1} - q_1(\bar{d}_{b,1} - \bar{d}_{b,0}) - \bar{b}_{b,1} r_1^{-1} \right] \\
& + \theta_l \lambda_{l,1}^{bud} \left[y_{l,1} - \bar{c}_{l,1} - q_1(\bar{d}_{l,1} - \bar{d}_{l,0}) - \bar{b}_{l,1} r_1^{-1} \right] \\
& + \theta_b \mu_{b,1} \left[\bar{b}_{b,1} + \gamma q_1 \bar{d}_{b,1} \right],
\end{aligned}
$$

where we use the market clearing conditions for debt, $\bar{b}_{l,1} = -\bar{b}_{b,1}$, and durables, $\bar{d} = \bar{d}_{l,1} + \bar{d}_{b,1}$. The planner takes into account that prices r_1 and q_1 are functions of the allocation and implicitly given by

$$
\begin{aligned}
r_1^{-1}(\bar{d}_1, \bar{b}_1, \bar{c}_{l,1}) &= \beta E_1 \left[u_c'(\bar{c}_{l,2}^{\omega}, \bar{d}_{l,2}^{\omega}) \right] / u_c'(\bar{c}_{l,1}, \bar{d}_{l,1}), \\
q_1(\bar{d}_1, \bar{b}_1, \bar{c}_{l,1}) &= \left(u_d'(\bar{c}_{l,1}, \bar{d}_{l,1}) + \beta E_1 \left[u_c'(\bar{c}_{l,2}^{\omega}, \bar{d}_{l,2}^{\omega}) q_2^{\omega} \right] \right) / u_c'(\bar{c}_{l,1}, \bar{d}_{l,1}).
\end{aligned}
$$

The social planner's first-order conditions with respect to $\bar{c}_{b,1}$, $\bar{c}_{l,1}$, $\bar{d}_{b,1}$ and $\bar{b}_{b,1}$ are given by

$$
0 = \theta_b u_c'(\bar{c}_{b,1}, \bar{d}_{b,1}) - \theta_b \lambda_{b,1}^{bud}, \tag{26}
$$

$$
\begin{aligned}
0 = \theta_l u_c'(\bar{c}_{l,1}, \bar{d}_{l,1}) &+ \theta_b \lambda_{b,1}^{bud} \left(-\frac{\partial q_1}{\partial \bar{c}_{l,1}} (\bar{d}_{b,1} - \bar{d}_{b,0}) - \bar{b}_{b,1} \frac{\partial r_1^{-1}}{\partial \bar{c}_{l,1}} \right) \\
+ \theta_l \lambda_{l,1}^{bud} & \left(-1 - \frac{\partial q_1}{\partial \bar{c}_{l,1}} (\bar{d}_{l,1} - \bar{d}_{l,0}) - \bar{b}_{l,1} \frac{\partial r_1^{-1}}{\partial \bar{c}_{l,1}} \right) + \theta_b \mu_{b,1} \gamma \frac{\partial q_1}{\partial \bar{c}_{l,1}} \bar{d}_{b,1},
\end{aligned} \tag{27}
$$

as well as

$$
\begin{aligned}
0 = {} & \theta_b u_d'(\bar{c}_{b,1}, \bar{d}_{b,1}) - \theta_l u_d'(\bar{c}_{l,1}, \bar{d}_{l,1}) + \beta E_1 \left[\theta_b \frac{\partial V_2^{b,\omega}}{\partial d_{b,1}} \right] - \beta E_1 \left[\theta_l \frac{\partial V_2^{l,\omega}}{\partial d_{l,1}} \right] \\
& + \beta \left(\theta_b E_1 \left[\frac{\partial V^{b,\omega}}{\partial \bar{d}_1} \frac{\partial \bar{d}_1}{\partial \bar{d}_{b,1}} \right] + \theta_l E_1 \left[\frac{\partial V^{l,\omega}}{\partial \bar{d}_1} \frac{\partial \bar{d}_1}{\partial \bar{d}_{b,1}} \right] \right) - \beta \left(\theta_b E_1 \left[\frac{\partial V^{b,\omega}}{\partial \bar{d}_1} \frac{\partial \bar{d}_1}{\partial \bar{d}_{l,1}} \right] + \theta_l E \left[\frac{\partial V^{l,\omega}}{\partial \bar{d}_1} \frac{\partial \bar{d}_1}{\partial \bar{d}_{l,1}} \right] \right) \\
& + \theta_b \lambda_{b,1}^{bud} \left(-\frac{\partial q_1}{\partial \bar{d}_{b,1}} (\bar{d}_{b,1} - \bar{d}_{b,0}) - q_1 - \bar{b}_{b,1} \frac{\partial r_1^{-1}}{\partial \bar{d}_{b,1}} \right) + \theta_l \lambda_{l,1}^{bud} \left(-\frac{\partial q_1}{\partial \bar{d}_{b,1}} (\bar{d}_{l,1} - \bar{d}_{l,0}) - \bar{b}_{l,1} \frac{\partial r_1^{-1}}{\partial \bar{d}_{b,1}} \right) \\
& - \theta_b \lambda_{b,1}^{bud} \left(-\frac{\partial q_1}{\partial \bar{d}_{l,1}} (\bar{d}_{b,1} - \bar{d}_{b,0}) - \bar{b}_{b,1} \frac{\partial r_1^{-1}}{\partial \bar{d}_{l,1}} \right) - \theta_l \lambda_{l,1}^{bud} \left(-\frac{\partial q_1}{\partial \bar{d}_{l,1}} (\bar{d}_{l,1} - \bar{d}_{l,0}) - q_1 - \bar{b}_{l,1} \frac{\partial r_1^{-1}}{\partial \bar{d}_{b,1}} \right) \\
& + \theta_b \mu_{b,1} \gamma \left(\frac{\partial q_1}{\partial \bar{d}_{b,1}} \bar{d}_{b,1} + q_1 \right) + \theta_b \mu_{b,1} \gamma \left(-\frac{\partial q_1}{\partial \bar{d}_{l,1}} \bar{d}_{b,1} \right),
\end{aligned} \tag{28}
$$

and

$$0 = \beta E_1 \left[\theta_b \frac{\partial V_2^{b,\omega}}{\partial b_{b,1}} \right] - \beta E_1 \left[\theta_l \frac{\partial V_2^{l,\omega}}{\partial b_{l,1}} \right] + \beta \left(\theta_b E_1 \left[\frac{\partial V^{b,\omega}}{\partial \bar{b}_1} \frac{\partial \bar{b}_1}{\partial \bar{b}_{b,1}} \right] + \theta_l E_1 \left[\frac{\partial V^{l,\omega}}{\partial \bar{b}_1} \frac{\partial \bar{b}_1}{\partial \bar{b}_{b,1}} \right] \right) \tag{29}$$
$$- \beta \left(\theta_b E \left[\frac{\partial V^{b,\omega}}{\partial \bar{b}_1} \frac{\partial \bar{b}_1}{\partial \bar{b}_{l,1}} \right] + \theta_l E_1 \left[\frac{\partial V^{l,\omega}}{\partial \bar{b}_1} \frac{\partial \bar{b}_1}{\partial \bar{b}_{l,1}} \right] \right) + \theta_b \lambda_{b,1}^{bud} \left(-\frac{\partial q_1}{\partial \bar{b}_{b,1}} (\bar{d}_{b,1} - \bar{d}_{b,0}) - r_1^{-1} - b_{b,1} \frac{\partial r_1^{-1}}{\partial \bar{b}_{b,1}} \right)$$
$$+ \theta_l \lambda_{l,1}^{bud} \left(-\frac{\partial q_1}{\partial \bar{b}_{b,1}} (\bar{d}_{l,1} - \bar{d}_{l,0}) - b_{l,1} \frac{\partial r_1^{-1}}{\partial \bar{b}_{b,1}} \right) - \theta_b \lambda_{b,1}^{bud} \left(-\frac{\partial q_1}{\partial \bar{b}_{l,1}} (\bar{d}_{b,1} - \bar{d}_{b,0}) - b_{b,1} \frac{\partial r_1^{-1}}{\partial \bar{b}_{l,1}} \right)$$
$$- \theta_l \lambda_{l,1}^{bud} \left(-\frac{\partial q_1}{\partial \bar{b}_{l,1}} (\bar{d}_{l,1} - \bar{d}_{l,0}) - r_1^{-1} - b_{l,1} \frac{\partial r_1^{-1}}{\partial \bar{b}_{l,1}} \right) + \theta_b \mu_{b,1} \left(1 + \gamma \frac{\partial q_1}{\partial \bar{b}_{b,1}} \bar{d}_{b,1} \right) + \theta_b \mu_{b,1} \left(-\gamma \frac{\partial q_1}{\partial \bar{b}_{l,1}} \bar{d}_{b,1} \right).$$

First, consider the first-order condition for durables (28). Note that $\partial V^{i,\omega} / \partial d_{i,1} = \lambda_{i,2}^{bud,\omega} q_2^{\omega}$. Since $-\theta_l u_d'(\bar{c}_{l,1}, \bar{d}_{l,1}) - \beta E_1 \left[\theta_l \lambda_{l,2}^{bud,\omega} q_2^{\omega} \right] - \theta_l \lambda_{l,1}^{bud} q_1 = 0$ holds because of the lender's first-order condition (see L1), the social planner's first-order condition for durables (28) can be written as

$$0 = \theta_b u_d'(\bar{c}_{b,1}, \bar{d}_{b,1}) + \beta E_1 \left[\theta_b \lambda_{b,2}^{bud,\omega} q_2^{\omega} \right] - q_1 \theta_b \lambda_{b,1}^{bud} + \theta_b \mu_{b,1} \gamma q_1 \tag{30}$$
$$+ \beta \left(\theta_b E_1 \left[\frac{\partial V^{b,\omega}}{\partial \bar{d}_1} \frac{\partial \bar{d}_1}{\partial \bar{d}_{b,1}} \right] + \theta_l E_1 \left[\frac{\partial V^{l,\omega}}{\partial \bar{d}_1} \frac{\partial \bar{d}_1}{\partial \bar{d}_{b,1}} \right] \right) - \beta \left(\theta_b E \left[\frac{\partial V^{b,\omega}}{\partial \bar{d}_1} \frac{\partial \bar{d}_1}{\partial \bar{d}_{l,1}} \right] + \theta_l E_1 \left[\frac{\partial V^{l,\omega}}{\partial \bar{d}_1} \frac{\partial \bar{d}_1}{\partial \bar{d}_{l,1}} \right] \right)$$
$$+ \left(\theta_b \lambda_{b,1}^{bud} - \theta_l \lambda_{l,1}^{bud} \right) \left(-\frac{\partial q_1}{\partial \bar{d}_{b,1}} (\bar{d}_{b,1} - \bar{d}_{b,0}) - \bar{b}_{b,1} \frac{\partial r_1^{-1}}{\partial \bar{d}_{b,1}} \right)$$
$$+ \theta_b \mu_{b,1} \gamma \left(\frac{\partial q_1}{\partial \bar{d}_{b,1}} - \frac{\partial q_1}{\partial \bar{d}_{l,1}} \right) \bar{d}_{b,1} - \left(\theta_b \lambda_{b,1}^{bud} - \theta_l \lambda_{l,1}^{bud} \right) \left(-\frac{\partial q_1}{\partial \bar{d}_{l,1}} (\bar{d}_{b,1} - \bar{d}_{b,0}) - \bar{b}_{b,1} \frac{\partial r_1^{-1}}{\partial \bar{d}_{l,1}} \right),$$

where we used the market clearing conditions $\bar{d}_{l,t} = \bar{d} - \bar{d}_{b,t}$ and $\bar{b}_{l,t} = -\bar{b}_{b,t}$ for $t = 1$. Define

$$\theta_b \Delta_1^d = \left(\theta_b \lambda_{b,1}^{bud} - \theta_l \lambda_{l,1}^{bud} \right) \left(-\frac{\partial q_1}{\partial \bar{d}_{b,1}} (\bar{d}_{b,1} - \bar{d}_{b,0}) - \bar{b}_{b,1} \frac{\partial r_1^{-1}}{\partial \bar{d}_{b,1}} \right) + \theta_b \mu_{b,1} \gamma \frac{\partial q_1}{\partial \bar{d}_{b,1}} \bar{d}_{b,1} \tag{31}$$
$$- \left(\theta_b \lambda_{b,1}^{bud} - \theta_l \lambda_{l,1}^{bud} \right) \left(-\frac{\partial q_1}{\partial \bar{d}_{l,1}} (\bar{d}_{b,1} - \bar{d}_{b,0}) - \bar{b}_{b,1} \frac{\partial r_1^{-1}}{\partial \bar{d}_{l,1}} \right) - \theta_b \mu_{b,1} \gamma \frac{\partial q_1}{\partial \bar{d}_{l,1}} \bar{d}_{b,1},$$

and

$$D_{d_{j,1}}^i = -\frac{\partial q_1}{\partial \bar{d}_{j,1}} (\bar{d}_{i,1} - \bar{d}_{i,0}) - \frac{\partial r_1^{-1}}{\partial \bar{d}_{j,1}} \bar{b}_{i,1}, \quad \text{and} \quad C_{d_{j,1}}^i = \gamma \bar{d}_{i,1}^1 \frac{\partial q_1}{\partial \bar{d}_{j,1}},$$

for $i, j \in \{b, l\}$, to get

$$\Delta_1^d = \left(\lambda_{b,1}^{bud} - \frac{\theta_l}{\theta_b} \lambda_{l,1}^{bud} \right) \left(D_{d_{b,1}}^b - D_{d_{l,1}}^b \right) + \mu_{b,1} \left(C_{d_{b,1}}^b - C_{d_{l,1}}^b \right). \tag{32}$$

Since the planner respects the lenders' first-order conditions in the first period (see L1), the shadow prices in the first period are asymmetric and given by (see 27 and 26)

$$\begin{aligned}
\lambda_{b,1}^{bud} &= u_c'(\bar{c}_{b,1}, \bar{d}_{b,1}) \\
\lambda_{l,1}^{bud} &= u_c'(\bar{c}_{l,1}, \bar{d}_{l,1}) + \frac{\theta_b}{\theta_l}\lambda_{b,1}^{bud}\left(-\frac{\partial q_1}{\partial \bar{c}_{l,1}}(\bar{d}_{b,1} - \bar{d}_{b,0}) - \bar{b}_{b,1}\frac{\partial r_1^{-1}}{\partial \bar{c}_{l,1}}\right) \\
&\quad + \lambda_{l,1}^{bud}\left(-\frac{\partial q_1}{\partial \bar{c}_{l,1}}(\bar{d}_{l,1} - \bar{d}_{l,0}) - \bar{b}_{l,1}\frac{\partial r_1^{-1}}{\partial \bar{c}_{l,1}}\right) + \frac{\theta_b}{\theta_l}\mu_{b,1}\gamma\frac{\partial q_1}{\partial \bar{c}_{l,1}}\bar{d}_{b,1} \\
&= u_c'(\bar{c}_{l,1}, \bar{d}_{l,1}) + \left(\frac{\theta_b}{\theta_l}\lambda_{b,1}^{bud} - \lambda_{l,1}^{bud}\right) D_{c_{l,1}}^b + \theta_b\mu_{b,1}C_{c_{l,1}}^b,
\end{aligned} \tag{33}$$

where we used $\bar{d}_{l,t} = \bar{d} - \bar{d}_{b,t}$ and $\bar{b}_{l,t} = -\bar{b}_{b,t}$ and defined

$$D_{c_{l,1}}^b = -\frac{\partial q_1}{\partial \bar{c}_{l,1}}(\bar{d}_{b,1} - \bar{d}_{b,0}) - \frac{\partial r_1^{-1}}{\partial \bar{c}_{l,1}}\bar{b}_{b,1}, \quad \text{and} \quad C_{c_{l,1}}^b = \gamma \bar{d}_{i,1}^1 \frac{\partial q_1}{\partial \bar{c}_{l,1}}.$$

The shadow price for the lender can be written as

$$\lambda_{l,1}^{bud} = \frac{u_c'(\bar{c}_{l,1}, \bar{d}_{l,1}) + \frac{\theta_b}{\theta_l}u_c'(\bar{c}_{b,1}, \bar{d}_{b,1})D_{c_{l,1}}^b + \frac{\theta_b}{\theta_l}\mu_{b,1}C_{c_{l,1}}^b}{1 + D_{c_{l,1}}^b}, \tag{34}$$

which reduces to $\lambda_{l,1}^{bud} = u_c'(\bar{c}_{l,1}, \bar{d}_{l,1})$ for $D_{c_{l,1}}^b = C_{c_{l,1}}^b = 0$. As in Davila and Korinek (2018), we use that

$$\frac{\partial V_2^{i,\omega}(\bar{d}_{i,1}, \bar{b}_{i,1}; \bar{d}_1, \bar{b}_1)}{\partial \bar{x}_{i,1}} = \frac{\partial V_2^{i,\omega}(\bar{d}_{i,1}, \bar{b}_{i,1}; \bar{d}_1, \bar{b}_1)}{\partial x_{i,1}} + \frac{\partial V_2^{i,\omega}(\bar{d}_{i,1}, \bar{b}_{i,1}; \bar{d}_1, \bar{b}_1)}{\partial \bar{x}_1}\frac{\partial \bar{x}_1}{\partial \bar{x}_{i,1}} \tag{35}$$

where $\partial V_2^{i,\omega}(\cdot)/\partial d_{i,1} = \lambda_{b,2}^{bud,\omega} q_2^\omega$ and $\partial V_2^{i,\omega}(\cdot)/\partial b_{i,1} = \lambda_{b,2}^{bud,\omega}$ are the derivatives of $V_2^{i,\omega}(\cdot)$ with respect to to individual states, evaluated at the equilibrium values, i.e. $x_{i,1} = \bar{x}_{i,1}$, $x \in \{b, d\}$, and the derivatives of $V_2^{i,\omega}(\cdot)$ with respect to type-specific aggregate variables are

$$\begin{aligned}
\frac{\partial V^{i,\omega}}{\partial \bar{b}_1}\frac{\partial \bar{b}_1}{\partial \bar{b}_{j,1}} &= \lambda_{i,2}^{bud,\omega} D_{b_{j,1}}^{i,\omega} + \mu_{i,2}^\omega C_{b_{j,1}}^{i,\omega}, \\
\frac{\partial V^{i,\omega}}{\partial \bar{d}_1}\frac{\partial \bar{d}_1}{\partial \bar{d}_{j,1}} &= \lambda_{i,2}^{bud,\omega} D_{d_{j,1}}^{i,\omega} + \mu_{i,2}^\omega C_{d_{j,1}}^{i,\omega},
\end{aligned} \tag{36}$$

and the un-internatlized price effects are

$$D_{x_{b,2}}^{i,\omega} = -\frac{\partial q_2^\omega}{\partial \bar{x}_{j,1}}(\bar{d}_{i,2}^\omega - \bar{d}_{i,1}) - \frac{\partial \left(r_2^\omega\right)^{-1}}{\partial \bar{x}_{j,1}} b_{i,2}^\omega, \quad \text{and} \quad C_{x_{j,2}}^{i,\omega} = \gamma \bar{d}_{i,2}^\omega \frac{\partial q_2^\omega}{\partial \bar{x}_{j,1}},$$

for $i, j \in \{b, l\}$ and for $x \in \{b, d\}$. Note that $D_{x_{j,2}}^{l,\omega} = -D_{x_{j,2}}^{b,\omega}$ holds as in Davila and Korinek (2018),

$$\begin{aligned}
D_{x_{j,2}}^{l,\omega} &= -\frac{\partial q_2^\omega}{\partial \bar{x}_{j,1}}(\bar{d}_{l,2}^\omega - \bar{d}_{l,1}) - \frac{\partial \left(r_2^\omega\right)^{-1}}{\partial \bar{x}_{j,1}} b_{l,2}^\omega = -\frac{\partial q_2^\omega}{\partial \bar{x}_{j,1}}(\bar{d} - \bar{d}_{b,2}^\omega - \bar{d} + \bar{d}_{b,1}) + b_{b,2}^\omega \frac{\partial \left(r_2^\omega\right)^{-1}}{\partial \bar{x}_{j,1}} \\
&= -D_{x_{j,2}}^{b,\omega},
\end{aligned}$$

which was used for $t = 1$ to get (30) by exploiting the market clearing conditions. Now define

$$\begin{aligned}
\theta_b \Delta_2^{d,\omega} &= \theta_b \frac{\partial V^{i,\omega}}{\partial \bar{d}_1} \frac{\partial \bar{d}_1}{\partial \bar{d}_{b,1}} - \theta_b \frac{\partial V^{i,\omega}}{\partial \bar{d}_1} \frac{\partial \bar{d}_1}{\partial \bar{d}_{l,1}} + \theta_l \frac{\partial V^{i,\omega}}{\partial \bar{b}_1} \frac{\partial \bar{b}_1}{\partial \bar{b}_{b,1}} - \theta_l \frac{\partial V^{i,\omega}}{\partial \bar{b}_1} \frac{\partial \bar{b}_1}{\partial \bar{b}_{l,1}} \\
&= \left(\theta_b \lambda_{b,2}^{bud,\omega} - \theta_l \lambda_{l,2}^{bud,\omega}\right) \left(-\frac{q_2^\omega}{\partial \bar{d}_{b,1}} (\bar{d}_{b,2}^\omega - \bar{d}_{b,1}) - b_{b,2}^\omega \frac{\partial \left(r_2^\omega\right)^{-1}}{\partial \bar{d}_{b,1}}\right) \\
&- \left(\theta_b \lambda_{b,2}^{bud,\omega} - \theta_l \lambda_{l,2}^{bud,\omega}\right) \left(-\frac{\partial q_2^\omega}{\partial \bar{d}_{l,1}} (\bar{d}_{b,2}^\omega - \bar{d}_{b,1}) - b_{b,2}^\omega \frac{\partial \left(r_2^\omega\right)^{-1}}{\partial \bar{d}_{l,1}}\right) \\
&+ \theta_b \mu_{b,2}^\omega \gamma \frac{\partial q_2^\omega}{\partial \bar{d}_{b,1}} \bar{d}_{b,2}^\omega - \theta_b \mu_{b,2}^\omega \gamma \frac{\partial q_2^\omega}{\partial \bar{d}_{l,1}} \bar{d}_{b,2}^\omega \\
&= \left(\theta_b \lambda_{b,2}^{bud,\omega} - \theta_l \lambda_{l,2}^{bud,\omega}\right) \left(D_{d_{b,2}}^{b,\omega} - D_{d_{l,2}}^{b,\omega}\right) + \theta_b \mu_{b,2}^\omega \left(C_{d_{b,2}}^{b,\omega} - C_{d_{l,2}}^{b,\omega}\right),
\end{aligned} \tag{37}$$

where the second equality uses (36). Using (32) and (37), condition (30) can now be written as $q_1 \lambda_{b,1}^{bud} = u_d'(\bar{c}_{b,1}, \bar{d}_{b,1}) + \beta E_1[\lambda_{b,2}^{bud,\omega} q_2^\omega] + \mu_{b,1}\gamma q_1 + \Delta_1^d + \beta E_1 \Delta_2^{d,\omega}$or

$$q_1 u_c'(\bar{c}_{b,1}, \bar{d}_{b,1}) = u_d'(\bar{c}_{b,1}, \bar{d}_{b,1}) + \beta E_1 \left[u_c'(\bar{c}_{b,2}^\omega, \bar{d}_{b,2}^\omega) q_2^\omega\right] + \mu_{b,1}\gamma q_1 + \Delta_1^d + \beta E_1 \left[\Delta_2^{d,\omega}\right], \tag{38}$$

with the first-period wedge

$$\begin{aligned}
\Delta_1^d &= \left(\lambda_{b,1}^{bud} - \frac{\theta_l}{\theta_b} \lambda_{l,1}^{bud}\right) \left(D_{d_{b,1}}^b - D_{d_{l,1}}^b\right) + \mu_{b,1} \left(C_{x_{b,1}}^b - C_{x_{l,1}}^b\right) \\
&= \left(\frac{u_c'(\bar{c}_{b,1}, \bar{d}_{b,1}) - \frac{\theta_l}{\theta_b} u_c'(\bar{c}_{l,1}, \bar{d}_{l,1}) - \mu_{b,1} C_{c_{l,1}}^b}{1 + D_{c_{l,1}}^b}\right) \left(D_{d_{b,1}}^b - D_{d_{l,1}}^b\right) + \mu_{b,1} \left(C_{x_{b,1}}^b - C_{x_{l,1}}^b\right),
\end{aligned}$$

and with the second-period wedge

$$\begin{aligned}
\Delta_2^{d,\omega} &= \left(\lambda_{b,2}^{bud,\omega} - \frac{\theta_l}{\theta_b} \lambda_{l,2}^{bud,\omega}\right) \left(D_{d_{b,2}}^{b,\omega} - D_{d_{l,2}}^{b,\omega}\right) + \mu_{b,2}^\omega \left(C_{d_{b,2}}^{b,\omega} - C_{d_{l,2}}^{b,\omega}\right), \\
&= \left(u_c'(\bar{c}_{b,2}^\omega, \bar{d}_{b,2}^\omega) - \frac{\theta_l}{\theta_b} u_c'(\bar{c}_{l,2}^\omega, \bar{d}_{l,2}^\omega)\right) \left(D_{d_{b,2}}^{b,\omega} - D_{d_{l,2}}^{b,\omega}\right) + \mu_{b,2}^\omega \left(C_{d_{b,2}}^{b,\omega} - C_{d_{l,2}}^{b,\omega}\right),
\end{aligned}$$

where we used $\lambda_{b,2}^{bud,\omega} = u_c'(\bar{c}_{b,2}^\omega, \bar{d}_{b,2}^\omega)$ and $\lambda_{l,2}^{bud,\omega} = u_c'(\bar{c}_{l,2}^\omega, \bar{d}_{l,2}^\omega)$. To rewrite the first-order conditions for bonds (29), we use $\partial V^{i,\omega}/\partial b_{i,1} = \lambda_{i,2}^{bud,\omega}$ and $\theta_l \lambda_{l,1}^{bud} r_1^{-1} = \beta E_1[\theta_l \lambda_{l,2}^{bud,\omega}]$.

Proceeding as above, then gives

$$\theta_b \lambda_{b,1}^{bud} r_1^{-1} = \beta E_1 \left[\theta_b \lambda_{b,2}^{bud,\omega}\right] + \theta_b \mu_{b,1} + \Delta_1^b + \beta E_1 \left[\Delta_2^{b,\omega}\right],$$

which completes the derivation of the results in Section 2.2.

C Definition of equilibrium for infinite-horizon model

Let $\Phi_t(x, y)$ denote the joint distribution of wealth and income across households in period t. An equilibrium for the infinite-horizon model of Section 3.1 is defined as:

Definition 1 *Given an initial distribution Φ_0, an equilibrium consists of a sequence of prices $\{r_t, q_t\}$, a sequence of household policy functions $\{b_t(x, y), c_t(x, y), d_t(x, y)\}$, a sequence of taxes $\{\tau_t^b, \tau_t^d\}$, and a sequence of joint distributions of wealth and income $\{\Phi_t\}$, such that*

(i) the policy functions $b_t(x, y)$, $c_t(x, y)$ and $d_t(x, y)$ solve the household problem given $\{r_t, q_t\}$ and $\{\tau_t^b, \tau_t^d\}$,

(ii) the distribution Φ_t is consistent with the household policy functions,

(iii) the markets for bonds and durables clear,

$$\begin{aligned} \sum b_t(x, y) d\Phi_t(x, y) &= 0, \\ \sum d_t(x, y) d\Phi_t(x, y) &= \bar{d}. \end{aligned}$$

The market for non-durable goods clears via Walras' Law, given that the policy functions ensure that all household budget constraints are satisfied. For a stationary equilibrium, we additionally require the distribution of wealth and income as well as prices to be constant over time, i.e. $\Phi_{t+1} = \Phi$, $r_t = r$ and $q_t = q$ for all t. Note that such an equilibrium requires that the tax rates are constant over time as well.

D Computational algorithm

This section presents how we solve the quantitative model from Section 3.1. First, we discuss how to solve for the stationary equilibrium of the model economy. Then, we show how to solve for the transition path between two different stationary equilibria.

D.1 Calculation of the stationary equilibrium

Solving for the stationary equilibrium involves finding time-invariant values for the real interest rate r and the price of durables q as well as a time-invariant joint distribution of wealth and income implied by the household policy functions such that the markets for durables and bonds clear (see previous section). The numerical procedure involves the following steps:

I. Choose initial values for r and q.

II. Given r and q, compute the policy functions for non-durable consumption $c(x, y)$, end-of-period bonds $b'(x, y)$, end-of-period durables $d'(x, y)$ and end-of-period wealth $x'(x, y) = b'(x, y) + qd'(x, y)$, using the endogenous grid point method (see Hintermaier and Koeniger, 2010) as outlined below.

III. Given the wealth policy function $x'(x,y)$, compute the implied stationary distribution $\lambda(x,y)$ (see below).

IV. Check whether markets for debt and durables clear. If $|\sum_{x,y}\lambda(x,y)b'(x,y)| < \epsilon^b$ and $|\sum_{x,y}\lambda(x,y)d'(x,y) - \bar{d}| < \epsilon^d$, with $\epsilon^b > 0$ and $\epsilon^d > 0$, stop: r and q are the equilibrium prices. Else, update prices (r,q) and go to Step II.

Solving the household problem via the endogenous grid method The endogenous grid point method used to solve the household problem for r and q involves the following steps:

1. Discretize next period's wealth space $\{x'_1, x'_2, ..., x'_m\}$, $x'_i < x'_{i+1}$. The discretized individual state space then is given by $\{x'_1, x'_2, ..., x'_m\} \times \{y'_1, y'_2, ..., y'_n\}$, where y'_k, $k = 1, ..., n$, are the income states that are possible next period.[31] Select a stopping rule parameter $\epsilon^{egm} > 0$.

2. Initialize the policy functions for non-durable and durable consumption $c_0(x'_i, y'_k)$ and $d'_0(x'_i, y'_k)$, $k \in \{1, ..., n\}$. Our guess is given by $c_0(x'_i, y'_k) = 0.5y'_k$ and $d'_0(x'_i, y'_k) = 0.5\bar{d}$ for all grid point combinations.

3. Update the consumption policy functions (using three auxiliary functions $\hat{c}_0(x'_i, y_k)$, $\hat{x}_0(x'_i, y_k)$ and $\hat{d}'_0(x'_i, y_k)$):

   - First, assume that the borrowing constraint does not bind in any state.
   - Use consumption policy functions $c_0(x'_i, y'_k)$ and $d'_0(x'_i, y'_k)$ to compute a guess for current-period non-durable and durable consumption at future wealth x'_i and today's income state y_k, i.e. $\hat{c}_0(x'_i, y_k)$ and $\hat{d}'_0(x'_i, y_k)$, by applying the Euler equations for bonds and durables:

$$\begin{aligned}
u_c(\hat{c}_0(x'_i, y_k), \hat{d}'_0(x'_i, y_k)) =& \beta r \sum_{j=1}^{n} \pi\,(y_j|y_k)\, u_c(c_0(x'_i, y'_j), d'_0(x'_i, y'_j)),\\
u_c(\hat{c}_0(x'_i, y_k), \hat{d}'_0(x'_i, y_k))q =& u_d(\hat{c}_0(x'_i, y_k), \hat{d}'_0(x'_i, y_k))\\
&+ \beta q \sum_{j=1}^{n} \pi\,(y_j|y_k)\, u_c(c_0(x'_i, y'_j), d'_0(x'_i, y'_j)),
\end{aligned}$$

     which are two equations in the two unknowns $\hat{c}_0(x'_i, y_k)$ and $\hat{d}'_0(x'_i, y_k)$ at given values of x'_i and y_k.

   - Now, find the states for which the borrowing constraint is violated. If the borrowing constraint is violated at given grid points x'_i and y_k, i.e. $\hat{d}'_0(x'_i, y_k) > x'_i/(q(1-\gamma))$, we set $\hat{d}'_0(x'_i, y_k) = x'_i/(q(1-\gamma))$. The corresponding value for non-durable consumption $\hat{c}_0(x'_i, y_k)$ can then be calculated via the two Euler equations after having combined them by eliminating the multiplier on the borrowing constraint, which now enters both Euler equations. If the

[31] The values for the income grid and the associated transition probabilities are listed in Section D.3.

constraint is not binding, i.e. $\hat{d}'_0(x'_i, y_k) \leq x'_i/(q(1-\gamma))$ holds, we keep the values of $\hat{d}'_0(x'_i, y_k)$ and $\hat{c}_0(x'_i, y_k)$ calculated in the step before for this state.

- Use the budget constraint and the auxiliary functions $\hat{c}_0(x'_i, y_k)$ and $\hat{d}'_0(x'_i, y_k)$ to compute current period wealth $\hat{x}$ for x'_i and y_k:

$$\hat{x}_0(x'_i, y_k) = \hat{c}_0(x'_i, y_k) + q\hat{d}'_0(x'_i, y_k) + \left(x'_i - q\hat{d}'_0(x'_i, y_k)\right)/r - y_k.$$

This implies $\hat{c}_0(x'_i, y_k) = \hat{c}_0(\hat{x}_0(x'_i, y_k), y_k)$ and $\hat{d}'_0(x'_i, y_k) = \hat{d}'_0(\hat{x}_0(x'_i, y_k), y_k)$.

- Calculate updates for the policy functions at $(x'_i, y'_k) \in \{x'_1, x'_2, ..., x'_m\} \times \{y'_1, y'_2, ..., y'_n\}$ by linearly interpolating $\hat{c}_0(\hat{x}_0, y_k)$ and $\hat{d}'_0(\hat{x}_0, y_k)$ at (x'_i, y'_k). This calculation yields the updated consumption policy functions $c_1(x'_i, y'_k)$ and $d'_1(x'_i, y'_k)$.

4. If $||c_1(x'_i, y'_k) - c_0(x'_i, y'_k)||_\infty < \epsilon^{egm}(1 + |c_0(x'_i, y'_k)||_\infty)$ and $||d'_1(x'_i, y'_k) - d'_0(x'_i, y'_k)||_\infty < \epsilon^{egm}(1 + |d'_0(x'_i, y'_k)||_\infty)$, stop and set $c(\cdot) = c_1(\cdot)$ and $d'(\cdot) = d'_1(\cdot)$.
Else, set $c_0(\cdot) = c_1(\cdot)$ and $d'_0(\cdot) = d'_1(\cdot)$ and go to Step 3.

Computing the stationary distribution For given policy functions, we compute the stationary distribution by calculating the normalized eigenvalue of the Markov transition matrix implied by the policy function for wealth and the income transition probabilities:

1. We add additional grid points for wealth relative the grid used for the calculation of the policy functions (we go from 10 to 50 thousand grid points for x) and calculate the wealth policy function values for these new states.

2. We calculate the transition probability of being in the state (x_j, y_l) in the next period conditional on currently being in state (x_i, y_k). We denote it as $\Pr((x_j, y_l)|(x_i, y_k))$. This probability is computed as $\Pr((x_j, y_l)|(x_i, y_k)) = \pi(y_l|y_k) \times I(x'(x_i, y_k) = x_j)$, where $I(x'(x_i, y_k) = x_j) = 1$ if $x'(x_i, y_k) = x_j$ and zero otherwise. The Markov transition matrix then consists of the individual transition probabilities $\Pr((x_j, y_l)|(x_i, y_k))$ for all grid point combinations.

3. Compute the eigenvector of this transition matrix that has the largest eigenvalue (which is equal to one). The stationary distribution of the model economy then is obtained by the normalizing this eigenvector.

Updating prices of debt and durables The prices are updated by using two nested bisection algorithms as follows: For a given price of durables q, we calculate the real interest rate r that clears the loan market, i.e. $|\sum_{x,y} \lambda(x, y)b'(x, y)| < \epsilon^b$, using bisection. If the market for durables is also cleared at this combination of q and r, i.e. $|\sum_{x,y}\lambda(x,y)d'(x,y)-\bar{d}|<\epsilon^d$, we can stop. If not, we update the price of durables q and then again calculate the real interest rate r that clears the loan market. The price q is updated by using bisection, too, to get the price that clears the durables market while the corresponding real interest rate at a given q is the value of r that clears the loan market.

D.2 Calculation of the transition path to the new stationary equilibrium

In period 0, the economy is in the laissez-faire stationary equilibrium without taxation. In the subsequent period 1, one or both tax rates are unexpectedly and permanently changed to the values τ^b_{new} and τ^d_{new}. Due to this change, the economy departs from the old stationary equilibrium in period 1 and gradually moves to the new stationary equilibrium under the tax rates τ^b_{new} and τ^d_{new}. The transition path to the new long-run equilibrium is computed by using the following steps (see e.g. Rios-Rull, 1999):

- Calculate the stationary equilibria for the laissez-faire economy and the economy with τ^b_{new} and τ^d_{new} as described above and denote the associated stationary distributions as Φ_{old} and Φ_{new}, respectively.
- The beginning-of-period distribution in period 0 is denoted Φ_0 and given by $\Phi_0 = \Phi_{old}$. The distribution of the economy once it has converged to the new stationary equilibrium is denoted as Φ_∞. It is given as $\Phi_\infty = \Phi_{new}$. Note that the beginning-of-period distribution of wealth in period 1 is not the same as in period 0 because the policy change will alter household wealth via durables price q. Since the distributions of bonds and durables at the beginning of period 1 are however not affected and the same as in period 0, we can calculate Φ_1 based on these distributions and price q_1.
- Compute the value function $V_0(x,y)$ in period 0, giving the expected lifetime utility of a household who is in the state (x,y) in period $t=0$, and the value function in the new stationary equilibrium $\mathcal{V}_\infty(\cdot)$.
- Computation of the transition path:
  1. Guess that the transition to the new stationary equilibrium takes $T>0$ periods. This implies that $\Phi_T=\Phi_\infty$ and $V_T=V_\infty$.
  2. Guess a sequence of interest rates $\{\hat{r}_t\}_{t=1}^{T-1}$ as well as durables prices $\{\hat{q}_t\}_{t=1}^{T-1}$. Choose stopping rule parameters $\epsilon^b>0$, $\epsilon^d>0$ and $\epsilon^\Phi>0$.
  3. With the known value $V_T(x,y)$ and guesses $\{\hat{r}_t,\hat{q}_t\}_{t=1}^{T-1}$, we can solve for $\left\{\hat{V}_t,\hat{c}_t,\hat{x}_{t+1},\hat{d}_{t+1},\hat{b}_{t+1}\right\}_{t=1}^{T-1}$ via backward induction.
  4. Use the policy functions $\{\hat{x}_{t+1}\}$ and $\Phi_1=\Phi_0$ to iterate the distribution forward to get $\hat{\Phi}_t$ for $t=2,...,T$.

5. Use the sequence $\left\{\hat{\Phi}_t\right\}_{t=1}^{T}$ to compute excess supply $\hat{A}_t^b = \sum \hat{b}_{t+1} \mathrm{d}\hat{\Phi}_t$ and $\hat{A}_t^d = \sum \hat{d}_{t+1} \mathrm{d}\hat{\Phi}_t - \bar{d}$ for periods $t = 1, ..., T$. If

$$\max_{1 \leq t < T} \left|\hat{A}_t^b\right| < \epsilon^b$$
$$\max_{1 \leq t < T} \left|\hat{A}_t^d\right| < \epsilon^d,$$

holds, go to Step 6. Else, adjust the guesses for $\{\hat{r}_t, \hat{q}_t\}_{t=1}^{T-1}$ and go to Step 3.

6. Check whether $\left\|\hat{\Phi}_T - \Phi_T\right\|_\infty < \epsilon^\Phi$. If it does, the model economy smoothly converges to the new stationary equilibrium and the algorithm ends. If not, go back to Step 1 and start again with a higher value for T.

- The obtained $V_1(\cdot)$ is the value function at time $t = 1$ after taxation has changed, such that $V_1(x, y)$ is the expected lifetime utility of a household with income y and beginning-of-period wealth x who has just been hit by the change in taxation. This value hence accounts for the transition of the economy to the new long-run equilibrium.

D.3 Transition probabilities and income values

The individual income transition probabilities are obtained as discussed in Section 3.1. The transition matrix is given as

$$\Pi = \begin{pmatrix} 0.9132 & 0.0867 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0001 \\ 0.0867 & 0.7870 & 0.1200 & 0.0001 & 0.0000 & 0.0000 & 0.0001 \\ 0.0000 & 0.1260 & 0.7355 & 0.1382 & 0.0001 & 0.0000 & 0.0001 \\ 0.0000 & 0.0001 & 0.1382 & 0.7355 & 0.1260 & 0.0000 & 0.0001 \\ 0.0000 & 0.0000 & 0.0001 & 0.1260 & 0.7870 & 0.0867 & 0.0001 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0867 & 0.9132 & 0.0001 \\ 0.0021 & 0.0021 & 0.0021 & 0.0021 & 0.0021 & 0.0021 & 0.9876 \end{pmatrix},$$

and the income grid values $(y_1, y_2, ..., y_7)$ are

$$(0.0123, 0.0250, 0.0376, 0.0544, 0.0819, 0.1667, 1).$$

Let i denote the row index and j the column index of matrix Π. The entry $\Pi(i, j) \equiv \pi(y_j|y_i)$ is the probability that next period's income y_{t+1} equals y_j, conditional on current income $y_t = y_i$.

E Additional figures

Figure 14: Bond holdings and welfare for y_6 (change in loan-to-value ratio γ)

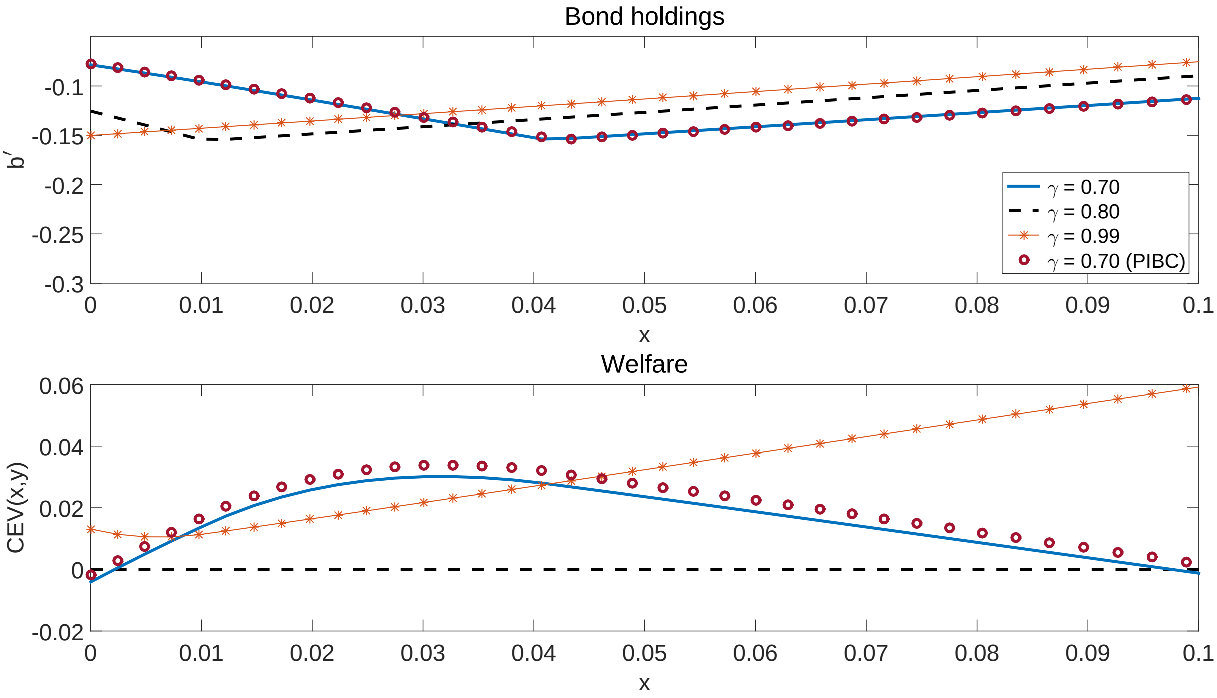

Figure 15: Durable holdings and welfare for y_6 (tax on durables))