Taxing versus subsidizing debt under financial frictions[1]

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Abstract

We examine optimal credit market policies in two models with durables/capital as collateral. Pecuniary externalities rationalize ex-ante debt taxes as macroprudential regulation, achieving constrained efficiency. Ex-post debt subsidies can implement first-best by stimulating collateral demand. Due to the same effect, debt subsidies that are constant over time can be superior to debt taxes. Saving subsidies can further enhance efficiency by addressing distributive effects of pecuniary externalities via interest rate reductions. The analysis shows that debt-increasing subsidies can outperform macroprudential regulation, and that constrained inefficiency caused by collateral externalities is insufficient to establish debt taxes as optimal credit market policies.

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1 Introduction

Pecuniary externalities under collateral constraints can lead to financial amplification and crises. The mechanism relies on price-dependent borrowing limits or margin constraints that tighten when asset prices fall. Agents do not internalize the impact of their decisions on asset prices, such that corrective policies can enhance efficiency. Macroprudential regulation, in form of ex-ante debt taxes and capital controls, can restore "constrained efficiency" – defined in the tradition of Stiglitz (1982) – by addressing "overborrowing", as shown by Jeanne and Korinek (2010, 2019, 2020), Bianchi (2011), Benigno et al. (2016), Korinek and Sandri (2016), Schmitt-Grohe and Uribe (2017), or Korinek (2018). These studies are based on a specific class of models where interest rates are exogenously determined and agents take borrowing limits as given. Thus, neither credit supply nor assets' collateralizability, which both seem to play a central role for the build-up of financial crises (see Geanakoplos, 2010, or Justiniano et al., 2019), are taken into account for the analysis of policies aimed to mitigate these crises. Is their neglect irrelevant?

This paper shows that endogenous interest rates and collateral premia, i.e. the valuation of assets to serve as collateral, are in fact decisive for optimal credit market policy. We apply two finite horizon models that exclusively contain conventional features; one model is taken from Davila and Korinek (2018).[3] Lack of commitment induces borrowing to be limited by borrowers' holdings of durables or capital, serving as collateral. Pecuniary externalities with regard to the collateral price and to the interest rate give rise to "collateral externalities" and "distributive externalities" (see Davila and Korinek, 2018). The former are responsible for the main mechanism in the above cited studies, whereas the latter are turned off therein. While state-contingent ex-post credit market interventions can achieve first best, we particularly focus on policies that are less challenging to be implemented than policies that are fully state-contingent. As the main novel contribution, we show that distributive effects and collateral premia are responsible for non-state-contingent credit market subsides that stimulate borrowing to be superior to debt-reduction policies. At large, inefficiencies due to externalities induced by collat-

[3] We first develop and analyze a model with durables as collateral under uncertainty. Davila and Korinek's (2018) model with capital formation under certainty is subsequently analyzed in Section 5.

eral constraints can be most effectively addressed by saving/debt subsidies that reduce interest rates and raise collateral prices. Overborrowing unambiguously prevails only in the special case where distributive effects and collateral premia are switched off or when the analysis is restricted to ex-ante debt policies, confining Davila and Korinek's (2018) conclusion that "collateral externalities generally lead to overborrowing" (p. 352). Our analysis implies that collateral externalities are neither sufficient to rationalize macroprudential regulation as an optimal policy nor as a policy that is preferable to other non fully state-contingent credit market policies.[4] Rather, our analysis highlights the effectiveness of non state-contingent policies that promote savings and lower borrowing costs. Real-world examples include home loan saving contracts or tax deductions for mortgage interest payments.

In a laissez faire equilibrium (of both models), agents do not internalize that the collateral price is too low and the interest rate is too high when borrowing constraints bind in a non-empty set of states. Following Davila and Korinek's (2018) classification, we distinguish collateral effects, which refer to uninternalized changes in the price of pledgeable assets affecting the collateral value, from distributive effects, which refer to differential effects of uninternalized interest rate changes on heterogenous agents. In contrast to the above cited studies, where agents take borrowing limits as given, the price of pledgeable assets is positively affected by a collateral premium, i.e. the valuation of assets to serve as collateral.[5] Corrective policies can address uninternalized changes in the collateral price, leveraging the fact that the collateral price increases with consumption and with agents' willingness to borrow, which raises the collateral premium. Agents further do not internalize that equilibrium interest rates relate to their consumption choices, which exerts a relevant impact on the equilibrium allocation under different marginal rates of substitution between dates/states (MRSs) of borrowers and lenders (see also Davila and Korinek, 2018). Corrective policies can address this externality and reduce the interest rate, which raises borrowers' consumption and narrows the distance between the MRSs.

[4] Precisely, we define macroprudential regulation as an ex-ante debt tax, following Bianchi and Mendoza (2018): "the macroprudential debt tax [...] is levied in good times when collateral constraints do not bind at date t but can bind with positive probability at $t+1$" (p.591).

[5] This asset price component is also known as the "collateral value" (see Fostel and Geanakoplos, 2008) or the "collateralizability premium" (see Ai et al., 2020).

Apparently, interest rates cannot be reduced when they are assumed to be exogenously determined, and collateral premia do not exist when agents take borrowing limits as given, like in the above cited studies. Then, a correction of the collateral price relies on debt reduction via ex-ante debt taxes.

To identify optimal credit market policies, we assume that the policy maker acts under full commitment and we apply the Ramsey approach to optimal policy, where the policy problem depends on the set of available instruments.[6] If state-contingent credit market instruments are available, first best is implementable via a Pigouvian debt subsidy that is introduced ex-post (i.e., in states where collateral constraints bind). The subsidy increases incentives to borrow and thus the willingness to pay for collateral, measured by the collateral premium. An ex-post debt subsidy can thereby raise the collateral price and the borrowing limit such that borrowing can in principle even get unconstrained, which has also been shown by Katagiri et al. (2017). Based on their quantitative analysis, they conclude that the optimal debt subsidy is practically infeasible, given the size and the frequency of required interventions. State-contingency in fact demands policies to be fine-tuned in response to any change in the state of the economy (see e.g. Bianchi and Mendoza, 2018). Given that the requirement to accurately track relevant conditions and to timely adjust policy tools can hardly be fulfilled in practice (see e.g. Cochrane, 2013), our analysis focusses on Pigouvian policies that are less complex and easier to implement than fully state-contingent policies.[7] Specifically, we examine ex-ante policies, which are imposed before borrowing constraints might become binding, and constant policies, where the tax/subsidy rate is held constant regardless of the state or period (see columns of Table 1). These policies take the form of taxes/subsidies on borrowing or saving (see rows of Table 1), which are non-equivalent under potentially binding borrowing constraints. Linear preferences of lenders further imply that debt policies do not alter the interest rate

[6]This property of the Ramsey approach is also relevant in Benigno et al. (2023), who show that a set of instruments that supports constrained efficiency can also implement an unconstrained allocation. In contrast to Bianchi and Mendoza (2018), who focus on optimal policy under discretion, we abstract from time inconsistency of policy plans.

[7]An alternative would be to adjust policy instruments with changes in variables that can easily be observed, like gdp or debt (see e.g. Bianchi and Mendoza, 2018). In our main model, the latter are however not correlated to the shock that cause borrowing constraints to bind, namely, an unexpected change in income inequality.

	Ex-ante	Fixed over time
Debt	1. **tax**/subsidy → collateral effects	2. tax/**subsidy** → collateral effects
Saving	3. tax/**subsidy** → collateral & distributive effects	4. tax/**subsidy** → collateral & distributive effects

Table 1: Non-state-contingent Pigouvian policies (numerical examples in bold)

and can only address collateral effects, enabling direct comparisons with related studies (see Section 2). In contrast, saving policies can endogenize interest rates and can thereby address distributive effects of pecuniary externalities (see arrows in Table 1).

The first optimal non-state-contingent policy is an ex-ante debt tax. It implements a constrained efficient allocation, as defined in Stiglitz (1982) or Davila et al. (2012). This allocation is chosen by a social planer who determines borrowing and maximizes social welfare subject to budget and borrowing constraints, conditional on maintaining equilibrium price relations under laissez faire. Given that ex-ante debt taxes leave the relevant price relations under laissez faire unchanged, the optimal Ramsey policy in both models implements constrained efficiency (like in Davila and Korinek, 2018). Concretely, it enhances efficiency by raising the collateral price via a debt reduction that increases the amount of funds available for consumption in states where the borrowing constraint binds. In contrast to ex-ante debt taxes, the other policies under consideration alter the price relations for the interest rate and the collateral price, which would not be possible when agents take borrowing limits as given and interest rates are exogenous.

The second non-state-contingent policy is a tax/subsidy on debt that is constant over time and influences borrowing regardless whether the constraint is binding or not. As shown by Bianchi and Mendoza's (2018) analysis of optimal debt policy, a policy maker can alleviate currently binding borrowing constraints by an ex-post subsidy and future borrowing constraints by an ex-ante tax. However, a constant debt subsidy, which tends to stimulate borrowing, reduces resources available for consumption when the constraint binds, while it raises agents' willingness to pay for collateral. It thus combines the inverse effects of an ex-ante debt tax on consumption with the effects of an ex-post debt subsidy on the collateral premium. A constant debt subsidy is superior to a constant debt tax if the effect on the collateral premium dominates. For the model with durables, this is the case when the loan-to-value ratio is sufficiently large (including values typically used in quantitative studies). In Davila and Korinek's (2018) model with capital, a constant debt subsidy is unconditionally superior to a constant debt tax.

The remaining two non-state-contingent policies impose taxes/subsidies on lenders. The third policy is an ex-ante saving tax/subsidy, which directly alters the price relation for the equilibrium interest rate. An ex-ante saving subsidy reduces the interest rate, which is inefficiently high under laissez faire, because borrowers do not internalize that higher current relative to future consumption lowers the interest rate. The reduction in borrowing costs narrows the distance between the MRS of lenders and borrowers; the latter engaging in precautionary saving under potentially binding borrowing constraints. Increased outstanding debt, however, tends to reduce consumption when the collateral constraint binds and thus to lower the collateral price. Hence, there is a trade-off between the effects on the prices of debt and of collateral. A policy maker decides to subsidize saving ex-ante and to reduce the costs of borrowing for potentially constrained agents when distributive effects dominate collateral effects. This is particularly the case in both models when the wealth distribution is sufficiently unequal. The fourth policy is a saving tax/subsidy that is constant over time. In contrast to the ex-ante saving subsidy, it tends to stimulate borrowing as well as consumption before and while the collateral constraint is binding by reducing the interest rate and by raising the collateral premium. It can thereby simultaneously address distributive and collateral effects in both models.

To unveil the role of the collateral premium and to reconstruct findings of the studies on macroprudential regulation cited above, we further refer to an alternative specification where the borrowing limit is assumed to depend on the aggregate stock of pledgeable assets. For this specification of the borrowing constraint, which is not consistent with the underlying imperfection (i.e. limited commitment), the price that alters the borrowing limit is not affected by the collateral premium, such that debt/saving subsidies can neither implement first best nor address adverse collateral effects. If distributive effects are further disregarded, optimal ex-ante and constant policies are debt taxes, implying that agents overborrow.

While the analytical results reveal the main principles, we further provide numerical results for the less stylized model with durables for illustrative purposes. The four optimal policies are 1) an ex-ante debt tax, 2) a constant debt subsidy, 3) an ex-ante saving subsidy, and 4) a constant saving subsidy (see Table 1). Except for the ex-ante debt tax, all policies tend to raise debt before the borrowing constraint binds, and the constant policies induce the largest increases in the collateral price, revealing the relevance of collateral premia. The ex-ante debt tax has the least impact on borrowers' consumption and leads to the smallest welfare gains relative to laissez faire, which are virtually negligible based on the distance to first best. Saving policies exert relatively large redistributive and social welfare effects via interest rate reductions.[8] An optimal constant saving subsidy, which leads to the largest welfare gains relative to laissez faire, even reduces welfare losses by about a half compared to first best.

By confining our analysis to models employing a linear utility function for lenders, a decision that facilitates the replication of established results on macroprudential regulation, we abstract from distributive effects under debt policies. If lenders' utility were instead a non-linear function of consumption like borrowers' utility, interest rates would depend on agents' endogenous MRSs. An ex-ante debt tax would lower borrowers' current relative to future consumption, such that lenders' current consumption would increase relative to future consumption and the period-1 interest rate would unambiguously fall. The reduction in the interest rate would mitigate (but not invert) the debt tax effect on borrowers' current relative to future consumption. The distributive effects would however demand an increase of borrowers' current consumption, which is depressed in a laissez faire equilibrium due to precautionary saving. The recommendation regarding an ex-ante debt policy is therefore less clear-cut when considering the relevance of distributive effects under non-linear lenders' utility. In contrast, saving subsidies can address distributive effects under non-linear lenders' utility, since they would reduce lenders' current consumption as well as interest rates. Both effects would induce borrowers to raise consumption relative to lenders, narrowing the distance between their MRSs. This mechanism is in

[8] Optimal saving policies lead to a redistribution of funds from lenders to borrowers, despite the absence of a redistributive motive of the social planner, owing to quasi-linear preferences.

principle relevant whenever borrowing constraints bind with a non-zero probability in a heterogeneous agent economy.

The remainder is structured as follows. Section 2 discusses the related literature. Section 3 develops the model with durables as collateral under uncertainty. Section 4 examines optimal policies. Section 4.5 presents numerical illustrations. Section 5 presents analytical results for Davila and Korinek's (2018) model with endogenous capital formation, where the borrowing constraint binds with certainty. Section 6 concludes.

2 Related literature

This paper is related to several studies on corrective policies under collateral externalities, like Jeanne and Korinek (2010, 2019), Bianchi (2011), Benigno et al. (2016), Korinek and Sandri (2016), Schmitt-Grohe and Uribe (2017), Bianchi and Mendoza (2018), or Korinek (2018). They focus on constrained efficient allocations, as defined in Stiglitz (1982), and macroprudential policies, like debt taxes or capital controls, that are imposed when borrowing constraints are not binding. In contrast to our analysis, these studies apply models where interest rates are exogenously determined and where – except of Bianchi and Mendoza (2018) – agents take borrowing limits as given, implying that there are neither distributive effects nor collateral premia on pledgeable assets. Bianchi and Mendoza (2018) focus on time-consistent policies under discretion, such that commitment to ex-post policies is not possible and first best cannot be implemented. They discuss how the collateral premium principally affects the price of collateral and optimal debt policy. In their quantitative analysis, they report results for macroprudential debt taxes that are imposed when the collateral constraint does not bind and the collateral premium equals zero. They further examine constant debt taxes and find that these lead to negligible welfare gains or even welfare losses, consistent with our results on constant debt policies. Bianchi (2011) finds that a constant debt tax can achieve sizable welfare gains, which we show to rely on the non-existence of collateral premia in our models. In addition to debt taxes, Benigno et al. (2016) analyze policies introduced in other markets, and show that an ex-post tax on non-tradables can raise the collateral price, such that the borrowing constraint does not bind. Bianchi (2016) and Jeanne and Korinek (2020) find welfare gains from ex-post policies in form of debt reliefs or liquidity provisions, which do not implement first best. In addition to these analyses, we examine time- and state-invariant debt/saving subsidies, and show that they can be superior to debt taxes.

Our finding that the stimulation of borrowing can enhance social welfare relates to the following studies: Benigno et al. (2013) examine the constrained efficient allocation of an economy where agents take into account that labor supply alters the borrowing limit, which compares to our analysis where agents internalize that borrowing limits depend on their holdings of eligible assets. They show that one should rather reallocate resources between (tradable and non-tradable goods) sectors to raise borrowing limits than subsidize borrowing. In a related model, Arce et al. (2023) show that an ex-ante debt tax is desirable even when ex-post labor market policies are applied and borrowing is enhanced in a constrained efficient allocation. Katagiri et al. (2017) apply a variant of Jeanne and Korinek's (2010) model where agents internalize collateral services of eligible assets. Like in our models, an ex-post debt subsidy can implement first best by raising the collateral price via the collateral premium such that the borrowing limit is not binding. Their analysis neither examines interest rate effects nor non-state-contingent policies, on which our analysis focusses. Schmitt-Grohe and Uribe (2021) establish the existence of multiplicity in the model examined by Bianchi (2011), giving rise to equilibria with underborrowing due to excessive precautionary savings. For a model with bank intermediation, Chi et al. (2022) show that agents borrow less under laissez faire compared to equilibria with ex-post expansions of bank reserves. Ottonello et al. (2022) shows that constrained inefficiency depends on whether borrowing limits depend on current or future collateral prices, and that debt subsidies can be optimal in the latter case.

In contrast to our analysis, none of the above cited studies considers distributive effects. In a seminal paper, Lorenzoni (2008) shows that distributive externalities under financial frictions cause agents to overinvest and to overborrow in an unregulated economy. Davila et al. (2012) show in a model with an endogenous wealth distribution that distributive effects can either lead to over- or underaccumulation of capital. Lanteri and Rampini (2023) develop a model of endogenous formation and reallocation of capital. They show that distributive effects of pecuniary externalities with regard to the capital price are larger than collateral externalities, such that a subsidy on new investment enhances efficiency. Both studies do not analyze credit market policies. Davila and Korinek (2018) apply a general framework with capital formation, for which they establish collateral and distributive effects. They show that pecuniary externalities can either cause over- or underinvestment, while they emphasize that "collateral externalities generally entail overborrowing" (p. 354). We show for their model that this conclusion holds only if the analysis is restricted to ex-ante debt policies.

The above cited studies focus on the analysis of constrained efficient allocations, which can either be derived from a problem of choosing initial allocations or from a Ramsey problem when equilibrium price relations are unaffected by policy instruments (see also Davila and Korinek, 2018). In contrast, the solutions to our policy problems differ from this type of constrained efficient allocation when the price relations for the collateral price or for the interest rate are affected by policy. Relatedly, Benigno et al. (2023) reexamine policy instruments used in Benigno et al. (2016), applying the Ramsey approach. Complementary to our analysis of different policy instruments, they show that a set of instruments that can implement a constrained efficient allocation can also be used to implement a superior allocation where borrowing constraints never bind. This possibility relies on the use of taxes/subsidies outside the credit market, while we show that first best is implementable with ex-post credit market policies.

3 A model with incomplete markets and limited commitment

In this Section, we develop a finite horizon model with durables in fixed supply. Section 5 presents Davila and Korinek's (2018) model with capital formation, which is slightly more stylized (without uncertainty and without discounting).[9] There exist two imperfections in both models: Only non state-contingent debt is available and agents are not able to commit to debt repayment. The latter leads to the key financial friction, i.e. a borrowing constraint with the borrower's asset serving as collateral.

[9] While the analysis of their model includes an additional (capital investment) tax/subsidy, the results on borrowing and saving taxes/subsidies correspond to the result for the model with durables.

3.1 Details

There are two mass-one groups $\{b, l\}$ with infinitely many agents, who live for three periods $t = 1, 2, 3$. In each period t, a household $i \in \{b, l\}$ derives utility from consumption of a non-durable good, $c_{i,t}$, and a durable good (or housing), $d_{i,t}$, as given by the function $u_{i,t} = u(c_{i,t}, d_{i,t})$. Agents maximize their expected lifetime utility, $E \sum_{t=1}^{3} \beta^{t-1} u(c_{i,t}, d_{i,t})$, where u is strictly increasing and concave, E denotes an expectations operator conditional on information in period 1, and $\beta \in (0, 1)$ is a discount factor. In each period, agents receive a potentially random endowment $y_{i,t}$ of non-durable goods and they exhibit an initial endowment of durables $d_{i,0}$. Agents can borrow and lend only in terms of non state-contingent one-period bonds $b_{i,t}$, which are issued at the price $1/r_t$. The budget constraint of an agent i for period t is given by

$$c_{i,t} + q_t(d_{i,t} - d_{i,t-1}) + (1 - \tau_{i,t})b_{i,t}/r_t = b_{i,t-1} + y_{i,t} + T_{i,t}, \tag{1}$$

where $\tau_{i,t}$ denotes distortionary taxes/subsidies on debt/saving. Specifically, we consider Pigouvian-type fiscal interventions, where budgetary effects of taxes/subsidies are (ex-post) neutralized in a non-distortionary way:

$$T_{i,t} = -\tau_{i,t} b_{i,t}/r_t, \tag{2}$$

which is not internalized by agents. There is no uncertainty in the periods 1 and 3, where total endowment with non-durables is equally distributed: $y_{b,1} = y_{l,1} = y/2$. Agents b (l) start with negative (positive) initial net financial wealth $b_{b,0} < 0$ ($b_{l,0} > 0$) and will be called borrowers (lenders). In period 2, endowments are randomly determined and can either take the same values as in period 1 (state L) or can be unequally distributed (state H). Specifically, both states are equally likely and endowment of borrowers in state H (with Higher inequality) is $y_{b,2} = y/(1 + \delta_H)$, where $\delta_H > 1$.

We assume that agents cannot commit to repay debt and that debt can be renegotiated after issuance in the same period. Borrowers can make a take-it-or-leave-it offer to reduce the value of debt. If a lender rejects the offer, she/he can seize a fraction γ of the borrower's durable goods, which she/he can sell at the market price q_t. Offers are therefore accepted when the repayment value of debt at least equals the current value of seizable assets. Without loss of generality, we assume that default and renegotiation never happen in equilibrium. When debt is issued, the amount of debt $-b_{i,t}$ is therefore constrained by

$$-b_{i,t} \leq \gamma q_t d_{i,t}, \tag{3}$$

where $\gamma \in (0, 1)$. According to (3), borrowing is constrained by the current market value of durables (e.g. housing), consistent with empirical evidence (see e.g. Cloyne et al., 2019). The borrowing constraint (3) can generate a feedback between agents' demand for durables and the debt limit, which is not internalized in individual decisions. Moreover, the borrowing constraint can lead to unequal marginal rates of substitutions between states and agents, giving rise to distributive effects, as discussed in Davila and Korinek (2018). Notably, agents will internalize that individual holdings of durables serve as collateral, such that the durables price increases with the collateral premium, i.e. the valuation of an asset for serving as collateral. To unveil the impact of this collateral premium on optimal policy choices, we refer to an alternative version of the borrowing constraint that is independent of the stock of durables held by the borrower. Under this ad-hoc constraint, which is inconsistent with the underlying inability to commit, the price of durables contains no collateral premium.

The available stock of durables equals d and the total non-durable endowment equals y. Since there is no borrowing/lending in the final period, the borrowing constraint is irrelevant and there are also no taxes/subsidies on debt/saving in $t = 3$, i.e. $\tau_{i,3} = T_{i,3} = 0$. A competitive equilibrium is then given by an allocation of durables, non-durables, and debt $\{c_{i,1}, d_{i,1}, b_{i,1}, c_{i,2}(s), d_{i,2}(s), b_{i,2}(s), c_{i,3}(s), d_{i,3}(s)\}$ for $i \in \{b, l\}$ and $s \in \{L, H\}$, a set of prices $\{r_1, r_2(s), q_1, q_2(s), q_3(s)\}$ satisfying agents' maximization problem s.t. the budget constraints (1) and the collateral constraints (3), and the market clearing conditions, $d_{b,t} + d_{l,t} = d$ and $b_{b,t} + b_{l,t} = 0$ $\forall t\{1, 2, 3\}$, given taxes/subsidies $\{\tau_{i,t}, T_{i,t}\}$ for $i \in \{b, l\}$ and $t \in \{1, 2\}$, and an initial distribution of debt and durables and sequences of non-durable endowments $\{y_{i,t}\}_{t=1}^{3}$ for $i \in \{b, l\}$.

3.2 Equilibrium properties

To further facilitate the analysis, we impose some simplifying assumptions on preferences, which correspond to those applied in Davila and Korinek's (2018) model (see Section 5), and on the relevance of the borrowing constraint.

Assumption 1 *Agents' preferences satisfy* $u_{l,t} = c_{l,t}$ *for* $t \in \{1,2,3\}$, $u_{b,t} = \log c_{b,t} + v(d_{b,t})$ *for* $t \in \{1,2\}$, *and* $u_{b,3} = c_{b,3} + v(d_{b,3})$, *where* $v_d > 0$ *and* $v_{dd} \leq 0$.

Assumption 2 *Initial debt* ($-b_{b,0}$) *is small enough that (3) is slack in* $t = 1$ *and inequality* (δ_H) *in* $t = 2$ *is large enough that (3) is binding in state* H *under laissez faire.*

The restrictions on agents' preferences in Assumption 1 facilitate the derivation of analytical results and allow isolating distinct effects of policy regimes. Specifically, as durables do not provide utility to lenders, which relates to studies on fire sales where borrowers have a superior use for assets, the distribution of durables will be degenerate and only borrowers will hold durables in equilibrium.[10] Due to linear utility of lenders, the interest rate is constant under laissez faire and under debt taxes/subsidies, like in Bianchi's (2011) seminal small open economy model. Here, it can however be adjusted under a saving tax/subsidy. We thus switch off distributive effects with regard to durables and focus on collateral effects when debt taxes/subsidies are applied, which facilitates comparisons with related studies. In contrast, a saving tax/subsidy might further address distributive effects via changes in the real interest rate. Assumption 2 ensures that the borrowing constraint is not binding in period 1, while there is a positive probability that it is binding in period 2. Policies that are exclusively imposed in period 1 (2) are therefore called ex-ante (ex-post) policies. In addition, we examine constant policies that are neither time- nor state-dependent and that apply equally in period 1 and 2.

We briefly describe the first best allocation, which serves as a reference case. Borrowers' and lenders' utility are linear in the terminal period according to Assumption 1, such that any allocation of available resources between both types of agents in period 3 is first

[10] Consistently, we restrict the initial endowment by $d_{b,0} = d$.

best. This property further implies that there is no justification for assigning different welfare weights to borrowers and lenders.[11] Without loss of generality, agents' welfare weights can therefore be set equal to one, leading to a utilitarian welfare function. It can easily be shown that the allocation would be efficient (even though financial markets are incomplete), if borrowing were not constrained. The allocation would then be identical to the allocation a social planer would choose who maximizes

$$W = E\sum_{t=1}^{3}\beta^{t-1}\left(u_{b,t}+u_{l,t}\right), \tag{4}$$

subject to the resource constraints. Specifically, the first best allocation is characterized by identical marginal utilities of consumption of borrowers and lenders for $t \in \{1,2,3\}$, $\partial u_{b,t}/\partial c_{b,t} = \partial u_{l,t}/\partial c_{l,t}$. The competitive equilibrium allocation can deviate from first best when the borrowing constraint binds in period 2 or when borrowers expect that the borrowing constraint might become binding and reduce consumption in period 1 due to a precautionary motive. Moreover, the borrowing constraint can induce inefficiencies caused by pecuniary externalities with regard to the durables price and the interest rate.

3.2.1 Laissez faire

Before we discuss welfare-enhancing policies, we describe the equilibrium under laissez faire, i.e. without policy interventions, which will serve as the main reference case, and the pecuniary externalities. Under Assumptions 1 and 2, the borrowers' optimality conditions can be summarized as $c_{b,1}^{-1}q_1 = v_d(d_{b,1}) + \beta E q_2 c_{b,2}^{-1}$,

$$c_{b,1}^{-1}/r_1 = \beta E[c_{b,2}^{-1}], \tag{5}$$

$$c_{b,2}^{-1}q_2 = v_d(d_{b,2}) + \beta q_3 + \mu_{b,2}\gamma q_2, \tag{6}$$

$$c_{b,2}^{-1}/r_2 = \beta + \mu_{b,2}, \tag{7}$$

$$-b_{b,2} = \gamma q_2 d_{b,2}, \text{ for } \mu_{b,2} > 0 \text{ or } -b_{b,2} \leq \gamma q_2 d_{b,2}, \text{ for } \mu_{b,2} = 0, \tag{8}$$

[11] A social welfare function $W = E\sum_{t=1}^{3}\beta^{t-1}\left(\phi_b u_{b,t} + \phi_l u_{l,t}\right)$ with welfare weights ϕ_i for $i \in \{b,l\}$ would imply the optimal allocation to satisfy $E\frac{u'(c_{l,t})}{u'(c_{b,t})} = \frac{\phi_b}{\phi_l}$ $\forall t \in \{1,2,3\}$ and thus $\phi_b = \phi_l$, since $u'(c_{i,3}) = 1$.

and $q_3 = v_d(d_{b,3})$, where $\mu_{b,2}$ denotes the multiplier on the borrowing constraint (3). Notably, the borrowers' optimality conditions for debt and durables in period 2, (6) and (7), would differ from corresponding optimality conditions of lenders even under identical preferences, since lenders do not face a (potentially binding) borrowing constraint. Given that the borrowing constraint (3) depends on the individual stock of durables, borrowers value durables also for their ability to serve as collateral and to raise the borrowing limit. This effect is captured by the multiplier $\mu_{b,2}$ entering the RHS of (6). The latter can be rewritten to get q_2 as a function of $c_{b,2}$ and of the collateral premium $\xi_2 = \mu_{b,2}\gamma$:

$$q_2 = \chi(c_{b,2}, \xi_2) = \frac{v_d(d)\,(1+\beta)}{c_{b,2}^{-1} - \xi_2}, \tag{9}$$
$$\text{with } \chi_c \equiv \partial\chi/\partial c_{b,2} > 0 \quad \text{and} \quad \chi_\xi \equiv \partial\chi/\partial\xi_2 > 0,$$

where we used $q_3 = v_d(d_{b,3})$ and $d_{b,t} = d$. Consumption $c_{b,2}$ tends to increase the durables price q_2 by reducing the marginal valuation of durables purchase costs (see LHS of 6). This effect is summarized by $\chi_c > 0$. The term ξ_2, which equals $\mu_{b,2}\gamma$, measures the collateral premium on durables and increases the durables price q_2 under a higher valuation of collateral services, $\chi_\xi > 0$. This implies that the borrowing decision (7) relates to the durables price via the tightness of the borrowing constraint (measured by $\mu_{b,2}$). The collateral premium for asset prices will be crucial when taxes/subsidies are imposed while borrowing constraints bind. Notably, the collateral premium would be equal to zero, $\xi_2 = 0$, if the borrowing constraint were independent of the individual stock of durables (see below). Lenders' optimal behavior satisfies

$$1/r_1 = \beta, \quad 1/r_2 = \beta. \tag{10}$$

It can be shown in a straightforward way that lenders will not hold durables under Assumption 1. The laissez faire equilibrium is thus characterized by the binding budget constraints $c_{b,3} = b_{b,2} + y_{b,3}$, $c_{b,2} = b_{b,1} + y_{b,2} - b_{b,2}/r_2$, and $c_{b,1} = b_{b,0} + y_{b,1} - b_{b,1}/r_1$, as well as $c_{l,3} = b_{l,2} + y_{l,3}$, $c_{l,2} = b_{l,1} + y_{l,2} - b_{l,2}/r_2$, and $c_{l,1} = b_{l,0} + y_{l,1} - b_{l,1}/r_1$.

Combining (7) with (10) shows that borrowers' marginal utility of non-durables consumption in period 2 exceeds lenders' marginal utility of non-durables consumption (=1) under a binding borrowing constraint: $(c_{b,2}^{-1}(H) - 1)\beta = \mu_{b,2}(H) > 0$. Due to Assumption

2, condition (5) implies non-durables consumption in period 1 and 2 to satisfy:

$$(c_{b,1}^{-1} - 1) = E[c_{b,2}^{-1} - 1] > 0. \tag{11}$$

Thus, agents' decisions are distorted by the borrowing constraint when there is a positive probability that it is binding. Pecuniary externalities can then affect the allocation in an adverse way. Subsequently, we will summarize collateral effects and distributive effects of externalities following the classification of Davila and Korinek (2018). Concretely, collateral effects refer to uninternalized changes in the collateral price affecting the borrowing limit. Distributive effects refer to uninternalized changes in the debt price, which are relevant under marginal rates of substitutions that differ between agents.

Like in Davila and Korinek's (2018) model (see Section 5), the collateral price is determined by the borrowers' optimality conditions. Condition (9) implies that the collateral price q_2 increases with borrowers' consumption in the same period, $\chi_c > 0$, and with the collateral premium, $\chi_\xi > 0$, which equals $\xi_2 = \gamma(c_{b,2}^{-1} - 1)\beta$ under laissez faire. The price relation for the collateral price under laissez faire is thus given by

$$q_2 = \frac{v_d(d)(1+\beta)}{c_{b,2}^{-1}(1-\beta\gamma) + \beta\gamma}, \tag{12}$$

implying $\partial q_2 / \partial c_{b,2} > 0$. In state L, where $\mu_{b,2}(L) = 0 \Rightarrow c_{b,2}(L) = 1$ holds under Assumption 2 (see 7 and 10), the price relation (12) simplifies to $q_2 = v_d(d)(1+\beta)$, such that $q_2(H) < q_2(L)$, given that $c_{b,2}(H) < 1$. Under a binding borrowing constraint in state H, the collateral price q_2 increases with the collateral premium ξ_2 (9), while the latter decreases with $c_{b,2}$ (see 7). These effects of their decisions on the durables price and thus on the borrowing limit are not internalized by borrowers, though by a policy maker.

Higher non-durables consumption of borrowers in $t = 1$ relative to $t = 2$ is associated with a lower interest rate (see 5), which is not internalized by individual agents. Yet, unconstrained lenders are only willing to lend at a constant rate under laissez faire (see 10). Hence, equilibrium interest rates are fixed at $1/\beta$, which also holds under taxes/subsidies on debt, leaving the lending conditions (10) unchanged. If, however, taxes/subsidies are imposed on saving, interest rates become endogenous to the policy maker, who internalizes the equilibrium relation between borrowers' MRS and the interest rate. Given that borrowers' marginal rates of substitution are distorted by the borrowing constraint (see 5 and 7), inefficiencies caused by distributive effects can therefore be addressed via interest rate changes induced by saving taxes/subsidies.

3.2.2 Efficiency and ex-post credit market policies

Under the Assumptions 1 and 2, the first best allocation, which maximizes (4) subject to resource constraints, satisfies $d_{b,t} = d$ for $t \in \{1, 2, 3\}$,

$$c_{b,1}^{fb} = c_{b,2}^{fb}(s) = 1, \quad c_{l,1}^{fb} = c_{l,2}^{fb}(s) = y - 1, \tag{13}$$

$c_{b,3}^{fb}(s) = ((b_{b,0} + y_{b,1} - c_{b,1}^{fb})\beta^{-1} + y_{b,2}(s) - c_{b,2}^{fb}(s))\beta^{-1} + y_{b,3}$ and $c_{l,3}^{fb}(s) = y - c_{b,3}^{fb}(s)$ for $s \in \{L, H\}$. Even though individual endowment with non-durables is random in period 2 and markets are incomplete, borrowers' first best consumption of non-durables is identical in $t = 1$ and $t = 2$. In contrast, borrowers' period-3-consumption is state-dependent.

Under a positive probability that the borrowing constraint is binding (see Assumption 2), the first best allocation, in particular (13), cannot be realized under laissez faire, where $c_{b,2}$ is state dependent, $c_{b,2}(L) = 1$ and $c_{b,2}(H) < 1$ (see 7), and $c_{b,1} < 1$ holds (see 11). Yet, first best can be implemented by a Pigouvian ex-post policy that raises the collateral price q_2 to a sufficiency high level such that the borrowing constraint is never binding, which corresponds to Katagiri et al.'s (2017) finding. To see this, consider a policy intervention in period 2 in form of a Pigouvian tax/subsidy on debt, $(1-\tau_{b,2})c_{b,2}^{-1}/r_2 = \beta + \mu_{b,2}$, which together with (10) leads to the following credit market equilibrium condition:

$$(1 - \tau_{b,2})c_{b,2}^{-1}\beta = \beta + \mu_{b,2}. \tag{14}$$

Using (14) to substitute out the multiplier $\mu_{b,2}$ in (6) and $q_3 = v_d(d)$, gives the following condition for the collateral price q_2 in state H:

$$q_2 = \frac{(1+\beta)\, v_d(d)}{c_{b,2}^{-1}(1-\beta\gamma) + \beta\gamma + \{\tau_{b,2} \cdot \beta\gamma c_{b,2}^{-1}\}}, \tag{15}$$

where the term in the curly brackets indicates that q_2 increases with a debt subsidy $\tau_{b,2} < 0$. Given that the latter does not affect other equilibrium conditions and that q_2

only matters for the equilibrium allocation due to its impact on the collateral constraint, the subsidy can in fact exclusively be used the raise the borrowing limit.[12] This effect is based on the collateral premium (see 9), which tends to increase when the multiplier on the collateral constraint $\mu_{b,2}$ is raised by agents' higher willingness to borrow under a debt subsidy (see 14). Implementation of first best can be summarized as follows.

Proposition 1 *The first best allocation is implemented in a competitive equilibrium under an ex-post Pigouvian debt subsidy satisfying*

$$\tau_{b,2}(H) \leq \left(1+\beta^{-1}\right) dv_d(d)[-b_{b,2}^{fb}(H)]^{-1} - (\beta\gamma)^{-1} < 0,$$

where $b_{b,2}^{fb}(H)$ denotes borrowing under a first best allocation, $b_{b,2}^{fb}(H) = \beta^{-2}(b_{b,0} + y_{b,1} - 1) + \beta^{-1}(y_{b,2}(H) - 1)$.

Proof. See Appendix. ■

Why is implementation of the first best allocation via ex-post credit market policies not examined in related studies on financial frictions? If the borrowing limit were assumed not to depend on individual borrowers' assets or income, like in Jeanne and Korinek (2010, 2019, 2020), Bianchi (2011), Benigno et al. (2016), Schmitt-Grohe and Uribe (2017), Korinek and Sandri (2016), or Korinek (2018), a Pigouvian ex-post debt subsidy would be ineffective. To see this, consider an ad-hoc borrowing constraint that depends on the aggregate stock of durables:

$$-b_{i,t} \leq \gamma q_t d. \tag{16}$$

In this case, condition (6) would reduce to $c_{b,2}^{-1} q_2 = v_d(d_{b,2}) + \beta q_3$. The collateral price would then be independent of agents' willingness to borrow, measured by the multiplier $\mu_{b,2}$ on the collateral constraint, and there would be no collateral premium effect on q_2, which satisfies $q_2 = c_{b,2} v_d(d)\,(1+\beta)$. Thus, under a borrowing limit (16) that is taken as given by borrowers, a Pigouvian ex-post debt tax/subsidy would exclusively alter the multiplier $\mu_{b,2}$ (see 14), leaving the laissez faire allocation unchanged.

Corollary 1 *Suppose that the borrowing constraint is given by (16), such that the borrowing limit is taken as given by borrowers. Then, an ex-post Pigouvian debt tax/subsidy does not affect the allocation in a competitive equilibrium.*

[12] This property does, evidently, not hold in general, in particular, under non-linear preferences.

Proof. See Appendix. ■

The subsequent analysis examines policies that cannot be made contingent on adverse states and can therefore not implement first best.

4 Non-state-contingent credit market policies

While state-contingent debt subsidies can implement first best, we now show that welfare can also be enhanced by non fully state-contingent corrective polices. We focus on ex-ante policies and on policies that are constant over time, which can be implemented without the complexities associated with fully state-contingent policies. Specifically, we examine four different types of Pigouvian credit market policies (see Table 1): 1) an ex-ante tax/subsidy on debt, 2) a constant tax/subsidy on debt, 3) an ex-ante tax/subsidy on saving, and 4) a constant tax/subsidy on saving. For this, we apply the Ramsey approach to optimal policy, where the policy maker acts under full commitment and internalizes equilibrium price relations. Under 1), the allocation is identical to a constrained efficient allocation, where the social planer respects budget and borrowing constraints as well as price relations that are unchanged compared to laissez faire. These price relations are in fact not forward-looking,[13] implying that the policy plan is time-consistent. In contrast, the price relation for the collateral price is altered under 2) and for the debt price under 3), while the price relations for both prices are simultaneously altered under 4). Given that relevant price relations under 2)-4) are forward-looking, the associated policy plans are in general not time-consistent.

4.1 An ex-ante Pigouvian tax on debt

We first consider the case, where a tax/subsidy on debt might be introduced in period 1, whereas no policy instrument is applied in period 2. Ex-ante debt taxes, which correspond to capital controls in open economies, have already been examined in several related studies (see Davila and Korinek, 2018, or Erten et al., 2021, for an overview), establishing that they can implement a constrained efficient allocation as defined by Stiglitz

[13] Notably, the price relation for durables in period 1 does not constrain the policy choice, since collateral constraints are not binding in period 1 and there is no durables trade in equilibrium.

(1982). Following Bianchi and Mendoza (2018), we will refer to an ex-ante debt tax as macroprudential regulation. Under such a policy, borrowers' optimality condition (5) changes to

$$(1-\tau_{b,1})c_{b,1}^{-1}/r_1 = \beta E c_{b,2}^{-1}. \tag{17}$$

In equilibrium, condition (17) and the optimal lending choice $1/r_1 = \beta$ imply $(1-\tau_{b,1}) = c_{b,1}Ec_{b,2}^{-1}$. By taxing debt in period 1, $\tau_{b,1} > 0$, agents can be induced to borrow less, which tends to raise $c_{b,2}$ relative to $c_{b,1}$ and the durables price q_2 via (12).[14] Given that borrowers do not internalize the adverse effect of period-1-borrowing on the durables/collateral price and thus the borrowing limit in period 2, a policy maker can enhance efficiency by addressing collateral effects of pecuniary externalities with an ex-ante debt tax. This mechanism is well-established in the literature on macroprudential regulation and capital controls, and has led to the notion of "overborrowing".

Proposition 2 *Suppose that the policy maker can apply a Pigouvian tax/subsidy on debt before the borrowing constraint might be binding. Then, the optimal allocation is constrained efficient and associated with a tax on debt, satisfying*

$$\tau_{b,1} = c_{b,1}\gamma d(1-\beta\gamma)E\left[\mu_2^{tb1}\cdot\chi_c\right] \geq 0, \tag{18}$$

where $\mu_2^{tb1} \geq 0$ denotes the multiplier on the borrowing constraint of the policy problem.

Proof. See Appendix. ■

The optimal ex-ante debt tax described in Proposition 2 implements the "constrained efficient allocation", which is chosen by a social planer respecting budget and borrowing constraints and allowing markets for durables and non-durables to clear in a competitive way (see Stiglitz, 1982, or Davila et al., 2012). Concretely, a constrained efficient allocation is chosen by a social planer who determines borrowing and maximizes social welfare W subject to budget and borrowing constraints, while taking the competitive equilibrium relations for interest rates (10) and the durables price (12) under laissez faire into account. The Ramsey optimal ex-ante debt tax leads to the same outcome, since it leaves

[14] This positive effect of higher net worth of borrowers on the durables/collateral price corresponds to the effect in Davila and Korinek (2018) imposed by their condition 1.

the pricing equations (10) and (12) unaffected.[15] In contrast, we will examine polices in the subsequent sections that alter (10) and (12), such that the prices q_2, $1/r_1$, and $1/r_2$ can be altered by policy in more direct ways. Under alternative credit market polices, competitive equilibrium allocations can thereby be implemented that are superior to the constrained efficient allocation under the ex-ante debt tax.

4.2 A constant Pigouvian tax/subsidy on debt

In this model, state contingency cannot simply be induced by cyclicality of policy instruments. We therefore consider that the debt tax/subsidy τ_b can neither be made contingent on specific periods nor on the state of the economy, i.e. on the distribution of agents' endowment, such that the debt tax/subsidy is constant and equally imposed in the periods $t = 1$ and $t = 2$. In this case, the tax/subsidy has ex-ante and ex-post effects relative to the state of the economy where the borrowing constraint might be binding. The borrowers' optimality conditions (5) and (7) then change to

$$(1 - \tau_b)c_{b,1}^{-1}/r_1 = \beta E[c_{b,2}^{-1}], \tag{19}$$

$$(1 - \tau_b)c_{b,2}^{-1}/r_2 = \beta + \mu_{b,2}, \tag{20}$$

where $1/r_1 = 1/r_2 = \beta$. Condition (19) and (20) imply that the multiplier on the collateral constraint satisfies $\mu_{b,2} = \beta(c_{b,2}^{-1}c_{b,1}E[c_{b,2}^{-1}] - 1)$, which differs from laissez faire $(\mu_{b,2} = (c_{b,2}^{-1} - 1)\beta)$. The collateral premium $\xi_2 = \gamma\mu_{b,2}$ and condition (9) then lead to the following price relation:

$$q_2 = \frac{v_d(d)\,(1 + \beta)}{c_{b,2}^{-1}(1 - \beta\gamma c_{b,1}E[c_{b,2}^{-1}]) + \beta\gamma}, \tag{21}$$

which simplifies in state L to $q_2 = c_{b,2}v_d(d)(1 + \beta)$. The durables price q_2 tends to be higher under a larger collateral premium ξ_2 (see 9), while a constant debt tax $\tau_b > 0$ tends to reduce the multiplier $\mu_{b,2}$ and thus ξ_2 (see 20). Due to this effect on q_2 and the negative effect of the debt tax on non-durables consumption $c_{b,1}$ relative to $c_{b,2}$ (see 19), the durables price q_2 is here characterized by a positive relation to $c_{b,1}$ in equilibrium (see

[15] Benigno et al. (2016) and Davila and Korinek (2018) also show that this approach can be equivalent to a Ramsey optimal policy where the policy maker chooses taxes ex-ante or on first period allocations.

21).

A constant debt tax tends to induce agents to borrow and to consume less in period 1 relative to period 2 (as in the case of the ex-ante tax), but also tends to reduce borrowing and consumption in period 2 when the borrowing constraint might be binding (see 20). Due to a lower collateral premium, a debt tax can induce a reduction in the durables price and in the borrowing limit in period 2. It might therefore be preferable to apply a subsidy rather than a tax on debt. These two effects of debt policies due to borrowing constraints that bind in the current period and in the subsequent period correspond to those discussed in Bianchi and Mendoza (2018) for an optimal state-contingent policy under discretion. In contrast to a policy maker under discretion, who can influence expectations about future policy makers' choices only via endogenous state variables, a policy maker under commitment fully accounts for agents conditioning their expectations on the policy choices. For the policy problem under commitment, the following proposition reveals when a debt subsidy (or tax) is preferable:[16]

Proposition 3 *Suppose that the policy maker can apply a constant Pigouvian tax/subsidy on debt in the periods 1 and 2. Then, the optimal allocation is associated with a tax/subsidy rate on debt satisfying*

$$\tau_b = c_{b,1}\gamma dE\left[\mu_2^{tb}\left(\chi_c - \left\{\chi_\xi \cdot \left(2\frac{c_{b,1}}{c_{b,2}} + \beta\right)\frac{\gamma\beta}{c_{b,2}^2}\right\}\right)\right], \tag{22}$$

where $\mu_2^{tb} \geq 0$ denotes the multiplier on the borrowing constraint of the policy problem, and the rate τ_b is negative if $\frac{c_{b,1}}{c_{b,2}(H)} > \frac{1-\beta^2\gamma}{2\beta\gamma}$.

Proof. See Appendix. ■

As revealed by Proposition 3, the collateral effects given on the RHS of (22) imply that an optimal non-contingent debt policy can either be a tax ($\tau_b > 0$) or a subsidy ($\tau_b < 0$). The reason is that a constant debt subsidy tends to raise q_2 via the collateral premium on durables (see χ_ξ), similar to an ex-post debt subsidy (see Section 3.2.2). At the same time, a constant debt subsidy tends to reduce $c_{b,2}$ and thus q_2 via increased debt (see χ_c), which

[16]Bianchi and Mendoza (2018) do not analytically or quantitatively identify the conditions under which a debt subsidy is optimal.

are the inverse effects of an ex-ante debt tax. If the impact on the collateral premium summarized by the positive term in the curly brackets in (22) dominates the latter effect, a debt subsidy is optimal, $\tau_b \leq 0$. The inequality at the end of the proposition, reveals that this holds if the ratio of period-1-consumption to period-2-consumption in state H, $c_{b,1}/c_{b,2}(H)$, is sufficiently large. Recall that this ratio is equal to one under first best (see 13), while it exceeds one under laissez faire when the collateral constraint binds (see 11). The inequality is therefore satisfied when the allocation under the constant tax/subsidy is (still) characterized by a binding collateral constraint, inducing the ratio $c_{b,1}/c_{b,2}(H)$ to exceed one, and the threshold $\frac{1-\beta^2\gamma}{2\beta\gamma}$ is smaller or equal to one. The latter is in fact the case when the liquidation value of collateral γ is sufficiently large, i.e. if $\gamma \geq 1/\left(\beta^2 + 2\beta\right)$, which can in principle be satisfied by empirically plausible loan-to-value-ratios (e.g. $\gamma = 0.8$).

Apparently, the term in the curly brackets in (22) is equal to zero if there were no collateral premium, like under a borrowing constraint that does not depend on the individual stock of durables (see 16). In this case, the RHS of (22) would be strictly positive, such that the optimal constant policy imposed on borrowers would be a debt tax. The latter relates to the welfare-enhancing constant debt tax in Bianchi (2011).

4.3 An ex-ante Pigouvian tax/subsidy on saving

We now consider a tax/subsidy on saving as a closely related policy instrument, which is however imposed on lenders. Given that borrowers and lenders structurally differ with regard to preferences and constraints, the impact of a tax/subsidy on saving will in general not be equivalent to the impact of a tax/subsidy on debt. Specifically, the analysis will reveal that distributive effects of pecuniary externalities play an important role for the policy maker's choice under a saving policy, which directly alters the interest rate. Notably, the absence of a redistributive motive for the social planner under quasi-linear preferences (see Section 3.2) implies that redistributive effects of interest rate changes exclusively stem from the mitigation of pecuniary externalities. In contrast, the interest rate was exogenous under the linear utility function of lenders (see Sections 4.1 and 4.2) as long as only borrowers were taxed.

Under an ex-ante tax/subsidy on saving, the interest rate in period 1 can directly be altered by policy, as shown by the lenders' optimal saving decision

$$(1-\tau_{l,1})/r_1 = \beta. \tag{23}$$

Combining (23) with the borrowers' optimality condition (5), gives $1/(1-\tau_{l,1}) = c_{b,1}Ec_{b,2}^{-1}$, implying that borrowers's period-1 non-durables consumption $c_{b,1}$ tends to decrease relative to $c_{b,2}$ with a saving tax, $\tau_{l,1} < 0$. Given that the interest rate now becomes endogenous via the policy maker's optimal choice, the relevant price relation is given by

$$r_1 = c_{b,1}^{-1}/(\beta Ec_{b,2}^{-1}), \tag{24}$$

while the collateral price satisfies the laissez faire price relation (12), like under the ex-ante debt tax. An ex-ante tax/subsidy on saving can indirectly alter the borrowing limit via the effect of $c_{b,2}$ on the collateral price similar to the ex-ante debt tax, while it can additionally affect the interest rate in a direct way via (23). The social planer can utilize the latter effect and lower the interest rate to address distributive effects of pecuniary externalities. In fact, the distributive effects call for a subsidy on saving and the collateral effects for a tax on saving. The sign of the optimal tax/subsidy rate therefore depends on the relative magnitudes of both effects.

Proposition 4 *Suppose that the policy maker can apply a Pigouvian tax/subsidy on saving before the borrowing constraint might be binding. Then, the optimal allocation is associated with a tax/subsidy rate on saving satisfying*

$$\tau_{l,1} = \underbrace{\left\{-b_{b,1}r_1E(\mu_2^{tl1})\left(E\left[\frac{\partial\phi_1^b}{\partial c_{b,1}}\right] - r_1E\left[\frac{\partial\phi_1^b}{\partial c_{b,2}}\right]\right)\right\}}_{\geq 0} - \underbrace{\left\{\frac{r_1\beta\gamma d}{(1-\beta\gamma)^{-1}}E(\mu_2^{tl1}\chi_c)\right\}}_{\geq 0}, \tag{25}$$

where $\partial\phi_1^b/\partial c_{b,1} > 0$, $\partial\phi_1^b/\partial c_{b,2} < 0$, and $\mu_2^{tl1} \geq 0$ denotes the multiplier on the borrowing constraint of the policy problem and $\phi_1^b = \beta(c_{b,1}/c_{b,2})$ the stochastic discount factor.

Proof. See Appendix. ■

The condition for the optimal ex-ante tax/subsidy rate (25) in Proposition 4 reveals that the sign of the tax/subsidy rate depends on two opposing effects: The first term (in curly brackets) on the RHS is strictly positive and summarizes the distributive effects induced by the borrowing constraint that is binding with a positive probability (see Assumption 2).[17] These effects, which would be inexistent without pecuniary externalities (see terms in the square brackets), call for a saving subsidy, $\tau_{l,1} > 0$, inducing a lower interest rate. Due to the higher debt price $1/r_1$, borrowers can increase their consumption of non-durables in period-1 relative to period 2 compared to laissez faire (see 11). The second term (in curly brackets) on the RHS is also strictly positive and summarizes the collateral effects, which can be addressed by reducing borrowing via a saving tax, $\tau_{l,1} < 0$, that tends to reduce the supply of debt (like an ex-ante debt tax tends to reduce the demand for debt, see Proposition 2). Evidently, the policy maker applies a saving subsidy, $\tau_{l,1} > 0$, when collateral effects are dominated by distributive effects, which is more likely for higher levels of debt $-b_{b,1}$ (see 25); the latter primarily depending on the exogenously given initial debt level $-b_{b,0}$.

4.4 A constant Pigouvian tax/subsidy on saving

Now suppose that the tax/subsidy on saving can neither be made contingent on particular periods nor on the state of the economy, such that the tax/subsidy rate is equally imposed in the periods $t = 1$ and $t = 2$. This policy regime would even be non-equivalent to a constant tax/subsidy on debt if all agents were ex-ante identical, because of the asymmetry of agents' problems in period 2 induced by the borrowing constraint. The lenders' optimality conditions are then given by

$$(1 - \tau_l)/r = \beta, \text{ where } r_1 = r_2 = r, \tag{26}$$

instead of (10), implying that the constant saving tax/subsidy alters the interest rate in both periods, 1 and 2. These interest rate effects of the constant saving tax/subsidy further affect the borrowing decisions in period 1 and 2

$$c_{b,1}^{-1}/(1 - \tau_l) = E[c_{b,2}^{-1}], \tag{27}$$

$$c_{b,2}^{-1}\beta/(1 - \tau_l) = \beta + \mu_{b,2}. \tag{28}$$

[17] Notably, the multiplier μ_2^{tl1} now depends on the difference between the marginal utilities of borrowers and lenders (see proof of Proposition 4).

The conditions (27) and (28) indicate that a constant saving subsidy $\tau_l > 0$ tends to raise borrowers' non-durable consumption in period 1 and 2. Simultaneously, it alters the valuation of the borrowing constraint, measured by the multiplier on the borrowing constraint $\mu_{b,2}$ and thereby the collateral premium ξ_2. Combining (27) and (28), gives $\mu_{b,2} = c_{b,2}^{-1} c_{b,1} \beta E[c_{b,2}^{-1}] - \beta$, which can be used to substitute out the multiplier $\mu_{b,2}$ in (6). Then, the durables price relation differs from the laissez faire version (12) and satisfies (21), like under the constant debt tax/subsidy. With these changes in the price relations for durables and debt, the policy maker can use a constant tax/subsidy on saving to simultaneously address collateral effects via the durables price q_2 as well as distributive effects via the interest rate r.

Proposition 5 *Suppose that the policy maker can apply a constant Pigouvian tax/subsidy on saving in the periods 1 and 2. Then, the optimal allocation is associated with a tax/subsidy rate on saving satisfying*

$$\tau_l = \Delta + \Psi, \quad \textit{with} \tag{29}$$

$$\Delta = \beta \left\{ -b_{b,1} r \beta E\left[\frac{\mu_2^{tl}}{\phi^b}\right] \left(E\left[\frac{\partial \phi^b}{\partial c_{b,1}}\right] - rE\left[\frac{\partial \phi^b}{\partial c_{b,2}}\right] \right) - E\left[b_{b,2} \frac{\mu_2^{tl}}{\phi^b} \left(\frac{\partial \phi^b}{\partial c_{b,1}} - r \frac{\partial \phi^b}{\partial c_{b,2}} \right) \right] \right\} \geq 0$$

$$\Psi = \beta \gamma d E\left[\mu_2^{tl} \left(\left\{ \chi_\xi \cdot \left(1 + 2r \frac{c_{b,1}}{c_{b,2}} \right) \gamma \beta c_{b,2}^{-2} \right\} - r\chi_c \right) \right],$$

and $\partial \phi^b / \partial c_{b,1} > 0$, $\partial \phi^b / \partial c_{b,2} < 0$, and $\mu_2^{tl} \geq 0$ denotes the multiplier on the borrowing constraint of the policy problem and $\phi^b = \beta(c_{b,1}/c_{b,2})$ the stochastic discount factor. The term Ψ is positive if $\frac{c_{b,1}}{c_{b,2}(H)} > \frac{1-\beta\gamma r^{-1}}{2\beta\gamma}$.

Proof. See Appendix. ■

According to Proposition 5, distributive effects, which are summarized by the term Δ in (29), can be addressed by a constant saving subsidy, which relates to the findings in Proposition 4. Compared to an ex-ante saving subsidy, a constant saving subsidy additionally reduces the interest rate in period 2, where the borrowing constraint might be binding. This additional effect is captured by the last term in the square brackets of Δ. In contrast to the terms referring to the distributive effect, the sign of the term Ψ, which summarizes the collateral effects, is ambiguous and depends on the effects on the collateral premium summarized in the curly brackets in Ψ. Given that a higher willingness to borrow increases the valuation of collateral, the collateral effects also call for a saving subsidy if the impact on the collateral premium is sufficiently large. Otherwise, the term Ψ is negative and calls for a saving tax, which tends to increase consumption by reducing debt (see χ_c). For $\Psi > 0$, the inequality $\frac{c_{b,1}}{c_{b,2}(H)} > \frac{1-\beta\gamma r^{-1}}{2\beta\gamma}$ has to hold, which differs from the corresponding condition for the constant debt subsidy by r^{-1} replacing β (see Proposition 3). As in Section 4.2, this inequality is likely to be satisfied when borrowing remains constrained even under the optimal policy (such that $c_{b,1}$ exceeds $c_{b,2}(H)$) and for sufficiently high loan-to-value ratios γ, which strengthen the effect via the collateral premium.

If the borrowing constraint were however independent of individual holdings of durables (see 16), such that there would be no collateral premium effect on q_2, the term Ψ would be strictly negative. Yet, even in this case the policy maker would apply a saving subsidy if the distributive effects Δ dominate; the latter being more likely under higher initial debt levels $-b_{b,0}$.

4.5 Prices, allocation and welfare

We now aim at illustrating the impact of corrective policies on prices, the allocation and social welfare, and the possibility to improve on the (constrained efficient) allocation implemented by an ex-ante debt tax via alternative non state-contingent policies. We introduce a functional form for $v(d_{b,t})$: $v(d_{b,t}) = \varkappa \log d_{b,t}$. We further assign values to model parameters that induce the – admittedly stylized – model to generate meaningful values of targeted variables. Specifically, we normalize y and set it equal to 2, and further set $d/y = 1.5$, $b_{b,0}/y = -0.25$, $\varkappa = 0.1$, and $\beta = 0.9$, leading to a housing-to-GDP ratio, debt-to-income ratios, interest rates, and tax/subsidy rates within reasonable ranges. The benchmark values for the inequality measure δ_H and for the share of seizable collateral γ are 1.1 and 0.8, respectively; the latter relating to commonly applied loan-to-value ratios. We then examine the sensitivity of the effects by altering the tightness of the borrowing constraint γ and income inequality δ_H. The solutions for the equilibrium objects under the four non-state-contingent policies summarized in Table 1 and under laissez faire are presented in the Figures 1-3. The first row in all Figures refers to a variation in γ, where an increase in γ reduces the tightness of the borrowing constraint and thereby the strength

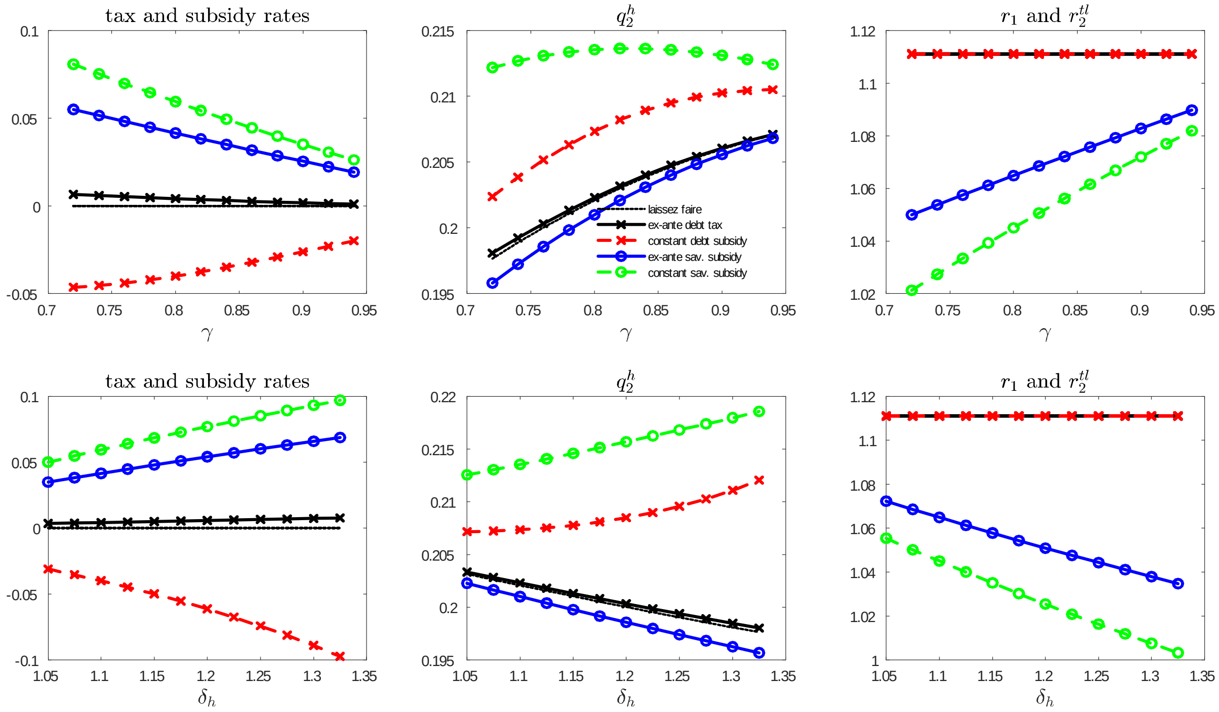

Figure 1: Instruments and prices (benchmark values $\gamma = 0.8$ and $\delta_H = 1.1$)

of the financial friction, which de-emphasizes the collateral effect. The second row in all Figures refers to a variation in δ_H, where an increase in δ_H increases the inequality of agents' non-durables endowment in state H in period 2 and thereby the relevance of the financial friction as well as of distributive effects.

The first column of Figure 1 shows the tax and subsidy rates under all five regimes. The laissez faire case (black dotted lines) exhibits zero tax/subsidy rates. The first policy regime (solid black lines with crosses) is the optimal ex-ante tax on debt $\tau_{b,1} > 0$ (see Proposition 2), which decreases with γ and increases with δ_H. The second policy regime (red dashed lines with crosses) is the optimal constant subsidy on debt $\tau_b < 0$ (see Proposition 3). The third (blue solid lines with circles) and the fourth regime (green dashed lines with circles) are the optimal ex-ante and the optimal constant saving subsidy, $\tau_{l,1} > 0$ and $\tau_l > 0$, as characterized in Propositions 4 and 5. The second column shows the durables (collateral) price in period 2, which is slightly increased compared to laissez faire under the ex-ante debt tax. The constant debt subsidy, which tends to raise borrowing in both periods 1 and 2 (see Figure 2), also leads to higher durables prices due to its impact on the collateral value. In contrast, the ex-ante saving subsidy, which raises

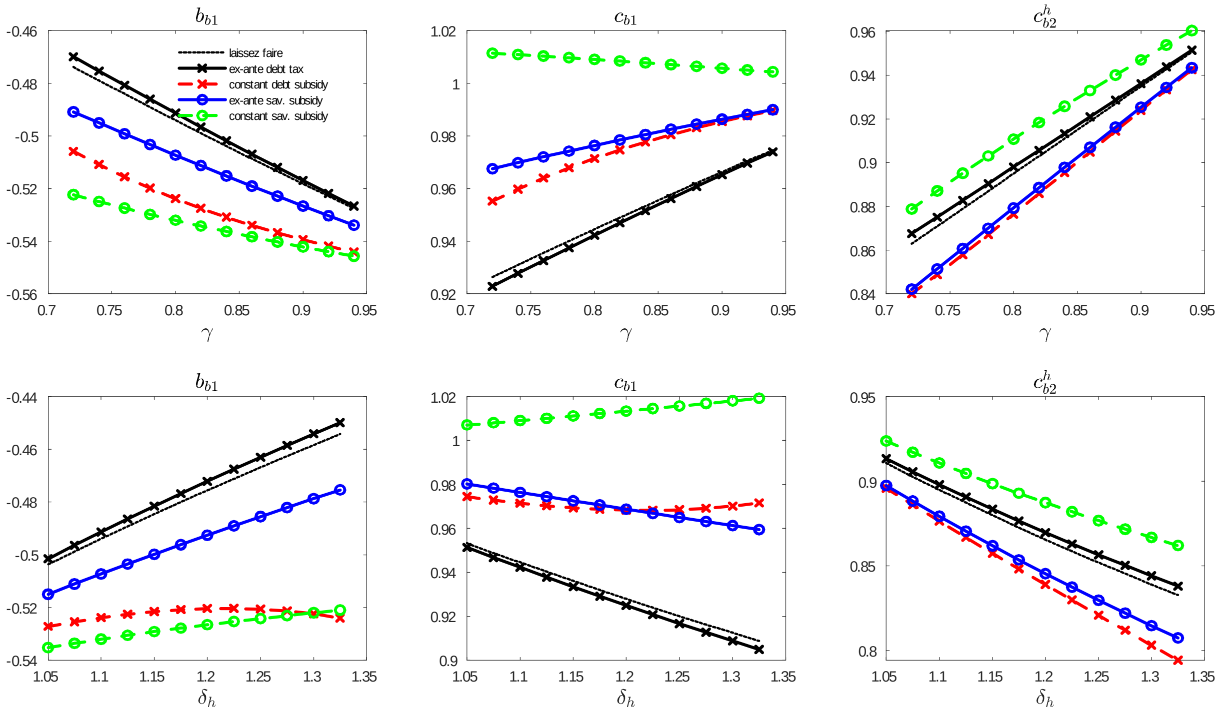

Figure 2: Debt and consumption (benchmark values $\gamma = 0.8$ and $\delta_H = 1.1$)

debt in period 1 and reduces non-durables consumption $c_{b,2}$ in period 2 (see Figure 2), leads to lower durables prices q_2. Simultaneously, it reduces the interest rate in period 1 below its laissez faire value (see third column), such that borrowing funds requires issuance of less debt $b_{b,1}$. The constant saving subsidy leads to the most pronounced increase in the durables price q_2. It further leads to a reduction in the interest rate r_1 in period 1 that is larger than under the ex-ante saving subsidy and it equally reduces the interest rate r_2 in period 2. Figure 2 further shows that all three subsidies raise debt $-b_{b,1}$ and lead to higher levels of non-durables consumption in period 1 compared to laissez faire, which in contrast decreases under the ex-ante debt tax. The constant debt subsidy and the ex-ante saving subsidy reduce consumption $c_{b,2}$ due to a higher debt burden in period 2. The opposite result for $c_{b,2}$ is induced by the ex-ante debt tax and by the constant saving subsidy, which lowers borrowing costs in both periods 1 and 2.

Figure 3 presents the welfare effects of the policy regimes. The welfare measure is based on W (see 4) and expressed in terms of equivalents of borrowers' non-durables consumption in period 1. The first column shows welfare effects of the four policy regimes relative to the laissez faire case. Evidently, the debt policies (ex-ante debt tax and

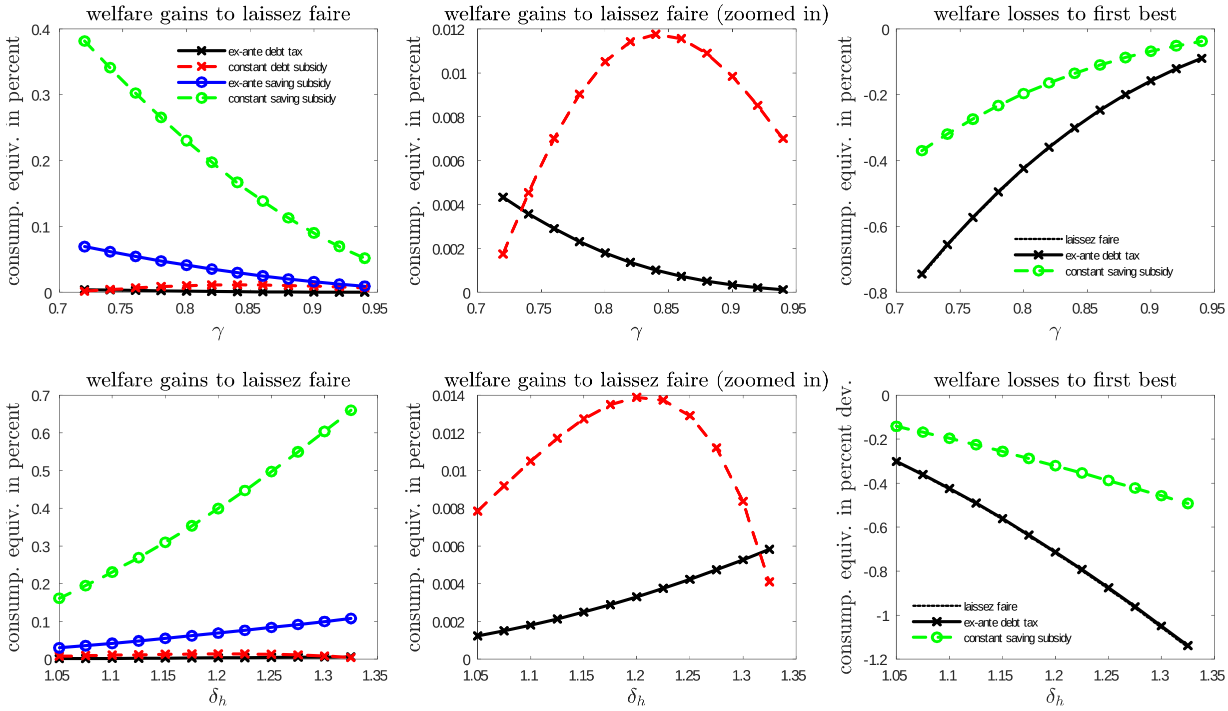

Figure 3: Social welfare (benchmark values $\gamma = 0.8$ and $\delta_H = 1.1$)

constant debt subsidy) lead to much smaller welfare gains than the saving subsidies. This results is simply due to the fact that the former policies can – by construction – not address distributive effects by changes in the interest rate.[18] In contrast, saving policies induce substantial interest rate reductions compared to laissez faire, which leads to a redistribution of resources in favor of borrowers (see also Figure 4). The second column of Figure 3 zooms in into the welfare effects of the debt policies, revealing that the ex-ante debt tax leads to the smallest welfare gains under the benchmark parameter values. It further shows that the ex-ante debt tax can principally be superior to the constant debt subsidy for tighter borrowing constraints, i.e. for lower loan-to-value ratios γ (see also Proposition 3), which reduce the positive impact of the constant debt subsidy on the collateral price via the collateral premium, $\xi_2 = \gamma\mu_{b,2}$.

A larger income inequality δ_H reduces borrowers' period-2 consumption $c_{b,2}$ under laissez faire (see Figure 2), lowering the collateral price according to (12). The welfare gains of ex-ante debt taxes, which raise $c_{b,2}$ via a debt reduction, therefore increase

[18] This would in principle be possible under alternative specifications of lenders' utility, for example, logarithmic utility, which is neglected here to keep the exposition transparent using polar cases.

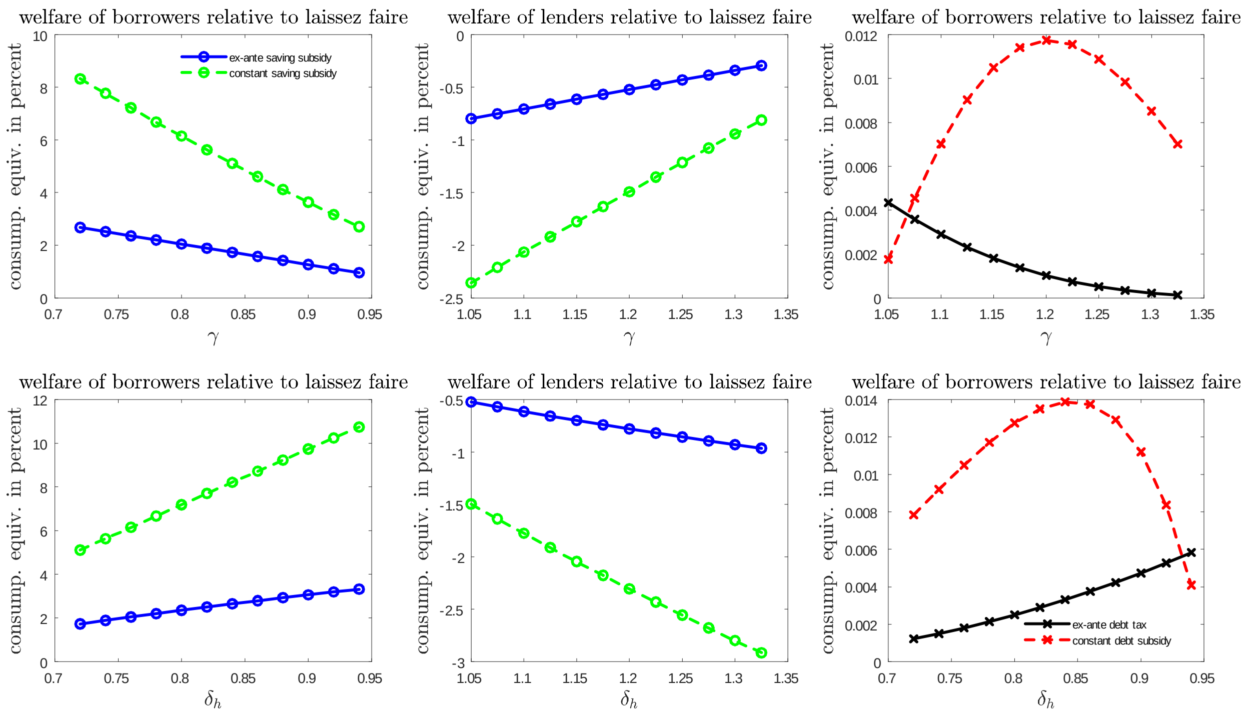

Figure 4: Welfare of borrowers and lenders (benchmark values $\gamma = 0.8$ and $\delta_H = 1.1$)

monotonically with δ_H (see Figure 3). Correspondingly, the welfare loss of an ex-ante debt subsidy, which lowers $c_{b,2}$, would increase with δ_H. The total welfare effect of a constant debt subsidy does however further depend on the effect on the collateral premium, which increases with the multiplier on the collateral constraint, given by $\mu_{b,2} = c_{b,2}^{-1} c_{b,1} \beta E[c_{b,2}^{-1}] - \beta$. When income gets more unequal, the adverse effect of the debt subsidy on $c_{b,2}$ (see Figure 2) dominates the positive effect on the collateral premium, such that the total welfare gain falls (see Figure 3). The constant debt subsidy can therefore be outperformed by the ex-ante debt tax for high δ_H values. The last column of Figure 3 presents welfare losses compared to first best. The values for laissez fare and the ex-ante debt tax are virtually identical, indicating that the total welfare gains of an ex-ante debt tax are negligible relative to first best. In contrast, the constant saving subsidy can substantially reduce the welfare loss in a competitive equilibrium compared to first best. For the benchmark values, it reduces the welfare loss by about a half. Finally, Figure 4 confirms the existence of redistributive effects under saving policies. Borrowers gain and lenders lose compared to laissez faire due to lower interest rates. The first and the second column of Figure 4 show that these redistributive effects monotonically increase with the tightness of the collateral constraint (lower γ) and the inequality of income (higher δ_H). Likewise, these welfare effects increase with initial debt $-b_{b,0}$ (not shown). In contrast, debt policies solely affect borrowers' welfare via intertemporal substitution, which is revealed in the last column of Figure 4, showing the same effects as for aggregate welfare (see second column of Figure 3).

5 A model with capital formation

To assess the robustness of our findings and to facilitate comparisons, we further apply a model with endogenous capital formation, like Bianchi and Mendoza (2018) or Davila and Korinek (2018). Concretely, we use Davila and Korinek's (2018) model applied for collateral externalities and replicate their results on an ex-ante debt policy that implements the constrained efficient allocation. In addition, we examine the other three policy regimes given in Table 1, like in the analyses of the previous model (see Section 4).

There is no uncertainty and there are no durable consumption goods in this economy. Agents' lifetime utility satisfies $u_l = c_{l,1} + c_{l,2} + c_{l,3}$ and $u_b = \log c_{b,1} + \log c_{b,2} + c_{b,3}$, which accords to Assumption 1 without durables ($v = 0$) and implies no discounting ($\beta = 1$). Borrowers have access to an investment technology, by which capital $k_{b,2}$ can be installed under convex costs $\alpha k_{b,2}^2/2$ in the first period. Capital can be traded in the second period at the price q^k and remains constant until it fully depreciates at the end of the last period. In the periods 2 and 3, borrowers use their full stock of capital to produce according to the technology $A_t k_{b,t}$ with $t \in (2, 3)$. Given that there is no uncertainty and no discounting, lenders' saving decision pins down the debt price at 1 under laissez faire. The borrowing constraint, which corresponds to (3), is given by

$$-b_{b,t} \leq \phi q^k k_{b,t}, \tag{30}$$

with $\phi > 0$. It is binding in period 2, while borrowing is (de facto) unconstrained in period 1. Since lenders have no use for capital, the entire stock of capital is held by borrowers: $k_{b,2} = k$. Under laissez faire, the borrowers' first order conditions can be written as $1/c_{b,1} = 1/c_{b,2}$, $1/c_{b,2} = 1 + \kappa_{b,2}$, $\alpha k(1/c_{b,1}) = (1/c_{b,2})\left(A_2 + q^k\right)$, and $q^k(1/c_{b,2}) = A_3 + \xi_2^k q^k$, where ξ_2^k denotes the collateral premium, $\xi_2^k = \kappa_{b,2}\phi$, and $\kappa_{b,2} \geq 0$

the multiplier on (30). Substituting out $\kappa_{b,2}$ with $\kappa_{b,2} = (1/c_{b,2}) - 1$, gives the following relation for the price of capital under laissez faire

$$q^k = A_3 \left[\phi + (1-\phi)/c_{b,2}\right]^{-1}, \tag{31}$$

while the stock of capital satisfies $k = \left(A_2 + q^k\right)/\alpha$. Like in the previous model, the collateral price q^k is inefficiently low under laissez faire, since agents do not internalize the impact of their decisions on q^k in a competitive equilibrium.

The first best allocation, which maximizes social welfare (4) for $\beta = 1$, satisfies $c_{b,1}^{fb} = c_{b,2}^{fb} = 1$ and $k^{fb} = (A_2 + A_3)/\alpha$. It cannot be realized in a laissez faire equilibrium under a binding borrowing constraint, where $\kappa_{b,2} > 0 \Rightarrow c_{b,2} = c_{b,1} < 1$. Yet, a Pigouvian debt subsidy that is only applied in the second period, $\tau_{b,2} < 0$ with (2), solely affects the borrowing condition $(1-\tau_{b,2})/c_{b,2} = 1 + \kappa_{b,2}$ and can implement the first best allocation by raising the collateral price q^k via the collateral premium according to $q^k = A_3[c_{b,2}^{-1}(1-\phi) + \phi + \tau_{b,2} \cdot \phi c_{b,2}^{-1}]^{-1}$. Hence, a sufficiently large ex-post debt subsidy can raise the borrowing limit such that it is not smaller than debt in a competitive equilibrium under the first best allocation, $-b_{b,2}^{fb}$, which corresponds to Proposition 1.

Proposition 6 *The first best allocation is implemented in a competitive equilibrium under a Pigouvian debt subsidy satisfying* $\tau_{b,2} \leq A_3 k_{b,2}[-b_{b,2}^{fb}]^{-1} - \phi^{-1} < 0$.

Proof. See Appendix. ■

Next, we again consider the four policies given in Table 1 with compensations satisfying (2), namely, an ex-ante debt tax/subsidy $\tau_{b,1}$, a constant debt tax/subsidy τ_b, an ex-ante saving tax/subsidy $\tau_{l,1}$, and a constant saving tax/subsidy τ_l. Like in Davila and Korinek (2018), the policy maker has further access to a Pigouvian tax/subsidy on capital investment, which we consider for all cases. For the first policy regime, we derive the optimal investment policy, confirming the results of Davila and Korinek (2018). For the other policy regimes, we do not further discuss the investment policy, since our focus is on corrective credit market policies. The following proposition summarizes the main results, which closely relate to the results derived in Section 4.

Proposition 7 *Consider the economy with capital formation. Suppose that the policy maker can apply a Pigouvian tax/subsidy on capital investment in the first period and*

1. *a Pigouvian tax/subsidy on debt in the first period. Then, the optimal allocation is constrained efficient and associated with a tax on debt satisfying*

$$\tau_{b,1} = (1 - c_{b,1})\, \phi k \left(\partial q^k / \partial c_{b,2}\right) \geq 0, \tag{32}$$

where $\partial q^k / \partial c_{b,2} > 0$*, and a subsidy on capital investment iff* $A_2 + \phi q^k \geq 0$*.*

2. *a constant Pigouvian tax/subsidy on debt. Then, the optimal allocation is associated with a subsidy on debt satisfying*

$$\tau_b = -\mu_2^{tb}[2\phi^2 k c_{b,1}^2 / c_{b,2}] \left(\partial q^k / \partial c_{b,1}\right) \leq 0, \tag{33}$$

where $\partial q^k / \partial c_{b,1} > 0$ *and* $\mu_2^{tb} \geq 0$ *denotes the multiplier on the borrowing constraint of the policy problem.*

3. *a Pigouvian tax/subsidy on saving in the first period. Then, the optimal allocation is associated with a tax/subsidy on saving satisfying*

$$\tau_{l,1} = \underbrace{\left\{ -\mu_2^{tl1} b_{b,1} r_1 \left(\frac{\partial \left(1/r_1\right)}{\partial c_{b,1}} - r_1 \frac{\partial \left(1/r_1\right)}{\partial c_{b,2}} \right) \right\}}_{\geq 0} - \underbrace{\left[r_1 \mu_2^{tl1} \phi k \frac{\partial q^k}{\partial c_{b,2}} \right]}_{\geq 0}, \tag{34}$$

where $\partial q^k / \partial c_{b,2} > 0$*,* $\partial \left(1/r_1\right) / \partial c_{b,1} > 0$*,* $\partial \left(1/r_1\right) / \partial c_{b,2} < 0$*, and* $\mu_2^{tl1} \geq 0$ *denotes the multiplier on the borrowing constraint of the policy problem.*

4. *a constant Pigouvian tax/subsidy on saving. Then, the optimal allocation is associated with a tax/subsidy on saving satisfying*

$$\tau_l = \underbrace{\left\{ -\mu_2^{tl} \left(b_{b,1} + \frac{b_{b,2}}{r} \right) \left(\frac{\partial \left(1/r\right)}{\partial c_{b,1}} - r \frac{\partial \left(1/r\right)}{\partial c_{b,2}} \right) \right\}}_{\geq 0} + \left[\mu_2^{tl} \frac{\phi k}{r^2} \frac{\partial q^k}{\partial c_{b,1}} \left(2\phi + 1 - r\right) \right], \tag{35}$$

where $r = r_1 = r_2$*,* $\partial \left(1/r\right) / \partial c_{b,1} > 0$*,* $\partial \left(1/r\right) / \partial c_{b,2} < 0$*,* $\partial q^k / \partial c_{b,1} > 0$ *and* $\mu_2^{tl} \geq 0$ *denotes the multiplier on the borrowing constraint of the policy problem.*

Proof. See Appendix. ■

Under the first regime (see part 1 of Proposition 7), the policy maker applies an ex-ante tax on debt $\tau_{b,1} > 0$ (see 32) and further imposes an investment subsidy for a sufficiently high productivity level, $A_2 + \phi q^k > 0$, which replicates Davila and Korinek's (2018) results for the constrained efficient allocation. Like in the previous model (see Proposition 2), an ex-ante debt tax leaves the laissez faire price relation (31) unaffected, where the collateral value satisfies $\xi_2^k = \phi(c_{b,2}^{-1} - 1)$. The collateral price q^k is raised by reducing borrowing ex-ante, which tends to increase funds available for consumption when the borrowing constraint binds. This rationalizes Davila and Korinek's (2018) claim that "collateral externalities cause overborrowing" in this model.

Under the <u>second</u> regime, the policy maker applies a constant debt subsidy $\tau_b < 0$ (see 33). In contrast to the corresponding case of the previous model (see Proposition 3), the potential trade off between raising the collateral premium on capital by stimulating borrowing (like under an ex-post debt subsidy) and raising the collateral price via higher consumption (like under an ex-ante debt tax) is here unambiguously solved in favor of a debt subsidy (see part 2 of Proposition 7). Specifically, the collateral premium ξ_2^k satisfies $\xi_2^k = \phi(c_{b,1}c_{b,2}^{-2} - 1)$ under a constant debt tax/subsidy, such that the price relation for q^k is given by $q^k = A_3[(1/c_{b,2}) - \phi(c_{b,1}c_{b,2}^{-2} - 1)]^{-1}$ (instead of 31). A constant debt subsidy tends to raise consumption in period 1 relative to period 2, which increases the collateral premium ξ_2^k and the collateral price q^k.

For the remaining cases, we consider taxes/subsidies imposed on lenders. Saving taxes/subsidies can cause the interest rate(s) to deviate from one, which can be used to address distributive effects of pecuniary externalities with regard to the interest rate. Like in the previous model (see Propositions 4 and 5), addressing distributive effects calls for an interest rate reduction by subsidizing saving, which is revealed by the positive terms in the curly brackets in (34) and (35) that increase with debt ($-b_{b,1}$ and $-b_{b,2}$). For the <u>third</u> policy regime, which is an ex-ante tax/subsidy on saving, the last term in (34) implies a trade-off, since collateral effects (in square brackets) call for a saving tax to raise consumption $c_{b,2}$ by a reduction of debt. Under the <u>fourth</u> policy regime, which is a constant tax/subsidy on saving, the policy choice implications of collateral effects – given by the term in square brackets in (35) – are ambiguous, since a saving subsidy tends to raise the collateral price via the collateral premium, whereas it tends to reduce funds available for consumption in $t = 2$ by increasing debt. Under a sufficiently large loan-to-value ratio, $\phi > (r-1)/2$, the former effect prevails, such that the collateral effect also calls for a saving subsidy (see also Proposition 5).

These results imply that agents do not overborrow in general, since the last three regimes may stimulate borrowing. However, optimal policy would unambiguously reduce debt, if the borrowing constraint were independent of individual asset position, e.g. $-b_{b,2} \leq \phi q^k k$ (which corresponds to 16), and distributive effects are disregarded. Likewise, implementation of first best by ex-post debt subsidies would then not be possible.

6 Conclusion

This paper derives optimal credit market policies in two incomplete market models with pecuniary externalities under collateral constraints. Collateral effects of pecuniary externalities can be addressed by a Pigouvian ex-ante debt tax, implementing the constrained efficient allocation. In contrast to the majority of studies on macroprudential regulation, we consider borrowers' assets as collateral and endogenous interest rates, giving rise to assets' collateral premia and distributive effects. We show that both are responsible for ex-post debt subsidies to be able to implement first best, for debt subsidies that are constant over time to outperform debt taxes, and for saving subsidies to enhance efficiency by reducing interest rates. Overall, we find that credit market policies that reduce interest rates and stimulate collateral premia by subsidizing debt or saving can outperform ex-ante debt taxes. The results imply that overborrowing unambiguously prevails only if collateral premia and distributive effects are neglected or if the analysis is restricted to ex-ante policies imposed on borrowers. Thus, our analysis shows that borrowing constraints that give rise to collateral effects are in general not sufficient to rationalize macroprudential regulation as an optimal credit market policy.

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8 Appendix

Proof of Proposition 1. In state H, the ex-post debt subsidy can be applied to raise the borrowing limit to a sufficiently high value such that the borrowing is unconstrained in a competitive equilibrium under the first best allocation, $-b_{b,2}^{fb}(H) \leq \gamma q_2 d$, where $b_{b,2}^{fb}(H) = \beta^{-2}(b_{b,0}+y_{b,1}-1)+\beta^{-1}(y_{b,2}(H)-1)$ (see 13). The price relation (15) implies that this requires the subsidy rate $\tau_{b,2}(H)$ to satisfy $\tau_{b,2}(H) \leq \left(1+\beta^{-1}\right) dv_d(d)[-b_{b,2}^{fb}(H)]^{-1} - (\beta\gamma)^{-1} < 0$. ■

Proof of Corollary 1. When (16) instead of (3) has to be satisfied, borrowers' housing decision (6) would change to $c_{b,2}^{-1} q_2 = v_d(d_{b,2}) + \beta q_3$, given that they do not consider their own stock of durables as collateral. Further using $q_3 = v_d(d_{b,3})$ and $d_{b,t} = d$, gives $q_2 = c_{b,2} v_d(d)\left(1+\beta\right)$, such that the price q_2 is not directly affected by the ex-post debt tax/subsidy $\tau_{b,2}$. Given that the multiplier $\mu_{b,2}$ just depends on the borrowing choice (14), the allocation is independent of $\tau_{b,2}$, which exclusively affects the valuation of the borrowing constraint, $\mu_{b,2}$. ■

Proof of Proposition 2. Using $c_{l,3} = -b_{b,2} + y_{l,3}$, $c_{l,2} - b_{b,2}\beta = -b_{b,1} + y_{l,2}$, $c_{l,1} - b_{b,1}\beta = b_{l,0} + y_{l,1}$, $c_{b,3} = b_{b,2} + y_{b,3}$ and $d_{b,t} = d$, the objective (4) can by be written as

$$\begin{aligned} W = & \log c_{b,1} + v(d) + (b_{l,0} + y_{l,1}) + \beta\left[\log c_{b,2} + v(d) + (y_{l,2} + b_{b,2}\beta)\right] \\ & + \beta^2\left[y_{b,3} + v(d) + y_{l,3}\right]. \end{aligned} \tag{36}$$

The primal problem of a policy maker who applies an ex-ante tax/subsidy on debt $\tau_{b.1}$ and a compensating lump-sum transfer/tax $T_{b,t} = -\tau_{b.1} b_{b,t}/r_t$ is identical to the problem of a social planer who determines period-1-borrowing, such that (5) does not hold, and maximizes social welfare W subject to budget and borrowing constraints taking the equilibrium price relations (10) and (12) under laissez faire into account, leading to a constrained efficient allocation. It can be summarized as

$$\begin{aligned} \max_{c_{b,1}, c_{b,2}, b_{b,1}, b_{b,2}} \; & E\{\log c_{b,1} + v(d) + (b_{l,0} + y_{l,1}) + \beta\left[\log c_{b,2} + v(d) + (y_{l,2} + b_{b,2}\beta)\right] \\ & + \beta^2\left[y_{b,3} + v(d) + y_{l,3}\right]\} \end{aligned} \tag{37}$$

$$\text{s.t. } 0 = b_{b,0} + y_{b,1} - c_{b,1} - b_{b,1}\beta, \quad 0 = b_{b,1} + y_{b,2} - c_{b,2} - b_{b,2}\beta, \quad 0 \leq \gamma q_2(c_{b,2})d + b_{b,2},$$

where $q_2(c_{b,2})$ satisfies (12), leading to the optimality conditions

$$\lambda_{b,1}^{tb1} = 1/c_{b,1}, \tag{38}$$

$$\lambda_{b,2}^{tb1} = (1/c_{b,2}) + \mu_2^{tb1}\gamma d\partial q_2(c_{b,2})/\partial c_{b,2}, \tag{39}$$

$$\lambda_{b,1}^{tb1} = E\lambda_{b,2}^{tb1}, \tag{40}$$

$$\mu_2^{tb1} = \beta(\lambda_{b,2}^{tb1} - 1) \geq 0, \tag{41}$$

where $\lambda_{b,1}^{tb1}$, $\lambda_{b,2}^{tb1}$, and μ_2^{tb1} are the multipliers for the constraints in order of their appearance in (37). Applying expectations conditional on period-1-information and substituting out the multipliers $\lambda_{b,1}^{tb1}$ and $\lambda_{b,2}^{tb1}$ in (38)-(40) leads to

$$c_{b,1}^{-1} = Ec_{b,2}^{-1} + E\left[\mu_2^{tb1}\gamma d\partial q_2(c_{b,2})/\partial c_{b,2}\right].$$

Combining the latter with the optimality condition (17) and $1/r_1 = \beta$, gives the following condition for the tax rate on debt

$$\tau_{b,1} = c_{b,1}E\left[\mu_2^{tb1}\gamma d(1-\beta\gamma)\chi_c\right] > 0,$$

where we used that $\partial q_2(c_{b,2})/\partial c_{b,2} = (1-\beta\gamma)\chi_c > 0$ and that $\mu_2^{tb1} = \beta(c_{b,1}^{-1} - 1) \geq 0$ further holds (see 38 and 41). ■

Proof of Proposition 3. For the policy maker's primal problem under commitment in Lagrangian form, we define $\phi_2^d(c_{b,1}, c_{b,2}) = \frac{v_d(d)(1+\beta)}{c_{b,2}^{-1}-(c_{b,1}c_{b,2}^{-2}-1)\beta\gamma}$ (see 21) and use (36)

$$\begin{aligned} L = E\{&\log c_{b,1} + v(d) + (b_{l,0} + y_{l,1}) + \beta\left[\log c_{b,2} + v(d) + (y_{l,2} + b_{b,2}\beta)\right] \\ &+\beta^2\left[y_{b,3} + v(d) + y_{l,3}\right] + \lambda_{b,1}^{tb}\left[b_{b,0} + y_{b,1} - c_{b,1} - b_{b,1}\beta\right] \\ &+\beta\lambda_{b,2}^{tb}\left[b_{b,1} + y_{b,2} - c_{b,2} - b_{b,2}\beta\right] + \beta\mu_2^{tb}[\gamma\phi_2^d(c_{b,1}, c_{b,2})d + b_{b,2}], \end{aligned}$$

leading to the optimality conditions

$$\lambda_{b,1}^{tb} = c_{b,1}^{-1} + \beta E[\mu_2^{tb}\gamma d \cdot \partial\phi_2^d/\partial c_{b,1}], \tag{42}$$

$$\lambda_{b,1}^{sp} = E\lambda_{b,2}^{tb}, \tag{43}$$

$$\lambda_{b,2}^{tb} = c_{b,2}^{-1} + \mu_2^{tb}\gamma d \cdot \partial\phi_2^d/\partial c_{b,2}, \tag{44}$$

$$\mu_2^{tb} = \beta(\lambda_{b,2}^{tb} - 1) \geq 0. \tag{45}$$

Taking expectations and substituting out the multipliers $\lambda_{b,1}^{tb}$ and $\lambda_{b,2}^{tb}$ in (42)-(44) gives

$$c_{b,1}^{-1} - Ec_{b,2}^{-1} = \gamma d E\left[\mu_2^{tb}\left((\partial\phi_2^d/\partial c_{b,2}) - \beta(\partial\phi_2^d/\partial c_{b,1})\right)\right].$$

Combining with $E\frac{1}{c_{b,2}} = (1-\tau_b)\frac{1}{c_{b,1}}$, which follows from (19) and $1/r_1 = \beta$, leads to the following condition for the optimal constant tax/subsidy rate

$$\tau_b = c_{b,1}\gamma d E\left[\mu_2^{tb}\left((\partial\phi_2^d/\partial c_{b,2}) - \beta(\partial\phi_2^d/\partial c_{b,1})\right)\right], \tag{46}$$

where the multiplier μ_2^{tb} satisfies $\mu_2^{tb} = \beta(c_{b,2}^{-1}-1)/(1-\beta\gamma d\partial\phi_2^d/\partial c_{b,2}) \geq 0$ (see 44 and 45). From (9), we now that $\partial\phi_2^d/\partial c_{b,1} = \chi_\xi\gamma\frac{\beta}{c_{b,2}^2}$ and $\partial\phi_2^d/\partial c_{b,2} = \chi_c - \chi_\xi 2\gamma\beta\frac{c_{b,1}}{c_{b,2}^3}$, such that condition (46) can be rewritten as

$$\tau_b = c_{b,1}\gamma d E\left[\mu_2^{tb}\left(\chi_c - \left\{\chi_\xi\frac{\gamma\beta}{c_{b,2}^2}\left(2\frac{c_{b,1}}{c_{b,2}}+\beta\right)\right\}\right)\right], \tag{47}$$

where $\chi_c > 0$ and $\chi_\xi > 0$. Applying $\chi_\xi = c_{b,2}^2\chi_c$ (see 9) to rewrite (47) as $\tau_b = c_{b,1}\gamma d E\left[\mu_2^{tb}\left(\chi_c\left(1-\gamma\beta\left(2c_{b,1}c_{b,2}^{-1}+\beta\right)\right)\right)\right]$, such that $\tau_b \leq 0$ if $\frac{c_{b,1}}{c_{b,2}(H)} > \frac{1-\beta^2\gamma}{2\beta\gamma}$. ■

Proof of Proposition 4. For the policy maker's primal problem under commitment in Lagrangian form, we define $\phi_1^b(c_{b,1}, c_{b,2}) = \beta(c_{b,1}/c_{b,2})$ and rewrite W with the goods market clearing conditions:

$$\begin{aligned} L = E\{&\log c_{b,1} + v(d) + (y - c_{b,1}) + \beta\left[\log c_{b,2} + v(d) + (y - c_{b,2})\right] + \beta^2\left[y + v(d)\right] \\ &+\lambda_{b,1}^{tl1}\left[b_{b,0} + y_{b,1} - c_{b,1} - b_{b,1}\phi_1^b(c_{b,1}, c_{b,2})\right] + \beta\lambda_{b,2}^{tl1}\left[b_{b,1} + y_{b,2} - c_{b,2} - b_{b,2}\beta\right] \\ &+\beta\mu_2^{tl1}[\gamma q_2(c_{b,2})d + b_{b,2}]\}, \end{aligned}$$

where $q_2(c_{b,2})$ satisfies (12), leading to the optimality conditions

$$\lambda_{b,1}^{tl1} = \left(c_{b,1}^{-1} - 1\right) / \left(1 + b_{b,1}E\left[\partial\phi_1^b/\partial c_{b,1}\right]\right), \tag{48}$$

$$\lambda_{b,1}^{tl1} = r_1\beta E\lambda_{b,2}^{tl1}, \tag{49}$$

$$\beta\lambda_{b,2}^{tl1} = \beta\left(c_{b,2}^{-1} - 1\right) - \lambda_{b,1}^{tl1}b_{b,1}\frac{\partial\phi_1^b}{\partial c_{b,2}} + \beta\mu_2^{tl1}\gamma\frac{\partial q_2}{\partial c_{b,2}}d, \tag{50}$$

$$\mu_2^{tl1} = \beta\lambda_{b,2}^{tl1} \geq 0. \tag{51}$$

Applying expectations and substituting out the multipliers $\lambda_{b,1}^{tl1}$ and $\lambda_{b,2}^{tl1}$ in (48)-(50), gives

$$r_1 \frac{1 + b_{b,1} E\left[\partial\phi_1^b/\partial c_{b,1}\right]}{1 + r_1 b_{b,1} E\left[\partial\phi_1^b/\partial c_{b,2}\right]} = \frac{c_{b,1}^{-1} - 1}{\beta(Ec_{b,2}^{-1} - 1) + \beta\gamma d E\left[\mu_2^{tl1}\partial q_2/\partial c_{b,2}\right]}.$$

Combining the latter with (23) and (24) and using $\frac{\partial q_2(c_{b,2})}{\partial c_{b,2}} = (1-\beta\gamma)\chi_c$ (see 12), leads to the following condition for the ex-ante tax/subsidy rate on saving

$$\tau_{l,1} = -b_{b,1} r_1 E\left[\mu_2^{tl1}\right]\left(E\left[\partial\phi_1^b/\partial c_{b,1}\right] - r_1 E\left[\partial\phi_1^b/\partial c_{b,2}\right]\right) - r_1\beta\gamma d E\left[\mu_2^{tl1}(1-\beta\gamma)\chi_c\right],$$

where $\partial\phi_1^b/\partial c_{b,1} = \beta/c_{b,2} > 0$ and $\partial\phi_1^b/\partial c_{b,2} = -\beta(c_{b,1}c_{b,2}^{-2}) < 0$. Combining (48), (50), and (51), shows that μ_2^{tl1} satisfies $E\mu_2^{tl1} = r_1^{-1}(c_{b,1}^{-1} - 1)/(1 + b_{b,1}E[\partial\phi_1^b/\partial c_{b,1}]) \geq 0$. ∎

Proof of Proposition 5. For the policy maker's primal problem under commitment in Lagrangian form, we define $\phi_1^b(c_{b,1}, c_{b,2}) = \beta\left(c_{b,1}/c_{b,2}\right)$ and $\phi_2^d(c_{b,1}, c_{b,2}) = \frac{v_d(d)(1+\beta)}{c_{b,2}^{-1} - (c_{b,1}c_{b,2}^{-2} - 1)\beta\gamma}$, and use the goods market clearing conditions to rewrite the welfare function, for convenience:

$$\begin{aligned}
L = & E\{\log c_{b,1} + v(d) + (y - c_{b,1}) + \beta\left[\log c_{b,2} + v(d) + (y - c_{b,2})\right] + \beta^2\left[y + v(d)\right] \\
& + \lambda_{b,1}^{tl}\left[b_{b,0} + y_{b,1} - c_{b,1} - b_{b,1}\phi_1^b(c_{b,1}, c_{b,2})\right] \\
& + \beta\lambda_{b,2}^{tl}\left[b_{b,1} + y_{b,2} - c_{b,2} - b_{b,2}\phi_1^b(c_{b,1}, c_{b,2})\right] + \beta\mu_2^{tl}[\gamma\phi_2^d(c_{b,1}, c_{b,2})d + b_{b,2}]\},
\end{aligned}$$

leading to the first order conditions

$$\begin{aligned}
& \lambda_{b,1}^{tl}\left(1 + b_{b,1}E\left[\partial\phi_1^b/\partial c_{b,1}\right]\right) \qquad (52) \\
& = \left(c_{b,1}^{-1} - 1\right) - \beta E\left[\lambda_{b,2}^{tl} b_{b,2}\left(\partial\phi_1^b/\partial c_{b,1}\right)\right] + \beta E\left[\mu_2^{tl}\gamma d\left(\partial\phi_2^d/\partial c_{b,1}\right)\right], \\
& \beta\lambda_{b,2}^{tl}\left(1 + b_{b,2}\left(\partial\phi_1^b/\partial c_{b,2}\right)\right) \qquad (53) \\
& = \beta\left(c_{b,2}^{-1} - 1\right) - \lambda_{b,1}^{tl} b_{b,1}\left(\partial\phi_1^b/\partial c_{b,2}\right) + \beta\left[\mu_2^{tl}\gamma d\left(\partial\phi_2^d/\partial c_{b,2}\right)\right], \\
& \lambda_{b,1}^{tl} = r\beta E\lambda_{b,2}^{tl}, \qquad (54) \\
& \mu_2^{tl} = \phi^b\lambda_{b,2}^{tl} \geq 0, \qquad (55)
\end{aligned}$$

where we used $E\phi_1^b(c_{b,1}, c_{b,2}) = 1/r$. Taking expectations and substituting out the multipliers $\lambda^{tl}_{b,1}$ and $\lambda^{tl}_{b,2}$ in (52)-(54), leads to

$$\begin{aligned}
&E\left[\mu_2^{tl}/\phi_1^b\right]\left(1+b_{b,1}E\left[\partial\phi_1^b/\partial c_{b,1}\right]\right)-E\left[\mu_2^{tl}/\phi_1^b\right]\left(1+rb_{b,1}E\left[\partial\phi_1^b/\partial c_{b,2}\right]\right)\\
=&\frac{1}{r\beta}\left(c_{b,1}^{-1}-1\right)-E\left(c_{b,2}^{-1}-1\right)-\frac{1}{r}E\left[\left(\mu_2^{tl}/\phi_1^b\right)b_{b,2}\left(\partial\phi_1^b/\partial c_{b,1}\right)\right]+\frac{1}{r}\gamma dE\left[\mu_2^{tl}\left(\partial\phi_2^d/\partial c_{b,1}\right)\right]\\
&+E\left[\left(\mu_2^{tl}/\phi_1^b\right)b_{b,2}\left(\partial\phi_1^b/\partial c_{b,2}\right)\right]-\gamma dE\left[\mu_2^{tl}\left(\partial\phi_2^d/\partial c_{b,2}\right)\right],
\end{aligned}$$

and by applying (26), to the following condition for the constant tax/subsidy rate

$$\begin{aligned}
\tau_l=&(-b_{b,1})\,r\beta E\left[\mu_2^{tl}/\phi_1^b\right]\left(E\left[\partial\phi_1^b/\partial c_{b,1}\right]-rE\left[\partial\phi_1^b/\partial c_{b,2}\right]\right)\\
&+\beta E\left[(-b_{b,2})\left(\mu_2^{tl}/\phi_1^b\right)\left((\partial\phi^b/\partial c_{b,1})-r(\partial\phi_1^b/\partial c_{b,2})\right)\right]+\Psi,
\end{aligned}\tag{56}$$

where $\partial\phi_1^b/\partial c_{b,1}>0$, $\partial\phi_1^b/\partial c_{b,2}<0$, and $\Psi=\beta\gamma dE[\mu_2^{tl}\left(\left(\partial\phi_2^d/\partial c_{b,1}\right)-r\partial\phi_2^d/\partial c_{b,2}\right)]$. The term Ψ on the RHS of (56) can by using $\frac{\partial\phi_2^d}{\partial c_{b,1}}=\chi_\xi\frac{\gamma\beta}{c_{b,2}^2}$ and $\frac{\partial\phi_2^d}{\partial c_{b,2}}=\chi_c-\chi_\xi 2\gamma\beta\frac{c_{b,1}}{c_{b,2}^3}$ be rewritten as

$$\Psi=\beta\gamma dE\left[\mu_2^{tl}\left(\left\{\chi_\xi\frac{\gamma\beta}{c_{b,2}^2}\left(1+2r\frac{c_{b,1}}{c_{b,2}}\right)\right\}-r\chi_c\right)\right],\tag{57}$$

Further applying $\chi_\xi=c_{b,2}^2\chi_c$ (see 9) to rewrite (57) as $\Psi=\beta\gamma drE[\mu_2^{tl}(\chi_c\{\gamma\beta(r^{-1}+2c_{b,1}c_{b,2}^{-1})-1\})]$, shows that $\Psi\geq 0$ if $\frac{c_{b,1}}{c_{b,2}(H)}>\frac{1-\beta\gamma r^{-1}}{2\beta\gamma}$. ■

Proof of Proposition 6. Consider the economy with capital formation. Under a Pigouvian debt subsidy in period 2, the capital price satisfies $q^k=A_3[c_{b,2}^{-1}(1-\phi)+\phi+\tau_{b,2}\cdot\phi c_{b,2}^{-1}]^{-1}$. Thus, the collateral constraint is slack under the first best allocation, $-b_{b,2}^{fb}\leq\phi q^k k^{fb}$, if the subsidy rate satisfies $\tau_{b,2}\leq A_3k_{b,2}[-b_{b,2}^{fb}]^{-1}-\phi^{-1}$, where we used $c_{b,1}^{fb}=c_{b,2}^{fb}=1$ and $k^{fb}=(A_2+A_3)/\alpha$ and $b_{b,2}^{fb}$ is given by $b_{b,2}^{fb}=y_{b,2}+y_{b,1}-2-(A_2+A_3)^2/(\alpha 2)+A_2(A_2+A_3)/\alpha$. ■

Proof of Proposition 7. Consider the economy with capital formation. In equilibrium, where capital is entirely held by borrowers, the budget constraints can be written as $c_{b,1}+\alpha k^2/2+b_{b,1}/r_1=y_{b,1}$, $c_{b,2}+b_{b,2}/r_2=y_{b,2}+b_{b,1}+A_2k$, $c_{b,3}=y_{b,3}+b_{b,2}+A_3k$, $c_{l,1}+b_{l,1}/r_1=y_{l,1}$, $c_{l,2}+b_{l,2}/r_2=y_{l,2}+b_{l,1}$, and $c_{l,3}=y_{l,3}+b_{l,2}$. The social welfare function (4) can thus for $\beta=1/r_1=1/r_2=1$ be rewritten as $W=\log c_{b,1}+y_{l,1}+[\log c_{b,2}+y_{l,2}]+[y_{b,3}+b_{b,2}+A_3k+y_{l,3}]$.

To establish the claims made in the first part of the proposition, consider that the policy maker introduces an investment tax/subsidy $\tau_{k,1}$ and an ex-ante debt tax/subsidy $\tau_{b,1}$, which are fully compensated (ex-post) by type-specific lump-sum transfers (like 2). The borrowers' optimality conditions then satisfy

$$(1-\tau_{b,1}) = c_{b,1}/c_{b,2}, \tag{58}$$

$$(1-\tau_{k,1})\alpha k\,(1/c_{b,1}) = (1/c_{b,2})\left(A_2+q^k\right). \tag{59}$$

The primal policy problem of the policy maker is identical to the problem of a social planer who determines period-1-borrowing as well as the capital investment decision and maximizes social welfare W subject to budget and borrowing constraints taking the equilibrium price relation (31) under laissez faire into account, leading to a constrained efficient allocation. The problem can be summarized as $\max W$ w.r.t. $c_{b,1}, c_{b,2}, b_{b,1}, b_{b,2}$, and k subject to $c_{b,1}+\alpha k^2/2+b_{b,1}=y_{b,1}$, $c_{b,2}+b_{b,2}=y_{b,2}+b_{b,1}+A_2k$, and $b_{b,2}+\phi q^k(c_{b,2})k \geq 0$, where $q^k(c_{b,2})$ satisfies (31) and thus $\partial q^k/\partial c_{b,2}>0$. The Lagrangian can be written as

$$\begin{aligned} L =& \log c_{b,1} + y_{l,1} + [\log c_{b,2} + y_{l,2}] + [y_{b,3} + b_{b,2} + A_3 k + y_{l,3}] \\ &+\lambda_1^{t1}\left[y_{b,1} - c_{b,1} - \alpha k^2/2 - b_{b,1}\right] + \lambda_2^{t1}\left[y_{b,2} + b_{b,1} + A_2 k - c_{b,2} - b_{b,2}\right] \\ &+\mu_2^{tb1}\left[b_{b,2} + \phi q^k(c_{b,2})k\right], \end{aligned}$$

leading to the first order conditions for $c_{b,1}, c_{b,2}, b_{b,1}, b_{b,2}$, and k

$$\lambda_1^{t1} = 1/c_{b,1},\quad \lambda_2^{t1} = (1/c_{b,2}) + \mu_2^{tb1}\phi k \partial q^k/\partial c_{b,2},\quad \lambda_1^{t1} = \lambda_2^{t1}, \tag{60}$$

$$\mu_2^{tb1} = \lambda_2^{t1} - 1 \geq 0, \tag{61}$$

$$\lambda_1^{t1}\alpha k = A_3 + \lambda_2^{t1} A_2 + \mu_2^{tb1}\phi q^k(c_{b,2}). \tag{62}$$

Substituting out the multipliers λ_1^{t1} and λ_2^{t1} using the three conditions in (60), gives $(1/c_{b,1}) = (1/c_{b,2}) + \mu_2^{tb1}\phi k\partial q^k/\partial c_{b,2}$. Using (58) to substitute out $1/c_{b,2}$ in the latter, leads to the following condition for the ex-ante debt tax/subsidy rate $\tau_{b,1}$:

$$\tau_{b,1} = \mu_2^{tb1} c_{b,1}\phi k \partial q^k/\partial c_{b,2} \geq 0,$$

where $\mu_2^{tb1} = (c_{b,1}^{-1} - 1) \geq 0$. Further substituting out the multipliers with $\lambda_1^{t1} = \lambda_2^{t1} = 1/c_{b,1}$ and $\mu_2^{tb1} = 1/c_{b,1} - 1$ in (62), gives $\alpha k\,(1/c_{b,1}) = A_3 + (1/c_{b,1})\,A_2 + (1/c_{b,1} - 1)\,\phi q^k$.

Rewriting it with the capital trading decision $q^k(1/c_{b,2}) = A_3+\kappa_{b,2}\phi q^k$ as $(1+\tau_{k,1})\alpha k\,(1/c_{b,1}) = \left((1/c_{b,2})\,A_2 + A_3 + ((1/c_{b,2}) - 1)\,\phi q^k\right)$ and combining with (59), implies that the investment tax/subsidy rate satisfies

$$\tau_{k,1} = -\frac{\{(1/c_{b,1}) - (1/c_{b,2})\}\left[A_2 + \phi q^k\right]}{A_3 + (1/c_{b,1})\,A_2 + \mu_2^{tb1}\phi q^k}.$$

Since $(1/c_{b,1}) = (1/c_{b,2}) + \mu_2^{tb1}\phi k\partial q^k/\partial c_{b,2}$ implies $1/c_{b,1} \geq 1/c_{b,2}$, the policy maker subsidizes capital $\tau_{k,1} \leq 0$ iff $A_2 + \phi q^k \geq 0$. This establishes the claims made in the first part of the proposition.

For the second part of the proposition, we consider a constant debt tax/subsidy and an investment tax/subsidy, which are fully compensated (ex-post) by lump-sum transfers. Agents' borrowing and investment decisions then satisfy

$$(1-\tau_b)/c_{b,1} = 1/c_{b,2}, \tag{63}$$

$$(1-\tau_b)/c_{b,2} = 1+\kappa_{b,2}, \tag{64}$$

and (59). Substituting out $\kappa_{b,2}$ in the capital trading condition $q^k(1/c_{b,2}) = A_3 + \kappa_{b,2}\phi q^k$ with (64) and then the tax/subsidy rate τ_b with (63), gives the price relation

$$q^k = \frac{A_3 c_{b,2}}{1-(c_{b,1}/c_{b,2})\,\phi + c_{b,2}\phi}\ , \tag{65}$$

implying that q^k relates to $c_{b,1}$ and $c_{b,2}$ by $\partial q^k/\partial c_{b,1} = \phi A_3 c_1^2\,(\phi c_{b,1} - c_{b,2} - \phi c_1^2)^{-2} > 0$ and $\partial q^k/\partial c_{b,2} = (1-2\phi c_{b,1}/c_{b,2})\,\partial q^k/\partial c_{b,1}$. The Lagrangian of the policy maker's problem can be written as

$$\begin{aligned}
L = &\log c_{b,1} + y_{l,1} + [\log c_{b,2} + y_{l,2}] + [y_{b,3} + b_{b,2} + A_3 k + y_{l,3}] \\
&+\lambda_1^{tb}\left[y_{b,1} - c_{b,1} - \alpha k^2/2 - b_{b,1}\right] \\
&+\lambda_2^{tb}\left[y_{b,2} + b_{b,1} + A_2 k - c_{b,2} - b_{b,2}\right] + \mu_2^{tb}[b_{b,2} + \phi q^k(c_{b,1}, c_{b,2})k],
\end{aligned}$$

where $q^k(c_{b,1}, c_{b,2})$ satisfies (65). The first order conditions for $c_{b,1}, c_{b,2}, b_{b,1}, b_{b,2}$, and k are

$$\lambda_1^{tb} = 1/c_{b,1} + \mu_2^{tb}\phi k\partial q^k/\partial c_{b,1}, \quad \lambda_2^{tb} = 1/c_{b,2} + \mu_2^{tb}\phi k\partial q^k/\partial c_{b,2}, \quad \lambda_1^{tb} = \lambda_2^{tb}, \tag{66}$$

$$\mu_2^{tb} = \lambda_2^{tb} - 1 \geq 0, \tag{67}$$

$$\lambda_1^{tb}\alpha k = A_3 + \lambda_2^{tb}A_2 + \mu_2^{tb}\phi q^k(c_{b,1}, c_{b,2}). \tag{68}$$

Substituting out the multipliers λ_1^{tb} and λ_2^{tb} using the first three conditions in (66), $(1/c_{b,1})+\mu_2^{tb}\phi k\partial q^k/\partial c_{b,1} = (1/c_{b,2})+\mu_2^{tb}\phi k\partial q^k/\partial c_{b,2}$, and substituting out $1/c_{b,2}$ with (63), gives the following condition for the debt tax/subsidy rate $\tau_b : \tau_b = \mu_2^{tb}c_{b,1}\phi k[(\partial q^k/\partial c_{b,2})-(\partial q^k/\partial c_{b,1})]$. Using that the capital price q^k satisfies $\partial q^k/\partial c_{b,2} = (1 - 2\phi c_{b,1}/c_{b,2})\,\partial q^k/\partial c_{b,1}$ and $\partial q^k/\partial c_{b,1} > 0$ (see 65), the latter can be rewritten as

$$\tau_b = -\mu_2^{tb}c_{b,1}\phi k\,(2\phi c_{b,1}/c_{b,2})\left(\partial q^k/\partial c_{b,1}\right) \leq 0.$$

For the third part of the proposition, we consider an ex-ante tax/subsidy on saving and an investment tax/subsidy, which are fully compensated (ex-post) by lump-sum transfers (see 2). Agents' saving and investment decisions then satisfy

$$(1 - \tau_{l,1})/r_1 = 1, \tag{69}$$

and (59). Given that (69) endogenizes the interest rate for the policy maker, $1/r_1 = c_{b,1}/c_{b,2}$ is a relevant restriction to the policy problem. Using the resource constraints to substitute out $c_{l,t}$, the Lagrangian of the policy maker's problem can be written as

$$\begin{aligned} L = &\log c_{b,1} + (y_1 - c_{b,1}) + [\log c_{b,2} + (y_2 - c_{b,2})] + [y_3 + A_3 k] \\ &+\lambda_1^{tl1}\left[y_{b,1} - c_{b,1} - \alpha k^2/2 - b_{b,1}\,(1/r_1)\right] \\ &+\lambda_2^{tl1}\left[y_{b,2} + b_{b,1} + A_2 k - c_{b,2} - b_{b,2}\right] + \mu_2^{tl1}[b_{b,2} + \phi q^k(c_{b,2})k], \end{aligned}$$

where we used $y_t = y_{b,t}+y_{l,t}$, and q^k and $1/r_1$ satisfy (31) and $1/r_1 = c_{b,1}/c_{b,2}$, respectively.

The first order conditions for $c_{b,1}, c_{b,2}, b_{b,1}, b_{b,2}$, and k are given by

$$\lambda_1^{tl1}(1 + b_{b,1}\partial\left(1/r_1\right)/\partial c_{b,1}) = 1/c_{b,1} - 1, \tag{70}$$

$$\lambda_1^{tl1} b_{b,1}[\partial\left(1/r_1\right)/\partial c_{b,2}] + \lambda_2^{tl1} = (1/c_{b,2}) - 1 + \mu_2^{tl1}\phi k \partial q^k/\partial c_{b,2}, \tag{71}$$

$$\lambda_1^{tl1}\left(1/r_1\right) = \lambda_2^{tl1} \geq 0, \tag{72}$$

$$\mu_2^{tl1} = \lambda_2^{tl1}, \tag{73}$$

and (62). Substituting out the multipliers λ_1^{tl1} and λ_2^{tl1} in (70)-(72), leads to

$$r_1 \frac{(1 + b_{b,1}\partial\left(1/r_1\right)/\partial c_{b,1})}{(1 + r_1 b_{b,1}\partial\left(1/r_1\right)/\partial c_{b,2})} = \frac{(1/c_{b,1} - 1)}{(1/c_{b,2}) - 1 + \mu_2^{tl1}\phi k \partial q^k/\partial c_{b,2}}.$$

Further using (69) as well as $1/r_1 = c_{b,1}/c_{b,2}$ and rearranging terms, gives

$$\begin{aligned}\tau_{l,1} = & -b_{b,1} r_1 \mu_2^{tl1}\left[\left(\partial\left(1/r_1\right)/\partial c_{b,1}\right) - \left(r_1 \partial\left(1/r_1\right)/\partial c_{b,2}\right)\right] \\ & - r_1 \mu_2^{tl1}\phi k \partial q^k/\partial c_{b,2},\end{aligned} \tag{74}$$

where $\partial\left(1/r_1\right)/\partial c_{b,1} > 0$ and $\partial\left(1/r_1\right)/\partial c_{b,2} < 0$.

For the fourth part of the proposition, we consider a constant tax/subsidy on saving and an investment tax/subsidy, which are fully compensated (ex-post) by lump-sum transfers (see 2). Agents' saving and investment decisions then satisfy

$$(1 - \tau_l)/r = 1, \tag{75}$$

where $r = r_1 = r_2$, and (59). Substituting out the interest rates in agents' borrowing decisions with (75), $c_{b,1}^{-1}/(1 - \tau_l) = 1/c_{b,2}$ and $c_{b,2}^{-1}/(1 - \tau_l) = 1 + \kappa_{b,2}$, and combining the latter to $c_{b,1}/c_{b,2} = c_{b,2}\left(1 + \kappa_{b,2}\right)$, implies that q^k satisfies the price relation (65). Proceeding as above, the Lagrangian of the policy maker's problem can be written as

$$\begin{aligned}L = & \log c_{b,1} + (y_1 - c_{b,1}) + [\log c_{b,2} + (y_2 - c_{b,2})] + [y_3 + A_3 k] \\ & + \lambda_1^{tl}\left[y_{b,1} - c_{b,1} - \alpha k^2/2 - b_{b,1}\left(1/r\right)\right] \\ & + \lambda_2^{tl}\left[y_{b,2} + b_{b,1} + A_2 k - c_{b,2} - b_{b,2}\left(1/r\right)\right] + \mu_2^{tl}[b_{b,2} + \phi q^k(c_{b,1}, c_{b,2})k],\end{aligned}$$

where q^k and $1/r$ satisfy (65) and $1/r = c_{b,1}/c_{b,2}$, respectively, leading to the following first order conditions for $c_{b,1}, c_{b,2}, b_{b,1}, b_{b,2}$, and k

$$\lambda_1^{tl}(1 + b_{b,1}\partial\,(1/r)\,/\partial c_{b,1}) + \lambda_2^{tl} b_{b,2}\partial\,(1/r)\,/\partial c_{b,1} = (1/c_{b,1}) - 1 + \mu_2^{tl}\phi k\partial q^k/\partial c_{b,1}, \quad (76)$$

$$\lambda_1^{tl} b_{b,1}\partial\,(1/r_1)\,/\partial c_{b,2} + \lambda_2^{tl}\,(1 + b_{b,2}\partial\,(1/r)\,/\partial c_{b,2}) = (1/c_{b,2}) - 1 + \mu_2^{tl}\phi k\partial q^k/\partial c_{b,2}, \quad (77)$$

$$\lambda_1^{tl}\,(1/r) = \lambda_2^{tl}, \quad (78)$$

$$\mu_2^{tl} = \lambda_2^{tl}\,(1/r) \geq 0, \quad (79)$$

and (68). Substituting out λ_1^{tl} and λ_2^{tl} in (76)-(78) and taking differences, leads to

$$\begin{aligned}
& r\mu_2^{tl} b_{b,1}\,(\partial\,(1/r)\,/\partial c_{b,1}) + \mu_2^{tl} b_{b,2}\,(\partial\,(1/r)\,/\partial c_{b,1}) \\
& - \left(rr\mu_2^{tl} b_{b,1}\,(\partial\,(1/r)\,/\partial c_{b,2}) + r\mu_2^{tl} b_{b,2}\,(\partial\,(1/r)\,/\partial c_{b,2})\right) \\
= {} & r^{-1}\,((1/c_{b,1}) - 1) - ((1/c_{b,2}) - 1) + r^{-1}\mu_2^{tl}\phi k\left(\partial q^k/\partial c_{b,1}\right) - \mu_2^{tl}\phi k\left(\partial q^k/\partial c_{b,2}\right),
\end{aligned} \quad (80)$$

where $\partial\,(1/r)\,/\partial c_{b,1} > 0$ and $\partial\,(1/r)\,/\partial c_{b,2} < 0$. Combining (80) with $c_{b,1}^{-1}/r = 1/c_{b,2}$ and $(1-\tau_l) = r$, leads to the following condition for the tax/subsidy rate

$$\begin{aligned}
\tau_l = {} & \left(r\mu_2^{tl} b_{b,1} + \mu_2^{tl} b_{b,2}\right)(\partial\,(1/r)\,/\partial c_{b,2}) - (\mu_2^{tl} b_{b,1} + \frac{1}{r}\mu_2^{tl} b_{b,2})\,(\partial\,(1/r)\,/\partial c_{b,1}) \\
& + \frac{1}{r}\mu_2^{tl}\phi k\left(\partial q^k/\partial c_{b,1}\right)[2\phi + 1 - r]\,/r,
\end{aligned} \quad (81)$$

where we used $\partial q^k/\partial c_{b,2} = (1 - 2\phi c_{b,1}/c_{b,2})\,\partial q^k/\partial c_{b,1}$ to derive the last term in (81). ∎