Taxing versus subsidizing debt under financial frictions[1]

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Abstract

We examine Pigouvian credit market policies in two incomplete markets models with durables/capital as collateral. Pecuniary externalities under binding collateral constraints rationalize ex-ante debt taxes, restoring constrained efficiency. Ex-post debt subsidies can even implement first-best by stimulating collateral demand. By the same effect, debt subsidies that are constant over time can be superior to debt taxes. Saving subsidies enhance efficiency by addressing distributive effects of pecuniary externalities via interest rate reductions. This shows that debt-increasing subsidies can outperform debt taxes, and that financial amplification based on collateral externalities is insufficient to derive prudential regulation as optimal credit market policy.

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1 Introduction

Pecuniary externalities under collateral constraints can lead to financial amplification and crises. The basic mechanism relies on price-dependent borrowing limits or margin constraints that tighten when asset prices fall. Agents do not internalize the impact of their decisions on asset prices, such that corrective policies can enhance efficiency. (Macro-)Prudential regulation, like ex-ante debt taxes and capital controls, can restore "constrained efficiency" – defined in the tradition of Stiglitz (1982) – by addressing "overborrowing", as shown by Jeanne and Korinek (2010, 2019, 2020), Bianchi (2011), Benigno et al. (2016), Schmitt-Grohe and Uribe (2017), Korinek and Sandri (2016), Bianchi and Mendoza (2018), or Korinek (2018). These studies apply a class of models where interest rates are exogenously determined and agents take borrowing limits as given.[3] The latter implies that collateral premia on pledgeable assets, which seem to play a decisive role for the build-up of financial crises (see, e.g., Geanakoplos, 2010, or Justiniano et al., 2019), are not taken into account for the design of policies aimed to mitigate these crises.

This paper shows that welfare-enhancing prudential regulation can be outperformed by debt-increasing policies when collateral premia exist and interest rates are endogenous. We apply two finite horizon models that exclusively contain conventional features; one model is taken from Davila and Korinek (2018). Lack of commitment induces borrowing to be limited by borrower's holdings of durables or capital, serving as collateral. Pecuniary externalities with regard to the collateral price and to the interest rate give rise to "collateral externalities" and "distributive externalities" (see Davila and Korinek, 2018). The former effects are responsible for the main mechanism in the above cited studies, whereas the latter are turned off therein. While it is known that assets' collateral premia and distributive effects can principally matter for corrective policies (see Bianchi and Mendoza, 2018, and Davila and Korinek, 2018), we show – as the main novel contribution – that they are responsible for Pigouvian credit market subsides that stimulate borrowing to be superior to debt taxes. Specifically, debt subsidies are preferable to

[3]In these models, borrowing limits depend on aggregate not on individual variables. An exception is Bianchi and Mendoza (2018), where agents' capital stocks serve as collateral, such that the capital price is affected by a collateral premium. Yet, their analysis focusses on policies that are implemented when the collateral constraint does not bind and the collateral premium equals zero.

debt taxes in Davila and Korinek's (2018) model when they are fixed over time, while an ex-post Pigouvian debt subsidy can even implement first best in both models. At large, inefficiencies due to externalities induced by collateral constraints can be most effectively addressed by debt and saving subsidies that raise asset prices via the collateral premium and reduce interest rates. Overborrowing unambiguously prevails only if collateral premia and distributive effects do not exist or if the analysis is restricted to ex-ante debt policies, corroborating previous findings. Our analysis reveals that the widely applied approach of modelling financial amplification via collateral externalities is in general not sufficient to rationalize prudential regulation as the most preferable credit market policy.

In a laissez faire equilibrium (of both models), agents do not internalize that the collateral price is too low and the interest rate is too high when borrowing constraints bind in a non-empty set of states. Following Davila and Korinek's (2018) classification, we distinguish collateral effects, which refer to uninternalized changes in the collateral price affecting the borrowing limit, from distributive effects, which refer to interest rate changes and are based on marginal rates of substitutions that differ between agents.[4] Corrective policies can utilize that the collateral price increases with consumption and with agents' willingness to borrow, which raises the price of durables via the so-called collateral premium, i.e. the valuation of assets to serve as collateral (see Fostel and Geanakoplos, 2008). Notably, the latter effect on the collateral price does not exist when agents take borrowing limits as given, like in the majority of the above cited studies on prudential regulation. Policies can further reduce the interest rate to correct for an inefficiently high borrowers' marginal rate of intertemporal substitution. The policies under consideration address collateral and distributive effects of the pecuniary externalities as well as the effects before and while borrowing constraints bind in different ways.

To identify optimal credit market policies, we assume that the policy maker acts under full commitment and we apply the Ramsey approach to optimal policy, where the policy problem depends on the set of available instruments.[5] If state-contingent credit market

[4] Given our focus on credit market policies, we rule out distributive effects with regard to the collateral price, which can principally be addressed by introducing further taxes/subsidies (on durables).

[5] We abstract from time inconsistency of policy plans, which can be relevant under endogenous collateral prices, as shown by Bianchi and Mendoza (2018).

	Ex-ante	Fixed over time
Debt	1. **tax**/subsidy	2. tax/**subsidy**
saving	3. tax/**subsidy**	4. tax/**subsidy**

Table 1: Non-state-contingent Pigouvian policies (numerical examples in bold)

instruments are available, first best can be implemented via a Pigouvian debt subsidy in states where collateral constraints bind, which is a key novel insight. The subsidy increases incentives to borrow and thus the willingness to pay for collateral, measured by the collateral premium. An ex-post debt subsidy can thereby raise the collateral price and the borrowing limit such that borrowing can even get unconstrained. For the subsequent part of analysis, we tie the policy maker's hands and introduce Pigouvian policies that are less complex than state-contingent policies; the latter being difficult to be implemented in practice. Specifically, we compare ex-ante policies, which are imposed before borrowing constraints might become binding, with constant policies, where the tax/subsidy rate is held constant regardless of the particular state or period (see columns of Table 1). These policies take the form of taxes/subsidies on borrowing or saving (see rows of Table 1), which are non-equivalent under potentially binding borrowing constraints.[6]

For the first non-state-contingent policy, i.e. the optimal ex-ante debt tax, we refer to the widely-applied concept of a "constrained efficient allocation", as defined in Stiglitz (1982) or Davila et al. (2012). This allocation is chosen by a social planer who determines borrowing and maximizes social welfare subject to budget and borrowing constraints, conditional on maintaining equilibrium price relations under laissez faire. An ex-ante debt tax leaves the relevant pricing equations under laissez faire unchanged and can implement the constrained efficient allocation, as in the above cited studies on prudential regulation.[7] Concretely, an ex-ante tax on debt enhances efficiency by indirectly raising the collateral price via a debt reduction that increases the amount of funds available for consumption in states where the borrowing constraint binds. In contrast to the ex-ante

[6] We further consider distinct preferences for borrowers and lenders, which are responsible for debt policies – in contrast to saving policies – not to induce interest rate changes, facilitating direct comparisons with the majority of studies on prudential regulation.

[7] As also shown by Davila and Korinek (2008), the allocation under the optimal Ramsey policy is then equivalent to the constrained efficient allocation.

debt tax, the other policy regimes under consideration alter the price relations for the interest rate and for the collateral price, which would not be possible when agents take borrowing limits as given and interest rates are exogenously determined.

The second non-state-contingent policy is a tax/subsidy on debt that is constant over time and influences borrowing regardless whether the constraint is binding or not. A constant debt subsidy tends to stimulate borrowing and to reduce consumption when the constraint binds, while it raises agents' willingness to pay for collateral. It thus combines the inverse effects of an ex-ante debt tax on consumption with the effects of an ex-post debt subsidy on the collateral premium. A constant debt subsidy is superior to a constant debt tax if the effect on the collateral premium dominates. For the model with durables, this is case when the loan-to-value ratio is sufficiently large (including values typically assigned in quantitative studies). In Davila and Korinek's (2018) model with capital formation a constant debt subsidy is unconditionally dominant to a constant debt tax.

The remaining two non-state-contingent policies impose taxes/subsidies on lenders. The third policy is an ex-ante saving tax/subsidy, which directly alters the price relation for debt and thus the equilibrium interest rate. An ex-ante saving subsidy further tends to stimulate borrowing ex-ante and to reduce consumption when the collateral constraint binds, which lowers the collateral price. A policy maker, who trades off these two price effects, decides to subsidize saving ex-ante and to reduce the costs of borrowing for potentially constrained agents when distributive effects dominate collateral effects, which is, for example, the case when the wealth distribution is sufficiently unequal. The fourth policy is a saving tax/subsidy that is constant over time. In contrast to the ex-ante saving subsidy, it tends to stimulate borrowing as well as consumption before and while the collateral constraint is binding by reducing the interest rate and by raising the collateral premium. It can thereby simultaneously address distributive and collateral effects.

To highlight the role of the collateral premium and to reconstruct findings of the studies on prudential regulation cited above, we repeatedly refer to an alternative specification where the borrowing limit is assumed to depend on the aggregate stock of pledgeable assets. In this case, the price that alters the borrowing limit is not affected by the collateral premium, implying that debt/saving subsidies can neither implement first best nor address adverse collateral effects. If distributive effects are further disregarded, optimal ex-ante and constant policies are debt taxes, implying that agents overborrow.

While the analytical results for both models reveal the main principles, we further provide numerical results for the model with durables for illustrative purposes. The four optimal policies are 1) an ex-ante debt tax, 2) a constant debt subsidy, 3) an ex-ante saving subsidy, and 4) a constant saving subsidy (see bold policies in Table 1). Except for the ex-ante debt tax, all policies tend to raise debt before the borrowing constraint binds, and the constant policies induce the largest increases in the collateral price, demonstrating the relevance of assets' collateral premium. The ex-ante debt tax has the least impact on borrowers' consumption and leads to the smallest welfare gains relative to laissez faire, which are virtually negligible based on the distance to first best. In contrast, a constant saving subsidy, which leads to the largest interest rate reduction, substantially reduces welfare losses (by about a half) compared to first best.

The remainder is structured as follows. Section 2 discusses the related literature. Section 3 develops the model with durables. Section 4 examines optimal policies. Section 4.5 presents numerical illustrations. Section 5 presents analytical results for Davila and Korinek's (2018) model with endogenous capital formation. Section 6 concludes.

2 Related literature

This paper is related to several studies on corrective policies under collateral externalities, like Jeanne and Korinek (2010, 2019), Bianchi (2011), Benigno et al. (2016), Korinek and Sandri (2016), Schmitt-Grohe and Uribe (2017), Bianchi and Mendoza (2018), or Korinek (2018). They focus on constrained efficient allocations, as defined in Stiglitz (1982), and prudential policies, like debt taxes or capital controls, that are imposed when borrowing constraints are not binding. In contrast to our analysis, these studies apply models where interest rates are exogenously determined and where – except of Bianchi and Mendoza (2018) – agents take borrowing limits as given, implying that there are neither distributive effects nor collateral premia on pledgeable assets. While Bianchi and Mendoza (2018) discuss that the collateral premium principally affects the price of capital and the policy choice, their analysis focusses on (prudential) policies that are imposed when the collateral constraint does not bind and the collateral premium equals zero. They further examine constant debt taxes and find that they lead to negligible welfare gains or even welfare losses, consistent with our results on constant debt policies. Bianchi (2011) finds that a constant debt tax can achieve sizable welfare gains, which we show to rely on the non-existence of collateral premia in our model. In addition to debt taxes, Benigno et al. (2016) analyze policies introduced in other markets, and show that an ex-post tax on non-tradables can raise the collateral price, such that the borrowing constraint does not bind. Bianchi (2016) and Jeanne and Korinek (2020) find welfare gains from ex-post policies in form of debt reliefs or liquidity provisions that do not implement first best, in contrast to ex-post Pigouvian debt subsidies applied in our paper. In addition to these analyses, we examine time- and state-invariant debt/saving subsidies, and show that they can be superior to debt taxes.

None of the above cited studies considers distributive effects or changes in intertemporal prices. In a seminal paper, Lorenzoni (2008) shows that distributive externalities under financial frictions cause agents to overinvest and to overborrow in an unregulated economy. Davila et al. (2012) show in a model with an endogenous wealth distribution that distributive effects can either lead to over- or underaccumulation of capital. Lanteri and Rampini (2021) develop a model of endogenous formation and reallocation of capital. They show that distributive effects of pecuniary externalities with regard to the capital price are larger than collateral externalities, such that a subsidy on new investment enhances efficiency. Both studies do not analyze credit market policies. Davila and Korinek (2018) develop a general framework with endogenous capital formation, for which they establish collateral and distributive effects. They restrict their analysis to ex-ante policies and show that these externalities can either cause over- or underinvestment, while they emphasize that "collateral externalities generally entail overborrowing". We show that this conclusion holds only if the perspective is restricted to ex-ante debt policies.

The above cited studies focus on the analysis of constrained efficient allocations, which can either be derived from a problem of choosing initial allocations or from a Ramsey problem when equilibrium price relations are unaffected by policy instruments (see also Davila and Korinek, 2018). In contrast, the solutions to our policy problems differ from this type of constrained efficient allocation when the price relations for the collateral price or for the interest rate are affected by policy. Relatedly, Benigno et al. (2020) re-examine policy instruments used in Benigno et al. (2016), applying the Ramsey approach. Complementary to our analysis of different policy instruments, they show that a set of instruments that can implement a constrained efficient allocation can also be used to implement a superior allocation where borrowing constraints never bind.

Our finding that agents borrow too much or too little under laissez faire depending on the policy regime relates to other studies showing that underborrowing can prevail. Benigno et al. (2013) examine the constrained efficient allocation – but not its implementation – of an economy where agents take into account that labor supply alters the borrowing limit, which compares to our analysis where agents internalize that borrowing limits depend on their holdings of eligible assets. They show that one should rather reallocate resources between (tradable and non-tradable goods) sectors to raise borrowing limits than subsidize borrowing. Schmitt-Grohe and Uribe (2021) establish the existence of multiplicity in the model examined by Bianchi (2011), giving rise to equilibria with underborrowing due to excessive precautionary savings. For a model with bank intermediation, Chi et al. (2022) show that agents borrow less under laissez faire compared to equilibria with ex-post expansions of bank reserves. Ottonello et al. (2022) shows that constrained inefficiency depends on whether borrowing limits depend on current or future collateral prices, and that debt subsidies can be optimal in the latter case.

3 A model with incomplete markets and limited commitment

In this Section, we develop an infinite horizon model with durables in fixed supply. Davila and Korinek's (2018) model with capital formation is presented in Section 5.[8] There exist two imperfections in both models: Only non-state contingent debt is available and agents are not able to commit to debt repayment. The latter leads to the key financial friction, i.e. a borrowing constraint with the borrower's asset serving as collateral.

[8]While the analysis of the latter model includes an additional (capital investment) tax/subsidy, the results on borrowing and saving taxes/subsidies correspond to the result for the former model.

3.1 Details

There are two mass-one groups $\{b, l\}$ with infinitely many agents, who live for three periods $t = 1, 2, 3$. In each period t, a household $i \in \{b, l\}$ derives utility from consumption of a non-durable good, $c_{i,t}$, and a durable good (or housing), $d_{i,t}$, as given by the function $u_{i,t} = u(c_{i,t}, d_{i,t})$. Agents maximize their expected lifetime utility, $E \sum_{t=1}^{3} \beta^{t-1} u(c_{i,t}, d_{i,t})$, where u is strictly increasing and concave, E denotes an expectations operator conditional on information in period 1, and $\beta \in (0, 1)$ is a discount factor. In each period, agents receive a potentially random endowment $y_{i,t}$ of non-durable goods and they exhibit an initial endowment of durables $d_{i,0}$. Agents can borrow and lend only in terms of non-state contingent one-period bonds $b_{i,t}$, which are issued at the price $1/r_t$. The budget constraint of an agent i for period t is given by

$$c_{i,t} + q_t(d_{i,t} - d_{i,t-1}) + (1 - \tau_{i,t})b_{i,t}/r_t = b_{i,t-1} + y_{i,t} + T_{i,t}, \tag{1}$$

where $\tau_{i,t}$ denotes distortionary taxes/subsidies on debt/saving. Specifically, we consider Pigouvian-type fiscal interventions, where budgetary effects of taxes/subsidies are (ex-post) neutralized in a non-distortionary way:

$$T_{i,t} = -\tau_{i,t} b_{i,t}/r_t, \tag{2}$$

which is not internalized by agents. There is no uncertainty in the periods 1 and 3, where total endowment with non-durables is equally distributed: $y_{b,1} = y_{l,1} = y/2$. Agents b (l) start with negative (positive) initial net financial wealth $b_{b,0} < 0$ ($b_{l,0} > 0$) and will be called borrowers (lenders). In period 2, endowments are randomly determined and can either take the same values as in period 1 (state L) or can be unequally distributed (state H). Specifically, both states are equally likely and endowment of borrowers in state H (with Higher inequality) is $y_{b,2} = y/(1 + \delta_H)$, where $\delta_H > 1$.

We assume that agents cannot commit to repay debt and that debt can be renegotiated after issuance in the same period. Borrowers can make a take-it-or-leave-it offer to reduce the value of debt. If a lender rejects the offer, he can seize a fraction γ of the borrower's durable goods, which he can sell at the market price q_t. Offers are therefore accepted when the repayment value of debt at least equals the current value of seizable assets.

Without loss of generality, we assume that default and renegotiation never happen in equilibrium. When debt is issued, the amount of debt $-b_{i,t}$ is therefore constrained by

$$-b_{i,t} \leq \gamma q_t d_{i,t}, \tag{3}$$

where $\gamma \in (0,1)$. According to (3), newly issued debt is constrained by the current market value of durables (e.g. housing), consistent with empirical evidence (see e.g. Cloyne et al., 2019). The borrowing constraint (3) can generate a feedback between agents' demand for durables and the debt limit, which is not internalized in individual decisions. Moreover, the borrowing constraint can lead to unequal marginal rates of substitutions between states and agents, giving rise to distributive effects, as discussed in Davila and Korinek (2018). Notably, agents will internalize that individual holdings of durables serve as collateral, such that the durables price increases with its collateral premium (see Fostel and Geanakoplos, 2008). To unveil the impact of this collateral premium on optimal policy choices, we introduce and repeatedly refer to an alternative version of the borrowing constraint that is independent of the individual stock of durables.

The available stock of durables equals d and the total non-durable endowment equals y. Since there is no borrowing/lending in the final period, the borrowing constraint is irrelevant and there are also no taxes/subsidies on debt/saving in $t = 3$, i.e. $\tau_{i,3} = T_{i,3} = 0$. A competitive equilibrium is then given by an allocation of durables, non-durables, and debt $\{c_{i,1}, d_{i,1}, b_{i,1}, c_{i,2}(s), d_{i,2}(s), b_{i,2}(s), c_{i,3}(s), d_{i,3}(s)\}$ for $i \in \{b, l\}$ and $s \in \{L, H\}$, a set of prices $\{r_1, r_2(s), q_1, q_2(s), q_3(s)\}$ satisfying agents' maximization problem s.t. the budget constraints (1) and the collateral constraints (3), and the market clearing conditions, $d_{b,t} + d_{l,t} = d$ and $b_{b,t} + b_{l,t} = 0 \; \forall t$, given taxes/subsidies $\{\tau_{i,t}, T_{i,t}\}$ for $i \in \{b, l\}$ and $t \in \{1, 2\}$, and an initial distribution of debt and durables and sequences of non-durable endowments $\{y_{i,t}\}_{t=1}^{3}$ for $i \in \{b, l\}$.

We briefly describe the first best allocation, which serves as a reference case. It can easily be shown that the allocation would be efficient (even though financial markets are incomplete), if borrowing were not constrained. The allocation would then be equivalent to the allocation a social planer would choose who maximizes utilitarian social welfare, given by

$$W = E \sum_{t=1}^{3} \beta^{t-1} \left(u_{b,t} + u_{l,t} \right), \tag{4}$$

subject to the resource constraints. Specifically, the first best allocation is characterized by identical marginal utilities of consumption of borrowers and lenders for $t \in \{1, 2, 3\}$

$$\partial u_{b,t} / \partial c_{b,t} = \partial u_{l,t} / \partial c_{l,t}. \tag{5}$$

The competitive equilibrium allocation can deviate from (5) when the borrowing constraint binds in period 2 or when borrowers expect that the borrowing constraint might become binding and reduce consumption in period 1 due to a precautionary motive. Moreover, the borrowing constraint can induce inefficiencies due to pecuniary externalities with regard to the durables price and the interest rate.

3.2 Pecuniary externalities and collateral premia

To further facilitate the analysis, we impose some simplifying assumptions on preferences, which correspond to those applied in Davila and Korinek's (2018) model (see Section 5), and on the relevance of the borrowing constraint.

Assumption 1 *Agents' preferences satisfy* $u_{l,t} = c_{l,t}$ *for* $t \in \{1, 2, 3\}$, $u_{b,t} = \log c_{b,t} + v(d_{b,t})$ *for* $t \in \{1, 2\}$, *and* $u_{b,3} = c_{b,3} + v(d_{b,3})$, *where* $v_d > 0$ *and* $v_{dd} \leq 0$.

Assumption 2 *Initial debt* ($b_{b,0}$) *is small enough that (3) is slack in* $t = 1$ *and inequality* (δ_H) *in* $t = 2$ *is large enough that (3) is binding in state* H *under laissez faire.*

The restrictions on agents' preferences in Assumption 1 facilitate the derivation of analytical results and allow isolating distinct effects of policy regimes. Specifically, as durables do not provide utility to lenders, which relates to studies on fire sales where borrowers have a superior use for assets, the distribution of durables will be degenerate and only borrowers will hold durables in equilibrium.[9] Due to linear utility of lenders, the interest rate is constant under laissez faire and under debt taxes/subsidies, like in Bianchi's (2011) seminal small open economy model. Here, it can however be adjusted under a

[9] Consistently, we restrict the initial endowment by $d_{b,0} = d$.

saving tax/subsidy. We thus switch off distributive effects with regard to durables and focus on collateral effects when debt taxes/subsidies are applied, which facilitates comparisons with related studies. In contrast, a saving tax/subsidy might further address distributive effects via changes in the real interest rate. Assumption 2 ensures that the borrowing constraint is not binding in period 1, while there is a positive probability that it is binding in period 2. Policies that are exclusively imposed in period 1 (2) are therefore called ex-ante (ex-post) policies. In addition, we examine constant policies that are neither time nor state dependent and that apply equally in period 1 and 2.

3.2.1 Laissez faire

Before we discuss welfare-enhancing policies, we describe the equilibrium under laissez faire, i.e. without policy interventions, which will serve as the main reference case, and the pecuniary externalities. Under Assumptions 1 and 2, the borrowers' optimality conditions can be summarized as $c_{b,1}^{-1}q_1 = v_d(d_{b,1}) + \beta E q_2 c_{b,2}^{-1}$,

$$c_{b,1}^{-1}/r_1 = \beta E[c_{b,2}^{-1}], \tag{6}$$

$$c_{b,2}^{-1}q_2 = v_d(d_{b,2}) + \beta q_3 + \mu_{b,2}\gamma q_2, \tag{7}$$

$$c_{b,2}^{-1}/r_2 = \beta + \mu_{b,2}, \tag{8}$$

$$-b_{b,2} = \gamma q_2 d_{b,2}, \text{ for } \mu_{b,2} > 0 \text{ or } -b_{b,2} \leq \gamma q_2 d_{b,2}, \text{ for } \mu_{b,2} = 0, \tag{9}$$

and $q_3 = v_d(d_{b,3})$, where $\mu_{b,2}$ denotes the multiplier on the borrowing constraint (3). Notably, the borrowers' optimality conditions for debt and durables in period 2, (7) and (8), would differ from corresponding optimality conditions of lenders even under identical preferences, since lenders do not face a (potentially binding) borrowing constraint. Given that the borrowing constraint (3) depends on the individual stock of durables, borrowers value durables also for their ability to serve as collateral and to raise the borrowing limit. This effect is captured by the multiplier $\mu_{b,2}$ entering the RHS of (7). The latter can be rewritten to get q_2 as a function of $c_{b,2}$ and of the <u>collateral premium</u> $\xi_2 = \mu_{b,2}\gamma$:

$$q_2 = \chi(c_{b,2}, \xi_2) = \frac{v_d(d)\,(1+\beta)}{c_{b,2}^{-1} - \xi_2}, \tag{10}$$

$$\text{with } \chi_c \equiv \partial\chi/\partial c_{b,2} > 0 \text{ and } \chi_\xi \equiv \partial\chi/\partial\xi_2 > 0,$$

where we used $q_3 = v_d(d_{b,3})$ and $d_{b,t} = d$. Consumption $c_{b,2}$ tends to increase the durables price q_2 by reducing the marginal valuation of durables purchase costs (see LHS of 7). This effect is summarized by $\chi_c > 0$. The term ξ_2, which equals $\mu_{b,2}\gamma$, measures the collateral premium on durables and increases the durables price q_2 under a higher valuation of collateral services, $\chi_\xi > 0$. This implies that the borrowing decision (8) relates to the durables price via the tightness of the borrowing constraint (measured by $\mu_{b,2}$). The collateral premium for asset prices will be crucial when taxes/subsidies are imposed while borrowing constraints bind. Notably, the collateral premium would be equal to zero, $\xi_2 = 0$, if the borrowing constraint were independent of the individual stock of durables (see below). Lenders' optimal behavior satisfies

$$1/r_1 = \beta, \quad 1/r_2 = \beta. \tag{11}$$

It can be shown in a straightforward way that lenders will not hold durables under Assumption 1. The laissez faire equilibrium is thus characterized by the binding budget constraints, $c_{b,3} = b_{b,2} + y_{b,3}$, $c_{b,2} = b_{b,1} + y_{b,2} - b_{b,2}/r_2$, and $c_{b,1} = b_{b,0} + y_{b,1} - b_{b,1}/r_1$, as well as $c_{l,3} = b_{l,2} + y_{l,3}$, $c_{l,2} = b_{l,1} + y_{l,2} - b_{l,2}/r_2$, and $c_{l,1} = b_{l,0} + y_{l,1} - b_{l,1}/r_1$.

Combining (8) with (11) shows that borrowers' marginal utility of non-durables consumption in period 2 exceeds of lenders' marginal utility of non-durables consumption (=1) under a binding borrowing constraint: $(c_{b,2}^{-1}(H) - 1)\beta = \mu_{b,2}(H) > 0$. Under Assumption 2, condition (6) implies non-durables consumption in period 1 and 2 to satisfy:

$$(c_{b,1}^{-1} - 1) = E[c_{b,2}^{-1} - 1] > 0. \tag{12}$$

Thus, agents' decisions are distorted by the borrowing constraint when there is a positive probability that it is binding. Pecuniary externalities can then affect the allocation in an adverse way. Subsequently, we will summarize collateral effects and distributive effects of externalities following the classification of Davila and Korinek (2018). Concretely, collateral effects refer to uninternalized changes in the collateral price affecting the borrowing limit. Distributive effects refer to uninternalized changes in the debt price, which are relevant under marginal rates of substitutions that differ between agents.

Like in Davila and Korinek's (2018) model (see Section 5), the collateral price is determined by the borrowers' optimality conditions. Condition (10) implies that the collateral price q_2 increases with borrowers' consumption in the same period, $\chi_c > 0$, and with the collateral premium, $\chi_\xi > 0$, which equals $\xi_2 = \gamma(c_{b,2}^{-1} - 1)\beta$ under laissez faire. The price relation for the collateral price under laissez faire is thus given by

$$q_2 = \frac{v_d(d)(1+\beta)}{c_{b,2}^{-1}(1-\beta\gamma) + \beta\gamma}, \tag{13}$$

implying $\partial q_2/\partial c_{b,2} > 0$. In state L, where $\mu_{b,2} = 0$ holds under Assumption 2, the price relation (13) simplifies to $q_2 = c_{b,2}v_d(d)(1+\beta)$, such that $q_2(H) < q_2(L)$, given that (8) holds. Under a binding borrowing constraint in state H, the collateral price q_2 tends to be higher due the collateral premium ξ_2, while the latter decreases with $c_{b,2}$ (see 10 and 13). These collateral effects of their decisions on the durables price and thus on the borrowing limit are not internalized by agents, though by a policy maker.

Higher non-durables consumption of borrowers in $t = 1$ relative to $t = 2$ tends to be associated with a lower interest rate (see 6), which is not internalized by agents. Yet, unconstrained lenders are only willing to lend at a constant rate under laissez faire (see 11). Hence, equilibrium interest rates are fixed at $1/\beta$, which also holds under taxes/subsidies on borrowing. If, however, taxes/subsidies are imposed on lenders, interest rates become endogenous to the policy maker. Given that borrowers' marginal rates of substitution are distorted by the borrowing constraint (see 6 and 8), inefficiencies due to distributive effects can be addressed via interest rate changes induced by saving taxes/subsidies.

3.2.2 Efficiency and ex-post credit market policies

Under the Assumptions 1 and 2, the first best allocation, which maximizes (4) and satisfies (5), is evidently characterized by $d_{b,t} = d$ for $t \in \{1, 2, 3\}$,

$$c_{b,1}^{fb}(s) = c_{b,2}^{fb}(s) = 1, \quad c_{l,1}^{fb}(s) = c_{l,2}^{fb}(s) = y - 1, \tag{14}$$

$c_{b,3}^{fb}(s) = ((b_{b,0} + y_{b,1} - c_{b,1}^{fb})\beta^{-1} + y_{b,2}(s) - c_{b,2}^{fb}(s))\beta^{-1} + y_{b,3}$ and $c_{l,3}^{fb}(s) = y - c_{b,3}^{fb}(s)$ for $s \in \{L, H\}$. Even though individual endowment with non-durables is random in period 2 and markets are incomplete, borrowers' first best consumption of non-durables is identical in $t = 1$ and $t = 2$. In contrast, period-3-consumption of non-durables is state-dependent.

Under a positive probability that the borrowing constraint is binding (see Assumption 2), the first best allocation, in particular (14), cannot be realized under laissez faire, where $c_{b,2}$ is state dependent (see 8) and $c_{b,1} < 1$ holds (see 12). Yet, first best can be implemented by a Pigouvian ex-post policy that raises the collateral price q_2 to a sufficiency high level such that the borrowing constraint is never binding. To see this, consider a policy intervention in period 2 in form of a Pigouvian tax/subsidy on debt, $(1 - \tau_{b,2})c_{b,2}^{-1}/r_2 = \beta + \mu_{b,2}$, which together with (11) leads to the following credit market equilibrium condition:

$$(1 - \tau_{b,2})c_{b,2}^{-1}\beta = \beta + \mu_{b,2}. \tag{15}$$

Using (15) to substitute out the multiplier $\mu_{b,2}$ in (7) and $q_3 = v_d(d)$, gives the following condition for the collateral price q_2 in state H (where $\mu_{b,2} > 0$):

$$q_2 = \frac{(1+\beta)\, v_d(d)}{c_{b,2}^{-1}(1-\beta\gamma) + \beta\gamma + \{\tau_{b,2} \cdot \beta\gamma c_{b,2}^{-1}\}}, \tag{16}$$

where the term in the curly brackets indicates that q_2 increases with a debt subsidy $\tau_{b,2} < 0$. Given that the latter does not affect other equilibrium conditions and that q_2 only matters for the equilibrium allocation due its impact on the collateral constraint, the subsidy can in fact exclusively be used the raise the borrowing limit.[10] This effect is based on the collateral premium (see 10), which tends to increase when the multiplier on the collateral constraint $\mu_{b,2}$ is raised by agents higher willingness to borrow under a debt subsidy (see 15). Implementation of first best can be summarized as follows.

Proposition 1 *The first best allocation is implemented in a competitive equilibrium under an ex-post Pigouvian debt subsidy satisfying*

$$\tau_{b,2}(H) \leq \left(1 + \beta^{-1}\right) dv_d(d)[-b_{b,2}^{fb}(H)]^{-1} - (\beta\gamma)^{-1} < 0,$$

where $b_{b,2}^{fb}(H)$ *denotes borrowing under a first best allocation,* $b_{b,2}^{fb}(H) = \beta^{-2}(b_{b,0} + y_{b,1} - c_{b,1}^{fb}) + \beta^{-1}(y_{b,2}(H) - c_{b,2}^{fb})$.

Proof. See Appendix. ■

Why is implementation of the first best allocation via ex-post credit market policies not examined in related studies on financial frictions? If the borrowing limit were assumed to

[10] This property does, evidently, not hold in general, in particular, under non-linear preferences.

depend on aggregate income and prices and thus to be exogenous to individual agents, like in Jeanne and Korinek (2010, 2019, 2020), Bianchi (2011), Benigno et al. (2016), Schmitt-Grohe and Uribe (2017), Korinek and Sandri (2016), or Korinek (2018), a Pigouvian ex-post debt subsidy would be ineffective. To see this, consider an alternative borrowing constraint that depends on the aggregate stock of durables:

$$-b_{i,t} \leq \gamma q_t d. \tag{17}$$

In this case, condition (7) would reduce to $c_{b,2}^{-1} q_2 = v_d(d_{b,2}) + \beta q_3$. The collateral price would then be independent of agents' willingness to borrow, measured by the multiplier $\mu_{b,2}$ on the collateral constraint, and there would be no collateral premium effect on q_2, given by $q_2 = c_{b,2} v_d(d)\,(1+\beta)$. Thus, under a borrowing limit that is taken as given by borrowers (see 17), a Pigouvian ex-post debt tax or subsidy would exclusively alter the multiplier $\mu_{b,2}$ (see 15), leaving the laissez faire allocation unchanged.

Corollary 1 *Suppose that the borrowing constraint is given by (17), such that the borrowing limit is taken as given by borrowers. Then, an ex-post Pigouvian debt tax/subsidy does not affect the allocation in a competitive equilibrium.*

The subsequent analysis examines policies that cannot be made contingent on adverse states and can therefore not implement first best.

4 Non-state-contingent credit market policies

While state-contingent debt subsidies can implement first best, we now show that welfare can also be enhanced by non-state-contingent corrective polices. We focus on ex-ante policies, which relate to prudential regulation, and on policies that are constant over time, which can be implemented without the complexities associated with state-contingent policies. Specifically, we examine four different types of Pigouvian credit market policies (see Table 1): 1) an ex-ante tax/subsidy on debt, 2) a constant tax/subsidy on debt, 3) an ex-ante tax/subsidy on saving, and 4) a constant tax/subsidy on saving. For this, we apply the Ramsey approach to optimal policy, where the policy maker acts under full commitment and internalizes equilibrium price relations. Under 1), the allocation is identical to a constrained efficient allocation, where the social planer respects budget and borrowing constraints as well as price relations that are unchanged compared to laissez faire. In contrast, the price relation for the collateral price is altered under 2) and for the debt price under 3), while the price relations for both prices are simultaneously altered under 4). The allocations under these policy regimes will thus differ and the policy maker can more directly address the pecuniary externalities under regimes 2)-4) than under 1).

4.1 An ex-ante Pigouvian tax on debt

We first consider the case, where a tax/subsidy on debt might be introduced in period 1, whereas no policy instrument is applied in period 2. Ex-ante debt taxes, which correspond to capital controls in open economies, have already been examined in several related studies (see Davila and Korinek, 2018, or Erten et al., 2021, for an overview), establishing that they can implement a constrained efficient allocation as defined by Stiglitz (1982). Under an ex-ante debt tax/subsidy, borrowers' optimality condition (6) changes to

$$(1-\tau_{b,1})c_{b,1}^{-1}/r_1 = \beta E c_{b,2}^{-1}. \tag{18}$$

In equilibrium, condition (18) and the optimal lending choice $1/r_1 = \beta$ imply $(1-\tau_{b,1}) = c_{b,1}Ec_{b,2}^{-1}$. By taxing debt in period 1, $\tau_{b,1} > 0$, agents can be induced to borrow less, which tends to raise non-durables consumption $c_{b,2}$ and the durables price q_2 (see 13).[11] Given that borrowers do not internalize the adverse effect of period-1-borrowing on the durables/collateral price and thus the borrowing limit in period 2, a policy maker can enhance efficiency by addressing collateral effects of pecuniary externalities with an ex-ante debt tax. This mechanism is well-established in the literature on prudential regulation and capital controls, which has led to the notion of "overborrowing".

Proposition 2 *Suppose that the policy maker can apply a Pigouvian tax/subsidy on debt before the borrowing constraint might be binding. Then, the optimal allocation is constrained efficient and associated with a tax on debt, satisfying*

$$\tau_{b,1} = c_{b,1}\gamma d(1-\beta\gamma)E\left[\mu_2^{tb1}\cdot\chi_c\right] > 0, \tag{19}$$

where $\mu_2^{tb1} \geq 0$ *denotes the multiplier on the borrowing constraint of the policy problem.*

[11]Notably, this positive effect of higher net worth of borrowers on the durables/collateral price corresponds to the effect in Davila and Korinek (2018) imposed by their condition 1.

Proof. See Appendix. ■

The optimal ex-ante debt tax described in Proposition 2 implements the "constrained efficient allocation", which is chosen by a social planer respecting budget and borrowing constraints and allowing markets for durables and non-durables to clear in a competitive way (see Stiglitz, 1982, or Davila et al., 2012). Concretely, a constrained efficient allocation is chosen by a social planer who determines borrowing and maximizes social welfare W subject to budget and borrowing constraints, while taking the competitive equilibrium relations for interest rates (11) and the durables price (13) under laissez faire into account. The Ramsey optimal ex-ante debt tax leads to the same outcome, since it leaves the pricing equations for debt (11) and durables (13) unaffected.[12] In contrast, we will examine polices in the subsequent sections that alter (11) and (13), such that the prices q_2, $1/r_1$, and $1/r_2$ can be altered by policy in more direct ways. Under alternative credit market polices, competitive equilibrium allocations can thereby be implemented that are superior to the constrained efficient allocation.

4.2 A constant Pigouvian tax/subsidy on debt

In this model, state contingency cannot simply be induced by cyclicality of policy instruments. We therefore consider that the debt tax/subsidy τ_b can neither be made contingent on specific periods nor on the state of the economy, i.e. on the distribution of agents' endowment, such that the debt tax/subsidy is constant and equally imposed in the periods $t = 1$ and $t = 2$. In this case, the tax/subsidy has ex-ante and ex-post effects relative to the state of the economy where the borrowing constraint might be binding. The borrowers' optimality conditions (6) and (8) then change to

$$(1 - \tau_b)c_{b,1}^{-1}/r_1 = \beta E[c_{b,2}^{-1}], \tag{20}$$

$$(1 - \tau_b)c_{b,2}^{-1}/r_2 = \beta + \mu_{b,2}, \tag{21}$$

where $1/r_1 = 1/r_2 = \beta$. Combining (20) with (21), shows that the multiplier $\mu_{b,2}$ satisfies $\mu_{b,2} = \beta(c_{b,2}^{-1}c_{b,1}E[c_{b,2}^{-1}] - 1)$, which differs from its laissez faire version ($\mu_{b,2} = (c_{b,2}^{-1} - 1)\beta$).

[12] Benigno et al. (2016) and Davila and Korinek (2018) also show that this approach can be equivalent to a Ramsey optimal policy where the policy maker chooses taxes ex-ante or on first period allocations.

The collateral premium $\xi_2 = \gamma\mu_{b,2}$ and the equilibrium condition for durables (10) then lead to the following price relation:

$$q_2 = \frac{v_d(d)\,(1+\beta)}{c_{b,2}^{-1}(1-\beta\gamma c_{b,1}E[c_{b,2}^{-1}]) + \beta\gamma}, \tag{22}$$

while the durables price in state L simplifies to $q_2 = c_{b,2}v_d(d)(1+\beta)$. The durables price q_2 tends to be higher under a larger collateral premium ξ_2 (see 10), while a constant debt tax $\tau_b > 0$ tends to reduce the multiplier $\mu_{b,2}$ and thus ξ_2 (see 21). Due to this effect on q_2 and the negative effect of the debt tax on non-durables consumption $c_{b,1}$ relative to $c_{b,2}$ (see 20), the durables price q_2 is here characterized by a positive relation to $c_{b,1}$ in equilibrium (see 22).

A constant debt tax tends to induce agents to borrow and to consume less in period 1 relative to period 2 (as in the case of the ex-ante tax), but also tends to reduce borrowing and consumption in period 2 when the borrowing constraint might be binding (see 21). Due to a lower collateral premium, a debt tax can induce a reduction in the durables price and in the borrowing limit in period 2. It might therefore be preferable to apply a subsidy rather than a tax on debt. The following proposition summarizes properties of an optimal constant tax/subsidy on debt:[13]

Proposition 3 *Suppose that the policy maker can apply a constant Pigouvian tax/subsidy on debt in the periods 1 and 2. Then, the optimal allocation is associated with a tax/subsidy rate on debt satisfying*

$$\tau_b = c_{b,1}\gamma d E\left[\mu_2^{tb}\left(\chi_c - \left\{\chi_\xi \cdot \left(2\frac{c_{b,1}}{c_{b,2}} + \beta\right)\frac{\gamma\beta}{c_{b,2}^2}\right\}\right)\right], \tag{23}$$

where $\mu_2^{tb} \geq 0$ denotes the multiplier on the borrowing constraint of the policy problem, and the rate τ_b is strictly negative when $\frac{c_{b,1}}{c_{b,2}(H)} > \frac{1-\beta^2\gamma}{2\beta\gamma}$.

Proof. See Appendix. ■

As revealed by Proposition 3, the collateral effects given on the RHS of (23) imply that an optimal non-contingent debt policy can either be a tax ($\tau_b > 0$) or a subsidy ($\tau_b < 0$).

[13] Notably, a policy maker under commitment fully accounts for agents conditioning their expectations, for example on the RHS of (22), on policy choices.

The reason is that a constant debt subsidy tends to raise q_2 via the collateral premium on durables (see χ_ξ), similar to an ex-post debt subsidy (see Section 3.2.2). At the same time, a constant debt subsidy tends to reduce $c_{b,2}$ and thus q_2 via increased debt (see χ_c), which are the inverse effects of an ex-ante debt tax. If the impact on the collateral premium summarized by the positive term in the curly brackets in (23) dominates the latter effect, a debt subsidy is optimal, $\tau_b < 0$. The inequality at the end of the proposition, reveals that this applies when the ratio of period-1-consumption to period-2-consumption in state H, $c_{b,1}/c_{b,2}(H)$, is sufficiently large. Recall that this ratio is equal to one under first best (see 14), while it exceeds one under laissez faire when the collateral constraint binds (see 12). The inequality is therefore likely to be satisfied when the allocation under the constant tax/subsidy is (still) characterized by a binding collateral constraint, inducing the ratio $c_{b,1}/c_{b,2}(H)$ to exceed one, and the threshold $\frac{1-\beta^2\gamma}{2\beta\gamma}$ is smaller or equal to one. The latter is in fact the case when the liquidation value of collateral γ is sufficiently large, i.e. if $\gamma \geq 1/\left(\beta^2 + 2\beta\right)$, which can in principle be satisfied by empirically plausible loan-to-value-ratios (e.g. $\gamma = 0.8$).

Apparently, the term in the curly bracket is equal to zero if there were no collateral premium, like under a borrowing constraint that does not depend on the individual stock of durables (see 17). In this case, the RHS of (23) would be strictly positive, such that the optimal constant policy imposed on borrowers would be a debt tax. The latter accords to welfare-enhancing constant debt taxes in Bianchi (2011).

4.3 An ex-ante Pigouvian tax/subsidy on saving

We now consider a tax/subsidy on saving as a closely related policy instrument, which is however imposed on lenders. Given that borrowers and lenders structurally differ with regard to preferences and constraints, the impact of a tax/subsidy on saving will in general not be equivalent to the impact of a tax/subsidy on debt. Specifically, the analysis will reveal that distributive effects can play an important role for the policy maker's choice under a saving policy. As long as only borrowers were taxed, the interest rate has been exogenous due to the linear utility function of lenders (see Sections 4.1 and 4.2), which relates to the specification in most studies on prudential regulation.

Under an ex-ante tax/subsidy on saving, the interest rate in period 1 can directly be altered by policy, as shown by the lenders' optimal saving decision

$$(1-\tau_{l,1})/r_1 = \beta. \tag{24}$$

Combining (24) with the borrowers' optimality condition (6), gives $1/(1-\tau_{l,1}) = c_{b,1}Ec_{b,2}^{-1}$, implying that borrowers's period-1 non-durables consumption $c_{b,1}$ tends to decrease relative to $c_{b,2}$ with a saving tax, $\tau_{l,1} < 0$. Given that the interest now becomes endogenous (for the policy maker), the relevant price relation is given by

$$r_1 = c_{b,1}^{-1}/(\beta E c_{b,2}^{-1}), \tag{25}$$

while the collateral price satisfies the laissez faire price relation (13), like under the ex-ante debt tax. An ex-ante tax/subsidy on saving can indirectly alter the borrowing limit via the effect of $c_{b,2}$ on the collateral price similar to the ex-ante debt tax, while it can additionally affect the interest rate in a direct way via (24). The social planer can utilize the latter effect and lower the interest rate to address distributive effects induced by the borrowing constraint. In fact, the distributive effects call for a subsidy on saving and the collateral effects for a tax on saving. The sign of the tax/subsidy rate therefore depends on the relative magnitudes of both effects.

Proposition 4 *Suppose that the policy maker can apply a Pigouvian tax/subsidy on saving before the borrowing constraint might be binding. Then, the optimal allocation is associated with a tax/subsidy rate on saving satisfying*

$$\tau_{l,1} = \underbrace{\left\{-b_{b,1}r_1E\left[\mu_2^{tl1}\right]\left(E\left[\frac{\partial\phi_1^b}{\partial c_{b,1}}\right]-r_1E\left[\frac{\partial\phi_1^b}{\partial c_{b,2}}\right]\right)\right\}}_{>0} - \underbrace{\left\{\frac{r_1\beta\gamma d}{(1-\beta\gamma)^{-1}}E\left[\mu_2^{tl1}\chi_c\right]\right\}}_{>0}, \tag{26}$$

where $\partial\phi_1^b/\partial c_{b,1} > 0$, $\partial\phi_1^b/\partial c_{b,2} < 0$, and $\mu_2^{tl1} \geq 0$ denotes the multiplier on the borrowing constraint of the policy problem and $\phi_1^b = \beta(c_{b,1}/c_{b,2})$ the stochastic discount factor.

Proof. See Appendix. ■

The condition for the optimal ex-ante tax/subsidy rate (26) in Proposition 4 reveals that the sign of the tax/subsidy rate depends on two opposing effects: The first term (in curly brackets) on the RHS is strictly positive and summarizes the distributive effects induced by the borrowing constraint that is binding with a positive probability (see Assumption 2).[14] These effects call for a saving subsidy, $\tau_{l,1} > 0$, inducing a lower interest rate. Due to the higher debt price $1/r_1$, borrowers can increase their consumption of non-durables in period-1 relative to period 2 compared to laissez faire (see 12). The second term (in curly brackets) on the RHS is also strictly positive and summarizes the collateral effects, which can be addressed by reducing borrowing via a saving tax, $\tau_{l,1} < 0$, that tends to reduce the supply of debt (like an ex-ante debt tax tends to reduce the demand for debt, see Proposition 2). Evidently, the policy maker applies a saving subsidy, $\tau_{l,1} > 0$, when collateral effects are dominated by distributive effects, which is more likely for higher levels of debt $-b_{b,1}$ (see 26) and this higher inequality.

4.4 A constant Pigouvian tax/subsidy on saving

Now suppose that the tax/subsidy on saving can neither be made contingent on particular periods nor on the state of the economy, such that the tax/subsidy rate is equally imposed in the periods $t = 1$ and $t = 2$. Notably, this policy regime would even be non-equivalent to a constant tax/subsidy on debt if all agents were ex-ante identical. The reason is the asymmetry of agents' problems in period 2 induced by the borrowing constraint, which is apparently irrelevant for saving decisions. Here, the lenders' optimality conditions are characterized by

$$(1 - \tau_l)/r = \beta, \text{ where } r_1 = r_2 = r, \tag{27}$$

instead of (11), implying that the constant saving tax/subsidy alters the interest rate in both periods, 1 and 2. These interest rate effects of the constant saving tax/subsidy further affect the borrowing decisions in period 1 and 2 satisfying

$$c_{b,1}^{-1}/(1 - \tau_l) = E[c_{b,2}^{-1}], \tag{28}$$

$$c_{b,2}^{-1}\beta/(1 - \tau_l) = \beta + \mu_{b,2}. \tag{29}$$

The conditions (28) and (29) indicate that a constant saving subsidy $\tau_l > 0$ tends to raise borrowers' non-durable consumption in period 1 and 2. Simultaneously, it alters

[14] Notably, the multiplier μ_2^{tl1} includes the difference between the marginal utilities of borrowers and lenders (see proof of Proposition 4).

the valuation of the borrowing constraint, measured by the multiplier on the borrowing constraint $\mu_{b,2}$ and thereby the collateral premium ξ_2. Combining (28) and (29) gives $\mu_{b,2} = c_{b,2}^{-1} c_{b,1} \beta E[c_{b,2}^{-1}] - \beta$, which can be used to substitute out the multiplier $\mu_{b,2}$ in (7). Then, the durables price relation differs from the laissez faire version (13) and satisfies (22), like under the constant debt tax/subsidy. Via these changes in the price relations for durables and debt, the policy maker can use a constant tax/subsidy on saving to simultaneously address collateral effects via the durables price q_2 as well as distributive effects via the interest rate r.

Proposition 5 *Suppose that the policy maker can apply a constant Pigouvian tax/subsidy on saving in the periods 1 and 2. Then, the optimal allocation is associated with a tax/subsidy rate on saving satisfying*

$$
\begin{aligned}
\tau_l &= \Delta + \Psi, \quad \textit{with} && (30)\\
\Delta &= \beta \left\{ -b_{b,1} r \beta E\left[\frac{\mu_2^{tl}}{\phi^b}\right] \left(E\left[\frac{\partial \phi^b}{\partial c_{b,1}}\right] - rE\left[\frac{\partial \phi^b}{\partial c_{b,2}}\right] \right) - E\left[b_{b,2} \frac{\mu_2^{tl}}{\phi^b} \left(\frac{\partial \phi^b}{\partial c_{b,1}} - r \frac{\partial \phi^b}{\partial c_{b,2}} \right) \right] \right\} > 0 \\
\Psi &= \beta \gamma d E\left[\mu_2^{tl} \left(\left\{ \chi_\xi \cdot \left(1 + 2r \frac{c_{b,1}}{c_{b,2}} \right) \gamma \beta c_{b,2}^{-2} \right\} - r \chi_c \right) \right],
\end{aligned}
$$

and $\partial \phi^b / \partial c_{b,1} > 0$, $\partial \phi^b / \partial c_{b,2} < 0$, and $\mu_2^{tl} \geq 0$ denotes the multiplier on the borrowing constraint of the policy problem and $\phi^b = \beta(c_{b,1}/c_{b,2})$ the stochastic discount factor. The term Ψ is strictly positive when $\frac{c_{b,1}}{c_{b,2}(H)} > \frac{1-\beta\gamma r^{-1}}{2\beta\gamma}$.

Proof. See Appendix. ■

According to Proposition 5, distributive effects, which are summarized by the term Δ in (30), can be addressed by a constant subsidy on saving, which relates to the findings in Proposition 4. In contrast to an ex-ante saving subsidy, a constant saving subsidy also reduces the interest rate in period 2, where the borrowing constraint might be binding. This additional effect is captured by the last term in the square brackets of Δ. In contrast to the terms referring to the distributive effect, the sign of the term Ψ, which summarizes the collateral effects, is ambiguous and, particularly, depends on the effects on the collateral premium summarized in the curly brackets in Ψ. Given that a higher willingness to borrow increases the valuation of collateral, the collateral effects also call for a saving subsidy if the impact on the collateral premium is sufficiently large. Otherwise, the term Ψ is negative and calls for a saving tax, which tends to increase consumption by reducing debt (see χ_c). For a saving subsidy, the inequality $\frac{c_{b,1}}{c_{b,2}(H)} > \frac{1-\beta\gamma r^{-1}}{2\beta\gamma}$ has to hold, which differs from the corresponding condition for the constant debt tax/subsidy just by r^{-1} replacing β (see Proposition 3). The inequality is likely to be satisfied when borrowing remains constrained even under the optimal policy (such that $c_{b,1}$ exceeds $c_{b,2}(H)$) and for sufficiently high loan-to-value ratios γ.

If the borrowing constraint were however independent of individual holdings of durables (see 17), such that there would be no collateral premium effect on q_2, the term Ψ would be strictly negative. Yet, even in this case the policy maker would apply a saving subsidy if the distributive effects Δ dominate.

4.5 Prices, allocation and welfare

Here, we aim at illustrating the impact of corrective policies on prices, the allocation and social welfare, and the possibility to improve on the (constrained efficient) allocation implemented by an ex-ante debt tax via alternative non-state-contingent policies. We introduce a functional form for $v(d_{b,t}) : v(d_{b,t}) = \varkappa \log d_{b,t}$. We further assign values to model parameters that induce the – admittedly stylized – model to generate meaningful values of targeted variables. Specifically, we normalize y and set it equal to 2, and further set $d/y = 1.5$, $b_{b,0}/y = -0.25$, $\varkappa = 0.1$, and $\beta = 0.9$, leading to a housing-to-GDP ratio, debt-to-income ratios, interest rates, and tax/subsidy rates within reasonable ranges. The benchmark values for the inequality measure δ_H and for the share of seizable collateral γ are 1.1 and 0.8, respectively; the latter relating to commonly applied loan-to-value ratios. We then examine sensitivity of the effects by altering the tightness of the borrowing constraint γ and income inequality δ_H. The solutions for the equilibrium objects under the four non-state-contingent policies summarized in Table 1 and under laissez faire are presented in the Figures 1-3. The first row in all Figures refers to a variation in γ, where an increase in γ reduces the tightness of the borrowing constraint and thereby the strength of the financial friction, which de-emphasizes the collateral effect. The second row in all Figures refers to a variation in δ_H, where an increase in δ_H increases the inequality of agents' non-durables endowment in state H in period 2 and thereby the relevance of the financial friction as well as of distributive effects.

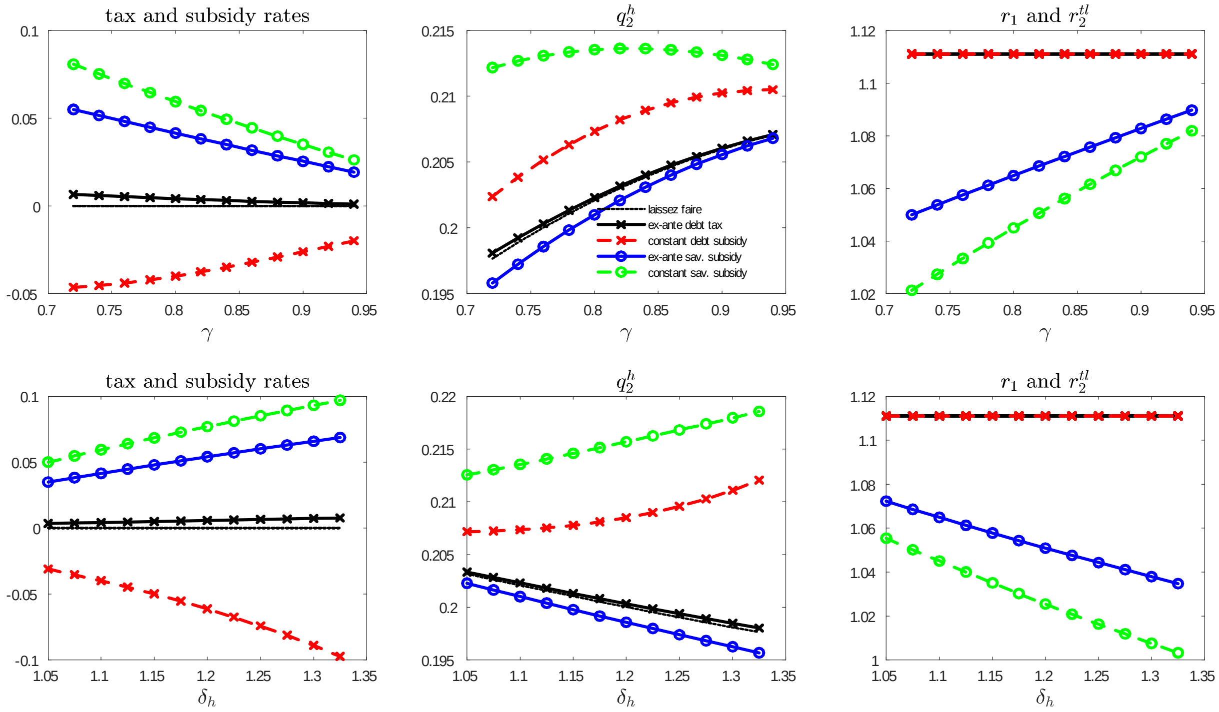

Figure 1: Instruments and prices (benchmark values $\gamma = 0.8$ and $\delta_H = 1.1$)

The first column of Figure 1 shows the tax and subsidy rates under all five regimes. The laissez faire case (black dotted lines) exhibits zero tax/subsidy rates. The first policy regime (solid black lines with crosses) is the optimal ex-ante tax on debt $\tau_{b,1} > 0$ (see Proposition 2), which decreases with γ and increases with δ_H. The second policy regime (red dashed lines with crosses) is the optimal constant subsidy on debt $\tau_b < 0$ (see Proposition 3). The third (blue solid lines with circles) and the fourth regime (green dashed lines with circles) are the optimal ex-ante and the optimal constant saving subsidy, $\tau_{l,1} > 0$ and $\tau_l > 0$, as characterized in Propositions 4 and 5. The second column shows the durables price in period 2, which is slightly increased compared to laissez faire under the ex-ante debt tax. The constant debt subsidy, which tends to raise borrowing in both periods 1 and 2 (see Figure 2), also leads to higher durables prices due to its impact on the collateral value. In contrast, the ex-ante saving subsidy, which raises debt in period 1 and reduces non-durables consumption $c_{b,2}$ in period 2 (see Figure 2), leads to lower durables prices q_2. Simultaneously, it reduces the interest rate in period 1 below its laissez faire value (see third column), such that borrowing funds requires issuance of less debt $b_{b,1}$. The constant saving subsidy leads to the most pronounced increase in the

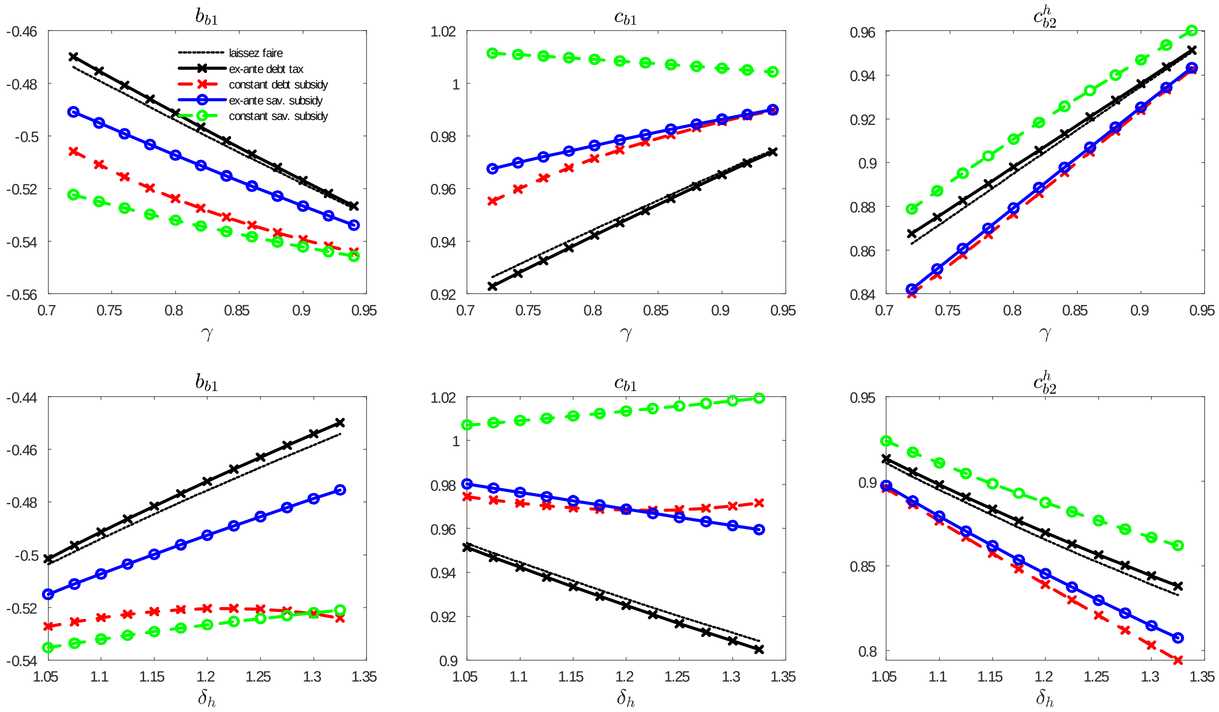

Figure 2: Debt and consumption (benchmark values $\gamma = 0.8$ and $\delta_H = 1.1$)

durables price q_2. It further leads to a reduction in the interest rate r_1 in period 1 that is larger than under the ex-ante saving subsidy and it reduces the interest rate r_2 in period 2. Figure 2 further shows that all three subsidies raise debt and lead to higher levels of non-durables consumption in period 1, which in contrast decreases under the ex-ante debt tax. The constant debt subsidy and the ex-ante saving subsidy reduce consumption $c_{b,2}$ due to a higher debt burden in period 2. The opposite result for $c_{b,2}$ is induced by the ex-ante debt tax and by the constant saving subsidy, which lowers borrowing costs in both periods 1 and 2.

Figure 3 presents the welfare effects of the policy regimes. The welfare measure is based on W (see 4) and expressed in terms of equivalents of borrowers' non-durables consumption in period 1. The first column shows welfare effects of the four policy regimes relative to the laissez faire case. Evidently, the debt policies (ex-ante debt tax and constant debt subsidy) lead to much smaller welfare gains than the saving subsidies. This results is simply due to the fact that the former policies can – by construction – not address distributive effects by changes in the interest rate, in contrast to the latter

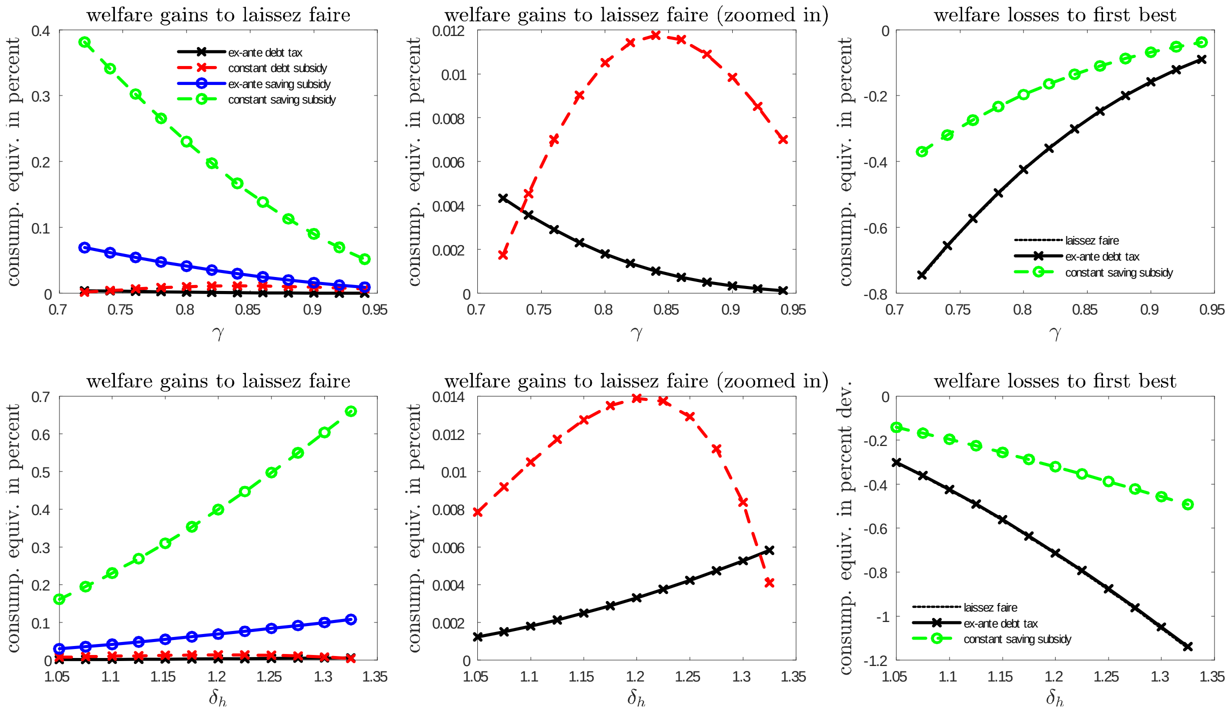

Figure 3: Social welfare (benchmark values $\gamma = 0.8$ and $\delta_H = 1.1$)

policies.[15] The second column of Figure 3 zooms in into the welfare effects of the debt policies, revealing that the ex-ante debt tax leads to the smallest welfare gains under the benchmark parameter values. It further shows that the ex-ante debt tax can principally be superior to the constant debt subsidy for tighter borrowing constraints, i.e. for lower loan-to-value ratios γ, which reduce the positive impact of the constant debt subsidy on the collateral price via the collateral premium, $\xi_2 = \gamma\mu_{b,2}$, and for higher inequality δ_H. The last column of Figure 3 presents welfare losses compared to first best, which can in principle be implemented by an ex-post debt subsidy (see Corollary 1). The values for laissez fare and the ex-ante debt tax can hardly be distinguished, indicating that the total welfare gains of an ex-ante debt tax are negligible relative to first best. In contrast, the constant saving subsidy can substantially reduce the welfare loss in a competitive equilibrium compared to first best. For the benchmark values, it reduces the welfare loss by about a half.

[15]This would in principle be possible under alternative specifications of lenders' utility, for example, logarithmic utility, which is neglected here to keep the exposition transparent using polar cases.

5 A model with capital formation

To assess the robustness of our findings and to facilitate comparisons, we further apply a model with endogenous capital formation, like Bianchi and Mendoza (2018) or Davila and Korinek (2018). Concretely, we use Davila and Korinek's (2018) model applied for collateral externalities and replicate their results on an ex-ante debt policy that implements the constrained efficient allocation. In addition, we examine the other three policy regimes given in Table 1, as in the analyses of the benchmark model (see Section 4).

There is no uncertainty and there are no durable consumption goods in this economy. Agents' lifetime utility satisfies $u_l = c_{l,1} + c_{l,2} + c_{l,3}$ and $u_b = \log c_{b,1} + \log c_{b,2} + c_{b,3}$, which accords to Assumption 1 without durables ($v = 0$) and implies no discounting ($\beta = 1$). Borrowers have access to an investment technology, by which capital $k_{b,2}$ can be installed under convex costs $\alpha k_{b,2}^2/2$ in the first period. Capital can be traded in the second period at the price q^k and remains constant until it fully depreciates at the end of the last period. In the periods 2 and 3, borrowers use their full stock of capital to produce according to the technology $A_t k_{b,t}$ with $t \in (2,3)$. Given that there is no uncertainty and no discounting, lenders' saving decision pins down the debt price at 1 under laissez faire. The borrowing constraint, which corresponds to (3), is given by

$$-b_{b,t} \leq \phi q^k k_{b,t}, \tag{31}$$

with $\phi > 0$. It is binding in period 2, while borrowing is (de facto) unconstrained in period 1. Since lenders have no use for capital, the entire stock of capital is held by borrowers: $k_{b,2} = k$. Under under laissez faire, the borrowers' first order conditions can be written as $1/c_{b,1} = 1/c_{b,2}$, $1/c_{b,2} = 1 + \kappa_{b,2}$, $\alpha k(1/c_{b,1}) = (1/c_{b,2})\left(A_2 + q^k\right)$, and $q^k(1/c_{b,2}) = A_3 + \xi_2^k q^k$, where ξ_2^k denotes the collateral premium, $\xi_2^k = \kappa_{b,2}\phi$, and $\kappa_{b,2} \geq 0$ the multiplier on (31). Substituting out $\kappa_{b,2}$ with $\kappa_{b,2} = (1/c_{b,2}) - 1$, gives the following relation for the price of capital under laissez faire

$$q^k = A_3 \left[\phi + (1-\phi)/c_{b,2}\right]^{-1}, \tag{32}$$

while the stock of capital satisfies $k = \left(A_2 + q^k\right)/\alpha$. Like in the benchmark model, the collateral price q^k is inefficiently low under laissez faire, since agents do not internalize the impact of their decisions on q^k in a competitive equilibrium.

The first best allocation, which maximizes utilitarian social welfare (4) for $\beta = 1$, satisfies $c_{b,1}^{fb} = c_{b,2}^{fb} = 1$ and $k^{fb} = (A_2 + A_3)/\alpha$. It cannot be realized in a laissez faire equilibrium under a binding borrowing constraint, where $\kappa_{b,2} > 0 \Rightarrow c_{b,2} = c_{b,1} < 1$. Yet, a Pigouvian debt subsidy that is only applied in the second period, $\tau_{b,2} < 0$ with (2), solely affects the borrowing condition $(1 - \tau_{b,2})/c_{b,2} = 1 + \kappa_{b,2}$ and can implement the first best allocation by raising the collateral price q^k via the collateral premium according to $q^k = A_3[c_{b,2}^{-1}(1-\phi)+\phi+\tau_{b,2}\cdot\phi c_{b,2}^{-1}]^{-1}$. Hence, a sufficiently large ex-post debt subsidy can raise the borrowing limit such that it is not smaller than debt in a competitive equilibrium under the first best allocation, $-b_{b,2}^{fb}$, which corresponds to Proposition 1.

Proposition 6 *The first best allocation is implemented in a competitive equilibrium under a Pigouvian debt subsidy satisfying* $\tau_{b,2} \leq A_3 k_{b,2}[-b_{b,2}^{fb}]^{-1} - \phi^{-1} < 0$.

Proof. See Appendix. ∎

Next, we again consider the four policies given in Table 1 with compensations satisfying (2), namely, an ex-ante debt tax/subsidy $\tau_{b,1}$, a constant debt tax/subsidy τ_b, an ex-ante saving tax/subsidy $\tau_{l,1}$, and a constant saving tax/subsidy τ_l. Like in Davila and Korinek (2018), the policy maker has further access to a Pigouvian tax/subsidy on capital investment $\tau_{k,1}$, which we consider for all cases.[16] For the first policy regime, we derive the optimal investment policy, confirming the results of Davila and Korinek (2018). For the other policy regimes, we do not further discuss the investment policy, since our focus is on corrective credit market policies. The following proposition summarizes the main results, which closely relate to the results derived in Section 4.

Proposition 7 *Consider the economy with capital formation. Suppose that the policy maker can apply a Pigouvian tax/subsidy on capital investment in the first period and*

1. *a Pigouvian tax/subsidy on debt in the first period. Then, the optimal allocation is constrained efficient and associated with a tax on debt satisfying*

$$\tau_{b,1} = (1 - c_{b,1})\,\phi k\left(\partial q^k/\partial c_{b,2}\right) > 0, \tag{33}$$

[16] Note that there is only one investment decision, which is made in the first period.

where $\partial q^k/\partial c_{b,2} > 0$, and a subsidy on capital investment, $\tau_{k,1} > 0$, iff $A_2 + \phi q^k > 0$.

2. *a constant Pigouvian tax/subsidy on debt. Then, the optimal allocation is associated with a subsidy on debt satisfying*

$$\tau_b = -\mu_2^{tb}[2\phi^2 k c_{b,1}^2/c_{b,2}]\left(\partial q^k/\partial c_{b,1}\right) < 0, \tag{34}$$

where $\partial q^k/\partial c_{b,1} > 0$ and $\mu_2^{tb} > 0$ denotes the multiplier on the borrowing constraint of the policy problem.

3. *a Pigouvian tax/subsidy on saving in the first period. Then, the optimal allocation is associated with a tax/subsidy on saving satisfying*

$$\tau_{l,1} = \underbrace{\left\{-\mu_2^{tl1} b_{b,1} r_1 \left(\frac{\partial(1/r_1)}{\partial c_{b,1}} - r_1 \frac{\partial(1/r_1)}{\partial c_{b,2}}\right)\right\}}_{>0} - \underbrace{\left[r_1 \mu_2^{tl1} \phi k \frac{\partial q^k}{\partial c_{b,2}}\right]}_{>0}, \tag{35}$$

where $\partial q^k/\partial c_{b,2} > 0$, $\partial(1/r_1)/\partial c_{b,1} > 0$, $\partial(1/r_1)/\partial c_{b,2} < 0$, and $\mu_2^{tl1} > 0$ denotes the multiplier on the borrowing constraint of the policy problem.

4. *a constant Pigouvian tax/subsidy on saving. Then, the optimal allocation is associated with a tax/subsidy on saving satisfying*

$$\tau_l = \underbrace{\left\{-\mu_2^{tl}\left(b_{b,1} + \frac{b_{b,2}}{r}\right)\left(\frac{\partial(1/r)}{\partial c_{b,1}} - r\frac{\partial(1/r)}{\partial c_{b,2}}\right)\right\}}_{>0} + \left[\mu_2^{tl}\frac{\phi k}{r^2}\frac{\partial q^k}{\partial c_{b,1}}(2\phi + 1 - r)\right], \tag{36}$$

where $r = r_1 = r_2$, $\partial(1/r)/\partial c_{b,1} > 0$, $\partial(1/r)/\partial c_{b,2} < 0$, $\partial q^k/\partial c_{b,1}$ and $\mu_2^{tl} > 0$ denotes the multiplier on the borrowing constraint of the policy problem.

Proof. See Appendix. ■

Under the first regime (see part 1 of Proposition 7), the policy maker applies an ex-ante tax on debt $\tau_{b,1} > 0$ (see 33) and further imposes an investment subsidy for a sufficiently high productivity level, $A_2 + \phi q^k > 0$, which replicates Davila and Korinek's (2018) results for the constrained efficient allocation. Like in our benchmark model (see Proposition 2), an ex-ante debt tax leaves the laissez faire price relation (32) unaffected, where the collateral value satisfies $\xi_2^k = \phi(c_{b,2}^{-1} - 1)$. The collateral price q^k is raised by reducing borrowing ex-ante, which tends to increase funds available for consumption when the borrowing constraint binds. This rationalizes Davila and Korinek's (2018) claim that "collateral externalities cause overborrowing" in this model.

Under the <u>second</u> regime, the policy maker applies a constant debt subsidy $\tau_b < 0$ (see 34). In contrast to the corresponding case in our benchmark model (see Proposition 3), the potential trade off between raising the collateral premium on capital by stimulating borrowing (like under an ex-post debt subsidy) and raising the collateral price via higher consumption (like under an ex-ante debt tax) is here unambiguously solved in favor of a debt subsidy (see part 2 of Proposition 7). Specifically, the collateral premium ξ_2^k satisfies $\xi_2^k = \phi(c_{b,1}c_{b,2}^{-2} - 1)$ under a constant debt tax/subsidy, such that the price relation for q^k is given by $q^k = A_3[(1/c_{b,2}) - \phi(c_{b,1}c_{b,2}^{-2} - 1)]^{-1}$ (instead of 32). A constant debt subsidy tends to raise consumption in period 1 relative to period 2, which increases the collateral premium ξ_2^k and the collateral price q^k.

For the remaining cases, we consider taxes/subsidies imposed on lenders. Saving taxes/subsidies can cause the interest rate(s) to deviate from one, which can be used to address distributive effects of pecuniary externalities with regard to the interest rate. Like in the benchmark model (see Propositions 4 and 5), addressing distributive effects calls for an interest rate reduction by subsidizing saving, which is revealed by the positive terms in the curly brackets in (35) and (36) that increase with debt ($-b_{b,1}$ and $-b_{b,2}$). For the <u>third</u> policy regime, which is an ex-ante tax/subsidy on saving, the last term in (35) implies a policy trade-off, since collateral effects (in square brackets) call for a saving tax to raise consumption $c_{b,2}$ by a reduction of debt. Under the <u>fourth</u> policy regime, which is a constant tax/subsidy on saving, the policy choice implication of collateral effects – given by the term in square brackets in (36) – are ambiguous, since a saving subsidy tends to raise the collateral price via the collateral premium, whereas it tends to reduce funds available for consumption in $t = 2$ by increasing debt. Under a sufficiently large loan-to-value ratio, $\phi > (r-1)/2$, the former effect prevails, such that the collateral effect also calls for a saving subsidy (see also Proposition 5).

These results imply that agents do not overborrow in general, since the last three regimes may stimulate borrowing. However, optimal policy would unambiguously reduce debt, if the borrowing constraint were independent of individual asset position, i.e. $-b_{b,2} \leq \phi q^k k$ (which corresponds to 17), and distributive effects are disregarded. Likewise, implementation of first best by ex-post debt subsidies would then not be possible.

6 Conclusion

This paper derives optimal credit market policies in two incomplete market models with pecuniary externalities under collateral constraints. Collateral effects of pecuniary externalities can be addressed by a Pigouvian ex-ante debt tax, implementing the constrained efficient allocation. In contrast to the majority of studies on macroprudential regulation, we consider agents' assets as collateral and endogenous interest rates, giving rise assets' collateral premia and distributive effects. We show that both are responsible for ex-post debt subsidies to be able to implement first best, for debt subsidies that are constant over time to outperform debt taxes, and for saving subsidies to enhance efficiency by reducing interest rates. Overall, we find that credit market policies that reduce interest rates and stimulate collateral premia by subsidizing debt or saving can outperform ex-ante debt taxes. The results imply that overborrowing unambiguously prevails only if collateral premia and distributive effects are neglected or if the analysis is restricted to ex-ante policies imposed on borrowers. Thus, our analysis shows that borrowing constraints that give rise to collateral effects are in general not sufficient to rationalize prudential regulation as an optimal credit market policy.

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8 Appendix

Proof of Proposition 1. In state H, the ex-post debt subsidy can be applied to raise the borrowing limit to a sufficiently high value such that the borrowing is unconstrained in a competitive equilibrium under the first best allocation, $-b_{b,2}^{fb}(H) \leq \gamma q_2 d$, where $b_{b,2}^{fb}(H) = \beta^{-2}(b_{b,0} + y_{b,1} - c_{b,1}^{fb}) + \beta^{-1}(y_{b,2}(H) - c_{b,2}^{fb})$. The price relation (16) implies that this requires the subsidy rate $\tau_{b,2}(H)$ to satisfy $\tau_{b,2}(H) \leq \left(1 + \beta^{-1}\right) dv_d(d)[-b_{b,2}^{fb}(H)]^{-1} - (\beta\gamma)^{-1} < 0$. $\blacksquare$

Proof of Proposition 2. Using $c_{l,3} = -b_{b,2} + y_{l,3}$, $c_{l,2} - b_{b,2}/r_2 = -b_{b,1} + y_{l,2}$, $c_{l,1} - b_{b,1}/r_1 = b_{l,0} + y_{l,1}$, $c_{b,3} = b_{b,2} + y_{b,3}$ and $d_{b,t} = d$, the objective (4) can by be written as

$$\begin{aligned} W = &\log c_{b,1} + v(d) + (b_{l,0} + y_{l,1}) + \beta \left[\log c_{b,2} + v(d) + (y_{l,2} + b_{b,2}\beta)\right] \\ &+\beta^2 \left[y_{b,3} + v(d) + y_{l,3}\right]. \end{aligned} \tag{37}$$

The primal problem of a policy maker who applies an ex-ante tax/subsidy on debt $\tau_{b.1}$ and a compensating lump-sum transfer/tax $T_{b,t} = -\tau_{b.1} b_{b,t}/r_t$ is identical to the problem of a social planer who determines period-1-borrowing, such that (6) does not apply, and maximizes social welfare W subject to budget and borrowing constraints taking the equilibrium price relations (11) and (13) under laissez faire into account, leading to a constrained efficient allocation. It can be summarized as

$$\begin{aligned} \max_{c_{b,1}, c_{b,2}, b_{b,1}, b_{b,2}} \; & E\{\log c_{b,1} + v(d) + (b_{l,0} + y_{l,1}) + \beta \left[\log c_{b,2} + v(d) + (y_{l,2} + b_{b,2}\beta)\right] \\ & + \beta^2 \left[y_{b,3} + v(d) + y_{l,3}\right]\} \end{aligned} \tag{38}$$

$$\text{s.t. } 0 = b_{b,0} + y_{b,1} - c_{b,1} - b_{b,1}\beta, \quad 0 = b_{b,1} + y_{b,2} - c_{b,2} - b_{b,2}\beta, \quad 0 \leq \gamma q_2(c_{b,2})d + b_{b,2},$$

where $q_2(c_{b,2})$ satisfies (13), leading to the optimality conditions

$$\lambda_{b,1}^{tb1} = 1/c_{b,1}, \tag{39}$$

$$\lambda_{b,2}^{tb1} = (1/c_{b,2}) + \mu_2^{tb1} \gamma d \partial q_2(c_{b,2})/\partial c_{b,2}, \tag{40}$$

$$\lambda_{b,1}^{tb1} = E\lambda_{b,2}^{tb1}, \tag{41}$$

$$\mu_2^{tb1} = \beta(\lambda_{b,2}^{tb1} - 1) \geq 0, \tag{42}$$

where $\lambda_{b,1}^{tb1}$, $\lambda_{b,2}^{tb1}$, and μ_2^{tb1} are the multipliers for the constraints in order of their appearance in (38). Applying expectations conditional on period-1-information and substituting out the multipliers $\lambda_{b,1}^{tb1}$ and $\lambda_{b,2}^{tb1}$ in (39)-(41) leads to

$$c_{b,1}^{-1} = Ec_{b,2}^{-1} + E\left[\mu_2^{tb1}\gamma d\partial q_2(c_{b,2})/\partial c_{b,2}\right].$$

Combining the latter with the optimality condition (18) and $1/r_1 = \beta$, gives the following condition for the tax rate on debt

$$\tau_{b,1} = c_{b,1}E\left[\mu_2^{tb1}\gamma d(1-\beta\gamma)\chi_c\right] > 0,$$

where we used that $\partial q_2(c_{b,2})/\partial c_{b,2} = (1-\beta\gamma)\chi_c > 0$ and that $\mu_2^{tb1} = \beta(c_{b,1}^{-1} - 1) > 0$ further holds (see 39 and 42). ■

Proof of Proposition 3. For the formulation of the planer's primal problem under commitment in Lagrangian form we define $\phi_2^d(c_{b,1}, c_{b,2}) = \frac{v_d(d)(1+\beta)}{c_{b,2}^{-1} - (c_{b,1}c_{b,2}^{-2} - 1)\beta\gamma}$ and use (37)

$$\begin{aligned}L = {} & E\left\{\log c_{b,1} + v(d) + (b_{l,0} + y_{l,1}) + \beta\left[\log c_{b,2} + v(d) + (y_{l,2} + b_{b,2}\beta)\right]\right. \\ & +\beta^2\left[y_{b,3} + v(d) + y_{l,3}\right] + \lambda_{b,1}^{tb}\left[b_{b,0} + y_{b,1} - c_{b,1} - b_{b,1}\beta\right] \\ & +\beta\lambda_{b,2}^{tb}\left[b_{b,1} + y_{b,2} - c_{b,2} - b_{b,2}\beta\right] + \beta\mu_2^{tb}[\gamma\phi_2^d(c_{b,1}, c_{b,2})d + b_{b,2}],\end{aligned}$$

leading to the optimality conditions

$$\lambda_{b,1}^{tb} = c_{b,1}^{-1} + \beta E[\mu_2^{tb}\gamma d \cdot \partial\phi_2^d/\partial c_{b,1}], \tag{43}$$

$$\lambda_{b,1}^{sp} = E\lambda_{b,2}^{tb}, \tag{44}$$

$$\lambda_{b,2}^{tb} = c_{b,2}^{-1} + \mu_2^{tb}\gamma d \cdot \partial\phi_2^d/\partial c_{b,2}, \tag{45}$$

$$\mu_2^{tb} = \beta(\lambda_{b,2}^{tb} - 1) \geq 0. \tag{46}$$

Taking expectations and substituting out the multipliers $\lambda_{b,1}^{tb}$ and $\lambda_{b,2}^{tb}$ in (43)-(45) gives

$$c_{b,1}^{-1} - Ec_{b,2}^{-1} = \gamma dE\left[\mu_2^{tb}\left((\partial\phi_2^d/\partial c_{b,2}) - \beta(\partial\phi_2^d/\partial c_{b,1})\right)\right].$$

Combining with $E\frac{1}{c_{b,2}} = (1-\tau_b)\frac{1}{c_{b,1}}$, which follows from (20) and $1/r_1 = \beta$, we get a condition for the optimal constant tax/subsidy rate

$$\tau_b = c_{b,1}\gamma dE\left[\mu_2^{tb}\left((\partial\phi_2^d/\partial c_{b,2}) - \beta(\partial\phi_2^d/\partial c_{b,1})\right)\right], \tag{47}$$

where the multiplier μ_2^{tb} satisfies $\mu_2^{tb} = \beta(c_{b,2}^{-1} - 1)/(1 - \beta\gamma d\partial\phi_2^d/\partial c_{b,2}) \geq 0$ (see 45 and 46). From (10), we now that $\partial\phi_2^d/\partial c_{b,1} = \chi_\xi\gamma\frac{\beta}{c_{b,2}^2}$ and $\partial\phi_2^d/\partial c_{b,2} = \chi_c - \chi_\xi 2\gamma\beta\frac{c_{b,1}}{c_{b,2}^3}$, such that condition (47) can be rewritten as

$$\tau_b = c_{b,1}\gamma dE\left[\mu_2^{tb}\left(\chi_c - \left\{\chi_\xi\frac{\gamma\beta}{c_{b,2}^2}\left(2\frac{c_{b,1}}{c_{b,2}} + \beta\right)\right\}\right)\right], \tag{48}$$

where $\chi_c > 0$ and $\chi_\xi > 0$. Applying $\chi_\xi = c_{b,2}^2\chi_c$ (see 10) to rewrite (48) as $\tau_b = c_{b,1}\gamma dE\left[\mu_2^{tb}\left(\chi_c\left(1 - \gamma\beta\left(2c_{b,1}c_{b,2}^{-1} + \beta\right)\right)\right)\right]$, shows that $\tau_b < 0$ when $\frac{c_{b,1}}{c_{b,2}(H)} > \frac{1-\beta^2\gamma}{2\beta\gamma}$. ■

Proof of Proposition 4. For the formulation of the planer's primal problem under commitment in Lagrangian form, we define $\phi_1^b(c_{b,1}, c_{b,2}) = \beta(c_{b,1}/c_{b,2})$ and use the goods market clearing conditions:

$$\begin{aligned}
L = E\{&\log c_{b,1} + v(d) + (y - c_{b,1}) + \beta\left[\log c_{b,2} + v(d) + (y - c_{b,2})\right] + \beta^2\left[y + v(d)\right] \\
&+\lambda_{b,1}^{tl1}\left[b_{b,0} + y_{b,1} - c_{b,1} - b_{b,1}\phi_1^b(c_{b,1}, c_{b,2})\right] + \beta\lambda_{b,2}^{tl1}\left[b_{b,1} + y_{b,2} - c_{b,2} - b_{b,2}\beta\right] \\
&+\beta\mu_2^{tl1}[\gamma q_2(c_{b,2})d + b_{b,2}]\},
\end{aligned}$$

where $q_2(c_{b,2})$ satisfies (13), leading to the optimality conditions

$$\lambda_{b,1}^{tl1} = \left(c_{b,1}^{-1} - 1\right) / \left(1 + b_{b,1}E\left[\partial\phi_1^b/\partial c_{b,1}\right]\right), \tag{49}$$

$$\lambda_{b,1}^{tl1} = r_1\beta E\lambda_{b,2}^{tl1}, \tag{50}$$

$$\beta\lambda_{b,2}^{tl1} = \beta\left(c_{b,2}^{-1} - 1\right) - \lambda_{b,1}^{tl1}b_{b,1}\frac{\partial\phi_1^b}{\partial c_{b,2}} + \beta\mu_2^{tl1}\gamma\frac{\partial q_2}{\partial c_{b,2}}d, \tag{51}$$

$$\mu_2^{tl1} = \beta\lambda_{b,2}^{tl1} \geq 0. \tag{52}$$

Applying expectations and substituting out the multipliers $\lambda_{b,1}^{tl1}$ and $\lambda_{b,2}^{tl1}$ in (49)-(51), gives

$$r_1\frac{1 + b_{b,1}E\left[\partial\phi_1^b/\partial c_{b,1}\right]}{1 + r_1 b_{b,1}E\left[\partial\phi_1^b/\partial c_{b,2}\right]} = \frac{c_{b,1}^{-1} - 1}{\beta(Ec_{b,2}^{-1} - 1) + \beta\gamma dE\left[\mu_2^{tl1}\partial q_2/\partial c_{b,2}\right]}.$$

Rewriting the latter with (24) and (25) and using $\frac{\partial q_2(c_{b,2})}{\partial c_{b,2}} = (1 - \beta\gamma)\chi_c$ (see 10), leads to the following condition for the ex-ante tax/subsidy rate on saving

$$\tau_{l,1} = -b_{b,1} r_1 E\left[\mu_2^{tl1}\right]\left(E\left[\partial\phi_1^b/\partial c_{b,1}\right] - r_1 E\left[\partial\phi_1^b/\partial c_{b,2}\right]\right) - r_1\beta\gamma d E\left[\mu_2^{tl1}(1-\beta\gamma)\chi_c\right],$$

where $\partial\phi^b/\partial c_{b,1} = \beta/c_{b,2} > 0$ and $\partial\phi^b/\partial c_{b,2} = -\beta(c_{b,1}c_{b,2}^{-2}) < 0$. Combining (49), (51), and (52), shows that μ_2^{tl1} satisfies $E\mu_2^{tl1} = r_1^{-1}(c_{b,1}^{-1} - 1)/(1 + b_{b,1}E[\partial\phi_1^b/\partial c_{b,1}])$. ■

Proof of Proposition 5. For the formulation of the planer's primal problem under commitment in Lagrangian form, we define $\phi^b(c_{b,1}, c_{b,2}) = \beta\left(c_{b,1}/c_{b,2}\right)$ and $\phi_2^d(c_{b,1}, c_{b,2}) = \frac{v_d(d)(1+\beta)}{c_{b,2}^{-1} - (c_{b,1}c_{b,2}^{-2} - 1)\beta\gamma}$, and use the goods market clearing conditions to rewrite the welfare function, for convenience:

$$\begin{aligned}
L = & E\{\log c_{b,1} + v(d) + (y - c_{b,1}) + \beta\left[\log c_{b,2} + v(d) + (y - c_{b,2})\right] + \beta^2\left[y + v(d)\right] \\
& + \lambda_{b,1}^{tl}\left[b_{b,0} + y_{b,1} - c_{b,1} - b_{b,1}\phi^b(c_{b,1}, c_{b,2})\right] \\
& + \beta\lambda_{b,2}^{tl}\left[b_{b,1} + y_{b,2} - c_{b,2} - b_{b,2}\phi^b(c_{b,1}, c_{b,2})\right] + \beta\mu_2^{tl}[\gamma\phi_2^d(c_{b,1}, c_{b,2})d + b_{b,2}]\},
\end{aligned}$$

leading to the first order conditions

$$\begin{aligned}
& \lambda_{b,1}^{tl}\left(1 + b_{b,1}E\left[\partial\phi^b/\partial c_{b,1}\right]\right) && (53)\\
& = \left(c_{b,1}^{-1} - 1\right) - \beta E\left[\lambda_{b,2}^{tl} b_{b,2}\left(\partial\phi^b/\partial c_{b,1}\right)\right] + \beta E\left[\mu_2^{tl}\gamma d\left(\partial\phi_2^d/\partial c_{b,1}\right)\right], \\
& \beta\lambda_{b,2}^{tl}\left(1 + b_{b,2}\left(\partial\phi^b/\partial c_{b,2}\right)\right) && (54)\\
& = \beta\left(c_{b,2}^{-1} - 1\right) - \lambda_{b,1}^{tl} b_{b,1}\left(\partial\phi^b/\partial c_{b,2}\right) + \beta\left[\mu_2^{tl}\gamma d\left(\partial\phi_2^d/\partial c_{b,2}\right)\right], \\
& \lambda_{b,1}^{tl} = r\beta E\lambda_{b,2}^{tl}, && (55)\\
& \mu_2^{tl} = \phi^b\lambda_{b,2}^{tl} \geq 0, && (56)
\end{aligned}$$

where we used $E\phi^b(c_{b,1}, c_{b,2}) = 1/r$. Taking expectations and substituting out the multipliers $\lambda_{b,1}^{tl}$ and $\lambda_{b,2}^{tl}$ in (53)-(55), leads to

$$\begin{aligned}
& E\left[\mu_2^{tl}/\phi^b\right]\left(1 + b_{b,1}E\left[\partial\phi^b/\partial c_{b,1}\right]\right) - E\left[\mu_2^{tl}/\phi^b\right]\left(1 + rb_{b,1}E\left[\partial\phi^b/\partial c_{b,2}\right]\right) \\
= & \frac{1}{r\beta}\left(c_{b,1}^{-1} - 1\right) - E\left(c_{b,2}^{-1} - 1\right) - \frac{1}{r}E\left[\left(\mu_2^{tl}/\phi^b\right) b_{b,2}\left(\partial\phi^b/\partial c_{b,1}\right)\right] + \frac{1}{r}\gamma d E\left[\mu_2^{tl}\left(\partial\phi_2^d/\partial c_{b,1}\right)\right] \\
& + E\left[\left(\mu_2^{tl}/\phi^b\right) b_{b,2}\left(\partial\phi^b/\partial c_{b,2}\right)\right] - \gamma d E\left[\mu_2^{tl}\left(\partial\phi_2^d/\partial c_{b,2}\right)\right],
\end{aligned}$$

and by applying (27), to the following condition for the constant tax/subsidy rate

$$\tau_l = (-b_{b,1})\, r\beta E\left[\mu_2^{tl}/\phi^b\right]\left(E\left[\partial\phi^b/\partial c_{b,1}\right] - rE\left[\partial\phi^b/\partial c_{b,2}\right]\right) \qquad (57)$$
$$+\beta E\left[(-b_{b,2})\left(\mu_2^{tl}/\phi^b\right)\left((\partial\phi^b/\partial c_{b,1}) - r(\partial\phi^b/\partial c_{b,2})\right)\right] + \Psi,$$

where $\partial\phi_1^b/\partial c_{b,1} > 0$, $\partial\phi_1^b/\partial c_{b,2} < 0$, and $\Psi = \beta\gamma dE[\mu_2^{tl}\left(\left(\partial\phi_2^d/\partial c_{b,1}\right) - r\partial\phi_2^d/\partial c_{b,2}\right)]$. The term Ψ on the RHS of (57) can by using $\frac{\partial\phi_2^d}{\partial c_{b,1}} = \chi_\xi \frac{\gamma\beta}{c_{b,2}^2}$ and $\frac{\partial\phi_2^d}{\partial c_{b,2}} = \chi_c - \chi_\xi 2\gamma\beta\frac{c_{b,1}}{c_{b,2}^3}$ (see 10) be rewritten as

$$\Psi = \beta\gamma dE\left[\mu_2^{tl}\left(\left\{\chi_\xi\frac{\gamma\beta}{c_{b,2}^2}\left(1 + 2r\frac{c_{b,1}}{c_{b,2}}\right)\right\} - r\chi_c\right)\right], \qquad (58)$$

Further applying $\chi_\xi = c_{b,2}^2\chi_c$ (see 10) to rewrite (58) as $\Psi = \beta\gamma drE[\mu_2^{tl}(\chi_c\{\gamma\beta(r^{-1} + 2c_{b,1}c_{b,2}^{-1}) - 1\})]$, shows that $\Psi > 0$ when $\frac{c_{b,1}}{c_{b,2}(H)} > \frac{1-\beta\gamma r^{-1}}{2\beta\gamma}$. ■

Proof of Proposition 6. Consider the economy with capital formation. Under a Pigouvian debt subsidy in period 2, the capital price satisfies $q^k = A_3[c_{b,2}^{-1}(1-\phi) + \phi + \tau_{b,2}\cdot\phi c_{b,2}^{-1}]^{-1}$. Thus, the collateral constraint is slack under the first best allocation, $-b_{b,2}^{fb} \leq \phi q^k k^{fb}$, if the subsidy rate satisfies $\tau_{b,2} \leq A_3 k_{b,2}[-b_{b,2}^{fb}]^{-1} - \phi^{-1}$, where we used $c_{b,1}^{fb} = c_{b,2}^{fb} = 1$ and $k^{fb} = (A_2 + A_3)/\alpha$ and $b_{b,2}^{fb}$ is given by $b_{b,2}^{fb} = y_{b,2} + y_{b,1} - 2 - (A_2 + A_3)^2/(\alpha 2) + A_2(A_2 + A_3)/\alpha$. ■

Proof of Proposition 7. Consider the economy with capital formation. In equilibrium, where capital is entirely held by borrowers, the budget constraints can be written as $c_{b,1} + \alpha k^2/2 + b_{b,1}/r_1 = y_{b,1}$, $c_{b,2} + b_{b,2}/r_2 = y_{b,2} + b_{b,1} + A_2 k$, $c_{b,3} = y_{b,3} + b_{b,2} + A_3 k$, $c_{l,1} + b_{l,1}/r_1 = y_{l,1}$, $c_{l,2} + b_{l,2}/r_2 = y_{l,2} + b_{l,1}$, and $c_{l,3} = y_{l,3} + b_{l,2}$. The social welfare function (4) can thus for $\beta = 1$ be rewritten as $W = \log c_{b,1} + y_{l,1} + [\log c_{b,2} + y_{l,2}] + [y_{b,3} + b_{b,2} + A_3 k + y_{l,3}]$.

To establish the claims made in the first part of the proposition, consider that the policy maker introduces an ex-ante capital investment tax/subsidy $\tau_{k,1}$ and an ex-ante debt tax/subsidy $\tau_{b,1}$, which are fully compensated (ex-post) by type-specific lump-sum transfers (like 2). The borrowers' optimality conditions then satisfy

$$(1-\tau_{b,1}) = c_{b,1}/c_{b,2}, \tag{59}$$

$$(1-\tau_{k,1})\alpha k\,(1/c_{b,1}) = (1/c_{b,2})\left(A_2+q^k\right). \tag{60}$$

The primal policy problem of the policy maker is then identical to the problem of a social planer who determines period-1-borrowing as well as the capital investment decision and maximizes social welfare W subject to budget and borrowing constraints taking the equilibrium price relations (32) under laissez faire into account, leading to a constrained efficient allocation. The problem can be summarized as $\max W$ w.r.t. $c_{b,1}, c_{b,2}, b_{b,1}, b_{b,2}$, and k subject to $c_{b,1}+\alpha k^2/2+b_{b,1} = y_{b,1}$, $c_{b,2}+b_{b,2} = y_{b,2}+b_{b,1}+A_2k$, and $b_{b,2}+\phi q^k(c_{b,2})k \geq 0$, where $q^k(c_{b,2})$ satisfies (32) and thus $\partial q^k/\partial c_{b,2} > 0$. The Lagrangian can be written as

$$\begin{aligned}L = &\log c_{b,1} + y_{l,1} + [\log c_{b,2} + y_{l,2}] + [y_{b,3} + b_{b,2} + A_3k + y_{l,3}]\\ &+\lambda_1^{t1}\left[y_{b,1} - c_{b,1} - \alpha k^2/2 - b_{b,1}\right] + \lambda_2^{t1}\left[y_{b,2} + b_{b,1} + A_2k - c_{b,2} - b_{b,2}\right]\\ &+\mu_2^{tb1}\left[b_{b,2} + \phi q^k(c_{b,2})k\right],\end{aligned}$$

leading to the first order conditions for $c_{b,1}, c_{b,2}, b_{b,1}, b_{b,2}$, and k

$$\lambda_1^{t1} = 1/c_{b,1},\ \ \lambda_2^{t1} = (1/c_{b,2}) + \mu_2^{tb1}\phi k\partial q^k/\partial c_{b,2},\ \ \lambda_1^{t1} = \lambda_2^{t1}, \tag{61}$$

$$\mu_2^{tb1} = \lambda_2^{t1} - 1, \tag{62}$$

$$\lambda_1^{t1}\alpha k = A_3 + \lambda_2^{t1}A_2 + \mu_2^{tb1}\phi q^k(c_{b,2}). \tag{63}$$

Substituting out the multipliers λ_1^{t1} and λ_2^{t1} using the three conditions in (61), gives $(1/c_{b,1}) = (1/c_{b,2}) + \mu_2^{tb1}\phi k\partial q^k/\partial c_{b,2}$. Using (59) to substitute out $1/c_{b,2}$ in the latter, leads to the following condition for the ex-ante debt tax/subsidy rate $\tau_{b,1}$:

$$\tau_{b,1} = \mu_2^{tb1}c_{b,1}\phi k\partial q^k/\partial c_{b,2} > 0,$$

where $\mu_2^{tb1} = (c_{b,1}^{-1} - 1) > 0$. Further substituting out the multipliers with $\lambda_1^{t1} = \lambda_2^{t1} = 1/c_{b,1}$ and $\mu_2^{tb1} = 1/c_{b,1} - 1$ in (63), gives $\alpha k\,(1/c_{b,1}) = A_3 + (1/c_{b,1})\,A_2 + (1/c_{b,1} - 1)\,\phi q^k$. Comparing the latter with (60) after rewriting it with the capital trading decision $q^k(1/c_{b,2}) = A_3 + \kappa_{b,2}\phi q^k$ as $(1+\tau_{k,1})\alpha k\,(1/c_{b,1}) = \left((1/c_{b,2})\,A_2 + A_3 + ((1/c_{b,2}) - 1)\,\phi q^k\right)$, shows that the ex-ante capital tax/subsidy rate satisfies

$$\tau_{k,1} = -\frac{\{(1/c_{b,1}) - (1/c_{b,2})\}\left[A_2 + \phi q^k\right]}{A_3 + (1/c_{b,1})\,A_2 + \mu_2^{tb1}\phi q^k}.$$

Since $(1/c_{b,1}) = (1/c_{b,2}) + \mu_2^{tb1}\phi k \partial q^k/\partial c_{b,2}$ implies $1/c_{b,1} > 1/c_{b,2}$, the policy maker subsidizes capital $\tau_{k,1} < 0$ iff $A_2 + \phi q^k > 0$. This establishes the claims made in the first part of the proposition.

For the second part of the proposition, we consider a constant debt tax/subsidy and an ex-ante capital investment tax/subsidy, which are fully compensated (ex-post) by lump-sum transfers. Agents' borrowing and investment decisions then satisfy

$$(1-\tau_b)/c_{b,1} = 1/c_{b,2}, \tag{64}$$

$$(1-\tau_b)/c_{b,2} = 1 + \kappa_{b,2}, \tag{65}$$

and (60). Substituting out $\kappa_{b,2}$ in the capital trading condition $q^k(1/c_{b,2}) = A_3 + \kappa_{b,2}\phi q^k$ with (65) and then the tax/subsidy rate τ_b with (64), gives the price relation

$$q^k = \frac{A_3 c_{b,2}}{1 - (c_{b,1}/c_{b,2})\,\phi + c_{b,2}\phi}\ , \tag{66}$$

implying that q^k relates to $c_{b,1}$ and $c_{b,2}$ by $\partial q^k/\partial c_{b,1} = \phi A_3 c_1^2 \left(\phi c_{b,1} - c_{b,2} - \phi c_1^2\right)^{-2} > 0$ and $\partial q^k/\partial c_{b,2} = (1 - 2\phi c_{b,1}/c_{b,2})\,\partial q^k/\partial c_{b,1}$. The Lagrangian of the planer's problem can be written as

$$\begin{aligned} L = {} & \log c_{b,1} + y_{l,1} + [\log c_{b,2} + y_{l,2}] + [y_{b,3} + b_{b,2} + A_3 k + y_{l,3}] \\ & + \lambda_1^{tb}\left[y_{b,1} - c_{b,1} - \alpha k^2/2 - b_{b,1}\right] \\ & + \lambda_2^{tb}\left[y_{b,2} + b_{b,1} + A_2 k - c_{b,2} - b_{b,2}\right] + \mu_2^{tb}[b_{b,2} + \phi q^k(c_{b,1}, c_{b,2})k], \end{aligned}$$

where $q^k(c_{b,1}, c_{b,2})$ satisfies (66). The first order conditions for $c_{b,1}, c_{b,2}, b_{b,1}, b_{b,2}$, and k are

$$\lambda_1^{tb} = 1/c_{b,1} + \mu_2^{tb}\phi k \partial q^k/\partial c_{b,1}, \quad \lambda_2^{tb} = 1/c_{b,2} + \mu_2^{tb}\phi k \partial q^k/\partial c_{b,2}, \quad \lambda_1^{tb} = \lambda_2^{tb}, \tag{67}$$

$$\mu_2^{tb} = \lambda_2^{tb} - 1, \tag{68}$$

$$\lambda_1^{tb}\alpha k = A_3 + \lambda_2^{tb} A_2 + \mu_2^{tb}\phi q^k(c_{b,1}, c_{b,2}). \tag{69}$$

Substituting out the multipliers λ_1^{tb} and λ_2^{tb} using the first three conditions in (67),

$(1/c_{b,1})+\mu_2^{tb}\phi k\partial q^k/\partial c_{b,1} = (1/c_{b,2})+\mu_2^{tb}\phi k\partial q^k/\partial c_{b,2}$, and substituting out $1/c_{b,2}$ with (64), gives the following condition for the debt tax/subsidy rate τ_b : $\tau_b = \mu_2^{tb}c_{b,1}\phi k[(\partial q^k/\partial c_{b,2})-(\partial q^k/\partial c_{b,1})]$. Using that the capital price q^k satisfies $\partial q^k/\partial c_{b,2} = (1-2\phi c_{b,1}/c_{b,2})\,\partial q^k/\partial c_{b,1}$ and $\partial q^k/\partial c_{b,1} > 0$ (see 66), the latter can be rewritten as

$$\tau_b = -\mu_2^{tb}c_{b,1}\phi k\left(2\phi c_{b,1}/c_{b,2}\right)\left(\partial q^k/\partial c_{b,1}\right) < 0.$$

For the third part of the proposition, we consider an ex-ante tax/subsidy on saving and an ex-ante capital investment tax/subsidy, which are fully compensated (ex-post) by lump-sum transfers (see 2). Agents' saving and investment decisions then satisfy

$$(1-\tau_{l,1})/r_1 = 1, \tag{70}$$

and (60). Given that (70) endogenizes the interest rate for the policy maker, $1/r_1 = c_{b,1}/c_{b,2}$ is a relevant restriction to the policy problem. Using the resource constraints to substitute out $c_{l,t}$, the Lagrangian of the planer's problem can be written as

$$\begin{aligned} L = &\log c_{b,1} + (y_1 - c_{b,1}) + [\log c_{b,2} + (y_2 - c_{b,2})] + [y_3 + A_3 k] \\ &+\lambda_1^{tl1}\left[y_{b,1} - c_{b,1} - \alpha k^2/2 - b_{b,1}\left(1/r_1\right)\right] \\ &+\lambda_2^{tl1}\left[y_{b,2} + b_{b,1} + A_2 k - c_{b,2} - b_{b,2}\right] + \mu_2^{tl1}[b_{b,2} + \phi q^k(c_{b,2})k], \end{aligned}$$

where we used $y_t = y_{b,t}+y_{l,t}$, and q^k and $1/r_1$ satisfy (32) and $1/r_1 = c_{b,1}/c_{b,2}$, respectively. The first order conditions for $c_{b,1}, c_{b,2}, b_{b,1}, b_{b,2}$, and k are given by

$$\lambda_1^{tl1}(1 + b_{b,1}\partial\left(1/r_1\right)/\partial c_{b,1}) = 1/c_{b,1} - 1, \tag{71}$$

$$\lambda_1^{tl1}b_{b,1}\partial\left(1/r_1\right)/\partial c_{b,2} + \lambda_2^{tl1} = (1/c_{b,2}) - 1 + \mu_2^{tl1}\phi k\partial q^k/\partial c_{b,2}, \tag{72}$$

$$\lambda_1^{tl1}\left(1/r_1\right) = \lambda_2^{tl1}, \tag{73}$$

$$\mu_2^{tl1} = \lambda_2^{tl1}, \tag{74}$$

and (63). Substituting out the multipliers λ_1^{tl1} and λ_2^{tl1} in (71)-(73), leads to

$$r_1\frac{(1 + b_{b,1}\partial\left(1/r_1\right)/\partial c_{b,1})}{(1 + r_1 b_{b,1}\partial\left(1/r_1\right)/\partial c_{b,2})} = \frac{(1/c_{b,1} - 1)}{(1/c_{b,2}) - 1 + \mu_2^{tl1}\phi k\partial q^k/\partial c_{b,2}}.$$

Further using (70) as well as $1/r_1 = c_{b,1}/c_{b,2}$ and rearranging terms, gives

$$\begin{aligned}\tau_{l,1} = & -b_{b,1} r_1 \mu_2^{tl1} \left(\left(\partial \left(1/r_1\right) / \partial c_{b,1} \right) - \left(r_1 \partial \left(1/r_1\right) / \partial c_{b,2} \right) \right) \\ & -r_1 \mu_2^{tl1} \phi k \partial q^k / \partial c_{b,2},\end{aligned} \tag{75}$$

where $\partial \left(1/r_1\right) / \partial c_{b,1} > 0$ and $\partial \left(1/r_1\right) / \partial c_{b,2} < 0$. The distributive effects (in the first line of 75) are positive and call for a subsidy $\tau_{l,1} > 0$, whereas the collateral effect (in the second line of 75) is negative and calls for a saving tax, $\tau_{l,1} < 0$.

For the fourth part of the proposition, we consider a constant tax/subsidy on saving and an ex-ante capital investment tax/subsidy, which are fully compensated (ex-post) by lump-sum transfers (see 2). Agents' saving and investment decisions then satisfy

$$(1 - \tau_l)/r = 1, \tag{76}$$

where $r = r_1 = r_2$, and (60). Substituting out the interest rate in agents' borrowing decisions with (76), $c_{b,1}^{-1}/(1 - \tau_l) = 1/c_{b,2}$ and $c_{b,2}^{-1}/(1 - \tau_l) = 1 + \kappa_{b,2}$, and combining the latter to $c_{b,1}/c_{b,2} = c_{b,2}\left(1 + \kappa_{b,2}\right)$, implies that q^k satisfies the price relation (66). Proceeding as above, the Lagrangian of the planer's problem can be written as

$$\begin{aligned}L = & \log c_{b,1} + (y_1 - c_{b,1}) + [\log c_{b,2} + (y_2 - c_{b,2})] + [y_3 + A_3 k] \\ & + \lambda_1^{tl} \left[y_{b,1} - c_{b,1} - \alpha k^2/2 - b_{b,1} \left(1/r\right) \right] \\ & + \lambda_2^{tl} \left[y_{b,2} + b_{b,1} + A_2 k - c_{b,2} - b_{b,2} \left(1/r\right) \right] + \mu_2^{tl} [b_{b,2} + \phi q^k (c_{b,1}, c_{b,2}) k],\end{aligned}$$

where q^k and $1/r$ satisfy (66) and $1/r = c_{b,1}/c_{b,2}$, respectively, leading to the following first order conditions for $c_{b,1}, c_{b,2}, b_{b,1}, b_{b,2}$, and k

$$\lambda_1^{tl} (1 + b_{b,1} \partial \left(1/r\right) / \partial c_{b,1}) + \lambda_2^{tl} b_{b,2} \partial \left(1/r\right) / \partial c_{b,1} = (1/c_{b,1}) - 1 + \mu_2^{tl} \phi k \partial q^k / \partial c_{b,1}, \tag{77}$$

$$\lambda_1^{tl} b_{b,1} \partial \left(1/r_1\right) / \partial c_{b,2} + \lambda_2^{tl} \left(1 + b_{b,2} \partial \left(1/r\right) / \partial c_{b,2}\right) = (1/c_{b,2}) - 1 + \mu_2^{tl} \phi k \partial q^k / \partial c_{b,2}, \tag{78}$$

$$\lambda_1^{tl} \left(1/r\right) = \lambda_2^{tl}, \tag{79}$$

$$\mu_2^{tl} = \lambda_2^{tl} \left(1/r\right), \tag{80}$$

and (69). Substituting out λ_1^{tl} and λ_2^{tl} in (77)-(79) and taking differences, leads to

$$\begin{aligned}
& r\mu_2^{tl} b_{b,1} \left(\partial \left(1/r\right)/\partial c_{b,1}\right) + \mu_2^{tl} b_{b,2} \left(\partial \left(1/r\right)/\partial c_{b,1}\right) \\
& - \left(rr\mu_2^{tl} b_{b,1} \left(\partial \left(1/r\right)/\partial c_{b,2}\right) + r\mu_2^{tl} b_{b,2} \left(\partial \left(1/r\right)/\partial c_{b,2}\right)\right) \\
= & r^{-1}\left(\left(1/c_{b,1}\right) - 1\right) - \left(\left(1/c_{b,2}\right) - 1\right) + r^{-1}\mu_2^{tl}\phi k \left(\partial q^k/\partial c_{b,1}\right) - \mu_2^{tl}\phi k \left(\partial q^k/\partial c_{b,2}\right),
\end{aligned}$$

where $\partial \left(1/r\right)/\partial c_{b,1} > 0$ and $\partial \left(1/r\right)/\partial c_{b,2} < 0$. Using $c_{b,1}^{-1}/r = 1/c_{b,2}$ and $(1-\tau_l) = r$, we can isolate the tax/subsidy rate as

$$\begin{aligned}
\tau_l = & \left(r\mu_2^{tl} b_{b,1} + \mu_2^{tl} b_{b,2}\right) \left(\partial \left(1/r\right)/\partial c_{b,2}\right) - \left(\mu_2^{tl} b_{b,1} + \frac{1}{r}\mu_2^{tl} b_{b,2}\right) \left(\partial \left(1/r\right)/\partial c_{b,1}\right) \\
& + \frac{1}{r}\mu_2^{tl}\phi k \left(\partial q^k/\partial c_{b,1}\right) \left[2\phi + 1 - r\right]/r,
\end{aligned} \tag{81}$$

where we used $\partial q^k/\partial c_{b,2} = \left(1 - 2\phi c_{b,1}/c_{b,2}\right) \partial q^k/\partial c_{b,1}$ to derive the last term in (81). The distributive effects in the first line of (81) are strictly positive and call for a subsidy $\tau_l > 0$, while the collateral effect in the second line of (81) is positive when $\phi > (r-1)/2$. ■