On the Role of MBS for the Macroeconomic Effects of QE1*

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Abstract
This paper studies the quantitative contribution of MBS purchases to the macroeconomic effects of the US Federal Reserve’s first round of quantitative easing, QE1. We develop a macroeconomic model with costly financial intermediation, collateralized lending, and an explicit specification of distinct central bank asset purchase programmes. We show that MBS purchases induce a fall in mortgage rates and increases in the price of collateral and in bank lending, which stimulates aggregate demand and real activity. Replicating the time paths of purchased asset and of associated price effects at the onset of the global financial crisis, our calibrated model predicts that MBS purchases accounted for more than 85\% of the (cumulated) output effects of QE1 and that an equally sized intervention without MBS purchases would have led to about 30\% smaller output effects.

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\textit{Keywords}: Unconventional monetary policy, mortgage debt, collateral constraints, costly financial intermediation

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1 Introduction

During the global financial crisis, US Federal Reserve (Fed) lowered the policy rate to its zero lower bound (ZLB) and introduced several unconventional policy measures. Large-scale asset purchase programs, also known as quantitative easing (QE), were one of the main measures. Within these programs the Fed mainly purchased treasuries and mortgage-backed securities (MBS), where the latter made up a large part of the purchased assets in the first round of quantitative easing (QE1).¹ Figure 1 illustrates the Fed’s policy response to the crisis with the federal funds rate hitting the ZLB and purchases of MBS and long-term treasury securities. The Fed argued that “housing and housing finance played a central role in precipitating the [...] crisis” and “steps that stabilize the housing market will help stabilize the economy” (see Bernanke, 2008). Specifically, MBS purchases were justified as they “should provide further support to the housing sector by encouraging home purchases and refinancing” (see Bernanke, 2012), which has been confirmed by observed reductions not only in MBS yields in secondary markets (see Krishnamurthy and Vissing-Jorgensen, 2011), but also in mortgage rates in the primary market (see Hancock and Passmore, 2011, 2015, and Fuster et al., 2017). Despite its central role in both, the crisis and the policy response, the theoretical literature on unconventional monetary policy so far ignored the specific role of mortgage debt and MBS purchases.²

In this paper, we assess the effects of central bank interventions accounting for frictions associated with mortgage loans and for purchases of MBS. We quantify the impact on macroeconomic aggregates and compare our results to related studies, in particular, to Del Negro et al. (2017, henceforth DEFK), who provide an assessment of the macroeconomic effects of QE1. For this, we apply a model with costly financial intermediation and collateralized lending, as well as with an explicit specification of central bank asset purchases against high powered money, allowing to precisely replicate the Fed’s asset purchases, as shown in Figure 2. Consistent with empirical evidence (see Krishnamurthy and Vissing-Jorgensen, 2011), we show that MBS purchases were more successful in reducing MBS yields and thus loan rates than equally-sized purchases of treasury securities.³ Our main novel finding is that more than 85% of the cumulated (one-year) output effect of QE1 that we found can in fact be attributed to MBS purchases, although MBS made up about half of the total volume of QE1, which highlights the importance of considering MBS purchases. Put differently, we show that if the Fed’s outright asset purchases were entirely conducted in terms

¹In November 2008, when QE1 started, the Fed announced that it would purchase MBS worth up to $500 billion issued by the government-sponsored enterprises Fannie Mae and Freddie Mac. On March 18, 2009 the Fed decided to expand the program and to purchase additional $750 billion of agency MBS, such that the announced total purchases added up to $1250 billion in QE1 (see Figure 1).
³The implied path of the spread between the yields on MBS and treasuries therefore accords to the observed spread between yields on MBS and treasuries as given in Figure 1.
of treasuries, the net output effect of QE1 would have been about 30% smaller.

Figure 1: MBS and long-term treasuries purchases in billions of USD (right axis), annual federal funds rate (red solid line), and spreads between average 30-year fix mortgage rate (dark blue dashed dotted line) / Fannie Mae 30-year current-coupon MBS yield (light blue dashed line) and 30-year treasury constant maturity rate (all three left axis), 2008Q1-2010Q4. [Source: FRED & Bloomberg.]

To account for the specific role of the mortgage market and housing, we follow Kiyotaki and Moore (1997) and consider two types of households, patient and impatient ones. While patient households hold deposits at financial intermediaries, the latter grant loans that are collateralized by their housing. We specify quantitative easing as secondary market purchases of eligible assets against central bank money, like in Hoermann and Schabert (2015). To induce demand for central bank reserves, we consider that they play a unique role for the settlement of transactions. To facilitate the calibration of the model, we do not explicitly model the settlement of bank deposit transactions (see Bianchi and Bigio, 2017) and specify the role of reserves in a parsimonious way by introducing an ad-hoc banking cost function (as in Curdia and Woodford, 2011).4 Asset purchases expand the supply of reserves, which reduces intermediation costs and stimulates bank lending. They further have a direct effect on prices of purchased assets when liquidity demand induces

4Concretely, the banking cost function is specified with two key parameters, a level parameter and an elasticity of banking costs with regard to the ratio of loans to reserves. The level parameter is calibrated to match average MBS yields between 1990Q1 and 2008Q3. The value for the elasticity is set to match the empirically observed spread effects between yields on MBS and treasuries at the start of the Fed’s intervention (see Krishnamurthy and Vissing-Jorgensen, 2011).
eligible assets to be scarce. Notably, these effects can occur even when the policy rate is at the zero lower bound, as long as the valuation of liquidity/reserves – that generally differs from the policy rate – is positive (see Hoermann and Schabert, 2015). When liquidity and eligible assets are scarce (as in the financial crisis) the central bank can influence MBS yields and thus loan rates by purchasing MBS at above-market prices.\(^5\) This is particularly beneficial for impatient borrowers, as interest rates on collateralized loans fall and banks supply more loans to households. The stimulation of private sector lending raises borrowers’ willingness to pay for housing and thus house prices,\(^6\) which further alleviates the borrowing constraint of impatient households with high marginal propensity to consume. While government bond purchases are also expansionary by accommodating banks’ liquidity demand, they exert substantially smaller effects on real activity, as they impact on private agents’ borrowing conditions to a much lesser extent than MBS purchases, consistent with empirical evidence (see Krishnamurthy and Vissing-Jorgensen, 2011).

Asset purchases are specified in the model by time series models that closely mimic the time paths of actual asset purchases of the US Fed, as shown in the right hand side panel of Figure 2, where the sizes of the interventions are given relative to 2008Q3. Thereby, we distinguish between MBS purchases and other central bank interventions, which we summarize in the model by money supply against short-term and long-term treasury debt. It should further be noted that purchases of MBS (green area) and long-term treasuries (orange area) are outright transactions that add

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\(^5\) Notably, we do not distinguish between rates on mortgage debt in the secondary and the primary market, implying a full pass-through of changes in MBS rates. While this assumption is made for convenience, it is in fact supported by empirical evidence, showing that the pass-through of MBS prices induced by QE announcement was almost complete (see Fuster et al. 2017), and consistent with the paths of the empirical spreads in Figure 2.

\(^6\) Guerrieri and Iacoviello (2016) also find a tight link between house prices, borrowing, and consumption depending on the tightness of borrowing constraints.
to the stock of assets held by the Fed, whereas money is temporarily supplied under liquidity facilities (light blue area, where only a small fraction consists of outright agency debt purchases). Similar to DEFK, we, firstly, aim at replicating the financial crisis and, secondly, at quantifying the (untargeted) effects of the purchases in QE1 on macroeconomic aggregates. To replicate core macroeconomic outcomes at the beginning of the financial crisis, we introduce shocks to financial intermediation costs, which has been found by Ajello (2016) to play an important role during the Great Recession. With this type of shock, the model is able to account for the fall in GDP and inflation as observed in the data while providing a reasonable match of the path of the expected ZLB duration. Moreover, it accounts for changes in the spread between the yields of MBS and treasury debt, as observed in the data. It should be noted that this strategy differs from DEFK, who introduce an (il-)liquidity shock that is calibrated to replicate an observed liquidity premium, whereas our banking cost shock allows to almost fully account for the macroeconomic effects at the onset of the financial crisis.

Regarding the asset purchase effects, we, firstly, find that the full QE1 program led to an increase in GDP by about 0.92% and in inflation by about 0.18% in 2008Q4 and in 2009Q1 to an increase in GDP by about 2.89% and in inflation by about 0.48%. This means that with the intervention the drop in output and inflation in 2008Q4 would have been 12% and 7% larger. In 2009Q1 the drop in output would have been even twice as large as observed and the drop in inflation 45% larger. Cumulating the effects over 4 quarters (2008Q4-2009Q3), we find that the output contraction would have been almost 60% and the drop in inflation almost 20% larger without the central bank intervention, which relates to the output (30% larger) and inflation (40% larger) effects without intervention found by DEFK. Secondly, with regard to the effects of MBS purchases within QE1, we find that for the horizon of 1 year 87% of the cumulative increase in GDP and 84% of the cumulative increase in inflation can be attributed to MBS purchases. Hence, MBS purchases amounting to about half (52%) of the entire QE1 program, contribute by about 85% to the overall effects of QE1, which highlights their importance. Thirdly, we find that purchases of MBS were much more effective than purchases of long-term government bonds. Specifically, we find that a counterfactual program of the same size as QE1, where all outright purchases consist of treasury debt (excluding MBS), leads to (cumulated) output and inflation effects that are 29% and 34% smaller than the effects of the correctly specified QE1 program (including MBS). Put differently, this means that if we had modeled QE1 only in terms of treasury securities, we would have underestimated the effects by about 30%. We, fourthly, highlight the importance of the pre-announcement of MBS purchases, as these were announced in November 2008, but started only in January 2009. We find that without the pre-announcement the cumulative increase in output and inflation would be about 25% and 30% smaller.

This paper is related to a growing body of literature analyzing macroeconomic effects of un-
conventional monetary policy in response to financial market disturbances. In a model, in which financial intermediation bears real resource costs, Curdia and Woodford (2011) find that under severe financial distress asset purchases may be beneficial, while apart from that they play no role for stabilization policy, even for a binding ZLB. Gertler and Karadi (2011) specify a macroeconomic model with endogenously leverage constrained banks and find that direct central bank lending is beneficial during a financial crisis, particularly, at the ZLB. Gertler and Karadi (2013) extend this model to allow for purchases of both government bonds and private securities and conclude that purchases of the latter are more effective, which relates to our conclusion. In a model with segmented bond markets, Chen et al. (2012) find small effects of purchases of long-term government debt relating to our findings with regard to long-term government bonds, which would be even smaller without a binding ZLB. Unlike these papers, where the central bank directly creates loans for private agents, Hoermann and Schabert (2015) specify central bank asset purchases in secondary markets. They analytically show that the size as well as the composition of the central bank’s balance sheet can exert macroeconomic effects, while the effects depend on the scarcity of eligible assets and hence are more pronounced during a crisis. Our analysis most closely relates to DEFK (2017), whose quantitative macroeconomic model builds upon Kiyotaki and Moore (2012). DEFK show that an adverse shock to the resellability of assets is able to replicate the large economic downturn during the financial crisis and to make the ZLB binding for several quarters. They further show that in the absence of asset purchases the negative effects of the crisis would have been even larger depending on the expected duration of the ZLB episode. Woodford (2016) examines the impact of purchases of long-term treasuries by the central bank on financial stability via banks’ incentives to issue short-term risky debt.

Our paper differs from these studies along several dimensions. First, the existing papers analyze the effects of central bank purchases of treasuries and/or corporate bonds and do not account for mortgage loans or MBS\(^7\), whereas our focus exactly lies on the specific effects of MBS purchases. Second, we use time series models to closely replicate observed paths of asset purchases. Third, we consider shocks to intermediation costs as the source of the adverse effects in 2008 and 2009, which accords to Ajello (2016).

The remainder of the paper is organized as follows. In section 2, the model is described. In section 3, we present our calibration strategy and the crisis scenario triggered by the banking cost shock. Moreover, we simulate asset purchases and particularly MBS purchases and discuss their effects during QE1. Finally, we compare MBS purchases to purchases of treasuries and discuss the role of the Fed pre-announcements in our model. Section 4 concludes.

\(^7\)An exception is Schabert (2018), who shows for an endowment economy that asset purchases can serve as corrective policies, comparable to Pigouvian subsidies, to address pecuniary externalities induced by financial constraints.
2 The Model

We follow Kiyotaki and Moore (1997) and consider two types of agents, patient and impatient ones. Intermediation between these two types is conducted by competitive banks which collect deposits from (patient) savers and grant loans to (impatient) borrowing households. We assume that debt contracts are not enforceable and are collateralized by housing (see Iacoviello, 2005); the supply of the latter being fixed. We assume that mortgage loans can be traded in a frictionless way, such that they are equivalent to mortgage-backed securities (MBS) in this model. Given that the rates in the primary and secondary market for mortgage debt are identical, MBS price effects of central bank interventions are fully passed through to the loan rate. This simplifying assumption is supported by empirical evidence, in particular, by Fuster et al. (2017). The treasury issues one-period and multi-period bonds which are held by financial intermediaries and the central bank. Following Hoermann and Schabert (2015), we assume that the central bank supplies money only against eligible assets, here, treasuries and MBS. The central bank sets the policy rate and can further control the amount of money supplied against eligible assets, e.g. it can increase the supply of reserves by purchasing MBS. For the simulation of the financial crisis and a binding zero lower bound, we apply shocks that render financial intermediation more costly.

2.1 Households

There is a continuum of households of mass 1 consisting of two types, patient ones indexed with $p$, who represent a share of $0 < s < 1$ of total population, and impatient ones indexed with $i$ (and share $1 - s$). They only differ with regard to their subjective discount factors: $1 > \beta^p > \beta^i > 0$. Both types of households derive utility from consumption $c_{*,t}$, housing $h_{*,t}$ and disutility from labor $n_{*,t}$ ($* = i, p$) and maximize the expected sum of discounted utility

$$E_0 \sum_{t=0}^{\infty} (\beta^*)^t \cdot u(c_{*,t}, h_{*,t}, n_{*,t}),$$

(1)

where $\beta^p$ is the discount factor for patient and $\beta^i$ for impatient households. We assume that the instantaneous utility function is separable in all arguments, strictly concave, increasing in housing and consumption, and decreasing in working time. The total stock of housing is assumed to be constant.

**Patient Households** A patient household $p$ enters a period $t$ with deposits $D_{p,t-1}$ held at financial intermediaries and real housing $h_{p,t-1}$. Neglecting borrowing from financial intermediaries (which will not occur in equilibrium), its budget constraint is given by

$$P_t c_{p,t} + P_t\rho_{p,t}(h_{p,t} - h_{p,t-1}) + D_{p,t}/R_t^D = D_{p,t-1} + P_t w_t n_{p,t} + P_t \tau_{p,t} + P_t \delta_{p,t},$$

(2)
where the left hand side contains expenditures for consumption, \( P_t c_{p,t} \), and housing, \( P_t h_{p,t} \), with the real house price \( p_{h,t} \) and new holdings of deposits \( D_{p,t} \) at the price \( 1/R_t \), while the right hand side shows deposits from the preceding period as well as labor income, \( P_t w_t n_{p,t} \), lump-sum transfers/taxes, \( P_t \tau_{p,t} \), and profits of firms and retailers \( P_t \delta_{p,t} \), due to the assumption that patient households are the owners of firms and retailers. A patient household chooses the values of \( c_{p,t}, h_{p,t}, n_{p,t} \) and \( d_{p,t} = D_{p,t}/P_t \) to maximize (1) subject to (2), leading to the first order conditions

\[
\begin{align*}
    u'(h_{p,t}) &= p_{h,t} u'(c_{p,t}) - \beta p E_{t} u'(c_{p,t+1}) p_{h,t+1}, \\
    -u'(n_{p,t}) &= w_t u'(c_{p,t}), \\
    \frac{1}{R_t} &= \beta p E_{t} \frac{u'(c_{p,t+1})}{u'(c_{p,t})} n_{t+1},
\end{align*}
\]

where \( u'(h_{p,t}), u'(c_{p,t}), \) and \( u'(n_{p,t}) \) denote the marginal utilities of housing, consumption and working time, and an associated transversality condition. Equation (3) describes housing demand of a patient household. In the optimum, marginal utility of current housing equals marginal utility of foregone consumption at the price of housing \( p_{h,t} \) less the discounted marginal utility of next period’s expected consumption \( \beta p E_{t} c_{p,t+1}^{-1} \) achieved from selling the house at the expected price. Equation (4) describes labor supply of a patient household and (5) is the optimality condition for holdings of deposits.

**Impatient Households** Since an impatient household \( i \) values current consumption more than a patient one, it will be a borrower in equilibrium. We assume that its debt is non-enforceable and is collateralized by housing. A household \( i \) can borrow from intermediaries in nominal terms an amount \( B_{i,t}^M / R_t^L < 0 \) in period \( t \) and pays back \( B_{i,t+1}^M \) in period \( t+1 \), where \( R_t^L \) is the gross nominal interest rate on these loans. We follow Kiyotaki and Moore (1997) and assume that borrowing is limited by a (fraction) of the expected value of housing at the beginning of the subsequent period when the loan matures

\[
B_{i,t}^M \geq -\phi E_t P_{t+1} h_{i,t+1},
\]

where \( \phi \) denotes the (exogenously given) pledgeable fraction of housing. An impatient household \( i \) enters a period \( t \) with mortgage debt \( B_{i,t-1}^M < 0 \) and real housing \( h_{i,t-1} \). It has expenditures for consumption, \( P_t c_{i,t} \), and housing, \( P_t h_{i,t} \), faces transfers/taxes, \( P_t \tau_{i,t} \), earns labor income \( P_t w_t n_{i,t} \), and borrows by issuing mortgage loans \( B_{i,t}^M / R_t^L \). Neglecting deposits held at financial intermediaries (which would never occur in equilibrium), the budget constraint of an impatient household \( i \) reads

\[
P_t c_{i,t} + P_t h_{i,t} [h_{i,t} - h_{i,t-1}] + B_{i,t}^M / R_t^L = B_{i,t-1}^M + P_t w_t n_{i,t} + P_t \tau_{i,t}.
\]
An impatient household $i$ chooses the values of $c_{i,t}$, $h_{i,t}$, $n_{i,t}$, and $b_{i,t}^M = B_{i,t}^M / P_t$ to maximize (1) subject to the collateral constraint (6) and the budget constraint (7) leading to the first order conditions

\[ u'(h_{i,t}) = u'(c_{i,t}) p_{h,t} - \beta^t E_t u'(c_{i,t+1}) p_{h,t+1} - \omega_t \phi E_t \pi_{t+1} p_{h,t+1}, \]  \hspace{1cm} (8)  
\[ -u'(n_{i,t}) = w_t u'(c_{i,t}), \]  \hspace{1cm} (9)  
\[ \frac{u'(c_{i,t})}{R^L_t} = \beta^t E_t \frac{u'(c_{i,t+1})}{\pi_{t+1}} + \omega_t, \]  \hspace{1cm} (10)  

where $\omega_t$ denotes the multiplier on the collateral constraint (6), and the complementary slackness conditions $\omega_t (b_{i,t}^M + \phi E_t \pi_{t+1} p_{h,t+1} h_{i,t}) = 0$, $b_{i,t}^M + \phi E_t \pi_{t+1} p_{h,t+1} h_{i,t} \geq 0$, and $\omega_t \geq 0$. Equation (8) describes housing demand of an impatient household. Here, the additional term $\omega_t \phi E_t \pi_{t+1} p_{h,t+1}$, indicates that housing has an additional value as collateral for loans for impatient agents. Equation (9) describes the labor supply decision of an impatient household and (10) describes the demand for debt.

2.2 Banks

There is a continuum of identical perfectly competitive banks of mass 1 indexed with $b$. A bank $b$ receives deposits from (patient) households, holds money $M^H_{b,t}$ and government bonds. It holds short-term government bonds $B_{b,t}$ (T-bills) with an interest rate $R^G_t$ and long-term (multi-period) government bonds $B_{b,t}^{LT}$, which are traded at the price $q_{b,t}^{LT}$ in $t$ and deliver a payoff $p_{b,t}^{LT}$ in $t+1$. Long-term bonds are modeled as perpetuities with coupon payments that exponentially decay at the rate $\rho \in (0, 1)$. Banks supply collateralized (mortgage) loans at the loan rate $R^L_t$. We assume that mortgage loans can be frictionlessly traded, such that they are equivalent to mortgage backed securities (MBS) traded at a price that equals the inverse of the loan rate. Notably, the implied full pass through of price effects in the secondary market for mortgage debt is consistent with empirical evidence of Fuster et al. (2017), who find an approximately full pass-through of price effects in the secondary market to the primary market, of QE-related monetary policy measures. Banks face costs of managing loans, for which we consider a stylized cost function $\Xi_t$, following Curdia and Woodford (2011). Bank $b$’s budget constraint is given by

\[ P_t \pi_{b,t}^B + D_{b,t-1} + \frac{B_{b,t}}{R^G_t} + q_{b,t}^{LT} B_{b,t}^{LT} + \frac{B_{b,t}^M}{R^L_t} + M^H_{b,t} + I_{b,t} (R^m_t - 1) + P_t \Xi (z_t, b_{b,t}, q_{b,t}) \]  \hspace{1cm} (11)

\[ = \frac{D_{b,t}}{R^G_t} + D_{b,t-1} + p_t^{LT} B_{b,t-1}^{LT} + B_{b,t-1}^M + M^H_{b,t-1}. \]

\[ ^8\text{Concretely, they find that “on average, a one dollar change in the MBS price leads to a 92 cent change in the rebate paid to the borrower” (see Fuster et al., 2017).} \]
where \( b_{b,t}^M = B_{b,t}^M/P_t \), \( q_{b,t} = Q_{b,t}/P_t \) and \( \pi_{b,t}^M \) denotes its profits. The term \( I_{b,t} (R_t^m - 1) \) in (11) denotes costs associated with the acquisition of new central bank money \( I_{b,t} \), as the central bank discounts eligible assets at the rate \( R_t^m \) (see below). To capture costs of providing financial intermediary, we introduce an ad-hoc cost function \( \Xi(\hat{z}_t^I, b_{b,t}, q_{b,t}) \), similar to Curdia and Woodford (2011). This function on the one hand captures the costs associated with loan creation, such that it is an increasing function of the volume of loans \( \partial \Xi_t/\partial b_{b,t} > 0 \). Further accounting for the specific role of central bank money for the settlement of banks’ transactions and liquidity management, banking costs are assumed to be decreasing in holdings of reserves, i.e. \( \partial \Xi_t/\partial q_{b,t} < 0 \) where \( Q_{b,t} = M_{b,t-1}^H + I_{b,t} \). Moreover, the banking costs satisfy \( \partial(\partial \Xi_t/\partial b_{b,t})/\partial q_{b,t} < 0 \) and \( \partial(\partial \Xi_t/\partial q_{b,t})/\partial b_{b,t} > 0 \), such that the marginal costs of loans decrease with reserves and the marginal gains of reserves decrease with loans. Notably, banking costs further depend on the stochastic component \( z_t^F \), which will serve as a shock that induces sufficiently severe effects to replicate main macroeconomic outcomes at the onset of the recent financial crisis.

Both types of bank assets, i.e. treasuries and MBS, are assumed to be in principle eligible and can, therefore, be used to get new reserves from the central bank. In accordance with the Fed’s pre-crisis money supply regime, we assume that short-term treasuries are in general eligible. Additionally, following QE1 practice, we consider purchases of MBS and/or long-term bonds. The central bank further sets the price of money in terms of eligible assets \( R_t^m \), which serves as the policy rate. Notably, the federal funds rate was almost identical to the rate on treasury repurchase agreements before the financial crisis, see e.g. Bech et al. (2012).9 New money injections \( I_{b,t} \) that a bank receives from the central bank are then limited by the following money supply constraint

\[
I_{b,t} \leq (1 + \epsilon_t^i) \frac{B_{b,t-1}}{R_t^m} + (z_t^{LT} - 1) \frac{P_t^{LT} B_{b,t-1}^{LT}}{R_t^m} + (z_t - 1) \frac{B_{b,t-1}^M}{R_t^m},
\]

(12)

where \( \epsilon_t^i, z_t \) and \( z_t^{LT} \) are exogenously determined by the central bank to conduct the QE1 program. Below we will describe in detail how the stochastic process for \( z_t \) is specified to match the size and the time pattern of the Fed’s MBS purchases and how \( z_t^{LT} \) is calibrated to replicate the long-term bond purchases during QE1.\(^{10}\) The term \( \epsilon_t^i \) is further fitted to match the whole size of the QE1 program, which further consists of temporary money supply under liquidity facilities and a small fraction of agency debt purchases. In the steady state, we set \( z = z^L = 1 \) and \( \epsilon^i = 0 \) such that only short-term government bonds are eligible, in accordance with the US Fed’s pre-crisis regime.

A bank \( b \) maximizes the present value of future profits \( \max E_0 \sum_{k=0}^{\infty} \vartheta_{t,t+k} \pi_{b,t+k}^B \) subject to its budget constraint (11) and the money supply constraint (12), where \( \vartheta_{t,t+k} \) denotes the stochastic discount factor of banks. The first order conditions with respect to deposits, short and long-term

\(^9\)Precisely, the average spread between the Fed’s treasury repo rate and the federal funds is smaller than 1 b.p.

\(^{10}\)Note that since the Fed only purchased Agency MBS in the program, the term \( (z_t - 1) \) will be measured by Agency MBS purchased by the Fed as share of total Agency MBS outstanding.
government bonds, MBS, money holdings and injections are given by

\[ \frac{1}{R_t^D} = E_t \vartheta_{t,t+1}^\ell \pi_{t+1}, \]  

\[ \frac{1}{R_t^G} = \frac{1}{R_t^D} \left( 1 + \frac{\vartheta_{t+1}^\ell 1 + \vartheta_{t+1}^r}{R_{t+1}^m} \right), \]  

\[ q_t^{LT} = E_t \frac{p_t^{LT}}{R_t^D} \left( 1 + \frac{\vartheta_{t+1}^\ell 1 + \vartheta_{t+1}^r}{R_{t+1}^m} \right), \]  

\[ \frac{1}{R_t^L} = \frac{1}{R_t^D} \left( 1 + \frac{\vartheta_{t+1}^\ell 1 + \vartheta_{t+1}^r}{R_{t+1}^m} \right) - \vartheta_t \frac{\partial \Xi_t}{\partial b_{b,t}^M}, \]  

\[ 1 = \frac{1}{R_t^D} - E_t \vartheta_{t,t+1}^\ell \vartheta_{t+1}^M, \]  

\[ R_t^m = 1 - \vartheta_t \frac{\partial \Xi_t}{\partial b_{b,t}^M}, \]  

where \( \vartheta_{t,t+1}^\ell = \vartheta_{t,t+1}^\ell \vartheta_{t+1}^r \) and \( \eta_t \) denotes the multiplier on (12), and the complementary slackness conditions \( \eta_t \left[ (1 + c_t^1) b_{b,t-k}^o + (z_{t+1}^T - 1) b_{b,t-k}^m \right] - \vartheta_{t,k}^r \leq 0 \), \( \left[ (1 + c_t^1) b_{b,t-k}^o + (z_{t+1}^T - 1) b_{b,t-k}^m \right] / (\pi_{t+k} R_{t+k}^m) \) is the stochastic discount factor of banks when it is expected to be binding, \( \vartheta_{t,1}^r > 0 \) (see 16). Equation (17) describes optimal holdings of money and the optimality conditions for new money (18) shows that the money supply constraint is binding when the marginal (negative) effect of injections on banking costs is sufficiently large, \( \eta_t > 0 \Leftrightarrow 1 - R_t^m > \vartheta_t / \vartheta_{b,t}^M. \)

### 2.3 Firms

A continuum of perfectly competitive identical firms indexed with \( j \) produce the intermediate good according to \( IO_{j,t} = (n_{j,t}^T)^\alpha \), where \( \alpha \in (0,1) \). The firm hires labor \( n_{j,t}^T \) at a common rate \( w_t \) to produce its output \( IO_{j,t} \), which it sells to the retailers at the price \( P_{j,t} \). Hence a firm \( j \) solves

\[ \max P_{j,t} (n_{j,t}^T) - P_t w_t n_{j,t}^T \]  

leading to the first order condition

\[ P_{j,t} d n_{j,t}^T = P_t w_t \]  

and to profits

\[ (1 - \alpha) P_{j,t} (n_{j,t}^T)^\alpha - P_t w_t (n_{j,t}^T)^{\alpha-1} = P_t w_t \]  

of (1 - \alpha) \( P_{j,t} (n_{j,t}^T)^\alpha \) that are distributed to the patient households, which own these firms.

There is further a continuum of monopolistically competitive retailers indexed with \( k \) who buy
intermediate goods at the price $P_{j,t}$, re-package them according to $IO_t = \int_0^1 IO_{j,t} dj$, differentiate them into $y_{k,t} = IO_{k,t}$, and sell the distinct goods $y_{k,t}$ at the price $P_{k,t}$ to perfectly competitive bundlers. They bundle them to the final good $y_t^{\frac{1}{1-\epsilon}} = \int_0^1 y_{k,t}^{\frac{1}{1-\epsilon}} dk$, where $\epsilon > 1$, which is sold at the price $P_t$. Hence a retailer $k$ faces the demand function $y_{k,t} = (P_{k,t}/P_t)^{-\epsilon} y_t$ and sets its own price $P_{k,t}$ accordingly taking $P_{j,t}$ as given. We assume that each period only a fraction $1 - \theta$ of retailers is allowed to change their price. The other fraction $\theta \in [0, 1)$ adjusts the price according to full indexation to the steady state inflation rate: $P_{k,t} = \pi P_{k,t-1}$. Defining $\tilde{Z}_t = P_{k,t}^\psi/P_t$ with the price of retailers $P_{k,t}^\psi$, optimal price setting satisfies $\tilde{Z}_t = \frac{\epsilon}{1-\epsilon} Z_{1,t}/Z_{2,t}$, with $Z_{1,t} = c_{p,t}^{-1} y_t m c_t + \theta \beta \pi E_t (\frac{\pi t}{\pi})^\epsilon Z_{1,t+1}$ and $Z_{2,t} = c_{p,t}^{-1} y_t + \theta \beta \pi E_t (\frac{\pi t}{\pi})^\epsilon - 1 Z_{2,t+1}$.

Due to perfectly competitive bundlers the aggregate price level $P_t$ for final goods is given by $P_t^{1-\epsilon} = \int_0^1 P_{k,t}^{1-\epsilon} dk$, implying $1 = (1 - \theta) \tilde{Z}_t^{1-\epsilon} + \theta \left( \frac{\pi t}{\pi} \right)^\epsilon$. Aggregate output is given by $y_t = (n_t^T)^a/n_t$ where $n_t = \int_0^1 (P_{k,t}/P_t)^{-\epsilon} dk$ is a measure of price dispersion, which can be written recursively as $n_t = (1 - \theta) \tilde{Z}_t^{1-\epsilon} + \theta \left( \frac{\pi t}{\pi} \right)^\epsilon n_{t-1}$ and where total labor demand equals total labor supply: $n_t = \int_0^1 n^T_{j,t} dj = sn_{p,t} + (1 - s) n_{i,t}$. Profits of intermediate goods producing firms and retailers that are distributed to patient households are collected in the term $P_t \delta_{p,t}$.

2.4 Public Sector

The treasury issues short (one-period) and long-term (multi-period) bonds. As in Hoermann and Schabert (2015), we assume that short-term bonds are supplied according to a constant growth rate $B_{T,t} = \Gamma B_{T,t-1}$, where $\Gamma > \beta \rho$ is the growth rate of total short-term government bonds, which are held by financial intermediaries and the central bank. Further, we assume that long-term debt is issued in form of perpetuities with exponentially decaying coupon payments. The rate of decay is given by $\rho \in (0, 1)$. Note that, bonds issued in period $t - s$ and $\rho^s$ bonds issued in $t$ are equivalent, which is why we assume –without loss of generality– that all long-term debt is of one type implying that the government redeems all old bonds in each period. Hence, a perpetuity issued in period $t$ at the price $q_{t,t}^{LT}$ pays out $1 + \rho E_t q_{t,t}^{LT}$ in period $t + 1$, such that $E_t p_{j,t}^{LT} = 1 + \rho E_t q_{t,t}^{LT}$. The budget constraint of the government reads

\[
B_{T,t-1} + (1 + \rho q_{t,t}^{LT}) B_{T,t-1}^{LT} + P_t \tau_t = (B_{T,t}/R_t^{LT}) + q_t^{LT} B_t^{LT} + P_t \tau_t^{m},
\]

where $P_t \tau_t^{m}$ are seigniorage revenues received from the central bank and $P_t \tau_t$ lump-sum transfers to households, which are assumed to be identical for both types, $\tau_{p,t} = \tau_{i,t}$. To ensure solvency, the government is assumed to follow a fiscal rule, similar to DEFK:

\[
\tau_t - \tau = \psi \cdot \left( [b_{T,t-1} + (1 + \rho q_{t,t}^{LT}) b_{T,t-1}^{LT}] \pi_t^{-1} - [b_T + (1 + \rho q_{T}^{LT}) b_T^{LT}] / \pi \right),
\]

where variables without time subscript are the associated steady state values and $\psi > 0$ governs the response of taxes to debt.
The central bank supplies money in regular open market operations either outright or temporarily via repos against short-term bonds (T-bills), $M_{t}^{H, TB} = \int_{0}^{1} M_{b,t}^{H, TB} \, db$ and $M_{t}^{R, TB} = \int_{0}^{1} M_{b,t}^{R, TB} \, db$, where $M_{b,t}^{H, TB} - M_{b,t-1}^{H, TB} + M_{b,t}^{R, TB} \leq B_{b,t-1}/R_{t}^{m}$, for each individual bank $b$. The central bank can further increase the supply of money by outright purchases of long-term government bonds or MBS (using $z_{t}$ and $z_{t}^{LT}$). The remaining part of the QE1, which mainly consists of a temporary money supply under various liquidity facilities as well as a small fraction of agency debt purchases, are – for convenience – modeled as additional repos against T-bills (see $\epsilon_{t}^{i}$ in 12). New money injections by the central bank in each period are given by $\int_{0}^{1} b_{t} \, db = I_{t} = M_{t}^{H, TB} - M_{t-1}^{H, TB} + M_{t}^{R, TB} + I_{t}^{LT} + I_{t}^{MBS} + I_{t}', \text{ where } I_{t}^{LT}$ and $I_{t}^{MBS}$ denote money supplied against long-term bonds and MBS, $I_{t}^{LT} \leq \int_{0}^{1} (z_{t}^{LT} - 1)(p_{t}^{LT} B_{b,t-1}^{LT}/R_{t}^{m}) \, db$, and $I_{t}^{MBS} \leq \int_{0}^{1} (z_{t} - 1) (B_{b,t-1}^{M} / R_{t}^{m}) \, db$ and where $I_{t}' = \epsilon_{t}^{i} \int_{0}^{1} B_{b,t-1}/R_{t}^{m} \, db$ (see 12). Summarizing the total stock of money supplied outright by $M_{t}^{H}$, i.e. $M_{t}^{H} = M_{t}^{H, TB} + I_{t}^{LT} + I_{t}^{MBS}$, the central bank budget constraint can be written as
\[
(B_{C,t}^{G} / R_{t}^{G}) + q_{t}^{LT} B_{C,t}^{LT} + (B_{C,t}^{M} / R_{t}^{L}) + P_{t} \epsilon_{t}^{m}
= B_{C,t-1} + (p_{t}^{LT} B_{C,t-1}^{LT}) + (M_{t}^{L} - M_{t-1}^{H}) + (R_{t}^{m} - 1) (M_{t}^{R, TB} + I_{t}^{LT} + I_{t}^{MBS} + I_{t}'),
\]
where $B_{C,t-1}, B_{C,t-1}^{LT}$, and $B_{C,t-1}^{M}$ denote central bank holdings of T-bills, long-term treasuries, and MBS under outright purchases. Assuming that the central bank transfers all its earnings from asset holdings and money supply facilities,
\[
P_{t} \epsilon_{t}^{m} = (1 - 1/R_{t}^{L}) B_{C,t}^{L} + (1 - 1/R_{t}^{L}) B_{C,t}^{M} + (E_{t} p_{t+1}^{LT} - q_{t}^{LT}) B_{C,t}^{LT}
+ (R_{t}^{m} - 1) (M_{t}^{H} - M_{t-1}^{L}) + (R_{t}^{m} - 1) (M_{t}^{R} + I_{t}^{LT} + I_{t}^{MBS} + I_{t}'),
\]
to the treasury. Thus, we get the following relationship between the evolution of assets held and money supplied by the central bank $M_{t}^{H} - M_{t-1}^{H} = B_{C,t} - B_{C,t-1} + B_{C,t}^{M} - B_{C,t-1}^{M} + E_{t} p_{t+1}^{LT} B_{C,t}^{LT} - p_{t}^{LT} B_{C,t-1}^{LT}$. Assuming that initial assets and liabilities satisfy, $M_{0}^{H} = B_{C,-1} + B_{C,-1}^{M} + p_{0}^{LT} B_{C,-1}^{LT}$, the balance sheet of the central bank reads
\[
M_{t}^{H-1} = B_{C,t-1} + B_{C,t-1}^{M} + p_{t}^{LT} B_{C,t-1}^{LT}.
\]
The policy rate is set by the central bank following a feedback rule respecting the zero lower bound
\[
R_{t}^{m} = \max\{1, (R_{t-1}^{m})^{\rho_{R}} (R_{m})^{1-\rho_{R}} (\pi_{t}/\pi)^{\rho_{\pi} (1-\rho_{R})} (y_{t}/y)^{\rho_{y} (1-\rho_{R})}\},
\]
where variables without time index denote steady state values, $1 > \rho_{R} \geq 0$, $\rho_{\pi} \geq 0$ and $\rho_{y} \geq 0$. Given that we do not model real growth and that there is thus no trend in real money demand (that would have to be accommodated by an increasing outright money supply), the central bank...
sets the ratio of money supplied under repos to money supplied outright against T-bills equal to one \( (M^R_{t,TB} = M^H_{t,TB}) \), which ensures non-negative injections in equilibrium.

2.5 Equilibrium

In equilibrium, agents’ optimal plans are satisfied and prices adjust such that all markets clear. Specifically, the market clearing conditions for aggregate output \( y_t = \xi_t + s\cdot c_{p,t} + (1 - s)\cdot c_{i,t} \), housing \( H = s\cdot h_{p,t} + (1 - s)\cdot h_{i,t} \), where \( H \) denotes the fixed housing supply, deposits \( \int_0^1 D_{b,t} db = sD_{p,t} \), loans \( \int_0^1 B_{h,t}^M db = (s - 1)B_{i,t}^M \), and treasury securities \( B_{T,t} = \int_0^1 B_{h,t} db + B_{C,t} \) hold. Notably, asset purchases will be non-neutral, if the money supply constraint (12) is binding, i.e. if \( \eta_t = -\left( \partial \xi_t / \partial i_{b,t} \right) - (R^m_t - 1) > 0 \) (see 18), which requires a sufficiently large marginal reduction of banking cost by additional reserves. This will be the case when we simulate the financial crisis, where banks liquidity demand is particularly high. Further note that the term, \( R^m_t - 1 \), equals zero when the policy rate is at the zero lower bound. For a scenario where the money supply constraint (12) is non-binding, \( \eta_t = 0 \), the conditions (17) and (18) imply that the deposit rate \( R^D_t \) equals the expected policy rate up to first order, \( R^D_t \approx E_t R^m_{t+1} \). In the subsequent analysis, we will however focus on a situation where the economy faces a financial crisis, such that reserves and eligible asset are scarce and the money supply constraint (12) is binding even for a policy rate at the zero lower bound, \( R^m_t = 1 \) (see Hoermann and Schabert, 2015, for further details). A definition of a competitive equilibrium is given in Appendix 5.1.

3 The Effects of MBS Purchases in QE1

In this section, we evaluate the Fed’s QE1 program and within this program the role of MBS purchases applying the model developed in the previous section. QE1 was initiated at the onset of the financial crisis when the federal funds rate reached its zero lower bound and large drops in GDP and inflation were observed. Therefore, we first show that in our model a shock to the cost of financial intermediation is able to generate a crisis in the range of what has been observed while inducing the ZLB to be binding for several quarters. Second, given this crisis scenario, we study the effects of the full QE1 program and elaborate the special role of MBS purchases within this program. We further show that MBS purchases are more effective than purchases of long-term bonds. Finally, we analyze the effects of the Fed’s announcements of MBS purchases. To derive a solution of the model with an occasionally binding zero lower bound, we compute the piecewise-linear perturbation solution suggested by Guerrieri and Iacoviello (2015).

The framework essentially features two channels by which MBS purchases affect the private sector behavior. As a precondition for the non-neutrality of central bank asset purchases at the ZLB, agents have to assign a positive value to the liquidity of eligible assets \( (\eta_t > 0) \). As discussed in detail in Hoermann and Schabert (2015), this does not rely on a non-zero monetary policy rate.
$R_t^m$ (see also (18)), such that liquidity premia also occur when the latter is at the zero lower bound. Given that the eligibility of assets tends to reduce the demanded interest rate (see (14) or (16)), the central bank influences, via the first channel, the price of the purchased asset, e.g. the MBS yield (and thus the loan rate) with MBS purchases or the long-term bond yield with purchases of long-term bonds. As the second channel, the central bank’s increased supply of high powered money due to asset purchases of any type tends to reduce the banks’ costs and thereby the loan rate (see (16) with $\partial(\partial\Xi_t/\partial b_{M^t})/\partial q_{b,t} < 0$). Thus, both channels tend to stimulate lending and real activity, while the effects of MBS purchases are relatively more pronounced (than the effects of treasuries purchases), since they affect the loan rate directly by the first channel.

3.1 Calibration

We use standard parameter values whenever it is possible and set the remaining parameters at values that allow matching selected targets. Specifically, we apply time series processes for the Fed’s MBS purchase programs and set the parameters of the processes using detailed information of the actual Fed’s MBS purchases. Further, the parameters of the banking cost function are set such that the model replicates unconditional moments of MBS yields as well as the estimated effects of QE1 on the spread between MBS yields and long-term bond yields reported by Krishnamurthy and Vissing-Jorgensen (2011).

One time period is assumed to be a quarter. To calculate the long run values, we use quarterly US data from the FRED database for the time period 1990Q1 to 2008Q3, excluding data of the recent financial crisis. The reason for not applying earlier data is that the housing finance system and the mortgage market has been greatly restructured in the 80’s. The total housing stock is normalized to $H = 1$ and the supply side parameters are set at standard values $\alpha = 2/3$ and $\epsilon = 21$. For the fraction $\theta$ of retailers who cannot flexibly adjust their prices we apply an intermediate value of $\theta = 0.85$. Estimating a closely related model with housing, Guerrieri and Iacoviello (2017) find a high value 0.915 for $\theta$. Further, Chen et al. (2012), who estimate their model used for studying long-term bond purchases, find an even higher value of 0.929, while other related studies use values of 0.78 (Gertler and Karadi, 2011 and 2013) and 0.75 (DEFK). The inflation rate, the policy rate, and the treasury rate are set at the empirical sample means for 1990Q1-2008Q3, leading to $\pi - 1 = 0.46\%$, $R^m - 1 = 1.06\%$, and $R^G - 1 = 1.09\%$. As discussed above, the loan rate $R^L$ is assumed to equal the MBS yield. Given that we do not model the mark-up of loan rates over MBS yields (see Figure 1), we set $R^L$ to the sample mean of the Fannie Mae 30-year current-coupon MBS yields implying $R^L - 1 = 1.68\%$. We calibrate the discount factor of patient households to achieve a real interest rate of 3.2\%, implying $\beta^p = 1/1.032^{1/4} \approx 0.992$.

See Appendix 5.2 for further details on the data and their sources.

We calculate the real interest rate as the average of the sample means for 1990Q1-2008Q3 of the deflated returns on 1-year treasury bills and 10-year treasuries, which implies a real interest rate of 3.2% (see DEFK).
<table>
<thead>
<tr>
<th>Description</th>
<th>Source/Target</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount factor patient households</td>
<td>real rate of 3.2%</td>
<td>$\beta^p$</td>
<td>0.992</td>
</tr>
<tr>
<td>Discount factor impatient households</td>
<td>Iacoviello (2005)</td>
<td>$\beta^i$</td>
<td>0.95</td>
</tr>
<tr>
<td>Production elasticity</td>
<td>Iacoviello (2005)</td>
<td>$\alpha$</td>
<td>2/3</td>
</tr>
<tr>
<td>Elasticity of substitution</td>
<td>Iacoviello (2005)</td>
<td>$\epsilon$</td>
<td>21</td>
</tr>
<tr>
<td>Pledgeable fraction of housing</td>
<td>$b^M_t/y = 0.93$</td>
<td>$\phi$</td>
<td>0.8854</td>
</tr>
<tr>
<td>Share of patient households</td>
<td>KVW (2014)</td>
<td>$s$</td>
<td>2/3</td>
</tr>
<tr>
<td>Weight of housing in utility</td>
<td>$p_h/y = 6.8$</td>
<td>$\gamma^h$</td>
<td>0.0711</td>
</tr>
<tr>
<td>Weight of labor in utility</td>
<td>$n^T = 0.33$</td>
<td>$\gamma^n$</td>
<td>6.5637</td>
</tr>
<tr>
<td>Price rigidity</td>
<td>see text</td>
<td>$\theta$</td>
<td>0.85</td>
</tr>
<tr>
<td>Banking cost function</td>
<td>$R^L - 1 = 1.68%$</td>
<td>$\kappa$</td>
<td>0.0519</td>
</tr>
<tr>
<td>Banking cost function</td>
<td>path of obs. MBS yields</td>
<td>$\iota$</td>
<td>0.033</td>
</tr>
</tbody>
</table>

Notes: KVW (2014) refers to Kaplan et al. (2014) in the text.

Given that the steady state deposit rate satisfies $R^D = \pi/\beta^p$ (see (5)), we get $R^D - 1 = 1.27\%$.

The value for the discount factor of impatient households $\beta^i$ is taken from Iacoviello (2005), $\beta^i = 0.95$. The fraction of impatient and thus constrained consumers is set at $1/3$, i.e. $s = 2/3$, which is the preferred estimate of Kaplan et al. (2014) for the share of hand-to-mouth consumers in the US. We use the following specification of the utility function

$$u(c_{*,t}, h_{*,t}, n_{*,t}) = \log c_{*,t} + \gamma^h \log h_{*,t} - \gamma^n n_{*,t}^2/2, \quad (19)$$

where $* = i, p$. The parameters $\gamma^n$ and $\gamma^h$ are calibrated such that total hours worked in the steady state is $n^T = 1/3$ and housing wealth to quarterly GDP equals $p_h/y = 6.8 = 4 \cdot 1.7$ as the sample mean of total housing wealth to annual GDP is 1.7, implying $\gamma^n = 6.5637$ and $\gamma^h = 0.0711$.

To account for the relative size of MBS purchases, we consider the sample mean of total Agency MBS outstanding to annual GDP of 23.25% and set the pledgeable fraction of housing $\phi$ such that $b^M_t/y = 4 \cdot 23.25$. This leads to a value of $\phi = 0.8854$, which closely relates to corresponding values in related studies, e.g. 0.9 in Guerrieri and Iacoviello (2017). Further, following its empirical counterpart prior to the crisis, total government debt in real terms is assumed to equal 60% of annual GDP in the steady state and the parameters of the policy rate rule and the tax rule are set to standard values: $\rho_R = 0.75$, $\rho_\pi = 2$, $\rho_y = 0.15$. Following DEFK we set $\psi = 0.1$ and the decay rate of coupon payments is set to $\rho = 0.967$ implying a duration of long-term bonds of 30 quarters as in Chen et al. (2012).

Regarding the banking cost function, which closely relates to the one applied in Curdia and
Woodford (2011), we consider the following functional form

\[ \Xi_t = z_t^{\Xi} \kappa \left( \frac{b_{h,t}^M}{m_t^H \pi_t + i_t} \right)^\iota, \]

where \( z_t^{\Xi} \) is a banking cost shock satisfying \( \log z_t^{\Xi} = \rho^{\Xi} \log z_{t-1}^{\Xi} + \varepsilon_t^{\Xi} \) with \( \varepsilon_t^{\Xi} \sim \text{i.i.d.} (0, \sigma^2_{\Xi}) \) and \( 0 < \rho^{\Xi} < 1 \). The parameters of the cost function, \( \kappa > 0 \) and \( \iota > 0 \), are set as follows. We calibrate the level parameter \( \kappa \) to match average MBS yields between 1990Q1 and 2008Q3, i.e. such that in the steady state \( R^L - 1 = 1.68\% \) is matched. The elasticity of banking costs with respect to the ratio of loans to reserves \( \iota \) is set such that Krishnamurthy and Vissing-Jorgensen’s (2011) estimates of QE1 effects on the spread between MBS yields and treasury yields are matched. Translated into quarterly spreads, they find that in 2008Q4 the spread between 30 year MBS yields and 30 year treasury yields fell by 7 basis points and in 2009Q1 by additional 1.5 basis points. These empirical findings are marked with ‘x’ in Figure 5. The value for \( \iota \) is set such that the difference between the model implied spread and these empirical findings is minimized. The resulting parameter values for the banking cost function are \( \kappa = 0.0519 \) and \( \iota = 0.033 \), which implies a drop in the spread of 5.1 basis points in 2008Q4, 2 basis points less than the corresponding estimate of 7, and of 10.5 in 2009Q1, 2 basis points above the corresponding estimate of 8.5. Our baseline calibration is summarized in Table 1.

3.2 Simulating the Fed’s Large Scale Asset Purchases

In this section, we describe how we specify asset purchases during QE1. For this, we first describe how we approximate the MBS purchase program and afterwards how the remaining part of QE1 purchases is simulated. Given that MBS purchases were conducted after a short pre-announcement period, we model the MBS purchases as announced changes in the instrument \( z_t \). In accordance with the implementation lag of one period, we specify the policy intervention as a shock that is observed in the announcement period \( \varepsilon_t > 0 \) that affects the instrument in period \( t + 1 \) but not in period \( t \). Thus, we allow for agents’ responses in the announcement period, even without any changes in the fraction of purchased MBS. The choice of the particular time series models for the instrument \( z_t \) is guided by the actual announcements and implementation of US Fed MBS purchases.

Based on the first announcement of QE1, we define the period 2008Q4 as the first quarter of QE1.\footnote{Table 2 in Appendix 5.4 shows the MBS purchases of the Fed in billions of dollars as well as relative to total agency MBS outstanding during QE1.} Total purchases at the end of the QE1 program actually exceeded the initially announced volume of $500 billion, as the US Fed expanded the program after several months. Specifically, in March 2009, the Fed announced an expansion by purchasing "up to an additional $750 billion of agency mortgage-backed securities" (FOMC, 2009). To approximate the actual MBS purchases
during QE1, given by the black dashed line in the left panel of Figure 3, we therefore consider two time series processes for $z_t$, an initially announced part (labeled with $A$) and the expansion (labeled with $B$). In accordance with the initial announcement regarding the volume and the duration ("expected to take place over several quarters", see Fed, 2008), we approximate part $A$ by an AR(1) process given by

$$\log z_t^A = \rho^A \log z_{t-1}^A + \epsilon_{t-1}^A,$$

where $\epsilon_1^A = 0.047$ such that the initial purchase equals the observed one. The coefficient of autocorrelation is set at $\rho^A = 0.54$ such that this part of the program in isolation leads to significant purchases over six quarters adding up to 10% of total US Agency MBS outstanding (which made up about $500 billion in 2008Q4) in line with the initial announcement. This process $A$ is depicted by the dotted line with circles in the left panel of Figure 3.

For the second part of the program that was announced at the end of 2009Q1 (labeled with $B$), we assume that actual purchases started in the subsequent quarter, namely 2009Q2, implying again a one-period implementation lag. We consider a second process to approximate the actual purchases from 2009Q2 to 2010Q2, given the AR(1) process of the first part. Specifically, we model part $B$ as an AR(2) process given by

$$\log z_t^B = \rho_1^B \log z_{t-1}^B + \rho_2^B \log z_{t-2}^B + \epsilon_{t-1}^B,$$

with $\epsilon_2^B = 0.0195$, $\rho_1^B = 1.45$, and $\rho_2^B = -0.6$. The process $B$ is shown by the dotted line with crosses in the left panel of Figure 3. In total, the announcements and implementation of MBS purchases during QE1, $z_t$, are specified as the sum of the two processes given by

$$\log z_t = \log z_t^A + \log z_t^B,$$

which is depicted by the solid line in the left panel of Figure 3 and closely approximates the observed purchases (dashed line) in a sufficient way.

Long-term bond purchases during QE1 are simulated with an AR(1) process following

$$\log z_t^{LT} = \rho^{LT} \log z_{t-1}^{LT} + \epsilon_{t-1}^{LT},$$

where $\epsilon_1^{LT} = 0.0162$ such that the initial purchase in 2009Q2 relative to MBS purchases in 2009Q1 equals the observed one, i.e. $\frac{182bn}{237bn} \approx 0.77$. The coefficient of autocorrelation is set at $\rho^{LT} = 0.445$ such that total long-term treasury purchases sum up to the observed volume until 2010Q2, as can be seen in the top-right panel of Figure 3. We simulate the remaining part of QE1 by the choice of $\epsilon_t$ (see (12)) to approximate other components of QE1. Precisely, QE1 can be separated into outright MBS and long-term bond purchases, other outright purchases, like purchases of agency debt, and non-outright liquidity facilities, like the Term Asset-Backed Securities Loan
Figure 3: Approximation of QE1 | Left: MBS purchases during QE1 in % of total US Agency MBS outstanding (black dashed line) and the approximation by $z_t$ (red solid line), $z^A_t$ (green circled dotted line) and $z^B_t$ (blue crossed dotted line). Top right: Long-term bond purchases relative to 2009Q1 MBS purchases (black dashed line) and the approximation by $z^LT_t$ (red solid line). Bottom right: Remaining elements of QE1 (relative to 2009Q1 MBS purchases, black dashed line) and their approximation through $\epsilon^i_t$ (red solid line).

Facility (TALF). Following DEFK (2017), we normalize the latter to 0 in 2008Q3 and consider the increase of these facilities during QE1. The dashed line in the right-hand panel of Figure 3 shows the sum of other outright purchases and non-outright liquidity facilities. In each period $\epsilon^i_t$ is set to approximate this part of QE1 exactly, as illustrated in the bottom-right panel of Figure 3. Concretely, the first impulse is set to a value such that injections through these facilities are 4.54 times ($\frac{1076}{237} = 4.54$) as large as injections in 2009Q1 through MBS purchases ($\epsilon^i_1 = 1.333$).

3.3 Financial Crisis and Zero Lower Bound

The first quantitative easing program (QE1) was implemented during the financial crisis, where output and inflation dropped and the policy rate hit the ZLB for the first time. Therefore, we implement – like DEFK – the breakout of the crisis and the announcement of the intervention within the same period, namely 2008Q4, which corresponds to period 1 in the simulations. In this subsection, we show that the banking cost shock $z^F_t$ is able to generate a crisis in our model that closely replicates the economic situation when QE1 was announced.

Before turning to our crisis scenario, consider first the effects of a small banking costs shock. To illustrate how this shock affects macroeconomic aggregates, the impulse response functions for a one percent increase in banking costs are given in Figure 8 in Appendix 5.3. In sum, the banking cost shock reduces banks’ loan supply and increases MBS yields, which lead to a fall in
house prices. Reduced lending, higher rates on loans, and the drop in the value of their collateral, worsen the borrowing conditions of impatient households, such that these households demand less consumption goods and housing. In total, the shock has contractionary effects on output and consumption as well as on wages and inflation. These reactions are similar to the ones described in Ajello (2016), who suggest this type of shock as the major impulse that triggered the financial crisis.

Figure 4: Comparison of core macroeconomic variables during QE1 [Left: Data on Output, Inflation, Federal Funds Rate, and Spread between MBS and Treasury Yields. Right: Model implied paths of corresponding variables.]
As DEFK, we aim at replicating the output and inflation paths at the onset of the financial crisis as well as the length of the ZLB period. In addition, we account for the spread between the yields of MBS and treasuries as a relevant financial market statistic to assess the predictive power of the model. For this, we have three parameters at our disposal, the size of the impulse $\varepsilon_1^x$ and the autocorrelation $\rho^x$ of the banking cost shock, and the elasticity of banking costs with respect to the ratio of loans to reserves $\xi$. As mentioned earlier, we set $\xi$ to match the initial reactions of the MBS and treasury yields to QE1 as identified by Krishnamurthy and Vissing-Jorgensen (2011). Further, we set the impulse $\varepsilon_1^x$ to replicate the observed drop in output and $\rho^x$ to achieve a binding ZLB period for several quarters.\footnote{The banking cost shock that leads to the scenario in Figure 4 has the impulse of $\varepsilon_0^x = 0.579$ and autocorrelation of $\rho^x = 0.948$.}

Figure 4 shows our crisis scenario and compares it to the evolution of selected variables in the data (see left column). The right column of Figure 4 shows the evolution of the corresponding variables in our simulation based on a scenario with the full QE1 intervention approximated by $z_{LT}^t$, $z_t$ and $\varepsilon_t^i$ as described in section 3.2, i.e. it replicates the situation as it was with crisis and intervention. The maximum drop of output in the data is $-7.6\%$ and is observed in the third quarter of the intervention. In the simulation, we replicate this maximum drop in GDP, while it occurs already one period before and is less persistent.\footnote{Our crisis scenario approximates the drop in GDP more closely than DEFK, whose simulation accounts for 56% of it. As DEFK, we can not replicate the hump-shaped reaction of output in the data.} The maximum drop in inflation relative to 2008Q3 was $-2.6\%$ and occurred in period 4 in the data, while the simulation generates a drop of $-2.4\%$ in period 1.\footnote{Note that the steady state inflation rate in our model is close to the annualized inflation target of 2% DEFK consider.} Further, our scenario replicates the timing and the recovery (about 30 b.p. drop over 6 quarters), while it overestimates the maximum hike of the MBS-treasury spread (about 30 b.p. vs. 60 b.p.).\footnote{The empirical spread is measured as the (quarterly) Fannie Mae 30-year current coupon MBS yields minus the yield of 30 year constant maturity treasury yields. As seen in Figure 1, the mortgage rate would imply a larger spread.} Finally, the duration of the ZLB episode is 5 quarters in our simulation, one quarter less than the ZLB episode that DEFK consider for their baseline specification and which is in the range of what Gust et al. (2013) and Kulish et al. (2014) estimate for the expected ZLB duration at that time. In sum, our crisis scenario replicates the core variables during the crisis reasonably well.

### 3.4 Effects of QE at the Zero Lower Bound

In this section, we present quantitative results regarding the macroeconomic effects of the approximated Fed's QE1 program and examine the specific role of MBS purchases within this program. The analysis is conducted separately for the full QE1 program and for just MBS purchases of the QE1 program, where we treat the policy interventions as realization of distinct specifications of...
data generating processes for the particular policy instruments $z_{LT}^t$, $z_t$, and $\epsilon_i^t$. Further, we conduct counterfactual analyses that show how the effects would be if the outright purchases consisted of purchases of long-term bonds or MBS exclusively. Finally, we study the role of pre-announcement of MBS purchases.

3.4.1 Benchmark Analysis: Full QE1 vs. MBS Only

Our crisis scenario described in section 3.3 provides responses of macroeconomic variables replicating the economic situation including the central bank intervention. To identify the macroeconomic effects of the Fed’s QE1 program and the MBS purchases within it, we compute in a counterfactual analysis responses of the same variables to the same crisis shock, but without the monetary policy intervention. We then compute the difference between both responses, i.e. responses including and excluding the policy intervention, which reveals how QE1 has affected macroeconomic variables at the ZLB. This difference between actual and counterfactual response is shown in Figure 5. Put differently, Figure 5 shows the isolated effects stemming from QE1. In particular, the black solid lines show the effects of the full QE1 program and the red dashed lines the effects of MBS purchases within QE1.

First of all, our results indicate that the isolated QE1 program increased GDP (relative to its steady state value) by about 0.92% and inflation by about 0.18% in 2008Q4, while the increases in 2009Q1 were 2.89% for output and 0.48% for inflation. This means that without QE1 the drop in output in 2008Q4 would have been 12% (-7.6% vs. -8.5%) and in 2009Q1 more than two times (-2.5% vs. -5.4%) larger. Moreover, we would have observed in 2008Q4 a 7% (-2.4% vs. -2.6%) and in 2009Q1 a 45% (-1.1% vs. -1.6%) larger drop in inflation. Cumulating the effects over 4 quarters (2008Q4-2009Q3), we get a cumulative increase in GDP (inflation) by 6.8% (0.89%) for the full QE1 program and by 5.9% (0.75%) for MBS purchases only. The cumulative effects indicate that without the Fed’s intervention the drop in output would have been about 60% and in inflation about 20% larger. These numbers relate to DEFK, who find for their baseline scenario that the contraction in output would have been at 30% and the decline in inflation 40% larger without interventions. Thus, our analysis implies larger output effects and smaller inflation effects, which might at least partially due to the property that supply-side (demand-side) borrowers specifically benefit from the central bank intervention in DEFK (our model).

Regarding the effects of isolated MBS purchases within QE1, Figure 5 shows that even in the first quarter of the intervention (2008Q4), where MBS purchases were announced but not yet conducted, they increased GDP by 1.19% and inflation by 0.19%. In 2009Q1, MBS purchases in isolation increased GDP by 2.3% and inflation by 0.38%, which make up about 80% of the increase in GDP and inflation. Moreover, for the horizon of 1 year, our results indicate that 87% of the cumulative increase in GDP and 84% of the cumulative increase in inflation can be attributed
Figure 5: Effects of Full QE1 Program (black solid line) and MBS Purchases only (red dashed line).
[Note: The lines show isolated effects of central bank interventions in terms of relative deviations from steady state, except for * showing level deviations from steady state and ** showing basis point deviations from steady state. Blue ‘x’ marks in the ‘Spread’ panel show spread effects of QE1 as estimated by Krishnamurthy and Vissing-Jorgensen (2012) to MBS purchases. Notably, MBS purchases contributed to roughly the half of the entire QE1 program (see Figure 2). Hence, we can conclude that MBS purchases were clearly more effective than the other elements of QE1 (see also below).

The main effects of MBS purchases can be summarized as follows: MBS purchases lead to a reduction in MBS yields and mortgage rates as they reduce the costs of financial intermediation and they increase lending. The fall in mortgage rates together with the increase in lending leads
to an increase in housing demand which raises house (collateral) prices and induces a relaxation of the collateral constraint of impatient households (with a relatively higher marginal propensity to consume). Hence, they increase their borrowing and thereby their consumption and housing, which stimulates aggregate demand. Moreover, borrowers benefit from debt deflation since the increased money supply lead to a rise in inflation. As the deposit rate also decreases with lower banking costs, patient households’ intertemporal substitution also contribute to the increase in current GDP.

3.4.2 Counterfactual Analysis: Purchases of MBS vs. Government Bonds

In our benchmark analysis, MBS purchases provide the largest effects of the full QE1 program, despite constituting just about 50% of the overall QE1 program, indicating that MBS purchases are particularly more effective than treasury purchases.\footnote{This finding relates to the results of Gertler and Karadi (2013), who find that, compared to treasuries, purchases of private securities have larger effects.} This provides a novel result on QE1, which has up to now been overlooked, as most theoretical studies on QE1 only consider one type of asset to be purchased by the central bank (see, e.g., DEFK) and have so far neglected MBS. In this section, we show that MBS purchases are more effective than treasury purchases and why it is important to account for the composition of QE1.

In our benchmark analysis we modeled purchases of public and private securities to approximate QE1 as precise as possible. In the following counterfactual analyses, we hold the liquidity facilities part of QE1 approximated by $\epsilon^i_t$ fixed and specify the remaining part of QE1 either by MBS or long-term treasuries only, i.e. we either consider the instrument $z^{LT}_t$ or $z_t$. In the first case, we abstract from MBS purchases and model the observed MBS purchases through long-term bond purchases. In the second case, we instead assume that all QE1 asset purchases were conducted in terms of MBS. The results of these analyses are shown in Figure 6. The black solid line shows our benchmark analysis with purchases of both types of securities. The blue dotted line shows the results for the first counterfactual purchase program where all QE1 asset purchases were assumed to be conducted in terms of long-term treasuries (which is achieved by setting $z_t = z = 1$ shutting down MBS purchases and by a compensating adjustment of $z^{LT}_t$). The red dashed line shows the results for the second counterfactual purchase program in which QE1 asset purchases were assumed to be only conducted in terms of MBS (for which $z^{LT}_t = z^{LT} = 1$ and $z_t$ is adjusted accordingly). The bottom row of Figure 6 illustrates that total injections are identical in both cases, as the blue dotted line in the middle panel coincides with the red dashed line in the right panel and vice versa, while the lines in the left panel are identical.

Given identical injections, what are the differences in total effects? If the total volume of QE1 was spent on long-term treasuries only, the output (inflation) effects of the intervention would have been 22% (25%) smaller in 2008Q4 and even 41% (50%) smaller in 2009Q1. The one-year
cumulated effects indicate a 29% (34%) smaller output (inflation) effect. In contrast, when only MBS were purchased in QE1, the output (inflation) effects of the intervention would have been identical in 2008Q4, as long-term bonds purchases were announced in 2009Q1, and would be only slightly larger afterwards due to the relatively small size of observed long-term bond purchases. Notably, the MBS-treasury spread shows the most substantial effect if only MBS were purchased, which reflects the direct price effect of asset purchases. Hence, if QE1 were only conducted in terms
of treasury securities in the model, one would underestimate its effects by about 30%. Notably, pure long-term treasury purchases lead to much smaller spread responses (60% in period 1 and 50% in period 2), given that they tend to directly lower the treasury yield and reduce the MBS yield only indirectly by the increase in money supply. In total, MBS purchases are more effective in improving borrowing conditions, and thereby real activity and inflation, than an equally sized purchase program in terms of treasuries.

3.4.3 The Role of Pre-announcements

In this final section, we demonstrate that the pre-announcement of purchases can already exert substantial macroeconomic effects. Specifically, we analyze the effects of this pre-announcement by looking at the counterfactual scenario that generates the effects of the same MBS purchase program without pre-announcement. In Figure 7, the red dashed lines show the effects of pre-announced MBS purchases within QE1 as it was considered before and the blue dotted lines show the effects of the same MBS purchase program if it were conducted without pre-announcement.

Consider for example the shock process for what we labeled as part A of QE1 MBS purchases: \( \log z_t^A = \rho_A \log z_{t-1}^A + \epsilon_{t-1}^A \), where we have set \( \epsilon_1^A = 0.047 \). This means that in 2008Q4 this shock is observed, but since \( z_t \) reacts with a delay, the purchase is made in 2009Q1. For the counterfactual scenario, we modify this such that \( z_t \) is affected contemporaneously by \( \epsilon_t : \log z_t^A = \rho_A \log z_{t-1}^A + \epsilon_t^A \), where we now set \( \epsilon_2^A = 0.047 \) implying that the initial purchase in 2009Q1 equals the observed one. The same is done for part B.

As one would expect, without pre-announcement the MBS purchase program has no effects in period 1 (2008Q4), and the MBS treasury yield spread starts to fall in period 2 and is almost identical to the case with a pre-announcement from period 3 onwards. The peak effect of the announced purchases reveals in 2009Q1, when purchases start and part B is announced. In contrast, if purchases were not pre-announced their maximum effect would be in 2009Q2, when purchases of part B start. The reason for this effect is that banks account for the possibility to liquidate today’s mortgage debt in the next period, which tends to lower the mortgage rate (see 16) and thus borrowing already today. With the pre-announcement the maximum increase in output (inflation) is 2.3% (0.38%), whereas without announcement it is 1.8% (0.29%), which is more than 20% smaller. The cumulative effects indicate a 25% smaller output and a 30% smaller inflation effect without pre-announcement. More importantly, with pre-announcement, the MBS purchase program was already effective in 2008Q4, without having conducted any MBS purchase. Thus, the price effects of pre-announced MBS purchases contributed substantially to the overall stimulating effects of QE1 at the onset of the financial crisis.
Figure 7: Pre-announcement effect [Red dashed line: pre-announced MBS purchases. Blue dotted line: MBS purchases without announcement. The lines show relative deviations from steady state, except for * showing level deviations from steady state and ** showing basis point deviations from steady state.

4 Conclusion

This paper analyzes the Fed’s purchases of mortgage-backed securities in QE1, which have so far been overlooked by the theoretical literature on unconventional monetary policy, despite the central role of mortgage debt in both, the outbreak of the crisis and the policy response. Specifically, we provide a quantitative analysis of the macroeconomic effects of the Fed’s asset purchases and, in particular of its MBS purchases, during QE1.
We show that with an adverse shock to financial intermediation costs our model is able to replicate the observed paths of output, inflation, the MBS-treasury spread, and of the expected ZLB duration at the beginning of the financial crisis. For the full QE1 program, we find that without the Fed’s intervention the cumulative drop in output (for 2008Q4-2009Q3) would have been 60% larger. For the isolated MBS purchases within QE1, which constitute about the half of the entire QE1 program, we find that they contributed by more than 80% to the overall output effects, highlighting their particular importance. We further find that purchases of MBS were much more effective than purchases of long-term government bonds, and that the output effects would have been 30% smaller if all purchases were conducted in terms of treasuries. Moreover, we show that without the pre-announcement of QE1 the cumulative output effects of MBS purchases would have been reduced by 25%. In summary, MBS purchases played a predominant role in the QE1 program and have successfully stimulated aggregate demand, output and prices.
References


Gertler, Mark and Peter Karadi. (2013) "QE 1 vs. 2 vs. 3...: A Framework for Analyzing Large-Scale Asset Purchases as a Monetary Policy Tool.” International Journal of Central Banking, 9, 5–53.

Guerrieri, Luca and Matteo Iacoviello. (2015) "Occbin: A Toolkit to Solve Models with

**Guerrieri, Luca and Matteo Iacoviello.** (2017) ”Collateral Constraints and Macroeconomic Asymmetries.” Journal of Monetary Economics, 90, 28–49.


**Hancock, Diana and Wayne Passmore.** (2011) ”Did the Federal Reserve’s MBS purchase program lower mortgage rates?” Journal of Monetary Economics, 58, 498–514.


**Krishnamurthy, Arvind, and Annette Vissing-Jorgensen.** (2011) ”The Effects of Quantitative Easing on Interest Rates: Channels and Implications for Policy” Brookings Papers on Economic Activity, Fall 2011.

**Kulish, Mariano, James Morley, and Tim Robinson.** (2014) ”Estimating DSGE Models with Forward Guidance” manuscript.

**Schabert, Andreas.** (2018) ”Central Bank Asset Purchases as a Corrective Policy.” manuscript.

5 Appendix

5.1 Equilibrium

Definition 1 A rational expectations equilibrium is a set of sequences \( \{c_{p,t}, h_{p,t}, n_{p,t}, c_{i,t}, h_{i,t}, n_{i,t}, p_{h,t}, w_t, \pi_t, \omega_t, b^M_{i,t}, R^m_t, R^c_{i,t}, \eta_t, \Xi_t, b^M_t, m^H_{i,t}, m^R_{TB}, m^H_{TB}, \vartheta_{i,t+1}, \vartheta_{i}, n_t, n^H_t, m^c_t, Z_t, Z_{2,t}, y_t, v_t, b_{T,t}, \tau_t \}_{t=0}^{\infty} \) satisfying the optimality conditions of patient households

\[
\begin{align*}
\gamma^h h_{p,t}^{-1} & = p_{h,t} c_{p,t}^{-1} - \beta^p E_t c_{p,t+1}^{-1} \phi_t, \\
\gamma^a n_{p,t} & = w_t c_{p,t}^{-1}, \\
1/R^D_t & = \beta^p E_t (c_{p,t+1}^{-1} c_{p,t}^{-1} \pi_t^{-1}),
\end{align*}
\]

impatient households

\[
\begin{align*}
\gamma^h h_{i,t}^{-1} & = c_{i,t}^{-1} p_{h,t} - \beta^i E_t c_{i,t+1}^{-1} p_{h,t} - \omega_t \phi_t \pi_t^{-1} + \omega_t, \\
\gamma^a n_{i,t} & = w_t c_{i,t}, \\
c_{i,t}^{-1}/R^L_t & = \beta^i E_t (c_{i,t+1}^{-1} \pi_t^{-1}) + \omega_t, \\
c_{i,t} + p_{h,t} [h_{i,t} - h_{i,t-1}] + b^M_i / R^L_t = b^M_{i,t-1} \pi_t^{-1} + w_t n_{i,t} + \pi_t, \\
b^M_{i,t} & = -\phi_t \pi_t^{-1} p_{h,t+1} h_{i,t}, \text{ if } \omega_t > 0, \\
or b^M_{i,t} > -\phi_t \pi_t^{-1} p_{h,t+1} h_{i,t}, \text{ if } \omega_t = 0,
\end{align*}
\]

banks

\[
\begin{align*}
1/R^D_t & = E_t (\vartheta_{i,t+1}^{-1} \phi_t), \\
1/R^G_t & = (1/R^D_t) (1 + E_t [\eta_{t+1} (1 + c^t_{i,t+1}) / R^m_{t+1}]), \\
q^L_t & = E_t [(c^L_{i,t+1} / R^D_t) (1 + \eta_{t+1} (z^L_{i,t+1} - 1) / R^m_{t+1})], \\
1/R^L_t & = (1/R^D_t) (1 + E_t [\eta_{t+1} (z^L_{i,t+1} - 1) R^m_{t+1}]) - \partial \Xi_t / \partial b^M_t, \\
1 & = (1/R^D_t) - E_t (\vartheta_{i,t+1} \partial \Xi_t / \partial m^H_t), \\
R^m_t & = 1 - \frac{\partial \Xi_t}{\partial \pi_t} - \eta_t, \\
\Xi_t & = \Xi_t \pi_t (b^M_t / (m^H_{t-1} / \pi_t + \pi_t))^t, \\
b^M_t & = (s - 1) b^M_{i,t},
\end{align*}
\]

\[
\begin{align*}
m^H_{TB} + m^R_{TB} & = m^H_{i,t-1} / \pi_t + b^M_{i,t-1} / \pi_t R^m_t, \text{ if } \eta_t > 0, \text{ or } m^H_{TB} + m^R_{TB} < m^H_{i,t-1} / \pi_t, \text{ if } \eta_t \neq 0, \\
i^L_{i,t} & = (z^L_{i,t} - 1) b^L_{i,t} / \pi_t R^m_t, \text{ if } \eta_t > 0, \text{ or } i^L_{i,t} < (z^L_{i,t} - 1) b^L_{i,t} / \pi_t R^m_t, \text{ if } \eta_t = 0 \text{ (36)} \\
i^M_{i,t} & = (z^L_{i,t} - 1) b^M_{i,t-1} / \pi_t R^m_t, \text{ if } \eta_t > 0, \text{ or } i^M_{i,t} < (z^L_{i,t} - 1) b^M_{i,t-1} / \pi_t R^m_t, \text{ if } \eta_t = 0 \text{ (37)}
\end{align*}
\]
the market clearing conditions

\[ y_t = \Xi_t + s \cdot c_{p,t} + (1 - s) c_{i,t} \]

\[ H = s \cdot h_{p,t} + (1 - s) h_{i,t} \]

and transversality conditions, given the with fixed supply \( H \), initial values \( b_{-1} > 0, b_{T,-1} > 0, m_{H,t} > 0, \pi_{-1} > 0, v_{-1} = 1 \), and the exogenous processes for \( \{z_{t}^{LT}, z_{t}, z_{t}^{\Xi}\}_{t=0}^{\infty} \) and i.i.d. innovations with mean zero \( \{\epsilon_{t}^{LT}, \epsilon_{t}, \tilde{\Xi}_{t}\}_{t=0}^{\infty} \) and \( \partial \Xi_{t}/\partial b_{i}^{M} = \epsilon_{i} \Xi_{t}/b_{i}^{M}, \partial \Xi_{t}/\partial m_{H,t}^{M} = -i \Xi_{t}/(\pi_{t}(m_{H,t}^{M} - i_{t})). \)

5.2 Data

In this section, we briefly describe the data used in this study. Our main source is the FRED database (https://research.stlouisfed.org/fred2/).

To calibrate the long run value of \( R^{m} \), we use the time series on the effective federal funds rate (FEDFUNDS), of \( R^{G} \) the one-year treasury constant maturity rate (DCS1), of \( R^{L} \) the Fannie Mae 30-year current-coupon MBS yields (MTGEFNCL.IND from Bloomberg) and of \( \pi \) the GDP.
implicit price deflator (GDPDEF). To calculate the real interest rate following DFGK, we use in addition to DGS1 the 10-year treasury constant maturity rate (GS10).

To calibrate the shock processes, we use the time series on mortgage-backed securities held by the Federal Reserve (MBST) from FRED and the time series on US Agency MBS outstanding from the website of the Securities Industry and Financial Markets Association (SIFMA) (www.sifma.org). Finally, we use the GDP time series from FRED to get MBS held by the Fed in percent on GDP as it is shown in Figure 1.

5.3 Impulse Response Functions

The impulse responses displayed in Figure 8 show deviations of each variable $x_t$ from its steady state value $x_t^{obs}$, i.e. $x_t^{obs} = 100 \log \frac{x_t}{x}$, in response to a banking cost shock with an impulse of $\varepsilon_{\Xi 1} = 0.0333^2$, which implies a 1 percent increase in banking costs, and an AR coefficient of $\rho_\Xi = 0.948$. As an exception variables with a star (*) show deviations in basis points and variables with two stars (**) deviations in levels. We consider shocks that are sufficiently small to ensure a binding borrowing constraint for impatient households (6) and a binding money supply constraint (12). The IRFs refer to the benchmark calibration given in Table 1.

As Figure 8 shows, the banking cost shock, $\varepsilon_{\Xi 1} > 0$, lets banks increase money holdings and reduce loan supply in order to offset the increase in banking costs. Hence, MBS yields rise and house prices fall. This tightens the collateral constraint of borrowers, $\omega_t$ increases, who reduce their consumption and housing. In total, the shock has contractionary effects on output and consumption as well as on wages and inflation.

5.4 MBS Purchase Programs

The MBS purchase programs in QE1 is summarized in Table 2. The table shows MBS purchases of the Fed in billions of dollars as well as relative to total agency MBS outstanding for the corresponding time periods.

<table>
<thead>
<tr>
<th>Quarter of QE1</th>
<th>09Q1</th>
<th>09Q2</th>
<th>09Q3</th>
<th>09Q4</th>
<th>10Q1</th>
<th>10Q2</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>MBS Purchases in:</td>
<td>$ billions</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$</td>
<td>0</td>
<td>236</td>
<td>231</td>
<td>225</td>
<td>216</td>
<td>160</td>
<td>49</td>
</tr>
<tr>
<td>% of total AMBS</td>
<td>0</td>
<td>4.7</td>
<td>4.5</td>
<td>4.3</td>
<td>4.0</td>
<td>2.9</td>
<td>0.9</td>
</tr>
<tr>
<td>$z_t^{QE1} - 1$</td>
<td>0</td>
<td>0.047</td>
<td>0.045</td>
<td>0.042</td>
<td>0.037</td>
<td>0.03</td>
<td>0.022</td>
</tr>
</tbody>
</table>
Figure 8: Impulse response functions to a banking cost shock with $\varepsilon_\Xi^2 = 0.0106^2$ and $\rho_\Xi = 0.948$. All variables are shown in percentage deviations from steady state, except for: * deviation in basis points, ** deviation in levels.