Fiscal rules, interest payments on debt, and the irrelevance of the Taylor principle\textsuperscript{1}

Ludger Linnemann  
TU Dortmund University

Andreas Schabert\textsuperscript{2}  
TU Dortmund University

\textbf{Abstract:} We show that in New Keynesian models with non-neutral government debt, the Taylor principle ceases to be relevant for equilibrium determinacy if the government follows a fiscal rule of levying taxes in proportion to its interest payments on existing debt. This is in contrast with previous studies which typically have assumed that taxes respond to the level of debt, and have found either a confirmation or reversal of the Taylor principle depending on the feedback from debt to taxes. We find, instead, that the equilibrium effect of the interest rate on debt is crucial for determinacy. If, as in our model, taxes are raised in response to debt interest payments, the range of indeterminacy monotonically decreases with the fiscal feedback parameter. When interest payments are completely tax financed, indeterminacy is ruled out without any restrictions on monetary policy.

\textsuperscript{1}The authors wish to thank Matt Canzoneri, Stefanie Schmitt-Grohe, and Sweder van Wijnbergen for valuable discussions. The usual disclaimer applies.

\textsuperscript{2}Corresponding author. TU Dortmund University, Department of Economics, Vogelpothsweg 87, D-44227 Dortmund, Germany, Email: andreas.schabert@udo.edu.
1 Introduction

Current New Keynesian macro models imply that the interest rate policy of the central bank has to be restricted to eliminate the possibility of instability or indeterminate equilibria. In the most basic model setup, adherence to the Taylor principle, that is monetary policy raising interest rates more than one-for-one in response to inflation, usually ensures local stability and uniqueness of the equilibrium (e.g. Woodford, 2003). However, this result is qualified if there is a role for government debt, as has been shown by many recent studies that integrate fiscal policy aspects into the New Keynesian paradigm and assess the role of monetary-fiscal interactions in preventing indeterminacy. Examples include Leith and Wren-Lewis (2000), Leith and von Thadden (2008), Canzoneri and Diba (2005), Canzoneri et al. (2006), Schmitt-Grohe and Uribe (2007), Gali et al. (2007), among others. In each of these studies, some model feature – like the presence of finite horizons, transaction services of government bonds, distortionary taxation, or intertemporally non-optimizing agents – leads to the breakdown of Ricardian equivalence, such that the time path of government debt becomes a factor affecting the equilibrium allocation and prices. As a consequence, in these models policy interactions can lead to qualifications of the Taylor principle or even to reversed determinacy results, depending on the fiscal stance. It thus appears that, if government debt is relevant at all, preventing local indeterminacy becomes a matter of policy coordination.

In this paper we show that simple, intuitive, and easily implementable fiscal rules can be designed to make the Taylor principle entirely irrelevant, and hence to free monetary policy from the burden of adjusting interest rates in a specific way to avoid indeterminacy or instability. To give some background, note that the majority of previous papers follow Leeper (1991) in formulating monetary policy as an interest rate feedback rule whereby nominal interest rates respond to inflation, and fiscal policy as a debt targeting rule whereby tax revenues respond to the excess of government debt over a target level. As Leeper (1991) showed, local determinacy is ensured if either interest rate policy follows the Taylor principle (or is ‘active’) and fiscal policy stabilizes government debt by strongly raising taxes in response to excess debt levels (or is ‘passive’, in Leeper’s terminology), or otherwise if interest rate policy is passive (responds weakly to inflation, thus violating the Taylor principle) and fiscal policy is active in the sense of not responding strongly to debt movements. As a general rule, thus, monetary policy must adhere to the Taylor principle as long as government debt grows at less than the real interest rate, and must abandon it otherwise. The choice of monetary policies is thus restricted by the fiscal stance. Leeper (1991) derived his result in a setting without any direct effect of government bonds on private sector behavior. While his central findings with respect to the policy coordination necessary to prevent indeterminacy have been qualified in models with fully specified departures from Ricardian equivalence in the literature quoted above, the central insight that the Taylor principle remains relevant for local determinacy if fiscal policy is
debt-stabilizing has been corroborated.

In the present paper, in contrast, we argue that the necessity for policy coordination is a direct consequence of the form of the particular fiscal rule used in virtually all previous studies. Instead, we analyze a fiscal rule specifying that tax revenues cover a fraction of the government’s interest payments on debt, rather than responding to the level of debt as in the standard approach. This simple policy prescription turns out to have far-reaching implications for determinacy. Contrary to the intuition of the Taylor principle that posits a lower bound on the interest rate response to inflation, under the fiscal rule proposed here there is an upper bound that increases with the fraction of the budget that is tax financed. If fiscal policy services a large enough part of its interest payments through taxes rather than through issuance of new debt, restrictions on monetary policy to implement a locally determined equilibrium cease to exist, such that either an active monetary policy or even a permanently pegged nominal interest rate ensure determinacy. We argue that the key to this result is the direct response of taxes to interest rates, which renders the Taylor principle irrelevant for local determinacy.

Our argument is organized as follows. In the next section, we set up a simple New Keynesian model in which Ricardian equivalence fails to hold. While we use Canzoneri et al.’s (2008) specification of liquidity services from government bonds as a specific example for concreteness, the model can be viewed as generic in the sense that leading alternative formulations of debt non-neutrality, in particular the overlapping generations model, would deliver very similar aggregate equilibrium conditions and qualitatively the same determinacy results. In section 3, we contrast the determinacy results between the standard approach of letting taxes respond to debt levels with our fiscal rule where taxes respond to interest rate payments. In particular, we show that local indeterminacy can be ruled out regardless of monetary policy. In particular, we show that local indeterminacy can be ruled out regardless of monetary policy. Section 4 concludes.

2 The model

2.1 Model setup

The class of models we have in mind are varieties of the standard New Keynesian model (see Clarida et al., 1999, or Woodford, 2003). The only difference to the latter model is that the stock of government bonds is allowed to affect private sector savings decisions, in order to incorporate a role for public debt in equilibrium determination. As it turns out, several popular models where Ricardian equivalence does not hold have a common structure in that government debt enters the household consumption Euler equation. This is, for example, true for the overlapping generations models by Leith and Wren-Lewis (2000) and Leith and von Thadden (2008), which build on Blanchard (1985), as well as for the model where bonds provide transaction services in a way similar to money as in Canzoneri and Diba (2005), Canzoneri et al. (2006), and Linnemann and Schabert (2009). In all of these models, a temporary rise in real public debt (either through an increase in
perceived wealth, or in the stock of means of transactions) makes households reduce their savings and raise current consumption.

Although our argument would apply in all of these structures with Ricardian non-equivalence, it is easiest to work with a simple specific example. We choose the bonds-in-the-utility-function model of Canzoneri et al. (2008). Households maximize expected lifetime utility, where the period utility function is

$$u_t = c_t^{1-\sigma} - n_t^{1+\eta} + \phi \ln B_t/P_t + \psi \ln M_t/P_t,$$

where $c_t$ is consumption, $n_t$ is labor supply, $B_t$ are nominal one period government bonds, $M_t$ is the nominal money stock, $P_t$ is the price level, and $\sigma$, $\eta$, $\phi$, and $\psi$ are positive parameters. Real bonds appear in the utility function as a shortcut for their role as a means of facilitating transactions, for example in providing collateral in credit transactions, similar to the real money stock. The household budget is

$$m_t + b_t + c_t \leq w_t n_t + R_t b_{t-1} \frac{b_t}{\pi_t} + m_{t-1} - \tau_t + \tau_t^c + d_t,$$

where lower case letters denote real variables (i.e. $b_t = B_t/P_t$), $w$ is the real wage, $R_t$ the gross nominal interest rate on government bonds, $\pi_t = P_t/P_{t-1}$ is the gross inflation rate, $\tau_t$ is real lump-sum taxes, $d_t$ are firm profits, and $\tau_t^c$ is the lump-sum rebate of seigniorage revenues transferred by the central bank (introduced in the household budget in order to keep the government budget in terms of debt only for ease of exposition).

The first order conditions are

$$w_t = c_t^{\sigma} n_t^{\eta},$$

$$c_t^{\sigma} = \beta c_{t+1}^{\sigma} \frac{R_t}{\pi_{t+1}} + \phi b_t^{-1},$$

where $\beta \in (0,1)$ is the subjective discount factor, along with a first order condition for money holdings which is inessential here as interest rate setting will be assumed, and transversality conditions for money and bonds.

Equation (1) is a standard optimality condition for the consumption-leisure decision, and (2) is the consumption Euler equation. The latter is the only non-standard specification here, since real government bonds appear on the rhs. A higher stock of real bonds reduces their marginal value in transactions, such that households lower their bond investments and, all else equal, choose higher consumption spending, which is why Ricardian equivalence fails to hold.

Note that an Euler equation of the same form as in (2) would arise in an overlapping generations model of the Blanchard (1985) type (see e.g. equation 4 in Leith and von Thadden, 2008). The only difference is the interpretation of the parameter $\phi$ which would then be related to the average expected remaining lifetime of households, whereas here it
indicates the relative importance of the transaction services of bonds. Importantly, none of our results depends on the magnitude of \( \phi \); indeed, the result holds with \( \phi \) being arbitrarily close to zero, which is reassuring given that empirically one would tend to expect \( \phi \) to be a rather small parameter. Nonetheless, (2) imposes a restriction on the long-run debt level. In particular, in a steady state (denoted here and henceforth by dropping time subscripts from variables), it must be the case that

\[
b = \frac{\phi \sigma}{(1 - \beta R/\pi)}
\]

which in principle can be used to calibrate \( \phi \).

The firms sector is modelled as in the standard New Keynesian model, i.e. with labor as the single production factor under constant returns to scale and monopolistically competitive intermediate goods firms setting prices in a staggered way. Aggregate supply can then, to a first order log-linear approximation denoted by carets over variables, be summarized by the New Keynesian Phillips curve

\[
\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \chi \hat{w}_t,
\]

with \( \chi = (1 - \xi)(1 - \beta \xi)/\xi > 0 \), where \( \xi \) is the probability of not adjusting prices in a given period in the Calvo (1983) model.

Finally, the public sector consists of the fiscal authority and the central bank. The fiscal authority issues risk-less one-period bonds, and collects lump-sum taxes \( \tau_t \) from households, and spends an exogenous amount of spending on goods. For expositional simplicity (but without loss of generality), we normalize government spending on goods to zero, such that the government’s flux budget reads

\[
R_{t-1}B_{t-1} = B_t + P_t \tau_t,
\]

such that the services on outstanding debt are the only flow that needs to be financed, either by issuing new debt or by raising taxes, where issuance of debt is limited by the solvency condition \( \lim_{t \to \infty} B_t \Pi_t R^{-1}_t = 0 \).

The central bank transfers seigniorage directly to the households, \( P_t \tau_t^c = M_t - M_{t-1} \), and controls the nominal interest rate \( R_t \) on government bonds as a simplified standard Taylor rule,

\[
R_t / R = (\pi_t / \pi)^\rho, \quad \rho \geq 0, \quad R_t \geq 1.
\]

where the average interest rate \( R > 1 \) and the inflation target \( \pi \) are chosen in a way consistent with a long-run equilibrium.

To close the model, a fiscal policy rule must be specified. We will consider two types of fiscal rules. For the first rule, we follow the previous literature which used rules like in Leeper (1991) or Bohn (1998),

\[
I. \quad (\tau_t - \tau) = \kappa_I (b_{t-1} - b),
\]

where \( \kappa_I \) is the feedback coefficient which relates deviations of debt levels \( b_t \) from their
constant long-run target $b$, which is assumed to be consistent with private sector behavior in the long-run, to deviations of tax revenues $\tau_t$ from their long-run value $\tau$. From the literature quoted in the introduction, it is well known that the Taylor principle ensures determinacy if $\kappa_I$ is large enough to prevent debt from growing (locally) at a rate higher than the real interest rate.

What interests us here is an alternative fiscal rule. We consider a simple and easily implementable tax rule, which specifies nominal tax revenues as a fraction of the nominal interest payments on outstanding debt:

$$II. \quad P_t \tau_t = \kappa_{II} (R_{t-1} - 1) B_{t-1}, \quad \kappa_{II} \in (0, 1].$$

Combining the fiscal rule (6) with (3), we have that $b_t = (1 + (1 - \kappa_{II}) (R_{t-1} - 1)) b_{t-1}/\pi_t$, which shows i.) that since $\kappa_{II} > 0$ – debt never grows with a rate larger or equal to the interest rate and that solvency is never violated, and ii.) that a non-zero constant debt level in the steady state implies that the real interest rate $r \equiv R/\pi$ has to satisfy $r = (1 - \kappa_{II}/\pi) / (1 - \kappa_{II})$.

2.2 Equilibrium

In equilibrium the aggregate resource constraint holds in that output $x_t$ is equal to consumption spending $c_t$, and household optimality conditions are compatible with firm’s supply behavior summarized in the New Keynesian Phillips curve and the monetary and fiscal policy rules. The equilibrium can be reduced to a set of four variables $(x_t, \pi_t, b_t, R_t)$. To assess local equilibrium determinacy, the model is log-linearized at the steady state $(x, b, R, \pi)$, which is characterized by an output level that is pinned down by (1), labor demand and the aggregate resource constraint, by the steady state version of (2), by monetary policy choosing a relation $R(\pi)$, and fiscal policy setting the parameter of either of the rules in (5) or (6).

Log-linearizing the model at a steady state, where monetary and fiscal policy are assumed to be consistent with the same real interest rate $r$ across all versions considered, the private sector of the economy and monetary policy can be summarized by

$$\sigma \hat{x}_t = \sigma (1 - \psi) E_t \hat{x}_{t+1} - (1 - \psi) \hat{R}_t + (1 - \psi) E_t \hat{\pi}_{t+1} + \psi \hat{b}_t$$

(7)

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \omega \hat{x}_t,$$

(8)

$$\hat{R}_t = \rho \hat{\pi}_t,$$

(9)

and the transversality condition for bonds, where $\omega = \chi(\sigma + \eta) > 0$, and $\psi = (1 - \beta r) \in (0, 1)$. Since debt enters (7), the government budget, which in linearized terms reads

$$\hat{b}_t = r \hat{R}_{t-1} - r \hat{\pi}_t + r \hat{b}_{t-1} - \frac{\tau}{b} \hat{\tau}_t,$$

(10)

3 A variant of this rule has also been used in Linnemann and Schabert (2009).
becomes a relevant equilibrium condition, with given \( \hat{b}_{t-1} \) and \( \hat{R}_{t-1} \). Using the log-linear versions of the two types of fiscal rules (5) and (6), which read \( I. \) \( \hat{\tau}_t = \kappa_I \hat{b}_{t-1} \), and \( II. \) \( \hat{\tau}_t = \rho \hat{R}_{t-1} + \hat{b}_{t-1} - \hat{\pi}_t \), respectively, in the linearized government budget constraint, we close the approximate model in (7) to (9) by one of the following two conditions for public debt

\[
I.) \quad \hat{b}_t = r \left( \hat{R}_{t-1} - \hat{\pi}_t \right) + (r - \kappa_I) \hat{b}_{t-1} \\
II.) \quad \hat{b}_t = (1 - \kappa_{II}) r \hat{R}_{t-1} - \hat{\pi}_t + \hat{b}_{t-1}
\]

Note that these fiscal rules differ in that the fiscal policy parameter affects the coefficient on beginning-of-period debt \( \hat{b}_{t-1} \) in (11), whereas it affects the coefficient on the interest rate \( \hat{R}_{t-1} \) in (12).

3 Results

In this section, we first present determinacy results under the traditional rule \( I. \) and our rule \( II. \), and then proceed to discuss the intuition for the difference in results in the following section. Both variants of the model, (7)-(9) and either (11) or (12), consist of two non-predetermined variables \( (\hat{x}_t, \hat{\pi}_t) \) and two predetermined variables \( (\hat{b}_{t-1}, \hat{R}_{t-1}) \) such that stability and uniqueness requires in both cases the existence of two stable and two unstable eigenvalues.

Determinacy under a standard debt rule

We begin with rule \( I. \); although it is standard in the literature, we need the analysis for comparison purposes here, and analytical results in the context of the example model have not been presented in published papers so far. For the following analysis, we restrict our attention to the case where the fiscal feedback parameter \( \kappa_I \) of rule (5) does not exceed one or the gross real interest rate, \( \kappa_I \leq \min \{1, r\} \), which seems to be the empirically relevant range (e.g. Bohn, 1998).

**Proposition 1** Consider equilibrium sequences for \( \{\hat{x}_t, \hat{\pi}_t, \hat{b}_t, \hat{R}_t\}_{t=0}^{\infty} \) satisfying (7)-(9) and (11) with \( \kappa_I \leq \min \{1, r\} \). Then

1. for \( \kappa_I < r - 1 \), the equilibrium is locally determined if and only if
   \[
   \rho < 1 - \frac{\sigma \psi (1-\beta)}{\omega(1+\psi(1+\kappa_I)/(r-1-\kappa_I))}.
   \]
2. for \( \kappa_I > r - 1 \), the equilibrium is locally determined only if \( (\rho - 1) \omega \left( 1 + \frac{\psi(\kappa_I+1)}{r-1-\kappa_I} \right) > -\sigma \psi (1-\beta) \).

**Proof.** The model (7)-(9) and (11) can be written as

\[
\begin{pmatrix}
E_t \hat{x}_{t+1} \\
E_t \hat{\pi}_{t+1} \\
\hat{R}_t \\
\hat{b}_t
\end{pmatrix} = A
\begin{pmatrix}
\hat{x}_t \\
\hat{\pi}_t \\
\hat{R}_{t-1} \\
\hat{b}_{t-1}
\end{pmatrix},
\]

where

\[
A = \begin{pmatrix}
\sigma (1-\psi) 1-\psi - (1-\psi) \psi & 0 & 0 & 0 \\
0 & \beta & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}^{-1} = \begin{pmatrix}
\sigma & 0 & 0 & 0 \\
-\omega & 1 & 0 & 0 \\
0 & \rho & 0 & 0 \\
0 & r & r & -\kappa_I
\end{pmatrix}.
\]
The characteristic polynomial of $A$ is given by $X \cdot F(X)$, where

$$F(X) = X^3 + \frac{\sigma + \omega + \sigma \beta - \sigma \psi - \psi \omega + r \sigma \beta - \kappa I \sigma \beta - r \sigma \beta \psi + \kappa I \sigma \beta \psi}{\sigma \beta (\psi - 1)} X^2$$

$$+ \frac{\kappa I \sigma - r \sigma - r \omega - \sigma + \kappa I \omega - \omega \rho - r \sigma \beta + r \sigma \psi + \kappa I \sigma \beta - \kappa I \sigma \psi - \kappa I \psi \omega + \psi \omega \rho}{\sigma \beta (\psi - 1)} X$$

$$+ \frac{r \sigma - \kappa I \sigma + r \omega \rho - \kappa I \omega \rho + \kappa I \psi \omega}{\sigma \beta (\psi - 1)},$$

indicating that one eigenvalue equals zero, which can be assigned to one predetermined variable. The other eigenvalues are the roots of $F(X) = 0$ with

$$F(0) = - \frac{(\sigma + \omega \rho) (r - \kappa I) + \omega \rho \kappa \psi}{\sigma \beta (1 - \psi)} < 0,$$

$$F(1) = - \frac{((\rho - 1) \omega + \sigma \psi (1 - \beta)) (r - 1 - \kappa I) + (\rho - 1) \omega \psi (\kappa I + 1)}{\sigma \beta (1 - \psi)},$$

$$F(-1) = - \frac{\omega (r + (1 - \psi) (1 - \kappa I)) (\rho + 1) + \sigma (2 - \psi) (\beta + 1) (1 + r - \kappa I)}{\sigma \beta (1 - \psi)} < 0.$$

Since there is one predetermined state variable and two non-predicted variables, the equilibrium is locally determined if and only if there are two unstable and one stable eigenvalue.

If $\kappa I < r - 1$, $F(0) < -1$, and there exists at least one unstable eigenvalue. Since $F(-1) < 0$, the equilibrium is locally determined only if $F(1) > 0$. Then, there would exist exactly one stable eigenvalue, which lies between zero and one. A necessary and sufficient condition for $F(1) > 0$ and for local equilibrium determinacy is $\rho < 1 - \frac{\omega (1 + \psi (1 + \kappa I)) (r - 1 - \kappa I)}{\sigma \psi (1 - \beta)}$.

If $\kappa I > r - 1$, $F(0)$ is still negative, $F(0) < 0$, since $\kappa I \leq r$. The existence of one stable eigenvalue again requires $F(1) > 0$, which does not rule out the existence of three stable eigenvalues. A necessary but not sufficient condition for a local equilibrium determinacy is therefore $F(1) > 0 \iff (\rho - 1) \omega \left( 1 + \frac{\psi (1 + \kappa I)}{\sigma \beta (1 - \psi)} \right) > - \sigma \psi (1 - \beta)$. ■

The determinacy conditions summarized in proposition 1 show that monetary policy is strongly constrained by determinacy requirements, which substantially change with the fiscal stance. The principle derived by Leeper’s (1991) roughly applies also in our environment with non-neutral debt: When fiscal policy is active, such that the feedback from debt on taxes is smaller than the average net real interest rate, $\kappa I < r - 1$, local determinacy necessarily requires monetary policy has to be passive, $\rho < 1$. When the feedback from debt to taxes exceeds the average real interest rate, $\kappa I > r - 1$, monetary policy should instead be active to support local determinacy.

The presence of direct debt effects on the private sector behavior, which is here summarized by the coefficient $\psi$ in (7), can in principle modify the latter result: If $\psi$ is sufficiently large, then local determinacy can also be consistent with some passive interest rate policies
if $\kappa_I > r - 1$. This result is one of the key points made by Canzoneri et al.’s (2006), who use a calibrated model with a fiscal rule of type I. and liquidity services of debt.

With the determinacy conditions for the general case presented in proposition 1, we can easily identify corresponding conditions for the limiting case $\psi \to 0$ to give a clearer exposition.

**Corollary 1** Consider equilibrium sequences for $\{\hat{x}_t, \hat{\pi}_t, \hat{b}_t, \hat{R}_t\}^\infty_{t=0}$ satisfying (7)-(9) and (11) and $\kappa_I \leq \min\{1, r\}$. For $\psi \to 0$ and

1. $\kappa_I < r - 1$, the equilibrium is locally determined if and only if $\rho < 1$,
2. $\kappa_I > r - 1$, the equilibrium is locally determined only if $\rho > 1$.

The conditions presented in corollary 1 accord with those derived by Leeper (1991). If fiscal policy is passive, $\kappa_I > r - 1$, the Taylor principle applies, while it is reversed if fiscal policy is active $\kappa_I < r - 1$.

**Determinacy under the interest payment rule** We now turn to the fiscal rule II, which links tax revenues to interest payments on debt rather than the debt level. The condition for local determinacy is presented in the following proposition.

**Proposition 2** Consider equilibrium sequences for $\{\hat{x}_t, \hat{\pi}_t, \hat{b}_t, \hat{R}_t\}^\infty_{t=0}$ satisfying (7)-(9) and (12). The equilibrium is locally determined if and only if

$$\rho < 1 + \frac{1 - r (1 - \kappa_{II})}{r (1 - \kappa_{II})}. \quad (13)$$

**Proof.** The model for the second fiscal rule (7)-(9) and (12) reads

$$\begin{bmatrix}
E_t \hat{x}_{t+1} \\
E_t \hat{\pi}_{t+1} \\
\hat{R}_t \\
\hat{b}_t
\end{bmatrix} = \tilde{A} \begin{bmatrix}
\hat{x}_t \\
\hat{\pi}_t \\
\hat{R}_{t-1} \\
\hat{b}_{t-1}
\end{bmatrix}, \text{ where } \tilde{A} = \begin{pmatrix}
\sigma (1 - \psi) & 1 - \psi & - (1 - \psi) \psi \\
0 & \beta & 0 \\
0 & 0 & 1 \\
0 & 0 & 0
\end{pmatrix}^{-1} \begin{pmatrix}
\sigma & 0 & 0 \\
- \omega & 1 & 0 \\
0 & \rho & 0 \\
0 & -1 & \eta
\end{pmatrix},$$

with $\eta = (1 - \kappa_{II}) r > 0$. The characteristic polynomial of $\tilde{A}$ reads $X \cdot H(X)$, where

$$H(X) = X^3 + \frac{(\sigma + \omega + 2\sigma\beta - \sigma\psi - \psi\omega - \sigma\beta\psi)}{\sigma\beta(\psi - 1)}X^2$$

$$+ \frac{(\sigma\psi - \omega - \sigma\beta - 2\sigma - \omega\rho + \psi\omega\rho)}{\sigma\beta(\psi - 1)}X + \frac{(\sigma + \omega\rho - \psi\omega\rho + \psi\eta\omega\rho)}{\sigma\beta(\psi - 1)},$$

indicating that one eigenvalue is $X = 0$, which can be assigned to one predetermined variable. The other eigenvalues are the roots of $H(X) = 0$, with

$$H(0) = - \frac{\sigma}{\sigma\beta(1 - \psi)} - \frac{\omega\rho(\psi\eta + 1 - \psi)}{\sigma\beta(1 - \psi)} < -1, \quad H(1) = \psi\omega \frac{1 - \eta\rho}{\sigma\beta(1 - \psi)},$$

$$H(-1) = - \frac{2\sigma(1 - \psi + 1 - \beta\psi)}{\sigma\beta(1 - \psi)} + \omega(2 - \psi) + 4\sigma\beta + 2\omega\rho(1 - \psi) + \psi\eta\omega\rho < 0.$$
Since there is one predetermined state variable and two non-predetermined variables, the equilibrium is locally determined if and only if there are two unstable and one stable eigenvalue. Given that $H(0) < -1$ there exists at least one unstable eigenvalue. Using $H(X)$ at $X = 1$ shows that there is at least one stable (and positive) eigenvalue lying between zero and one if $1 > \eta \rho r$. Given that $H(-1) < 0$ the third eigenvalue is unstable. Using $\eta = (1 - \kappa_{II}) r$, the equilibrium is locally determined if and only if $\rho < 1/ [(1 - \kappa_{II}) r] \iff \rho < 1 + \frac{1-r(1-\kappa_{II})}{r(1-\kappa_{II})}$. \[\Box\]

It should be noted that the determinacy condition (13) does not depend on the magnitude of direct debt effects $\psi$. The result summarized in the following corollary shows that fiscal policy can be conducted in a way such that monetary policy can be implemented without any restrictions if $\kappa_{II} = 1$, i.e. if interest payments on debt are entirely tax financed and the government budget is balanced.

**Corollary 2** If fiscal policy is conducted according to (12) with $\kappa_{II} = 1$, the equilibrium is locally determined for any monetary policy of the form (9).

For illustration, we plot the regions of determinacy (marked with a plus sign), instability (marked with circles), and indeterminacy (marked with squares) in figure 1. In constructing the figure, we have chosen the following parameters as examples: $\sigma = \eta = 1$, a standard Calvo price rigidity of $\xi = 0.75$, a discount factor of $\beta = 0.99$, and a quarterly real interest rate of $r = 1.0075$, implying $\psi = 0.002575$ (note that by the steady state version of the Euler equation, $b = \phi x^\sigma / (1 - \beta r)$, the choice $\beta < 1/r$ implies a positive steady state debt level). It should be noted that the central result appears to be very robust with respect to different parameter choices.
Figure 1: Determinacy (plus signs), instability (circles), and indeterminacy (squares) for rule I. (left panel) and rule II. (right panel)

As figure 1 shows, under rule I, the standard Taylor principle holds unless the tax response to debt is very weak, in which case the Taylor principle is turned on its head (since monetary policy must then help to avoid debt explosion). Under rule II, equilibrium might not exist due to unstable debt dynamics if the tax response to interest payments on debt is too weak, but for even moderate fiscal reaction coefficients this does not seem to be relevant. Most interestingly, there can be no indeterminacy under this rule.

In order to understand the role of fiscal rules, note that there are two opposing effects on the debt dynamics: higher inflation tends to reduce debt and higher nominal interest rates tend to increase debt, for a given tax policy (see 10). Under the standard tax rule (5) that responds to debt levels, both jointly affect debt via the real interest rate. When the feedback from debt to taxes exceeds the steady state real interest rate, the real debt sequence tends to be stabilized irrespective of the particular monetary policy stance (as long as $\psi$ is sufficiently small). The task of stabilizing inflation expectations and thus of ruling out local indeterminacy then falls on the central bank’s interest rate setting. In the limiting case, where debt is neutral, the pure Taylor principle applies (as in Leeper, 1991). If taxes are raised by less than the average real rate in response to higher debt, the real debt sequence can in principle be stabilized by higher inflation. If the central bank conducts an active interest rate policy, higher inflation would increase the real rate and would in fact destabilize debt. Thus, the Taylor principle is reversed, such that a passive interest rate policy is required for determinacy in this case.
If, instead, our interest payment rule (6) is applied, the effects of interest rates and inflation on debt can be decoupled such that the real interest rate is not the only decisive factor for debt dynamics (see 12). If the fraction of tax financed interest rate payments $\kappa_{II}$ is very small, the debt sequence can be stabilized if monetary policy does not raise the interest rate actively with higher inflation, like in the case of a standard tax rule. When the share of tax financed interest payments is higher, the effects of inflation and the nominal rate on debt are different. The interest rate effect on debt is then reduced, such that neither an active nor a passive interest rate policy will destabilize debt. At the same time equilibrium inflation rates are constrained to be consistent with a stable debt sequence. Due to this requirement arbitrary inflation expectations are ruled out regardless of the particular monetary policy responsiveness, such that determinacy obtains and the Taylor principle becomes irrelevant.

4 Conclusion

Several recent studies have emphasized the importance of monetary and fiscal policy interactions for macroeconomic stability. These studies broadly confirmed the Taylor principle, which also applies in economies without fiscal policy, or found a reversal of the Taylor principle for very low fiscal feedback from debt to taxes. These results are based on a fiscal rule that is common for almost all of these studies. It demands tax revenues to be increased with the level of public debt. In this paper we instead propose an alternative fiscal rule, which requires that the government pays at least a part of its interest payments on debt through taxation. This rule is not only intuitive and simple to implement, but has, as we show, the virtue that it can render the Taylor principle irrelevant and thus free monetary policy from consideration of determinacy requirements. In particular, any type of feedback from inflation to interest rates, i.e. passive and active monetary policy rules, is consistent with a locally determined equilibrium if interest payments on debt are fully tax financed. In this sense, our results show that a balanced government budget can be a positive contribution to macroeconomic stability.

5 References

Canzoneri, M.B., R.E. Cumby, B. Diba, and D. Lopez-Salido (2008), Monetary Aggregates and Liquidity in a Neo-Wicksellian Framework, Journal of Money, Credit and Banking 40,
Canzoneri, M.B., R.E. Cumby, B. Diba, and D. Lopez-Salido (2006), Monetary and Fiscal Policy Coordination when Bonds Provide Transactions Services, Georgetown University.