# An interest rate peg might be better than you think*

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**Abstract**

Active interest rate policy is frequently recommended based on its merits in reducing macroeconomic volatility and being a simple and transparent policy device. In a standard New Keynesian model, we show that an even simpler policy, namely an interest rate peg, can be welfare enhancing: The minimum state variable solution and an autoregessive solution under a peg can lead to lower welfare losses than the unique solution under an active interest rate rule. Given that a peg is usually blamed to facilitate endogenous fluctuations, we further show that a peg can be implemented in a way that ensures equilibrium determinacy.

## 1 Introduction

Recent macroeconomic research on monetary policy, which is based on New Keynesian models, has led to a simple advice for central bankers: Interest rates should be set in an active way. Though this device for interest rate setting is not exactly implied by welfare-maximization, it is commonly viewed as a useful short-cut for the latter. By raising the nominal interest rate by more than one for one in case inflation is (expected to be) increasing, the real interest rate rises causing agents to save more and to consume less such that aggregate demand and firms' costs decline. By applying this strategy, monetary policy can stabilize prices, which reduces welfare costs of imperfect price adjustments.

Theoretical analysis of monetary policy has further shown that this is not the main virtue of an active policy: It rules out the possibility of multiple equilibria and thereby endogenous fluctuations (see Benhabib, Schmitt-Grohe, and Uribe (2001), or Woodford

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(2003)). Due to this property, an active interest rate setting is widely viewed as a prerequisite for macroeconomic stability. Consequently, passive policies are usually dismissed given that they in principle allow for self-fulfilling expectations, or, sunspot equilibria. However, this view on active vs. passive policies is not necessarily justified on welfare grounds, since both types of policies are not derived from a welfare maximizing policy plan.

This paper takes a closer look at the welfare effects of simple policy rules. Thereby, we consider a prominent (passive) monetary policy regime, namely a peg, which is banned from the recent literature, probably due to its failure to guarantee equilibrium uniqueness.[1] A welfare comparison in the workhorse macro model (a standard New Keynesian model) surprisingly shows that a peg can outperform a simple active interest rate rule. This result holds for the minimum state variable solution under a peg as well as for an autoregressive solution, where lagged inflation rates serve as an endogenous state variable. Finally, we demonstrate that a peg can be implemented by the central bank in a way that ensures the existence of a unique solution, i.e. it can *uniquely* implement the autoregressive solution or the minimum state variable solution under a peg. While the main purpose of the paper is to demonstrate that some simple rules are better (or worse) than one commonly thinks, the analysis also contributes to the debate on the alleged problems associated with constant interest rate projections (see Honkapohja and Mitra (2005a), and Galí (2007)).

The remainder is organized as follows. Section 2 presents the framework, introduces different policy specifications, and derives welfare effects. Section 3 demonstrates how an equilibrium under a peg can be implemented in a unique way. Section 4 concludes.

## 2 A consensus model

Consider the following simple New Keynesian model, which can be derived from a microfounded sticky price framework and is for example also applied in Clarida, Gali, and Gertler (1999):

$$\begin{aligned} \pi_t &= \beta E_t \pi_{t+1} + \kappa x_t + u_t, \qquad (1) \\ x_t &= E_t x_{t+1} - \frac{1}{\sigma} R_t + \frac{1}{\sigma} E_t \pi_{t+1}, \\ u_t &= \rho u_{t-1} + \varepsilon_t, \quad \rho \in (0,1), \end{aligned}$$

where $\pi_t$ denotes the gross inflation rate, $x_t$ the output-gap, $R_t$ the gross interest rate, and $u_t$ an autoregressive cost push shock. $\varepsilon_t$ is i.i.d. with $E_{t-1}\varepsilon_t = 0$ and a constant variance $\sigma_\varepsilon^2$. All variables are expressed in terms of percentage deviations from their respective values at an efficient steady state (see Woodford (2003)). The composite coefficient $\kappa$ of the Phillips curve is defined as $\kappa = (1-\phi)(1-\phi\beta)(\sigma+\eta)/\phi$ where $\beta$ is the household's

[1] An exception is the recent discussion on the usefulness of constant interest rate projections (Honkapohja and Mitra (2005b) and Galí (2007)).

constant discount factor, $1/\sigma$ the elasticity of intertemporal substitution, $1/\eta$ the Frisch labor supply elasticity, and $\phi$ the fraction of firms that do not adjust their prices in each period.

Monetary policy is specified in form of simple feedback rules for the nominal interest rate. We thus refrain from deriving an optimal monetary policy. In particular, we consider three different simple rules for monetary policy: *i.*) The rule proposed by Taylor (1993), *ii.*) an active interest rate policy that is consistent with monetary policy acting under discretion, and *iii.*) an interest rate peg.

### 2.1 Solutions under different simple rules

Following common practice, we restrict our attention to convergent equilibrium sequences. The model is simple enough to derive closed form solutions.

**An active interest rate policy** can be described with interest rate rules of the form

$$R_t = w_\pi \pi_t + w_x x_t, \tag{2}$$

where $w_\pi > 1$ and $w_x \geq 0$. Specifically, the feedback coefficients equal $w_\pi = 1.5$ and $w_x = 0.5$ in case of the Taylor rule, and $w_\pi = \rho + (1-\rho)\sigma\varepsilon$ and $w_x = 0$ in the case of a simple active rule consistent with discretionary policy (see appendix A.1). In both cases, the model given in (1) can be reduced to the two-dimensional system

$$\begin{aligned} \pi_t &= \beta E_t \pi_{t+1} + \kappa x_t + u_t \\ x_t &= E_t x_{t+1} - \frac{w_\pi}{\sigma}\pi_t - \frac{w_x}{\sigma} x_t + \frac{1}{\sigma} E_t \pi_{t+1} \end{aligned}$$

It is well-known that this system exhibits exactly one solution when $w_\pi > 1$ and $w_x \geq 0$ (since the system then exhibits two unstable eigenvalues) which allows for convergent equilibrium sequences only if the solution exhibits no history dependence (see e.g. Woodford (2001)). Thus, we know that both simple rules will lead to a linear solution of the following form

$$\pi_t = a_u u_t, \quad x_t = b_u u_t.$$

Using the method of undetermined coefficients the coefficients $a_u$ and $b_u$ of this minimum state variable (MSV) solution can easily be derived:

$$a_u = \frac{\sigma\left(1-\rho+\frac{w_x}{\sigma}\right)}{\sigma(1-\beta\rho)\left(1-\rho+\frac{w_x}{\sigma}\right)+\kappa(w_\pi-\rho)}, \qquad b_u = -\frac{a_u}{\sigma}\frac{w_\pi-\rho}{1-\rho+\frac{w_x}{\sigma}}.$$

**An interest rate peg,** i.e. a policy characterized by keeping the nominal interest rate at its long run efficient level ($\bar{R} = 1/\beta > 1$): $R_t = 0$, leads to the following conditions for the equilibrium sequences of inflation and the output-gap:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u_t,$$
$$x_t = E_t x_{t+1} + \frac{1}{\sigma} E_t \pi_{t+1}.$$

It is well known that this policy gives rise to multiple equilibrium solutions. Precisely, equilibrium conditions can be solved by other solutions than the MSV solution, since under a peg there exist one stable and one unstable eigenvalue. Hence, there exist additional stable solutions that exhibit endogenous state variables.

One type of solution features artificial state variables (like past expectations of today's non-predetermined variables, $E_{t-1}\pi_t$ or $E_{t-1}y_t$). These solutions are well-known to support the existence of sunspot-equilibria, where arbitrary changes in expectations (non-fundamental shocks) can affect macroeconomic variables. Here, we disregard these types of solutions and exclusively apply "well-behaved" solutions, namely, the MSV solution and an autoregressive (AR) solution. By construction, both *cannot* support sunspot equilibria, i.e., welfare reducing endogenous fluctuations.

1. Minimum state solution: As the first type of solution we consider the MSV solution under the peg, which takes the form

$$\pi_t = a_u^{peg} u_t, \quad x_t = b_u^{peg} u_t.$$

Applying the method of undetermined coefficients delivers

$$a_u^{peg} = \frac{1-\rho}{\kappa}\left[(1-\rho)(1-\beta\rho)\frac{1}{\kappa} - \frac{\rho}{\sigma}\right]^{-1}, \tag{3}$$
$$b_u^{peg} = \frac{a_u}{\kappa}(1-\beta\rho) - \frac{1}{\kappa}.$$

2. Autoregressive solution: As the second type of solution we consider an AR solution, where lagged inflation serves as an additional state variable. The solution form is given by

$$\pi_t = a_\pi \pi_{t-1} + a_u^{peg,AR} u_t, \tag{4}$$
$$x_t = b_\pi \pi_{t-1} + b_u^{peg,AR} u_t,$$

where the method of undetermined coefficients yields

$$a_\pi = \frac{1+\frac{\kappa}{\sigma}}{2\beta} + \frac{1}{2} - \sqrt{\left(\frac{1+\frac{\kappa}{\sigma}}{2\beta} + \frac{1}{2}\right)^2 - \frac{1}{\beta}}, \qquad b_\pi = \frac{a_\pi}{\kappa} - \frac{\beta}{\kappa} a_\pi^2, \tag{5}$$

$$a_u^{peg,AR} = \frac{1-\rho}{\kappa}\left[(1-\rho)(1-\beta\rho)\frac{1}{\kappa} - \frac{\beta(1-\rho)a_\pi}{\kappa} - \frac{\rho}{\sigma} - \frac{a_\pi}{\sigma} - b_\pi\right]^{-1},$$

$$b_u^{peg,AR} = \frac{a_u}{\kappa}(1-\beta\rho) - \frac{\beta}{\kappa}a_\pi a_u - \frac{1}{\kappa}.$$

Note that one of the two solutions for $a_\pi$ lies inside the unit circle, while the other lies outside the unit circle. A stable solution requires to pick the stable solution for $a_\pi$, which – since $\frac{1+\frac{\kappa}{\sigma}}{2\beta} > \frac{1}{2}$ – *must* contain the root with a negative sign.

## 2.2 Welfare effects

In this section we compute welfare effects under the alternative policies and solutions. Following large parts of the literature, we apply a second order approximation of household welfare of the underlying model with optimizing agents. In particular, we adopt Woodford's (2003) approach, leading to a quadratic loss function that measures welfare losses of deviations from an efficient steady state (where long run distortions are eliminated by fiscal transfer and long-run price stability is ensured by an inflation target equal to one):

$$L = -E_0 \sum_{t=0}^{\infty} \beta^t (\pi_t^2 + \lambda x_t^2) = -\frac{1}{1-\beta}\left(Var\ \pi + \lambda Var\ x\right), \tag{6}$$

where we assumed that the economy is initially in its steady state and we used that the equilibrium sequences under all solutions are covariance stationary. $Var\ \pi$ then denotes the unconditional variance (here, conditional on the information available at the beginning of period 0) of $\pi_t$. The weight on output gap fluctuations satisfies $\lambda = \kappa/\epsilon$, where $\epsilon$ is the price elasticity that price setting firms face (see Woodford (2003)). The unconditional variances are given by

$$Var\ \pi = \frac{1}{1-a_\pi^2}\left[a_u^2 Var\ u + 2a_\pi a_u Cov(u,\pi)\right],$$

$$Var\ x = b_\pi^2 Var\pi + b_u^2 Var\ u + 2b_\pi b_u Cov(u,\pi),$$

where $Var\ u = \frac{1}{1-\rho}\sigma_\varepsilon^2$ and $Cov(u,\pi) = a_u \frac{\rho}{(1-\rho^2)(1-a_\pi\rho)}\sigma_\varepsilon^2$ (see appendix A.2). For the computation of the variances we apply a set of standard parameter values given in table 1, which lead to $\kappa = 0.1717$ and $\lambda = 0.029$. The normalization of $\sigma_\varepsilon^2$ does evidently not affect the relative welfare effects. These parameter values lead to a policy rule under discretionary optimization that is characterized by $w_\pi = 1.5$ (and $w_x = 0$). For this benchmark calibration we obtain the following results for the solutions and the variances under different rules:

Taylor rule: $\pi_t = 3.563 u_t, \;\; x_t = -3.563 u_t$

$$Var\ \pi = 66.814, \quad Var\ x = 66.814, \quad \text{and} \quad L^{Taylor} = -6872.52.$$

Policy under discretion, $w_\pi = 1.5$: $\pi_t = 0.878 u_t, \quad x_t = -5.268 u_t$

$$Var\ \pi = 4.057, \quad Var\ x = 146.05\ ; \quad \text{and} \quad L^{active} = -823.56.$$

Interest rate peg MSV: $\pi_t = 0.696 u_t, \quad x_t = -6.267 u_t,$

$$Var\ \pi = 2.552, \quad Var\ x = 206.739, \quad \text{and} \quad L^{peg,MSV} = -846.74.$$

Interest rate peg AR: $\pi_t = 0.665\pi_{t-1} - 0.182 u_t, \;\; x_t = 1.323\pi_{t-1} - 5.244 u_t,$

$$Var\ \pi = 1.241, \quad Var\ x = 176.639, \quad \text{and} \quad L^{peg,AR} = -629.45.$$

The results show that the Taylor rule yields the worst welfare result, which is due to the most effective output gap stabilization that comes at the cost of the highest inflation variance. Evidently, an active policy under discretionary optimization performs much better than the Taylor rule. The MSV solution under the peg leads to an even lower inflation variance, but slightly higher welfare losses caused by a less stabilized output gap. Notably, the AR solution under the peg clearly leads to the lowest welfare losses, which is mainly due to the smallest inflation variance. The latter property is hardly surprising, since inflation under the AR solution exhibits inertia that helps to smooth inflation fluctuations.[2]

Yet, this welfare ranking is by far not robust to changes of parameter values, as can be seen in table 2. Most of all, the degree of autocorrelation of the exogenous state (i.e. the cost push shock) matters for the relative welfare effects. As argued above, an additional state variable ($\pi_{t-1}$) can contribute to less volatile sequences of macroeconomic variables (here, in particular, inflation). However, additional state variables extend the state space and thus the support of the variables, which tends to raise unconditional variances. As long as the autocorrelation $\rho$ of the exogenous state is large, the latter effect will be less important. But, for smaller values of $\rho$, here $\rho \leq 0.8$ (see table 2), welfare losses can be higher for the AR solution than for the active policy under discretion ($w_\pi = \rho + (1-\rho)\sigma\varepsilon$ and $w_x = 0$). For a high value of $\rho$ ($\rho = 0.95$), both solutions under the peg outperform both active policies.

Finally, variations of the elasticity of intertemporal substitution and of the degree of price stickiness show that the relative performance of the solutions also depends on the

[2] Under both solutions to the peg, a cost push shock causes output and inflation to decrease. This outcome can differ from the transmission of cost push shocks under solutions with artificial state variables, where inflation rises and – due to a lower real interest rate – output too. Such a solution is known to support self-fulfilling inflation expectations.

relative welfare costs of output and inflation fluctuations. Lowering the fraction of non-price-adjusting firms ($\phi = 0.7$) tends to lower welfare losses in general. Then, aggressive (active) responses to changes in inflation are less desirable, such that even the MSV solution under the peg outperforms the active discretionary policy for $\rho = 0.9$. If the private sector is less willing to substitute consumption intertemporally ($\sigma = 2$), central bank passiveness tends to lead to higher inflation and welfare losses.[3]

## 3 Uniqueness under an interest rate peg

As argued above, the two solutions analyzed for the interest rate peg are not the only possible solutions. In particular, non-fundamental solutions (with artificial state variables) are possible that might lead to endogenous fluctuations. A policy regime that facilitates the latter is evidently not desirable. Thus, we aim at designing policy rules for the central bank that implement a peg in a way that renders multiple solutions impossible. In particular, we show that both solutions to the peg can be implemented by an appropriately designed interest rate rule.

### 3.1 Minimum state solution

The MSV solution under a peg, which is characterized by $\pi_t = a_u^{peg} u_t$ and $x_t = b_u^{peg} u_t$, can be implemented by a rule of the form (2). To implement a peg, $R_t = 0$, the coefficients have to satisfy $\frac{w_x}{w_\pi} = -\frac{\pi_t}{x_t}$. Using that the MSV solution implies $\frac{\pi_t}{x_t} = \frac{a_u^{peg}}{b_u^{peg}}$, we get the following conditions for the policy rule coefficients

$$w_\pi = \alpha \text{ and } w_x = -\alpha \left(a_u^{peg}/b_u^{peg}\right)$$

where $\alpha$ is an arbitrary constant. We can easily assess the equilibrium determinacy conditions of the New Keynesian model closed with this interest rate rule. The model in matrix form is given by

$$E_t y_{t+1} = \mathbf{A} y_t + B u_t \text{ with } y_t = (x_t, \pi_t)' \text{ and } \mathbf{A} = \begin{pmatrix} 0 & \beta \\ \sigma & 1 \end{pmatrix}^{-1} \begin{pmatrix} -\kappa & 1 \\ \sigma + w_x & w_\pi \end{pmatrix}$$

To ensure determinacy, the matrix A must exhibit two unstable eigenvalues which is guaranteed by the three following conditions

$$\begin{aligned} \det(\mathbf{A}) > 1 &\Leftrightarrow \frac{\sigma + w_x + \kappa w_\pi}{\sigma\beta} > 1 \\ \det(\mathbf{A}) - \operatorname{trace}(\mathbf{A}) > -1 &\Leftrightarrow w_\pi + w_x \frac{1-\beta}{\kappa} > 1 \\ \det(\mathbf{A}) + \operatorname{trace}(\mathbf{A}) > -1 &\Leftrightarrow \kappa (1 + w_\pi) + (2\sigma + w_x)(1+\beta) > 0 \end{aligned}$$

[3] The unconditional variances tend to rise with $\rho$ under active policies, whereas $\rho$ exerts an ambiguous effect on the variances under the peg.

Under the benchmark calibration of table 1 and, for instance, $\alpha = 1.5$, these conditions are fulfilled and the MSV solution is the unique solution. The interest rate rule then reads $R_t = 1.5\pi_t - 0.1667x_t$. This rule uniquely implements sequences $\{R_t, x_t, \pi_t\}_{t=0}^{\infty}$ satisfying $\pi_t = a_u^{peg} u_t$, $x_t = b_u^{peg} u_t$, and $R_t = 0$.

### 3.2 Autoregressive solution

Similarly, the autoregressive solution can uniquely be implemented by an interest rate rule of the form[4]

$$R_t = r_x x_t + r_\pi \pi_t + r_l \pi_{t-1} \tag{7}$$

We set the parameters in (7) so as to implement the autoregressive solution. Eliminating $\pi_t$ in (7) by $\pi_t = a_\pi \pi_{t-1} + a_u^{peg,AR} u_t$ yields $R_t = r_x x_t + (r_l + r_\pi a_\pi)\,\pi_{t-1} + r_\pi a_u^{peg,AR} u_t$, which further has to imply a sequence of constant interest rates, $R_t = 0$. For this, we use the output solution, $x_t = b_\pi \pi_{t-1} + b_u^{peg,AR} u_t \Leftrightarrow \alpha(-x_t + b_\pi \pi_{t-1} + b_u^{peg,AR} u_t) = 0$ for an arbitrary $\alpha \neq 0$, which implies $R_t = r_x x_t + (r_l + r_\pi a_\pi)\,\pi_{t-1} + r_\pi a_u^{peg,AR} u_t = 0$ if $r_x = -\alpha$; $r_l + r_\pi a_\pi = \alpha b_\pi$; and $r_\pi a_u^{peg,AR} = \alpha b_u^{peg,AR}$. Thus, (7) implements an interest rate peg if (but not only if) the policy rule coefficients satisfy

$$r_x = -\alpha, \qquad r_l = \alpha\,(b_\pi - a_\pi \varpi)\,, \qquad r_\pi = \alpha\varpi \tag{8}$$

where $\varpi = b_u^{peg,AR}/a_u^{peg,AR}$. Setting $\alpha = 0.25$, for instance and applying the parameter values in table 1, leads to $r_x = -0.25$, $r_l = -4.47$, and $r_\pi = 7.22$. Eliminating the interest rate in (1), by a policy rule satisfying (7) and (8) leads to a $3 \times 3$ system in $x_t$, $\pi_t$, and $\pi_{t-1}$. Since the latter is relevant for monetary policy, the equilibrium solution takes the form (4). Determinacy then requires that there are two eigenvalues outside the unit circle and one stable eigenvalue.

For $\alpha = 0.25$ and the parameter values in table 1, the eigenvalues under (8) are given by $\lambda_1 = 0.665$; $\lambda_2 = \lambda_3 = 1.080$ (modulus), ensuring that the AR solution is the unique solution.[5] The single stable eigenvalue 0.665, resembles the autoregressive coefficient in the inflation solution: $\pi_t = 0.665\pi_{t-1} - 0.182u_t$. Thus, the central bank can construct an interest rate rule in a way, which implies 1) a constant interest rate (i.e. $R_t = 0$) and 2) unique equilibrium sequences that are identical to the equilibrium sequences under the autoregressive solution.

## 4 Conclusion

This paper shows that a popular monetary policy device, namely, an active interest rate policy (or a Taylor rule), can easily be outperformed by an even simpler monetary policy

[4] Galí (2007) proceeds in a similar way to implement a peg in a unique way: He induces equilibrium determinacy in a New Keynesian model by implementing a peg with a rule, where the central bank reacts to inflation and current and lagged output.

[5] Further details on the determinacy conditions are available from the authors upon request.

strategy: an interest rate peg. While we do not seek to identify optimal policies, we want to demonstrate that central bankers should not apply recipes just because of their appealing simplicity. Once a policy maker departs from a fully optimal (commitment) strategy, it is ex-ante not clear which kind of simple rule most closely resembles the outcome under the commitment plan. To our surprise, even an interest rate peg can be more desirable (in welfare terms) than an active interest rate policy consistent with an optimal plan under discretion. Finally, it is shown that the common argument in favor of active interest rate policies, i.e. the absence of endogenous fluctuations, is not incompatible with a constant interest rate.

## Tables

**Table 1** Benchmark parameter values

| $\sigma$ | $\eta$ | $\beta$ | $\phi$ | $\epsilon$ | $\rho$ | $\sigma_\varepsilon^2$ |
|---|---|---|---|---|---|---|
| 1 | 1 | 0.99 | 0.75 | 6 | 0.9 | 1 |

**Table 2** Unconditional variances and welfare for various parameter values

| | Peg (AR) | | Peg (MSV) | | Discretion | | Taylor | |
|---|---|---|---|---|---|---|---|---|
| | Welfare loss | | Welfare loss | | Welfare loss | | Welfare loss | |
| | Var$\pi$ | Var$x$ | Var$\pi$ | Var$x$ | Var$\pi$ | Var$x$ | Var$\pi$ | Var$x$ |
| $\rho = 0.75$ | -452.81 | | -4334.86 | | -279.91 | | -1276.50 | |
| | 2.36 | 75.86 | 34.47 | 310.25 | 1.38 | 49.64 | 12.41 | 12.41 |
| $\rho = 0.8$ | -464.98 | | -1767.35 | | -367.92 | | -1982.18 | |
| | 2.02 | 91.95 | 12.12 | 193.98 | 1.81 | 65.25 | 19.27 | 19.27 |
| $\rho = 0.85$ | -508.73 | | -1042.81 | | -517.89 | | -3400.34 | |
| | 1.66 | 119.85 | 5.43 | 174.52 | 2.55 | 91.84 | 33.06 | 33.06 |
| $\rho = 0.9$ | **-629.45** | | **-846.74** | | **-823.56** | | **-6872.52** | |
| | 1.24 | 176.64 | 2.55 | 206.74 | 4.06 | 146.05 | 66.81 | 66.81 |
| $\rho = 0.95$ | -1065.44 | | -1133.14 | | -1754.03 | | -19742.23 | |
| | 0.71 | 347.40 | 1.00 | 361.09 | 8.64 | 311.06 | 191.93 | 191.93 |
| $\phi = 0.7$* | -392.31 | | -469.40 | | -476.43 | | -3967.05 | |
| | 0.59 | 75.92 | 1.03 | 83.52 | 1.85 | 66.51 | 38.00 | 38.00 |
| $\sigma = 2$** | -535.00 | | -892.69 | | -489.62 | | -4994.05 | |
| | 2.01 | 77.74 | 4.78 | 96.72 | 1.92 | 69.26 | 48.41 | 35.57 |

Notes: The parameters $\sigma$ and $\phi$ equal 1 and 0.75, except for * and ** where $\rho$=0.9.

# A Appendix

## A.1 Discretionary policy

Under discretion, the CB minimizes its loss function w.r.t. $\pi_t$ and $x_t$, treating expectations as given: $\min_{\pi_t, x_t} 1/2 E_t \sum_{i=0}^{\infty} \beta^i \left\{ \pi_{t+i}^2 + \lambda x_{t+i}^2 \right\}$, subject to $\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u_t$. It is well established that the first order conditions for $\pi_t$ and $x_t$ under discretion lead to the targeting rule $\pi_t = -\frac{\lambda}{\kappa} x_t$ (see Woodford (2003)). Using the generic solution form $\pi_t = a_u u_t$ and $x_t = b_u u_t$ and plugging it into the Euler equation leads to $R_t = [b_u \sigma(\rho - 1) + \rho a_u] u_t$. Further using $u_t = \frac{\pi_t}{a_u}$ and that the target rule implies $\frac{a_u}{b_u} = -\frac{\lambda}{\kappa}$ yields $R_t = \left( \frac{\kappa}{\lambda} \sigma(1 - \rho) + \rho \right) \pi_t$, which with $\frac{\kappa}{\lambda} = \epsilon$ leads to $R_t = [\epsilon \sigma(1 - \rho) + \rho] \pi_t$ which is the policy used in section 2.1.

## A.2 Unconditional moments

Given the state space form for $\pi_t$ in (4), the unconditional variance of $\pi_t$, $Var\pi_t$, can be computed by

$$Var\pi_t = a_\pi^2 Var\pi_{t-1} + a_u^2 Var\ u_t + 2a_\pi a_u Cov(u_{t,}\pi_{t-1}).$$

We use that the unconditional expectation of $u_t$ satisfies $E(u_t) = 0$, so that $Cov(u_{t,}\pi_{t-1}) = E(u_t \pi_{t-1})$. We thus need to derive $E(u_t \pi_{t-1})$. First iterate $u_t$ and $\pi_{t-1}$ backwards to obtain

$$u_t = \rho^t u_0 + \sum_{s=0}^{t-1} \rho^s \varepsilon_{t-s}$$

$$\pi_{t-1} = a_\pi^t \pi_{-1} + a_\pi^{t-1} a_u u_0 + \sum_{s=0}^{t-2} a_\pi^s a_u \left( \rho^{t-1-s} u_0 + \sum_{j=0}^{t-2-s} \rho^j \varepsilon_{t-1-s-j} \right)$$

Computing $E(u_t \pi_{t-1})$ from these expressions and using $E(\varepsilon_n \varepsilon_m) = 0\ \forall\ n \neq m$ yields

$$E(u_t \pi_{t-1}) = \rho^t a_\pi^t E(u_0 \pi_{-1}) + \rho^t a_\pi^{t-1} a_u E\left(u_0^2\right) + \rho^{2t-1} a_u \frac{1 - (a_\pi/\rho)^{t-1}}{1 - (a_\pi/\rho)} E\left(u_0^2\right) \quad (9)$$
$$+ E \sum_{s=0}^{t-2} a_\pi^s a_u \sum_{j=0}^{t-2-s} \varepsilon_{t-1-s-j}^2 \rho^{s+1+2j}$$

We now simplify the term in the square brackets by applying the formula for finite geometric sums. This yields

$$\sum_{s=0}^{t-2} a_\pi^s a_u \sum_{j=0}^{t-2-s} \varepsilon_{t-1-s-j}^2 \rho^{s+1+2j} = \left( a_u \rho \sum_{s=0}^{t-2} (a_\pi \rho)^s \sum_{j=0}^{t-2-s} \rho^{2j} \right) \sigma_\varepsilon^2$$
$$= \sigma_\varepsilon^2 a_u \rho \sum_{s=0}^{t-2} (a_\pi \rho)^s \frac{1-\rho^{2(t-1-s)}}{1-\rho^2} = \sigma_\varepsilon^2 \frac{a_u \rho}{1-\rho^2} \left( \frac{1-(a_\pi\rho)^{t-1}}{1-a_\pi\rho} - \rho^{2t-2} \frac{1-\left(\frac{a_\pi}{\rho}\right)^{t-1}}{1-\frac{a_\pi}{\rho}} \right)$$

where we used that the unconditional expectation $E\left(\varepsilon_t^2\right) = Var\ \varepsilon = \sigma_\varepsilon^2$. Substituting out the sums in (9) and using that stationarity of $u$ and $\pi$ implies $E(u_0\pi_{-1}) = E(u_t\pi_{t-1})$, we obtain

$$E(u_t\pi_{t-1}) = \frac{1}{1-\rho^t a_\pi^t} \left\{ \left[ \frac{\rho^t a_\pi^{t-1} a_u}{1-\rho^2} \sigma_\varepsilon^2 + \frac{\rho^{2t-1} a_u}{1-\rho^2} \frac{1-\left(\frac{a_\pi}{\rho}\right)^{t-1}}{1-\left(\frac{a_\pi}{\rho}\right)} \sigma_\varepsilon^2 \right] \right.$$
$$\left. + \left[ \sigma_\varepsilon^2 \frac{a_u\rho}{1-\rho^2} \left( \frac{1-(a_\pi\rho)^{t-1}}{1-a_\pi\rho} - \rho^{2t-2} \frac{1-\left(\frac{a_\pi}{\rho}\right)^{t-1}}{1-\frac{a_\pi}{\rho}} \right) \right] \right\}$$

where we used $E\left(u_0^2\right) = Var\ u = \frac{\sigma_\varepsilon^2}{1-\rho^2}$. Cancelling identical terms and expanding the first term by $(1-a_\pi\rho)$ produces

$$E(u_t\pi_{t-1}) = \frac{a_u\rho}{1-\rho^t a_\pi^t} \left\{ \left[ \frac{(\rho a_\pi)^{t-1} - (\rho a_\pi)^t}{1-\rho^2} \frac{1}{1-a_\pi\rho} + \frac{1}{1-\rho^2} \frac{1-(a_\pi\rho)^{t-1}}{1-a_\pi\rho} \right] \sigma_\varepsilon^2 \right\}$$
$$= \frac{1}{1-\rho^t a_\pi^t} \frac{a_u\rho}{(1-\rho^2)(1-a_\pi\rho)} \left\{ \left[1-\rho^t a_\pi^t\right] \sigma_\varepsilon^2 \right\}$$

This expression can be simplified to yield

$$Cov(u,\pi) = a_u \frac{\rho}{(1-\rho^2)(1-a_\pi\rho)} \sigma_\varepsilon^2, \quad \text{where } Cov(u,\pi) = E(u_t\pi_{t-1}). \tag{10}$$

Using (10) and that stationarity of $\pi$ implies $Var\ \pi_t = Var\ \pi_{t-1} = Var\ \pi$, we can compute the unconditional variance of inflation as $Var\ \pi = \frac{1}{1-a_\pi^2}\left[a_u^2 Var\ u + 2a_\pi a_u Cov(u,\pi)\right]$.

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