Financial Constraints, Externalities, and Corrective Monetary Policy[1]

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Abstract

Pecuniary externalities induced by financial frictions can lead to amplification of macroeconomic outcomes. We show that monetary policy can address these externalities via interventions in secondary markets for debt securities. Central bank asset purchases alter prices by driving a wedge between the borrowing rate and lenders' rate of return. Monetary policy can thereby correct for uninternalized asset price effects, for which conventional monetary policy instruments are unsuited. Contrary to equivalent Pigouvian policies, implementation of corrective asset purchases does not rely on compensating taxes/transfers. We show that countercyclical asset purchases enhance social welfare by mitigating financial amplification of aggregate shocks.

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1 Introduction

Central banks have included large-scale purchases of private debt securities in secondary markets into their set of policy instruments. Empirical evidence shows that these interventions have altered asset prices not only during but also after the recent financial crisis.[3] This raises the question if these instruments can beneficially be used on a regular basis.[4] As the main novel contribution, we show that central bank asset purchases can serve as a corrective policy that addresses pecuniary externalities and can thereby mitigate financial amplification when financial constraints are binding (see Jeanne and Korinek, 2020).[5] We show that a central bank interest rate policy cannot serve for this purpose, whereas asset purchases can in principle be mimicked by a Pigouvian tax/subsidy. Yet, the latter rely on the availability of compensating (non-distortionary) fiscal transfers/taxes, which are not required for welfare-enhancing asset purchases. Notably, the analysis neither relies on central bank interventions to be conducted at a large scale nor outright, implying that the results also apply to central bank collateral policies including private debt securities.

In contrast to central bank adjustments of a short-term risk-free interest rate, asset purchases can be directed towards (secondary) markets for specific debt securities, which might be characterized by laissez faire prices that do not fully capture externalities. We show that price effects, which have been aimed and observed for recently conducted asset purchases programmes,[6] can particularly be used to address externalities stemming from financial frictions. We establish that the rich set of instruments under a variety of money supply facilities and the possibility to produce central bank money in a costless way enables central banks to conduct operations independently of fiscal taxes/transfers schemes.[7] From this perspective, asset purchases are a superior corrective instrument under limited availability of compensating (non-distortionary) taxes/transfers, which are necessary for the implementation of Pigouvian taxes/subsidies. In addition to cor-

[3] See Dell'Ariccia et al. (2018) and Kuttner (2018), who survey evidence on effects of asset purchase programmes conducted by the BoE, the ECB and the Fed.

[4] Several studies have already shown that asset purchases can be beneficial in financial crisis (e.g., Curdia and Woodford, 2011, Gertler and Karadi, 2011, or Del Negro et al., 2017).

[5] Financial amplification is induced by a collateral constraint that leads to a positive feedback between asset demand, prices, and borrowing capacities, like in Lorenzoni (2008), Stein (2012), Bianchi and Mendoza (2018), or Davila and Korinek (2018). Our paper complements this literature on prudential policies by focussing on gains of (ex-post) corrective policies conducted by the central bank.

[6] ECB purchases of ABS in 2014 were expected to "facilitate credit conditions" and corporate bond purchases of the BoE are amined at "reducing the costs of borrowing for companies".

[7] Put differently, the central bank can perfectly control the price level without relying on taxes/transfers from/to the treasury or the private sector.

rective asset purchases, central banks can further use regular money supply operations (e.g. in terms of treasury debt) to implement its inflation target.

We examine central bank purchases of private debt securities in an environment where financial markets are not extraordinarily stressed. Instead, we acknowledge vast empirical evidence that financial markets are regularly distorted by frictions, i.e. by financial constraints that always bind for a positive mass of agents. This induces permanent deviations from first best and leads to inefficiencies due to pecuniary externalities (see Davila and Korinek, 2018). These externalities are based on a positive feedback between collateral demand, prices, and the borrowing capacity. Such a feedback mechanism has already been examined in several studies on prudential policies, which alleviate adverse effects of deleveraging in a crisis.[8] Our focus is instead on ex-post interventions, which can serve as a substitute for ex-ante regulations, as shown by Bornstein and Lorenzoni (2018) and Jeanne and Korinek (2020).[9] In contrast to a central bank interest rate policy, asset purchases can be used to correct for this pecuniary externality by driving a wedge between the effective real rates for borrowers and lenders. Specifically, lenders can be incentivized to increase their supply of funds by offering an above-market price for purchases of debt securities. This raises the lenders' effective real return on debt, while the real interest rate for borrowers falls in equilibrium.[10] We further show that state-contingent asset purchases can mitigate financial amplification of aggregate shocks.

The analysis is conducted in a framework that is sufficiently stylized to isolate the main mechanism, while it includes essential elements of monetary policy implementation. It is not intended to provide a description of a policy measure implemented in a specific episode, but it is rather aimed at identifying efficiency gains from applying an established policy instrument. To account for the special role of central bank money for the settlement of transactions, we assume, for convenience, that it serves as the unique means of payment for non-durable consumption goods (see Lucas and Stokey, 1987). We abstract from financial intermediation, endogenous production, and price rigidities. This facilitates isolating asset purchase effects, since conventional monetary policies are then neutral with regard to the equilibrium allocation. Agents randomly differ by

[8] Lorenzoni (2008), Bianchi (2011), Stein (2012), Bianchi and Mendoza (2018), and Schmitt-Grohe and Uribe (2018) focus on ex-ante corrective policies under financial constraints that bind only in crises states.

[9] Prudential regulation is helpful when borrowing ex-ante depresses asset prices ex-post under binding financial constraints. Yet, ex-post interventions can directly induce higher asset prices, such that there is less need for ex-ante restrictions on (over-)borrowing.

[10] Specifically, the loan rate falls by a reduction in the (il-)liquidity premium, which accords to empirical evidence on price effects of US Federal Reserve asset purchases (see Gagnon et al., 2011).

their valuation of non-durables, giving rise to borrowing/lending in terms of money. We consider limited commitment to repay debt, leading to an endogenous collateral constraint (as in Kiyotaki and Moore, 1997). The central bank supplies money only against eligible assets, which basically consist of treasury securities. In addition, we consider central bank purchases of private debt securities in secondary markets,[11] which can be non-neutral by affecting borrowers and lenders in different ways. When the monetary policy rate, i.e. the price of money in terms of eligible assets, is set below the marginal rate of intertemporal substitution, eligible assets are scarce and money supply is effectively rationed.[12] Then, Wallace's (1981) irrelevance result for open market operations does not apply. Lenders participate in asset purchases programs when the central bank offers an above-market price, leading to a lower real loan rate than in the laissez faire equilibrium (which prevails under conventional monetary policy).

We establish the existence of inefficiencies due to pecuniary externalities induced by financial frictions, which correspond to the "collateral effects" of externalities in Davila and Korinek (2018) or in Bianchi and Mendoza (2018). We follow Lucas and Stokey (1987) and assume that agents pool end-of period funds (within households), such that distracting complexities induced by an endogenous wealth distribution and "distributive effects" of externalities (see Davila and Korinek, 2018) are avoided. In the spirit of the classical Ramsey-approach to optimal policy, we consider a policy problem where non-distortionary taxes/transfers are not available, such that the first best allocation is not implementable.[13] As a by-product, we show that the central bank can use its instruments to ensure intertemporal solvency neither relying on fiscal backing or transfers nor losing its ability to control the price level. Our main analytical results are then derived in four steps: Firstly, we follow Bianchi and Mendoza (2018) and examine the problem of a social planer who maximizes social welfare, while choosing debt on the behalf of private agents subject to the equilibrium conditions.[14] Secondly, we show that the resulting constrained efficient long-run allocation can in principle be implemented by a (Pigouvian) subsidy on debt, if non-distortionary

[11] This can alternatively be interpreted as a central bank collateral framework where private debt securities are eligible. In contrast to related studies on unconventional policies (see Curdia and Woodford, 2011, or Gertler and Karadi, 2011), the central bank does not directly trade with borrowers.

[12] Under money rationing, the central bank can simultaneously control the price and the amount of money, and can thereby implement welfare dominating allocations compared to policy regimes that satiate money demand (see also Schabert, 2015).

[13] Precisely, a feasible consumption sequence relies on positive initial public liabilities. Without (lump-sum) tax revenues the present value of central bank earnings have to be positive. This rules out zero interest rates and complete self-ensurance of agents, which is required first best allocation.

[14] This corresponds to the problem of identifying an optimal Ramsey policy when an instrument is available by which the policy-maker can influence agents' willingness to borrow in a budget-neutral way.

taxes were available. Thirdly, we show that a central bank can exactly replicate the Pigouvian debt subsidy by a suitably sized asset purchase programme (without relying on taxes/transfers). Fourthly, we demonstrate that asset purchases can further implement allocations that welfare-dominate the constrained efficient allocation.[15]

We calibrate the model to provide numerical examples for the analytical results as well as short-run effects of asset purchases. We find that asset purchases can increase ex-ante social welfare (measured in consumption equivalents) in the long run by up to 1% compared to the laissez faire case, while the largest contribution (about 0.9%) stems from restoring constrained efficiency. Under a stochastic aggregate income, asset purchases should be conducted in a countercyclical way, such that borrowing is particularly stimulated in low income states. The reason is that borrowers suffer not only from a reduction in income, but also from an uninternalized decline in the price of collateral. Countercyclical asset purchases then stimulate (dampen) borrowing and thus borrowers' consumption in situations where the borrowing capacity is reduced (enhanced). Likewise, asset purchases should particularly improve borrowing conditions when financial constraints tighten stochastically. Thereby, asset purchases can mitigate financial amplification of aggregate shocks.

Related literature Our analysis relates to studies on unconventional monetary policies under price rigidities. Curdia and Woodford (2011) and Gertler and Karadi (2011) show that direct central bank lending to ultimate borrowers can be beneficial if financial market frictions are sufficiently severe. Chen et al. (2012) find that changing the composition of treasury debt as under US Federal Reserve large scale asset purchase programmes during the financial crisis had moderate GDP growth and inflation effects. Del Negro et al. (2017) examine government purchases of equity in response to an adverse shock to resaleability and show that the introduction of this type of policy after 2008 have prevented a repeat of the Great Depression. Woodford (2016) extends Stein's (2012) fire sale model to assess the impact of central bank purchases of long-term treasuries on financial stability when crises are exogenously triggered. Our paper further relates to Araújo et al. (2015), who show that asset purchases exert ambiguous welfare effects under endogenous collateral constraints. In contrast to our paper, they do not examine pecuniary

[15] The reason is that asset purchases tend to increase the price of collateral relative to non-durables, which alleviates the borrowing constraint in terms of non-durables. This effect stems from the increased lenders' willingness to pay for housing relative to non-durables when they receive a higher effective return on lending. In principle, this can alternatively be generated by an additional Pigouvian subsidy on durables, which relates to Benigno et al.'s (2020) findings under multiple fiscal instruments

externalities, consider "no special uses of currency", and assume that all central bank trades occur at market prices.[16] Finally, our paper relates to the studies by Bianchi (2016), Benigno et al. (2016), and Jeanne and Korinek (2020) on ex-ante (prudential) policies and ex-post policies in non-monetary economies with a single endogenous price and pecuniary externalities induced by financial constraints. In contrast to our analysis, they consider *real* ex-post policies, like bailouts, subsidies, or liquidity provision, that are financed by taxes or deposits. None of these nor other studies examine the difference between monetary and fiscal instruments for correcting pecuniary externalities.

In Section 2, we present the model. Section 3 provides analytical results on welfare-enhancing financial market interventions. In Section 4, we present numerical examples and analyze state-contingent asset purchases under aggregate risk. Section 5 concludes.

2 The model

In this Section, we develop a model with idiosyncratic preference shocks, incomplete asset markets, and limited commitment. To isolate the main mechanisms, we abstract from implications of an endogenous distribution of agents' net wealth. For this, we follow Lucas and Stokey (1987) and assume that funds are pooled within households at the end of each period, such that household members are identical at the beginning of each period before they split up into borrowers and lenders. Given that we restrict our attention to the analysis of ex-post (monetary) policy interventions, we focus on intraperiod (collateralized) loans, implying that borrowers do not build up debt that has to be repaid in subsequent periods. Acknowledging core properties of central bank money, we consider its specific role for transactions and that it can be costlessly produced. To suitably take account of central bank asset purchase programmes, we specify regular treasury open market operations and additionally consider secondary market interventions of the central bank.[17] We disregard endogenous production and price rigidities, such that conventional monetary policy measures are neutral.

2.1 Overview and timing of events

The economy consists of household members, a central bank, and a government. To account for the special role of central bank money, we assume that it is essential because of its unique role

[16] Correia et al. (2016) show that credit subsidies are desirable and superior to monetary policy in a model with costly enforcement (without pecuniary externalities).

[17] The specification of central bank operations closely to Schabert (2015), where financial frictions and idiosyncratic shocks are neglected.

for the settlement of transactions. For simplicity, we assume that money serves as the exclusive means of payment for non-durable goods. Money is supplied by the central bank in exchange for eligible assets, which basically consist of short-term treasuries. Debt contracts are only available in nominal terms. Due to limited enforcement of debt contracts, agents can borrow money among each other only against collateral. The central bank sets the price of money in terms of eligible assets and decides on the amount of money supplied via treasury open market operations. In addition, it can supply money via purchases of collateralized loans in secondary markets. When the central bank trades assets at an above-market price, eligible assets are scarce and money supply is rationed. In this case, asset purchases can be non-neutral. The government issues short-term debt and receives central bank remittances. Fiscal taxes/transfers are not available. The timing of events in each period is as follows:

i.) Households enter a period with holdings of money, government bonds and durable goods, and receive an exogenously determined endowment of non-durable goods.

ii.) Open market operations are conducted, where the central bank sells or purchases assets outright or supplies money under repurchase agreements (repos) against treasury securities at the policy rate.

iii.) Idiosyncratic preference shocks are realized and durable goods are traded. Agents with a high realization of the preference shock are willing to consume more non-durables than agents with a low realization, such that the former tend to borrow money from the latter against collateral. Collateralized loans might be purchased by the central bank and the proceeds are available to extend loan supply.

iv.) Non-durable goods are traded and repos are settled.

v.) The asset market opens, where agents repay collateralized loans, the government issues bonds, and the central bank reinvests earnings from maturing bonds.

2.2 Private sector

There are infinitely many households of measure one that consist of infinitely many members i, which are characterized by identical initial stocks of wealth. In each period, household members draw an idiosyncratic shock. Like Lucas and Stokey (1987), we assume that at the end of each period (after loans are repaid) household members obtain equal shares of total household wealth, such that they are again equally endowed before new idiosyncratic shocks are drawn in the next period. Members' utility increases with consumption $c_{i,t}$ of a non-durable good and holdings of a durable good, i.e. housing $h_{i,t}$; total supply of the latter being normalized to one. Member i receives an endowment of non-durable goods in each period $y_{i,t}$, where $y_{i,t} = y_t$ and y_t denotes aggregate endowment that is stochastic with mean one. They can differ with regard to their marginal valuation of consumption of the non-durable good due to preference shocks $\epsilon_i > 0$, which are i.i.d. across agents and time. The instantaneous utility function $u_{i,t}$ is

$$u_{i,t} = u(\epsilon_i, c_{i,t}, h_{i,t}), \tag{1}$$

where $h_{i,t}$ denotes the end-of-period stock of housing. We assume that $u_{i,t}$ is strictly increasing, concave, separable in consumption of non-durables and in housing, and satisfies the usual Inada conditions. The idiosyncratic shock ϵ_i exhibits two possible realizations, $\epsilon_i \in \{\epsilon_l, \epsilon_b\}$, with mean one, equal probabilities $\pi_\epsilon = 0.5$, and $\epsilon_l < \epsilon_b$. Agents rely on money for purchases of non-durable goods, whereas we treat housing as a "credit good" (see Lucas and Stokey, 1987). They hold money $M^H_{i,t-1}$ at the beginning of each period and they can acquire additional money $I_{i,t}$ from the central bank, for which they rely on eligible assets. Specifically, agents can get money $I_{i,t}$ from the central bank in open market operations, where money is supplied in exchange for treasury securities discounted with the policy rate R^m_t in either outright transactions or repos. Hence, acquisition of money is constrained by holdings of government bonds $B_{i,t-1}$,

$$0 \leq I_{i,t} \leq \kappa^B_t B_{i,t-1}/R^m_t, \tag{2}$$

where the central bank controls the maximum amount of money supplied against treasuries by choosing the fraction of bonds that are accepted in open market operations, $\kappa^B_t \in [0, 1]$, which can be interpreted as an allotment rate.[18] In contrast to purchases of private debt, purchases of public debt can affect the allocation only via an increase in the supply of money, while associated effects on treasury rates will be irrelevant for the equilibrium allocation.

Idiosyncratic preference shocks ϵ_i materialize after treasury open market transactions are conducted. An agent drawing the realization ϵ_b (ϵ_l) is willing to consume more (less) than agents who draw ϵ_l (ϵ_b). Hence, ϵ_b-type agents tend to borrow an additional amount of money from ϵ_l-type agents. We assume that borrowing and lending among agents only takes place in form of short-term nominal debt at the price $1/R^L_t$. Like Jermann and Quadrini (2012) and Woodford

[18] This specification of open market operations follows Schabert (2015).

(2016), we assume that loan contracts are signed at the beginning of the period and repaid at the end of each period.

The crucial element of the model is a financial constraint, which can be microfounded by limited commitment and the possibility of debt renegotiation as follows: We assume that borrowers can threaten to repudiate the debt contract and that lenders protect themselves by collateralizing borrowers' housing. Following a repudiation, they can seize a fraction z of borrowers' housing and can sell it at the current price Q_t in the housing market. We consider the case where borrowers have all the bargaining power and are able to negotiate the loan down to the liquidation value of their housing (see Hart and Moore, 1994). Lenders take this possibility into account, such that the debt repayment does not exceed the value of the seizable collateral. Hence, debt $-L_{i,t} > 0$ of a borrower i with housing h_i is constrained by

$$-L_{i,t} \leq zQ_t h_{i,t}, \tag{3}$$

where Q_t denotes the housing price and $z \in (0,1)$ the exogenous liquidation share of collateral. Notably, treasuries held after open market operations $(1-\kappa_t^B)B_{i,t-1}$ cannot be used as collateral, which simplifies the exposition and the analysis. Allowing for this or for intraperiod bond trades, would affect the treasury rate, but would leave the results qualitatively unaffected.

As a main feature of our analysis, we consider the possibility that the central bank purchases collateralized loans in addition to treasuries: After the preference shocks are realized and while loans are supplied, the central bank offers money in exchange for a fraction $\kappa_t \in [0,1]$ of loans at the price $1/R_t^m$:

$$0 \leq I_{i,t}^L \leq \kappa_t L_{i,t}/R_t^m. \tag{4}$$

Notably, a loan price that is set differently from the open market price of bonds $(1/R_t^m)$ would not provide an additional degree of freedom, since it just matters in combination with the independent instrument κ_t. By purchasing loans, the central bank can thus influence lenders' valuation of loans. Asset purchases ($\kappa_t > 0$) increase the amount of loanable funds, such that lenders can raise their supply of loans. Asset purchases are conducted in form of repos, where loans are repurchased by lenders before they mature (such that lenders earn the interest on loans). After loans are issued and asset purchases are conducted, the market for non-durables opens. Money is assumed to serve as the means of payment for non-durable goods, for which agent i can use money holdings $M_{i,t-1}^{H}$ as well as new injections $I_{i,t}$ and $I_{i,t}^{L}$ plus/minus loans:

$$P_t c_{i,t} \leq I_{i,t} + I_{i,t}^{L} + M_{i,t-1}^{H} - L_{i,t}/R_t^{L}. \tag{5}$$

where P_t denotes the price of non-durables. It should be noted that the constraints (2)-(5) depend on by various prices, which are taken as given by private agents. Precisely, they do not internalize that their behavior affects the real price of housing $q_t = Q_t/P_t$ (see 3) and of loans $1/R_t^{L}$ (see 5). These pecuniary externalities matter for the liquidity and borrowing constraints, and they relate to the "collateral externalities" in Davila and Korinek (2018). After consumption goods are traded, repurchase agreements are settled, i.e. agents buy back loans and treasuries under repos from the central bank. Then, the asset market opens, where agents repay intraperiod loans, invest in treasuries, and might trade assets among each other. Thus, the budget constraint of agent i is

$$\begin{aligned} & M_{i,t-1}^{H} + B_{i,t-1} + L_{i,t}\left(1 - 1/R_t^{L}\right) + P_t y_t - P_t c_{i,t} \\ & \geq M_{i,t}^{H} + (B_{i,t}/R_t) + \left(I_{i,t} + I_{i,t}^{L}\right)(R_t^{m} - 1) + P_t q_t \left(h_{i,t} - (1-\delta_h) h_{i,t-1}\right), \end{aligned} \tag{6}$$

where $1/R_t$ denotes the price of treasuries in period t. In each period, housing depreciates at a fixed rate δ_h and housing is newly constructed at the same rate. This assumption implies a constant supply of housing, while it avoids a unit root under the social planer solution. Household members maximize expected lifetime utility $E\sum_{t=0}^{\infty}\beta^t u_{i,t}$, where E denotes an expectations operator and $\beta \in (0,1)$ a discount factor, subject to (1)-(6), $M_{i,t-1}^{H} \geq 0$, $B_{i,t-1} \geq 0$, where $M_{i,t-1}^{H} > 0$ or $B_{i,t-1} > 0$, and taking prices as given. The resulting first order conditions are given in Appendix A.

The liquidity constraint (5) binds if the nominal marginal rate of intertemporal substitution exceeds one,[19] which is common with standard cash-in-advance models. When lenders are willing to refinance loans at the central bank to the maximum amount, the money supply constraint (4) is binding. This is the case when $R_t^{m} < R_t^{L}$, i.e. when the central bank offers a price for loans $1/R_t^{m}$ that exceeds the market price $1/R_t^{L}$. Precisely, the multiplier $\mu_{i,t} \geq 0$ on the money supply constraint (4) satisfies[20]

$$\frac{\mu_{i,t}}{u_c(\epsilon_i, c_{i,t})} = \frac{1}{1-\kappa}\left(\frac{1}{R_t^{m}} - \frac{1}{R_t^{L}}\right), \tag{7}$$

[19] This is shown in Appendix A.

[20] To derive this equality, we combined conditions (41), (43), and (49), given in Appendix A.

where u_c denote the derivative of u with respect to c. If, however, the policy rate equals the loan rate, $R_t^m = R_t^L$, the money supply constraint (4) becomes slack, $\mu_{i,t} = 0$ (see 7). Loan demand and loan supply further imply that the credit market allocation can be affected by the borrowing constraint and by loan purchases (for $\mu_{i,t} > 0$), respectively. The borrowers' loan demand condition can be written as[21]

$$\frac{1}{R_t^L} = \beta \frac{E_t(u_c(\epsilon_i, c_{i,t+1})/\pi_{t+1})}{u_c(\epsilon_i, c_{i,t})} + \frac{\zeta_{i,t}}{u'(\epsilon_i, c_{i,t})}. \tag{8}$$

Hence, the multiplier $\zeta_{i,t} \geq 0$ on the borrowing constraint (3) tends to raise the RHS of (8), implying a relative increase in current marginal utility of consumption, which can be mitigated by a lower loan rate. Put differently, under a binding borrowing constraint (3) the borrowers' nominal marginal rate of intertemporal substitution exceeds the loan rate R_t^L. Lenders' loan supply condition can be written as $(1/R_t^L) = \kappa_t \cdot (1/R_t^m) + (1-\kappa_t) \cdot \beta E_t \left[u_c(\epsilon_i, c_{i,t+1})/\pi_{t+1} \right] / u_c(\epsilon_i, c_{i,t})$.[22] Hence, the loan rate R_t^L depends on the lender's nominal marginal rate of intertemporal substitution as well as on the policy rate R_t^m, under positive loan purchases, $\kappa_t > 0$, [23] which can alternatively be seen from

$$\frac{1}{R_t^L} = \left[\frac{1-\kappa_t}{1-\kappa_t R_t^L / R_t^m} \right] \cdot \beta \frac{E_t(u_c(\epsilon_i, c_{i,t+1})/\pi_{t+1})}{u_c(\epsilon_i, c_{i,t})}. \tag{9}$$

The loan supply condition (9) shows that the term in the square brackets drives a policy-induced wedge between the effective real returns for borrowers and lenders. Further note that a policy rate satisfying $R_t^m < R_t^L$ ensures that money is scarce (see (50) in Appendix A), such that the liquidity constraint (5) is binding. Then, agents liquidate assets as far as possible, such that the money supply constraints (2) and (4) are binding. As a consequence, access to money supply will effectively be constrained by the available amount of eligible assets, i.e. bonds and loans, and asset purchases can be non-neutral.

2.3 Public sector

The treasury issues one-period nominal bonds at the price $1/R_t$ and receives remittances τ_t^m from the central bank, such that its budget constraint reads $(B_t^T/R_t) + P_t \tau_t^m = B_{t-1}^T$. As an essential part of our analysis, we will show that monetary policy implementation does not rely

[21] It can be obtained by combining conditions (41), (43), (45), and (49), given in Appendix A.

[22] It can be obtained by combining conditions (41), (43), and (47), given in Appendix A.

[23] Hence, a higher share of purchased loans κ_t for a given policy rate $R_t^m < R_t^L$, or a lower policy rate R_t^m for a given share of purchased loans, $\kappa_t > 0$, tend to reduce the loan rate, while the loan rate approaches the policy rate, $R_t^L \to R_t^m$, for $\kappa_t \to 1$.

on fiscal backing. We therefore disregard further taxes and transfers for the analysis of corrective asset purchases. As described above, short-term government bonds serve as eligible assets for central bank operations. Hence, sufficiently large holdings of treasuries can in principle support self-insurance against illiquidity risk (see also Woodford, 1990). If interest rates were equal to zero, money holdings would be costless and the first best allocation could be implemented. Yet, non-availability of taxes renders implementation of first best impossible (see Section 2.4). For comparative purposes, we will below consider Pigouvian taxes/subsidies as alternative corrective policies (see Section 3.1.2), which however rely on the availability of non-distortionary transfers/taxes.

The central bank supplies money in open market operations either outright or temporarily via repos against treasuries, M_t^H and M_t^R, where $I_t = M_t^H - M_{t-1}^H + M_t^R$. It can further increase the supply of money by purchasing loans from lenders, I_t^L, i.e. it supplies money under repos against loans. At the beginning of each period, its holdings of treasuries and the stock of outstanding money are given by B_{t-1}^c and M_{t-1}^H. It then receives treasuries and loans in exchange for money. Before the asset market opens, where the central bank rolls over maturing assets, repos in terms of treasuries and collateralized loans are settled. Accounting for earnings in money supply operations $(R_t^m - 1)\left(I_t^L + M_t^R + M_t^H - M_{t-1}^H\right)$, its budget constraint reads $(B_t^c/R_t) - B_{t-1}^c + P_t\tau_t^m = R_t^m\left(M_t^H - M_{t-1}^H\right) + (R_t^m - 1)\left(I_t^L + M_t^R\right)$. Remittances $P_t\tau_t^m$ (in units of bonds) to the treasury consist of interest earnings from money supply as well as asset holdings,

$$P_t\tau_t^m = (R_t^m - 1)\left(M_t^H - M_{t-1}^H\right) + (R_t^m - 1)\left(I_t^L + M_t^R\right) + (1 - 1/R_t)\, B_t^c. \tag{10}$$

Central bank asset holdings therefore evolve according to $B_t^c - B_{t-1}^c = M_t^H - M_{t-1}^H$. Further assuming that initial values satisfy $B_{-1}^c = M_{-1}^H$, gives the central bank balance sheet $B_t^c = M_t^H$.

The central bank has four instruments at its disposal. It sets the policy rate R_t^m and can decide how much money to supply against treasuries, for which it can adjust $\kappa_t^B \in (0, 1]$. It further decides on the fraction of loans $\kappa_t \in [0, 1]$ that it is willing to exchange for money. Finally, the central bank can choose how much money to supply outright in exchange for treasuries or temporarily via repos, by controlling the ratio of treasury repos to outright purchases $\Omega_t > 0$: $M_t^R = \Omega_t M_t^H$. Thereby, the central bank can adjust its earnings from outright money supply, where it reinvests payoffs in treasuries, and from repos for a given level of the policy rate and for a given money demand. Consider for example the case where nominal money demand is constant and the policy rate is positive $R_t^m > 1$. Under outright purchases, the central bank has no earning, whereas they are strictly positive under repos.

2.4 Equilibrium properties

In equilibrium, agents' optimal plans are satisfied and prices adjust such that all markets clear: $0 = \sum_i l_{i,t}$, $h = \sum_i h_{i,t}$, $y = \sum_i c_{i,t}$, $m_t^H = \sum_i m_{i,t}^H$, $m_t^R = \sum_i (i_{i,t} - m_{i,t}^H)$, $b_t = \sum_i b_{i,t}$, and $b_t^T = b_t^c + b_t$, where $l_{i,t} = L_{i,t}/P_t$, $m_{i,t}^H = M_{i,t}^H/P_t$, $m_t^R = M_t^R/P_t$, $b_{i,t} = B_{i,t}/P_t$, $b_t = B_t/P_t$, $b_t^c = B_t^c/P_t$, and $b_t^T = B_t^T/P_t$. As a reference case, we consider the first best allocation, which maximizes welfare of ex-ante identical agents

$$E \sum_{t=0}^{\infty} \beta^t \sum_i u_{i,t}, \tag{11}$$

s.t. $h = \sum_i h_{i,t}$, and $y = \sum_i c_{i,t}$. Applying the law of large numbers and indexing all agents drawing ϵ_l (ϵ_b) in period t with l (b), we can summarize first best as a set of sequences $\{c_{b,t}^*, c_{l,t}^*, h_{b,t}^*, h_{l,t}^*\}_{t=0}^{\infty}$ satisfying $h_{b,t}^* + h_{l,t}^* = h$, $c_{l,t}^* + c_{b,t}^* = y$,

$$u_c(\epsilon_b, c_{b,t}^*) = u_c(\epsilon_l, c_{l,t}^*), \text{ and } h_{b,t}^* = h_{l,t}^*. \tag{12}$$

Under the first best allocation, the marginal utilities of non-durables and of the end-of-period stock of housing are identical for borrowers and lenders (see 12). This will not be the case in a competitive equilibrium where the borrowing constraint (3) is binding. Only if the equilibrium lending rate R_t^L is equal to one and the supply of eligible assets is sufficiently large, such that money is abundantly available, agents would be able to self-ensure against liquidity risk (see Woodford, 1990). Given that positive initial non-durables consumption requires strictly positive initial public liabilities, the public sector relies on a positive present value of revenues to satisfy intertemporal solvency. Since taxes are not available, positive interest earnings of the central bank are thus required, ruling out that interest rates are permanently equal to zero. To see this, use (10) to get the public sector budget constraint $B_{t-1} + R_t^m M_{t-1}^H = (R_t^m - 1)\left(I_t^L + M_t^R\right) + (B_t/R_t) + R_t^m M_t^H$, iterate it forward from $t = 0$ onwards, and apply period-0-expectations,

$$\begin{aligned} &\left(B_{-1} + R_0^m M_{-1}^H\right)/P_0 \\ &= E_0 \sum_{t=0}^{\infty} \left(\prod_{k=1}^{t} \frac{\pi_k}{R_{k-1}}\right) \left\{\left[(R_t^m - 1)\left(i_t^L + \Omega_t m_t^H\right)\right] + \left[\left(R_t^m - R_{t+1}^m/R_t\right) m_t^H\right]\right\}, \end{aligned} \tag{13}$$

where we used that public sector solvency requires $\lim_{t\to\infty} E_0(\Pi_{k=1}^t \frac{\pi_k}{R_{k-1}})(\frac{b_t}{R_t} + R_t^m m_t^H) = 0$ and that $m_t^R = \Omega_t m_t^H$. First best requires consumption to be unconstrained, such that the liquidity constraint (5) has to be slack. The first order conditions for money and bonds, which are given by (42), (43), and (44) in Appendix A, then imply the treasury rate and the policy rate to satisfy $R_t = 1$ and $R_t^m = 1$. In this case, the term in the curly brackets on the RHS of (13) would be equal zero, whereas the LHS of (13) has to be strictly positive under a non-zero initial consumption of non-durables, which can be seen from the constraints (2) and (5) for $t = 0$.[24]

Proposition 1 *The first best allocation cannot be implemented.*

The two terms in square brackets on the RHS of the intertemporal budget constraint (13) summarize central bank earnings from temporary and from outright money supply operations. It can be satisfied for $R_t^m > 1$, given that $i_t^L \geq 0$, (5) requires $m_t^H > 0$, and that $R_t^m - R_{t+1}^m/R_t$ is positive if $R_t^m > 1$.[25] Thus, the central bank can set the policy rate $R_t^m > 1$ and the ratio of repos to outright money $\Omega_t > 0$ to satisfy (13). Put differently, by setting its instruments R_t^m and Ω_t, the central bank can control the initial price level $P_0 > 0$, regardless of asset purchases, i.e. for $\kappa_t \geq 0 \Rightarrow i_t^L \geq 0$.

Proposition 2 *Suppose that the central bank does not purchases any loans, $\kappa_t = 0$. For given initial public sector liabilities, B_{-1} and M_{-1}^H, and sequences for interest rates $\{R_t, R_t^m > 1\}_{t=0}^{\infty}$ and real balances $\{m_t^H\}_{t=0}^{\infty}$, the initial price level decreases with higher values for the ratio of repos to outright money Ω_t.*

Since multiple instruments $\{R_t^m, \Omega_t, \kappa_t^B\}$ are available, the central bank can ensure (13) to hold independently of fiscal policy and can control the price level for a given equilibrium allocation (see Proposition 4 below).

To facilitate the derivation of analytical results, we further assume that the collateral constraint (3) is always binding for agents drawing ϵ_b, i.e. the multiplier $\zeta_{b,i,t}$ is strictly positive, which can be guaranteed by a sufficiently large difference in agents' valuation of consumption relative to the liquidation value of collateral, $(\epsilon_b - \epsilon_l)/z$.

Assumption 1 *The ratio $(\epsilon_b - \epsilon_l)/z$ is sufficiently large such that the borrowing constraint (3) is binding for all agents drawing ϵ_b, $-L_{b,t} = zQ_t h_{b,t}$.*

A definition of a competitive equilibrium under Assumption 1 is given in Appendix A. For asset purchases to be non-neutral, the policy rate has to be lower than the lender's marginal rate of

[24] Given Inada conditions, non-durable consumption has to be strictly positive in equilibrium.

[25] To show this, one can apply the first order conditions in Appendix A: Substituting out $\psi_{i,t+1} + \lambda_{i,t+1}$ in (43) with (44) and combining with (42), gives $E_t\left[\left(\lambda_{i,t+1} + \kappa_{t+1}^B \eta_{i,t+1}\right)/\pi_{t+1}\right] = E_t\left[(R_{t+1}^m/R_t)(\lambda_{i,t+1} + \eta_{i,t+1})/\pi_{t+1}\right]$. The latter implies $0 < R_t^m - 1 = \frac{E_t\left[\left(R_t^m - R_{t+1}^m/R_t\right)(\lambda_{i,t+1}+\eta_{i,t+1})/\pi_{t+1}\right]}{E_t\left[\left(\lambda_{i,t+1}+\eta_{i,t+1}\right)/\pi_{t+1}\right]}$ if $\kappa_{t+1}^B = 1$, and an even larger value for $(R_t^m - R_{t+1}^m/R_t)$ if $\kappa_{t+1}^B < 1$.

intertemporal substitution, implying $R_t^m < R_t^L$. Then, money supply is rationed by the available amount of assets eligible for central bank operations. If, however, the policy rate equals nominal marginal rate of intertemporal substitution, $R_t^m = R_t^L$, money supply is non-rationed. Agents are then indifferent in the level of additional money acquired in exchange for bonds and loans. In particular, when the central bank offers money against loans, lenders have no incentive to sell them. To avoid an indetermined demand for central bank asset purchases, which is irrelevant for the main results, we introduce the following assumption.

Assumption 2 *When money supply is not rationed, $R_t^m = R_t^L$, agents do not sell loans to the central bank, $I_{i,t}^L = 0$.*

3 Analytical results

In this Section, we examine pecuniary externalities and policy effects in an analytical way. In the first part of this Section, we examine the case of non-rationed money supply and show that conventional monetary policies are neutral. In the second part of this Section, we consider the case of rationed money supply, where asset purchases can be non-neutral. We show how asset purchases can enhance welfare by addressing pecuniary externalities and by easing borrowing conditions. Based on the laissez faire case, we start with identifying a constrained efficient allocation that can be implemented by a planer who decides on agents' borrowing. We then show that the long-run constrained efficient allocation can in principle be implemented by central bank asset purchases and, equivalently, by a Pigouvian debt subsidy. We finally demonstrate that asset purchases can even implement allocations that welfare-dominate this constrained efficient allocation.

3.1 Non-rationed money supply

A prerequisite for the non-neutrality of asset purchases is rationing of money. We first consider the standard case of non-rationed money supply: Suppose that the central bank sets the policy rate such that it equals the equilibrium loan rate, $R_t^m = R_t^L$, and the money supply constraint (4) is not binding ($\mu_{i,t} = 0$). Thus, it sets the policy rate contingent on an equilibrium object, which relates to monetary policy studies using interest rate feedback rules. Precisely, the central bank can set the policy rate R_t^m at a level, where the implied demand for money is not rationed by the available amount of eligible assets, such that arbitrage-freeness leads to the equality $R_t^L = R_t^m$. Under non-rationed money supply and Assumption 2, the monetary policy instruments κ, κ^B, and Ω are irrelevant for the equilibrium allocation and can be set to satisfy (13).

Proposition 2 *A competitive equilibrium under non-rationed money supply is a set of sequences* $\{c_{b,t},\ c_{l,t},\ h_{b,t},\ \widetilde{q}_t,\ r_{t+1}\}_{t=0}^{\infty}$ *satisfying*

$$u_c(\epsilon_l, c_{l,t}) = \beta 0.5 E_t \left[\left(u_c(\epsilon_l, c_{l,t+1}) + u_c(\epsilon_b, c_{b,t+1}) \right) \cdot r_{t+1} \right], \tag{14}$$

$$\begin{aligned}(1/\widetilde{q}_t) \cdot [u_h(h - h_{b,t}) - u_h(h_{b,t})]/z \\ = u_c(\epsilon_b, c_{b,t}) - \beta 0.5 E_t \left[\left(u_c(\epsilon_l, c_{l,t+1}) + u_c(\epsilon_b, c_{b,t+1}) \right) \cdot r_{t+1} \right],\end{aligned} \tag{15}$$

$$u_c(\epsilon_l, c_{l,t}) \cdot \widetilde{q}_t = u_h(h - h_{b,t}) + \beta E_t (1 - \delta_h)[u_c(\epsilon_l, c_{l,t+1}) \cdot \widetilde{q}_{t+1}], \tag{16}$$

$$c_{b,t} - c_{l,t} = 2z\widetilde{q}_t h_{b,t}, \tag{17}$$

$$y_t = c_{b,t} + c_{l,t}, \tag{18}$$

where $r_{t+1} = R_t^L/\pi_{t+1}$, $\widetilde{q}_t = q_t/R_t^L$, *and* $R_t^L = R_t^m$, *for* $\{y_t\}_{t=0}^{\infty}$ *and a sequence* $\{R_t^m > 1\}_{t=0}^{\infty}$ *set by the central bank.*

Proof. See Appendix B. ■

Due to wealth sharing, cash holdings and loans rather than individual net wealth positions are relevant for agents' consumption and housing choices. Thus, the combined cash-in-advance and collateral constraint is decisive for consumption of non-durables and end-of-period housing decisions of the representative borrower and the representative lender (see 17). Therefore, the relative price of collateral in terms of the cash good (non-durables), $\widetilde{q}_t = q_t/R_t^L$, is relevant. Further note that the multiplier $\zeta_{b,t}$ on the borrowing constraint (3) satisfies

$$\zeta_{b,t} = [u_c(\epsilon_b, c_{b,t}) - u_c(\epsilon_l, c_{l,t})]/R_t^L = [u_h(h_{l,t}) - u_h(h_{b,t})]/(zq_t) > 0, \tag{19}$$

indicating that both, the housing and the consumption choice (that would ideally satisfy $h_b = h_l$ and $u_c(\epsilon_b, c_{b,t}) = u_c(\epsilon_l, c_{l,t})$, see 12), are distorted by a binding borrowing constraint ($\zeta_{b,t} > 0$). On the one hand, the marginal utility of consumption is then larger for borrowers than for lenders, $u_c(\epsilon_b, c_{b,t}) > u_c(\epsilon_l, c_{l,t})$. On the other hand, borrowers' housing exceeds lenders' housing, $h_{b,t} > h/2$, as the former is characterized by a relatively higher valuation of housing due to its ability to serve as collateral.

3.1.1 Neutrality of monetary policy

Proposition 2 reveals that the nominal interest rate and thus the policy rate matter jointly with either the housing price or the inflation rate. Precisely, the conditions (14)-(18) impose restrictions on the allocation, $c_{b,t}, c_{l,t}$, and $h_{b,t}$, the relative price of housing $\widetilde{q}_t = q_t/R_t^L$, and the real interest rate $r_{t+1} = R_t^L/\pi_{t+1}$ (see 14-17), but not separately on q_t, π_{t+1}, and R_t^L. Thus, monetary policy, i.e. a change in the policy rate R_t^m ($= R_t^L$), leaves relative prices and the allocation unaffected, while the future inflation rate and the housing price increase with the nominal loan rate.[26]

Corollary 1 *Under non-rationed money supply, relative prices and the equilibrium allocation are unaffected by monetary policy.*

The reason for the neutrality of policy rate changes is that monetary policies can only affect equilibrium prices that are equally relevant for both agents, while the aggregate supply of durable and non-durable goods is exogenously determined. For the subsequent analysis, we define a laissez faire equilibrium as follows.

Definition 1 *A laissez faire equilibrium is a competitive equilibrium under non-rationed money supply,* $R_t^m = R_t^L$*.*

3.1.2 Constrained efficiency

The financial friction considered in this model, i.e., the collateral constraint, can induce inefficiencies due to pecuniary externalities. Several (non-monetary) studies that apply a related friction have shown that credit market interventions can correct for these externalities and can thereby implement a constrained efficient allocation (see Davila and Korinek, 2018, for an overview). In this section, we follow the same strategy and consider the problem of social planer who chooses a welfare-maximizing credit market allocation subject to the laissez faire equilibrium conditions (see e.g. Bianchi and Mendoza , 2018).

We unveil efficiency gains that can be reaped by an optimal credit market allocation of a social planer, who takes into account how prices that are relevant for financial constraints are affected by agents' decisions. Specifically, the price of housing relative to non-durables $\widetilde{q}_t = q_t/R_t^L$ is related to lenders' consumption and to lenders' housing choices (see 16). An increase in the relative housing price $\widetilde{q}_t$ tends to raise the difference between non-durable consumption of borrowers and lenders (see 17), since it increases borrowers' consumption possibilities by raising the price of collateral in terms of consumption. Yet, the impact of the demand for housing and non-durables on the relative housing price $\widetilde{q}_t$ is not internalized by individuals, giving rise to inefficiencies induced by pecuniary externalities. This correspond to Davila and Korinek's (2018) "collateral effects" of externalities.[27]

[26] The latter effect is due to the liquidity constraint and the well-known inflation tax on cash goods, implying that higher interest rates reduce the demand for non-durables and raise housing demand.

[27] In contrast, "distributional effects" of externalities are not relevant for the consumption and housing choices of the representative borrower and the representative lender, due to the irrelevance of individual net asset positions (see Proposition 2).

For example, borrowers do not internalize that an increase in their housing (thus a decrease in lenders' housing) tends to increase the relative housing price $\widetilde{q}_t$ as lenders' willingness to pay for housing increases. Hence, there exists an uninternalized positive feedback loop between collateral demand, prices, and borrowing, as in Bianchi (2011), Stein (2012), or Bianchi and Mendoza (2018). Notably, these studies focus on prudential financial regulation and restrict their attention on financial constraints that are only binding in crisis states. They show that ex-ante interventions should reduce debt to mitigate adverse deleveraging effects. Considering regularly binding constraints (like Kiyotaki and Moore, 1997), we instead examine (ex-post) interventions that are introduced while borrowers are constrained. In these cases, policies should support borrowing, as for example shown by Bianchi (2016) and Jeanne and Korinek (2020) for bailouts and borrowing subsidies.

To facilitate comparisons with this literature on pecuniary externalities under financial frictions, we assume that the social planer chooses debt on the behalf of private agents, implying that the borrowers' credit demand condition (15) is not binding and that the lenders' credit supply condition (14) determines the real loan rate R_t^L/π_{t+1}. The social planer maximizes (11) subject to the remaining equilibrium conditions:

$$\max_{\{c_{b,t},c_{l,t},h_{b,t},\widetilde{q}_t\}_{t=0}^{\infty}} E\sum_{t=0}^{\infty}\beta^t[u(\epsilon_b,c_{b,t},h_{b,t})+u(\epsilon_l,c_{l,t},h-h_{b,t})],\text{ s.t. (16), (17), and (18).} \tag{20}$$

For the purpose of the analysis, we consider the solution to (20) under commitment and disregard the issue of time-inconsistency.[28] Thus, we restrict our attention to time-invariant processes of the solution to the policy plan, which consists of the following first order conditions (see Appendix B)

$$\psi_t-\psi_{t-1}(1-\delta_h)=-\frac{u_c(\epsilon_b,c_{b,t})-u_c(\epsilon_l,c_{l,t})}{-u_{cc}(\epsilon_l,c_{l,t})\cdot\widetilde{q}_t+u_c(\epsilon_l,c_{l,t})/(zh_{b,t})}, \tag{21}$$

$$\frac{u_c(\epsilon_b,c_{b,t})-u_c(\epsilon_l,c_{l,t})}{u_h(h_{l,t})-u_h(h_{b,t})}z\widetilde{q}_t=\left(1+\frac{\psi_t\cdot(-u_{hh}(h_{l,t}))}{u_h(h_{l,t})-u_h(h_{b,t})}\right)\left(1+\frac{-u_{cc}(\epsilon_l,c_{l,t})}{u_c(\epsilon_l,c_{l,t})}zh_{b,t}\widetilde{q}_t\right), \tag{22}$$

where ψ_t denotes the multiplier of (16), as well as the constraints (16), (17), and (18). This plan can in principle be implemented by a policy that is commonly used to correct for pecuniary externalities induced by the financial constraints, namely, a Pigouvian debt tax/subsidy.[29] Specifically,

[28] An analysis of time consistent corrective policies in an environment with an occasionally binding financial constraint is provided by Bianchi and Mendoza (2018).

[29] In contrast to single agent (open economy) models (e.g., Benigno et al., 2016, Bianchi and Mendoza, 2018), this type of corrective policy has to be type-specific in our model (see also Davila and Korinek, 2018).

suppose there exists a tax/subsidy on debt $\tau_t^L \gtreqless 0$ and a lump-sum transfer/tax for borrowers $\tau_{b,t}^R = \tau^L l_t / R_t^L$. Then, the borrower's effective real interest rate is $r_{b,t+1}^\tau = R_t^L \pi_{t+1}^{-1}/(1-\tau_t^L)$, and the borrower's loan price net of taxes is $(1-\tau_t^L)/R_t^L$. Notably, the rate τ_t^L alters the budget constraint (6) and the cash-in-advance constraint of borrowers (5), which has to be the case for the compensating lump-sum transfer/tax as well.[30] Then, condition (17) remains unchanged, while the marginal costs of borrowing are altered by τ_t^L entering (15). Hence, the condition $(1-\tau_t^L) u_c(\epsilon_b, c_{b,t}) = \beta 0.5 E_t \left[(u_c(\epsilon_l, c_{l,t+1}) + u_c(\epsilon_b, c_{b,t+1})) r_{t+1} \right] + (1/\widetilde{q}_t)[u_h(h_{l,t}) - u_h(h_{b,t})]/z$, determines the associated debt tax/subsidy rate τ_t^L.

The solution to the problem (20) cannot be characterized in a transparent way without further restrictions. Yet, the properties of the constrained efficient allocation under a logarithmic utility function can be derived in a straightforward way for the long-run (for which time indices are omitted).

Proposition 3 *Suppose that $u(\epsilon_i, c_{i,t}, h_{i,t}) = \epsilon_i \log(c_{i,t}) + \log(h_{i,t})$ and that type-specific Pigouvian debt taxes/subsidies are available. Then, the long-run allocation under the solution to the planer problem (20) can be implemented by a Pigouvian debt subsidy if*

$$\delta_h < (1 - \beta(1-\delta_h))^2 (\epsilon_l / z) \tag{23}$$

Compared to the laissez faire equilibrium, the Pigouvian subsidy raises (reduces) borrowers' (lenders') durables as well as non-durables and raises the real interest rate R^L/π.

Proof. See Appendix B. ■

Proposition 3 implies that a corrective policy that stimulates borrowing can enhance social welfare in the long-run if (23) holds. This condition is hardly restrictive, given that the depreciation rate of housing is typically very small (see Section 4.1). An increase in borrowing, which is induced by a Pigouvian debt subsidy, tends to increase borrowers' non-durable consumption and has to be supported by a larger stock of borrowers' housing. As implied by the long-run version of (16), $\widetilde{q} = \frac{u_h(h-h_b)}{(1-\beta(1-\delta_h))u_c(\epsilon_l, y-c_b)}$, and $c_l = y - c_b$, the relative housing price $\widetilde{q}$ increases with the latter, $\partial \widetilde{q}/\partial h_b > 0$, and decreases with the former, $\partial \widetilde{q}/\partial c_b < 0$. Given that housing is a durable good, permanent adjustments in housing demand are associated with price changes that tend to dominate the price effects of non-durables under reasonable depreciation rates.

Under (23), borrowing is (constrained) inefficiently low under laissez faire in a long-run equilibrium, given that private agents do not internalize the favorable effects of increased housing

[30] Concretely, the representative borrower's cash-in-advance constraint then changes to $P_t c_{b,t} \leq I_{b,t} + M_{b,t-1}^H - L_{b,t}[(1-\tau^L)/R_t^L] - P_t \tau_{b,t}^R$, while lenders are not taxed/subsidized.

demand on the relative housing price $\widetilde{q}$. The social planer can then correct for this pecuniary externality by a Pigouvian subsidy, which causes agents to borrow more and leads to an increase in borrowers' consumption and housing (see Proposition 3). Notably, the subsidy tends to reduce borrowing costs $r_b^{\tau} = R^L \pi^{-1}/(1-\tau^L)$, while it raises the real interest rate $r = R^L/\pi$, causing an increase in loan supply and a decline in lenders' non-durables and housing.

3.2 Money rationing and asset purchases

We now turn to the case where money supply is rationed, such that central bank purchases of collateralized loans, $\kappa_t > 0$ (see 4), can be non-neutral. Firstly, we will show that an asset purchase policy can be equivalent to a Pigouvian subsidy on debt by driving a wedge between the relevant real returns for borrowers and lenders (see 9). Thus, asset purchases can enhance social welfare by addressing pecuniary externalities, which is not possible under a conventional monetary policy regime (see Corollary 1). Secondly, we will show that asset purchases can enhance welfare even further by increasing the relative price of collateral, which can equivalently be induced by a Pigouvian subsidy on durables if non-distortionary taxes were available.

3.2.1 Monetary policy instruments under money rationing

For asset purchases to be non-neutral, the central bank has to ration money supply, i.e. it has to implement a policy regime of scarce money. For this, it has to set the policy rate below the price agents are willing to pay for money, i.e., the market loan rate, $R_t^m < R_t^L$. Precisely, the central bank can set the price such that agents' demand money against their entire holdings of eligible collateral (see 7). Thus, the money supply constraints in terms of treasuries (2) and collateralized loans (4) are binding,

$$i_{i,t} = \kappa_t^B 0.5 b_{t-1}/(\pi_t R_t^m) \text{ for } i \in \{l, b\} \text{ and } i_{l,t}^L = \kappa_t l_t / R_t^m. \tag{24}$$

When the central bank offers an above-market price for loan purchases, $R_t^m < R_t^L$, there exists a wedge between the borrowers' and the lenders' effective real interest (loan) rate. This wedge can be used to address the pecuniary externalities in the credit market like with a Pigouvian debt subsidy (see Section 3.1.2). Concretely, under an asset purchase regime, the effective real return for a lender is distorted by the wedge $\frac{1-\kappa_t}{1-\kappa_t R_t^L/R_t^m} \geq 1$ (see 9), which increases with the fraction of purchased loans κ_t and with the price discount R_t^L/R_t^m.

Proposition 4 *A competitive equilibrium under money rationing $R_t^m \in (1, R_t^L)$ can be characterized as follows:*

1. *A competitive equilibrium is given by a set of sequences* $\{c_{b,t}, c_{l,t}, h_{b,t}, (q_t/\pi_{t+1}), (R_t^L/\pi_{t+1})\}_{t=0}^{\infty}$ *satisfying (15), (18),*

$$u_c(\epsilon_l, c_{l,t}) = \beta 0.5 E_t\left[\left[(u_c(\epsilon_l, c_{l,t+1}) + u_c(\epsilon_b, c_{b,t+1}))\left\{\frac{R_t^L}{\pi_{t+1}}\frac{1-\kappa_t}{1-\kappa_t R_t^L/R_t^m}\right\}\right]\right], \tag{25}$$

$$\beta E_t\left[0.5(u_c(\epsilon_l, c_{l,t+1}) + u_c(\epsilon_b, c_{b,t+1}))\,(q_t/\pi_{t+1})\right] \tag{26}$$

$$= u_h(h_{l,t}) + \beta^2(1-\delta_h)E_t\left[0.5(u_c(\epsilon_l, c_{l,t+2}) + u_c(\epsilon_b, c_{b,t+2}))\,(q_{t+1}/\pi_{t+2})\right],$$

$$c_{b,t} - c_{l,t} = z h_{b,t}\left[2 - \kappa_t R_t^L/R_t^m\right]\left\{q_t/R_t^L\right\}, \tag{27}$$

and the transversality conditions, given $\{y_t\}_{t=0}^{\infty}$ *and* $\{0 \leq \kappa_t < R_t^m/R_t^L,\ 1 < R_t^m < R_t^L\}_{t=0}^{\infty}$.

2. *For a particular set of equilibrium sequences* $\{c_{b,t}, c_{l,t}, h_{b,t}, (q_t/\pi_{t+1}), (R_t^L/\pi_{t+1})\}_{t=0}^{\infty}$ *and for the policy instruments* $\{0 \leq \kappa_t < R_t^m/R_t^L,\ 1 < R_t^m < R_t^L\}_{t=0}^{\infty}$*, the sequences* $\{m_t^H, b_t, R_t, \pi_t\}_{t=0}^{\infty}$ *and the initial price level* $P_0 > 0$ *satisfy (13),*

$$c_{b,t} = 0.5(1+\Omega_t)m_t^H + z(q_t/\pi_{t+1})h_{b,t}/(R_t^L/\pi_{t+1}), \tag{28}$$

$$(1+\Omega_t)m_t^H = \kappa_t^B b_{t-1}\pi_t^{-1}/R_t^m + m_{t-1}^H\pi_t^{-1}, \tag{29}$$

$$(b_t/R_t) = b_{t-1}\pi_t^{-1} - (R_t^m - 1)\left(i_t^L + \Omega_t m_t^H\right) - R_t^m(m_t^H - m_{t-1}^H\pi_t^{-1}), \tag{30}$$

$$(1/R_t)E_t\left[u_c(\epsilon_l, c_{l,t+1})/\pi_{t+1}\right] \tag{31}$$

$$= E_t\left[\frac{(1-\kappa_{t+1}^B)0.5(u_c(\epsilon_l, c_{l,t+2}) + u_c(\epsilon_b, c_{b,t+2}) + \kappa_{t+1}^B u_c(\epsilon_l, c_{l,t+1})}{R_{t+1}^m \pi_{t+1}}\right]$$

where $i_t^L = \kappa_t z q_t h_{b,t}/R_t^m$*, given the repo ratio and the fraction of eligible treasuries* $\{\kappa_t^B > 0, \Omega_t > 0\}_{t=0}^{\infty}$ *and initial public sector liabilities* $M_{-1}^H > 0$*, and* $B_{-1} > 0$.

The first part of Proposition 4 shows that the competitive equilibrium can be characterized by sequences for the allocation and relative prices $\{c_{b,t}, c_{l,t}, h_{b,t}, (q_t/\pi_{t+1}), (R_t^L/\pi_{t+1})\}_{t=0}^{\infty}$. The property that only relative prices rather than absolute prices can be determined at this stage relates to the case of non-rationed money supply (see Corollary 1). Notably, the allocation, $c_{b,t}$, $c_{l,t}$, and $h_{b,t}$, and the relative prices, q_t/π_{t+1} and R_t^L/π_{t+1}, are only affected by the two policy instruments κ_t and R_t^m. Yet, the central bank has more instruments at its disposal, namely, the repo ratio Ω_t and the fraction of eligible treasuries κ_t^B. The second part of Proposition 4 shows that these two additional instruments are relevant for the determination of the set of equilibrium sequences for public sector liabilities, the treasury rate, and the inflation rate $\{m_t^H, b_t, R_t, \pi_t\}_{t=0}^{\infty}$ and of the initial price level P_0. Put differently, the central bank can set the fraction of eligible treasuries κ_t^B and the repo share Ω_t to implement a particular inflation rate $\{\pi_t\}_{t=0}^{\infty}$ and a particular price level P_0 (via 13) for a given equilibrium allocation.

Corollary 2 *Under money rationing, the central bank can implement a particular sequence for the price level* $\{P_t\}_{t=0}^{\infty}$ *by setting its instruments* $\{\kappa_t^B > 0, \Omega_t > 0\}_{t=0}^{\infty}$.

Notably, by purchasing eligible assets the central bank increases the amount of loanable funds. Comparing (17) with (27), suggests that asset purchases rather reduce than raise the difference between borrowers' and lenders' consumption relative to borrowers' housing. This relates to the direct effect of loan purchases, endowing lenders with additional funds. Yet, there is an additional indirect effect on the relative price housing, q_t/R_t^L. Asset purchases raise the lenders' return on loans and, thus, their willingness to pay for housing relative to non-durables. Hence, an increase in the wedge $\frac{1-\kappa_t}{1-\kappa_t R_t^L/R_t^m}$ induced by asset purchases tends to raise the relative price q_t/R_t^L.[31]

3.2.2 Credit market effects: Mimicking a Pigouvian debt subsidy

Let the price discount s_t be the ratio of the loan price paid by the central bank $1/R_t^m$ and the market price $1/R_t^L$, $s_t = (1/R_t^m)\,/\,\left(1/R_t^L\right) \Leftrightarrow$

$$s_t = R_t^L/R_t^m \geq 1.$$

As shown in Proposition 4, positive asset purchases $\kappa_t > 0$ at an above market price $1/R_t^m > 1/R_t^L \Leftrightarrow s_t = R_t^L/R_t^m > 1$ induce a wedge $\frac{1-\kappa_t}{1-\kappa_t s_t} > 1$ that raises lenders willingness to supply loans (see 25). Thereby, a central bank intervention can influence the loan rate and the credit market outcome similar to a credit market intervention via a subsidy on debt. An apparent difference between both policies is that the latter alters borrowers' perceived debt price, whereas the former shifts the lenders' total return from supplying loans. Hence, these policies imply different responses of the loan rate R_t^L/π_{t+1}. Nevertheless, an asset purchase policy can implement an credit market allocation that is identical to the allocation under a Pigouvian credit market intervention.

Yet, corrective taxes as well as asset purchases have further consequences. For the implementation of Pigouvian debt taxes/subsidies non-distortionary transfers/taxes are typically introduced, which neutralize the effects on agents' income and the fiscal authorities budget. Relatedly, the implementation of an asset purchase policy implies that agents' cash balances and the central banks' net earnings will be affected. The central bank can simply neutralize the impact of additional asset purchase earnings on the intertemporal public sector budget constraint via adjustments in κ_t^B and in the repo share Ω_t (see 13). The impact of asset purchases on agents' cash holdings can further be neutralized by fine-tuning both instruments, i.e. the price of loans

[31] This can be seen from combining (25) and (26), to $u_c(\epsilon_l, c_{l,t})\widehat{q}_t = u_h(h_{l,t}) + \beta(1-\delta_h)E_t[u_c(\epsilon_l, c_{l,t+1})\widehat{q}_{t+1}]$ and its long-run equilibrium version $\widehat{q} = u_h(h_l)/[(1-\beta(1-\delta_h))u_c(\epsilon_l, c_l)]$, where $\widehat{q}_t = (q_t/R_t^L)\cdot[(1-\kappa_t R_t^L/R_t^m)/(1-\kappa_t)]$.

$1/R_t^m$ and the fraction of eligible loans κ_t, which both affect the RHS of (17) directly and via the relative housing price q_t/R_t^L.

For the long-run equilibrium, one can directly relate the monetary policy instruments to the Pigouvian tax/subsidy and show how an asset purchase policy can replicate the allocation under a Pigouvian tax/subsidy. For convenience, we consider that the central bank controls $s_t > 1$ as a policy instrument instead of the policy rate R_t^m, i.e. the central bank sets R_t^m contingent on the equilibrium realization of the loan rate R_t^L.

Proposition 3 *Suppose that money supply is rationed, $s_t > 1$, and (23) is satisfied. Then, the long-run allocation under a Pigouvian debt subsidy at the rate $\tau^L \in (-1, 0)$ can be implemented by the central bank via asset purchases if*

$$\kappa = -\tilde{\tau}^L > 0 \text{ and } s = 2/(1-\tilde{\tau}^L). \tag{32}$$

Proof. Substituting out the relative prices q_t/π_{t+1} and R_t^L/π_{t+1} with (25) and (26) in (15) and (27), a long-run competitive equilibrium under money rationing can be reduced to a set $\{c_b, c_l, h_b\}$ satisfying $y = c_l + c_b$,

$$\left[\frac{1-\kappa}{1-\kappa s}\right] = \frac{u_c(\epsilon_l, c_l)}{u_c(\epsilon_b, c_b)} \left(1 + (1-\beta(1-\delta_h)) \frac{[u_h(h_l) - u_h(h_b)]/z}{u_h(h_l)}\right), \tag{33}$$

$$\frac{c_b - c_l}{h_b} = \frac{z}{1-\beta(1-\delta_h)} \frac{u_h(h_l)}{u_c(\epsilon_l, c_l)} \cdot \left[\frac{(2-\kappa s)(1-\kappa)}{1-\kappa s}\right], \tag{34}$$

given the policy instruments $\kappa \in (0, 1/s)$ and $s = R^L/R^m > 1$. Proceeding analogously with (14)-(17), the corresponding conditions under a Pigouvian debt subsidy with $\tau^L \in (-1, 0)$ are $y = c_l + c_b$,

$$\left[(1-\tau^L)\right] = \frac{u_c(\epsilon_l, c_l)}{u_c(\epsilon_b, c_b)} \left(1 + (1-\beta(1-\delta_h)) \frac{(u_h(h_l) - u_h(h_b))/z}{u_h(h_l)}\right), \tag{35}$$

$$\frac{c_{b,t} - c_{l,t}}{h_{b,t}} = \frac{z}{1-\beta(1-\delta_h)} \frac{u_h(h_l)}{u_c(\epsilon_l, c_l)} \cdot [2]. \tag{36}$$

The comparison of the terms in square brackets in (33)-(34) with the corresponding terms in (35)-(36) immediately reveals that the implementation of an equilibrium allocation under a Pigouvian debt subsidy requires both monetary policy instruments, $s > 1$ and $\kappa \in (0, 1/s)$, to simultaneously satisfy $\frac{1-\kappa}{1-\kappa s} = 1 - \tilde{\tau}^L$ and $\frac{(1-\kappa)(2-\kappa s)}{(1-\kappa s)} = 2$, and therefore $\kappa = -\tilde{\tau}^L \in (0, 1)$ and $s = 2/(1-\tilde{\tau}^L) > 1$. ■

The central bank can implement the (constrained efficient) allocation under a Pigouvian debt subsidy $\tau^L \in (-1, 0)$ by purchasing loans up to a fraction κ that equals the subsidy rate and by offering a price $1/R^m = (1/R^L) \cdot 2/(1+\kappa)$ (see 32). As in the case of the Pigouvian debt subsidy, asset purchases tend to reduce the equilibrium real loan rate R^L/π, which is the relevant rate for borrowers, compared to laissez faire (see also Section 4.2). At the same time the lenders' effective real rate $r_l^{ap} = \frac{R^L}{\pi}\frac{1-\kappa}{1-\kappa s}$ increases, such that the representative borrower (lender) consumes more non-durables (less) than under laissez faire (see Proposition 3).

3.2.3 Goods market effects: Further welfare enhancement

An asset purchase policy can, however, not only affect the real interest rates of borrowers and lenders in different ways (like the Pigouvian debt subsidy), but can further relax borrowing conditions. Nonetheless, first best cannot be implemented with an asset purchase policy, because an increase in borrowing has to be associated with more non-durables holdings of borrowers or a higher price of non-durables.[32] However, an asset purchase policy can implement allocations that welfare-dominate the allocation under a Pigouvian debt subsidy. Importantly, the two instruments $\{\kappa_t, s_t\}$ do not affect the private sector behavior in identical ways (see 33 and 34). A higher price discount s_t (e.g. induced induced by a lower policy rate R_t^m) increases the amount of money supplied per loan, whereas a higher κ_t increases the fraction of purchased loans.

To enhance social welfare in the long-run compared to the allocation under a Pigouvian debt subsidy, the central bank can act as follows: On the one hand, it can ease the constraint imposed on the non-durable consumption differential relative to borrowers' housing $c_b - c_l$ (see 34) compared to the Pigouvian debt subsidy (36) by setting $\{\kappa, s\}$ to satisfy

$$s > 2/(1+\kappa) \tag{37}$$

instead of $s = 2/(1+\kappa)$ as implied by (32). Under (37), the borrowing constraint is effectively relaxed (see proof of Proposition 3), such that the consumption differential relative to borrowers' housing (which is inefficiently small) can be increased compared to the allocation under a Pigouvian debt subsidy. On the other hand, any change in the instruments also affects the wedge $\frac{1-\kappa}{1-\kappa s}$ between the effective real returns for lenders and borrowers (see 15 and 25). Yet, the central bank can use two distinct channels for its two instrument $\{\kappa, s\}$. Specifically, it can relax borrowing conditions by ensuring (37) and simultaneously steer relative prices to address the pecuniary externality associated with the borrowing constraint.[33] To implement a long-run allocation that

[32] To see this, recall that under first best $u_{c_b} = u_{c_l}$ holds (see 12). Condition (33) would in this case imply $\frac{1-\kappa}{1-\kappa s} > 1$ to equal $1 + \frac{1-\beta}{z}\frac{u_{h_l} - u_{h_b}}{u_{h_l}}$ and thus $u_{h_l} > u_{h_b} \Rightarrow h_b > h_l$, which violates the second requirement for a first best allocation (see 12).

[33] In fact, an optimal choice of both instruments would in fact be associated with an infinitely large value for s,

welfare-dominates the constrained efficient allocation under the policy plan (see 20), the central bank can manipulate relative prices via choices for $\kappa \in (0, 1/s)$ and $s > 1$ to satisfy (37) and

$$\frac{u_c(\epsilon_b, c_b) - u_c(\epsilon_l, c_l)}{u_h(h - h_b) - u_h(h_b)} = \frac{\sigma(h_{b,t}/c_l) + h_{b,t}\left(2 - \kappa s\right)/(c_b - c_l)}{1 - \delta_h^{-1}\left(1 - \beta(1 - \delta_h)\right)\sigma^h\left(1 - h/h_b\right)^{-1}\left(1 - \kappa s\right)/\left(1 - \kappa\right)}, \tag{38}$$

which is the long-run version of the planers' optimality condition (22), where $\sigma^h = -u_{hh}h/u_h$ and $\sigma = -u_{cc}c/u_c$ and the multiplier ψ_t has been substituted out with (21) and prices with conditions (25)-(27).

Proposition 4 *Suppose that money supply is rationed, $s_t > 1$, and (23) is satisfied. Then, the central bank can implement long-run equilibrium allocations via asset purchases that welfare-dominate the long-run constrained efficient allocation under (20).*

When the central bank sets the price discount s according to (37), it relaxes the borrowing constraint (see 34) compared to the constrained efficient allocation under (20). To avoid distorting relative prices, the wedge $\frac{1-\kappa}{1-\kappa s}$ that affects loan supply then has to take a lower value than under (32). Social welfare can thus be enhanced compared to the constrained efficient allocation under (20), by increasing s and reducing κ compared to (32).

Such a welfare improvement can also be induced by non-monetary instruments that influence the relative housing price q/R^L. Evidently, a Pigouvian subsidy on durables can in principle exactly replicate this additional effect of asset purchases: Consider the laissez faire equilibrium and a subsidy $\tau_t^h < 0$ on housing, where effects on agents' budgets are neutralized by type-specific and non-distortionary taxes. Then, the long-run relative price for durables satisfies $q/R^L = \frac{1}{1+\tau^h}\frac{u_h(h-h_b)}{(1-\beta(1-\delta_h))u_c(\epsilon_l, y-c_b)}$, such that the term in the square brackets on the RHS of (36) equals $2/(1+\tau^h)$ instead of 2. Further using (33)-(35), shows that any combination of the monetary policy instruments κ and s can in principle be replicated by a combination of Pigouvian subsidies on debt and durables satisfying $\widetilde{\tau}^L = 1 - \frac{1-\kappa}{1-\kappa s}$ and $\widetilde{\tau}^h = 0.5\frac{(2-\kappa s)(1-\kappa)}{1-\kappa s} - 1$. Yet, this policy mix requires the availability of type-specific and non-distortionary taxes, which are not necessary for corrective asset purchases.

4 Numerical results

In this section, we provide numerical examples illustrating the analytical results as well as results for the short-run. The latter shows that asset purchases should be conducted in a countercyclical way.

while κ has to be adjusted to avoid adverse effects of price distortions (see 33). This is demonstrated below in the numerical analysis (see Section 4.2).

4.1 Calibration

For the numerical analysis, we apply a separable CRRA utility function for household member i

$$u(\epsilon_i, c_{i,t}, h_{i,t}) = \epsilon_i \frac{c_{i,t}^{1-\sigma} - 1}{1-\sigma} + \gamma \frac{h_{i,t}^{1-\sigma^h} - 1}{1-\sigma^h}, \text{ where } \gamma, \sigma^{(h)} > 0, \tag{39}$$

We further introduce aggregate risk in form of a random process for y_t, which will be used in Section 4.3. We assume that log aggregate endowment with non-durables follows an AR1 process

$$\log y_t = \rho \log y_{t-1} + \varepsilon_{y,t}. \tag{40}$$

Hence, we have to assign values for the elasticity of intertemporal substitution σ, the discount factor β, the utility weight for housing γ, the liquidation value of collateral z, the degree of heterogeneity $\Delta\epsilon = \epsilon_b - \epsilon_l$, the housing depreciation rate δ_h, the autocorrelation coefficient ρ of the AR1 process, and the standard deviation of the innovations σ_ε. For the computation of the non-durable price q and of the loan rate, we further assign specific values to the growth rate of treasuries Γ and for the repo share Ω. Given that both are not relevant for the equilibrium allocation (see Proposition 4), we set $\Gamma = \Omega = 1$, for convenience. We interpret a model period as one year and calibrate the laissez faire equilibrium of the model consistent with postwar US data. Estimating the process (40) using (linearly detrended) annual US data for real GDP per capita (for 1947-2008), leads to $\rho = 0.752$ and $\sigma_\varepsilon = 0.0216$. The inverse of the elasticity of intertemporal substitution $\sigma^{(h)}$ is set equal to 2, which is a typical value applied in business cycle studies. The constant liquidation value of collateral z is set equal to 0.55, which is similar to values applied in related studies (see Iacoviello, 2005). For these parameter values, condition (23) is already satisfied for a relatively high value for the housing depreciation rate of 7.7%, while we assign a value of 2.2% to δ_h, following Berger et al. (2018) choice based on BEA data.

For the remaining three parameters, β, γ, and $\Delta\epsilon$, we apply values that allow to match three targets for the reference case without financial market interventions. Notably, the data samples are not aligned due to limited availability. The first target is the mean share of installment loans to income of 21% (1998-2004, Survey of Consumer Finances), which correspond to the specification in our model, where loans are demanded for non-durables rather than for housing. The second target is the mean yield on MBS of 6.6% for pre-2009 US data, taken from Hancock and Passmore (2011), which corresponds to the rate on collateralized loans $R^L - 1$. The third target is the cross sectional standard deviation of real log consumption of 0.64 (see De Giorgi and

Gambetti, 2012). While it is not possible to exactly match all three targets, our choice $\beta = 0.8$, $\gamma = 0.002$, and $\Delta\epsilon = 0.76$ yields to a reasonable match given by an interest rate on collateralized debt 1.06, a loan to income share of 0.2, and a standard deviation of real log consumption of 0.6.

4.2 Welfare gains of asset purchases without aggregate risk

We first compute the long-run equilibrium allocation and associated prices for different policy regimes. As a reference case, we consider a laissez-faire regime, i.e. where no corrective policy is applied and monetary policy is neutral (see Corollary 1). Figure 1 shows how asset purchases, which require monetary policy instruments to satisfy $\kappa \in (0, 1/s)$ and $s > 1$, affect relative prices and the equilibrium allocation. The effects are computed for the range of values $\kappa \in (0, 0.75)$ and $s \in (1, 1.2)$, while inflation is fixed at a relatively high value that guarantees a positive equilibrium loan rate ($\pi - 1 = 7\%$). All variables are expressed in terms of percentage deviations from their corresponding laissez faire values. and amplifies the increase in the lenders' real rate

Higher values for the share of purchased loans κ as well as a larger price discount s enhance the effects of the central bank intervention. As revealed in the first row of Figure 1, higher values for κ and s reduce the real rate for borrowers $r_b^{ap} = R^L/\pi$, while they tend to raise the real rate for lender $r_l^{ap} = \frac{R^L}{\pi}\frac{1-\kappa}{1-\kappa s}$. Notably, the latter is not the case for combinations of a low s and a high κ, where lenders receive a large amount of money at a small discount, such that they tend to consume more and their real rate r_l^{ap} is below the laissez faire case (< 0). Overall, the price effects of the policy interventions can be substantial, in particular for the relative housing price q/R^L, which can be more than twice as large as in the laissez faire case for a combination of high κ and high s. Consistently, borrowers' consumption of non-durables also tends to increase in these cases, except for low values of s and κ (see above). The effects on borrowers' housing reveal that a larger share of purchased loans κ tend to raise collateral demand, which is not generally the case for a larger discount s, where the central bank supplies more money per loan unit.

The last two rows of Figure 1 present the welfare effects of asset purchases. For this, we compute the utility values for a representative borrower and a representative lender, $u(\epsilon_b, c_b, h_b)$ and $u(\epsilon_l, c_l, h_l)$, and present the permanent consumption equivalents for borrowers' and lenders' welfare, $v_b^{ce} = [(1-\sigma)\, u(\epsilon_b, c_b, h_b)]^{1/(1-\sigma)}$ and $v_l^{ce} = [(1-\sigma)\, u(\epsilon_l, c_l, h_l)]^{1/(1-\sigma)}$, as well as for ex-ante social welfare, $v^{ce} = [(1-\sigma)\, 0.5\{u(\epsilon_l, c_l, h_l) + u(\epsilon_b, c_b, h_b)\}]^{1/(1-\sigma)}$. Apparently, welfare of lenders falls with larger values for κ and s, whereas welfare of borrowers tends to increase. Notably, welfare for borrowers increases by more than 30% compared to the laissez faire case.

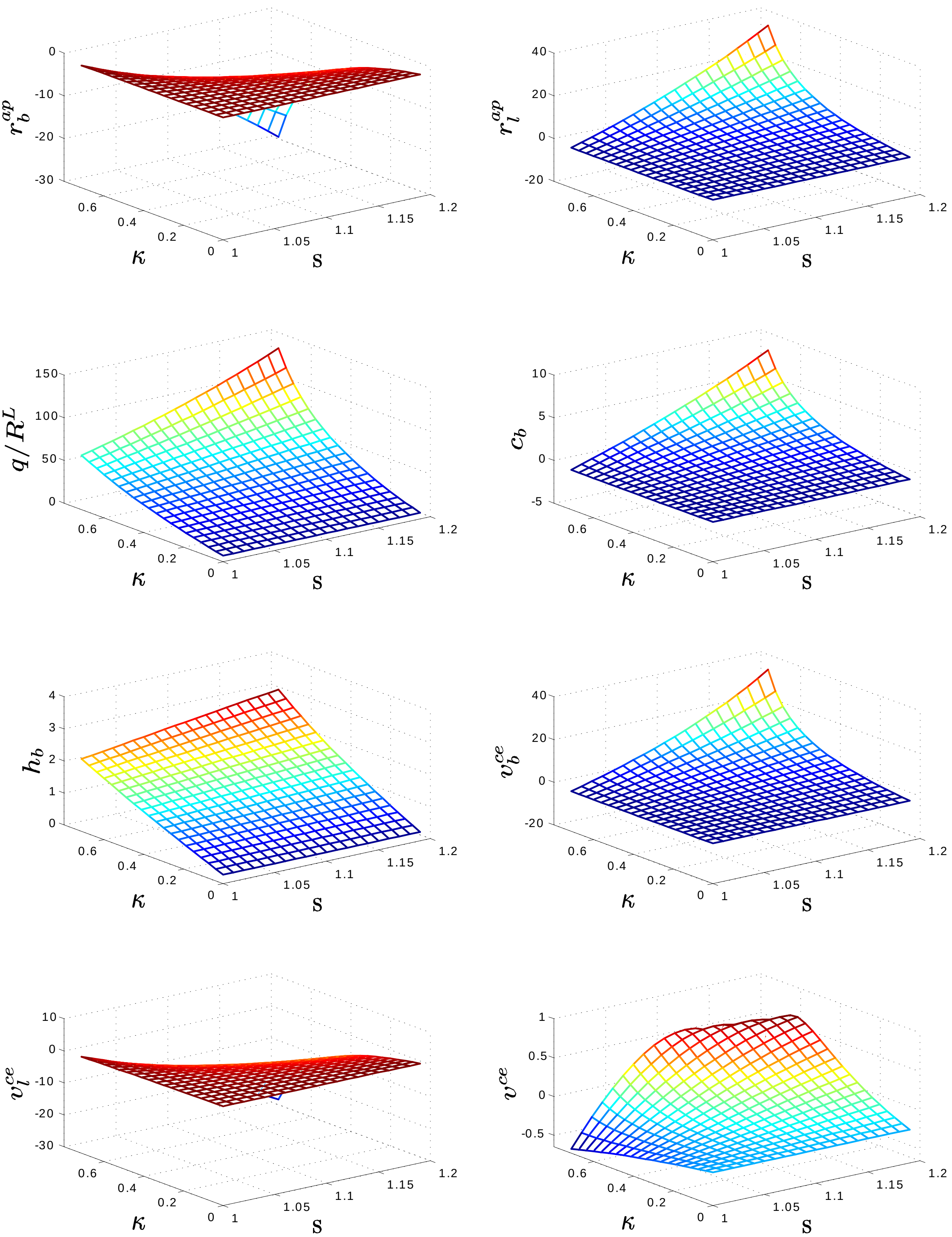

Figure 1: Effects of κ and s for $\pi = 1.07$ (in % deviations from laissez faire values)

Yet, the impact on ex-ante social welfare is much smaller, since the two welfare components mostly offset each other. For small values of s, the distortive effects of asset purchases dominate and social welfare decreases with κ compared to the laissez faire case. For larger values of s, social welfare tends to increase in κ and s, while changes in social welfare are non-monotonic as the loss of lenders strongly increases for combinations of large κ and s values. In total, welfare does increase by more than 1% of social welfare under laissez faire.

To see how monetary policy instruments under asset purchases can enhance efficiency (see also Section 3.2), we vary the price discount $s > 1$ and compute the optimal fraction of purchased loans κ according to the efficiency condition (38) as well as to (33), (34), and $y = c_l + c$. Figure 2 shows the effects for various values of s for the benchmark value for the liquidation share of collateral, $z = 0.55$ (black solid line) and a lower value $z = 0.50$ (red dashed line), while we mark the values of the long-run constrained efficient allocation under (20) with (blue) circles ($\kappa = -\widetilde{\tau}^L = 0.31$ for $z = 0.55$ and $\kappa = -\widetilde{\tau}^L = 0.38$ for $z = 0.45$). All values are now given in absolute terms, except for social welfare v^{ce}, which is again given in terms of percentage deviations from the corresponding laissez faire value.

As shown in the first line of Figure 2, a higher price discount s is accompanied by a lower share of purchased loans κ, in accordance with the efficiency condition (38), and by a higher value for the term $\frac{(1-\kappa)(2-\kappa s)}{1-\kappa s}$, implying a relaxation of the borrowing constraint (see 34). This is reflected by an increase in the consumption differential relative to borrowers' housing, $(c_b - c_l)/h_b$. The lower value of κ in fact reduces the wedge $\frac{1-\kappa}{1-\kappa s}$ to correct for distortionary effects on market prices and to address the pecuniary externality. Then, the relative price of housing q/R^L, which is larger than in the laissez faire case, decreases with s. These effects are accompanied by a decrease in borrowers' housing and thus an increase in lenders' housing induced by the enhanced lenders' willingness to save. Overall, an increase in s and the associated reduction in κ enhance ex-ante social welfare compared to the constrained efficient allocation under (20) by relaxing the collateral requirement (see also Proposition 4). Concretely, the welfare gain under the pure credit market intervention (i.e. the Pigouvian debt subsidy) of about 0.9% compared to laissez faire can be increased up to 1%, with diminishing gains for larger values for s.[34] For a more severe collateral constraint, (see red dashed line for $z = 0.50$), the effects of changes in s are analogous, while, intuitively, the maximum welfare gain from policy interventions is larger (about 1.2%).

[34] Notably, these welfare gains closely relate to the values found by Benigno et al. (2016).

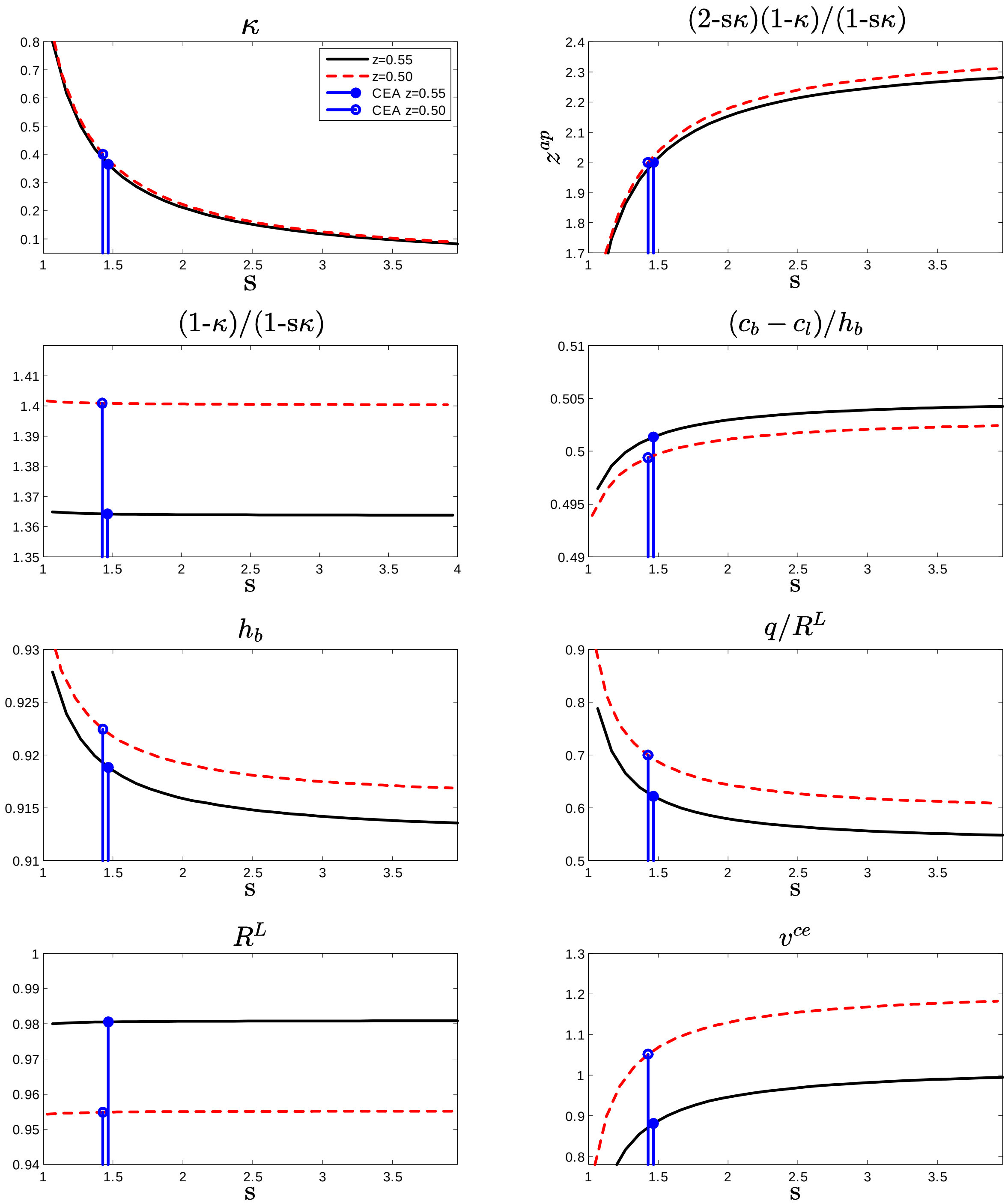

Figure 2: Equilibrium objects under asset purchases and the constrained efficient allocation (CEA) for variations in s [Note: All values are given in absolute terms, except for social welfare, which is in % deviations from laissez faire values.]

4.3 State contingent credit market interventions

We now consider aggregate risk and examine short-run effects of state contingent asset purchases. Due to aggregate shocks, welfare losses stemming from financial frictions can be amplified in the short-run, i.e. when the economy deviates from a stationary equilibrium due to $\varepsilon_t \neq 0$. To understand the welfare-enhancing role of state contingent interventions, consider first the laissez faire case. When the economy is hit by an adverse income shock, $\varepsilon_t < 0$, non-durable consumption of all agents decreases. When lenders reduce their demand for housing, the decline in the housing price and in the collateral value, which tends to tighten agents' borrowing capacity, are not internalized. In such a situation, a corrective policy that tends to increase the collateral price and stimulates borrowing can enhance welfare, which can be achieved by asset purchases at above-market prices.

As discussed in Section 3.2.3, an optimal choice for the fraction of purchased loans κ_t and the price discount s_t would lead to an infinitely large value for the latter, while the welfare gains beyond those of the constrained efficient allocation under (20) are limited (see Figure 2). We therefore focus on the optimal state contingent asset purchases, which can equivalently be implemented by a Pigouvian debt subsidy, and show how this policy responds to aggregate shocks. Precisely, we set the means κ and s to implement the long-run constrained efficient allocation under (20), i.e. to satisfy (32). The model is solved applying a second order perturbation method.[35]

Figure 3 presents impulse responses to a negative income shock by one standard deviation, which hits all agents equally. The black solid line shows the responses under the state contingent asset purchase policy, while the red dashed line with crosses shows the responses for the time invariant policy (both with the same long-run equilibrium allocations). Apparently, the adverse shock reduces non-durable consumption of borrowers and lenders. Compared to the time-invariant policy, state contingent interventions reduce the increase in the borrowers' real rate $r^{ap}_{b,t+1} = R^L_t/\pi_{t+1}$ and amplifies the increase in the lenders' real rate $r^{ap}_{l,t+1} = \frac{R^L_t}{\pi_{t+1}} \cdot \frac{1-\kappa_t}{1-\kappa_t R^L_t/R^m_t}$.[36] State contingent asset purchases mitigate adverse income shock effects on borrowers' non-durable consumption by stimulating borrowing via a relaxation of the borrowing constraint, i.e. asset purchases raise the effective liquidation value of collateral $\widetilde{z}_t = \frac{z}{2}\frac{(1-\kappa)(2-\kappa s)}{(1-\kappa s)}$ (see last panel of

[35] We use dynare's implementation of a second order perturbation.

[36] While the differences in the responses of housing and consumption under both regimes are relatively small, differences in the interest rates are much more pronounced.

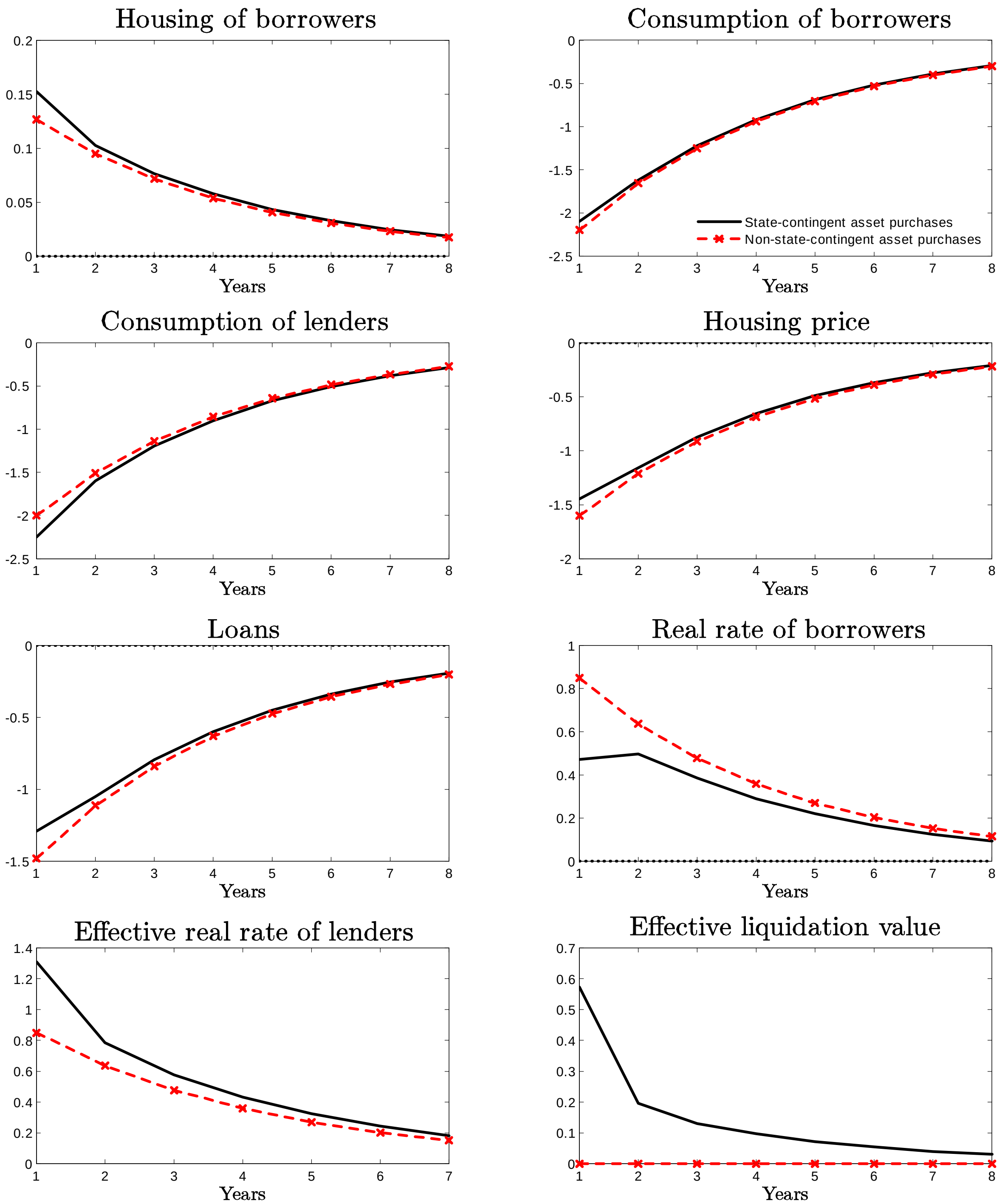

Figure 3: Responses to a minus one st.dev. aggregate income shock (in % deviations from a non-stochastic mean)

Figure 3). Hence, state contingent asset purchases should be countercyclical in the sense that they stimulate (dampen) borrowing in downturns (upturns).

We finally assess how an asset purchase policy should respond to an exogenous worsening of financial conditions and examine an unexpected fall in the liquidation value of collateral (or loan-to-value ratio) z_t, which can be interpreted as a situation with stressed financial markets.[37] The state contingent asset purchase policy mitigates the adverse effects of the liquidation value shock on loans and on borrowers' non-durable consumption (see Figure 4). By raising the price discount s_t, which tends to lower (raise) the borrowers' (lenders') real interest rate, the central bank reduces the fall in loans, which is associated with an increase in borrowers' housing and thus in the price of housing. While borrowers' non-durable consumption decreases under a time-invariant policy, state contingent asset purchases can stabilize borrowers' consumption subsequent to the z_t-shock. Overall, responses to both types of shocks show that asset purchases should stimulate borrowing in adverse states and, symmetrically, mitigate the build-up of debt in favorable states. Thereby, a state contingent asset purchases reduce the shock amplification due to a positive feedback loop between collateral demand, prices, and borrowing.

5 Conclusion

Can central bank asset purchases serve as corrective policies? This paper shows that central bank asset purchases in secondary markets can enhance social welfare as a corrective policy that mitigates financial amplification. The central bank can incentivize lenders to enhance the supply of funds by purchasing debt at an above-market price. This causes the borrower's real interest rate to fall relative to the effective real interest of lenders, which allows addressing pecuniary externalities induced by a collateral constraint. The effects of asset purchases can in principle also be generated by a combination of Pigouvian subsidies on debt and housing. In contrast to the latter, asset purchases do not rely on the availability of non-distortionary sum taxes. Under limited availability of fiscal instruments, asset purchases are a superior corrective policy, which is based on specific properties of central bank money: It can costlessly be produced and exhibits a unique role for the settlement of transactions.

We further show that state-contingent asset purchases should be conducted in a countercyclical way to reduce amplification of aggregate shocks via financial frictions. Asset purchases can

[37] Concretely, we assume that z_t is generated by $\log z_t = \rho \log z_{t-1} + (1-\rho)\log z + \varepsilon_{z,t}$, where $\varepsilon_{z,t}$ is i.i.d. with mean zero and standard deviation σ_ε.

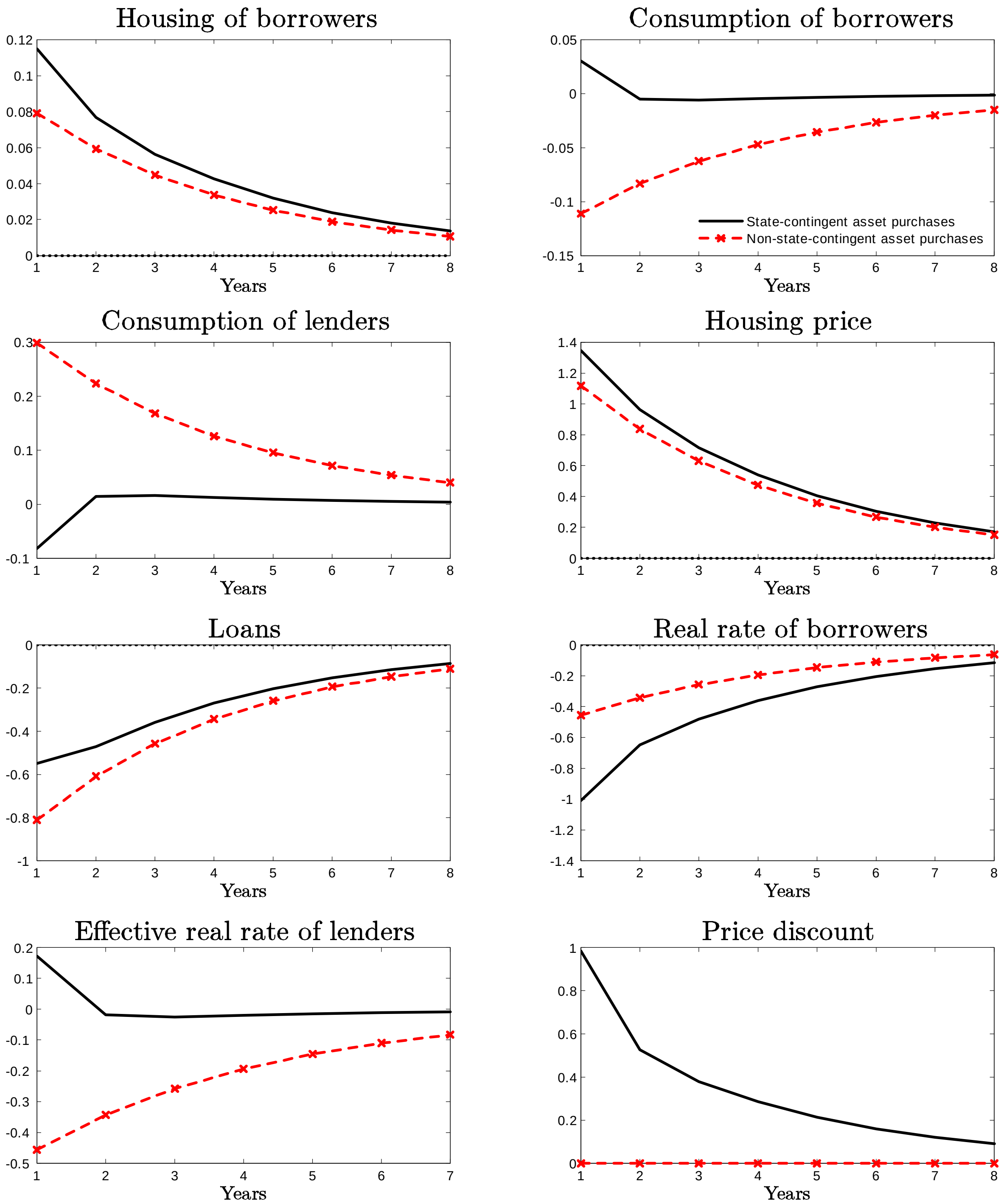

Figure 4: Responses to a minus one st.dev. liquidation value shock (in % deviations from a non-stochastic mean)

thereby contribute to financial stability, in addition to macroprudential financial regulation. In this paper, we do not analyze (ex-post) asset purchases together with ex-ante regulation to isolate novel effects of unconventional monetary policy, leaving a joint analysis for future research.

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Appendix

A Appendix to Section 2

Problem of private agents Maximizing $E\sum_{t=0}^{\infty}\beta^t u_{i,t}$, where the discount factor satisfies $\beta \in (0,1)$, subject to (1)-(6), $M_{i,t-1}^H > 0$, $B_{i,t-1} > 0$, and taking prices as given, leads to the following first order conditions for non-durables, holdings of treasuries and money, and additional money from treasury open market operations $\forall i \in \{b,l\}$:

$$u_c(\epsilon_i, c_{i,t}) = \lambda_{i,t} + \psi_{i,t}, \tag{41}$$

$$\lambda_{i,t} = \beta R_t E_t \left[\left(\lambda_{i,t+1} + \kappa_{t+1}^B \eta_{i,t+1} \right) / \pi_{t+1} \right], \tag{42}$$

$$\lambda_{i,t} = \beta E_t \left[\left(\lambda_{i,t+1} + \psi_{i,t+1} \right) / \pi_{t+1} \right], \tag{43}$$

$$\overline{E}_t \psi_{i,t} = (R_t^m - 1)\, \overline{E}_t \lambda_{i,t} + \overline{E}_t R_t^m \eta_{i,t}, \tag{44}$$

where $\pi_t = P_t/P_{t-1}$ denotes the inflation rate and $\overline{E}_t$ the expectations at the beginning of period t before individual shocks are drawn. Further, $\lambda_{i,t} \geq 0$ is the multiplier on the asset market constraint (6), $\eta_{i,t} \geq 0$ the multiplier on the money supply constraint (2), and $\psi_{i,t} \geq 0$ the multiplier on the cash-in-advance constraint (5), where all constraints are expressed in real terms. Condition (44) reflects that idiosyncratic shocks are not revealed before treasury open market operations are initiated. The following type-specific first order conditions for loans and housing have to be satisfied, for borrowers

$$\lambda_{i,t} \left(1 - 1/R_t^L \right) - (\psi_{i,t}/R_t^L) + \zeta_{i,t} = 0, \tag{45}$$

$$u_h(h_{i,t}) + \zeta_{i,t} z q_t + \beta E_t (1 - \delta_h) q_{t+1} \lambda_{i,t+1} - q_t \lambda_{i,t} = 0, \tag{46}$$

and for lenders, where we additionally consider the first order condition for money acquired from loan purchases $I_{l,t}^L$,

$$\lambda_{i,t} \left(1 - 1/R_t^L \right) - (\psi_{i,t}/R_t^L) + \mu_{i,t} \kappa_t = 0, \tag{47}$$

$$u_h(h_{i,t}) + \beta E_t q_{t+1} \lambda_{i,t+1} - q_t \lambda_{i,t} = 0, \tag{48}$$

$$-\lambda_{i,t} \left(1 - 1/R_t^m \right) + (\psi_{i,t}/R_t^m) - \mu_{i,t} = 0. \tag{49}$$

where $\mu_{i,t} \geq 0$ denotes the multiplier on the money supply constraint (4). Condition (49) describes lenders' willingness to sell loans to the central bank. The conditions (45) and (47) further show that the multiplier on the cash-in-advance constraint (5) is positive if the loan rate R_t^L exceeds one, as the latter measures the price of cash goods (non-durables). Further, the associated complementary slackness conditions,

$$\eta_{i,t}[\kappa_t^B b_{i,t-1}(\pi_t R_t^m)^{-1} - i_{i,t}] = 0,\ \zeta_{i,t}\,[zq_t h_{i,t} + l_{i,t}] = 0,$$
$$\mu_{i,t}\left[\kappa_t l_{i,t}/R_t^m - i_{i,t}^L\right] = 0,\ \psi_{i,t}\left[i_{i,t} + i_{i,t}^L + m_{i,t-1}^H - (l_{i,t}/R_t^L) - c_{i,t}\right] = 0,$$

where $b_{i,t} = B_{i,t}/P_t$, $l_{i,t} = L_{i,t}/P_t$, $m_{i,t}^H = M_{i,t}^H/P_t$, $i_{i,t} = I_{i,t}/P_t$, $i_{i,t}^L = I_{i,t}^L/P_t$, as well as (2)-(5), (6) as an equality, and the associated transversality conditions hold.

Conditions (41) and (43) can be combined to $\frac{\psi_{i,t}}{u'(\epsilon_i,c_{i,t})} = 1 - \beta\frac{E_t[u_c(\epsilon_i,c_{i,t+1})/\pi_{t+1}]}{u_c(\epsilon_i,c_{i,t})}$. Hence, the liquidity constraint (5) binds if the nominal marginal rate of intertemporal substitution $\frac{u_c(\epsilon_i,c_{i,t})}{\beta E_t(u_c(\epsilon_i,c_{i,t+1})/\pi_{t+1})}$ exceeds one. The conditions (41), (43), and (44) further imply

$$\frac{\overline{E}_t\eta_{i,t}}{\overline{E}_t u_c(\epsilon_i, c_{i,t})} = \frac{1}{R_t^m} - \beta\frac{\overline{E}_t[u_c(\epsilon_i, c_{i,t+1})/\pi_{t+1}]}{\overline{E}_t u_c(\epsilon_i, c_{i,t})}, \tag{50}$$

where the last $\frac{\beta\overline{E}_t[u_c(\epsilon_i,c_{i,t+1})/\pi_{t+1}]}{\overline{E}_t u_c(\epsilon_i,c_{i,t})}$ cannot be larger than the inverse of the loan rate $1/R_t^L$ (see 8 and 9). A policy rate satisfying $R_t^m < R_t^L$ therefore ensures that money is scarce, such that the liquidity constraint (5) is binding, and that agents liquidate bonds as far as possible, such that the money supply constraint (2) is binding as well as (4).

Competitive equilibrium

Definition 2 *Under Assumption 1, a competitive equilibrium is a set of sequences* $\{c_{i,t}, h_{i,t}, l_{i,t}, i_{i,t}, i^L_{i,t}, \zeta_{i,t}, \lambda_{i,t}, m^H_{i,t}, b_{i,t}, \pi_t, R^L_t, R_t, q_t\}_{t=0}^{\infty}$ *and* P_0 *satisfying for* $i \in \{b, l\}$

$$m^H_{i,t-1}\pi_t^{-1} + b_{i,t-1}\pi_t^{-1} + l_{i,t}\left(1 - 1/R^L_t\right) + y_t$$

$$= m^H_{i,t} + (b_{i,t}/R_t) + \left(i_{i,t} + i^L_{i,t}\right)(R^m_t - 1) + c_{i,t} + q_t\left(h_{i,t} - h_{i,t-1}\right), \tag{51}$$

$$\lambda_{i,t} = \beta E_t[u'(\epsilon_i, c_{i,t+1})/\pi_{t+1}], \tag{52}$$

$$\beta E_t\left[\left(\lambda_{i,t+1} + \psi_{i,t+1}\right)/\pi_{t+1}\right] = \beta R_t E_t\left[\left((1 - \kappa^B_{t+1})\lambda_{i,t+1} + \kappa^B_{t+1}\frac{\psi_{i,t+1} + \lambda_{i,t+1}}{R^m_{t+1}}\right)/\pi_{t+1}\right] \tag{53}$$

$$\frac{1}{R^L_t} = \beta\frac{E_t\left[u'(\epsilon_i, c_{i,t+1})/\pi_{t+1}\right]}{u'(\epsilon_i, c_{i,t})} + \frac{\zeta_{i,t}}{u'(\epsilon_i, c_{i,t})} \tag{54}$$

$$\text{or } \frac{1}{R^L_t} = \beta\frac{E_t\left[u'(\epsilon_i, c_{i,t+1})/\pi_{t+1}\right]}{u'(\epsilon_i, c_{i,t})} \cdot \frac{1 - \kappa_t}{1 - \kappa_t R^L_t/R^m_t},$$

$$c_{i,t} = i_{i,t} + i^L_{i,t} + m^H_{i,t-1}\pi_t^{-1} - l_{i,t}/R^L_t \text{ if } \psi_{i,t} > 0, \tag{55}$$

$$\text{or } c_{i,t} \leq i_{i,t} + i^L_{i,t} + m^H_{i,t-1}\pi_t^{-1} - l_{i,t}/R^L_t \text{ if } \psi_{i,t} = 0,$$

$$R^m_t i_{i,t} = \kappa^B_t b_{i,t-1}\pi_t^{-1} \quad \text{if } \eta_{i,t} > 0, \tag{56}$$

$$\text{or } R^m_t i_{i,t} < \kappa^B_t b_{i,t-1}\pi_t^{-1} \quad \text{if } \eta_{i,t} = 0,$$

$$0 \leq R^m_t i^L_{i,t} = \kappa_t l_{i,t} \quad \text{if } \mu_{i,t} > 0 \tag{57}$$

$$\text{or } 0 \leq R^m_t i^L_{i,t} \leq \kappa_t l_{i,t} \quad \text{if } \mu_{i,t} = 0,$$

$$-l_{i,t} = z q_t h_{i,t} \quad \text{if } \zeta_{i,t} > 0, \tag{58}$$

$$\text{or } -l_{i,t} \leq z_t q_t h_{i,t} \quad \text{if } \zeta_{i,t} = 0,$$

$$q_t\lambda_{i,t} = u_{h,i,t} + \zeta_{i,t} z q_t + \beta E_t(1 - \delta_h)q_{t+1}\lambda_{i,t+1}, \tag{59}$$

$$i_t = (1 + \Omega_t)m^H_t - m^H_{t-1}\pi_t^{-1}, \tag{60}$$

$$0 = \Sigma_i l_{i,t}, \tag{61}$$

$$h = \Sigma_i h_{i,t}, \tag{62}$$

$$b_{t-1}\pi_t^{-1} = (R^m_t - 1)\left(i^L_t + \Omega_t m^H_t\right) + (b_t/R_t) + R^m_t(m^H_t - m^H_{t-1}\pi_t^{-1}), \tag{63}$$

$$\frac{\left(B_{-1} + R^m_0 M^H_{-1}\right)}{P_0} = E_0\sum_{t=0}^{\infty}\left(\prod_{k=1}^{t}\frac{\pi_k}{R_{k-1}}\right)\left\{(R^m_t - 1)\left(i^L_t + \Omega_t m^H_t\right) + \left(R^m_t - \frac{R^m_{t+1}}{R_t}\right)m^H_t\right\}, \tag{64}$$

where $b_{t-1} = \sum_i b_{i,t-1}$, $i_t = \sum_i i_{i,t}$, $i^L_t = \sum_i i^L_{i,t}$, *and* $m^H_{t-1} = \sum_i m^H_{i,t-1}$, *and the multipliers* $\psi_{i,t}$, $\mu_{i,t}$, *and* $\eta_{i,t}$ *satisfy*

$$\psi_{i,t} = u'(\epsilon_i, c_{i,t}) - \beta E_t\left[u'(\epsilon_i, c_{i,t+1})/\pi_{t+1}\right] \geq 0, \tag{65}$$

$$\mu_{i,t} = [(1/R^m_t) - (1/R^L_t)]u'(\epsilon_i, c_{i,t})/(1 - \kappa) \geq 0, \tag{66}$$

$$\eta_{i,t} = [\left(\overline{E}_t u'(\epsilon_i, c_{i,t})\right)/R^m_t] - \beta E_t\left[u'(\epsilon_i, c_{i,t+1})/\pi_{t+1}\right] \geq 0, \tag{67}$$

a monetary policy setting $\{R^m_t \geq 1,\ \kappa^B_t > 0,\ \kappa_t \in [0, R^m_t/R^L_t),\ \Omega_t > 0\}_{t=0}^{\infty}$, *given* $\{y_t\}_{t=0}^{\infty}$, *and initial values* $h_{i,-1} > 0$, $M^H_{i,-1} > 0$, *and* $B_{i,-1} > 0$.

B Appendix to Section 3

Proof of Proposition 2. Consolidating agents' budget constraints (51) for $i \in \{b, l\}$ with the public sector budget constraint (63) and using the market clearing conditions (61)-(62) as well as (60), leads to

$$y_t = c_{b,t} + c_{l,t}, \tag{68}$$

Suppose that the central bank commits to set its policy instruments contingent on equilibrium outcomes, such that $R_t^m = R_t^L \; \forall t \geq 0$. Then, the credit supply wedge $\frac{1-\kappa_t}{1-\kappa_t R_t^L/R_t^m}$ in condition (54) equals one and the multiplier $\mu_{i,t}$ equals zero (see 66). Using that household member face equal probabilities to draw ϵ_l or ϵ_b, we can write condition (54) for $i = l$ as

$$u_c(\epsilon_l, c_{l,t}) = \beta 0.5 E_t \left[(u_c(\epsilon_l, c_{l,t+1}) + u_c(\epsilon_b, c_{b,t+1})) \left\{ R_t^L / \pi_{t+1} \right\} \right] \tag{69}$$

where we used that $\zeta_{l,t} = 0$. Condition (69) and condition (52) imply that the multiplier $\lambda_{i,t}$ satisfies $\lambda_{i,t} = u_c(\epsilon_l, c_{l,t})/R_t^L$ for $i \in \{b, l\}$. Substituting out $\lambda_{i,t}$ with the latter in condition (59) for $i = l$ and using condition (62), leads to

$$u_c(\epsilon_l, c_{l,t}) \left\{ q_t / R_t^L \right\} = u_h(h - h_{b,t}) + \beta E_t (1 - \delta_h) [u_c(\epsilon_l, c_{l,t+1}) \left\{ q_{t+1} / R_{t+1}^L \right\}], \tag{70}$$

and $\zeta_{b,t} = [u_h(h - h_{b,t}) - u_h(h_{b,t})]/(z q_t) > 0$, where the inequality is ensured under Assumption 1. Substituting out $\zeta_{b,t}$ in condition (54) for $i = b$, leads to

$$\begin{aligned} &\left\{ R_t^L / q_t \right\} [u_h(h - h_{b,t}) - u_h(h_{b,t})]/z \\ &= u_c(\epsilon_b, c_{b,t}) - \beta 0.5 E_t \left[(u_c(\epsilon_l, c_{l,t+1}) + u_c(\epsilon_b, c_{b,t+1})) \left\{ R_t^L / \pi_{t+1} \right\} \right], \end{aligned} \tag{71}$$

Given that $\zeta_{b,t} > 0$, condition (58) holds with equality for $i = b$, $-l_{b,t} = z q_t h_{b,t}$. Combining condition (65) with (69), shows that $\psi_{i,t} = u'(\epsilon_i, c_{i,t}) - u_c(\epsilon_l, c_{l,t})/R_t^L > 0$ for $i \in \{b, l\}$, such that condition (55) holds with equality. Using (61), condition (55) for $i \in \{b, l\}$ can be combined to

$$c_{b,t} - c_{l,t} = 2 z h_{b,t} \{ q_t / R_t^L \} - i_{l,t}^L, \quad \text{where } i_{l,t}^L \in [0, \kappa_t z h_{b,t} \{ q_t / R_t^L \}] \tag{72}$$

where we used $i_{l,t}^L \leq \kappa_t l_{l,t}/R_t^m$ if $R_t^m = R_t^L$ and that beginning-of-period asset holdings and injections from treasury open market operations are identical for all household members. Hence, (68)-(72) provide a set of five condition that have to be satisfied by the sequences $\{c_{b,t}, \, c_{l,t}, \, h_{b,t}, \, q_t, \, \pi_{t+1}\}_{t=0}^{\infty}$ in a competitive equilibrium. ■

Problem of the social planer Suppose that the policy maker can choose a debt tax/subsidy financed by a non-distortionary tax. Then, the borrowers' optimal loan demand condition

$$(1-\tau)\epsilon_b c_{b,t}^{-\sigma}/R_t^L = \beta E_t[0.5(\epsilon_b c_{b,t+1}^{-\sigma} + \epsilon_l c_{l,t+1}^{-\sigma})/\pi_{t+1}] + \gamma((h-h_{b,t})^{-\sigma} - h_{b,t}^{-\sigma})/[q_t z]$$

can always be satisfied by the policy maker. Further, the lenders' optimal loan supply condition $\epsilon_l c_{l,t}^{-\sigma} = \beta E_t[0.5(\epsilon_b c_{b,t+1}^{-\sigma} + \epsilon_l c_{l,t+1}^{-\sigma}) \cdot \left(R_t^L/\pi_{t+1}\right)]$ can be satisfied by an appropriate real interest rate. This is equivalent to the problem of a social planer who takes the laissez faire equilibrium conditions as given, while choosing debt on the behalf of private agents. Maximizing ex-ante social welfare under commitment

$$\begin{aligned}
&\max_{\{c_{b,t},c_{l,t},h_{b,t},(q_t/R_t^L)\}_{t=0}^{\infty}} E\sum_{t=0}^{\infty}\beta^t[u(\epsilon_b,c_{b,t},h_{b,t}) + u(\epsilon_l,c_{l,t},h-h_{b,t})] \\
&+\lambda_t\left[y_t - c_{l,t} - c_{b,t}\right] \\
&+\mu_t\left[2zh_{b,t}\cdot(q_t/R_t^L) - c_{b,t} + c_{l,t}\right] \\
&+\psi_t[u_c(\epsilon_l,c_{l,t})\cdot(q_t/R_t^L) - u_h(h-h_{b,t}) - \beta(1-\delta_h)E_t\left[u_c(\epsilon_l,c_{l,t+1})\cdot(q_{t+1}/R_{t+1}^L)\right]]
\end{aligned}$$

where we neglect the conditions for period $t=0$, leads to the following first order conditions

$$c_{b,t} : u_c(\epsilon_b,c_{b,t}) - \lambda_t - \mu_t = 0, \tag{73}$$

$$\begin{aligned} c_{l,t} : 0 = {} & u_c(\epsilon_l,c_{l,t}) - \lambda_t + \mu_t - \psi_{t-1}(1-\delta_h)\left[u_{cc}(\epsilon_l,c_{l,t})\cdot(q_t/R_t^L)\right] \\ & +\psi_t\left[u_{cc}(\epsilon_l,c_{l,t})\cdot(q_t/R_t^L)\right], \end{aligned} \tag{74}$$

$$h_{b,t} : 0 = u_h(h_{b,t}) - u_h(h-h_{b,t}) + \mu_t\left(2z(q_t/R_t^L)\right) + \psi_t u_{hh}(h-h_{b,t}), \tag{75}$$

$$(q_t/R_t^L) : \mu_t 2zh_{b,t} + \psi_t u_c(\epsilon_l,c_{l,t}) - \psi_{t-1}(1-\delta_h)u_c(\epsilon_l,c_{l,t}) = 0, \tag{76}$$

where λ_t, μ_t, and ψ_t denote the multiplier on the constraints (68), (72), and (70), respectively. Substituting out λ_t with condition (73) in condition (74), gives

$$0 = u_c(\epsilon_l,c_{l,t}) - u_c(\epsilon_b,c_{b,t}) + 2\mu_t + \left(\psi_t - \psi_{t-1}(1-\delta_h)\right)\left[u_{cc}(\epsilon_l,c_{l,t})(q_t/R_t^L)\right] \tag{77}$$

and substituting out μ_t with condition (76), gives condition (21). Further combine (77) with (76) to

$$2\mu_t = \frac{u_c(\epsilon_b,c_{b,t}) - u_c(\epsilon_l,c_{l,t})}{1 + zh_{b,t}(q_t/R_t^L)\left[-u_{cc}(\epsilon_l,c_{l,t})/u_c(\epsilon_l,c_{l,t})\right]}$$

which can be used to substitute out μ_t in (75), leading to (22).

Proof of Proposition 3. In a long-run equilibrium, where $x_t = x_{t-1} = x$ for a generic variable

x_t, the social planer optimality conditions (21) and (22) imply

$$\frac{u_c(\epsilon_b, c_b) - u_c(\epsilon_l, c_l)}{u_h(h - h_b) - u_h(h_b)} z(q/R^L) = \left(1 + zh_b(q/R^L)\frac{-u_{cc}(\epsilon_l, c_l)}{u_c(\epsilon_l, c_l)}\right)\left(1 + \psi\frac{-u_{hh}(h - h_b)}{u_h(h - h_b) - u_h(h_b)}\right),$$

$$\psi = -\delta_h^{-1}\frac{u_c(\epsilon_b, c_b) - u_c(\epsilon_l, c_l)}{-u_{cc}(\epsilon_l, c_l) \cdot (q/R^L) + u_c(\epsilon_l, c_l)/(zh_b)}.$$

Substituting out the multiplier ψ, leads to

$$\frac{u_c(\epsilon_b, c_b) - u_c(\epsilon_l, c_l)}{u_h(h - h_b) - u_h(h_b)} = \frac{1 + zh_b(q/R^L)\frac{-u_{cc}(\epsilon_l,c_l)}{u_c(\epsilon_l,c_l)}}{z(q/R^L) + \delta_h^{-1}\left(-u_{hh}(h - h_b)\right)(zh_b)/u_c(\epsilon_l, c_l)} \tag{78}$$

We further use that condition (70) implies

$$(q/R^L) = \frac{u_h(h - h_b)}{(1 - \beta(1 - \delta_h))\, u_c(\epsilon_l, c_l)} \tag{79}$$

to substitute out the relative price q/R^L, leading to

$$\frac{u_c(\epsilon_b, c_b) - u_c(\epsilon_l, c_l)}{u_h(h - h_b) - u_h(h_b)} = \frac{(1 - \beta(1 - \delta_h))\, u_c(\epsilon_l, c_l) + zh_b u_h(h - h_b)\frac{-u_{cc}(\epsilon_l,c_l)}{u_c(\epsilon_l,c_l)}}{zu_h(h - h_b) + \delta_h^{-1}(1 - \beta(1 - \delta_h))\left(-u_{hh}(h - h_b)\right)(zh_b)} \tag{80}$$

Under a Pigouvian debt tax/subsidy condition (15) changes to

$$\begin{aligned}&(1 - \tau_t^L)u_c(\epsilon_b, c_{b,t})\\ &= \beta 0.5 E_t\left[\left(u_c(\epsilon_l, c_{l,t+1}) + u_c(\epsilon_b, c_{b,t+1})\right)\left\{R_t^L/\pi_{t+1}\right\}\right] + \left\{R_t^L/q_t\right\}\left[u_h(h_{l,t}) - u_h(h_{b,t})\right]/z,\end{aligned}$$

Combining the latter with condition (14), leads to the following long-run competitive equilibrium condition

$$\frac{(1 - \tau_t)u_c(\epsilon_b, c_{b,t}) - u_c(\epsilon_l, c_{l,t})}{[u_h(h - h_{b,t}) - u_h(h_{b,t})]} = \frac{1}{z}\left\{R_t^L/q_t\right\}$$

Considering a long-run equilibrium and substituting out the relative price q/R^L with (79), leads to

$$\frac{(1 - \tau^L)u_c(\epsilon_b, c_b) - u_c(\epsilon_l, c_l)}{u_h(h - h_b) - u_h(h_b)} = \frac{(1 - \beta(1 - \delta_h))\, u_c(\epsilon_l, c_l)}{zu_h(h - h_b)} \tag{81}$$

Apparently, the LHS of (81) differs from the LHS of (80) only by the tax rate τ^L, while the RHSs differ due to the derivatives of the relative price (q/R^L). When the implementation of the constrained efficient allocation requires a subsidy $\tau^L < 0$, the RHSs (81) and (80) have to satisfy

$$\begin{aligned}&\frac{(1 - \beta(1 - \delta_h))\, u_c(\epsilon_l, c_l) + zh_b u_h(h - h_b)\frac{-u_{cc}(\epsilon_l,c_l)}{u_c(\epsilon_l,c_l)}}{zu_h(h - h_b) + \delta_h^{-1}(1 - \beta(1 - \delta_h))\left(-u_{hh}(h - h_b)\right)(zh_b)} < \frac{1}{z}\frac{(1 - \beta(1 - \delta_h))\, u_c(\epsilon_l, c_l)}{u_h(h - h_b)}\\ \Leftrightarrow &\left(\frac{-u_{cc}(\epsilon_l, c_l)}{u_c(\epsilon_l, c_l)u_c(\epsilon_l, c_l)}\right) < \frac{(1 - \beta(1 - \delta_h))^2}{\delta_h}\frac{1}{z}\left(\frac{-u_{hh}(h - h_b)}{u_h(h - h_b)u_h(h - h_b)}\right)\end{aligned}$$

For the utility function $u(\epsilon_i, c_{i,t}, h_{i,t}) = \epsilon_i \log(c_{i,t}) + \log(h_{i,t})$ the latter inequality can be rewritten as

$$\delta_h < (1-\beta(1-\delta_h))^2 (\epsilon_l/z)$$

To identify the impact on the allocation of durable and non-durable goods, we apply the competitive equilibrium conditions (18), (81), and $c_b - c_l = 2zh_b \frac{u_h(h-h_b)}{(1-\beta(1-\delta_h))u_c(\epsilon_l,c_l)}$, and substitute out c_l with $c_l = y - c_b$ to get $F(\boldsymbol{\tau}^L, h_b, c_b) = 0$ and $G(h_b, c_b) = 0$, where

$$F(\boldsymbol{\tau}^L, h_b, c_b) = \frac{(1-\boldsymbol{\tau}^L)\cdot u_c(\epsilon_b, c_b) - u_c(\epsilon_l, y-c_b)}{(u_h(h-h_b) - u_h(h_b))\,(1-\beta(1-\delta_h))\,u_c(\epsilon_l, y-c_b)(1/z)} - \frac{1}{u_h(h-h_b)},$$
$$G(h_b, c_b) = 2zh_b \frac{u_h(h-h_b)}{(1-\beta(1-\delta_h))\,u_c(\epsilon_l, y-c_b)} - 2c_b + y.$$

The partial derivatives of $G(h_b, c_b)$, where G_x abbreviates $\partial G/\partial x$, are given by

$$G_{h_b} = \frac{2zu_h(h-h_b) - 2zh_b u_{hh}(h-h_b)}{(1-\beta(1-\delta_h))\,u_c(\epsilon_l, y-c_b)} > 0$$
$$G_{c_b} = -2\left(\frac{zh_b u_h(h-h_b)\cdot(-u_{cc}(\epsilon_l, y-c_b))}{u_c(\epsilon_l, y-c_b)^2} + 1\right) < 0$$

implying $\partial h_b/\partial c_b = -G_{c_b}/G_{h_b} > 0$. The partial derivatives of $F(\boldsymbol{\tau}^L, h_b, c_b)$ are given by

$$F_{\boldsymbol{\tau}^L} = -\frac{u_c(\epsilon_b, c_b) - u_c(\epsilon_l, y-c_b)}{(u_h(h-h_b) - u_h(h_b))\,(1-\beta(1-\delta_h))\,u_c(\epsilon_l, y-c_b)(1/z)} < 0$$
$$F_{h_b} = -\frac{\left[(1-\tau^L)\cdot u_c(\epsilon_b, c_b) - u_c(\epsilon_l, y-c_b)\right]\cdot\left[(-u_{hh}(h-h_b) - u_{hh}(h_b))\right]}{(u_h(h-h_b) - u_h(h_b))^2\,(1-\beta(1-\delta_h))\,u_c(\epsilon_l, y-c_b)(1/z)} - \frac{-u_{hh}(h-h_b)}{u_h(h-h_b)} < 0$$
$$F_{c_b} = \frac{\left\{\begin{array}{c}\left[(1-\tau^L)u_{cc}(\epsilon_b, c_b) - (-u_{cc}(\epsilon_l, y-c_b))\right] - \left[(1-\tau^L)u_c(\epsilon_b, c_b) - u_c(\epsilon_l, y-c_b)\right] \\ \cdot(-u_{cc}(\epsilon_l, y-c_b))\,z^{-1}/u_c(\epsilon_l, y-c_b)\end{array}\right\}}{(u_h(h-h_b) - u_h(h_b))\,(1-\beta(1-\delta_h))\,u_c(\epsilon_l, y-c_b)(1/z)} < 0.$$

Thus, consumption of the representative borrower decreases with the tax rate, since $\partial c_b/\partial \tau^L = -(G_{h_b}F_{\tau^L})/(F_{c_b}G_{h_b} - F_{h_b}G_{c_b}) < 0$. Hence, introducing a subsidy $\tau^L < 0$ increases consumption and housing of the representative borrower (by $\partial h_b/\partial c_b > 0$). Given that consumption (housing) of lenders decreases for a given aggregate supply (stock of housing), the lenders' consumption Euler equation (14), which can be written as $1 = \beta 0.5[1 + \epsilon_b(\delta - c_b)/(\epsilon_l(\delta - c_l))]\left(R^L/\pi\right)$, further implies that the real interest rate R^L/π increases with the subsidy. ■

Proof of Proposition 4. Suppose that the central bank commits to set its policy instruments contingent on equilibrium outcomes, such that $R_t^m < R_t^L\ \forall t \geq 0$. Compared to the case of non-rationed money supply (where $R_t^m = R_t^L\ \forall t \geq 0$), non-durable consumption is again restricted by (68). For $R_t^m < R_t^L$, the multiplier $\mu_{i,t}$ is strictly positive (see 66) and the credit supply wedge

$\frac{1-\kappa_t}{1-\kappa_t R_t^L/R_t^m}$ in condition (54) may exceed one, such that condition (54) for $i = l$ is

$$u_c(\epsilon_l, c_{l,t}) = \beta 0.5 E_t[(u_c(\epsilon_l, c_{l,t+1}) + u_c(\epsilon_b, c_{b,t+1}))(R_t^L/\pi_{t+1})]\frac{1-\kappa_t}{1-\kappa_t R_t^L/R_t^m} \tag{82}$$

Substituting out $\lambda_{i,t}$ with condition (52) in condition (59) implies

$$\begin{aligned} &\beta E_t\left[0.5(u_c(\epsilon_l, c_{l,t+1}) + u_c(\epsilon_b, c_{b,t+1}))\left(q_t/\pi_{t+1}\right)\right] \\ &= u_h(h_{l,t}) + \beta^2(1-\delta_h)E_t\left[0.5(u_c(\epsilon_l, c_{l,t+2}) + u_c(\epsilon_b, c_{b,t+2}))\left(q_{t+1}/\pi_{t+2}\right)\right], \end{aligned} \tag{83}$$

and that the multiplier $\zeta_{b,t}$ again satisfies $\zeta_{b,t} = [u_h(h - h_{b,t}) - u_h(h_{b,t})]/(zq_t) > 0$ where the inequality is ensured under Assumption 1. Substituting out $\zeta_{b,t}$ in condition (54) for $i = b$, again leads to (71). Combining condition (65) with (82), shows that $\psi_{i,t} = u'(\epsilon_i, c_{i,t}) - u_c(\epsilon_l, c_{l,t})\frac{1-\kappa_t R_t^L/R_t^m}{1-\kappa_t}/R_t^L > 0$ for $i \in \{b, l\}$, such that condition (55) holds with equality. Using (61), condition (55) for $i \in \{b, l\}$ can be combined to

$$c_{b,t} - c_{l,t} = zh_{b,t}\left[2 - \kappa_t R_t^L/R_t^m\right]\{q_t/R_t^L\} \tag{84}$$

where we used $i_{l,t}^L = \kappa_t l_{l,t}/R_t^m$ if $R_t^m < R_t^L$ (see 57) and that beginning-of-period asset holdings and injections from treasury open market operations are identical for all household members. Hence, (68), (71), (82), (83), and (84) provide a set of five condition that have to be satisfied by the sequences $\{c_{b,t},\, c_{l,t},\, h_{b,t},\, (q_t/\pi_{t+1}), (R_t^L/\pi_{t+1})\}_{t=0}^{\infty}$, for given sequences $\{y_t\}_{t=0}^{\infty}$ and $\{0 \leq \kappa_t < R_t^m/R_t^L,\, 1 < R_t^m < R_t^L\}_{t=0}^{\infty}$ in a competitive equilibrium.

Using that identical beginning-of-period asset holdings and injections from treasury open market operations for all household members, we know that $m_{b,t}^H = 0.5m_t^H$ and (55) implies $i_{b,t} = 0.5[(1+\Omega_t)m_t^H - m_{t-1}^H\pi_t^{-1}]$. Given that $\psi_{b,t} > 0$, condition (60) thus implies for $i = b$

$$c_{b,t} = 0.5(1+\Omega_t)m_t^H + zq_t h_{b,t}/R_t^L. \tag{85}$$

where we further used $i_{b,t}^L = 0$ and that condition (57) is binding, $-l_{b,t} = zq_t h_{b,t}$ for $\zeta_{i,t} > 0$. Under $R_t^m < R_t^L$, condition (82) and (67) imply $\eta_{i,t} = [\left(\overline{E}_t u'(\epsilon_i, c_{i,t})\right)/R_t^m] - u_c(\epsilon_l, c_{l,t})\frac{1-\kappa_t R_t^L/R_t^m}{1-\kappa_t}/R_t^L > 0$ for $i \in \{b, l\}$. Hence, condition (56) is binding for $i \in \{b, l\}$ such that condition (60) can be written as

$$\kappa_t^B b_{t-1}\pi_t^{-1}/R_t^m = (1+\Omega_t)m_t^H - m_{t-1}^H\pi_t^{-1} \tag{86}$$

Substituting out the multipliers $\lambda_{i,t+1}$and $\psi_{i,t+1}$ with condition (52) and condition (65) in condition (53),

$$
\begin{aligned}
&(1/R_t)E_t\left[u_c(\epsilon_l, c_{l,t+1})/\pi_{t+1}\right] \qquad (87)\\
&= E_t[((1-\kappa^B_{t+1})0.5(u_c(\epsilon_l, c_{l,t+2}) + u_c(\epsilon_b, c_{b,t+2})) + \kappa^B_{t+1}u_c(\epsilon_l, c_{l,t+1})/R^m_{t+1})/\pi_{t+1}]
\end{aligned}
$$

and substituting out i^L_t with the binding condition (57) in condition (63) leads to

$$
(b_t/R_t) = b_{t-1}\pi_t^{-1} - (R^m_t - 1)\left(\kappa_t z q_t h_{b,t}/R^m_t + \Omega_t m^H_t\right) - R^m_t(m^H_t - m^H_{t-1}\pi_t^{-1}). \qquad (88)
$$

Hence, conditions (85)-(88) provide a set of four condition $\forall t \geq 0$ and condition (13) with $i^L_t = \kappa_t z q_t h_{b,t}/R^m_t$ a further condition that have to be satisfied the sequences $\{m^H_t, b_t, R_t, \pi_t\}^\infty_{t=0}$ and the initial price level $P_0 > 0$ in a competitive equilibrium for a particular set of equilibrium sequences $\{c_{b,t}, c_{l,t}, h_{b,t}, (q_t/\pi_{t+1}), (R^L_t/\pi_{t+1})\}^\infty_{t=0}$ and the policy instruments $\{\kappa_t,\ R^m_t\}^\infty_{t=0}$ as well as $\{\kappa^B_t > 0, \Omega_t > 0\}^\infty_{t=0}$, a sequence $\{y_t\}^\infty_{t=0}$, and initial public sector liabilities $M^H_{-1} > 0$, and $B_{-1} > 0$. ■