

# Default Risk Premia on Government Bonds in a Quantitative Macroeconomic Model<sup>1</sup>

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This version: 12th April 2012

## **Abstract**

We develop a macroeconomic model where the government does not guarantee to repay debt. We ask whether movements in the prices of government bonds can be rationalized by lenders' unwillingness to roll over debt when the outstanding debt level exceeds a government's repayment capacity. Default occurs if a worsening state of the economy leads to a build-up of debt that exceeds the government's ability to repay. Investors are unwilling to engage in a Ponzi game and withdraw lending in this case and thus force default at an endogenously determined fractional repayment rate. Interest rates on government bonds reflect expectations of this event. We analytically show that there exist two equilibrium bond prices. Our numerical analysis shows that, at moderate debt-to-gdp levels, default premia hardly emerge in the low risk equilibrium. High risk premia can either arise at high debt-to-gdp ratios, where even small changes in fundamentals lead to steeply rising interest rates, or as realizations of the high risk equilibrium.

**Keywords:** Sovereign default; fiscal policy; government debt; multiple equilibria

**JEL:** E62, G12, H6, E32

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<sup>1</sup>The authors would like to thank Christian Bayer, Huixin Bi, Eric Leeper, Juha Kilponen, and Dirk Niepelt for helpful suggestions; the usual disclaimer applies. A previous version circulated under the title "Understanding default risk premia on public debt" (2010). Financial support from the Deutsche Forschungsgemeinschaft (SFB 823) is gratefully acknowledged.

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## 1 Introduction

The recent financial crisis has turned into a fiscal crisis in several European countries. An unusually large adverse shock has reduced tax revenues and has led to higher government spending in an attempt to mitigate the consequences of the shock for aggregate output and employment. The resulting boost in public deficits has produced unprecedented levels of government debt, which are already above 100% of annual gdp in some countries and are predicted to rise to even higher levels in the near future. Sizeable yield spreads between government bonds of member countries of the European Monetary Union have emerged and for some countries (in particular for Greece, Ireland, and Portugal) yield spreads in relation to comparatively safe German bonds increased dramatically since 2008.

It is hardly controversial that these spreads reflect the risk that governments might default on their debt obligations. The purpose of the present paper is to analyze the emergence of default risk premia on government bonds in a general equilibrium context. In the literature, a common approach is to determine default as an outcome of an optimizing sovereign borrower who is willing to default if the gains from non-repayment of external debt exceed the costs of autarky and potential resource losses in case of default (see Eaton and Gersovitz, 1981, or Arellano, 2008, among others). In contrast to this, we consider an alternative approach where default is initiated by the decision of lenders to withdraw financing of the government when outstanding public debt exceeds the government's debt repayment capacity. The focus on the lender side of the credit relationship allows us to remain agnostic with respect to the sovereign borrower's objectives and, in particular, its incentives to default.

A government's debt repayment capacity equals the maximum present value of government surpluses, such that its ability to raise future revenues is essential for the probability of default.<sup>3</sup> Lenders' expectations about the government's future surpluses are therefore decisive for the premia they demand as a compensation for the risk of sovereign default. We show analytically that this set-up can give rise to multiple bond market equilibria, such that changes in bond prices can in principle be driven by self-fulfilling expectations. If, for example, default is expected to be very likely, lenders demand a high interest rate premium to be compensated for default risk, which raises the debt burden even more such that the probability of default actually increases.

We apply a dynamic general equilibrium framework and consider a government with limited commitment. It levies an exogenously determined proportional tax on labor in-

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<sup>3</sup>This approach is similar to Bi (2012) and Bi and Leeper (2012) where default also depends on whether current outstanding debt exceeds a "fiscal limit", which consists of the discounted sum of future maximum surpluses.

come and issues non-state contingent one-period bonds to finance a given stream of real government expenditures, while failing to guarantee repayment of debt. If an adverse productivity shock makes the present value of future surpluses fall short of covering the level of outstanding debt, even if the surplus maximizing tax rate (which is well defined due to a tax Laffer curve with an interior maximum) were levied for the entire future, the government's debt repayment capacity is exceeded. A potential household-lender who realizes that he would support a Ponzi game if he invested in government bonds is assumed to stop lending to the government. In this case, default becomes inevitable and available funds are distributed to bond holders, who therefore experience only a partial redemption of their investments at an endogenously determined rate. Each individual lender assesses the probability that this event will occur in the next period and, consequently, demands a default risk premium as a compensation for expected losses. Given that lenders can seize current net revenues from the government in the case of default (a situation that differs from credit relations where the lender may become a claimant on future profit streams), government bonds are then priced like standard debt contracts with limited liability.

We neither consider governments' incentives to default on external debt nor real costs of default, which might cause governments to raise surpluses that preclude costly defaults. Instead, the essence of the analysis is to explore the impact of a potential lending boycott on equilibrium risk premia and bond prices. To isolate the implications of this approach in the most transparent way and to show analytically how multiple bond market equilibria can arise, we assume that tax rates and government spending are constant (like in the literature on the fiscal theory of price level, see Sims, 1994, and Woodford, 1994, or on the fiscal theory of sovereign default, see Uribe, 2006). These simplifying assumptions obviously come at the cost that we may underestimate the full impact of fiscal policy on default probabilities to the extent that in reality governments behave in a less mechanical way.

The main results are as follows. We show that there may exist multiple equilibrium prices for government debt. In particular, two bond prices can exist in equilibrium: both a combination of a low price (a high interest rate), high default risk, and high public debt, as well as one of a high price (a low interest rate), low default risk, and low public debt are compatible with the expected rate of return of investors and with the government's demand for external funds.<sup>4</sup> Default is likely to occur if the lenders coordinate their expectations on the high risk equilibrium, thereby imposing an unsustainable financing burden on the government through high risk premia in the period of maturity. If we focus on the lower

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<sup>4</sup>The existence of two equilibrium bond prices relates to Calvo (1988), who also considers sovereign default and shows that two equilibria (with high and low interest rates) can exist.

of the equilibrium interest rates, we find that default premia are monotonically increasing in the initial debt level and depend negatively on productivity. We show that the relation between the debt-to-gdp ratio and the default risk premium can be very steep above certain critical levels of debt-to-gdp, consistent with the observation that risk premia may rise suddenly and very strongly when fiscal positions worsen. In a calibrated version intended to capture relevant quantitative features of average European Monetary Union member countries in a stylized way, non-negligible interest rate spreads emerge only for very high levels of debt in the neighborhood of 200% of gdp. Thus, for the average Eurozone country the model predicts that fiscal spare capacity is ample and, consequently, that risk premia are negligible at observed debt-to-gdp levels, which is consistent with empirical evidence for countries such as France, Germany, or the Netherlands. The emergence of high risk premia observed in other countries can be rationalized in the context of the calibrated model in either of two ways: first, default expectations may be self-fulfilling and lead to the high interest rate equilibrium; second, the relevant maximum debt capacity may be much lower than the one used in our baseline model if investors believe that the maximum politically feasible tax rate (which we do not model explicitly) is lower than the surplus maximizing tax rate.

Our approach to model sovereign default is related to Uribe's (2006) "Fiscal Theory of Sovereign Default". He considers nominal debt and exogenous surpluses in an endowment economy to demonstrate that default is inevitable under certain monetary-fiscal policy regimes. Our strategy to determine default differs substantially from his approach. As shown in Schabert (2010), the intertemporal budget constraint is neither sufficient as a criterion to determine the default rate, i.e. the fractional rate of repayment of outstanding debt in the case of default, nor the interest rate on risky bonds. For the case which is most closely related to our set-up, Uribe (2006) introduces an additional "fiscal policy constraint restricting the behavior of the default rate" (p. 1869), which allows to uniquely determine the equilibrium interest rate. In the present paper, in contrast, we instead introduce the assumptions of a sudden lending stop in the case where a Ponzi-game of the government becomes inevitable and that the government serves lenders with current surpluses, which allows us to endogenously determine an entire sequence of default rates without any additional restriction on the government's behavior.<sup>5</sup> This is an aspect which differs from Bi (2012), where default can in principle occur in every state and at every debt level (though with different probabilities). In contrast to our approach, in her paper default depends on a random draw from a fiscal limit distribution that induces default if it

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<sup>5</sup>Without such an assumption or Uribe's (2006) fiscal closing rules (like a rule whereby the government decides to default if the tax-to-debt ratio falls below a certain threshold), default rates can only be determined in the initial period, as shown in Schabert (2010).

exceeds the current debt level, while the particular default rate is exogenously determined through a draw from a historical default rate distribution.

The remainder is organized as follows. Section 2 introduces the model. Section 3.1 describes the determination of equilibrium bond prices in a simplified version where analytical results are available, whereupon section 3.2 presents quantitative results for a calibrated model version. Section 4 concludes.

## 2 The model

In this section, we present a simple real dynamic general equilibrium model where the government levies income taxes and issues non-state contingent one period debt. Labor supply is endogenous, which gives rise to a Laffer curve that bounds equilibrium tax revenues. We consider the case where fiscal policy does not guarantee that the government never runs a Ponzi-game.<sup>6</sup> Individual households will stop lending to the government when they realize that a Ponzi scheme is inevitable. Without further access to credit, the government then defaults while lenders can seize current net revenues. Households rationally consider that this event is possible when adverse productivity shocks lead to a build-up of public debt. They form expectations of the future fractional rate of repayment of government debt. Accordingly, in an arbitrage-free equilibrium, risk premia exist that compensate household-lenders for the risk of government default.

### 2.1 The private sector

There exists a continuum of infinitely lived and identical households of mass one. Their utility increases in consumption  $c_t$  and decreases in working time  $l_t$ , the latter variable being bounded by a unit time endowment such that  $l_t \in (0, 1)$ . The objective of a representative household is given by

$$\max E_s \sum_{t=0}^{\infty} \beta^t \left[ \ln c_{t+s} + \frac{1 - l_{t+s}}{\gamma} \right], \quad \text{with } \beta \in (0, 1), \gamma > 0, \quad (1)$$

where  $\beta$  denotes the subjective discount factor. Households borrow and lend among each other via one-period private debt contracts. Private debt is introduced here to define a risk-free interest rate  $R_t^{rf}$ . Let  $d_{t-1}$  denote the beginning of period net private asset position and  $1/R_t^{rf}$  the period- $t$ -price for a payoff of one unit of output in period  $t + 1$ . We restrict our attention to the case where private debt contracts are enforceable and

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<sup>6</sup>This assumption is analogous to the fiscal policy specification in Uribe (2006) and in the fiscal theory of the price level (see Sims, 1994, and Woodford, 1994). In contrast to these studies, in our purely real model the price level is irrelevant.

households satisfy the borrowing constraint

$$\lim_{t \rightarrow \infty} \left( d_{t+s} / R_{t+s}^{rf} \right) \prod_{i=1}^t 1 / R_{s+i-1}^{rf} \geq 0. \quad (2)$$

Utility maximization subject to the borrowing constraint (2) requires the following first order condition for borrowing and lending in terms of private debt (i.e. the consumption Euler equation) to be satisfied,

$$c_t^{-1} = R_t^{rf} \beta E_t (c_{t+1}^{-1}), \quad (3)$$

as well as the transversality condition

$$\lim_{t \rightarrow \infty} E_s \left( d_{t+s} / R_{t+s}^{rf} \right) \prod_{i=1}^t 1 / R_{s+i-1}^{rf} = 0. \quad (4)$$

Households can further invest in one-period government bonds  $b_t$ , subject to  $b_{-1} > 0$  and  $b_t \geq 0$ . The government offers one-period debt contracts at the price  $1/R_t$  in period  $t$  that promise to deliver one unit of output in period  $t+1$ . In contrast to private borrowers, the government does not guarantee full debt repayment. In case of default, the lenders will proportionally be served out of current net revenues. Hence, we assume limited liability in the sense that the lenders have no claims to future surpluses.

If current and discounted future surpluses are expected to be large enough to repay outstanding debt, the household optimality condition for investment in government bonds is the analogue to the Euler equation (3), namely,  $c_t^{-1} = R_t \beta E_t (c_{t+1}^{-1})$ . The constraint  $b_t \geq 0$  further requires that in the household optimum the transversality condition

$$\lim_{t \rightarrow \infty} E_s (b_{t+s} / R_{t+s}) \prod_{i=1}^t 1 / R_{s+i-1}^{rf} = 0, \quad (5)$$

holds, where  $R_{t+s} = R_{t+s}^{rf}$  when the government fully services its debt obligations. If beginning-of-period public debt exceeds a level that is too high to be repayable even for the maximum present value of budget surpluses (see section 2.2 for a definition), the government runs into a Ponzi game, which would be inconsistent with the households' transversality condition (5). In this case, households are assumed to stop lending to the government, which necessarily implies that the government defaults in period  $t$ , i.e. can honor only a fraction of its debt obligations out of current surpluses. Note that we neglect the possibility that the default rate can be reduced if individual households coordinate their period  $t$  lending in a way that the government can roll over debt to an amount that is repayable.

Since households are assumed to have rational expectations, they realize the possibility

of partial default on government bonds and account for the probability of default (of course, since households are atomistic, an individual investor does not take into consideration the influence of his behavior on the probability of default). Let  $1 - \delta_t$  denote the fraction of government bonds that is redeemed and  $\delta_t$  the default rate. The household flow budget constraint then reads

$$c_t + (b_t/R_t) + \left(d_t/R_t^{rf}\right) \leq (1 - \tau_t)w_t l_t + (1 - \delta_t) b_{t-1} + d_{t-1} + \pi_t,$$

where  $\pi_t$  are firms' profits and labor income  $w_t l_t$  (with the real wage rate  $w_t$ ) is subject to a proportional tax rate  $\tau_t \in (0, 1)$ . For simplicity, we neglect non-negative lump-sum government transfers, by which the government redistributes resources when current debt is lower than the present value of future surpluses. Assuming that these transfers will be paid at some random point in the future, we can abstract from them in the remainder of the analysis (see Aiyagari et al., 2002, for a similar assumption). The household optimum is characterized by the first order conditions (3),

$$c_t = \gamma (1 - \tau_t) w_t, \tag{6}$$

$$c_t^{-1} = R_t \beta E_t (c_{t+1}^{-1} (1 - \delta_{t+1})), \tag{7}$$

and the transversality conditions (4) and (5). Note that the Euler equation for risky government debt, (7), differs from the one for risk-free private debt (3), in that the pricing of government bonds is affected by the fact that repayment is expected to be only partial because of possible future default.

Perfectly competitive firms produce the output good  $y_t$  with a simple linear technology

$$y_t = a_t l_t, \tag{8}$$

where labor productivity  $a_t$  is generated by a stationary process with mean  $\bar{a} = 1$ . Labor demand satisfies

$$w_t = a_t. \tag{9}$$

## 2.2 The public sector

The government does not have access to lump-sum taxation and does not guarantee full debt repayment. It raises revenues by issuing debt at price  $q_t$  and taxing labor income, and it purchases an exogenously given amount  $g_t$  of the final good in each period. Throughout, we assume government spending to be constant,  $g_t = g > 0$ . The underlying assumption is that political constraints make a certain amount of government spending inevitable. The

flow budget constraint is given by

$$b_t R_t^{-1} + s_t = (1 - \delta_t) b_{t-1}, \quad (10)$$

where  $R_t = 1/q_t$  is the gross real interest rate and the surpluses  $s_t$  equal tax revenues net of expenditures,

$$s_t = \tau_t w_t l_t - g. \quad (11)$$

We assume that the government does not guarantee to fully service debt and does not preclude that public debt might evolve on a path that implies a Ponzi scheme. Since households are not willing to engage in such schemes, they may stop lending and (temporarily) disrupt the government from access to credit.

To see this, consider, for a moment, the default free case, i.e. presume the non-repayment rate  $\delta_{t+k}$  were equal to zero for all  $k \geq 0$ . In this case, one would obtain by iterating the government flow budget constraint (10) forward and taking expectations,  $\delta_{t+k} = 0 \quad \forall k \geq 0 \Rightarrow$

$$b_{t-1} = E_t \sum_{k=0}^{\infty} s_{t+k} \prod_{i=1}^k (1/R_{t+i-1}) + \lim_{k \rightarrow \infty} E_t b_{t+k} R_{t+k}^{-1} \prod_{i=1}^k \frac{1}{R_{t+i-1}}. \quad (12)$$

Now suppose that outstanding debt at the beginning of period  $t$ ,  $b_{t-1}$ , exceeds the present value of future surpluses, i.e. the first term on the right-hand side of (12). Then, the limit term would exceed zero,  $\lim_{k \rightarrow \infty} E_t b_{t+k} R_{t+k}^{-1} \prod_{i=1}^k 1/R_{t+i-1} > 0$ . By definition, the government would then run into a Ponzi game. But this, together with  $R_{t+k} = R_{t+k}^{rf} \quad \forall k \geq 0$  for  $\delta_{t+k} = 0$  (see 3 and 7) would be inconsistent with the households' transversality condition (5). Given that households will then stop lending to the government, end-of-period debt equals zero,  $b_t = 0$ . The only way for the government budget constraint (10) to be satisfied in this case is through default, which endogenously determines  $\delta_t > 0$  (see below, 15).

As a specific way to implement a fiscal policy that entails default risk in this sense, we assume that the government keeps the tax rate constant,  $\tau_t = \tau$ . This is a prominent example of a large class of fiscal rules that do not incorporate enough self-corrective behavior on the part of the government as to avoid Ponzi schemes in each period of time. In principle, the government could always prevent default by cutting spending or raising tax rates. If this possibility were deemed credible by private market participants, there would be no default risk and observed risk premia could not be explained. Therefore, we use the assumptions of constant government spending and tax rates as a simple way to implement the idea that governments may not be willing or flexible enough, probably due to political pressures, to adjust spending quickly enough as to eliminate the risk that

deteriorating business cycle conditions engender unstable debt dynamics.<sup>7</sup>

There exists a maximum value for the present value of future surpluses, which we call the maximum debt repayment capacity. The latter is the maximum amount of debt that the government would be able to repay if it imposed the surplus maximizing tax rate for the entire future. A well defined surplus maximizing tax rate,  $\tau_t^*$ , exists because with proportional labor income taxation there is a tax Laffer curve with an interior maximum (see page 11 for details). We denote the period  $t$  value of the maximum debt repayment capacity by  $\Psi_t$ , defined as

$$\Psi_t = E_t \sum_{k=0}^{\infty} s_{t+k}^* \prod_{i=1}^k 1/R_{t+i-1}^{rf}. \quad (13)$$

Here,  $s_{t+k}^* = \tau_t^* w_{t+k}^* l_{t+k}^* - g$  is the maximum period surplus that is obtained if the surplus maximizing tax rate  $\tau_t^*$  is applied. This leads to corresponding levels of labor income denoted  $w_{t+k}^* l_{t+k}^*$  and the risk-free rate  $R_{t+k}^{rf}$  is applied for discounting.<sup>8</sup> When households lend to the government, they take the maximum debt repayment capacity into account. We thereby allow for the case where the current tax rate differs from the surplus maximizing tax rate, which could in principle be implemented by future governments.

The maximum initial debt level that can be expected to be repaid without default is thus characterized by  $b_{t-1} = \Psi_t$ . The government will fully serve debt obligations if  $b_{t-1} \leq \Psi_t$ . As long as this is the case, no government default occurs. Default, however, becomes inevitable if the current stock of debt exceeds the maximum repayment capacity:

$$b_{t-1} > \Psi_t. \quad (14)$$

If this is the case, the government is not able to generate enough current and future revenues to enable full repayment of outstanding debt.

In the case where (14) is satisfied, (12) with  $R_{t+k} = R_{t+k}^{rf} \forall k \geq 0$  is inconsistent

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<sup>7</sup>A constant tax rate can further be seen as a natural benchmark in this framework: if government bonds were state contingent, it is well-established that in this type of model an optimal income tax rate under commitment (and without default) would have to be constant and sufficiently large to finance initial outstanding debt and future expenditures (see e.g. Ljungqvist and Sargent, 2004). Here, government bonds are however non-state contingent, which implies that this type of tax policy is in general not consistent with a set of "measurability constraints" for each period that relate the present value of future surpluses to the beginning-of-period stock of public debt to rule out Ponzi games (see Aiyagari et al., 2002). The choice of a constant tax rate can thus be viewed as the strategy of a government that ignores this subtle difference and sets the tax rate as if debt were state contingent.

<sup>8</sup>Note that the maximum debt repayment capacity bears a resemblance to Aiyagari's (1994) natural debt limit for consumers. Private households cannot accumulate more debt than would be expected to be repaid by pledging the entire stream of future incomes. While households are assumed to respect the natural private debt limit (as a borrowing constraint) for their plans, the government sets its instruments without internalizing the maximum debt repayment capacity, which is why default may occasionally occur in our model.

with the transversality condition (5) and no individual household is willing to lend to the government. The consequence is that aggregate lending to the government comes to a halt, such that end-of-period debt equals zero,  $b_t = 0$ , in the current period. The government is then unable to fully honor its obligations and redeems as much as possible of its outstanding debt out of current surpluses. As a consequence, repayment will only be partial. The non-repayment or default rate  $\delta_t$  for the case (14) satisfies (see 10)

$$\delta_t = 1 - s_t/b_{t-1}. \quad (15)$$

To sum up, if beginning-of-period debt  $b_{t-1}$  is smaller than  $\Psi_t$ , households are willing to lend to the government according to (7), while the government does not default in period  $t$ ,  $\delta_t = 0$ , and borrows to balance its budget such that end-of-period debt equals  $b_t = (b_{t-1} - s_t) R_t$ . The price of debt,  $1/R_t$ , then reflects the probability of default in  $t+1$ . If, however, beginning-of-period debt is too high such that (14) is satisfied, households stop lending. The government defaults and repays debt, with a default rate given by (15). In the period subsequent to a default event, the stock of government debt is zero and default is not possible in the next period, such that households are again willing to lend to the government. As mentioned above, we assume that the government transfers back resources in some future period, which guarantees that the intertemporal budget constraint will be satisfied in all periods (see Aiyagari et al., 2002).

### 2.3 Equilibrium

In equilibrium, prices adjust to clear markets for goods, labor, and assets and the net stock of risk-free private debt  $d_t$  is zero in the aggregate. Households' initial asset endowments are assumed to be positive, i.e. the government is initially indebted. An equilibrium is a set of sequences  $\{c_t, l_t \in [0, 1], y_t, w_t, b_t \geq 0, \delta_t \in [0, 1], R_t^{rf}, R_t, s_t\}_{t=0}^{\infty}$  satisfying (3)-(6), (7), (8), (9), (11), (13), and

$$y_t = c_t + g_t, \quad (16)$$

$$b_t = \begin{cases} (b_{t-1} - s_t) R_t & \text{if } \Psi_t \geq b_{t-1} \\ 0 & \text{if } \Psi_t < b_{t-1} \end{cases}, \quad (17)$$

$$\delta_t = \begin{cases} 0 & \text{if } \Psi_t \geq b_{t-1} \\ 1 - s_t/b_{t-1} & \text{if } \Psi_t < b_{t-1} \end{cases}, \quad (18)$$

a fiscal policy setting  $\tau \in [0, 1]$ , given  $\{a_t\}_{t=0}^{\infty}$ ,  $g > 0$ , and initial debt  $b_{-1} > 0$ .

The equilibrium allocation is not directly affected by public debt and the (expected) default rate. The first property is due to the fact that the current labor income tax rate is assumed not to be state contingent. The second property follows from the fact that

default does not lead to resource losses or distortions. Of course, the price of government bonds will depend on the expected default rate, which can be seen from the asset pricing equation (7). This reflection of the probability of future default in the interest rate on government bonds is our main object of study.

The equilibrium sequences of consumption, working time, output, the wage rate, the risk-free rate and government surpluses  $\{c_t, l_t, y_t, w_t, R_t^{rf}, s_t\}_{t=0}^{\infty}$  are determined for given  $g$  and  $\{a_t\}_{t=0}^{\infty}$  by (6), (8), (9), (11) and (16), which can be summarized by

$$c_t = c(a_t, \tau) := \gamma(1 - \tau)a_t, \quad (19)$$

$$l_t = l(a_t, \tau) := (c(a_t, \tau) + g)/a_t, \quad (20)$$

$$s_t = s(a_t, \tau) := \tau c(a_t, \tau) - (1 - \tau)g, \quad (21)$$

$$R_t^{rf} = c(a_t, \tau)^{-1} \beta^{-1} / E_t \left( c(a_{t+1}, \tau)^{-1} \right), \quad (22)$$

as well as  $w_t = a_t$  and  $y_t = a_t l(a_t, \tau)$ .<sup>9</sup>

While the equilibrium sequences  $\{c_t, l_t, y_t, w_t, s_t\}_{t=0}^{\infty}$  are not affected by sovereign default, these variables are of course correlated with the default rate  $\delta_t$  due to changes in the state  $a_t$ . In any case, they will be stationary, given that the state  $a_t$  is stationary. With the above solutions, we can compute the surplus maximizing tax rate  $\tau_t^* \in (0, 1)$ , which depends on the state of the business cycle  $a_t$ . Using the solutions (19)-(21), we can identify a time-varying tax rate  $\tau_t^* \in (0, 1)$  that maximizes government surpluses  $s_t = \tau_t \gamma(1 - \tau_t)a_t - (1 - \tau_t)g$ . This state contingent tax rate can immediately be shown to be unique and to satisfy  $\tau_t^* = \tau^*(a_t)$  with  $\tau^*(a_t) = \frac{1}{2} + \frac{g}{2\gamma a_t}$ .

In order to determine the bond prices we need to compute expectations about future defaults. We substitute out the risk-free rate in (13), to get  $\Psi_t = E_t \sum_{k=0}^{\infty} \beta^k \frac{c_{t+k}^{*-1}}{c_t^{*-1}} s_{t+k}^*$ , and rewrite it by using the solutions (19) and (21):

$$\Psi_t = \gamma(1 - \tau_t^*) a_t E_t \sum_{k=0}^{\infty} \beta^k [\tau_{t+k}^* - a_{t+k}^{-1} \gamma^{-1} g]. \quad (23)$$

The expected default rate, public debt, and the bond price have to be determined simultaneously using the equilibrium conditions (7), (17), and (18). In order to identify these solutions, we have to consider the probabilities of the two distinct cases  $\Psi_t \geq b_{t-1}$  and  $\Psi_t < b_{t-1}$ .

Let  $a_t^*$  be the productivity level that leads to a maximum debt repayment capacity  $\Psi_t$

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<sup>9</sup>If default occurs ( $\delta_t = 1 - s_t/b_{t-1}$ ) the budget constraints imply  $c_t = (1 - \tau_t)w_t l_t + (1 - \delta_t)b_{t-1} = (1 - \tau_t)w_t l_t + s_t$  and thus  $y_t = a_t l_t = c_t + g$ .

that exactly equals beginning-of-period debt  $b_{t-1}$ ,

$$a_t^* : \Psi(a_t^*) = b_{t-1}. \quad (24)$$

Thus,  $a_t^*$  is the minimum productivity level that allows full debt repayment and thus precludes default; we will refer to this as the productivity threshold. Further, let  $\pi_t(a_{t+1}) = \pi(a_{t+1}|a_t)$  be the probability of a particular value  $a_{t+1}$  conditional on  $a_t$ . Then, the probabilities of default and of non-default in  $t+1$  conditional on the information in  $t$  are

$$\begin{aligned} \text{prob}(\Psi_{t+1} < b_t | a_t, b_t) &= \int_{-\infty}^{a_{t+1}^*} \pi_t(a_{t+1}) da_{t+1}, \\ \text{prob}(\Psi_{t+1} \geq b_t | a_t, b_t) &= \int_{a_{t+1}^*}^{\infty} \pi_t(a_{t+1}) da_{t+1}. \end{aligned}$$

These probabilities can be used to rewrite the asset pricing equation (7), which includes the expectation term  $E_t[c_{t+1}^{-1}(1 - \delta_{t+1})]$ , where we account for the possibility that consumption and the default rate are not independent. According to the assumptions in section 2.2, the default rate  $\delta_{t+1}$  equals zero if  $\Psi_{t+1} \geq b_t$ , and  $\delta_{t+1} = 1 - s_{t+1}/b_t$  if  $\Psi_{t+1} < b_t$ . Hence,  $E_t[c_{t+1}^{-1}(1 - \delta_{t+1})]$  is given by

$$\begin{aligned} &E_t[c_{t+1}^{-1}(1 - \delta_{t+1})] \\ &= \int_{-\infty}^{a_{t+1}^*} \pi_t(a_{t+1}) [c_{t+1}^{-1} \cdot (s_{t+1}/b_t)] da_{t+1} + \int_{a_{t+1}^*}^{\infty} \pi_t(a_{t+1}) [c_{t+1}^{-1} \cdot (1 - 0)] da_{t+1}. \end{aligned}$$

Using the solutions (19) and (21), the asset pricing equation (7) can thus be written as

$$1/R_t = \frac{\beta}{c_t^{-1}} \left[ b_t^{-1} \int_{-\infty}^{a_{t+1}^*} \pi_t(a_{t+1}) \left[ c(a_{t+1}, \tau)^{-1} s(a_{t+1}, \tau) \right] da_{t+1} + \int_{a_{t+1}^*}^{\infty} \pi_t(a_{t+1}) \left[ c(a_{t+1}, \tau)^{-1} \right] da_{t+1} \right]. \quad (25)$$

Risk premia can then be computed as follows (further details can be found in appendix A.2): At the beginning of period  $t$ ,  $b_{t-1}$  is known and the stochastic productivity level  $a_t$  realizes. We get solutions  $\{c_t, s_t\}$  from (19) and (21). Then, we can compute the maximum debt repayment capacity using (23), where the conditional expectation in (23) is calculated using a discrete transition probability matrix for productivity. If  $\Psi_t < b_{t-1}$ , the government defaults, while bonds are not traded. For  $\Psi_t \geq b_{t-1}$ , the government does not default in period  $t$ . The bond price  $1/R_t$ , end-of-period debt  $b_t$ , and the productivity threshold  $a_{t+1}^*$  then simultaneously solve (25), the updated version of (24) which reads  $b_t = \Psi(a_{t+1}^*)$ , and the government's flow budget identity

$$b_t/R_t = b_{t-1} - s_t. \quad (26)$$

After the equilibrium bond price  $1/R_t$  is derived, we compute the sovereign risk premium  $R_t - R_t^{rf}$  (using 22).

### 3 Results

#### 3.1 Multiple bond market equilibria

In this section, we examine the determination of bond prices and show analytically that multiple equilibrium bond prices can exist. To be able to derive analytical results, we apply a simplified version of the model. We assume that  $a_t$  is a serially uncorrelated random draw from the uniform distribution with support  $[a_l, a_h]$  where  $a_l$  and  $a_h$  are positive constants. To further simplify the derivation of analytical results, we assume that only the first-order terms of the maximum debt capacity (23) are non-negligible, and that the surplus maximizing tax rate is constant and equals the maximizer of the surplus maximizing tax rate for average productivity,  $\tau^* = \frac{1}{2} + \frac{g}{2\bar{a}\gamma}$ . We further restrict our attention to the upward sloping part of the Laffer curve,  $\tau \leq \tau^*$ .

With these assumptions, consumption, surpluses, and maximum repayable debt are linear functions of the current exogenous state  $a_t$ :

$$\Psi(a_t) = (1 - \tau^*)(\gamma a_t \tau^* - g) + (1 - \tau^*) a_t \frac{\beta}{1 - \beta} (\gamma \tau^* - (1/\bar{a})g), \quad (27)$$

$$c(a_t) = \gamma(1 - \tau)a_t = \theta_2 a_t, \quad (28)$$

$$s(a_t) = (1 - \tau)\gamma \tau a_t - (1 - \tau)g = \theta_3 a_t - \theta_4, \quad (29)$$

where in each line the second equality sign defines the composite parameters  $\theta_{2,3,4} > 0$ . Further, end-of-period debt satisfies  $b_t = \Psi(a_{t+1}^*)$  (see 24) and the government budget (26) demands  $1/R_t = (b_{t-1} - s_t)/b_t = (b_{t-1} - \theta_3 a_t + \theta_4)/\Psi(a_{t+1}^*)$ . The asset pricing equation (25) can then be written as

$$1/R_t = \beta a_t \left\{ \Psi(a_{t+1}^*)^{-1} \left[ \theta_3 \int_{a_l}^{a_{t+1}^*} \pi(a_{t+1}) da_{t+1} - \theta_4 \int_{a_l}^{a_{t+1}^*} \pi(a_{t+1}) (1/a_{t+1}) da_{t+1} \right] + \int_{a_{t+1}^*}^{a_h} \pi(a_{t+1}) (1/a_{t+1}) da_{t+1} \right\}.$$

With uniformly distributed productivity levels, this simplifies to

$$1/R_t = \beta \frac{a_t}{a_h - a_l} \left\{ \frac{\theta_3 (a_{t+1}^* - a_l) - \theta_4 (\log a_{t+1}^* - \log a_l)}{\Psi(a_{t+1}^*)} + (\log a_h - \log a_{t+1}^*) \right\}. \quad (30)$$

Thus, condition (30), which can be interpreted as a credit supply condition, describes the bond price  $1/R_t = q_t$  as a function of end-of-period debt  $b_t = \Psi(a_{t+1}^*)$  for a given exogenous state  $a_t$ .

The government's demand for credit is described by the period budget constraint (26),

which reads  $b_t/R_t = (b_{t-1} - s_t)$ . Using (29) it can be written as  $1/R_t = (b_{t-1} - \theta_3 a_t + \theta_4)/b_t \Leftrightarrow$

$$1/R_t = (b_{t-1} - \theta_3 a_t + \theta_4) / \Psi(a_{t+1}^*). \quad (31)$$

Credit supply (30) and demand (31) provide two conditions that determine the price  $1/R_t$  and the quantity of debt  $b_t = \Psi(a_{t+1}^*)$  issued in period  $t$ . It can be shown that there are either a unique, no, or two equilibrium bond prices, which is summarized in the following proposition.

**Proposition 1** *Suppose that productivity is uniformly distributed with  $a_t \in [a_l, a_h]$  and credit demand exceeds a level below which debt is risk-free,  $b_{t-1} - s_t > \Psi(a_l)/R_t^{rf}$ .*

1. *There exists a unique equilibrium if credit demand satisfies  $b_{t-1} - s_t \leq \Omega_{1,t}$ , where  $\Omega_{1,t} \equiv a_t \beta (1 - \tau) \gamma \tau - (1/R_t^{rf}) (1 - \tau) g$ .*
2. *For  $\Omega_{1,t} < b_{t-1} - s_t$ , two equilibria exist if (but not only if)  $b_{t-1} - s_t \leq \Omega_{2,t}$ , where  $\Omega_{2,t} \equiv a_t \beta \frac{1}{a_h - a_l} \{(1 - \tau) \gamma \tau (a_h e^{-\Xi(a_l)} - a_l) - (1 - \tau) g \log(a_h/a_l) + \Xi(a_h) [(1 - \tau) g + \Psi(a_h e^{-\Xi(a_l)})]\}$  and  $\Xi$  as defined in (36, see appendix A.1).*

**Proof.** See appendix A.1. ■

Credit demand (31) implies end-of-period debt to be proportional to the interest rate for a given stock of debt at the beginning-of-period  $b_{t-1}$  and the exogenous state  $a_t$ , which reflects the fact that the government has to issue more bonds  $b_t$  if the price  $q_t$  is lower, or, the interest rate  $R_t = 1/q_t$  is higher. At the same time, credit supply (30) also implies a positive relation between  $b_t$  and  $R_t$ , since future surpluses that suffice to repay debt become less likely for higher thresholds  $a_{t+1}^*$  ( $= \Psi^{-1}(b_t)$ ). This tends to reduce the expected return from bonds (since it increases the probability of default) and induces investors to demand a higher risk premium as a compensation. Yet, with higher end-of-period debt levels the risk premium and, thus, the interest rate increase more than proportionally. Due to this feedback mechanism, there might exist more than one bond market equilibrium, which is more likely when credit demand is larger, i.e. if  $\Omega_{1,t} < b_{t-1} - s_t$ . In fact, we find that the latter condition is likely to be satisfied for reasonable parameter values, such that there are typically two bond market equilibria. When credit demand is extremely large, there exists no equilibrium, which can be ruled out if (but not only if) credit demand satisfies  $b_{t-1} - s_t < \Omega_{2,t}$ . Hence, depending on how investors coordinate their expectations, a high or a low equilibrium bond price can be realized and self-fulfilling default expectations are possible. This result is, for example, similar to the outcome in Calvo's (1988) two period model, where the government might voluntarily default on its debt and two equilibria (with high and low interest rates) can exist.

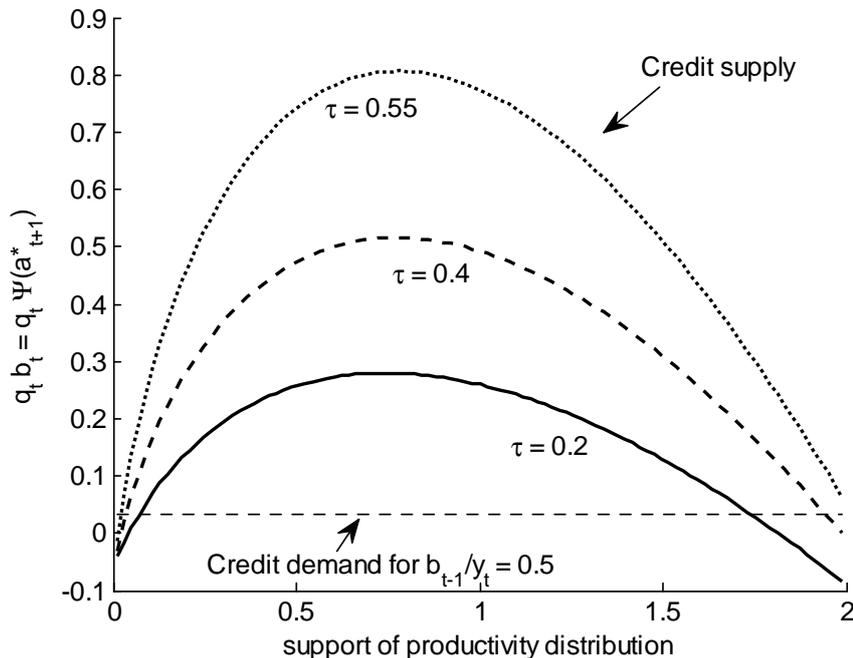


Figure 1: Multiple bond market equilibria (uniformly distributed productivity shocks)

To illustrate the determination of equilibrium bond prices  $q_t = 1/R_t$  and the potential non-uniqueness of equilibria, we apply the parameter values  $\beta = 0.95$ ,  $\gamma = 2/3$  (and thus  $l(\bar{a}) = 1/3$ ), and  $g/y(\bar{a}) = 0.2$ , while we consider alternative values for the tax rate  $\tau \in \{0.2, 0.4, 0.55\}$  and different values for the debt-to-gdp ratio  $b_{t-1}/y_t$  to examine equilibrium properties and comparative statics. We assume that the uniform distribution for the productivity level is characterized by  $a_l = 0.01$  and  $a_h = 1.99$ . These parameter values are merely illustrative examples; see the next section for a parameterization intended to match certain characteristics of European data in the context of a more realistic specification of the process governing productivity.

Figure 1 illustrates how one or two bond market equilibria can exist.<sup>10</sup> The figure shows credit supply (30) and demand (31), whereas both functions are multiplied by next period's debt,  $b_t$ . Thus, the figure shows the supplied and demanded volume of credit  $q_t b_t$  for all possible realizations of  $a_{t+1}^*$  (over the entire support of the productivity distribution). The intersection of credit demand and supply determine the (future) productivity threshold  $a_{t+1}^*$  (and thus  $q_t b_t = q_t \Psi(a_{t+1}^*)$ ). The figure shows three credit supply curves for different

<sup>10</sup>Note that the bond prices in this simplified model version hardly take realistic values due to the extreme assumption of uniformly distributed productivity shocks.

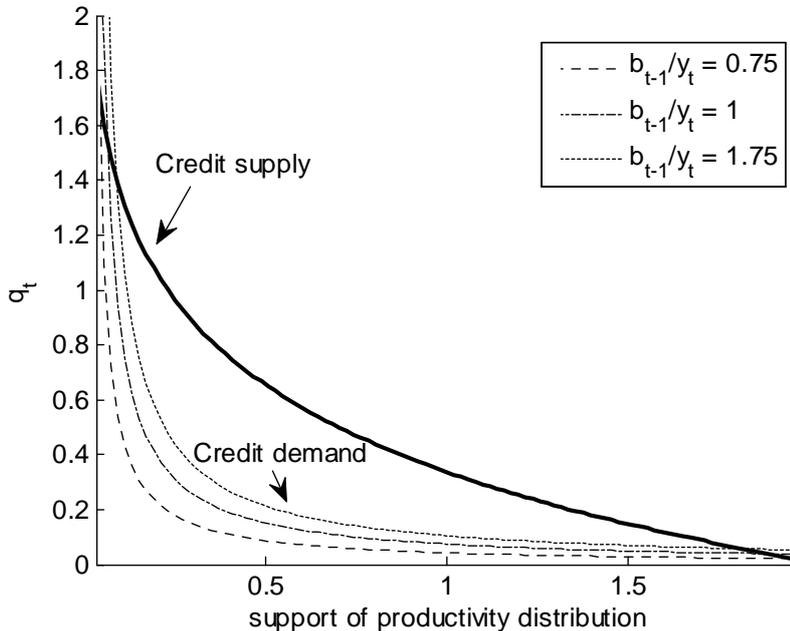


Figure 2: Bond prices in "low risk" and "high risk" equilibrium

tax rates (using  $\tau \in \{0.2, 0.4, 0.55\}$ , leaving credit demand for a fixed government share unaffected), as well as a horizontal (dashed) credit demand line.

For the highest tax rate,  $\tau = 0.55$ , which implies a substantial average primary surpluses, there exists exactly one bond market equilibrium within the feasible set of values for the threshold  $a_{t+1}^*$ . The productivity threshold is close to the lower bound of the support,  $a_l$ , which implies a low equilibrium risk premium, since debt issued in  $t$  is already sustainable at small productivity values  $t + 1$ . For less extreme values of the tax rate (0.4 and 0.2), there exist two bond market equilibria, one with a low productivity threshold close to the lower bound and one with a high productivity threshold close to the upper bound,  $a_h$ . The latter is obviously associated with a high risk premium, given that only few productivity realizations in  $t + 1$  are sufficiently high to avoid default in  $t + 1$ .

As figure 2 shows, the bond price  $q_t$  at the "low risk" equilibrium decreases with a higher stock of initially outstanding debt  $b_{-1}$  to gdp  $y_t$ . In contrast, the bond price in the "high risk" equilibrium increases with higher initial debt. Given this implausible comparative static property of the high equilibrium rate, we will first examine the lower equilibrium interest rate in the subsequent analysis.<sup>11</sup>

<sup>11</sup>In simulations, we found that realizations of the high equilibrium interest rate directly leads to default (see Juessen et al., 2010).

Table 1: Summary of parameter values

$\beta$ (discount factor)	0.97
$\gamma$ (utility parameter)	0.332
$g/y(\bar{a})$ (government share)	0.405
$\tau$ (tax rate)	0.436
$\rho$ (autocorrelation)	0.9
Implied output volatility	0.075

### 3.2 Quantitative results

In this section, we relax the simplifications made in the previous section and solve the model numerically for a more realistic parametrization. In particular, we move away from the assumption of uniformly distributed productivity shocks that was only used for analytical tractability above. While model periods are typically interpreted as quarters in the business cycle literature, we deviate from this assumption and interpret one period as a year, which facilitates computing theoretical predictions for government bonds with one-year maturity (without considering multi-period bonds). Throughout the quantitative analysis, we set the discount rate at  $\beta = 0.97$  to match a standard average value for a risk-free annual real interest rate and the mean working time share equal to  $l(\bar{a}) = 1/3$  (such that the utility parameter  $\gamma$  is given by  $\gamma = [(1 - (g/y(\bar{a}))) \cdot l(\bar{a})] / (1 - \tau)$ ).

As a point of departure, we consider a baseline scenario for the group of Euro-16 countries, i.e. for the 16 countries forming the Eurozone before Estonia's accession. For these countries, we calculate the average share of government spending in output  $g/y$  using data from the European Commission's Annual Macroeconomic Database (AMECO) over the period 1995-2009. Specifically, we calculate  $g/y$  as the ratio of total current expenditure (excluding interest payments) of the general government over gdp at current market prices (thus, our measure of government expenditure includes transfers). For the group of Euro-16 countries, the sample average is  $g/y = 0.405$ . For this  $g/y$ , we set the tax rate  $\tau$  so that an initial debt-to-gdp ratio of 100% ( $b_{-1}/y_0 = 1$ ) would be sustainable in the absence of default risk and the risk-free interest rate being  $1/\beta$  (which gives  $\tau = 1 - \beta + g/y(\bar{a})$ ). This ensures that our model calibration does not violate the transversality condition in the model.

We assume that productivity levels follow a first order Markov process with an autocorrelation of 0.9 and normally distributed innovations. To calibrate the standard deviation of productivity shocks, we regress the log of annual real gdp for the Euro-16 countries on a constant and a linear time trend, which avoids filtering out lower frequency output

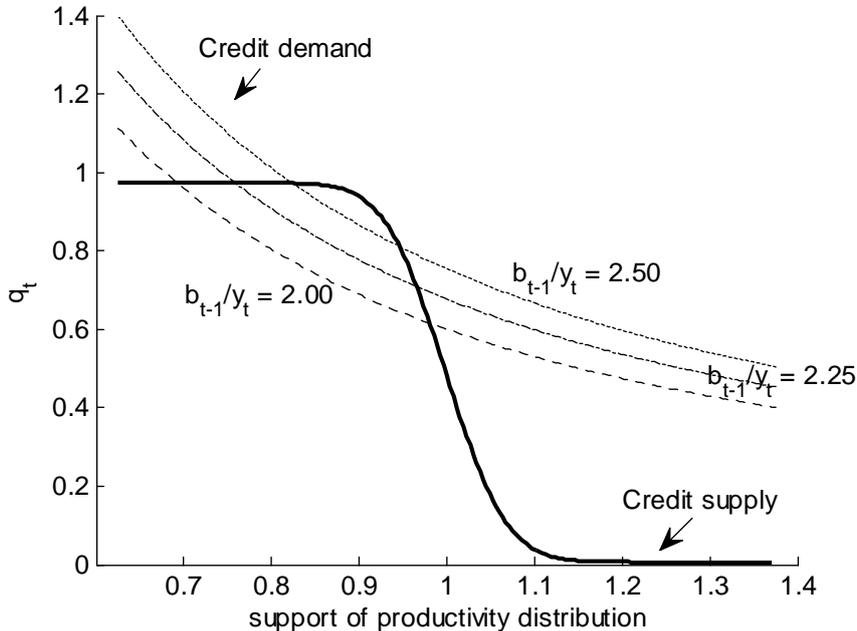


Figure 3: Multiple bond market equilibria

fluctuations that might be relevant for debt dynamics and sovereign default.<sup>12</sup> The estimated standard deviations of real output range from  $\sigma_y = 3.9\%$  for the Netherlands to  $\sigma_y = 15.7\%$  for Greece, with an average of  $7.5\%$ . We choose the innovation variance in our model such that the realized standard deviation of logged demeaned output from stochastically simulated model runs conforms with the average value over the output standard deviations in our sample. Table 1 summarizes the parameter choices.

In our benchmark calibration, the debt repayment capacity  $\Psi_t$  determined by (13) is based on the surplus maximizing tax rate  $\tau_t^* = 0.5 + g/(2\gamma a_t)$ . We compute the value of  $\Psi_t = E_t \sum_{k=0}^{\infty} \beta^k \frac{c_{t+k}^{*-1}}{c_t^{*-1}} s_{t+k}^*$  using simulation techniques, where  $c_t^* = c(a_t, \tau_t^*)$  and  $s_t^* = s(a_t, \tau_t^*)$  denote consumption and surpluses as functions of the state and the state-dependent surplus maximizing tax rate  $\tau_t^*$ . Specifically, we draw time series of productivity realizations for a sample length of 1,000 (conditional on the starting value  $a_t$ ) and then determine the associated sequences of consumption and surpluses using the decision rules, taking into account the time-varying Laffer tax rates  $\tau_t^*$ . These series are used to approximate the infinite sum of future discounted surpluses. We further approximate the

<sup>12</sup>Data on output (which are available over a longer time span than the tax and spending data) are from AMECO and cover the longest available time period in each case, which is 1960-2008 for 11 out of the 16 countries, but shorter for Malta, Cyprus, Slovakia, Slovenia, and unified Germany.

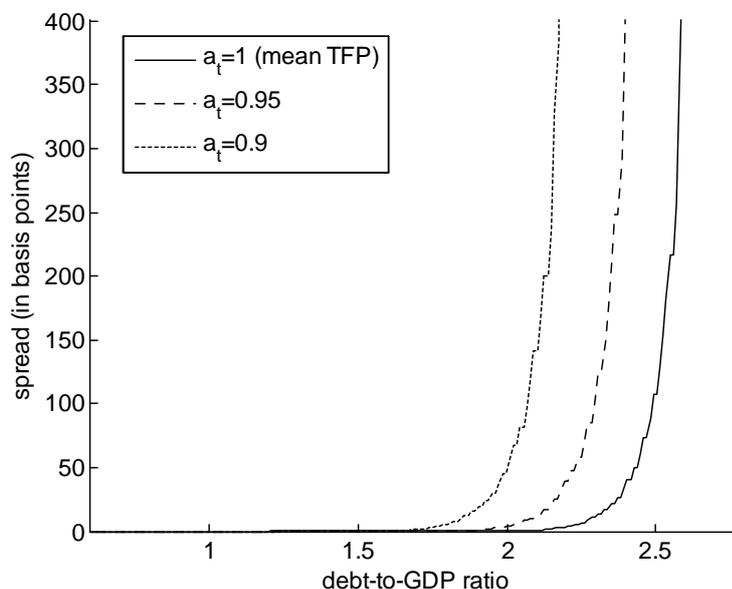


Figure 4: Interest rate spreads on government bonds,  $R_t - R_t^{rf}$

conditional expectation by repeating this simulation 5,000 times and taking the mean over all repetitions.

Below, we will also capture the case (that might arguably be relevant during the recent debt crisis in Greece) that the government is believed not to be willing to increase taxes to the surplus maximizing level in each period of time. This allows us to highlight the quantitative implications of scenarios where investors base their expectations on tax rates that are smaller than the Laffer curve maximizers, the latter being viewed as politically infeasible.

Figure 3 shows the determination of bond market equilibria. Like in the simplified model version (with uniformly distributed productivity shocks), we typically find two bond market equilibria: a "low risk" equilibrium at a high bond price and a "high risk" equilibrium with a low bond price. Though the credit supply curve is almost flat for the low productivity threshold, it is in fact monotonically decreasing over the entire support.

We first focus on the left equilibrium ("low risk") that exhibits intuitive comparative static properties. Figure 4 shows the model's implied bond pricing rule, i.e. the equilibrium relation between the interest rate spread of risky government bonds over the riskless interest rate,  $R_t - R_t^{rf}$ , and the beginning-of-period ratio of debt to output for a given productivity level.<sup>13</sup> In the figure, the solid line displays risk premia when the current

<sup>13</sup>Appendix A.2 presents details on the computation of equilibrium bond prices.

state is at the mean productivity level ( $a_t = 1$ ), while the dashed and dotted lines correspond to lower productivity levels which reduce government surpluses and thus the debt capacity and shift the equilibrium bond pricing rule to the left.

Figure 4 shows several key properties of the model for a parameterization intended to capture the average EU country. First, sizeable risk premia on government bonds are expected to emerge only at very high (and hitherto unobserved) debt-to-output ratios. This is due to the fact that the maximum debt capacity is calculated using the surplus maximizing tax rate, which is very high with a mean of just over 70%. Thus, the model implies that to the extent that investors expect that the government is indeed able to tax at the surplus maximizing rate if necessary, there is ample debt capacity such that risk spreads would not occur at debt-to-gdp levels below 200%. This can be seen as characterizing the situation in countries like France, Germany, or the Netherlands. Second, the figure shows that a severe business cycle slump (as depicted by the dashed and dotted lines in the figure), can lead to the emergence of risk premia as future surpluses are expected to be small in this case. Third, the pricing curves tend to become very steep at high debt ratios, suggesting that above a certain critical value – that itself depends on the current aggregate state of the economy – even small further increases in debt can lead to rapidly increasing risk spreads. The reason is that, if a high debt level is reached, not only would any additional adverse shock lead to a high probability of partial default, but also the endogenous repayment rate would be progressively smaller (see 15). Thus, the model predicts that market participants may become extremely sensitive to changes in debt at some point, and can thus be seen as an illustration of what may be called a rational market panic.

We note, however, that this point is reached only at very high debt-to-gdp ratios when the maximum debt capacity is based on  $\tau_t^*$  and when we focus on the equilibrium with the lower of the possibly two bond interest rates, as we did in figure 4. However, we have shown above that multiple equilibria may exist, which is also the case for the parameterization chosen here. In figure 5, we add the second possible equilibrium risk spread (shown by the bold lines), again for different values of the current aggregate state. The figure shows that the existence of multiple equilibria implies that, depending on which equilibrium materializes, risk spreads may differ by more than 10 percentage points (1,000 basis points). We interpret the existence of multiple equilibria as entailing the possibility of self-fulfilling default expectations. A given debt-to-gdp ratio may imply low spreads if investors coordinate their expectations on the low interest rate equilibrium, but may also lead to extremely high spreads for the same fundamentals if investors command higher risk premia due to bearish expectations. This opens up the possibility of explaining a sudden

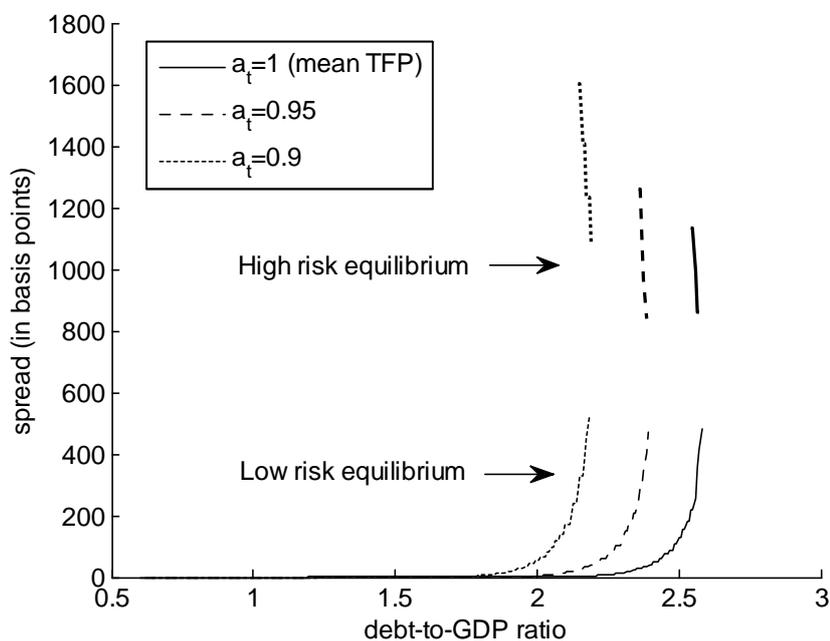


Figure 5: Low and high risk equilibrium

emergence of large risk premia as a "sunspot event", in which expectations may force default since in the high interest rate equilibrium the fiscal stance becomes immediately unsustainable.

Figure 6 shows the model's implied bond pricing curves for the case that the tax rate that investors use in computing the debt capacity falls short of the surplus maximizing tax rate  $\tau_t^*$ . As mentioned above, the surplus maximizing tax rate has a mean of above 70% in the model. This is much larger than empirically observed tax rates, such that it seems reasonable to assume that investors might not believe the government to be able to raise taxes to such a high level (the apparent difficulties of increasing tax rates experienced by the current Greek government being a case in point). Figure 6 shows the relation between debt-to-output ratios and risk spreads where  $\Psi_t$  is based on tax rates that are fractions of the mean surplus maximizing rate  $\tau^*$ . Current productivity is assumed to equal mean productivity in all cases. One can see that tax rates smaller than the Laffer curve maximizer can dramatically shift the position of the bond pricing curve, because of the implied reduction in the debt capacity. Thus, if expectations on the debt capacity are based on smaller tax rates than the Laffer curve tax, high risk premia can emerge for much lower levels of indebtedness, including those recently reached by many European countries. This can be interpreted as an indication that investors' confidence in the political ability of

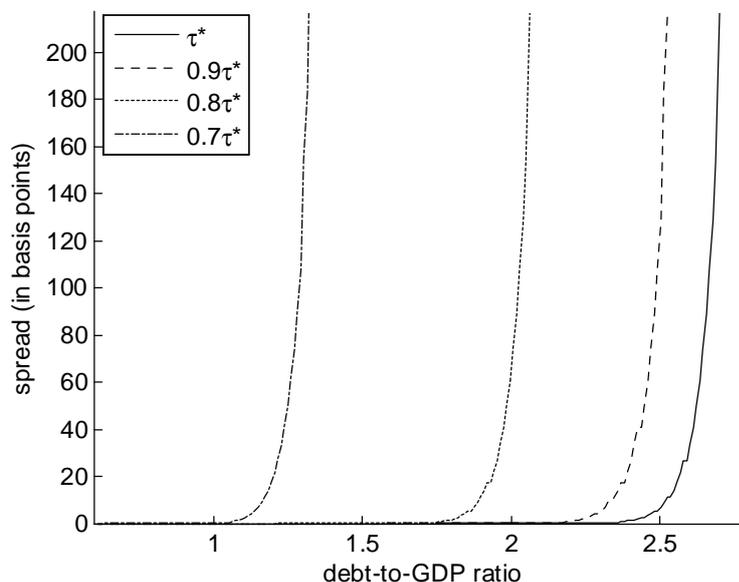


Figure 6: Spreads for different debt capacities (with current productivity at its mean)

the government may be crucial for the level of debt above which default risk becomes large. Hence – on top of the possibility of multiple equilibria – this may be a second mechanism which can tentatively explain why countries with similar debt-to-gdp ratios (like Spain and Germany) experience large differences in risk premia on government bonds.

#### 4 Conclusion

This paper shows that risk premia on government bonds can be rationalized by lenders' unwillingness to roll over debt when outstanding public debt exceeds a government's debt repayment capacity. The government does not commit to fully repay debt, such that default occurs when the state of the economy worsens in a way that leads to a build-up of public debt which exceeds the maximum present value of future surpluses. The default risk premium on public bonds reflects the probability of this event and the expected rate of partial repayment in case of default. We have shown the possibility of multiple equilibrium bond prices (i.e. a high and a low risk equilibrium), which can give rise to self-fulfilling default expectations, since the probability of default depends on the fiscal burden of interest payments, which themselves depend on the expected probability of default.

Considering the case where lenders expect the low equilibrium interest rate to prevail, we have presented a calibrated version of the model to analyze its quantitative implications. The model predicts that above a critical debt ratio, risk premia begin to rise

steeply, since market participants expect that further adverse business cycles shocks may lead to a situation where the government's debt repayment capacity is exceeded. However, for parameter values characterizing an average Eurozone member country, the model predicts negligible risk premia unless the government debt-to-gdp ratio rises beyond values which have hitherto been observed. Large premia can, in the context of the model, be explained either as a materialization of the high risk equilibrium, or as (unmodelled) political constraints that make the surplus maximizing tax rate unfeasible, and thus restrict the maximum debt capacity.

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## A Appendix

### A.1 Proof of proposition 1

To lighten the notation in this section, we drop the time index and define  $b = b_t$ ,  $b_{-1} = b_{t-1}$ ,  $a = a_t$ ,  $a' = a_{t+1}$ , and  $a^* = a_{t+1}^*$ . For  $a^* \rightarrow a_l$ , the RHS of (30) is given by  $a\beta \frac{\log a_h - \log a_l}{a_h - a_l}$ , which is the inverse of the risk-free rate  $R^{rf} = 1 / \left( a\beta \frac{\log(a_h/a_l)}{a_h - a_l} \right)$ . Now combine credit supply (30) with credit demand  $1/R = (b_{-1} - \theta_3 a + \theta_4) / \Psi(a^*)$  to get the following condition for  $a^*$ :

$$b_{-1} - (\theta_3 a - \theta_4) = \beta \frac{a}{a_h - a_l} \{ \theta_3 (a^* - a_l) - \theta_4 (\log a^* - \log a_l) + \Psi(a^*) (\log a_h - \log a^*) \}. \quad (32)$$

To examine the solution(s) for  $a^*$ , we define  $F(a^*)$  as

$$F(a^*) \equiv \kappa \{ \theta_3 (a^* - a_l) - \theta_4 (\log a^* - \log a_l) + \Psi(a^*) (\log a_h - \log a^*) \} - (b_{-1} - (\theta_3 a - \theta_4)),$$

where  $\kappa = \beta \frac{a}{a_h - a_l}$  and solve for  $F(a^*) = 0$ . At the lower bound of the support,  $F(a^*)$  is given by  $F(a_l) = \frac{1}{R^{rf}} \Psi(a_l) - (b_{-1} - (\theta_3 a - \theta_4))$  implying that  $F(a_l) < 0$  if credit demand is sufficiently large

$$b_{-1} - (\theta_3 a - \theta_4) > \frac{1}{R^{rf}} \Psi(a_l). \quad (33)$$

At the upper bound of the support,  $F(a_h)$  satisfies  $F(a_h) = \beta a \left( \theta_3 - \frac{\log a_h - \log a_l}{a_h - a_l} \theta_4 \right) + \theta_3 a - \theta_4 - b_{-1}$ , implying that  $F(a_h) > 0$  if credit demand does not exceed a particular threshold

$$b_{-1} - (\theta_3 a - \theta_4) < a\beta (1 - \tau) \gamma \tau - \frac{1}{R^{rf}} (1 - \tau) g, \quad (34)$$

where we used  $\theta_3 = (1 - \tau) \gamma \tau$  and  $\theta_4 = (1 - \tau) g$ . Next, we examine the slope of  $F(a^*)$ , which is given by

$$F'(a^*) = -\kappa \Xi(a^*) + \kappa (\log a_h - \log a^*), \quad (35)$$

where the second term is strictly positive and decreasing in  $a^*$  and  $\Xi(a^*)$  is defined as  $\Xi(a^*) \equiv \frac{\Psi(a^*) - (\theta_3 a^* - \theta_4)}{a^* \Psi'(a^*)}$ . Since feasible values of  $\Psi(a^*)$  are positive,  $\Psi(a^*) = (1 - \tau^*) (\gamma a^* \tau^* - g) + (1 - \tau^*) a^* \frac{\beta}{1 - \beta} (\gamma \tau^* - g) > 0$ , we know that  $\Xi(a^*)$  is also positive (since  $\tau \leq \tau^*$ ):

$$\Xi(a^*) = \frac{(1 - \tau^*) (\gamma a^* \tau^* - g) - (1 - \tau) (\gamma \tau a^* - g) + a^* (1 - \tau^*) \frac{\beta}{1 - \beta} (\gamma \tau^* - g)}{a^* (1 - \tau^*) \gamma \tau^* + a^* (1 - \tau^*) \frac{\beta}{1 - \beta} (\gamma \tau^* - g)} > 0, \quad (36)$$

and its derivative is strictly negative,  $\Xi'(a^*) = -\frac{g(\tau^* - \tau)(1 - \beta)}{a^{*2}(1 - \tau^*)(\gamma \tau^* - g\beta)} < 0$ . At the upper bound, the derivative of  $F(a^*)$  equals  $F'(a_h) = -\kappa \Xi(a_h) < 0$  and is strictly negative. For smaller values of  $a^*$ , the second term in (35) monotonically increases, while the first term,  $-\kappa \Xi(a^*)$ , monotonically decreases. Hence, we know that  $F'(a^*) = 0$  cannot have more than one solution, such that  $F(a^*) = 0$  has either no, one, or two solutions for

$a^* \in (a_l, a_h)$ . To assess the existence of two equilibria, we examine  $F(a^*)$  at its maximum, which is characterized by  $F'(\tilde{a}) = 0 \Rightarrow \log \tilde{a} = \log a_h - \Xi(\tilde{a}) \Leftrightarrow \tilde{a} = a_h e^{-\Xi(\tilde{a})}$  and

$$F(\tilde{a}) = \kappa \left\{ \theta_3 \left( a_h e^{-\Xi(\tilde{a})} - a_l \right) - \theta_4 (\log a_h - \log a_l) + \Xi(\tilde{a}) \left( \theta_4 + \Psi \left( a_h e^{-\Xi(\tilde{a})} \right) \right) \right\} - (b_{-1} - (\theta_3 a - \theta_4)).$$

Iff  $F(\tilde{a}) > 0 \Leftrightarrow (b_{-1} - (\theta_3 a - \theta_4)) < \kappa \{ \theta_3 (a_h e^{-\Xi(\tilde{a})} - a_l) - \theta_4 (\log a_h - \log a_l) + \Xi(\tilde{a}) [\theta_4 + \Psi(a_h e^{-\Xi(\tilde{a})})] \}$ , there exists two equilibria. Given that  $e^{-\Xi(\tilde{a})}$  is increasing in  $\tilde{a}$  and  $\Xi(\tilde{a})$  is decreasing in  $\tilde{a}$ , we know that there exist two equilibria if but not only if

$$(b_{-1} - (\theta_3 a - \theta_4)) < \kappa \left\{ \theta_3 \left( a_h e^{-\Xi(a_l)} - a_l \right) - \theta_4 \log(a_h/a_l) + \Xi(a_h) \left( \theta_4 + \Psi \left( a_h e^{-\Xi(a_l)} \right) \right) \right\}. \quad (37)$$

We can therefore conclude that there exists a unique equilibrium if (33) and (34) are satisfied. When (33) is satisfied and (34) is violated, there exist two equilibria if but not only if (37) is satisfied. ■

## A.2 Computation of equilibrium bond prices

We replace the original problem presented in section 2 by a discrete valued problem, i.e. we assume that the model's state space consists of a finite number of discrete points.<sup>14</sup> We use Tauchen's (1982) algorithm to approximate the first order Markov process for productivity by a discrete-valued Markov chain. We provide the size of the interval  $I_a = [a_1, a_n]$  and the number of grid points,  $n$  (we use  $n = 401$ ). Given the autocorrelation  $\rho$ , the interval  $I_a$  is chosen to include  $\pm 4$  standard deviations of the productivity process. Tauchen's algorithm then delivers the exogenous state space of the model  $S = \{a_1, a_2, \dots, a_n\}$ ,  $a_i < a_{i+1}$ ,  $i = 1, 2, \dots, n - 1$ , and the associated transition probability matrix  $P = (p_{ij})$ , whose row  $i$  and column  $j$  element is the probability of moving from state  $a_i$  state to state  $a_j$ .

For a given combination of initial debt  $b_{t-1}$  and current productivity level  $a_t$ , the equilibrium interest rate spread on government bonds is determined as follows:

- For a given current productivity level,  $a_t$ , current consumption  $c_t = c(a_t)$ , surpluses  $s_t = s(a_t)$ , and the maximum debt repayment capacity of the current period,  $\Psi_t = \Psi(a_t)$  are computed with (19), (21), and (23).  $\Psi_t$  is calculated using simulation techniques as described in section 3.2, taking into account the time-varying tax rates  $\tau_t^*$ . The risk-free rate is determined using  $R_t^r = c_t^{-1} / (\beta E_t c_{t+1}^{-1})$ , where the conditional expectation  $E_t c_{t+1}^{-1}$  is computed by  $E_t c_{t+1}^{-1} = \sum_{j=1}^n p_{ij} \cdot c(a_j)^{-1}$  and  $i$  denotes the index number for today's stochastic state,  $a_t$ .

<sup>14</sup>As an alternative to the discretization, one can solve the integrals in the asset pricing equation (25) using numerical integration. Both approaches yield almost identical results.

- Then, we check whether the government defaults in period  $t$  or not. If  $\Psi_t < b_{t-1}$ , the government defaults, end-of-period debt equals zero,  $b_t = 0$ , and there is no borrowing. If  $\Psi_t > b_{t-1}$ , the government does not default in period  $t$  and the bond market equilibrium price is determined as follows:

- The bond market equilibrium consists of a price  $1/R_t$  and end-of-period debt  $b_t$ . Replacing the integrals in (25) by sums over the finite number of states, the asset pricing equation reads

$$\frac{b_{t-1} - s_t}{b_t} = \frac{\beta}{c_t^{-1}} \left[ b_t^{-1} \sum_{a_{t+1}=a_1}^{a_{t+1}^*} \pi_t(a_{t+1}) \left[ c(a_{t+1})^{-1} s(a_{t+1}) \right] + \sum_{a_{t+1}=a_{t+1}^*}^{a_n} \pi_t(a_{t+1}) \left[ c(a_{t+1})^{-1} \right] \right] \quad (38)$$

Use the updated version of (24),  $b_t = \Psi(a_{t+1}^*)$ , to replace  $b_t$  in (38) :

$$b_{t-1} - s_t = \frac{\Psi(a_{t+1}^*) \beta}{c_t^{-1}} \left[ \Psi(a_{t+1}^*)^{-1} \sum_{a_{t+1}=a_1}^{a_{t+1}^*} \pi_t(a_{t+1}) \left[ c(a_{t+1})^{-1} s(a_{t+1}) \right] + \sum_{a_{t+1}=a_{t+1}^*}^{a_n} \pi_t(a_{t+1}) \left[ c(a_{t+1})^{-1} \right] \right] \quad (39)$$

Equation (39) is then solved for the unknown productivity threshold in the next period,  $a_{t+1}^*$ . The lower value  $a_{t+1}^*$  equilibrium corresponds to the low interest rate equilibrium (see the discussion on multiple equilibria in section 3.1).

- Given the solution for  $a_{t+1}^*$ , next-period's debt level  $b_t$  and the asset price  $1/R_t$  are determined by  $b_t = \Psi(a_{t+1}^*)$  and

$$\frac{1}{R_t} = \frac{\beta}{c_t^{-1}} \left[ \Psi(a_{t+1}^*)^{-1} \sum_{a_{t+1}=a_1}^{a_{t+1}^*} \pi_t(a_{t+1}) \left[ c(a_{t+1})^{-1} s(a_{t+1}) \right] + \sum_{a_{t+1}=a_{t+1}^*}^{a_n} \pi_t(a_{t+1}) \left[ c(a_{t+1})^{-1} \right] \right]$$

- The risk premium on government bonds for given states  $b_{t-1}$  and  $a_t$  is calculated as  $R_t - R_t^{rf}$ .