Can Monetary Policy be Superior for Financial Stabilization?[1]

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Abstract

This paper proposes asset purchases as a financial stabilization instrument. We develop a monetary model with collateral constraints, where an ex-ante Pigouvian debt tax restores constrained efficiency. Central bank asset purchases in secondary markets, which exert effects equivalent to ex-post Pigouvian loan subsidies, can likewise implement constrained efficiency and even first best. As a novel argument, we show that welfare-enhancing asset purchases can also be conducted when taxes/transfers are not available. Corrective price effects can be separated from inflation targets and can enhance efficiency regardless of distributive effects or of an inferior value extraction from central bank asset holdings.

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1 Introduction

Central banks have included large-scale purchases of private debt securities in secondary markets into their set of policy instruments. Empirical evidence shows that these interventions are able to alter asset prices, not only during the great financial crisis.[3] Asset purchase programs were further (re-)introduced or scaled-up at the onset of the Covid-19 pandemic to mitigate adverse price responses in financial markets (see e.g. Haddad et al., 2021, Vissing-Jorgensen, 2021). This suggests that ex-post monetary policy interventions are useful for the correction of price effects that are at the heart of amplification mechanisms which threaten financial stability.[4] The latter objective, however, does typically not lie in the domain of monetary policy. This raises the question why corrective price effects are not induced by ex-post interventions of other policy branches/actors, like asset purchases by the government or Pigouvian subsidies. This paper addresses this question by providing a novel argument in favor of monetary policy instruments to be applied for financial stabilization, referring to the funding and the compensation of corrective policy interventions.

The analysis is conducted in a framework with financial market imperfections, which provide a rationale for financial regulation,[5] and with reserves being essential for banks. We show that corrective price effects induced by ex-post central bank asset purchases in secondary markets can enhance efficiency in a way that obviates prudential regulation, and that asset purchases can even implement the first best allocation. Yet, price effects of asset purchases can equivalently be induced by a Pigouvian subsidy on loan supply that is financed and compensated by a non-distortionary tax. We consider the case where taxes/transfers are not available to finance and compensate corrective policy interventions, such that first best cannot be achieved and fiscal policy cannot fund Pigouvian subsidies. As the main novel result, we show that asset purchases can enhance social welfare compared to any competitive equilibrium without asset purchases regardless of the availability of taxes. This result relies on the existence of sufficiently many central bank instruments, which allow *i.*) isolating price effects of asset purchases, *ii.*) guaranteeing non-reliance on fiscal backing, and *iii.*) achieving common central bank targets. We further show that these conclusions also hold when we account for distributive effects or for an inferior ability

[3] See Dell'Ariccia et al. (2018) and Kuttner (2018), who survey evidence on effects of asset purchase programmes conducted by the BoE, the ECB and the Fed.

[4] See Bornstein and Lorenzoni (2018) and Jeanne and Korinek (2020).

[5] Financial amplification is induced by a collateral constraint that leads to a positive feedback between asset demand, prices, and borrowing capacities, like in Lorenzoni (2008), Jeanne and Korinek (2010), Bianchi (2011), Stein (2012), Bianchi and Mendoza (2018), or Davila and Korinek (2018).

of central banks to extract value from asset holdings, which are argued to limit the efficacy of monetary policy interventions (see Jeanne and Korinek, 2020).

We develop a finite horizon model with limited commitment and where central bank money is essential. Banks intermediate funds between borrowers and depositors. To account for the special role of central bank money, we assume that it serves as the unique means to settle banks' deposit transactions. The central bank supplies reserves only against eligible assets. These are treasury securities in regular money supply facilities where the central bank controls the terms of trade, i.e. the repo rate, serving as the policy rate. In addition, the central bank may conduct asset purchases, i.e. purchases of bank loans against reserves.[6] Since borrowers cannot commit to repay debt, loans are required to be collateralized by borrowers' assets. Given that the collateral constraint might be binding, precautionary saving induces inefficiency of the competitive equilibrium allocation. Binding collateral constraints are further associated with an inefficiently low price of pledgeable assets, which is not internalized by private agents, giving rise to a financial amplification mechanism (as in Bianchi, 2011, or Davila and Korinek, 2018). Moreover, costly central bank money, i.e. a positive policy rate, induces inefficiently low holdings of reserves and deposits (while leaving the allocation of commodities unaffected).

For the analysis of welfare-enhancing policies, we first consider – as common in related studies – the idealized case where non-distortionary taxes/transfers are available. The latter can be used to repay initial public sector liabilities and to neutralize budgetary effects of corrective (distortionary) taxes. Given that positive central bank earnings are then not required for public sector solvency, we assume that the central bank costlessly supplies reserves such that they are abundant (related to the Friedman-rule). The allocation of commodities is then equivalent to an allocation under laissez faire in a corresponding real (non-monetary) economy that exclusively consists of common elements. Based on this laissez faire allocation, we show that an ex-ante Pigouvian tax on debt can address the pecuniary externality with regard to the collateral price and can implement a constrained efficient allocation (defined as in Stiglitz, 1982, or Davila et al., 2012), which confirms well-established results in studies on prudential regulation (see e.g. Jeanne and Korinek, 2010, Bianchi, 2011).[7]

[6] Related to Dreze and Polemarchakis (2000), reversed treasury open market operations are conducted in the terminal period, overcoming the Hahn's (1965) paradox in finite horizon models. Further note that the analysis neither relies on asset purchases to be conducted at a large scale nor outright, implying that the effects can principally be induced by all central bank facilities including private debt.

[7] In contrast to Bianchi (2011), the constraint efficient allocation (under a Pigouvian debt tax) cannot equivalently be implemented by introducing margin requirements. The reason is that the market price of collateral tends

As an alternative corrective policy, we then introduce ex-post central bank asset purchases. Specifically, the central bank offers banks to purchase collateralized loans under binding collateral constraints. Given that reserves are not scarce, banks voluntarily sell loans to the central bank only at an above-market price. Banks then tend to earn profits from supplying and selling loans, leading to a lower loan rate under banking competition. With lower borrowing costs, agents are willing to raise debt and their payments for collateral, which tends to increase the price of eligible assets via the collateral premium, i.e. the valuation of assets to serve as collateral (see Fostel and Geanakoplos, 2008). Given that the collateral price in adverse states is too low relative to borrowers' exposure under laissez faire, asset purchases can thereby directly address this inefficiency which implies ex-ante excessive borrowing. Thus, asset purchases and their impact on the profitability of loan supply, which is fully internalized by banks, do not create moral hazard.[8] In contrast to asset purchases, ex-post transfers of funds to borrowers or bailouts only exert indirect (general equilibrium) price effects and cannot correct collateral prices to avoid adverse effects of pecuniary externalities. Via their corrective price effects, asset purchases can enhance welfare by raising the borrowing limit and can implement a constrained efficient allocation. Hence, they serve as a substitute for prudential regulation. In principle, asset purchases can even increase the collateral price to a sufficiently high level such that the borrowing limit is never reached and the competitive equilibrium is characterized by the first best allocation. However, the corrective price effects of asset purchases can equivalently be induced by a Pigouvian subsidy on eligible assets, which is funded and compensated by non-distortionary taxes.[9] Thus, the beneficial effects of asset purchases do per-se not rationalize why they are preferable to non-monetary interventions.

We then consider the case without non-distortionary taxes/transfers, which can in principle be motivated empirically or theoretically (see e.g. Atkinson and Stiglitz, 1976). In this case, first best cannot be achieved and a Pigouvian subsidy that is equivalent to asset purchases cannot be implemented. To accentuate the argument, we assume that also other taxes are not available in this case, such that corrective policies cannot be financed via fiscal policy. Given that public

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to decrease with the latter policy, accelerating the adverse effects of pecuniary externalities. This effect does not exist in Bianchi (2011) due to the specification of the borrowing limit, which is taken as given by individual agents.

[8] Corrective asset purchases induce a higher valuation of borrowers' assets in adverse states, whereas moral hazard can be created if interventions (like bailouts or debt reliefs) induce borrowers to value their wealth less in adverse states (see Jeanne and Korinek, 2020, or Stavrakeva, 2020).

[9] Given that money is abundant under laissez faire, asset purchases by the central bank are then equivalent with regard to the allocation of commodities to asset purchases conducted by the treasury.

sector solvency requires revenues to be raised in an alternative way, we consider positive costs of central bank money, leading to central bank interest earnings and scarcity of reserves.[10] Banks are then principally willing to acquire reserves via asset purchases even when the purchase price of loans is less than the market price. We show that corrective asset purchases can be conducted regardless of tax revenues and can enhance efficiency of the commodity allocation by raising the collateral price. Yet, asset purchases can principally also alter social welfare via their impact on monetary aggregates. The latter effect can however be completely neutralized by adjustments of regular money supply facilities, i.e. treasury open market operations. Due to the possibility to isolate corrective price effects of asset purchases via further monetary instruments, efficiency can be enhanced compared to any competitive equilibrium allocation without asset purchases even when taxes are not available. From this perspective, asset purchases are a superior corrective instrument compared to Pigouvian policies. We further show that conducting asset purchases for financial stabilization does not imply that the central bank needs to sacrifice other targets or objectives, specifically, regarding the inflation rate, which can equally be implemented as under a monetary policy regime without asset purchases.[11]

We further examine whether asset purchases induce effects that limit their efficacy compared to Pigouvian taxes/subsidies, for which we augment the model accordingly. Firstly, we consider that the central bank is characterized by an inferior ability (compared to banks) to extract value from assets held under asset purchase programs. Consistent with the specification of the fundamental imperfection that underlies the collateral constraint (i.e. limited commitment), we assume that the central bank can seize a smaller fraction of borrowers' collateral than banks. While this property can be addressed with haircuts, it does not hinder the central bank to exert welfare-enhancing price effects via asset purchases. Secondly, we assess if distributive effects, which were ruled out by construction in the previous analysis, render asset purchases less desirable. The analysis of the augmented model shows that inefficiencies due to distributive effects can in fact most effectively be addressed by raising the collateral price via asset purchases. Thus, neither distributive effects nor an inferior value extraction from central bank asset holdings invalidate the conclusions drawn above.

[10] We disregard positive interest payments on reserves, for convenience. Under interest on reserves (IOR), all results would equally hold for the spread between the policy rate and IOR instead for the policy rate level.

[11] For the analysis of inflation, we refer to different types of fiscal policy regimes that either guarantee zero public sector liabilities in the terminal period or not (see e.g. Nakajima and Polemarchakis, 2005). The conclusions regarding the desirability of asset puchases are robust to the introduction of price rigidities, which are neglected to facilitate the derivation of analytical results.

Section 2 describes the related literature. In Section 3, we present the model. Section 4 provides analytical results on welfare-enhancing ex-ante and ex-post policy interventions. In Section 5, we augment the model to assess the robustness of the main results. Section 6 examines money and inflation under asset purchases. Section 7 concludes.

2 Related literature

Our paper is most closely related to the two strands in the literature on the macroeconomic effects of unconventional monetary policies (i.e. public sector acquisitions of non-short-term-treasury debt securities or other private sector assets) and on prudential regulation under pecuniary externalities. Regarding the first strand, several studies examine effects of unconventional monetary policies under financial market imperfections and price rigidities. Curdia and Woodford (2011) and Gertler and Karadi (2011) show that direct central bank lending to ultimate borrowers can be beneficial when financial intermediation via banks is costly. While ad-hoc costs of credit origination by the central bank renders financial intermediation entirely by the central bank undesirable, costly central bank credit provision can be beneficial to mitigate financial crises. Chen et al. (2012) consider segmented financial markets and find that changing the composition of treasury debt as under US Federal Reserve large scale asset purchase programmes during the financial crisis had moderate GDP growth and inflation effects. Del Negro et al. (2017) examine government purchases of equity in response to an adverse shock to assets' resaleability and show that introducing this policy have prevented a repeat of the Great Depression. Correia et al. (2021) compare welfare effects of tax-financed credit subsidies to those of monetary policy instruments in a cash-in-advance model with banking and costly enforcement. They show that credit subsidies are superior to central bank interest rate policy, which is constrained by the zero lower bound, as well as to costly credit provision by the public sector modelled as in Gertler and Karadi (2011). Our paper further relates to Araújo et al. (2015), who examine asset purchases in an economy with flexible prices and collateral constraints. They show that welfare effects are ambiguous, since purchases of assets that serve as collateral can loosen or tighten borrowing constraints, particularly, when the fraction of the asset held by the central bank is large. In contrast to our paper, they assume that money and bonds are perfect substitutes and that all central bank trades occur at market prices. None of these papers consider pecuniary externalities induced by financial frictions or prudential policies.

The second strand of the literature, to which our paper is related, focusses on a financial amplification mechanism via effects of pecuniary externalities induced by collateral constraints (see Lorenzoni, 2008, Jeanne and Korinek, 2010, 2019, Bianchi 2011, Benigno et al., 2016, Schmitt-Grohe and Uribe, 2017, Bianchi and Mendoza, 2018, Davila and Korinek, 2018, or Korinek, 2018).[12] A common conclusion drawn in these studies is that the amplification of adverse shocks and welfare losses under potentially binding collateral constraints can be mitigated by prudential (ex-ante) policies that reduce borrowing, which has established the view that a laissez-faire equilibrium in these types of economies suffer from overborrowing.[13] Like in these studies, we show that prudential regulation, i.e. ex-ante Pigouvian taxes on debt, can enhance social welfare and implement a constraint efficient allocation. Within the same class of models, Benigno et al. (2016) show the desirability of ex-post tax on non-tradables that raise the collateral price, and Bianchi (2016) shows that debt reliefs lead to welfare gains, which relate to our analysis of ex-post policies. None of these studies considers monetary policy.

Few studies merge the two aforementioned topics. Bornstein and Lorenzoni (2018) and Jeanne and Korinek (2020) analyze the relation between ex-ante and ex-post policies; the latter being identified with monetary policy interventions. While Jeanne and Korinek's (2020) specification of ex-ante policies in form of a Pigouvian tax on debt closely relates to ours, the crucial difference to our approach is the specification of ex-post policy. For the latter, they distinguish untargeted from targeted ex-post "liquidity provisions", i.e. open market purchases of borrowers' assets or loans proportional to the recipient's debt; both being financed by real funds borrowed from lenders/depositors. Given that ex-post liquidity provisions are not associated with financial constraints, they can principally implement first best. For the analysis of the optimal mix of ex-ante and ex-post policies, Jeanne and Korinek (2020) consider additional ad-hoc social costs of liquidity provisions. They show that costly ex-post policies do not obviate ex-ante regulation, while a more generous liquidity provision allows relaxing ex-ante regulation. This main conclusion relates to Bornstein and Lorenzoni (2018), who develop a framework with pre-set nominal prices and aggregate demand externalities. They show that prudential regulation is unnecessary when the central bank sets the real interest rate in a state contingent way, while a prudential regulation is useful when monetary policy is not perfectly state contingent. They further examine asset purchases, which are associated with inefficiency costs, and conclude that

[12] Davila and Korinek (2018) further examine distributive effects – which we adress in Section 5 – in a comprehensive analysis of effects induced by pecuniary externalities based on financial frictions.

[13] The generality of the latter conclusion has been questioned by Benigno et al. (2013) and Schmitt-Grohe and Uribe (2021).

they seem to be substitutes for ex-ante regulation. Like in Jeanne and Korinek (2020), purchases of assets are financed by funds raised from and repaid to lenders. In both studies, policy enforces that funds are transferred from lender/depositors to borrowers, reducing their demand to borrow against collateral. In contrast to our model, neither Bornstein and Lorenzoni (2018) nor Jeanne and Korinek (2020) consider any role of money.

An essential role for money is assumed in Woodford (2016) and Chi et al. (2021), which both disregard central bank acquisition of non-treasury debt or borrowers' assets. Woodford (2016) considers interest rate policy, reserve requirements, and "quantitative easing" (identified with an extension of the central bank balance sheet via purchases of long-term treasuries) as monetary policy instruments. The focus of the analysis is whether these independent dimensions of monetary policies increase or decrease financial stability risk. To address this question, Woodford (2016) embeds Stein's (2012) fire sale model in a dynamic general equilibrium model where financial crises are exogenously triggered. The paper shows that lowering interest rates tend to raise financial stability risks whereas quantitative easing should rather lower them. Chi et al. (2022) augment the model of Bianchi (2011) by introducing banks which require reserves. In contrast to Woodford (2016) and related to our approach, Chi et al. (2022) consider that households instead of banks are financially constrained. By paying interest on reserves in a state contingent way, monetary policy can raise funds that are provided to constrained borrowers via non-distortionary fiscal taxes/transfers. Combined with a capital control policy, interest on reserves can thereby implement a constraint efficient allocation. Contrary to the policies in Woodford (2016) and Chi et al. (2022), the main monetary policy measure considered in this paper, i.e. price-correcting asset purchases, neither increases the central bank balance sheet nor alters the interest rate on reserves.

None of all above cited studies considers corrective price effects of central bank asset purchases or examine the case where taxes are not available to finance policy interventions directly or indirectly by balancing the public sector budget.

3 The model

In this Section, we develop a finite-horizon model with incomplete asset markets and limited commitment. On the one hand, the model is structured to facilitate comparisons with the studies on prudential regulation. On the other hand, we account for a specific role of money, which is typically neglected in the latter studies. Precisely, we acknowledge that central bank money is required for the settlement of transactions and we explicitly specify central bank money supply against eligible assets in secondary markets.[14] To embed these features in a realistic way, we include banks in the model, though they are not relevant for allocation of commodities.

The economy further consists of households, who borrow from and deposit funds at banks, and a government. Deposits and bank loans are non-contingent and in nominal terms with money serving as the unit of account. Due to limited enforcement, agents can borrow exclusively against collateral (housing). Banks hold government bonds, which are eligible for regular central bank open market operations. In addition, the central bank can supply money via purchases of loans in secondary markets. The government receives central bank remittances. For the first part of the analysis, we assume that non-distortionary taxes/transfers are available.[15] We then examine the case where taxes are not available. The timing of events in each period is as follows: 1. Stochastic income is realized. 2. Agents borrow and lend in terms of loans and deposits. 3. Treasury open market operations are conducted and asset purchases might be offered. 4. Goods are traded and bank transactions are settled. 5. Government bonds mature and are issued.

3.1 Households

There are infinitely many agents i who live from period $t = 0$ to $t = T$. In each period, agents receive a stochastic income, i.e. a stochastic endowment of non-durable goods $y_{i,t}$. We will restrict the endowments of agents to ensure that there are exactly two types of agents $i \in (b, l)$, both being of mass one. Their utility increases with consumption $c_{i,t}$ of a non-durable good and of the housing stock $h_{i,t}$. We further assume that holdings of deposits $d_{i,t}$ (in real terms) increases utility as a short-cut for modelling transaction services of deposits. The instantaneous utility function $u_{i,t}$ satisfies

$$u_{i,t} = u(c_{i,t}, d_{i,t}, h_{i,t}), \quad \text{for } t \in [t, T-1] \text{ and } \; u_{i,T} = u(c_{i,T}, h_{i,T}), \tag{1}$$

where the utility function is separable in all arguments. Following conventional textbook specifications of money-in-the-utility function (see e.g. Woodford, 2003), we assume that there exists a satiation level in real deposits at a finite positive value $\overline{d} > 0$, such that $u_d(\overline{d}) = 0$ and $u_d(d_{i,t}) > 0$ if $d_{i,t} > \overline{d}$. Corresponding to studies on fire sales, borrowers have a superior use for pledgeable

[14] The specification of central bank operations closely relates to Schabert (2015), who analyses optimal monetary policy under frictionless financial markets.

[15] Precisely, we consider lump-sum taxes/transfers. Given that income is exogenously determined in our model, an income tax would also be non-distortionary. Since exogeneity of income is solely assumed for simplicity, we disregard income taxes, which are usually distortionary (e.g. under an endogenous labor supply).

assets. Here, housing is available as collateral and we assume that housing provides utility only to borrowers, i.e. $u_{l,t} = u(c_{l,t}, d_{l,t})$.

Income shocks y_i materialize at the beginning of each period. From $t = 0$ to $t = T - 1$, agents of type b draw relatively low realizations of income and borrow from banks, whereas agents of type l draw higher income realizations and deposit funds at banks. Income of agents b in $t = T$ is sufficiently high to repay debt. We assume that loans and deposits are contracted in nominal terms and non-state contingent. The interest rate on bank loans $L_{b,t}$ is R_t^L and of deposits $D_{l,t}$ is R_t^D. Type-b-agents do not hold deposits, such that their budget constraint is

$$P_t y_{b,t} \geq -L_{b,t} + R_{t-1}^L L_{b,t-1} + P_t c_{b,t} + P_t q_t \left(h_{b,t} - h_{b,t-1}\right) + P_t \tau_{b,t}, \tag{2}$$

where $\tau_{i,t}$ denotes type-specific non-distortionary transfers/taxes, P_t the price of non-durables, and q_t the real price of housing. The budget constraint of agents of type l, who will never borrow from banks and will not hold housing, is given by

$$P_t y_{l,t} \geq D_{l,t} - R_{t-1}^D D_{l,t-1} + P_t c_{l,t} - P_t \omega_{l,t} + P_t \tau_{l,t}, \tag{3}$$

where $\omega_{l,t}$ denotes profits of banks, owned by type-l-agents.

The central element is a financial constraint, which can be microfounded by limited commitment and the possibility of debt renegotiation as follows: We assume that borrowers can threaten to repudiate the debt contract and that lenders protect themselves by collateralizing borrowers' housing. Following a repudiation, they can seize a fraction z of borrowers' housing and can sell it at the current price $P_t q_t$ in the housing market. We consider the case where borrowers have all the bargaining power and are able to negotiate the loan down to the liquidation value of their housing (see Hart and Moore, 1994). Lenders take this possibility into account, such that debt repayment does not exceed the value of the seizable collateral. Hence, debt $L_{b,t} > 0$ of a borrower b with housing h_b is constrained by

$$L_{b,t} \leq z P_t q_t h_{b,t}, \tag{4}$$

where the fraction $z \in (0, 1)$ depends on enforcement and thus affects the ability to extract value from their assets (see Section 5). Maximizing lifetime utility $E \sum_{t=0}^{T} \beta^t u_{i,t}$, where E denotes the expectation operator, subject to the budget constraints (2) and the collateral constraint (4) leads to the following optimality conditions for borrowers for $t = 0$ to $t = T - 1$

$$u'(c_{b,t})\, q_t = u'(h_{b,t}) + \beta E_t u'(c_{b,t+1})\, q_{t+1} + \{\zeta_{b,t} z q_t\}, \tag{5}$$

$$u'(c_{b,t}) = \beta R_t^L E_t u'(c_{b,t+1})\, \pi_{t+1}^{-1} + \zeta_{b,t}, \tag{6}$$

and $L_{b,T} = 0$, where $\pi_{t+1} = P_{t+1}/P_t$ denotes the inflation rate and $\zeta_{b,t} \geq 0$ denotes the multiplier on the collateral constraint (4). Notably, the term in the curly brackets in (5) summarizes the collateral premium of the asset/housing, i.e. the valuation of asset to serve as collateral (see Fostel and Geanakoplos, 2008).[16] In period T, the optimal behavior of borrowers satisfies $u'(c_{b,T})\, q_T = u'(h_{b,T})$. The first order conditions of lenders for $t = 0$ to $t = T - 1$, who are restricted by the budget constraint (3), are for $t = 0$ to $t = T - 1$

$$u'(c_{l,t}) = u'(d_{l,t}) + \beta R_t^D E_t u'(c_{l,t+1})\, \pi_{t+1}^{-1}, \tag{7}$$

as well as $d_{l,T} = 0$, and $\lambda_{l,t} = u'(c_{l,t})$, where $d_{l,t} = D_{l,t}/P_t$ and $\lambda_{l,t}$ denotes the multiplier on (3).

3.2 Banks

We consider a continuum of perfectly competitive banks $j \in [0, 1]$, which are equally endowed and will behave in an identical way. They receive deposits $D_{j,t}$ from type-l-agents and supply loans $L_{j,t}$ to type-b-agents. They hold short-term government bonds (i.e., treasury bills) $B_{j,t}$, which are issued at the period-t-price $1/R_t$. Banks further hold central bank money in form of reserves $M_{j,t}$ because of their unique ability to settle deposit transactions. Reserves are supplied via open market transactions, which are carried out as outright transactions or as temporary sales or purchases (repos). For both, treasury bills serve as eligible assets and the price of reserves in treasury open market operations is R_t^m, which serves as the main policy rate. Specifically, reserves supplied to back j in treasury open market operations $I_{j,t}^B$ satisfy

$$I_{j,t}^B \leq \widetilde{\kappa}_t^B B_{j,t-1}/R_t^m, \tag{8}$$

where $\widetilde{\kappa}_t^B \in [0, 1]$ denotes a share of randomly selected treasuries accepted as eligible assets and $\widetilde{\kappa}_t^B = 1$ indicates full allotment. In addition to these regular open market operations, we consider the possibility that the central bank temporarily purchases loans from banks, which we summarize as asset purchases. Specifically, the central bank announces to offer money under

[16] Notably, the collateral premium would be absent if the borrowing limit were solely a function of aggregate variables, as for example specified in Jeanne and Korinek (2020). We will examine consequences of the latter type of borrowing constraints for policy effects in the subsequent analysis.

repos in exchange for a share $\widetilde{\kappa}_t^L \in [0,1]$ of bank loans, which are randomly chosen to avoid differential treatment borrowers. The purchase price offered by the central bank equals $1/R_t^A$:

$$I_{j,t}^L \leq \widetilde{\kappa}_t^L L_{j,t}/R_t^A. \quad (9)$$

Notably, the purchase price $1/R_t^A$ might differ from the open market price of bonds ($1/R_t^m$) and from the market price of loans (1), and can be higher or lower than the latter. We assume that central bank money is required for the settlement of deposit transactions, which are not explicitly modelled. We assume that settlement requires banks to hold central bank money equal to a fixed fraction $\widetilde{\mu} \in (0,1)$ of deposits

$$\widetilde{\mu} D_{j,t} \leq I_{j,t}^B + I_{j,t}^L + M_{j,t-1}. \quad (10)$$

Given that bank i transferred T-bills to the central bank under outright sales and that it repurchases a fraction of T-bills, $B_{j,t}^R = R_t^m M_{j,t}^R$, from the central bank, bank i's holdings of T-bills before it enters the bond market equals $B_{j,t-1} + B_{j,t}^R - \widetilde{B}_{j,t}$ and its money holdings equals $M_{j,t-1} - R_t^m M_{j,t}^R + I_{j,t}^B + I_{j,t}^L$. Thus, bank j's profits $P_t \omega_{j,t}$ are given by

$$\begin{aligned} P_t \omega_{j,t} = {} & D_{j,t} - R_{t-1}^D D_{j,t-1} - L_{j,t} + R_{t-1}^L L_{j,t-1} - B_{j,t}/R_t + B_{j,t-1} - M_{j,t} + M_{j,t-1} \\ & - I_{j,t}^B (R_t^m - 1) - I_{j,t}^L (R_t^A - 1), \end{aligned} \quad (11)$$

where the terms in second line of (11) give the costs of reserves acquisition. Notably, the last term tends to raise profits if the purchase price of loans, $1/R_t^A$, exceeds the market price, 1. The aggregate stock of reserves only changes via central bank money supply facilities (8) and (9), while demand deposits can be created subject to (10). Banks can in principle trade reserves among each other at the end of each period, after bank transactions and repos are settled. We abstract from interest payments on reserves (IOR_t) to avoid complicating the notation even further. In fact, all results/conditions referring to the policy rate R_t^m would then apply to the difference between R_t^m and IOR_t.

We assume that bankers maximize profits, $E_t \sum_{k=0}^{T} p_{t,t+k} \omega_{j,t+k}$, where $p_{t,t+k}$ denotes the shareholders' stochastic discount factor $p_{t,t+k} = \beta^k \lambda_{j,t+k}/\lambda_{j,t}$, subject to (8)-(11). The first order conditions with respect to loans, deposits, reserves from treasury open market operations and from asset purchases, treasury securities, and money holdings for $t = 0$ to $t = T - 1$ are

$$l_{j,t} : \lambda_{j,t} = \beta E_t R_t^L \lambda_{j,t+1} \pi_{t+1}^{-1} + \kappa_{j,t}^L \widetilde{\kappa}_t^L / R_t^A, \tag{12}$$

$$d_{j,t} : \lambda_{j,t} = \beta E_t \left(R_t^D \lambda_{j,t+1} \right) \pi_{t+1}^{-1} + \mu_{j,t} \widetilde{\mu}, \tag{13}$$

$$i_{j,t}^B : \mu_{j,t} = \kappa_{j,t}^B + \lambda_{j,t}(R_t^m - 1), \tag{14}$$

$$i_{j,t}^L : \kappa_{j,t}^L = \mu_{j,t} - \lambda_{j,t}(R_t^A - 1), \tag{15}$$

$$b_{j,t} : \lambda_{j,t}/R_t = \beta E_t \lambda_{j,t+1} \pi_{t+1}^{-1} + \beta E_t \kappa_{j,t+1}^B \widetilde{\kappa}_{t+1}^B \pi_{t+1}^{-1} / R_{t+1}^m, \tag{16}$$

$$m_{j,t} : \lambda_{j,t} = \beta E_t \pi_{t+1}^{-1} \left(\lambda_{j,t+1} + \mu_{j,t+1} \right), \tag{17}$$

where $\kappa_{j,t}^B$, $\kappa_{j,t}^L$ and $\mu_{j,t}$ denote the multipliers on (8), (9), and (10), respectively. The first order condition (12) shows that asset purchases tend to reduce the loan rate if the money supply constraint (9) is binding, $\kappa_{j,t}^L > 0$, which either requires reserves to be scarce ($\mu_{j,t} > 0$) or an above-market-price for loan purchases, $1/R_t^A > 1$ (see 15). The conditions (14) and (16) show that the bond rate is similarly affected by a binding money supply constraint for treasury operations, $\kappa_{j,t}^B > 0$. Moreover, the first order conditions (14), (16), and (17) relate the treasury rate to the expected monetary policy rate. Under full allotment $\widetilde{\kappa}_t^B = 1$, they imply

$$\beta E_t \pi_{t+1}^{-1} \left(\lambda_{j,t+1} + \mu_{j,t+1} \right) = \beta E_t \pi_{t+1}^{-1} [(\mu_{j,t+1} + \lambda_{j,t+1}) \cdot R_t / R_{t+1}^m], \tag{18}$$

and thus $R_t = R_{t+1}^m$ for a non-state-contingent monetary policy rate. Given that banks are perfectly competitive, bank profits defined as (11) will be driven down to zero in equilibrium.

In the terminal period T, where lenders receive no utility from deposit holdings, banks will neither supply further loans $L_{j,T} = 0$ nor create new deposits $D_{j,T} = 0$. Likewise, their money and bond holdings will equal zero at the end of period $T : M_{j,T} = B_{j,T} = 0$. Accordingly, there will be no asset purchases in period T, $I_{T,t}^L = 0$, and treasury open market operations are conducted as outright sales of bonds held by the central bank against central bank money outstanding, $I_{j,T}^B = -M_{j,T-1}$. This relates to Dreze and Polemarchakis's (2000) specification and overcomes the Hahn's (1965) paradox of maintaining a positive value for money under a finite horizon.

3.3 Public sector

The treasury issues bills, i.e. one-period bonds, at the price $1/R_t$ and receives remittances τ_t^m from the central bank. For the first part of the analysis, we assume that the treasury has type-specific non-distortionary taxes/transfers $P_t \tau_{i,t}$ at its disposal. This assumption is changed in the subsequent part of the analysis, where we assess policy options when these taxes/transfers are not available. The treasury's budget constraint reads

$$(B_t^g/R_t) + P_t\tau_{b,t} + P_t\tau_{l,t} + P_t\tau_t^m = B_{t-1}^g. \tag{19}$$

Notably, non-distortionary taxes/transfers $\tau_{i,t}$ will be used for two purposes: Firstly, they are source of revenues to balance the budget and to repay initial public sector liabilities. Secondly, they might serve for financing and compensating policy instruments that are introduced below to correct prices. For the analysis of inflation in Section 6, we characterize tax regimes in a more specific way and refer to Ricardian and non-Ricardian regimes (see e.g. Benhabib et al., 2001).

The central bank supplies money in open market operations either outright or temporarily via repos against treasuries, M_t and M_t^R, where $I_t^B = M_t - M_{t-1} + M_t^R$. Both types of transactions are introduced for realism and they provide an additional central bank instrument (see below). Yet, only one type is open market transaction is in fact required for the analysis. The central bank can further increase the supply of money by temporarily purchasing loans from lenders, I_t^L, i.e. it supplies money under repos against loans. At the beginning of each period, its holdings of treasuries and the stock of outstanding money are given by B_{t-1}^c and M_{t-1}. It then receives treasuries and loans in exchange for money. Before the asset market opens, where the central bank rolls over maturing assets, repos in terms of treasuries and collateralized loans are settled. Its budget constraint therefore reads $(B_t^c/R_t)-B_{t-1}^c+P_t\tau_t^m = R_t^m(M_t - M_{t-1})+(R_t^m-1)M_t^R+\left(R_t^A-1\right)I_t^L$. Remittances $P_t\tau_t^m$ to the treasury consist of interest earnings from money supply facilities as well as from asset holdings,

$$P_t\tau_t^m = (R_t^m - 1)(M_t - M_{t-1}) + (R_t^m - 1)M_t^R + \left(R_t^A - 1\right)I_t^L + (1 - 1/R_t)B_t^c. \tag{20}$$

Substituting out remittances in the central bank budget constraint shows that central bank asset holdings evolve according to $B_t^c - B_{t-1}^c = M_t - M_{t-1}$. Further assuming that initial values satisfy $B_{-1}^c = M_{-1}$, gives the central bank balance sheet $B_t^c = M_t$. The central bank has five instruments at its disposal: It sets the policy rate R_t^m as well as the price for loan purchases $1/R_t^A$, and can decide how much money to supply against treasuries, $\widetilde{\kappa}_t^B \in (0,1]$, and against loans, $\widetilde{\kappa}_t^L \in [0,1]$. Finally, the central bank can choose how much money to supply outright or temporarily via repos in exchange for treasuries, by controlling the ratio of treasury repos to outright purchases $\Omega_t \geq 0 : M_t^R = \Omega_t M_t$. Thereby, the central bank can adjust its earnings from outright money supply, where it reinvests payoffs in treasuries, and from repos.

3.4 Equilibrium

Before we identify welfare-enhancing policies in an analytical way, for which we apply some simplifying assumptions, we summarize some main properties of the model to demonstrate that they do not rely on further restrictions. Throughout, we assume the endowment and income to ensure that there are two types of agents, borrowers and lenders, who are identical within each group and who never switch their roles. Initial endowment and income of lenders suffice to meet borrowers' demand for external funds, while period T income of borrowers suffices to repay debt. In equilibrium, agents' optimal plans are satisfied and prices adjust such that all markets clear: $\int l_{j,t}dj = \int l_{b,t}db$, $\int d_{j,t}dj = \int d_{l,t}dl$, $h = \int h_{b,t}db$, $\int y_{b,t}db + \int y_{l,t}dl = \int c_{b,t}db + \int c_{l,t}dl$, $m_t = \int m_{j,t}dj$, $m_t^R = \int (i_{j,t}^B - m_{j,t} + m_{j,t-1}\pi_t^{-1})dj$, $b_t = \int b_{j,t}dj$, and $b_t^g = b_t^c + b_t$, where $l_{i,t} = L_{i,t}/P_t$, $m_{i,t}^H = M_{i,t}^H/P_t$, $m_t^R = M_t^R/P_t$, $b_{i,t} = B_{i,t}/P_t$, $b_t = B_t/P_t$, $b_t^c = B_t^c/P_t$, and $b_t^T = B_t^T/P_t$. Moreover, $\lambda_{j,t} = \lambda_{l,t}$, since lenders are the shareholder of banks. Consolidating the budget constraints of the central bank and of the treasury, gives

$$(b_t/R_t) + \tau_{b,t} + \tau_{l,t} + R_t^m \left(m_t - m_{t-1}\pi_t^{-1}\right) + \left(R_t^m - 1\right)\Omega_t m_t + \left(R_t^A - 1\right) i_t^L = b_{t-1}\pi_t^{-1}, \quad (21)$$

where we used $b_t^g = b_t^c + b_t$. Note that the central bank balance sheet $B_t^c = M_t$ further implies $b_t^g - b_t = m_t$. Iterating (21) forward and applying banks' terminal conditions $m_T = b_T = 0$, leads to the intertemporal budget constraint of the consolidated public sector

$$\left(b_{-1} + R_0^m m_{-1}\right)/\pi_0 - \sum_{t=0}^{T}\left(\prod_{k=1}^{t}\frac{\pi_k}{R_{k-1}}\right)\left(\tau_{l,t} + \tau_{b,t}\right) \quad (22)$$
$$= \sum_{t=0}^{T-1}\left(\prod_{k=1}^{t}\frac{\pi_k}{R_{k-1}}\right)\left\{\left[\left(R_t^m - 1\right)\Omega_t m_t + \left(R_t^A - 1\right) i_t^L\right] + \left[\left(R_t^m - R_{t+1}^m/R_t\right) m_t\right]\right\}.$$

Using (21), bank $j's$ profits satisfy $P_t\omega_{j,t} = D_{j,t} - R_{t-1}^D D_{j,t-1} - L_{j,t} + R_{t-1}^L L_{j,t-1} + P_t\tau_{b,t} + P_t\tau_{l,t}$. Substituting out dividends with the latter, the representative depositor's budget constraint (3) can be rewritten as

$$y_{l,t} = l_{b,t} - R_{t-1}^L\pi_t^{-1} l_{b,t-1} + c_{l,t} - \tau_{b,t}. \quad (23)$$

We can then define a competitive equilibrium for two types of agents, where we neglect the reference to individual banks (j), for convenience.

Definition 1 *A competitive equilibrium of the economy with two types of agents consists of a set of sequences* $\{c_{b,t}, c_{l,t}, h_{b,t}, q_t, \zeta_{b,t}, \mu_t, \kappa_t^L, l_{b,t}, d_{l,t}, \pi_t, \kappa_t^B, R_t, R_t^L, R_t^D, m_t, b_t, i_t^B, i_t^L\}_{t=0}^{T-1}$ *and*

$\{c_{b,T}, c_{l,T}, q_T, \pi_T\}$ *satisfying* $y_{l,t} + y_{b,t} = c_{b,t} + c_{l,t}$ *and (23) for* $t \in \{0, T\}$, $h_{b,t} = h$, *(5)-(7), (21),*

$$l_{b,t} \le zq_t h_{b,t}, \text{ if } \zeta_{b,t} = 0, \text{ or } l_{b,t} = zq_t h_{b,t}, \text{ if } \zeta_{b,t} > 0, \tag{24}$$

$$u'(c_{l,t}) = \beta R_t^L E_t u'(c_{l,t+1}) \pi_{t+1}^{-1} + \kappa_t^L \widetilde{\kappa}_t^L / R_t^A, \tag{25}$$

$$u'(c_{l,t}) = \beta R_t^D E_t u'(c_{l,t+1}) \pi_{t+1}^{-1} + \widetilde{\mu}\mu_t, \tag{26}$$

$$\kappa_t^L = \mu_t - u'(c_{l,t})(R_t^A - 1) \ge 0, \tag{27}$$

$$\kappa_t^B = \mu_t - u'(c_{l,t})(R_t^m - 1) \ge 0, \tag{28}$$

$$u'(c_{l,t})/R_t = \beta E_t u'(c_{l,t+1}) \pi_{t+1}^{-1} + \beta E_t \kappa_{t+1}^B \widetilde{\kappa}_{t+1}^B \pi_{t+1}^{-1} / R_{t+1}^m, \tag{29}$$

$$u'(c_{l,t}) = \beta E_t \pi_{t+1}^{-1} \left(u'(c_{l,t+1}) + \mu_{t+1}\right), \tag{30}$$

$$i_t^B \le \widetilde{\kappa}_t^B b_{t-1} \pi_t^{-1} / R_t^m \text{ if } \kappa_t^B = 0, \text{ or } i_t^B = \widetilde{\kappa}_t^B b_{t-1} \pi_t^{-1} / R_t^m \text{ if } \kappa_t^B > 0, \tag{31}$$

$$i_t^L \le \widetilde{\kappa}_t^L l_{b,t} / R_t^A \text{ if } \kappa_t^L = 0, \text{ or } i_t^L = \widetilde{\kappa}_t^L l_{b,t} / R_t^A \text{ if } \kappa_t^L > 0, \tag{32}$$

$$\widetilde{\mu} d_t \le i_t^B + i_t^L + m_{t-1}\pi_t^{-1} \text{ if } \mu_t = 0, \text{ or } \widetilde{\mu} d_t = i_t^B + i_t^L + m_{t-1}\pi_t^{-1} \text{ if } \mu_t > 0, \tag{33}$$

$$i_t^B = m_t - m_{t-1}\pi_t^{-1} + \Omega_t m_t, \tag{34}$$

for $t \in \{0, T-1\}$, $u'(c_{b,T}) q_T = u'(h)$, *and (22), given* $\{y_{l,t}, y_{b,t}\}_{t=0}^T$, *monetary policy* $\{\widetilde{\kappa}_t^B, \widetilde{\kappa}_t^L, R_t^m, R_t^A, \Omega_t\}_{t=0}^{T-1}$, *fiscal policy* $\{\tau_{l,t}, \tau_{b,t}\}_{t=0}^T$, *and initial values* $R_{-1}^L > 0$, $l_{b,-1} \ge 0$, $b_{-1} > 0$, *and* $m_{-1} > 0$.

As implied by Definition 1, the competitive equilibrium including the initial inflation rate π_0 (and thus the initial price level) can in principle be fully determined and the issue of money holdings in the terminal period T is addressed by reversed treasury open market operations (see Section 6). Regarding the subsequent efficiency analyses, we apply utilitarian welfare as a measure for social welfare

$$W = E \sum_{t=0}^{T} \beta^t (u_{b,t} + u_{l,t}). \tag{35}$$

The first best allocation can easily be identified: A social planer who aims at maximizing (35) subject to the resource constraint, $y_{l,t} + y_{b,t} = c_{b,t} + c_{l,t}$, will chose consumptions levels of agents such that the marginal utilities are identical and deposits are held at the satiation level.

Corollary 1 *A first best allocation is characterized by* $u'(c_{l,t}) = u'(c_{b,t})$ $\forall t \in [0, T]$ *and* $d_{l,t} = \overline{d}$ $\forall t \in [0, T-1]$.

Combining the lenders' optimality condition for deposits (7) with the one of banks (26), shows that the multiplier μ_t on the banks' liquidity constraint (10) satisfies

$$u'(d_{l,t}) = \mu_t \widetilde{\mu}. \tag{36}$$

Thus, lenders are satiated with deposits, $d_{l,t} = \overline{d}$, when the multiplier on the banks' liquidity constraint (10) equals zero, $\mu_t = 0$. If the central bank sets $R_t^m = 1$, acquisition of money in treasury open market operations is costless. Banks are then not unwilling to hold more central bank money than required by (10) and are thus indifferent with regard to the size of open market transactions. To see this, $\mu_t = \widetilde{\kappa}_t^B = 0$, in a more detailed way, consider that money is supplied under full allotment in treasury open market operations $\widetilde{\kappa}_t^B = 1$ at the (market) price $1/R_t^m = 1$. According to (28), the multipliers on the liquidity constraint (10) and on the money supply constraint (8) are then identical, $\mu_t = \widetilde{\kappa}_t^B$, while (18) implies that the treasury rate satisfies, $R_t = 1$. Hence, holdings of public sector liabilities are neither associated with costs (in the case of money) nor with positive interest earnings (in the case of bonds). When no asset purchases are offered, $\widetilde{\kappa}_t^L = 0$, credit supply (25) equals $u'(c_{l,t}) = \beta R_t^L E_t u'(c_{l,t+1}) \pi_{t+1}^{-1}$. Now suppose that the multiplier on the reserve requirement (10) were positive $\mu_t > 0$. Then, the deposit rate would be strictly lower than the loan rate (see 26). Since money holdings are costless, banks would then be able to generate positive profits. Given that banks are perfectly competitive, this can be ruled out, implying that the multiplier μ_t has to be equal to zero. Then, condition (36) implies that deposit demand is satiated, $u'(d_{l,t}) = 0 \Rightarrow d_{l,t} = \overline{d}$. Money demand (30) further simplifies to $u'(c_{l,t}) = \beta E_t u'(c_{l,t+1}) \pi_{t+1}^{-1}$, such that interest rates on loans and deposits satisfy, $R_t^L = R_t^D = 1$.

Corollary 2 *If $R_t^m = 1$ and $\widetilde{\kappa}_t^B = 1$, the liquidity constraint (10) and the money supply constraint (8) are slack, while deposit demand is satiated.*

Given that monetary policy is irrelevant for the allocation of commodities and for social welfare under $R_t^m = \widetilde{\kappa}_t^B = 1$, we refer to this case without asset purchases as a laissez faire equilibrium.

Definition 2 *A laissez faire equilibrium is a competitive equilibrium under monetary policy satisfying $R_t^m = \widetilde{\kappa}_t^B = 1$ and $\widetilde{\kappa}_t^L = 0$.*

Now consider that the central bank offers purchases of loans, $\widetilde{\kappa}_t^L > 0$. Then, banks would be indifferent between selling loans or not if the central bank offers the market price $1/R_t^A = 1/R_t^L = 1$. If however the central bank offers loan purchases at an above-market-price $1/R_t^A > 1$, banks are willing to sell as much loans as possible and the multiplier κ_t^L on (9) is strictly positive even if $\mu_t = 0 : \kappa_t^L = u'(c_{l,t})(1 - R_t^A) > 0$ (see 27). The potential profits from asset sales are then competed away by reductions of the loan rate R_t^L, which is then less than one (see 25). Thus, asset purchases might be non-neutral even when central bank money is not scarce, $\mu_t = 0$.

When (non-distortionary) taxes/transfers are not available, $\tau_{i,t} = 0$, public sector solvency can nonetheless be guaranteed if the central bank raises revenues. For this, a monetary policy rate above zero, $R_t^m > 1$, is required at least for one period $t = k < T$ (see 22). Notably, asset purchases at a price $1/R_t^A < 1 \Leftrightarrow R_t^A > 1$ is not sufficient for this purpose, since banks would not sell loans at this below-market price if $R_t^m = 1$. Given that money acquisition is not costless in $t = k$ for $R_k^m > 1$, the money supply constraint (10) is binding, $\mu_k > 0$, which follows from (28), and deposit demand is not satiated, $u'(d_{l,k}) > 0 \Rightarrow d_{l,k} < \overline{d}$, which follows from (36).

Corollary 3 *If taxes/transfers are not available, the public sector is solvent (22) if and only if $R_t^m > 1$ for at least one period $t = k$. Then, deposit demand is not satiated in $t = k$.*

A particularly useful case, on which we will focus in Section 4.5, is a utility function with constant marginal utility of deposits. In this case, we can exploit the possibility to separate two subsets of equilibrium objects. Summarizing the competitive equilibrium shows that it is independent of conventional monetary policy, specifically, of the policy rate R_t^m, whereas the asset purchases instruments R_t^A and $\widetilde{\kappa}_t^L$ can affect the allocation of commodities:

Corollary 4 *For $\partial u'(d_{l,t})/\partial d_{l,t} = 0$, a competitive equilibrium is a set of sequences $\{c_{b,t}, c_{l,t}, q_t, h_{b,t}, \zeta_{b,t}, \mu_t, \kappa_t^L, l_{b,t}, r_{t+1}^L = R_t^L \pi_{t+1}^{-1}, r_{t+1}^D = R_t^D \pi_{t+1}^{-1}\}_{t=0}^{T-1}$ and $\{c_{b,T}, c_{l,T}, q_T\}$ satisfying $y_{l,t} + y_{b,t} = c_{b,t} + c_{l,t}$ and (23) for $t \in \{0, T\}$, $h = h_{b,t}$, (5)-(7), (24)-(27) for $t \in \{0, T-1\}$, and $u'(c_{b,T}) q_T = u'(h)$, given policies $\{R_t^A, \widetilde{\kappa}_t^L\}_{t=0}^{T-1}$, fiscal policy $\{\tau_{b,t}\}_{t=0}^T$, and initial values $\pi_0 > 0$, $R_{-1}^L > 0$ and $l_{b,-1} \geq 0$.*

For a constant marginal utility of deposits, which includes the case of satiated deposit demand $u'(d_{l,t}) = 0$, the competitive equilibrium can thus be summarized in a way that is equivalent to a non-monetary version of the model, except of the possibility of loan purchases by the central bank. Specifically, equilibrium objects summarized in Corollary 4 are independent of the policy rate R_t^m, the bond rate R_t, deposits d_t, bonds b_t, and money, m_t, $i_{j,t}^B$, and $i_{j,t}^L$. Notably, determination of the real loan rate in period 0, $R_{-1}^L \pi_0^{-1}$, is based on an initial nominal loan rate R_{-1}^L and the initial inflation rate π_0, which is assumed to be implemented at a level consistent with policy targets. As summarized in the following corollary, initial inflation, the subsequent inflation rates and other monetary equilibrium objects, i.e. for monetary aggregates, bonds, and the bond rate are functions of the monetary instruments $\widetilde{\kappa}_t^B$, Ω_t, and R_t^m. Given that the latter instruments do not affect the allocation of consumption, housing, and loans (see Corollary 4), they can freely be set, for example, in accordance with other policy objectives (see Section 6).

Corollary 5 *Suppose that $\partial u'(d_{l,t})/\partial d_{l,t} = 0$. Given $\{l_{b,t}, \mu_t\}_{t=0}^{T-1}$, $\{c_{l,t}\}_{t=0}^T$, $\{\widetilde{\kappa}_t^L, R_t^A\}_{t=0}^{T-1}$ and $\{\tau_{b,t}\}_{t=0}^T$, the set of sequences $\{d_t, \pi_t, R_t, \kappa_t^B, m_t, b_t, i_t^B, i_t^L\}_{t=0}^{T-1}$ and π_T can be determined by (21), (28)-(34) for $t \in \{0, T-1\}$ and (22), for a monetary policy $\{\widetilde{\kappa}_t^B, \Omega_t, R_t^m\}_{t=0}^{T-1}$ and R_T^m, and fiscal policy $\{\tau_{l,t}\}_{t=0}^T$.*

Notably, the separation of monetary aggregates will be particularly helpful for the analysis of the case where non-distortionary taxes/transfers are not available and the net policy rate exceeds zero, $R_t^m > 1$ (see Corollary 3). In this case, deposit demand will not be satiated, such that social welfare depends on the prevailing level of deposits held by lenders. Asset purchases can alter the loan rate (see 12), on the one hand, and can change the supply of money, on the other hand (see 9). We will show that the former price effect is crucial for the welfare-enhancing role of asset purchases via their impact on financial frictions, whereas the latter, i.e. the money supply effect of asset purchases, might affect deposits and can potentially matter for non-financial targets of the central bank. According to Corollary 4, the asset purchase instruments, R_t^A and $\widetilde{\kappa}_t^L$, are in general relevant for the allocation of commodities. Corollary 5 implies that monetary aggregates, including deposits, are further affected by other monetary policy instruments. In fact, the money supply effects of asset purchases and the effects on deposits can be completely neutralized by adjustments in treasury money supply operations (see Section 6), without affecting the allocation of commodities (see Corollary 4).

4 Efficiency analysis

In this Section, we derive the main results on welfare-enhancing policies in an analytical way. To facilitate this analysis and to provide transparent results, we restrict our attention to three periods and impose some common simplifying assumptions on preferences and endowments (see e.g. Davila and Korinek, 2018), which will hold throughout the analysis unless stated otherwise. We start the efficiency analysis by summarizing some main properties of the first best allocation and the laissez faire equilibrium, which shows that the latter is inefficient due to a potentially binding collateral constraint and a pecuniary externality with regard to the collateral price. We continue by considering a well-established prudential policy, namely an ex-ante Pigouvian tax on debt, and show that it can enhance welfare by addressing the pecuniary externality and can implement a constrained efficient allocation as defined by Stiglitz (1982) and Davila et al. (2012). As the main part of the analysis, we then examine the welfare-enhancing role of ex-post policies, in particular, of central bank loan purchases. For the idealized case where non-distortionary taxes/transfers are available, we show that the latter policy is equivalent to a Pigouvian loan subsidy and can implement a constrained efficient allocation as well as the first best allocation. We then turn to the case where non-distortionary taxes/transfers and thus the Pigouvian policies are not available. As the main novelty, we show that introducing asset purchases can enhance efficiency compared to any competitive equilibrium without asset purchases even when no taxes are available.

4.1 Simplifying assumptions

For the remainder of the analysis, we focus on the common specification with $T = 2$, implying three periods, $t = 0$, 1, and 2. The two types of agents are endowed with different initial debt/wealth levels. There is no uncertainty in the periods 0 and 2. Initial endowment with wealth/debt and income (in terms of non-durables) is restricted to ensure that agents b borrow $0 < l_{b,0} < zq_0h_{b,0}$ in an unconstrained way and that debt can be repaid. In period 1, income is random and can either take the values $y_{b,1}(u)$ and $y_{l,1}(u)$ such that borrowing in period 1 is *u*nconstrained or the values $y_{b,1}(c)$ and $y_{l,1}(c)$ such that $y_{b,1}(c) < y_{l,1}(c)$ and borrowing in period 1 is *c*onstrained. Both states u and c are equally likely. Aggregate income satisfies $y_{b,0} + y_{l,0} = y_{b,2} + y_{l,2} = y = 2$, where $y_{b,0} \leq y_{l,0}$, and might be risky in $t = 1$, $y_{b,1}(u) + y_{l,1}(u) = y_1(u) \geq y_{b,1}(c)+y_{l,1}(c) = y_1(c)$ where $0.5(y_1(u)+y_1(c)) = y$. We introduce type- and time-specific utility functions, and assume that borrowers do not face net taxes/transfers.

Assumption 1 *Agents live for three periods and have preferences satisfying*

$$u_{b,t} = \log(c_{b,t}) + \log(h_{b,t}), \text{ for } t \in (0,1), \text{ and } u_{b,2} = c_{b,2} + \log(h_{b,2}),$$
$$u_{l,t} = c_{l,t} + f(d_{l,t}), \text{ for } t \in (0,1), \text{ and } u_{l,2} = c_{l,2},$$

with $f_d(\overline{d}) = 0$ and $f_d(d_{l,t}) > 0$ if $d_{l,t} > \overline{d}$. *There are no net taxes/transfers on/to borrowers.*

Assumption 2 *Initial net wealth of borrowers* $n_{b,0} = y_{b,0} - R^L_{-1}\pi_0^{-1}l_{b,-1}$ *ensures that* $l_{b,0} \geq 0$ *and that the borrowing constraint is slack in period 1,* $\zeta_{b,0} = 0$. *Borrowers' income in state c in period 1 is small enough that the borrowing constraint is binding under laissez faire.*

The assumptions of a three-period time horizon and linear or quasi-linear utility of borrowers follows common practice in studies on prudential regulation (see Lorenzoni, 2008, Jeanne and Korinek, 2010, 2020, or Davila and Korinek, 2018). Linearity induces irrelevance of the allocation of non-durable goods in the final period for social welfare (35). Linear utility of lenders further implies that the real loan rate r^L_t in a laissez faire equilibrium satisfies

$$E_0 r^L_1 = r^L_2 = 1/\beta, \quad \text{with } r^L_t = R^L_{t-1}\pi_t^{-1}, \tag{37}$$

while it can be lowered via asset purchases (see 25). Under a constant real loan rate (37), there are no distributive effects with regard to debt/savings. For the analysis of distributive effects under asset purchases, we augment the model, in particular, the preferences of lenders (see

Section 5.2). It should further be noted that we will consider non-distortionary taxes/transfers on/to borrowers only as compensations for distortionary tax/subsidies that are introduced to correct prices under Pigouvian policies, such that borrowers do not face net taxes or transfers (see Assumption 1).

4.2 First best and laissez faire

As a reference case, we describe the first best allocation and the associated asset price q in an efficient competitive equilibrium under Assumption 1.

Proposition 1 *Under Assumption 1, the first best allocation satisfies* $d_{l,0} = d_{l,1} = \overline{d}$, *and*

$$c_{i,0}^{fb} = c_{i,1}^{fb}(s) = 1, \tag{38}$$

for $i \in \{b, l\}$ *and* $s \in \{u, c\}$. *This allocation is implemented in a laissez faire equilibrium without borrowing constraints, where the associated asset price* q_1^{fb} *satisfies* $q_1^{fb}(s) = h^{-1}(1 + \beta)$.

Proof. See Appendix. ■

While a laissez faire equilibrium without borrowing constraints leads to the first best allocation, a laissez faire allocation under constrained borrowing (see Assumption 2) is inefficient. Specifically, borrowers' consumption in $t = 0$ is lower than under first best due a potentially binding borrowing constraint leading to precautionary saving. When the borrowing constraint binds in $t = 1$, consumption in $t = 1$ is also lower than under first best as well as the housing price q_1.

Proposition 2 *Under Assumption 1 and 2, a laissez faire allocation satisfies*

$$c_{b,0}^{lf} < 1, \quad c_{b,1}^{lf}(u) = 1, \;\; c_{b,1}^{lf}(c) < 1,$$

and the asset price $q_1^{lf}(u) = q_1^{fb}(s)$ *for* $s \in \{c, u\}$ *and*

$$q_1^{lf}(c) = \frac{(1 + \beta)\, h^{-1}}{(1 - z)(1/c_{b,1}^{lf}(c)) + z} < q_1^{fb}(s). \tag{39}$$

Proof. See Appendix. ■

Under laissez faire, the collateral (housing) price q_1^{lf} relates to borrowers' consumption $c_{b,1}$ under a binding borrowing constraint (see 39), which is not internalized by individual agents. Efficiency can thus be enhanced under higher consumption and a higher collateral price level; the latter enabling agents to increase borrowing. This effect of the pecuniary externality can be addressed by a social planer with corrective policies, which will be subsequently examined. Throughout the remainder of the analysis, Assumptions 1 and 2 hold unless stated otherwise.

4.3 Prudential regulation and constrained efficiency

As a well-established policy intervention in studies on prudential regulation under collateral constraints (see Jeanne and Korinek, 2010, Bianchi, 2011, or Bianchi and Mendoza, 2018), we examine a Pigouvian tax on debt $\tau^d_{b,t}$, which is imposed before the borrowing constraint might be binding. To avoid further effects of these taxes, type-specific tax revenues are rebated in a type-specific and non-distortionary way $\tau_{b,0} = -\tau^d_{b,0} l_{b,0}$. Then, the set of equilibrium conditions is exclusively affected by the corrective tax via the borrowing condition

$$\left(1 - \tau^d_{b,0}\right) c^{-1}_{b,0} = \beta E_0 \left[r^L_1 \cdot c^{-1}_{b,1} \right], \tag{40}$$

which replaces the corresponding optimality condition of borrowers under laissez faire, i.e. $c^{-1}_{b,0} = \beta E_0 [r^L_1 c^{-1}_{b,1}]$. Importantly, the equilibrium relation between the asset price q_1 and borrowers' consumption $c_{b,1}$ under laissez faire (39) is unaffected by the debt tax. It can be shown that an ex-ante tax on debt, $\tau_{b,0} > 0$, can enhance welfare by addressing the pecuniary externality via a reduction of debt $l_{b,0}$ that allows increasing $c_{b,1}$. This intervention can thereby implement a constrained efficient allocation as defined in Stiglitz (1982) or Davila et al. (2012), confirming findings in studies on prudential regulation (see e.g. Jeanne and Korinek, 2010, or Bianchi, 2011).

Proposition 3 *Consider a laissez faire equilibrium. A constrained efficient allocation can be implemented by introducing an ex-ante Pigouvian tax on debt satisfying*

$$\tau_{b,0} = c_{b,0} \beta z h E_0 \left[\mu^{ce}_1 r^L_1 (\partial q_1 / \partial c_{b,1}) \right] > 0, \tag{41}$$

where $\mu^{ce}_1 \geq 0$ denotes the multiplier on the borrowing constraint of the policy problem.

Proof. See Appendix. ■

For the limiting case where the full market value of housing serves as collateral, i.e. $z \to 1$, the asset price q_1 would be equal to q^{fb}_1 even in state $s = c$, i.e. $q_1(c) = (1 + \beta) h^{-1}$, and would then be independent of consumption $c_{b,1}$. Then, the laissez faire equilibrium would be constrained efficient. Yet, this property does not in general imply that other policies, specifically, ex-post policies, cannot enhance welfare even further.

Non-equivalence to a loan-to-value ratio reduction Bianchi (2011) argues that prudential regulation can equivalently be implemented by a planer via margin requirements. Following his notation, this could be specified in our model by choosing a value $\theta_t \in [0, 1)$ such that the collateral constraint changes from (4) to $l_{b,t} \leq (1 - \theta_t) \cdot z q_t h_{b,t}$, which effectively leads to a contingent loan-to-value ratio $\widetilde{z}_t = (1 - \theta_t) z \leq z$. Thus, raising θ_t is indeed able to lower debt. Yet, such a policy measure does not only reduce the maximum level of debt that can be raised, but it also exerts an adverse effect on the collateral price. To see this, recall that borrowers take into account that their own housing serves as collateral. A reduction in the de-facto collaterizable fraction of housing from z to $\widetilde{z}_t$ tends to reduce the collateral premium of housing (see 5). Accordingly, the collateral price tends to decrease with a reduction of the effective loan-to-value ratio $\widetilde{z}_t$ in the same period, as implied by the positive impact of z on the RHS of the laissez faire price relation for the collateral price in period 1 (39). This adverse collateral price effect of lowering the effective loan-to-value ratio does not exist under an ex-ante debt tax, which rather tends to increase the collateral price via a debt reduction.

Corollary 6 *Introducing margin requirements is not equivalent to a Pigouvian tax on debt and, ceteris paribus, reduces the contemporaneous collateral price.*

Notably, the adverse price effect of margin requirements is absent in Bianchi (2011), where the borrowing limit is taken as given by borrowers (since collateral solely consists of aggregate values) and no collateral premium exists. We abstain from a rigorous analysis of margin requirements, for which substantial changes of the model would be necessary. Specifically, a reduction of beginning-of-period debt in $t = 1$ via margin requirements requires the collateral constraint also to be binding in $t = 0$, since changes in the effective loan-to-value ratio would be ineffective otherwise. This would however cause the adverse effects of the pecuniary externality with regard to the collateral price also to be relevant in period 0. These period-0-effects would be accelerated by the price effect of margin requirements within our three period framework. Thus, a comprehensive analysis should allow for state-contingent margin requirements in an analysis of a recursive framework, which is beyond the scope of this paper.

4.4 Ex-post asset purchases

Now consider ex-post asset purchases as a policy that is applied to address inefficiencies under laissez faire (see Definition 2). Specifically, let the central bank purchase loans contingent on the state c where the borrowing constraint is binding, i.e. $\widetilde{\kappa}_1^L(c) > 0$. Then (25) and (27) imply the real loan rate $r_2^L = R_1^L \pi_2^{-1}$ to satisfy

$$\beta r_2^L(c) = 1 - [(1/R_1^A(c)) - 1]\widetilde{\kappa}_1^L(c). \tag{42}$$

Thus, asset purchases lower the real loan rate if the purchase price exceeds the market price, $1/R_1^A(c) > 1$. Evidently, such a policy exerts effects on the loan rate that are equivalent to effects of an ex-post Pigouvian loan supply subsidy paid to banks. Specifically, the effects of asset purchases on the real interest rate can be mimicked by a subsidy at rate $\tau_1^s(c) > 0$ that affects loan supply by $(1 + \tau_1^s(c))\,\beta r_2^L(c) = 1$, and that is financed and compensated by a non-distortionary tax on depositors, $\tau_{l,1}(c) = \tau_1^s(c) l_{j,1}(c)$.

Corollary 7 *Under $R_t^m = \kappa_t^B = 1$, asset purchases satisfying $1/R_1^A(c) > 1$ and $\widetilde{\kappa}_1^L(c) > 0$ and a Pigouvian loan supply subsidy are equivalent with regard to their effects on the real loan rate r_2^L and the allocation of commodities.*

Given that $R_t^m = R_t = 1$, monetary policy does not earn any interest income from money supply or maturing assets. Thus, when assets are purchased at an above market price, transfers to the fiscal authority τ_t^m are negative (see 20) and asset purchases require funds to be raised by the fiscal authority, like in the case of loan supply subsidies. Thus, both policies rely on non-distortionary taxes.

We now show that an asset purchase policy can enhance welfare and can implement a constrained efficient allocation, obviating prudential regulation, i.e. ex-ante debt taxes. To understand the effects of this ex-post policy, combine (5) with (7), to get the following price relation for $s = c$

$$q_1 = \frac{(1+\beta)\,h^{-1}}{(1-z)c_{b,1}^{-1} + z\beta r_2^L}. \tag{43}$$

The reason for the impact of the real loan rate on the asset price q_1 is the fact that borrowers take the collateral premium of housing into account. Thus, a lower loan rate, which raises agents' willingness to borrow and thereby the multiplier $\zeta_{b,t}$ on the collateral constraint (see 6), tends to enhance the valuation of collateral (see 5). In a laissez faire equilibrium, the loan rate is exogenously fixed by $r_2^L = 1/\beta$, such that the collateral price q_1^{lf} does not seem to depend on the loan rate (see 39). Yet, ex-post asset purchases can reduce the real loan rate below $1/\beta$ (see 42) and thereby tend to raise q_1 (see 43). Via this effect, ex-post asset purchases can enhance welfare in a straightforward way by raising the borrowing limit. Moreover, asset purchases can address the pecuniary externality and can implement constrained efficiency, such that the allocation cannot be improved by altering initial period decisions. We demonstrate this properties for the case where the asset purchase instruments R_1^A and $\widetilde{\kappa}_1^L$ are set contingent on an equilibrium object (i.e., $c_{b,1}$), which corresponds to state-contingent adjustments of the monetary policy rate (the so-called Taylor-rule) in many monetary policy studies.

Proposition 4 *Consider a laissez faire equilibrium. Introducing ex-post asset purchases that* *satisfy*

$$[(1/R_1^A(c)) - 1]\widetilde{\kappa}_1^L(c) = \frac{1-z}{z}\left(c_{b,1}^{-1}(c) - 1\right), \tag{44}$$

which requires the asset purchase price to exceed the market price, $1/R_1^A > 1$*, for* $c_{b,1} \neq c_{b,1}^{fb}$*, enhances welfare and implements a constraint efficient allocation.*

Proof. See Appendix. ■

The fact that the allocation under asset purchases is constrained efficient implies that it cannot improved by an additional ex-ante tax on debt.[17] Yet, the collateral constraint can still be binding under an asset purchase policy satisfying (44). In fact, asset purchases can enhance welfare even further. Specifically, a sufficiently large loan rate reduction can raise the housing price q_1 to a level that ensures that the borrowing limit exceeds the debt level under the first best allocation $l_{b,1}^{fb}$ that can be implemented in a competitive equilibrium with $R_t^m = \widetilde{\kappa}_t^B = 1$.

Proposition 5 *Consider a laissez faire equilibrium. Introducing ex-post asset purchases implements the first best allocation (38) if*

$$[(1/R_1^A(c)) - 1]\widetilde{\kappa}_1^L(c) \geq \frac{1}{z} - \frac{1+\beta}{l_{b,1}^{fb}(c)} > 0, \tag{45}$$

where $l_{b,1}^{fb}(c) = R_1^L\pi_1^{-1}(c)\left(R_{-1}\pi_0^{-1}l_{b,-1} - y_{b,0} + 1\right) - y_{b,1}(c) + 1$*, or if but not only if* $\left[(1/R_1^A(c)) - 1\right]\cdot \widetilde{\kappa}_1^L(c) \geq \frac{1}{z} - \frac{1+\beta}{(1-n_{b,0})2\beta^{-1} - y_{b,1}(c)+1} > 0$*, ensuring that the collateral constraint is not binding under the first best allocation.*

Proof. See Appendix. ■

Notably, the RHS of condition (45) depends on the endogenous equilibrium object $l_{b,1}^{fb}(c)$, which is a function of the inflation rates $\pi_1(c)$ and π_0. Up to now, we ignored these inflation rates in the analysis, for convenience. In Section 6, we show that both can be determined in equilibrium and can be isolated from asset purchase effects, in particular, by the remaining monetary policy instruments. In the last part of Proposition 5, we provide – as an alternative – a sufficient condition, which depends on lagged ($n_{b,0}$) or exogenous variables ($y_{b,1}(c)$).

For the limiting case $z \to 1$, where the asset price q_1 satisfies $q_1 = (1+\beta)\,h^{-1}/(\beta R_1^L \pi_2^{-1})$, asset purchases would – in contrast to an ex-ante debt tax – still have a direct impact on q_1 and would be able to enhance welfare relative to laissez faire.

[17] This differs from Jeanne and Korinek's (2020) result on the desirability of prudential policies when ex-post liquidity injections are costly, which will be discussed in Section 5.

On the role of the collateral premium The previous results have shown that loan rate reductions via asset purchases (or loan supply subsidies) can enhance efficiency through their effect on the asset price q_1. This mechanism is based on agents' increased willingness to spend for collateral, measured by the collateral premium, which can be enhanced by raising their incentives to borrow via a loan rate reduction. Yet, if the borrowing limit were independent of the individual stock of collateral, like in Jeanne and Korinek (2020), this effect would not exist. If for example, the borrowing limit rather depends on the aggregate than the individual level of housing, i.e. $l_{b,1} \leq zq_1 h$, there would be no collateral premium. In this case, borrowers' optimality condition for housing satisfies $c_{b,1}^{-1} q_1 = h^{-1} + \beta q_2$, implying – with $q_2 = h^{-1}$ – a collateral price q_1 equal to $q_1 = c_{b,1} h^{-1} (1 + \beta)$, such that it is independent of the real loan rate r_2^L. Changes in r_2^L, induced by ex-post asset purchases (see 42), then solely affect the multiplier $\zeta_{b,1}$ on (4), via the borrowers' optimality condition $\zeta_{b,1} = c_{b,1}^{-1} - \beta r_2^L$, and the allocation of commodities between borrowers and lenders in period 2; the latter being irrelevant for social welfare due to linear utility.

Corollary 8 *Consider an economy where the borrowing limit is independent of the individual housing stock and given by $l_{b,1} \leq zq_1 h$, instead of (4). Then, ex-post asset purchases and Pigouvian loan subsidies neither affect the collateral price nor social welfare.*

A comparison of our set-up to Jeanne and Korinek's (2020) shows that they consider a transfer of funds to borrowers as ex-post policies rather than a pure manipulation of prices, which we consider in our analysis. Thus, the type of ex-post policies which they consider are effective in their model even though there are no collateral premia or direct price effects.

4.5 Non-availability of taxes

We now examine the case where non-distortionary taxes/transfers are not available. We do not endogenize a justification for this assumption, but rather refer to feasibility and efficiency based on unobservable characteristics and/or unmodelled distributional objectives (see e.g. Atkinson and Stiglitz, 1976, Hammond, 1979). Non-existence of non-distortionary taxes/transfers suffices to rule out implementation of first best and of the type of Pigouvian policies (debt tax and loan subsidy) discussed above. In fact, we abstract from introducing alternative taxes, which would tend to reduce efficiency by distorting agents' decisions, and assume that there are no taxes available, such that neither corrective policies can be tax-financed nor initial liabilities can be repaid via tax revenues. As summarized in Corollary 3, public sector solvency then requires that revenues are raised by setting $R_t^m > 1$ for at least one period 0, 1, or 2. For that period, agents are not willing to hold deposits at the satiation level, $d_{l,t} < \overline{d}$. Given the latter, we simplify the analysis by assuming that the marginal utility of deposits is constant.

Assumption 3 *The marginal utility of deposits is constant,* $f'(d_{l,t}) = \gamma > 0$ for $d_{l,t} < \overline{d}$.

Under $R_t^m > 1$ and Assumption 3, the competitive equilibrium conditions (7), (25), (26), and (27) imply that $\mu_t = \widetilde{\gamma} = \gamma/\widetilde{\mu} > 0$ and that the real loan rate under ex-post asset purchases satisfies

$$\beta r_2^L(c) = 1 - \left[(\widetilde{\gamma}/R_1^A(c)) + ((1/R_1^A(c)) - 1)\right] \widetilde{\kappa}_1^L(c), \tag{46}$$

Notably, asset purchases can now be effective even if the central bank offers a purchase price that is below the market price of loans, $1/R_1^A < 1$, because money is scarce, $\mu_1 > 0$, when the policy rate satisfies $R_1^m > 1$ (see 28). Thus, banks might be willing to acquire money via asset purchases even if this is costly, $R_1^A > 1$.

Corollary 9 *Suppose that taxes are not available. Then, asset purchases can be non-neutral even when the purchase price is below the market price of loans,* $1/R_1^A < 1$.

Given that money is costly, $R_t^m > 1$, when taxes are not available (see Corollary 3), the equilibrium level of deposits, is not efficient. Given a constant marginal utility of deposits, we can however separate the competitive equilibrium, as summarized in the Corollaries 4 and 5, such that deposits are independent of asset purchases. It can then easily be shown that asset purchases can implement allocations of commodities that are welfare superior to laissez faire and constrained efficient or even identical with first best, following the same strategy as in the previous section.

Lemma 1 *Consider the laissez faire allocation under Assumption 3 and suppose that taxes are not available. Then, ex-post asset purchases can implement*

1. *a constrained efficient allocation of non-durable goods* $\{c_{b,0}^{ce}, c_{b,1}^{ce}(s)\}$ *if* $R_1^m > 1$ *and*

$$\left[(\widetilde{\gamma}/R_1^A(c)) + (1/R_1^A(c)) - 1\right] \cdot \widetilde{\kappa}_1^L(c) = \frac{1-z}{z}\left(c_{b,1}^{-1}(c) - 1\right), \tag{47}$$

2. *the first best allocation of non-durable goods* $\{c_{b,0}^{fb}, c_{b,1}^{fb}(s)\}$ *if* $R_1^m > 1$ *and*

$$\left[(\widetilde{\gamma}/R_1^A(c)) + (1/R_1^A(c)) - 1\right] \cdot \widetilde{\kappa}_1^L(c) \geq \frac{1}{z} - \frac{1+\beta}{(1-n_{b,0})2\beta^{-1} - y_{b,1}(c) + 1} > 0. \tag{48}$$

Proof. See Appendix. ■

Once it has been shown that asset purchases can enhance efficiency of the commodity allocation (see Lemma 1), we can use that any level of deposits $d_{l,t}$ in a competitive equilibrium without asset purchases can also be implemented in a competitive equilibrium with asset purchases. The reason is that the money supply effect of asset purchases can be completely neutralized by adjusting the size and the price of treasury open market operations. Due to the separation properties summarized in the Corollaries 4 and 5, this neutralization does not affect the allocation of commodities. Thus, the central bank can purchase assets in a desirable way, while keeping a particular level of deposits unchanged. Thereby, it can enhance efficiency compared to any competitive equilibrium without asset purchases.

Proposition 6 *Suppose that taxes are not available. Then, central bank asset purchases can nevertheless enhance welfare compared to any competitive equilibrium without asset purchases.*

Proof. See Appendix. ■

Notably, the Assumption 3 and the implied separability (see Corollary 4 and 5) are helpful for the derivation of the result summarized in Proposition 6. Yet, they in fact not necessary for asset purchases to be able to enhance welfare in an unambiguous way. This property actually relies on the availability of a sufficiently large set of monetary policy instruments, by which the corrective price effects of asset purchases can be isolated (see also Section 6). Put differently, effects of asset purchases on monetary aggregates, in particular, on deposits, can be neutralized by additional monetary policy instruments, which relates to the neutralization of the budgetary effects of distortionary taxes/subsidies via non-distortionary taxes/transfers under Pigouvian policies.

5 Robustness

In this Section, we address factors that might limit the efficacy and the desirability of asset purchases. Specifically, we consider arguments for potential costs of central bank asset purchases or, more generally, ex-post liquidity-providing policy interventions raised by Curdia and Woodford (2011), Gertler and Karadi (2011), Bornstein and Lorenzoni (2018), Jeanne and Korinek (2020), and Chi et al. (2021).[18] To motivate these costs, we found in total six factors that are suggested to lead to distortions or deadweight losses,

1. inferiority of the central bank to extract value from asset holdings,
2. distributive effects between borrowers and lenders,

[18] Jeanne and Korinek (2020) list six alternative interpretations of ex-post liquidity-providing policy interventions, namely, open market purchases, non-distorionary transfers, interest rate cuts, targeted liquidity provision, debt relief, and recapitalizations.

3. raising taxes and/or issuing government debt to finance policy interventions,
4. creating credit and identifying preferred private sector investments,
5. inefficient investments due to interest rates below the natural rate,
6. producing central bank money.

In the subsequent analysis, we will explicitly analyze the factors 1 and 2, which are stressed by Bornstein and Lorenzoni (2018) and Jeanne and Korinek (2020). The reasons for the remaining factors not to be relevant for our analysis can be summarized as follows: We disregard the factor 3, which is considered by Gertler and Karadi (2011) and Jeanne and Korinek (2020), because costs of financing corrective policies in principle apply to all policy interventions and not solely to ex-post asset purchases. In fact, this argument has already been addressed in Section 4.5, where we assumed that non-distortionary taxes/transfers are not available and where asset purchases do not rely on government financing. Factor 4, which is considered by Curdia and Woodford (2011) and Gertler and Karadi (2011), refers to costs of credit creation by the central bank, which do not apply in our framework where the central bank rather purchases already existing debt securities in secondary markets than originates loans. Because neither capital investment nor reductions in the overall level of interest rates are considered in our analysis, inefficiencies related to factor 5, which is mentioned by Jeanne and Korinek (2020), do not exist in our framework. Due to assets' imperfect substitutability, asset purchases rather change the spread between the loan rate and other interest rates (e.g. on bonds or deposits) than the level of interest rates on other assets. This property would be unchanged even when investment possibilities in productive capital were introduced. Finally, we can neglect costs of producing reserves, which are considered by Chi et al. (2021), since our suggested asset purchase programs have been shown to be neutral with regard to the total supply of reserves and deposits.

While we focus here on the analysis of asset purchases, it should be noted that the factors 2, 3 and 5 are in general also relevant for Pigouvian taxes/subsidies, which is most obvious for potential effects from distortionary taxes and transfers that are required for compensation/financing. Moreover, Pigouvian taxes/subsidies would also be associated with effects of endogenous interest rates and distributive effects, when less restrictive assumptions on preferences are imposed, as in the second part of the subsequent analysis.

5.1 Inferior value extraction from central bank asset holdings

When the central bank purchases assets, it supplies money under repurchase agreements against bank loans. As long as these repos are always settled, inferior value extraction from asset holdings is not relevant. Yet, suppose that uncommitted banks do not meet their promise to repurchase assets/loans that were originally purchased by the central bank at the price $1/R_t^A$. In this case, the central bank will hold loans until maturity, while the value of these loans $L_{b,t}^c$ ultimately depends on the central bank's ability to collateralize borrowers' housing.

To account for an inferior ability of the central bank to extract value from their assets in a way that is consistent with our assumption on the underlying imperfection, we assume that the central bank can only seize the fraction z^c of borrowers' housing and that z^c is strictly smaller than the fraction z that banks can seize. Notably, this does not directly affect banks' optimal loan supply decisions based on the collateral requirement (4). To address potential losses due to insufficient collateralization, the central bank can adjust the size of its asset purchase program by offering loan purchases at a haircut $(1 - z^c/z)$, such that the money supply restriction (9) changes to

$$I_{j,t}^L \leq (z^c/z) \cdot \widetilde{\kappa}_t^L \cdot L_{j,t}/R_t^A.$$

To achieve identical effects of central bank interventions compared to the case without inferior value extraction, the central bank can evidently apply a suited haircut and simultaneously raise the purchase price $1/R_t^A$, such that the total impact of asset purchases on the real loan rate is unchanged. This strategy might lead to increased funding requirements or to lower revenues of asset purchase programs, which do however not affect social welfare either because this solely alters the amount of revenues raised by non-distortionary taxes or central bank interest earnings that are transferred to the treasury. Thus, none of the results derived in the previous sections is altered by the inferior value extraction of central bank assets.

Corollary 10 *The ability of ex-post asset purchases to enhance efficiency is not affected by an inferior value extraction from central bank asset holdings, regardless of the availability of taxes.*

A higher purchase price, which the central bank offers to compensate for haircuts, will evidently lead to higher budgetary costs of asset purchase programs. When non-distortionary taxes are available, this effect will not be relevant for any other equilibrium object. When non-distortionary taxes are not available, a higher purchases price can be financed by increased central bank interest earnings by raising the policy rate, which are irrelevant for the allocation of commodities and social welfare in our model (see Corollary 4). Notably, Corollary 10 implies that asset purchases can implement a constrained efficient allocation even under an inferior value extraction from central bank asset holdings and regardless of the availability of taxes. Then, asset purchases obviate prudential regulation. This property is not shared by costly ex-post liquidity injections to borrowers, as considered in Jeanne and Korinek's (2020), which cannot not correct for the pecuniary externality.

5.2 Distributive effects

We now assess if distributive effects might alter the effectiveness and desirability of ex-post interventions. For the previous analysis, we assumed that utility of depositors/lenders is linear in consumption as well as utility of borrowers in the terminal period. As a consequence, distributional effects of changes in the loan rate induced by asset purchases were not relevant for social welfare. Here, we relax the extreme assumption on preferences and assume that utility is always non-linear in consumption and take the same form for lenders/depositors and borrowers. To simplify the analysis, we disregard uncertainty and assume that borrowers will be constrained in period 1 with certainty. This is summarized in the following Assumption, which replaces Assumption 1 and 2.

Assumption 4 *Agents' preferences satisfy*

$$u_{b,t} = \log c_{b,t} + \log(h_{b,t}), \text{ for } t \in (0,1,2),$$
$$u_{l,t} = \log c_{l,t} + f(d_{l,t}), \text{ for } t \in (0,1), \text{ and } u_{l,T} = \log c_{l,T},$$

with $f_d(\overline{d}) = 0$ *and* $f_d(d_{l,t}) > 0$ *if* $d_{l,t} > \overline{d}$. *There are no net taxes/transfers on/to borrowers,* $y_t = y$, *and the borrowing constraint is slack in* $t = 0$ *and binds in* $t = 1$.

As shown in Section 4.4, a reduction in the real loan rate r_2^L due to asset purchases reduces the rate of return on lending and thus the costs of debt repayment. Thus, asset purchases tend to induce a redistribution of resources in period 2 from lenders/depositors to borrowers (see 23). Under linear utility in period 2, this redistribution was irrelevant for social welfare. This irrelevance does however not hold under Assumption 4, which might alter the effectiveness and desirability of asset purchases.

Yet, even if the loan rate r_2^L affects social welfare via redistribution of funds in period 2, asset purchases are desirable. The reason is that borrowers suffer from being constrained in period 1 and are characterized by a higher marginal utility of consumption. The latter gives rise to distributive effects of pecuniary externalities with regard to the lending rate (see Davila and Korinek, 2020), which are based on a binding borrowing constraint. Hence, corrective policies should be applied in a way that addresses both effects of externalities, i.e. collateral effects and distributive effects. An asset purchase policy can in fact be applied to induce the borrowing constraint to be slack, as shown in Proposition 5 and Lemma 1, such the joint source of adverse effects is switched off.

Proposition 7 *Suppose that Assumption 4 holds and that either non-distortionary taxes/transfers are available or that Assumption 3 holds. Then, an optimal asset purchase policy reduces the loan rate such that the collateral constraint is not binding.*

Proof. See Appendix. ■

The result summarized in Proposition 7 implies that an asset purchase policy that leads to slack collateral constraints remains desirable even when distributive effects are considered. Specifically, adverse distributive effects (of the financial friction), i.e. unequal marginal utilities of consumption between borrowers and lenders, are then rather mitigated/eliminated by asset purchases than amplified.

6 Money and inflation

The previous analysis has focussed on the efficiency of the allocation of commodities. To facilitate the analysis, we imposed assumptions which allowed separating the allocation of commodities and loans from monetary variables, specifically, monetary aggregates and inflation rates (see Corollary 4 and 5). In this Section, we focus on the latter. Specifically, we will show that inflation rates are not necessarily affected by asset purchases, implying that monetary policy can be conducted such that asset purchases do not interfere with common central bank targets.

As described above, there is no utility from deposit holdings and no trade in financial markets in period 2. Hence, end of period asset holdings equal zero, $m_2 = b_2 = d_2 = l_2 = 0$. According to (34), treasury open market operations satisfy $I_2^B = -M_1$, such that money is redeemed by the central bank in exchange for bonds, which mature at the end of period 2. As implied by Corollary 5, the set of monetary variables $\{i_1^L, d_0, d_1, R_0, R_1, \pi_0, \pi_1, \pi_2, m_0, m_1, b_0, b_1, i_0^B, i_1^B, i_2^B\}$ can be determined for a given allocation of commodities and loans, and for policies $\{\widetilde{\kappa}_0^B, \widetilde{\kappa}_1^B, \Omega_0, \Omega_1, R_0^m, R_1^m, R_2^m, \tau_{l,1}, \tau_{l,2}, \tau_{l,3}\}$ under Assumption 1 and 2. Hence, the central bank has several instruments at its disposal to influence the inflation rates π_0, π_1, and π_2, and to neutralize potential effects of asset purchases on the inflation rates. Following the structure of the efficiency analysis, we separately discuss the case where non-distortionary taxes are available, such that first best can be implemented and deposit demand is satiated, and the case where taxes are not available and the marginal utility of deposits is constant (see Assumption 3).

In Section 4.4, we have shown how asset purchases can enhance welfare and even implement first best (see Proposition 4 and 5). For the latter, the asset purchase instruments $R_1^A(c)$ and

$\widetilde{\kappa}_1^L(c)$ are assumed to satisfy (45), where the RHS depends – via $l_{b,1}^{fb}(c)$ – on the inflation rates π_0 and $\pi_1(c)$. The following proposition therefore focusses on the determination of the inflation rates π_0, π_1, and π_2 under first best, where deposits are held at the satiation level. We therefore distinguish three different fiscal policy regimes: *i.*) a *Ricardian* regime that unconditionally guarantees public sector solvency, i.e. zero end-of-period public sector liabilities in $t = 2$, *ii.*) a *conditional non-Ricardian* regime where tax revenues net of costs for corrective policies are specified regardless of public sector solvency, and *iii.*) an *unconditional non-Ricardian* regime where gross tax revenues are specified regardless of public sector solvency. In addition to the well-known regimes *i.*) and *iii.*) (see e.g. Benhabib et al., 2001), we introduce the regime *ii.*), to separate financing costs of corrective policies from repayment of liabilities, which equally applies to the costs of Pigouvian subsidies and to the costs of asset purchases $[1/R_1^A) - 1]\widetilde{\kappa}_1^L l_{b,1}$.

Proposition 8 *Suppose that Assumptions 1 and 2 hold and that asset purchases satisfying (45) are introduced in a laissez faire equilibrium, such that first best is implemented. Then, the inflation rate in* $t = 2$ *satisfies* $\pi_2 = \beta$, *while inflation rates in* $t = 0$ *and* $t = 1$ *are i.) indetermined under a Ricardian fiscal policy regime regardless of asset purchases, can ii.) be determined and are independent of asset purchases under a conditional non-Ricardian regime, and can iii.) be determined and depend on the costs of asset purchases under an unconditional non-Ricardian regime.*

Proof. See Appendix. ■

Proposition 8 establishes the determination of the initial inflation rate under asset purchases satisfying (45) with equality, which is the least expensive policy that implements first best. Corresponding to well-established results in the literature (see e.g. Nakajima and Polemarchakis, 2005), inflation determination depends on the type of fiscal policy regime. Specifically, inflation rates cannot be determined under a Ricardian regime (see *i.*)), regardless of asset purchases. Under the conditional non-Ricardian regime, inflation rates can be determined and are independent of corrective policies, i.e. asset purchases. When fiscal policy is unconditionally non-Ricardian (see *iii.*)), the inflation rates can be determined and they depend on the costs of corrective policies. Higher costs of asset purchases, which reduce the net revenues of the public sector, then tend to increase initial inflation, which would likewise be the case under a Pigouvian subsidy.

For the case where taxes are not available, such that fiscal policy is evidently unconditional non-Ricardian, our focus shifts towards the neutralization of asset purchase effects on deposits, which has already been applied in Proposition 6. Here, we additionally establish that there are sufficiently many instruments to further influence the inflation rates independently of asset purchases.

Proposition 9 *Suppose that Assumptions 1-3 hold and that taxes are not available. Then, state-contingent adjustments of the monetary policy instruments* $\widetilde{\kappa}_0^B$, $\widetilde{\kappa}_1^B$, R_0^m, R_1^m, Ω_0, *and* Ω_1 *can implement feasible values for deposits* d_0 *and* d_1 *and for inflation rates* π_0, π_1 *and* π_2 *independently of asset purchase programs.*

Proof. See Appendix. ■

Proposition 9 confirms that asset purchases can be conducted without affecting the equilibrium values of deposits, which has already been used for Proposition 6, and implies that asset purchases do not impede implementing targeted values for the inflation rate. The simple reason is that the central bank has enough instruments related to treasury open market operations, $\widetilde{\kappa}_t^B$, R_t^m, and Ω_t, at its disposal to offset the impact of asset purchases on monetary aggregates and inflation. Thus, the central bank need not sacrifice common objectives, e.g. regarding broad money or inflation, when it corrects prices via asset purchases.

7 Conclusion

Financial stability has been threatened by the great financial crisis as well as by the Covid-19 pandemic, where central banks intervened at a large scale and seemed to exert beneficial effects on asset prices. This raises the question if (ex-post) monetary policy might be superior to (ex-ante) prudential regulation. This paper develops a monetary model with a financial amplification mechanism based on pecuniary externalities, where prudential regulation implements a constrained efficient allocation. We show that central bank asset purchases in secondary markets can also implement a constrained efficient allocation by addressing adverse effects of pecuniary externalities, obviating prudential regulation, and can even implement first best. While these effects can equivalently be induced by a Pigouvian loan supply subsidy, we show that the central bank can conduct welfare-enhancing asset purchases even when taxes are not available for financing/compensating policy interventions. This property is based on the central bank's ability to raise revenues via interest earnings from money supply and asset holdings. We further show that asset purchases do not impede achieving other (conventional) central bank targets/objectives, and that neither distributive effects nor inferior value extraction from central bank asset holdings limit the efficacy of asset purchases.

8 References

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Appendix

Proof of Proposition 1. Corollary 1 implies that $c_{b,0}^{fb} = c_{b,1}^{fb} = c_{b,1}^{fb}(s) = c_{l,1}^{fb}(s) = 1$ hold under Assumption 1, and that $d_{l,0} = d_{l,1} = \overline{d}$. In a laissez faire equilibrium, $\mu_{j,t} = 0$ holds (see Definition 2). For unrestricted borrowing, $\zeta_{b,t} = 0$, the competitive equilibrium conditions (5), (6), and (25) for $t = 0, 1$, and 2 simplify to $c_{b,1}^{-1} q_1(s) = h^{-1} + \beta q_2$, $q_2 = h^{-1}$, $c_{b,0}^{-1} = \beta E_0[r_1^L(s) c_{b,1}^{-1}(s)]$, $c_{b,1}^{-1}(s) = \beta r_2^L$, and (37), implying that $c_{b,0}^{-1} = 1$, $c_{b,1}^{-1}(s) = 1$ and $q_1(s) = (1+\beta)h^{-1}$. ■

Proof of Proposition 2. In a laissez faire equilibrium (see Definition 2) under Assumption 1 and 2, borrowers' credit demand (6) and banks' credit supply (25) for $t = 2$ and $t = 3$ satisfy (37), $c_{b,0}^{-1} = \beta E_0[r_1^L(s) c_{b,1}^{-1}(s)]$, $c_{b,1}^{-1}(s) = \beta r_2^L + \zeta_{b,1}(s)$, and therefore $c_{b,1}^{-1}(u) = 1$ and $c_{b,1}^{-1}(c) = 1 + \zeta_{b,1}(c) > 1$, since $\zeta_{b,1}(c) > 0$. Substituting out $c_{b,1}^{-1}(u)$ and $c_{b,1}^{-1}(c)$ in $c_{b,0}^{-1} = \beta 0.5[r_1^L(u) \cdot c_{b,1}^{-1}(u) + r_1^L(c) \cdot c_{b,1}^{-1}(c)]$, shows that $c_{b,0}^{-1} = \beta 0.5 \left[r_1^L(u) \cdot 1 + r_1^L(c) \cdot \left(1 + \zeta_{b,1}(c)\right)\right] = \beta E r_1^L + \beta 0.5 r_1^L(c) \zeta_{b,1}(c) > 1$ and thus $c_{b,0} < 1$. The borrowers' optimality condition for housing (5) further implies $c_{b,1}^{-1}(s) q_1(s) = h^{-1} + \beta q_2 + \zeta_{b,1}(s) z q_1(s)$ and $q_2 = h^{-1}$, such that $q_1(u) = c_{b,1}(u)\,(1+\beta)\,h^{-1}$ and $q_1(c) = (1+\beta)\,h^{-1}/(c_{b,1}^{-1}(c) - \zeta_{b,1}(s) z)$. Substituting out $\zeta_{b,1}(c)$ with $c_{b,1}^{-1}(c) = 1 + \zeta_{b,1}(c)$ in the latter, gives $q_1(c) = (1+\beta)\,h^{-1}/[(1-z) c_{b,1}^{-1}(c) + z]$, which is strictly smaller than $q_1^{fb}(s) = h^{-1}\,(1+\beta)$ since $c_{b,1}(c) < 1$. ■

Proof of Proposition 3. Under laissez faire, one can rewrite social welfare W (see 35) by using $h_{b,t} = h$, the lenders' budget constraints, and that agents are satiated with deposits, $d_{l,t} = \overline{d}$:

$$W = E\left\{\begin{array}{c} \log c_{b,0} + \log(h) + \left(y_{l,0} + r_0^L l_{b,-1} - l_{b,0}\right) + f(\overline{d}) \\ +\beta\left[\log c_{b,1} + \log(h) + y_{l,1} - l_{b,1} + r_1^L l_{b,0} + f(\overline{d})\right] + \beta^2\left[y_{b,2} + \log(h) + y_{l,2} + f(\overline{d})\right] \end{array}\right\}.$$

The primal problem of a policy maker who applies an ex-ante tax on debt $\tau_{b,0}$ and a compensating non-distortionary transfer is identical to the problem of a social planer who determines period-0-borrowing and maximizes social welfare W subject to budget and borrowing constraints taking the equilibrium price relations $E_0 r_1^L = 1/\beta$ and (39) into account, leading to a constrained efficient allocation. It can be summarized as

$$\begin{aligned} \max_{c_{b,1},c_{b,2},l_{b,1},l_{b,2}} \quad & E\{\log c_{b,0} + \log(h) + \left(y_{l,0} + r_0^L l_{b,-1} - l_{b,0}\right) + f(\overline{d}) \qquad (49) \\ & + \beta\left[\log c_{b,1} + \log(h) + y_{l,1} - l_{b,1} + r_1^L l_{b,0} + f(\overline{d})\right] + \beta^2\left[y_{b,2} + \log(h) + y_{l,2} + f(\overline{d})\right]\} \\ \text{s.t.} \quad & 0 = y_{b,0} + l_{b,0} - r_0^L l_{b,-1} - c_{b,0},\ 0 = y_{b,1} + l_{b,1} - r_1^L l_{b,0} - c_{b,1},\ 0 \leq z q_1 h - l_{b,1}, \end{aligned}$$

where q_1 satisfies $q_1^{lf}(u) = q_1^{fb}$ or (39), leading to the optimality conditions $\lambda_{b,0}^{ce} = 1/c_{b,0}$,

$$\lambda_{b,1}^{ce} = (1/c_{b,1}) + \mu_1^{ce} z h \partial q_1/\partial c_{b,1}, \qquad (50)$$

$$1 = \beta E_0 r_1^L + \lambda_{b,0}^{ce} - \beta E_0\left(r_1^L \lambda_{b,1}^{ce}\right), \qquad (51)$$

$$\mu_1^{ce} = \beta(\lambda_{b,1}^{ce} - 1) \geq 0, \qquad (52)$$

where $\lambda_{b,0}^{ce}$, $\lambda_{b,1}^{ce}$, and μ_1^{ce} are the multipliers for the constraints in order of their appearance in (49). Using $E_0 r_1^L = 1/\beta$, condition (51) simplifies to $\lambda_{b,0}^{ce} = \beta E_0 r_1^L \lambda_{b,1}^{ce}$. Multiplying (50) with βr_1^L, applying expectations conditional on period-0-information $E_0 \beta r_1^L \lambda_{b,1}^{ce} = E_0\left(\beta r_1^L / c_{b,1}\right) + E_0[\beta r_1^L \mu_1^{ce} z h(\partial q_1/\partial c_{b,1})]$, and using $1/c_{b,0} = \lambda_{b,0}^{ce} = \beta E_0 r_1^L \lambda_{b,1}^{ce}$ as well as (40), gives

$$1/c_{b,1} = (1 - \tau_{b,0})\, c_{b,1}^{-1} + \beta E_0[r_1^L \mu_1^{ce} z h(\partial q_1/\partial c_{b,1})],$$

leading to the condition (41) for the tax rate on debt, where the positive sign follows from $\partial q_1(c)/\partial c_{b,1} > 0$ and $\mu_1^{ce}(c) > 0$ (see 39 and 52). ■

Proof of Proposition 4. Consider the price relation (43) and $s = c$. The impact of consumption $c_{b,1}$ on q_1, which is not internalized by private agents, can be offset if r_2^L ensures that $\partial q_1/\partial c_{b,1} = 0$. For this, suppose that asset purchases satisfy (44). Combining the latter with

(42), gives $\beta r_2^L = 1 + \frac{1-z}{z}(1 - c_{b,1}^{-1})$ and substituting out βr_2^L in (43), leads to

$$q_1 = \frac{(1+\beta)\,h^{-1}}{(1-z)c_{b,1}^{-1} + z(1 + \frac{1-z}{z}(1 - c_{b,1}^{-1}))} = (1+\beta)\,h^{-1}$$

Given that $q_1 = q_1^{fb} > q_1^{lf}$ (see Proposition 1 and 2), welfare is enhanced by an increase in the borrowing limit. To show that the allocation is then constrained efficient, we set up the following problem of a social planer who determines period-1-borrowing and maximizes social welfare W subject to budget and borrowing constraints taking the equilibrium price relations $E_0 r_1^L = 1/\beta$ and $q_1 = (1+\beta)\,h^{-1}$ into account:

$$\begin{aligned}\max_{c_{b,0},c_{b,1},l_{b,0},l_{b,1}} \quad & E\{\log c_{b,0} + \log(h) + \left(y_{l,0} + r_0^L l_{b,-1} - l_{b,0}\right) + (1+\beta+\beta^2) f(\overline{d}) + \\ & \beta\left[\log c_{b,1} + \log(h) + y_{l,1} - l_{b,1} + r_1^L l_{b,0}\right] + \beta^2\left[y_{b,2} + \log(h) + y_{l,2}\right]\} \\ \text{s.t.} \quad & 0 = y_{b,0} + l_{b,0} - r_0^L l_{b,-1} - c_{b,0},\ 0 = y_{b,1} + l_{b,1} - r_1^L l_{b,0} - c_{b,1},\ 0 \le z\,(1+\beta) - l_{b,1},\end{aligned} \tag{53}$$

leading to the optimality conditions $\lambda_{b,0}^{ce} = 1/c_{b,0}$, $\lambda_{b,1}^{ce} = (1/c_{b,1})$, $\lambda_{b,0}^{ce} = \beta E_0 r_1^L \lambda_{b,1}^{ce}$, and $\mu_1^{ce} = \beta(\lambda_{b,1}^{ce} - 1) \ge 0$, where $\lambda_{b,0}^{ce}$, $\lambda_{b,1}^{ce}$, and μ_1^{ce} are the multipliers for the constraints in order of their appearance in (53). Substituting out $\lambda_{b,0}^{ce}$ and $\lambda_{b,1}^{ce}$, leads to $1/c_{b,0} = \beta E_0\left(r_1^L/c_{b,1}\right)$, which equals the optimality condition for borrowing under laissez faire in period 0 and thus establishes that the allocation under (44) is constrained efficient, i.e. cannot be improved by changes in period-0-decisions. ■

Proof of Proposition 5. Borrowing under first best satisfies in $t = 1$

$$l_{b,1}^{fb}(s) = r_1^L(s)\left(r_0^L l_{b,-1} - y_{b,0} + 1\right) - y_{b,1}(s) + 1, \tag{54}$$

where we used $y_{b,0} = -l_{b,0} + r_0^L l_{b,-1} + c_{b,0}$, $y_{b,1} = -l_{b,1} + r_1^L l_{b,0} + c_{b,1}$, and (38). Since borrowing is unconstrained under first best, implementation of a first best allocation in a competitive equilibrium requires the asset price q_1 to satisfy $q_1(c) \ge l_{b,1}^{fb}(c)/(zh)$. Using that $q_1(c)$ satisfies (43) in a competitive equilibrium, this requirement can be re-rewritten as

$$\frac{z\,(1+\beta)}{(1-z) + z\beta r_2^L(c)} \ge l_{b,1}^{fb}(c), \tag{55}$$

where we used that $c_{b,1}^{fb} = 1$. Using (42), the inequality (55) can be ensured to hold by asset purchases satisfying (45). Alternatively, asset purchases can be conditioned on lagged or exogenous variables. For this, use (54) for $s = c$, and add $r_1^L(u)\left(r_0^L l_{b,-1} - y_{b,0} + 1\right) = r_1^L(u) l_{b,0}^{fb} > 0$ on the

RHS of (54), to get

$$l_{b,1}^{fb}(c) < \left(r_0^L l_{b,-1} - y_{b,0} + 1\right)\left(r_1^L(c) + r_1^L(u)\right) - y_{b,1}(c) + 1$$
$$= (1 - n_{b,0})\, 2\beta^{-1} - y_{b,1}(c) + 1$$

where $n_{b,0} = y_{b,0} - r_0^L l_{b,-1}$ and we used $E_0 r_1^L = 0.5\left(r_1^L(c) + r_1^L(u)\right) = 1/\beta$. The borrowing constraint will therefore not be binding if but not only if the real loan rate satisfies $\frac{z(1+\beta)}{(1-z)+z\beta r_2^L(c)} \geq (1 - n_{b,0})\, 2\beta^{-1} - y_{b,1}(c) + 1 \Leftrightarrow$

$$\beta r_2^L(c) \leq \frac{(1+\beta)}{(1 - n_{b,0})\, 2\beta^{-1} - y_{b,1}(c) + 1} - \frac{1-z}{z}.$$

Using (42), this inequality can be ensured by conducting asset purchases, i.e. by setting the instruments $1/R_1^A$ and $\widetilde{\kappa}_1^L$, according to $\left[(1/R_1^A(c)) - 1\right]\widetilde{\kappa}_1^L(c) \geq \frac{1}{z} - \frac{1+\beta}{(1-n_{b,0})2\beta^{-1} - y_{b,1}(c)+1} > 0$. ■

Proof of Lemma 1. When taxes are not available, the policy rate has to satisfy $R_t^m > 1$ for at least $t = 0$, 1, or 2 (see Corollary 3). Suppose that $R_1^m > 1$, such that (28) and (36) imply $u'(d_{l,1}) > 0$ with $u'(d_{l,1}) = \gamma$ under Assumption 3. Given that the marginal utility of deposits is constant, consumption, debt, and the asset price do not depend on deposits and on monetary policy instruments, except of asset purchases, as summarized in Corollary 4. Exactly like in the case of satiated deposit demand (see Proposition 4), asset purchases can then implement a constrained efficient allocation of non-durable goods by ensuring that the real loan rate for $s = c$ satisfies $\beta r_2^L = 1 + \frac{1-z}{z}(1 - c_{b,1}^{-1})$. From (46), it follows that this requires asset purchases under $R_1^m > 1$ to satisfy (47). Correspondingly, asset purchases can implement the first best allocation of non-durable goods by ensuring that the real loan rate for $s = c$ satisfies $\beta r_2^L \leq \frac{1+\beta}{(1-n_{b,0})2\beta^{-1} - y_{b,1}+1} - \frac{1-z}{z}$. From (46), it follows that the latter inequality requires asset purchases to satisfy (48). ■

Proof of Proposition 6. Under Assumption 3, the policy rate has to satisfy $R_t^m > 1$ in at least one period t (see Corollary 3), such that (28) and (36) imply that the liquidity constraint (33) is binding $\mu_t > 0$ and with (36) that $u'(d_{l,t}) > 0$, where $\mu_t = \widetilde{\gamma} = \gamma/\widetilde{\mu}$. Now suppose that the central bank sets R_1^m and R_1^A according to $R_1^m - 1 < \widetilde{\gamma}$ and $R_1^A - 1 < \widetilde{\gamma}$. Then, (27) and (28) imply $\kappa_1^L > 0$ and $\kappa_1^B > 0$, such that (31) and (32) are binding and the liquidity constraint (33) can be written as

$$\widetilde{\mu} d_1 = \widetilde{\kappa}_1^B \left(b_0 \pi_1^{-1}/R_1^m\right) + \widetilde{\kappa}_1^L \left(l_1/R_1^A\right) + m_0 \pi_1^{-1}. \quad (56)$$

Condition (56) determines d_1 for a set of equilibrium values for π_1, b_0, l_1, m_0, and monetary policy instruments R_1^A, $\widetilde{\kappa}_1^L$, R_1^m and $\widetilde{\kappa}_1^B$, where the last two instruments are irrelevant for the allocation of non-durables and housing (see Corollary 4). Since $u'(d_{l,1}) = \gamma$, social welfare W can be separated as $W = W_{c,h} + W_d$, where $W_{c,h} = \log c_{b,0} + c_{l,0} + \beta E[\log c_{b,1} + c_{l,1}] + \beta^2 y + (1+\beta+\beta^2)\log(h)$ and $W_d = \gamma d_{l,0} + \beta E \gamma d_{l,1}$. Now use that asset purchases can enhance efficiency of the non-durable goods allocation according to Lemma 1 and can thus yield a higher value $W_{c,h}$ compared to a competitive equilibrium without asset purchases, $\widetilde{\kappa}_1^L = 0$. According to (56), the impact of $\widetilde{\kappa}_1^L(l_1/R_1^A)$ on the equilibrium value for deposit d_1 can be offset by adjustments of $\widetilde{\kappa}_1^B$ and R_1^m such that W_d is unaffected by asset purchases and social welfare W is strictly larger than without asset purchases. ■

Proof of Proposition 7. Under Assumption 4, the following optimality conditions hold

$$c_{b,0}^{-1} = \beta r_1^L c_{b,1}^{-1}, \tag{57}$$

$$c_{l,0}^{-1} = \beta r_1^L c_{l,1}^{-1}, \tag{58}$$

$$c_{l,1}^{-1} = \beta r_2^L c_{l,2}^{-1} + \kappa_1^L \widetilde{\kappa}_1^L / R_1^A, \tag{59}$$

$$\kappa_1^L = \mu_1 - c_{l,1}^{-1}(R_1^A - 1), \tag{60}$$

$$c_{b,1}^{-1} = \beta r_2^L c_{b,2}^{-1} + \zeta_{b,1}, \tag{61}$$

$$c_{b,1}^{-1} q_1 = u'(h) + \beta c_{b,2}^{-1} q_2 + \{\zeta_{b,1} z q_1\}. \tag{62}$$

Substituting out κ_1^L and $\zeta_{b,1}$ with (60) and (61) in (59) and (62), leads to

$$c_{l,1}^{-1} = \beta r_2^L c_{l,2}^{-1} + \left(u'(d_{l,1})\,\widetilde{\mu}^{-1} - c_{l,1}^{-1}(R_1^A - 1)\right) \widetilde{\kappa}_1^L / R_1^A, \tag{63}$$

$$q_1 = \frac{(1+\beta)u'(h)}{c_{b,1}^{-1}(1-z) + z\beta r_2^L c_{b,2}^{-1}}, \tag{64}$$

where we used $u'(d_{l,t})\,\widetilde{\mu}^{-1} = \mu_1$. Combining (57) and (58), further gives $c_{l,0}^{-1}/c_{l,1}^{-1} = c_{b,0}^{-1}/c_{b,1}^{-1}$. Using $c_{l,t} = y - c_{b,t}$, we get $\frac{y - c_{b,1}}{y - c_{b,0}} = \frac{c_{b,1}}{c_{b,0}}$ and thus

$$c_{b,0} = c_{b,1} \text{ and } \beta^{-1} = r_1^L. \tag{65}$$

If non-distortionary taxes/transfers are available a welfare-maximizing policy would implement the satiation level of deposits $\overline{d}$. If not and Assumption 3 holds, any equilibrium level of deposits can be implemented regardless of asset purchases (see Corollary 5). In either case, the allocation of deposits and housing is independent of asset purchases, such that an optimal asset purchase policy can be identified by maximizing $W_c = \sum_{t=0}^{2} \beta^t (u_{b,t} + u_{l,t})$. Using (65) and that condition (63) is the single equilibrium condition that includes the asset purchase instruments $(\widetilde{\kappa}_1^L, R_1^A)$ and does not impose a constraint to a policy problem, the latter can be summarized as

$$\max_{c_{b,1},c_{b,2},l_{b,0},l_{b,1},r_2^L} \{(1+\beta)[\log c_{b,1} + \log(y - c_{b,1})] + \beta^2 [\log(c_{b,2}) + \log(y - c_{b,2})]\} \tag{66}$$
$$\text{s.t. } 0 = y_{b,0} + l_{b,0} - r_0^L l_{b,-1} - c_{b,1},\; 0 = y_{b,1} + l_{b,1} - \beta^{-1} l_{b,0} - c_{b,1},\; 0 = y_{b,2} - r_2^L l_{b,1} - c_{b,2},$$
$$0 \leq z q_1(c_{b,1}, c_{b,2}, r_2^L) h - l_{b,1},$$

where $q_1(c_{b,1}, c_{b,2}, r_2^L)$ satisfies (64). The first order conditions with respect to $c_{b,1}$, $c_{b,2}$, $l_{b,0}$, $l_{b,1}$ and r_2^L are

$$\lambda_{b,0}^{\text{lg}} = (1+\beta)\left(c_{b,1}^{-1} - (y - c_{b,1})^{-1}\right) - \beta \lambda_{b,1}^{\text{lg}} + \mu_1^{\text{lg}} z h \partial q_1 / \partial c_{b,1}, \tag{67}$$
$$\lambda_{b,2}^{\text{lg}} = c_{b,2}^{-1} - (y - c_{b,2})^{-1} + \beta^{-2} \mu_1^{\text{lg}} z h \partial q_1 / \partial c_{b,2}, \tag{68}$$
$$\lambda_{b,0}^{\text{lg}} = \lambda_{b,1}^{\text{lg}}, \qquad \lambda_{b,1}^{\text{lg}} = \beta \lambda_{b,2}^{\text{lg}} r_2^L + \mu_1^{\text{lg}}, \tag{69}$$
$$\lambda_{b,2}^{\text{lg}} l_{b,1} = \beta^{-2} \mu_1^{\text{lg}} z h \partial q_1 / \partial r_2^L, \tag{70}$$

where $\lambda_{b,0}^{\text{lg}}$, $\lambda_{b,1}^{\text{lg}}$, $\lambda_{b,2}^{\text{lg}}$, and μ_1^{lg} are the multipliers for the constraints in order of their appearance in (66). Since $\partial q_1 / \partial r_2^L < 0$ (see 64) and $\lambda_{b,2}^{\text{lg}}$ as well as μ_1^{lg} are non-negative, condition (70) requires, $\lambda_{b,2}^{\text{lg}} = \mu_1^{\text{lg}} = 0$. Thus, the real loan rate has to be lowered by asset purchases such that the collateral constraint is not binding for the social planer, which is feasible as shown in Proposition 5 and Lemma 1. Then, the conditions in (69) imply $\lambda_{b,1}^{\text{lg}} = \lambda_{b,0}^{\text{lg}} = 0$, such that (67) and (68) require the marginal utilities of consumption of borrowers and lenders are equated. Accordingly, the multiplier $\zeta_{b,1}$ of the borrowers' optimization problem satisfies $\zeta_{b,1} = (u'(d_{l,1})\widetilde{\mu}^{-1} - c_{l,1}^{-1}(R_1^A - 1))\widetilde{\kappa}_1^L / R_1^A$ (see 61 and 63). ■

Proof of Proposition 8. When asset purchases satisfying (45) are introduced in a laissez faire equilibrium, where $R_t^m = \widetilde{\kappa}_t^B = 1$, $\mu = 0$, and $d_1 = d_2 = \overline{d}$, first best is implemented (see Proposition 5). Then, the money supply constraints (31) and (32), and the liquidity constraint (33) are slack, such that i_0^B, i_1^B, m_0 and m_1 are indetermined. According to (25) and (30), the loan rates R_0^L and R_1^L and the inflation rates π_1 and π_2 then satisfy $R_0^L = 1$, $R_1^L(c) = 1 - (1/R_1^A(c) - 1)\widetilde{\kappa}_1^L(c)$, $R_1^L(u) = 1$, $1/\beta = E_0 \pi_1^{-1} \Leftrightarrow$

$$1/\beta = 0.5(\pi_1^{-1}(c) + \pi_1^{-1}(u)), \tag{71}$$

and $\pi_2 = \beta$. Using the terminal conditions, $m_2 = b_2 = 0$, absence of asset purchases in $t = 0$

and $t=2$, $i_0^L = i_2^L = 0$, and that $R_t^m = 1$ and $\widetilde{\kappa}_t^B = 1$ imply $R_t = 1$ (see 18), the public sector budget constraints (21) for $t = 0, 1$, and 2 reduce to

$$b_0 + \tau_{l,0} + \left(m_0 - m_{-1}\pi_0^{-1}\right) = b_{-1}\pi_0^{-1},$$
$$b_1 + \tau_{l,1} + \left(m_1 - m_0\pi_1^{-1}\right) + \left(R_1^A - 1\right) i_1^L = b_0\pi_1^{-1},$$
$$\tau_{l,2} + \left(-m_1\pi_2^{-1}\right) = b_1\pi_2^{-1},$$

which can – by substituting out b_0 and b_1 – be integrated to get

$$(m_{-1} + b_{-1})\,\pi_0^{-1}\pi_1^{-1} = \tau_{l,0}\pi_1^{-1} + \tau_{l,1} + \tau_{l,2}\beta - \left((1/R_1^A) - 1\right)\widetilde{\kappa}_1^L l_{b,1}, \tag{72}$$

where we used that (32) is binding, $i_1^L = \widetilde{\kappa}_1^L l_{b,1}/R_1^A$, under (45). Given that (72) holds for all states s and that asset purchases in $t = 1$ are state contingent, we apply (72) separately for both states $s = c$ and $s = u$. For $s = c$, we suppose that (45) holds with equality. We further use that loans under first best in $s = c$ satisfy $l_{b,1}^{fb}(c) = R_0^L\pi_1^{-1}(c)\left(R_{-1}^L\pi_0^{-1}l_{b,-1} - y_{b,0} + 1\right) + 1 - y_{b,1}(c)$ and $R_0^L = 1$, to rewrite (72) for $s = c$ as follows

$$\pi_1^{-1}(c) = \frac{\tau_{l,1}(c) + \tau_{l,2}\beta + (1+\beta) - \frac{1}{z}\left(1 - y_{b,1}(c)\right)}{(m_{-1} + b_{-1})\,\pi_0^{-1} + \left[\frac{1}{z}\left(R_{-1}^L\pi_0^{-1}l_{b,-1} - y_{b,0} + 1\right)\right] - \tau_{l,0}}. \tag{73}$$

Likewise, we apply (72) for $s = u$, and rewrite it as follows

$$\pi_1^{-1}(u) = \frac{\tau_{l,1}(u) + \tau_{l,2}\beta}{(m_{-1} + b_{-1})\,\pi_0^{-1} - \tau_{l,0}}, \tag{74}$$

where we used $i_1^L(u) = 0$. We now consider three fiscal policy regimes: Under a Ricardian regime, where zero public sector liabilities $m_2 = b_2 = 0$ are unconditionally guaranteed, (72) does not constitute an additional restriction, such that π_0, $\pi_1(c)$ and $\pi_1(u)$ cannot be determined. On the contrary, (72) imposes a restriction on the inflation rates if the fiscal policy regime is non-Ricardian, i.e. tax revenues do not guarantee zero public sector liabilities in period 2. Under a conditional non-Ricardian regime, taxes are adjusted to cover asset purchase costs $\left((1/R_1^A) - 1\right)\widetilde{\kappa}_1^L l_{b,1}$. Let taxes $\tau_{l,0}$, $\widetilde{\tau_{l,1}}$ and $\tau_{l,2}$ be non-Ricardian, where $\widetilde{\tau_{l,1}}$ are period-1-taxes $\tau_{l,1}$ net of asset purchase costs. Then, (72) and (73) can be written as $(m_{-1} + b_{-1})\,\pi_0^{-1}\pi_1^{-1} = \tau_{l,0}\pi_1^{-1} + \widetilde{\tau_{l,1}} + \tau_{l,2}\beta$ and

$$\pi_1^{-1}(c) = \frac{\widetilde{\tau_{l,1}}(c) + \tau_{l,2}\beta}{(m_{-1} + b_{-1})\,\pi_0^{-1} - \tau_{l,0}}. \tag{75}$$

Using (71), the inflation rates π_0, $\pi_1(c)$ and $\pi_1(u)$ can be then determined by

$$\pi_0 = \frac{m_{-1} + b_{-1}}{\tau_{l,0} + 0.5\beta\left(\widetilde{\tau_{l,1}}(c) + \tau_{l,1}(u)\right) + \beta^2\tau_{l,2}},$$

(74) and (75), and are independent of asset purchases. Under an unconditional non-Ricardian regime, taxes are not adjusted to cover asset purchase costs. Using (71), the inflation rates π_0, $\pi_1(c)$ and $\pi_1(u)$ can then be determined by

$$2\beta^{-1} = \frac{\tau_{l,1}(u) + \tau_{l,2}\beta}{(m_{-1} + b_{-1})\,\pi_0^{-1} - \tau_{l,0}} + \frac{\tau_{l,1}(c) + \tau_{l,2}\beta + (1+\beta) + \frac{1}{z}\left(1 - y_{b,1}(c)\right)}{\left(m_{-1} + b_{-1} + \frac{1}{z}R^L_{-1}l_{b,-1}\right)\pi_0^{-1} + \frac{1}{z}\left(1 - y_{b,0}\right) - \tau_{l,0}},$$

(73) and (74), and depend on the costs of asset purchases. ■

Proof of Proposition 9. Suppose that taxes are not available and Assumption 3 holds. Under Assumption 1, (30) implies $\pi_2 = \beta$. Suppose further that $\widetilde{\kappa}_2^B = 0$, $R_2^m = 1$, $R_0^m \in (1, 1+\widetilde{\gamma})$ and $R_1^m \in (1, 1+\widetilde{\gamma})$, such that $\mu_0 = \mu_1 = \widetilde{\gamma} > 0$, $R_1 = 1$ (see 29), and $\mu_2 = 0$ hold (see 7 and 26) and that the money supply constraint (31) binds in $t=0$ and $t=1$ (see 28). Likewise, suppose that $R_1^A < 1+\widetilde{\gamma}$, such that (32) binds in $t=1$ (see 27). Then, substitute out i_0^B and i_1^B with (34) in the binding money supply constraint (31) and in the binding liquidity constraint (33), to get for $t=0$ and $t=1$

$$\widetilde{\kappa}_0^B b_{-1}\pi_0^{-1}/R_0^m = (1+\Omega_0)m_0 - m_{-1}\pi_0^{-1}, \quad \widetilde{\kappa}_1^B b_0\pi_1^{-1}/R_1^m = (1+\Omega_1)m_1 - m_0\pi_1^{-1}, \qquad (76)$$

$$\widetilde{\mu}d_0 = (1+\Omega_0)\,m_0, \quad \widetilde{\mu}d_1 = (1+\Omega_1)\,m_1 + i_1^L. \qquad (77)$$

Without taxes, the consolidated public sector budget constraint (21) reduces to $(b_t/R_t) + R_t^m(m_t - m_{t-1}\pi_t^{-1}) + (R_t^m - 1)\,\Omega_t m_t + \left(R_t^A - 1\right)i_t^L = b_{t-1}\pi_t^{-1}$. Substituting out b_0 and b_1 with the latter for $t=1$ and $t=2$, $b_0\pi_1^{-1} = (b_1/R_1) + R_1^m\left(m_1 - m_0\pi_1^{-1}\right) + (R_1^m - 1)\,\Omega_1 m_1 + \left(R_t^A - 1\right)i_t^L$ and $-m_1R_2^m = b_1$, leads to the following versions of the integrated public sector budget constraint and the money supply condition (76) for $t=1$

$$b_{-1}\pi_0^{-1} + R_0^m m_{-1}\pi_0^{-1} = \left(R_0^m - (R_1^m/R_0) + (R_0^m - 1)\,\Omega_0\right)m_0 + (R_1^m - 1)\,(1+\Omega_1)m_1\,(\pi_1/R_0) + \left(R_1^A - 1\right)i_1^L\,(\pi_1/R_0), \qquad (78)$$

$$m_0\pi_1^{-1}\left(1 - \widetilde{\kappa}_1^B R_1^m\right) = (1+\Omega_1)\left(1 - \widetilde{\kappa}_1^B\,(R_1^m - 1)\right)m_1 + \widetilde{\kappa}_1^B\,(1/R_1^m - 1)\,i_t^L. \qquad (79)$$

Now use $i_1^L = \widetilde{\kappa}_1^L l_{b,1}/R_1^A$ and the binding liquidity constraints (77) to substitute out i_1^L, m_0 and m_1 in (78), (79), and in the money supply condition (76) for $t=0$. Defining the bond price $q_0 = 1/R_0$ and the bond price function $q_0(R_1^m, \widetilde{\kappa}_1^B, \pi_1) = 1/\left(1+\widetilde{\gamma}\right) + \beta E_0(\widetilde{\gamma} - (1 - 1/R_1^m))\widetilde{\kappa}_1^B\pi_1^{-1}$, we get a system of four equations which determine the set $\{\pi_0,\, \pi_1,\, d_0,\, d_1\}$

$$
\begin{aligned}
b_{-1}\pi_0^{-1} + R_0^m m_{-1}\pi_0^{-1} = & \left(R_0^m - (R_1^m q_0(R_1^m, \widetilde{\kappa}_1^B, \pi_1)) + (R_0^m - 1)\,\Omega_0\right) \frac{\widetilde{\mu}}{1+\Omega_0} d_0 \qquad (80)\\
& + (R_1^m - 1)\left(\widetilde{\mu} d_1 - (1+\Omega_1)\left\{\frac{\widetilde{\kappa}_1^L l_{b,1}}{R_1^A}\right\}\right)\left(\pi_1 q_0(R_1^m, \widetilde{\kappa}_1^B, \pi_1)\right)\\
& + \left\{\left(1 - \frac{1}{R_1^A}\right)\widetilde{\kappa}_1^L l_{b,1}\right\} \cdot \left(\pi_1 q_0(R_1^m, \widetilde{\kappa}_1^B, \pi_1)\right),\\
\frac{\widetilde{\mu}}{1+\Omega_0} d_0 \pi_1^{-1}\left(1 - \widetilde{\kappa}_1^B R_1^m\right) = & \left(1 - \widetilde{\kappa}_1^B (R_1^m - 1)\right)\left(\widetilde{\mu} d_1 - (1+\Omega_1)\left\{\frac{\widetilde{\kappa}_1^L l_{b,1}}{R_1^A}\right\}\right) \qquad (81)\\
& + \widetilde{\kappa}_1^B (1/R_1^m - 1) \cdot \left\{\frac{\widetilde{\kappa}_1^L l_{b,1}}{R_1^A}\right\},\\
\widetilde{\kappa}_0^B b_{-1}\pi_0^{-1}/R_0^m = & \widetilde{\mu} d_0 - m_{-1}\pi_0^{-1}, \qquad (82)\\
[(1+\widetilde{\gamma})\,\beta]^{-1} = & E_0 \pi_1^{-1}, \qquad (83)
\end{aligned}
$$

given a set of monetary policies instruments $\{R_0^m,\, R_1^m,\, \Omega_0,\, \Omega_1,\, \widetilde{\kappa}_1^L,\, R_1^A,\, \widetilde{\kappa}_0^B,\, \widetilde{\kappa}_1^B\}$ and an equilibrium value for loans $l_{b,1}$. Suppose that the central bank targets specific values for deposits, $d_0 = \widehat{d_0}$ and $d_1 = \widehat{d_1}$, and for the inflation rates, $\pi_0 = \widehat{\pi}_0$ and $\pi_1 = \widehat{\pi}_1$ within the range of feasible values, satisfying (83), and implements the targeted values in a competitive equilibrium without asset purchases ($\widetilde{\kappa}_1^L = 0$) via its six instruments $\widetilde{\kappa}_0^B \in [0,1]$, $\widetilde{\kappa}_1^B \in [0,1]$, $R_0^m \in (1, 1+\widetilde{\gamma})$, $R_1^m \in (1, 1+\widetilde{\gamma})$, $\Omega_0 \geq 0$, and $\Omega_1 \geq 0$. To ensure that these values are realized, the central bank can adjust the six instruments to neutralize the effects of asset purchases on (80)-(82). Specifically, the asset purchase instruments enter equation (80) twice via $1/R_1^A$ and – jointly with loans – via $\widetilde{\kappa}_1^L l_{b,1}$, and further alter equation (81) via the term $\widetilde{\kappa}_1^L l_{b,1}/R_1^A$. These effects can be neutralized by changes in R_0^m, R_1^m, Ω_0, Ω_1, and $\widetilde{\kappa}_1^B$. Implied effects of changes in R_0^m on equation (82) can further be neutralized by adjusting $\widetilde{\kappa}_0^B$. ■