Debt, Default, and Commitment*

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Abstract

This paper extends Eaton and Gersovitz (1981)'s model of sovereign debt with incomplete markets and equilibrium default to explore the role of commitment for debt, default and welfare. While the government makes an ex-ante optimal state-contingent plan for its future actions, including debt repayment, it will re-optimize its plan ex post with an exogenously given probability, nesting the standard Markov case without commitment and the full-commitment Ramsey case if the probability is one and zero, respectively. If and to what extend the government commits to default in some states is found to depend on preferences, default costs and the degree of commitment. Model versions with full (or high intermediate) commitment are shown to have some advantages relative to the usually studied no-commitment case, which is illustrated for a quantitative application to the recent European debt crisis.

Keywords: Sovereign Debt, Default, Incomplete Markets, Commitment, Discretion

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1 Introduction

In the last few years a large literature has evolved that uses quantitative versions of Eaton and Gersovitz (1981)'s incomplete markets model of sovereign debt with equilibrium default to study questions in international macroeconomics (see e.g. Aguiar and Gopinath, 2006; Arellano, 2008) and perform policy experiments (see e.g. Anzoategui, 2016; Hatchondo et al., 2017b; Bocola and Dovis, 2018).[1] In the basic model, the government of a small open economy can sell non-state contingent one-period bonds to foreign investors at a price that reflects its risk of default. Two key assumptions in the literature are the government's ability to refuse debt repayment and its inability to commit to future actions. Without commitment, the presence of direct costs of default ensures that the government has an incentive to repay its debt in some states of the world, such that positive debt can be sustained at positive risk of default.[2] Under incomplete markets, the no-commitment assumption is however not necessary to generate equilibrium default since even a fully-committed (Ramsey) policy maker might want to make use of costly default in some states (see Adam and Grill, 2017). This raises the question: How does the policy maker's degree of commitment affect the positive and welfare predictions of models in the spirit of Eaton and Gersovitz (1981)?

In this paper, I will address this question by extending an otherwise standard model of sovereign debt and default to allow for "loose commitment" (see Debortoli and Nunes, 2010).[3] While the government can make a state-contingent plan for its future actions, including debt repayment, it will re-optimize ex post with an exogenously given probability that is used as a measure of the government's degree of commitment (or credibility) in this paper. If the probability is one, the model reduces to the well-known standard Markov case without any commitment (see Eaton and Gersovitz, 1981; Arellano, 2008). If the re-optimization probability is equal to zero, the government acts like a Ramsey planner in the optimal policy literature (see Chari and Kehoe, 1999). It therefore chooses an ex-ante optimal plan, which specifies debt issuance and repayment for all future contingencies, and is fully committed to implement its prescriptions even when it is not optimal to do so ex post (see Adam and Grill, 2017).[4] Nesting the two polar commitment cases as well as intermediate ones, the model framework allows to isolate the role of commitment for debt, default and welfare in a transparent way.

Clarifying the role of commitment in the context of sovereign debt and default matters for a number of reasons. First, it is not clear how a government's degree of commitment affects the positive predictions of the canonical sovereign default model. More specifically, whether complete lack of commitment is necessary to generate many of the features highlighted in the literature, like countercyclical interest rates and capital outflows, is not known. Second, the welfare implications of many policy measures that attempt to change borrowing and default incentives crucially depend on the

[1] See Aguiar et al. (2016) for a survey of this literature. There is also a closely related literature that uses versions of the model to study unsecured consumer credit and private bankruptcy (see e.g. Chatterjee et al., 2007; Livshits et al., 2007). Livshits (2015) provides a recent summary of this literature.

[2] There is also a literature that studies unsecured debt in models with incentive-compatibility constraints (see Kehoe and Levine, 1993) or endogenous debt limits (see Zhang, 1997) that rule out equilibrium default.

[3] In the literature, loose commitment is also referred to as "stochastic replanning" (Roberds, 1987), "quasi-commitment" (Schaumberg and Tambalotti, 2007) and "imperfect credibility" (Bodenstein et al., 2012). The underlying concept is however the same.

[4] In the language of game theory: The full-commitment case considers open-loop strategies, whereas the Markov case involves closed-loop strategies.

government's degree of commitment. If, for instance, a third party can increase the cost of default, doing so could be beneficial if the government does not have any commitment but would reduce welfare if it is fully committed to its promises.[5] Clarifying the role of commitment is therefore relevant for many policy-related applications of the canonical default model. Third, in the literature, the costs associated with lack of commitment are usually computed by comparing welfare of a Markov economy to that of an identical economy with a government that cannot default by assumption (see e.g. Schmitt-Grohé and Uribe, 2017). Such a calculation will however only provide a lower bound for the actual welfare costs of lack of commitment since a fully-committed policy maker might want to default as well. By nesting varying degrees of commitment, the model studied in this paper allows to directly quantify the welfare gains of commitment in a correct way. Fourth, the loose-commitment framework enables me to study how a policy maker optimally responds to the possibility of not being able to keep his promises in the context of debt repayment, which introduces novel trade-offs not present in the full- and no-commitment cases.

When the model is specified and calibrated as is common in the literature, i.e. to match certain short- and long-run features of a typical emerging economy under the no-commitment assumption, the government would never default under full commitment since this would interfere with its strong desire for front-loading consumption and low degree of risk aversion. If the government exhibits however a higher degree of risk aversion, which hardly affects the quantitative predictions of the no-commitment model, the insurance role of default gains in importance and equilibrium default occurs. Indeed, if the government is sufficiently risk averse, it may even default more often under commitment than under discretion. As in the well-studied no-commitment case (see Arellano, 2008), the full-commitment case features default in low-income states and countercyclical default risk. Under commitment, borrowing is however countercyclical, which reflects the government's ability to effectively smooth consumption across states. As a result, budget surpluses are procyclical and consumption is less volatile than income, which is in contrast to the no-commitment case that predicts countercyclical surpluses and excess consumption volatility (see Arellano, 2008).[6] The reason for these different outcomes is that, unlike a government without commitment, a government with full commitment does not face a bond price schedule that is responsive to debt issuance and current income because risk of default is not determined by future default incentives in this case. Instead, by committing to default (or repay) in specific future states, the default probability (and therefore the bond price) can be chosen directly and jointly with the borrowing position. In equilibrium, the government will however choose to borrow and default in a way that results in a positive comovement between debt and default risk.

Under loose commitment, i.e. for intermediate degrees of commitment, the government faces additional trade-offs relative to the two polar commitment cases. If income is persistent, there now are in principle two additional and opposing motives for making default promises. On the one hand, the government might want to promise repayment in bad states to improve current borrowing conditions which are adversely affected by the possibility of re-optimized default. On the other

[5] Another example is the introduction of a non-defaultable debt instrument (see Hatchondo et al., 2017a) or a third party credit (Fink and Scholl, 2016), like a loan from the IMF, which would be inferior borrowing instruments from the perspective of a Ramsey planner (unless they increase an exogenously imposed debt limit) but could be beneficial for a Markov economy.

[6] Under commitment, when the economy is very impatient but also very risk averse, surpluses can become almost acyclical and consumption can become almost as volatile as output. Borrowing will however remain countercyclical.

hand, it might want to promise to not repay in states in which a default will take place under re-optimization to avoid situations where the promise to repay has to be kept even though income is low and debt is high. Being trapped in such a situation can be very costly since rolling over maturing debt is prohibitively expensive due to high risk of re-optimized default, requiring large reductions in consumption to honor debt payments. When the model is calibrated as is usual in the literature, i.e. for a low degree of risk aversion and without any commitment, the first motive dominates and default almost exclusively occurs under re-optimization regardless of the size of the re-optimization probability. For high degrees of commitment, the loose-commitment government then effectively behaves like a government that cannot choose to default but defaults with an exogenous probability. When considering a higher degree of risk aversion, the second motive becomes relevant for some degrees of commitment as well. Whereas average debt is found to increase with the degree of commitment, there is a hump-shaped relationship between commitment and the default probability. The cyclical properties of a loose-commitment economy can be described as a mixture of the two polar commitment cases. Qualitatively, it however only behaves similar to a commitment economy for very high degrees of commitment. Otherwise, the commitment problem dominates the behavior of the government. The main welfare finding of this paper is that the welfare gains of commitment mostly accrue at high degrees of commitment, such that a no-commitment economy would hardly benefit from obtaining an intermediate degree of commitment.

Are the predictions of the full-commitment model (as well as the loose-commitment model with a high degree of commitment) empirically plausible? Just like the no-commitment case, the full-commitment case predicts countercyclical interest rate spreads. This feature is consistent with empirical evidence (see Neumeyer and Perri, 2005; Uribe and Yue, 2006) and an important requirement for any serious model of sovereign debt and default. What about the remaining model predictions? In this case, the answer is not as obvious. In models à la Eaton and Gersovitz (1981), the fiscal (budget) surplus coincides with the trade surplus because only the government borrows from abroad. Usually, papers in the default literature interpret the surplus as the trade surplus, most likely because standard no-commitment models generate countercyclical net exports, which is consistent with empirical observations. In the data, the fiscal surplus is however not countercyclical but procyclical in developed economies (as well as emerging economies in East Asia) and acyclical in Latin American economies (see Gavin and Perotti, 1997; Lane, 2003; Kaminsky et al., 2004). The model will therefore fail according to one of the two possible interpretations regardless of the assumed degree of commitment. Another feature of no-commitment models à la Arellano (2008) highlighted in the literature is their ability to generate excess consumption volatility, which has been documented for many emerging economies (see Aguiar and Gopinath, 2007) and - at least in an endowment economy - requires countercyclical surpluses. For developed economies, excess consumption volatility is however not observed, consistent with the commitment model version. Does this mean that the no-commitment model outperforms the alternative commitment cases for applications to emerging economies along this dimension? The answer is again not as obvious as one might think at first. Alvarez-Parra et al. (2013) show that excess volatility of aggregate consumption in emerging economies is driven by highly volatile durable consumption. Non-durable consumption by contrast is not more volatile than output in emerging but also in developed economies. Since the default literature only considers non-durable consumption, it is therefore fair to ask whether excess consumption volatility is actually a desirable model prediction.

Another way to judge the plausibility of the different commitment cases is to compare their predictions for the cyclical behavior of public debt to the empirical evidence. Judging the different model versions based on this criterion implies that the full- and loose-commitment cases are empirically plausible due to their potential to generate countercyclical debt, which is observed for developed economies but also for several emerging economies (see e.g. Ottonello and Perez, 2019). By contrast, the no-commitment model fails according to this criterion as it predicts strongly procyclical public debt, which is needed to generate a countercyclical trade surplus. The full-commitment model can therefore not only successfully replicate the cyclical behavior of the fiscal surplus but also that of public debt, whereas the no-commitment model can only replicate the observed cyclicality of the trade surplus.[7]

Given that the different commitment cases have advantages and disadvantages, one might ultimately want to select the model version that provides the best fit for the application of interest. For applications to developed economies with non-negligible sovereign risk, full commitment (or a high intermediate degree of commitment) might be the more appropriate modeling assumption. The recent European debt crisis provides an example of such an application. In this context, Bocola et al. (2019) point out the difficulty that the standard no-commitment model has in generating the countercyclical borrowing behavior observed for crisis countries in the Euro area. To address this issue, they propose a model calibration that relies on the inclusion of a minimum consumption level (or non-discretionary government spending), which effectively increases the economy's aversion to very low income states. This property makes the government's borrowing behavior less responsive to debt-elastic interest rates and thereby allows the no-commitment model to jointly generate countercyclical debt and default risk. This paper shows that the authors' recalibration strategy effectively forces the no-commitment economy to behave like a full-commitment one naturally does. Using the same calibration strategy employed by the authors, I show that the full-commitment model version provides a better fit to their targeted moments, especially regarding the cyclicality of debt.

Related Literature This paper contributes to two strands of literature. First, it contributes to the quantitative literature on sovereign debt with incomplete markets and equilibrium default (see Aguiar and Amador, 2014; Aguiar et al., 2016). As mentioned in the introduction, this literature usually assumes that the borrower does not have any commitment. A notable exception is the paper by Adam and Grill (2017) who were the first to highlight that a fully-committed Ramsey planner can find it optimal to make use of costly default as a policy instrument.[8] While the present paper builds on this insight and therefore shares some similarities with this study, there are a number of crucial differences. Most importantly, the model studied in this paper allows for different degrees of commitment, nesting the full-commitment as well as the standard no-commitment case within one framework. Indeed, for the special without any commitment, my model economy is identical to the one studied in Aguiar and Gopinath (2006) or Arellano (2008). This property allows me to directly isolate the role of commitment for the predictions of the canonical default model and calculate the

[7] A further argument in favor of the fiscal-surplus interpretation is that the trade-surplus interpretation is already somewhat difficult to defend based on the observation that current account dynamics are not entirely driven by changes in public debt.

[8] For a closed economy model, Pouzo and Presno (2016) consider a policy maker who can commit to future taxes but not to future debt repayment.

welfare gains of commitment. By contrast, Adam and Grill (2017) consider the special case of full commitment for a small open economy that has a very distinct structure relative to the canonical sovereign default model à la Arellano (2008), making it difficult to relate their findings to previous studies. For instance, in their full-commitment model, default is treated as a continuous decision that is associated with a temporary resource loss which is assumed to be proportional to the default rate and the stock of debt. This modeling choice makes the use of default quite similar to that of inflation in models of optimal policy with nominal public debt (see e.g. Chari and Kehoe, 1999; Schmitt-Grohé and Uribe, 2004). The canonical default model treats default however as a binary decision that results in temporary exclusion from financial markets and a resource loss that depends on current output (see Aguiar and Amador, 2014). Since these costs of default are known to be crucial for many of the qualitative and quantitative predictions in the sovereign default literature, deviating from the conventional specification is not innocuous for the results.[9] Another assumption made by Adam and Grill (2017), which is in contrast to the default literature, is to model the small open economy and foreign investors as equally patient. In this paper, I will consider both, a patient as well as an impatient economy, and illustrate the resulting differences. In their paper, Adam and Grill (2017) argue that a random disaster state, i.e. a large and persistent but infrequent output decline, is needed to generate equilibrium default under commitment. The quantitative analysis in this paper demonstrates that this is not the case when considering the economic environment studied in Aguiar and Gopinath (2006) or Arellano (2008), regardless of whether the model is calibrated to a volatile emerging economy or a more stable developed one. Lastly, although Adam and Grill (2017) perform some stylized quantitative exercises, the focus of their paper is on normative and analytical results, whereas I focus on positive and quantitative ones. Indeed, while Adam and Grill (2017) do not report first or second moments for simulations of their model, this paper provides details about the cyclical (and long-run) properties of relevant model variables for the optimal policy under full, loose and no commitment.[10]

As already mentioned above, within the quantitative sovereign default literature, this paper is also related to Bocola et al. (2019) who recognize that the standard no-commitment model cannot generate countercyclical borrowing as observed for crisis countries in the Euro area and argue that the inclusion of a minimum consumption level helps to address this issue. In this paper, I show that a full-commitment model naturally gives rise to the observed properties even without such positive subsistence consumption. This paper also relates to recent studies in the quantitative default literature that consider political uncertainty, such as Cuadra and Sapriza (2008), Hatchondo et al. (2009), Scholl (2017) and Chatterjee and Eyigungor (2019). These papers model exogenous or endogenous turnover between policy makers with different objectives, resulting in the implementation of non-benevolent policies. In contrast to these studies, this paper considers uncertainty about a policy maker's ability to keep his promises as the only source of political uncertainty.

[9] In the full-commitment economy studied in Adam and Grill (2017), a government without commitment would either always or never default on any positive amount of debt, depending on the default cost parametrization, which prevents a fruitful quantitative and welfare comparison of the two extreme commitment cases.

[10] Work in progress by Mateos-Planas and Rios-Rull (2016) also briefly considers the case of full commitment in the context of a simplified model of unsecured debt and equilibrium default. However, the authors only consider this case to obtain first-order conditions that those derived for a no-commitment model can be compared to. Importantly, they (i) do not propose a unifying model framework that allows for varying degrees of commitment, and (ii) do not explore the qualitative and quantitative implications that different degrees of commitment, including the two extreme cases, have for the predictions of models à la Eaton and Gersovitz (1981) and Arellano (2008). These are the two main contributions of this paper.

Second, this paper is related to recent studies that allow for loose commitment in models of optimal public policy. Building on the pioneering work of Roberds (1987), Schaumberg and Tambalotti (2007) introduce intermediate degrees of commitment into a linear-quadratic framework of optimal monetary policy. By contrast, Debortoli and Nunes (2010) develop a more general approach for models of optimal policy, labeled ”loose commitment” by the authors, that is based on the recursive Lagrangian-based method of Marcet and Marimon (2011), and apply it to study the relationship between commitment and capital taxes. As in this paper, the authors find that the loose-commitment case involves additional trade-offs relative to the cases with full and without commitment. Furthermore, the authors show that sizable welfare gains of commitment only accrue for high degrees of commitment, which is consistent with the findings of this paper as well. Other recent applications with loose commitment include Bodenstein et al. (2012), Debortoli and Nunes (2013), Debortoli and Lakdawala (2016), Du et al. (2017) and Davis et al. (2018).

Layout The remainder of the paper is organized as follows. Section 2 presents the model. Section 3 offers a quantitative model analysis. Section 4 concludes.

2 Model

The economic environment is as in Aguiar and Gopinath (2006) or Arellano (2008). Time is discrete and indexed by $t \in \{0, 1, 2, ..., \infty\}$. The model features a small open economy and a continuum of risk-neutral foreign investors. The benevolent government of the small open economy can trade non-state contingent one-period bonds with investors to smooth and/or front-load consumption of a risk-averse representative household who inhabits the economy. Importantly, the government can choose whether to repay its debt or not, which is taken into account by its creditors who charge an interest rate that reflects the risk of default. When the government invests in a bond, it is assumed to be risk free.

2.1 Environment

The domestic household has preferences over consumption $\{c_t\}_{t=0}^{\infty}$ given by $\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_t)\right]$, with period utility function $u(c) = (c^{1-\gamma} - 1)/(1-\gamma)$, $0 < \gamma \neq 1$, and discount factor $\beta \in (0,1)$.[11] In each period, the economy receives a random income y_t which follows a stationary first-order Markov process with conditional transition probabilities $\pi(y_{t+1}|y_t)$ and finite support $\mathbb{Y} = \{y_1, ..., y_Y\}$. The income values are assumed to satisfy $y_1 < y_2 < ... < y_Y$. The economy faces the minimum consumption level $\underline{c} \geq 0$, which can be interpreted as subsistence consumption (see Adam and Grill, 2017) or exogenous government spending (see Bianchi et al., 2018; Bocola et al., 2019). To smooth and/or front-load consumption of the domestic household, the government can trade a non-state contingent one-period bond b_t on financial markets at the unit price q_t. The set of feasible bond positions is given as $\mathbb{B} \equiv [\underline{b}, \overline{b}]$, with $-\infty < \underline{b} \leq 0$ and $0 < \overline{b} \leq (y_1 - \underline{c})/r$, where r is the constant real risk-free rate on financial markets. Since the focus of this paper is on debt and default, a positive b-value will

[11]For the quantitative analysis, I will allow for iso-elastic recursive preferences to distinguish between household attitudes towards time and risk (see Section 3.1), nesting the additively separable specification above as a special case.

denote a debt position, whereas a negative one implies a savings position. As in Arellano (2008), the government can choose to default on debt payments, leading to temporary exclusion from financial markets, during which the economy faces an income loss $\phi(y_t) \geq 0$ and is neither able to borrow nor save. Conditional on being in financial autarky at the end of a period, the government regains access to financial markets in the subsequent period with probability θ.

Bonds are traded with risk-neutral investors who can borrow or save on financial markets at the risk-free rate r. Risk neutrality and expected profit maximization imply that a bond issued by the government in period t is priced according to $q_t = \mathbb{E}_t[1 - d_{t+1}]/(1+r)$, reflecting investors' rational expectations of future default $d_{t+1} \in \{0,1\}$, with $d_{t+1} = 1$ denoting default and $d_{t+1} = 0$ debt repayment. As is common in the literature, in each period, investors are assumed to act after the government's policies are determined, such that the policy maker can internalize the impact of his actions on the bond price.

2.2 Full Commitment

Before moving to the policy problem under loose commitment, it is helpful to start with the ex-ante optimal Ramsey policy plan. In sequential notation, the decision problem of the government under full commitment is given as

$$\max_{\{b_t, c_t, d_t, h_t, q_t\}_{t=0}^{\infty}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_t) \right]$$

subject to

$$\begin{aligned}
c_t &= y_t - \underline{c} - h_t \phi(y_t) + (1-h_t)(q_t b_t - b_{t-1}), && (1)\\
h_t &= (d_t \xi_t + 1 - \xi_t) h_{t-1} + d_t (1 - h_{t-1}), && (2)\\
q_t &= \frac{1 - \mathbb{E}_t[d_{t+1}]}{1+r}, && (3)\\
b_t &\in \mathbb{B}, \quad c_t \geq \underline{c}, \quad d_t \in \{0,1\} && (4)\\
0 &= b_t h_t, \quad 0 = d_t(1-\xi_t) h_{t-1}, \quad 0 = q_t h_t, && (5)
\end{aligned}$$

given initial values $b_{-1} \in \mathbb{B}$ and $h_{-1} \in \{0,1\}$.

The indicator variable $h_t \in \{0,1\}$ denotes the credit status of the government at the end of period t. If it is equal to zero, the government has access to financial markets, whereas it is excluded for $h_t = 1$. The random variable $\xi_t \in \{0,1\}$ denotes whether the economy regains access to financial markets in period t ($\xi_t = 1$) or not ($\xi_t = 0$). Constraint (1) is the economy's budget constraint, (2) is the law of motion for the government's credit status, (3) is the bond pricing condition and (4) lists restrictions for debt, consumption and default choices. The three constraints listed in (5) ensure that no debt is issued in periods of autarky, that the government can only default if it has access to financial markets and that the bond price equals zero in periods of autarky, respectively.

Recursive Formulation Since the bond price condition (3) features investors' expectations of future government actions, the policy maker faces a time-inconsistency problem for $t \geq 1$ and the decision problem under full commitment cannot be formulated recursively by only using the natural state variables $(h_{t-1}, b_{t-1}, y_t, \xi_t)$ (see Kydland and Prescott, 1977). As in Kydland and Prescott (1980), or more recently Farhi (2010), I therefore decompose the policy problem into two subproblems, one for $t = 0$ and one for $t \geq 1$, and augment the natural state space to obtain a recursive formulation for the decision problem from $t = 1$ onward.[12] Inspection of the bond pricing condition (3) reveals that it is sufficient to add the default promise $d \in \{0, 1\}$ as a (co-)state variable to recursify the government problem for $t \geq 1$.

Conditional on having a good credit status, default or repayment takes place according to the promise d, with $d = 0$ denoting debt repayment and $d = 1$ default. The associated value of commitment $\mathcal{V}^c(\cdot)$, which serves as a promise-keeping constraint, is given as

$$\mathcal{V}^c(b, d, y) = (1 - d)\,\mathcal{V}^r(b, y) + d\mathcal{V}^d(y), \tag{6}$$

with $\mathcal{V}^r(\cdot)$ denoting the value of repayment and $\mathcal{V}^d(\cdot)$ the value of default.

In the repayment case, the government solves

$$\mathcal{V}^r(b, y) = \max_{b' \in \mathbb{B},\, \mathbf{d}_{y'} \in \{0,1\}^Y} \left\{ u\left(y + q\left(\mathbf{d}_{y'}, y\right) b' - b - \underline{c}\right) + \beta \mathbb{E}_{y'|y}\left[\mathcal{V}^c(b', d_{y'}, y')\right] \right\}, \tag{7}$$

where $\mathbf{d}_y = (d_{y_1}, d_{y_2}, ..., d_{y_Y}) \in \{0, 1\}^Y$ is the vector of state-contingent default promises.

Conditional on debt repayment, the government chooses next period's debt position b' and makes promises about whether it is going to default in income state y', as given by $d_{y'}$. The bond price schedule is given as

$$q\left(\mathbf{d}_{y'}, y\right) = \frac{1 - \mathbb{E}_{y'|y}\left[d_{y'}\right]}{1 + r}. \tag{8}$$

In the default case, the government solves

$$\mathcal{V}^d(y) = \max_{\mathbf{d}_{y'} \in \{0,1\}^Y} \left\{ \begin{array}{c} u\left(y - \phi(y) - \underline{c}\right) \\ + (1 - \theta)\,\beta \mathbb{E}_{y'|y}\left[\mathcal{V}^d(y')\right] \\ + \theta \beta \mathbb{E}_{y'|y}\left[\mathcal{V}^c(0, d_{y'}, y')\right] \end{array} \right\}, \tag{9}$$

where θ is the probability that the government leaves financial autarky in the next period. Note that in the default (and autarky) case, the government can only promise to repay or default conditional on regaining access to financial markets, consistent with the second constraint in (5).[13]

For periods $t \geq 1$, a recursive equilibrium for the Ramsey problem is defined as follows:

Definition 1 *A recursive Ramsey equilibrium consists of value and policy functions* $\mathcal{V}^c : \mathbb{B} \times \{0, 1\} \times \mathbb{Y} \to \mathbb{R}$, $\{\mathcal{V}^r, \mathcal{B}^r, \mathbf{d}^r_{y'}\} : \mathbb{B} \times \mathbb{Y} \to \mathbb{R} \times \mathbb{B} \times \{0, 1\}^Y$*, and* $\{\mathcal{V}^d, \mathbf{d}^d_{y'}\} : \mathbb{Y} \to \mathbb{R} \times \{0, 1\}^Y$*, that satisfy conditions (6), (7), and (9), as well as bond price function* $q : \{0, 1\}^Y \times \mathbb{Y} \to [0, 1/(1 + r)]$ *that satisfies condition (8).*

In period $t = 0$, the government does not face a time-inconsistency problem because its actions do not affect outcomes in previous periods. When having access to financial markets in the initial period,

[12] Due to the discrete nature of default, the Lagrangian-based approach of Marcet and Marimon (2011), which is usually employed in the optimal policy literature, cannot be employed in this paper.

[13] In equilibrium, the government will not make promises to default when it is in autarky.

i.e. $h_{-1} = 0$ or $h_{-1} = 1 \wedge \xi_0 = 1$, the government then solves

$$\mathcal{V}^o(b,y) = \max_{d \in \{0,1\}} \mathcal{V}^c(b,d,y), \tag{10}$$

with $(b,y) = (b_{-1}, y_0)$ and $\mathcal{V}^o(\cdot)$ denoting the option value of default. In the initial period, the default decision is a jump variable as would be the case in every period under discretion. When the government is excluded from financial markets in the initial period, i.e. $h_{-1} = 1 \wedge \xi_0 = 0$, no default choice is made for $t = 0$ and the government's value is given by (9) with $y = y_0$.

2.3 Loose Commitment

Now suppose that the government can promise a state-contingent default policy for the future but, as in Schaumberg and Tambalotti (2007) or Debortoli and Nunes (2010), at the start of a period, may renege on promises made in the past with constant probability $1 - \lambda \in [0,1]$ and choose whether to default or repay in a discretionary way.[14] At the beginning of a period, promises made in the past are hence kept with certainty only with probability λ. Conditional on ignoring past commitments, the government can immediately announce a new state-contingent policy plan for the future.

Before the default and borrowing decisions are carried out in a given period, a re-optimization shock is realized, which determines whether the government is committed to implement past promises. If this is the case, default or repayment takes place according to the promise d as in the Ramsey case. If, however, the government is not committed to keep its promises, debt repayment is determined in a discretionary way, as given by (10). Note that the re-optimized default decision can of course coincide with the inherited default promise.

Regardless of whether repayment takes place as a prior commitment or under re-optimization, the government now solves

$$\mathcal{V}^r(b,y) = \max_{b' \in \mathbb{B},\, \mathbf{d}_{y'} \in \{0,1\}^Y} \left\{ \begin{array}{c} u\left(y + q\left(b', \mathbf{d}_{y'}, y\right) b' - b - \underline{c}\right) \\ +\lambda\beta\mathbb{E}_{y'|y}\left[\mathcal{V}^c(b', d_{y'}, y')\right] \\ +(1-\lambda)\beta\mathbb{E}_{y'|y}\left[\mathcal{V}^o(b', y')\right] \end{array} \right\}, \tag{11}$$

in the repayment case.

As in the Ramsey case, conditional on repayment, the government chooses borrowing and makes default promises. Under loose commitment, with probability $1 - \lambda$, the promise might however be broken in the subsequent period, which is understood by the government and its creditors.

The bond price schedule now is given as

$$q\left(b', \mathbf{d}_{y'}, y\right) = \frac{1 - \mathbb{E}_{y'|y}\left[\lambda d_{y'} + (1-\lambda)\mathcal{D}(b', y')\right]}{1 + r}. \tag{12}$$

The policy function $\mathcal{D}(\cdot)$ solves the repayment decision problem under discretion for the next period, which like the first-period problem in the Ramsey case is given by (10), and hence depends on tomorrow's beginning-of-period debt position b' and income y'.

[14] In Appendix A.1, I provide a micro-foundation for λ based on endogenous re-optimization with random costs.

In the default case, the government solves

$$\mathcal{V}^d(y) = \max_{\mathbf{d}_{y'} \in \{0,1\}^Y} \left\{ \begin{array}{c} u(y - \phi(y) - \underline{c}) + (1-\theta)\beta \mathbb{E}_{y'|y}\left[\mathcal{V}^d(y')\right] \\ +\theta\lambda\beta\mathbb{E}_{y'|y}\left[\mathcal{V}^c(0, d_{y'}, y')\right] \\ +\theta(1-\lambda)\beta\mathbb{E}_{y'|y}\left[\mathcal{V}^o(0, y')\right] \end{array} \right\}. \tag{13}$$

For $t \geq 1$, the recursive equilibrium for the loose-commitment case is defined as follows:

Definition 2 *A recursive equilibrium under loose commitment consists of value and policy functions* $\mathcal{V}^c : \mathbb{B} \times \{0,1\} \times \mathbb{Y} \to \mathbb{R}$, $\{\mathcal{V}^o, \mathcal{D}\} : \mathbb{B} \times \mathbb{Y} \to \mathbb{R} \times \{0,1\}$, $\{\mathcal{V}^r, \mathcal{B}^r, \mathbf{d}^r_{y'}\} : \mathbb{B} \times \mathbb{Y} \to \mathbb{R} \times \mathbb{B} \times \{0,1\}^Y$, *and* $\{\mathcal{V}^d, \mathbf{d}^d_{y'}\} : \mathbb{Y} \to \mathbb{R} \times \{0,1\}^Y$, *that satisfy condition (6), (10), (11), and (13), as well as bond price function* $q : \mathbb{B} \times \{0,1\}^Y \times \mathbb{Y} \to [0, 1/(1+r)]$ *that satisfies condition (12).*

It is easy to see that for $\lambda = 0$, the model collapses to the model studied in Arellano (2008) as a special case, whereas for $\lambda = 1$, the model reduces to the full-commitment case from Section 2.2. The initial-period problem is again given by (9) if the government is in financial autarky and by (10) if it has access to financial markets.

2.4 Discussion

In this section, I briefly comment on some model features to clarify the focus of this paper.

Economic Environment To illustrate how commitment affects debt and default in a transparent way, I purposefully restrict attention to the most basic version of the model by Eaton and Gersovitz (1981), following the work of Aguiar and Gopinath (2006) and Arellano (2008). By doing so, I abstract from many model features that the subsequent literature has introduced, like endogenous debt recovery (see Yue, 2010; Hatchondo et al., 2016), risk-averse investors (see Lizarazo, 2013), or long-maturity debt (see Hatchondo and Martinez, 2009; Chatterjee and Eyigungor, 2012). While exploring such features in a loose-commitment setting might be interesting, they introduce additional time-inconsistency problems that deserve special attention.[15] Studying these issues in a model with loose commitment is therefore left for future research. In addition, although the aforementioned features are relevant for some applications, many past and current studies in the literature use the basic one-period model of sovereign debt and default without debt recovery as their starting point. The insights of this paper should therefore be informative for many other applications.

Discrete Default In principle, the government might want to promise to repay a positive fraction of its debt conditional on default because - under (loose) commitment - it internalizes the adverse impact of default on borrowing conditions in previous periods (see Adam and Grill, 2017). In this paper, I exclude such behavior by assumption due to the binary nature of default. The reason for doing so is threefold. First, given the fixed costs associated with default, it is quite likely that the government is only willing to promise default when the gains (omitted debt payments) are sufficiently large anyway. Second, if the government optimally promises to default in equilibrium even when the

[15] Importantly, modeling these features under loose commitment would be much more demanding computationally since the recursive model formulation would have to feature additional endogenous state variables to keep track of the government's promises.

internalized indirect cost is high due to the binary nature of debt repayment, then it would certainly also be optimal to do so with non-full default. In this sense, I only make default less attractive from an ex-ante perspective. Third, allowing (promised) default to have an extensive and an intensive margin would enormously increase the computational burden if the number of income states Y is low and make it computationally infeasible for even a moderate number of states.[16] Given that a low number of states might bias the quantitative (cyclical) predictions of the no-commitment model (see Hatchondo et al., 2010), assuming discrete default in this paper allows to consider a much higher number of states relative to a version with voluntary partial repayment under commitment.

Loose Commitment Following the optimal policy literature, one can avoid assigning a deeper meaning to the re-optimization probability $1-\lambda$ and simply treat it as a model parameter that allows to smoothly change the government's degree of commitment to gain insights about the role of commitment for policy outcomes. Alternatively, one may view the re-optimization probability as a reflection of exogenously given features of the economy that allow the government to sustain outcomes that go beyond those of the discretionary Markov equilibrium. As shown by Auclert and Rognolie (2016), the model by Eaton and Gersovitz (1981) features a unique Markov equilibrium, which also happens to be the only subgame-perfect equilibrium.[17] While the standard model of sovereign debt and default does not offer an explanation for why governments can credibly commit to non-Markov policies, it is fair to say that, in reality, there are mechanisms not captured by the standard environment that affect policy makers' credibility. The re-optimization probability can be interpreted as capturing features that can give rise to such mechanisms, like details of a country's political system or social norms. Following Debortoli and Nunes (2010), a natural interpretation of the loose-commitment environment views λ as the political survival probability of an incumbent politician who may lose his position in office. Once he is replaced by another (identical) politician, the latter does not care about promises made by his predecessor, such that policy makers can only commit to future policies conditional on remaining in power.[18] This interpretation does however not explain why politicians are committed to implement their own past promises. A plausible reason for this could be the existence of private costs that politicians in office experience when not honoring promises made in the past. It might, for instance, be quite costly for a politician to break his promises ex post because it hurts his political reputation and reduces popularity among voters and/or financial supporters.[19] More generally, social stigma associated with not keeping one's promises could explain why individuals (and hence governments) possess some degree of commitment.[20] Providing a deep theory for the loose-commitment environment is however beyond the scope of this paper.

[16]Adam and Grill (2017) assume three states for their quantitative model exercise, including a disaster state.

[17]See Aguiar and Amador (2019) for an alternative proof of uniqueness and Bloise et al. (2017) for a related proof in a more general model.

[18]In the context of sovereign debt repayment, such behavior can occasionally be observed in practice when a new government takes over and decides to default on debt issued by a previous government that it considers illegitimate. A recent example of such an event is Ecuador's default in 2007 (see Mansell and Openshaw, 2009).

[19]In Appendix A.1, I show that the loose-commitment model can be reformulated based on this idea.

[20]Alternatively, the re-optimization probability can also be interpreted as the probability of an unforeseen and severe event that requires the government to review its plan because it cannot condition its policies on such an event (see Debortoli and Nunes, 2010).

2.5 Policy Trade-Offs

To guide the quantitative analysis, it is useful to first illustrate the key trade-offs faced by the government and how they depend on the degree of commitment λ.[21] To understand how borrowing is affected by λ, take a look at the government's Euler equation,

$$\underbrace{(1-\lambda)u_c(c)\frac{\partial \mathbb{E}_{y'|y}[\mathcal{D}(b',y')]}{\partial b'}b'}_{\text{time inconsistency}} \underbrace{-\mu_{\underline{b}}+\mu_{\overline{b}}}_{\text{debt/savings constraints}} = \underbrace{\lambda \mathbb{E}_{y'|y}\left[(1-d_{y'})\left(u_c(c)-\tilde{\beta}u_c(c')\right)\right]}_{\text{consumption smoothing (commitment part)}} + \underbrace{(1-\lambda)\mathbb{E}_{y'|y}\left[(1-d')\left(u_c(c)-\tilde{\beta}u_c(c')\right)\right]}_{\text{consumption smoothing (no-commitment part)}}, \tag{14}$$

with $\tilde{\beta} \equiv \beta(1+r)$ and multipliers $\mu_{\underline{b}}, \mu_{\overline{b}} \geq 0$ that are associated with the debt constraints $b' \geq \underline{b}$ and $-b' \geq \overline{b}$, respectively.[22]

The government borrows (or saves) to smooth consumption across income states, which is captured by the RHS of (14). Depending on the country's impatience relative to its creditors, measured by $\tilde{\beta}$, the government also uses debt (savings) to front-load (back-load) consumption. The extent to which the government can achieve these two goals is limited in a number of ways.

As reflected by the multipliers on the LHS of the Euler equation, exogenous borrowing and saving limits may directly constrain the government's debt choice. Moreover, since bonds are non-state contingent, consumption smoothing can only be achieved in expectation. Default can help to provide at least some degree of state-contingency by allowing the government to refuse debt repayment in selected income states (see the RHS of (14)). Foreign investors will however anticipate the possibility of default and demand a lower bond price q in return. Whether the government internalizes this price effect when choosing to default depends on its ability to commit, making the effectiveness of default crucially depend on the degree of commitment λ, which can be seen by looking at the RHS of bond pricing condition (12).

To understand how the default decision interacts with the borrowing decision, first consider the no-commitment part of the default probability, which is weighted by $1-\lambda$. This part depends on the discretionary default choice made in the next period, $d' = \mathcal{D}(b',y')$, and reflects how the government's incentive to default will vary with future income y' and debt b'. More specifically, the optimal discretionary default choice will satisfy

$$\mathcal{D}(b',y') = \begin{cases} 1, & \text{if} \quad \mathcal{V}^d(y') - \mathcal{V}^r(b',y') > 0, \\ 0, & \text{if} \quad \mathcal{V}^d(y') - \mathcal{V}^r(b',y') \leq 0. \end{cases} \tag{15}$$

While $\mathcal{D}(b',y')$ and $d_{y'}$ both depend on future income y', the relationship between $\mathcal{D}(b',y')$ and y' will be established ex post in the next period. Under discretion, the government will therefore not internalize the effect that default has on borrowing conditions in the previous period. The dependence

[21] See Mateos-Planas and Rios-Rull (2016) for a related project on unsecured debt and equilibrium default that compares optimality conditions separately derived for cases with full- and no-commitment. These optimality conditions can however only indirectly be compared due the way the full-commitment problem is formulated. By contrast, the loose-commitment formulation used in this paper provides a unifying framework that nests the two polar commitment cases for $\lambda \in \{0,1\}$, allowing a transparent comparison solely based on first-order conditions derived for general $\lambda \in [0,1]$.

[22] For readability, the multipliers are normalized as $\mu_x = (1+r)\tilde{\mu}_x$, where $\tilde{\mu}_x$ denotes the true multiplier for $x \in \{\underline{b}, \overline{b}\}$.

of $\mathcal{D}(b',y')$ on b' furthermore implies that, under loose commitment $\lambda \in [0,1)$, the bond price q varies with the chosen level of debt because investors anticipate its impact on the default probability, giving rise to the first term on the LHS of the Euler equation (14).

As in Eaton and Gersovitz (1981) and Arellano (2008), it will be more attractive for the government to default ex post for higher levels of debt, such that the no-commitment part of the default probability will increase with b', i.e. $(1-\lambda)\partial\mathbb{E}_{y'|y}[\mathcal{D}(b',y')]/\partial b' \geq 0$. Although default can be used to address the problem of market incompleteness, the time-inconsistency problem raised by it can therefore severely limit the government's ability to smooth (and front-load) consumption when its commitment is low. To illustrate this, suppose that the government is hit by a negative income shock. Since income is persistent and it will be more attractive to default for lower levels of income, such a shock raises the default probability for a given debt choice b', making it particularly hard to borrow for a low-commitment government when it is most useful to do so.

Now consider the commitment part of the default probability, which is weighted by λ. In contrast to the no-commitment part, which depends on the discretionary decision rule (15) that is taken as given by the government when choosing b', the commitment part, which depends on the state-contingent default promises $d_{y'}$, $y' \in \mathbb{Y}$, is fully controlled by the government and jointly chosen with the borrowing level. To illustrate the trade-off faced by the government when committing to default in certain states, suppose that the government chooses its promises by selecting an income threshold $\hat{y} \in \mathbb{Y}$, such that

$$d_{y'} = \begin{cases} 1, & \text{if} \quad y' < \hat{y}, \\ 0, & \text{if} \quad y' \geq \hat{y}, \end{cases} \tag{16}$$

for all $y' \in \mathbb{Y}$.[23] Under the assumption of a continuous and bounded income support $\mathbb{Y} \equiv \left[\underline{y}, \overline{y}\right]$, with $\underline{y} > \overline{y}$,[24] the optimal choice of $\hat{y}$ satisfies the intertemporal optimality condition

$$\underbrace{u_c(c)b'}_{\text{marginal cost}} = \underbrace{\tilde{\beta}\left[\mathcal{V}^d(\hat{y}) - \mathcal{V}^r(b',\hat{y})\right]}_{\text{marginal benefit}} \underbrace{-\mu_{\underline{y}} + \mu_{\overline{y}}}_{\text{inequality constraints}}, \tag{17}$$

where $\mu_{\underline{y}}, \mu_{\overline{y}} \geq 0$ are multipliers associated with the inequality constraints $\hat{y} \geq \underline{y}$ and $-\hat{y} \geq \overline{y}$, respectively.[25]

The LHS of (17) measures the marginal cost of increasing default threshold $\hat{y}$, which raises the risk of default and lowers consumption c by reducing bond price q (see (12)). This is the indirect cost of default that is not internalized by the discretionary default choice. The first term on the RHS captures the marginal benefit of an increase in $\hat{y}$. As in the no-commitment case, the gains from default depend on the difference between the value of default and the value of repayment. Since the default decision is however made from an ex-ante perspective, these gains are discounted by the (effective) discount factor $\tilde{\beta}$. Interestingly, for a given debt choice b', a lower discount factor β will reduce the attractiveness of default under commitment and hence the default probability, whereas

[23] In the quantitative analysis, I find that the optimal default policy can indeed be characterized this way.

[24] In this case, one can write the commitment part of the continuation value in Bellman equation (11) as $\mathbb{E}_{y'|y}\left[\mathcal{V}^c(b',d_{y'},y')\right] = \int_{\underline{y}}^{\hat{y}} \mathcal{V}^d(y')f(y'|y)dy' + \int_{\hat{y}}^{\overline{y}} \mathcal{V}^r(b',y')f(y'|y)dy'$, with $f(\cdot)$ denoting the conditional probability density function for income $y' \in \mathbb{Y}$. The assumption of a continuous income support is only made for illustration purposes in this section. Consistent with the model description in Section 2.3, the quantitative analysis will feature a discrete income support.

[25] The multipliers are again normalized by the risk-free (gross) interest rate.

under discretion, a lower discount factor will tend to make default more attractive because the costs associated with financial autarky are discounted more. As illustrated by Euler equation (14), the government of a more impatient economy will however also want to borrow more, which raises $\mathcal{V}^d(\hat{y}) - \mathcal{V}^r(b', \hat{y})$ for a given $\hat{y}$. Whether a higher β-value raises or lowers the risk of default under commitment in equilibrium is hence not clear a priori.

For intermediate degrees of commitment $\lambda \in (0,1)$, the government faces the risk of not being able to honor its promises ex post with a positive probability. How does it respond to the threat of re-optimization? First note that the degree of commitment λ affects the marginal cost of default via bond price q, which matters for consumption c, and the marginal benefit of default via continuation values $\mathcal{V}^d(\cdot)$ and $\mathcal{V}^r(\cdot)$. The government might in principle respond in two opposing ways. On the one hand, it might want to commit to service its debt in future states of the world to counteract the adverse effect that the possibility of re-optimized default has on bond price q. On the other hand, the government might promise to not repay in states in which default will take place under re-optimization to avoid situations in which the promise to repay has to be kept even though output is low. Being stuck with the promise to repay in such a situation can be very costly because borrowing might be endogenously constrained due to high risk of re-optimized default, requiring large reductions in consumption to service debt payments.

3 Quantitative Analysis

Given the negative implications of (re-optimized) default under loose commitment, how large are the welfare gains of commitment? How does the degree of commitment affect consumption, debt and default dynamics? What does the optimal policy under full commitment prescribe for a no-commitment calibration as commonly used in the literature? In this section, I will answer these questions (and others) by using calibrated model versions.

3.1 Recursive Preferences

For some parts of the quantitative analysis, it will be useful to distinguish between the domestic household's willingness to substitute consumption over time and his risk aversion.[26] Following Epstein and Zin (1989, 1991) and Weil (1990), from now on, I will therefore assume that the household has iso-elastic recursive preferences described by the utility recursion

$$\mathcal{V}_t = u(c_t) + \beta \frac{\left(\mathbb{E}_t\left[(1+(1-\beta)(1-\gamma)\mathcal{V}_{t+1})^{\frac{1-\alpha}{1-\gamma}}\right]\right)^{\frac{1-\gamma}{1-\alpha}} - 1}{(1-\beta)(1-\gamma)}. \tag{18}$$

While the intertemporal elasticity of substitution is still given by $1/\gamma$, the coefficient of relative risk aversion now equals α, with $0 < \alpha \neq 1$. For $\alpha = \gamma$, this non-expected utility specification reduces to the more common expected utility version from Section 2.1. The Bellman equations for the government problem with recursive preferences and the associated first-order conditions are shown

[26] See Tourre (2017) and Rebelo et al. (2018) for continuous-time models of sovereign debt and default without commitment that also use recursive preferences for the borrowing country.

Parameter	Description	Value
α	Coefficient of relative risk aversion	2
β	Discount factor	0.966
γ	Inverse elast. of intertemp. substitution	2
ρ	Persistence of income	0.945
σ	Std. dev. of income shock	0.025
ϕ_1	Default cost parameter	-1.187
ϕ_2	Default cost parameter	1.228
λ	Degree of commitment	0
θ	Probability of exiting autarky	0.250
$\overline{b}$	Debt limit	0.250
$\underline{b}$	Saving limit	0
$\underline{c}$	Minimum consumption level	0
r	Risk-free rate	0.010

Table 1: Baseline model parameters

in Appendix A.2.

3.2 Model Specification

To perform a quantitative evaluation of the model, it is necessary to specify functional forms and parameters.

Functional Forms As in Chatterjee and Eyigungor (2012), I assume convex income costs of default,

$$\phi(y) = \max\left\{0, \phi_1 y + \phi_2 y^2\right\},$$

with $\phi_2 \geq 0$.[27]

For $\phi_2 > 0$, this specification implies that $\phi(y)/y$ increases in income y, such that the income costs of default are overproportionately high in good states. This property makes default particularly attractive when income is low, which is a well-known requirement for generating countercyclical risk of default in models without commitment (see Aguiar and Amador, 2014) and in line with empirical evidence (see Tomz and Wright, 2007).

The income support $\mathbb{Y}$ and transition probabilities $\pi(y_{t+1}|y_t)$ are obtained by discretizing a log-normal AR(1)-process,

$$\ln y_t = \rho \ln y_{t-1} + \sigma\varepsilon_t, \ \varepsilon_t \overset{i.i.d.}{\sim} \mathcal{N}(0,1),$$

with $\rho \in [0,1)$, via the method proposed by Tauchen (1986).

Parameters The baseline model calibration closely follows the sovereign default literature. I therefore set $\gamma = \alpha = 2$ as well as $\underline{c} = 0$, assume complete lack of commitment ($\lambda = 0$) and set the remaining model parameters to match certain short- and long-run properties of the Argentine

[27] For calibrations with high minimum consumption $\underline{c}$, it could be that this cost function yields negative consumption in default periods with high output. For $\underline{c} > 0$, I therefore use the modified specification $\phi(y) = \min\left\{y_Y - \underline{c}, \max\left\{0, \phi_1 y + \phi_2 y^2\right\}\right\}$ to avoid this problem, as suggested by Chatterjee and Eyigungor (2012). For the calibrations considered in this paper, this problem did however not arise.

	$\lambda = 0$	$\lambda = 0.4$	$\lambda = 0.7$	$\lambda = 0.9$	$\lambda = 0.97$	$\lambda = 1$
Mean						
Def. prob. overall (annual)	0.030	0.026	0.049	0.113	0.056	0.000
Def. prob. comm. (annual)	-	0.000	0.001	0.000	0.000	0.000
Def. prob. no comm. (annual)	0.030	0.042	0.152	0.754	0.927	-
Debt-service-to-output	0.055	0.061	0.075	0.157	0.226	0.251
Interest rate spread (annual)	0.033	0.029	0.059	0.156	0.066	0.000
Standard deviation						
Output	0.079	0.079	0.079	0.079	0.079	0.079
Consumption	0.081	0.082	0.082	0.082	0.080	0.079
Surplus-to-output	0.012	0.013	0.014	0.018	0.014	0.000
Correlation with output						
Consumption	0.989	0.988	0.985	0.976	0.986	1.000
Interest rate spread (annual)	-0.506	-0.052	-0.320	-0.820	-0.883	-
Debt–issuance-to-output	0.890	0.888	0.819	0.217	-0.063	-0.998
Surplus-to-output	-0.167	-0.173	-0.161	-0.057	-0.040	-0.602

Table 2: Selected moments for simulated model economies with different degrees of commitment λ (baseline calibration)

economy. Argentina is a typical emerging economy that exhibits many of the features highlighted in the literature, such as countercyclical interest rates and net exports (see Neumeyer and Perri, 2005; Uribe and Yue, 2006), which the standard no-commitment model can successfully replicate (see Arellano, 2008). By changing the degree of commitment λ and keeping everything else unchanged, the baseline calibration then enables me to isolate the role of commitment for the predictions of the canonical sovereign default model and to assess the welfare gains of commitment.

The parameter values of the baseline calibration are summarized in Table 1. One model period corresponds to one quarter. The parameters for Argentina's output process are taken from Arellano (2008). The probability of reentry θ is set to 0.25, which is line with the short duration of financial autarky after default observed during the last few decades (see Cruces and Trebesch, 2013). The risk-free rate r is set to 1% (see Aguiar and Gopinath, 2006). The default cost parameters (ϕ_1, ϕ_2) are set to $(-1.1871, 1.2280)$ to match an annual default probability of 3% (see Arellano, 2008) and an unconditional output cost of 7%, which is in line with Zarazaga (2012)'s estimate for Argentina's default in 2001. The discount factor β is set to 0.966, targeting Argentina's average external-debt-service-to-GDP ratio of 5.53% (see Arellano, 2008). The lower bound on debt $\underline{b}$ is set to zero, i.e. the country cannot save (see Chatterjee and Eyigungor, 2012). The upper bound on debt $\overline{b}$ is set to 0.25, which is just high enough to avoid scenarios with a binding debt constraint for $\lambda = 0$.

3.3 Results

The model is solved numerically via value function iteration (without interpolation) and is then simulated for $500,000$ periods, using 500 burn-in periods to reduce the impact of initial conditions. The results for the baseline calibration are reported in Table 2, which lists selected model moments for different degrees of commitment λ.[28]

[28] Given that the full-commitment case does not feature a debt-elastic bond price, one may perhaps worry about the existence of an ergodic distribution for the model variables in this case. For the debt bounds considered in this paper, I do

The No-Commitment Case Without commitment ($\lambda = 0$), the model replicates the predictions of the canonical sovereign default model (see Arellano, 2008). The no-commitment economy hence features countercyclical interest rate spreads, $p_t \equiv (1/q_t)^4 - (1+r)^4$, and surpluses, $f_t \equiv b_t - q_t b_{t+1}$, as well as excess consumption volatility. The mechanism that generates these predictions has already briefly been discussed in Section 2.5 and is as follows. Without commitment, the bond price is responsive to debt issuance and current income since these variables predict the probability of default in the next period. This property makes it prohibitively expensive for the government to increase its debt position or even roll over maturing debt in response to a negative income shock, resulting in procyclical borrowing, countercyclical surpluses and consumption being more volatile than income.

The Full-Commitment Case What would the government like to do in the baseline model economy if it had the ability to commit? The answer is given in the last column of Table 2, which reports model moments for the full-commitment case ($\lambda = 1$), keeping all other model parameters unchanged. Under full commitment, the government never defaults and borrows up to its debt limit $\overline{b}$ (see panel (a) in Figure 1). The main reason why this is the desired policy for the baseline model economy is its high impatience and low degree of risk aversion. The latter implies that the economy has a rather limited desire to smooth consumption across states. The former implies that the economy has a large preference for front-loading consumption and that the already low gains of default are additionally discounted at a high rate. The full-commitment policy accommodates these preferences by borrowing as much as possible and committing to never default since this would only interfere with the strong front-loading motive.[29] To illustrate this, suppose that the government has borrowed up to its debt limit, such that consumption is given as $c = y - \underline{c} - (q-1)\overline{b}$. The only way to further increase consumption is to promise repayment for all states, which pushes the bond price q to its maximum value $1/(1+r)$.

The Loose-Commitment Case Now consider the case with intermediate degrees of commitment $\lambda \in (0,1)$. Table 2 shows that the degree of commitment λ has a positive effect on average debt and a hump-shaped one on the average (overall) default probability. To understand the mechanism underlying these effects, Table 2 also lists average values for the (non-weighted annualized) full-commitment and no-commitment components of the default probability, $\mathbb{E}_{y'|y}\left[d_{y'}(b,y)\right]$ and $\mathbb{E}_{y'|y}\left[\mathcal{D}(b'(b,y),y')\right]$. It turns out that, for the no-commitment baseline calibration, the government not only promises to always repay under full commitment but in almost all loose-commitment cases as well. Only the first of the two motives for making default promises discussed at the end of Section 2.5 hence matters for the baseline calibration.

The government of the baseline economy has a strong incentive to front-load consumption by persistently accumulating debt. As argued in detail in Section 2.5, the debt elasticity of the bond price restricts the government's debt issuance. Increasing the degree of commitment λ reduces the weight on the debt-elastic component of the bond price (12), which the impatient government

however find that the unconditional model moments are well-defined regardless of the assumed degree of commitment λ: Drawing different values for the income shocks and/or further increasing the number of simulation periods does not affect the computed statistics for the model versions considered in this paper.

[29] Note that this is in contrast to the no-commitment case, where a higher degree of impatience increases the likelihood of default for a given debt position.

	$\lambda = 0$	$\lambda = 0.4$	$\lambda = 0.7$	$\lambda = 0.9$	$\lambda = 0.97$	$\lambda = 1$
Mean						
Def. prob. overall (annual)	0.030	0.036	0.067	0.141	0.079	0.020
Def. prob. comm. (annual)	-	0.007	0.024	0.042	0.027	0.020
Def. prob. no comm. (annual)	0.030	0.052	0.144	0.726	0.914	-
Debt-service-to-output	0.042	0.048	0.060	0.139	0.217	0.243
Interest rate spread (annual)	0.032	0.049	0.092	0.212	0.099	0.022
Standard deviation						
Output	0.079	0.079	0.079	0.079	0.079	0.079
Consumption	0.081	0.081	0.081	0.080	0.080	0.079
Surplus-to-output	0.009	0.010	0.012	0.018	0.016	0.010
Correlation with output						
Consumption	0.994	0.992	0.988	0.975	0.980	0.993
Interest rate spread (annual)	-0.385	-0.150	-0.234	-0.747	-0.775	-0.674
Debt-issuance-to-output	0.878	0.861	0.763	0.246	0.088	-0.238
Surplus-to-output	-0.163	-0.163	-0.126	0.019	0.031	0.063

Table 3: Selected moments for simulated model economies with different degrees of commitment λ (baseline calibration with $\alpha = 10$)

gladly takes advantage of by borrowing more. While a higher λ-value reduces the risk of default for a given debt position b, the increase in average debt raises the overall default frequency for low and intermediate λ-values since the incentive to default under discretion increases with debt. For some (high) critical degree of commitment, re-optimized default will become sufficiently rare, such that the overall default frequency starts to decline with λ. It is important to highlight that the hump-shaped impact of λ on the average default probability does not rely on promised default being (almost) completely absent in equilibrium, which will be shown below.

Promised Equilibrium Default The baseline calibration studied so far features only a low degree of risk aversion which - together with a high degree of impatience - implies that the economy's demand for insurance via default is quite low. Table 3 presents model statistics for a model parametrization that only differs from the baseline calibration because of a higher coefficient of relative risk aversion $\alpha = 10$, which Mehra and Prescott (1985) consider the highest value that can plausibly be assumed.[30] First, notice how the no-commitment economy is affected by a higher degree of risk aversion. As can be seen in the first column of Table 3, the qualitative model properties are unaffected and - with the exception of the average debt position - the model moments hardly differ quantitatively relative to the baseline scenario either.

Now consider the statistics for the full-commitment case listed in the last column of Table 3. When the economy is more risk averse, the policy maker still borrows up to the debt limit but now insures against bad income states by promising to default in such instances (see panel (b) in Figure 1). With a default probability of 2%, the full-commitment economy defaults almost as often as the no-commitment economy. For α-values in excess of 10, default can even occur more frequently

[30]For the baseline calibration, it is not sufficient to exclusively change the direct default costs, the degree of impatience or any other single model component except for the economy's risk aversion to generate sizable default frequencies for the full- and the no-commitment cases. Section 3.4 provides a detailed discussion of the impact that different model parameters have on the full-commitment solution.

(a) Low risk aversion ($\alpha = 2$) (b) High risk aversion ($\alpha = 10$)

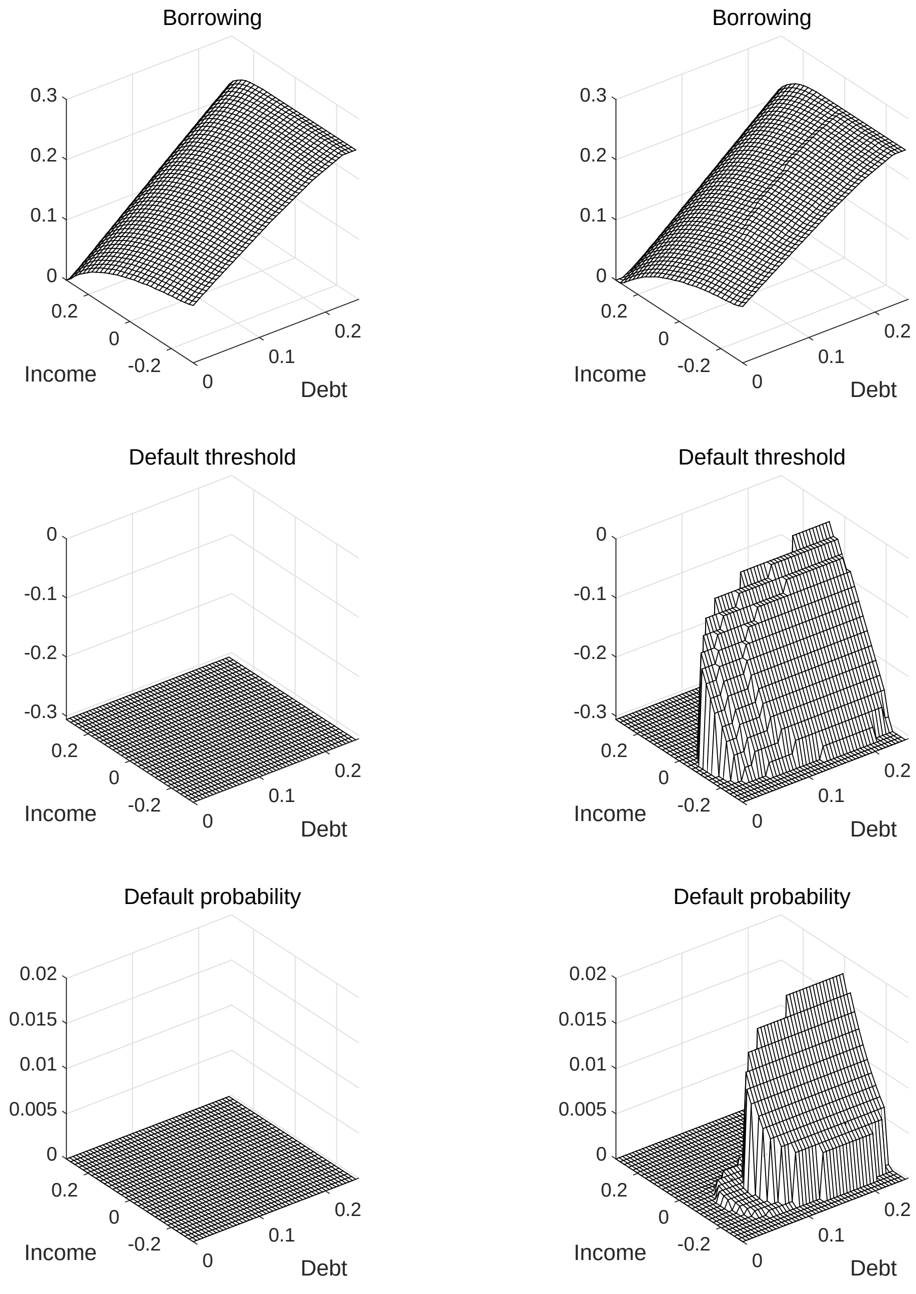

Figure 1: Optimal policies under full commitment and implied default probability

	$\lambda = 0.4$	$\lambda = 0.7$	$\lambda = 0.9$	$\lambda = 0.97$	$\lambda = 1$
Baseline calibration	0.006	0.018	0.114	0.318	0.484
Baseline calibration with $\alpha = 10$	0.004	0.018	0.129	0.341	0.508

Table 4: Welfare-equivalent consumption variation Δ (in %) for different degrees of commitment λ

under commitment than under discretion. While lack of commitment might be a reason why certain economies are not able to borrow as much as others, it can therefore not necessarily be blamed for why some countries are more likely to default. As in the no-commitment case, default under commitment occurs in bad times. Although default is promised to occur in low-income states, in principle, the default probability could nevertheless be procyclical since the government might want to commit to default in bad states when it is in a boom. Such a policy would effectively buy insurance against adverse shocks when the economy can afford to face a lower bond price the most, i.e. when the LHS of (17) is low. Table 3 shows that interest rate spreads, reflecting the risk of default, are however clearly countercyclical.

The important qualitative difference relative to the optimal policy under discretion is that, under commitment, borrowing is countercyclical and surpluses are procyclical, which reflects the government's ability to effectively smooth consumption across states via debt issuance. Consumption is therefore less volatile than income. Such a policy is feasible in this case because the government does not face a debt-elastic bond price schedule as in the no-commitment case. Instead, the bond price only reflects default promises (see (8)), which the government can perfectly control when it issues debt, such that the borrowing position is not constrained by the possibility of future default.

Table 3 shows that the qualitative impact of the degree of commitment on average debt and default remains for higher risk aversion. The hump-shaped relationship between commitment and the frequency of default is therefore not driven by a commitment policy that always involves repayment. Interestingly, for high but not full commitment, the government promises to default more often than in the full-commitment case, such that the hump-shaped relationship between default risk and the degree of commitment can also be observed for the commitment part of the overall default probability. The second motive for making default promises under loose commitment discussed in Section 2.5 now is therefore visibly operative. Under loose commitment, the economy's cyclical properties are closer to the no-commitment case than the full-commitment case, demonstrating that even small deviations from full commitment can lead to strong changes in public policy.

The Welfare Gains of Commitment How large are the welfare gains of commitment? To answer this question, I compute the welfare-equivalent per-period consumption variation

$$\Delta = \frac{\sum_y \left(1+(1-\beta)(1-\gamma)\mathcal{V}^o_\lambda(0,y)\right)^{1/(1-\gamma)}\Pi(y)}{\sum_y \left(1+(1-\beta)(1-\gamma)\mathcal{V}^o_0(0,y)\right)^{1/(1-\gamma)}\Pi(y)} - 1, \tag{19}$$

which measures how much households in a no-commitment economy have to be compensated to achieve the same expected lifetime utility as in an economy with degree of commitment λ, where $\mathcal{V}^o_\lambda(\cdot)$ denotes the associated option value and $\Pi(y)$ the unconditional probability of income y implied by the Markov chain.[31]

Table 4 shows the results for the baseline calibration with low risk aversion ($\alpha = 2$) as well as the same model calibration with high risk aversion ($\alpha = 10$). Unsurprisingly, the welfare gains increase with λ. Interestingly however, the gains of commitment largely accrue at high degrees of commitment, whereas going from $\lambda = 0$ to 0.4 or even 0.7 hardly increases welfare. Since the optimal policy under commitment does not involve equilibrium default for the baseline calibration, the welfare gains of commitment are only due to improvements in the ability to front-load consumption in this case. With high risk aversion, in which case the insurance role of default becomes more important, the welfare gains however hardly change. The front-loading motive therefore appears to matter more in terms of welfare than the insurance motive.

3.4 Recalibrating the Model

So far, the role of commitment has only been investigated for a model calibration that was tailored to the no-commitment case ($\lambda = 0$). To understand the commitment case in a more transparent way, this section will recalibrate the model economy to match the same long-run targets for debt and default as in the baseline case but for $\lambda = 1$. Table 6 displays statistics for different recalibrated model versions. The parameter values used for these calibrations are summarized in Table 5. Parameter values not explicitly mentioned there are kept from the baseline calibration.

Overview Before discussing the recalibrated model versions step by step, I want to give a brief overview of how individual model parameters can in principle be expected to affect policy outcomes under commitment. First, consider the debt limit $\overline{b}$. As shown in the previous section, the debt limit directly matters for policy under commitment by affecting the long-run debt position, especially when the economy is quite impatient. Furthermore, if $\overline{b}$ is low, the debt limit will tend to bind more frequently, which in turn affects the role of default as a shock absorber. Now consider the default cost parameters (θ, ϕ_1, ϕ_2). As in the no-commitment case, the direct costs of default are of course relevant for the attractiveness of default under commitment as well. More specifically, lower resource costs and a higher reentry probability will tend to increase the probability of default. As highlighted in Section 2.5, the more impatient the economy is, i.e. the lower β is, the more a Ramsey planner wants to issue debt to front-load consumption, which raises average debt. If the same average debt position is targeted, e.g. by adjusting debt limit $\overline{b}$, a more impatient policy maker wants to default less often because the gains of default are discounted more (see the RHS of (17)). As illustrated in the previous section, under commitment, a higher degree of risk aversion α raises the economy's demand for ex-ante insurance via default which, keeping the intertemporal elasticity of substitution γ unchanged, increases the average default probability. In the remainder of this section, I will illustrate how the model parameters affect the model predictions quantitatively by considering various recalibrated model versions.

[31] Using the transformation $\tilde{\mathcal{V}}_t \equiv (1+(1-\beta)(1-\gamma)\mathcal{V}_t)^{1/(1-\gamma)}$, the recursion (18) can be written as $\tilde{\mathcal{V}}_t = \left[(1-\beta)c_t^{1-\gamma} + \beta\left(\mathbb{E}_t\left[\tilde{\mathcal{V}}_{t+1}^{1-\alpha}\right]\right)^{(1-\gamma)/(1-\alpha)}\right]^{1/(1-\gamma)}$ (see Karantounias, 2018). The welfare measure Δ then is given by (19) due to homogeneity of $\tilde{\mathcal{V}}(\cdot)$.

	Model I	Model II	Model III	Model IV	Model V
α	10	10	-	-	-
β	0.980	$1/(1+r)$	$1/(1+r)$	$1/(1+r)$	0.980
ϕ_1	-1.410	-1.261	-2.183	0.006	-0.902
ϕ_2	1.440	1.298	2.165	0	0.955
$\overline{b}$	0.068	0.144	0.119	0.111	$(y_1-\underline{c})/r$
$\underline{c}$	-	-	-	-	0.728
θ	-	-	0.333	1	-

Table 5: Changed parameter values for recalibrated full-commitment model versions ($\lambda = 1$)

The Role of Preferences I will first look at calibrations that allow to illustrate the role of household preferences for policy outcomes. To understand the role of the economy's (im-)patience, I separately calibrate the model for moderate discounting $\beta = 0.98$ (Model I) as well as $\beta = 1/(1+r)$ (Model II), in which case the government is as patient as the investors and hence does not have an incentive to front-load consumption.[32] The degree of risk aversion is again set to $\alpha = 10$ to ensure a sufficiently strong motive for insurance via default. The parameters $\left(\phi_1, \phi_2, \overline{b}\right)$ are then chosen to match an annual default probability of 3%, an average default cost of 7% and a quarterly debt-service-to-GDP ratio of 5.53%. The respective parameter values are listed in Table 5. The remaining (non-target) model parameters are kept as in the baseline calibration. Table 6 presents the results. Relative to the exercise from the previous section, debt becomes strongly countercyclical, surpluses clearly procyclical and - as a consequence - consumption is less volatile than income. Note that the cyclicality of borrowing and surpluses changes with the economy's degree of impatience. Reflecting the economy's front-loading motive, the consumption-smoothing role of borrowing becomes less important and debt is used less as a shock absorber.

The Role of Default Costs Without commitment, the output cost and the exclusion cost of default have to be quite high since the government would otherwise default for even very low debt levels, which results in too low debt levels in equilibrium. Under full commitment, the government internalizes the indirect cost of default that arises because not repaying debt in some states today increases the cost of borrowing in the previous period. The total cost of default implied by the baseline parametrization might thus simply be too high for default to be an attractive policy under commitment. This reasoning is indeed confirmed by the third column of Table 6 (Model III), which lists selected moments for a model for which parameters $\left(\phi_1, \phi_2, \overline{b}\right)$ have been recalibrated, assuming a higher re-entry probability ($\theta = 1/3$) and a high discount factor of $1/(1+r)$.[33] Note that the cyclical model properties do not differ much relative to the other full-commitment calibrations, demonstrating the robustness of the presented results. I also consider a recalibrated model version with proportional resource costs of default as in Aguiar and Gopinath (2006) (see Model IV), which implies $\phi_2 = 0$. Again I assume that the economy is as patient as the investors and target the same long-run statistics as before. In addition, I assume that re-entry to financial markets now occurs with

[32] Without impatience, the lower bound $\underline{b}$ will bind for some states since the government has an incentive to accumulate assets. Qualitatively, the model properties under commitment are however not affected much by particular $\underline{b}$-values.

[33] One can alternatively also reduce the target for the average default cost instead of using a lower duration of financial exclusion. Patience is however needed to obtain non-negligible default probabilities for $\alpha = 2$.

	Model I	Model II	Model III	Model IV	Model V
Mean					
Def. prob. (annual)	0.030	0.029	0.031	0.029	0.030
Debt-service-to-output	0.056	0.055	0.055	0.056	0.056
Interest rate spread (annual)	0.034	0.033	0.034	0.032	0.034
Standard deviation					
Output	0.079	0.079	0.079	0.079	0.079
Consumption	0.078	0.077	0.077	0.077	0.076
Surplus-to-output	0.005	0.009	0.008	0.007	0.008
Correlation with output					
Consumption	0.998	0.994	0.996	0.996	0.995
Interest rate spread (annual)	-0.657	-0.543	-0.402	-0.170	-0.454
Debt-issuance-to-output	-0.561	-0.682	-0.730	-0.760	-0.674
Surplus-to-output	0.148	0.277	0.293	0.304	0.310

Table 6: Selected moments for different full-commitment calibrations (see Table 5)

certainty in the period after a default, i.e. $\theta = 1$. Given that there is one parameter less to specify now, I do not target the average cost of default as before. Table 6 shows that the model predictions are again close to those of the other recalibrated version. While it would be possible to match the long-run statistics also with $\theta < 1$, doing so requires a very low value for ϕ_1 and would result in procyclical interest rate spreads, although default still take place in below-average income states.[34] This feature is also present in the no-commitment model studied by Aguiar and Gopinath (2006), who also assume proportional resource costs of default.

The Role of Minimum Consumption Under full commitment, the long-run model predictions reflect the debt limit $\overline{b}$, which so far has been imposed in an ad-hoc fashion.[35] As in Adam and Grill (2017), one can alternatively set this upper bound equal to the natural debt limit, which in the endowment economy studied in this paper implies $\overline{b} = (y_1 - \underline{c})/r$. Instead of directly choosing $\overline{b}$ to match the average debt position, one can now set $\underline{c} > 0$ to change the debt capacity of the economy. Besides rationalizing the debt limit, this calibration strategy has an additional advantage: a positive $\underline{c}$-value makes the household effectively more averse to low-income states, increasing the demand for insurance via promised default even for moderate α-values and high direct costs of default. The last column of Table 6 lists results for a recalibration of the baseline case with $(\phi_1, \phi_2, \underline{c})$ now chosen to match the average output loss of 7% as well as the long-run targets for debt and default, assuming a moderate degree of impatience ($\beta = 0.98$). The results are again very similar to those of the other model calibrations.

3.5 Model Predictions and Empirical Evidence

To summarize the findings of the previous section: The optimal policy under full commitment implies countercyclical debt and default risk, procyclical surpluses and no excess consumption volatil-

[34] Alternatively, it is also possible to increase the degree of risk aversion to make ex-ante insurance via default more attractive.

[35] To avoid binding debt limits, one could introduce an endogenous discount factor or portfolio adjustment costs into the model (see Schmitt-Grohé and Uribe, 2003).

ity.[36] What do these properties imply for the empirical plausibility of the full-commitment case? The quantitative sovereign default literature usually focuses especially on emerging economies and shows that the no-commitment model can successfully replicate certain stylized facts of emerging market business cycles, like countercylical interest rate rates and net exports as well as excess consumption volatility (see Aguiar et al., 2016). Except for the countercyclicality of interest rates, these features are at odds with the Ramsey model predictions, which might suggest that the full-commitment model is not a good fit for applications to emerging economies.

This conclusion is however not necessarily correct. First, consider the excess volatility of consumption. Alvarez-Parra et al. (2013) show that aggregate consumption volatility in emerging economies is due to highly volatile durable consumption, whereas non-durable consumption is not more volatile than output. Given that the model economy only features a non-durable consumption good, the full-commitment model might in fact make a prediction about the behavior of consumption that is more in line with the data compared to the no-commitment case. Now consider the countercyclicality of public debt. In developed economies, but also many emerging economies (see e.g. Ottonello and Perez, 2019), public debt is countercyclical which again is predicted by the full-commitment case but not by the no-commitment one. Given that net exports clearly are countercyclical in emerging economies (as well as many developed ones), does the no-commitment model at least dominate the full-commitment one with regard to this stylized fact? In the model, the trade surplus coincides with fiscal (budget) surplus. While the trade surplus is countercyclical in the data, the fiscal surplus is not but acyclical in Latin American emerging economies and procyclical in East Asian ones (see Gavin and Perotti, 1997; Lane, 2003; Kaminsky et al., 2004). This behavior can again be explained by the full-commitment specification but not by the no-commitment one. The two polar commitment cases thus both fail according to one of the two surplus interpretations.[37] Which failure is less serious will ultimately depend on the particular application that one is interested in. For the fiscal surplus interpretation, the predictions of the full-commitment specification are consistent with the behavior of most developed economies. The discussion above suggests that it can also not be dismissed as a reasonable model for emerging economies in this case.

3.6 An Application to the European Debt Crisis

Interestingly, a recent paper by Bocola et al. (2019) recognizes the inability of the standard no-commitment model to jointly match the behavior of public debt and interest rate spreads observed for several countries during the European debt crisis. Bocola et al. (2019) show that by including a large minimum consumption level, which they interpret as non-discretionary public spending, the no-commitment model can also be applied to developed economies with non-negligible default risk.[38] As explained in detail by Bocola et al. (2019), the reason why, under discretion, a sufficiently high minimum consumption level together with a high discount factor can give rise to countercyclical

[36]Since the loose-commitment case exhibits the qualitative properties of the full-commitment (no-commitment) case for high (low) degrees of commitment, the discussion in this section will focus on the empirical plausibility of the full-commitment case.

[37]By allowing for shocks to the growth trend of income as in Aguiar and Gopinath (2006, 2007), it should be possible to also generate a countercyclical trade balance (fiscal surplus) under full commitment.

[38]By contrast, Anzoategui (2016) addresses the problem of procyclical public debt in the absence of commitment by assuming that, except for the default decision, all fiscal policies, including debt issuance, are governed by exogenous fiscal rules that are estimated from the data.

	Data	Bocola et al. (2019)	Calibration with $\lambda = 1$
Average interest rate spread	0.32	0.09	0.19
Average debt-service-to-output	8.43	8.52	8.52
Interest rate spread volatility	0.88	0.83	0.88
Debt service cyclicality	-0.87	-0.29	-0.75

Table 7: Selected moments (in percent) for application to European debt crisis

debt in the presence of countercyclical credit risk is that the government effectively becomes more averse to bad income states. As a result, the government acts "myopically" (Bocola et al., 2019) in the sense that its borrowing behavior does not respond very much to the debt-elastic interest rate, which allows to generate scenarios in which public debt and default risk both increase in bad times. Calibrating the model to match the behavior of public debt and interest rate spreads observed for Spain, Bocola et al. (2019) provide a much better fit of the model to the data relative to a standard calibration without minimum consumption. Table 7 displays the predictions of Bocola et al. (2019)'s model as well as the data moments that they target and report in their paper.[39]

Given that the full-commitment model naturally gives rise to the properties that Bocola et al. (2019) force the no-commitment model to have via $\underline{c}$, it is interesting to see how the Ramsey policy compares to the augmented Markov case. Since the loose-commitment framework in this paper nests the no-commitment specification studied by Bocola et al. (2019) and the full-commitment version as special cases, it is possible to directly compare the models' performances using the same calibration strategy. To assess the model's ability to match the targets listed in Table 7, I choose the same values for the non-target parameters used by Bocola et al. (2019), which are $(r, \gamma, \theta, \rho, \sigma) = (0.0045, 2, 0.282, 0.97, 0.01)$, and set the four parameters $(\beta, \underline{c}, \phi_1, \phi_2)$ to match the four target statistics. Their respective values are $(0.98, 0.8743, -0.5086, 0.567)$.[40] As can be seen in the third column of Table 7, relative to the no-commitment calibration, the full-commitment case $(\lambda = 1)$ provides a better fit for the average spread level as well as the cyclicality of debt and matches the spread volatility and the average debt position equally well. Since the government of the full-commitment economy does not have to be forced to borrow in a countercyclical fashion, the model can match the negative correlation between debt and output very well, whereas the no-commitment case generates a correlation that is negative but much less so compared to the data.

4 Conclusion

This paper has extended Eaton and Gersovitz (1981)'s model of sovereign debt with incomplete markets and equilibrium default to explore the role of commitment for debt, default and welfare. In the model, a benevolent government can sell non-state contingent one-period bonds at a price that reflects its risk of default. While the government makes an ex-ante optimal state-contingent plan for

[39] While the model in Bocola et al. (2019) is isomorphic to the model by Arellano (2008) augmented by a minimum consumption level, the authors interpret income y as tax revenues, consumption c as public spending and the public debt position b as total government debt held by risk-neutral foreign or domestic residents, whose utility from private consumption does not enter the government's objective.

[40] As in Bocola et al. (2019), I assume time-separable iso-elastic preferences in this case, i.e. $\alpha = \gamma$. The debt limit is given as $\bar{b} = (y_1 - \underline{c})/r$.

its future actions, including debt repayment, it will re-optimize its plan ex post with a given probability. The model nests the standard Markov case without commitment as well as the full-commitment Ramsey case if the probability is one and zero, respectively, and allows to study how a government's degree of commitment affects its decisions in a transparent way. For intermediate degrees of commitment, the government faces additional trade-offs that change its behavior relative to the two polar commitment cases. If and to what extend a government commits to default in some states of the world is found to depend on preferences, default costs and the degree of commitment. Model versions with full or intermediate commitment have been shown to have some advantages relative to the usually studied no-commitment case, which makes them attractive for several applications. This point was illustrated for a quantitative exercise related to the recent European debt crisis in which the full-commitment case outperformed the no-commitment one.

A Appendix

A.1 Costly Re-Optimization

Suppose that in contrast to Section 2.3, the government can now choose in any period whether to honor its promises in a discretionary fashion. Not keeping a promise ex post is however associated with utility cost κ, which is assumed to be an i.i.d. random variable with support $\mathbb{K}$. It is furthermore assumed that the government cannot condition its promises on future values of κ. This assumption is necessary to have re-optimization occur in equilibrium as in the loose-commitment case. Without uncertainty about κ or with the ability to condition promises on future values of κ, the government effectively faces a credibility constraint which ensures that only ex-post optimal promises can be made ex ante.

In this model version, the government's decision problem in the repayment case changes to

$$\mathcal{V}^r(b,y) = \max_{b' \in \mathbb{B},\, \mathbf{d}_{y'} \in \{0,1\}^Y} \left\{ \; u\left(y + q\left(b', \mathbf{d}_{y'}, y\right) b' - b - \underline{c}\right) + \beta \mathbb{E}_{y',\kappa'|y} \left[\mathcal{V}^e(b', d_{y'}, y', \kappa')\right] \; \right\}, \tag{20}$$

where $\mathcal{V}^e(\cdot)$ denotes the option value of discretion,

$$\mathcal{V}^e(b,d,y,\kappa) = \max_{e \in \{0,1\}} \left\{ e\mathcal{V}^c(b,d,y) + (1-e)\left(\mathcal{V}^o(b,y) - \kappa\right) \right\}. \tag{21}$$

The bond price schedule (12) now is replaced by

$$q\left(b', \mathbf{d}_{y'}, y\right) = \frac{1 - \mathbb{E}_{y'|y}\left[\lambda\left(b', d_{y'}, y'\right) d_{y'} + \left(1 - \lambda\left(b', d_{y'}, y'\right)\right)\mathcal{D}(b', y')\right]}{1+r}, \tag{22}$$

where the probability of keeping promise $d_{y'}$ now is endogenous and given as

$$\lambda\left(b', d_{y'}, y'\right) = \mathbb{E}_{\kappa'}\left[\mathcal{E}(b', d_{y'}, y', \kappa')\right], \tag{23}$$

with $\mathcal{E}(b,d,y,\kappa)$ denoting the policy function that solves the promise-keeping problem (21).

In the default case, the value is

$$\mathcal{V}^d(y) = \max_{\mathbf{d}_{y'} \in \{0,1\}^Y} \left\{ \begin{array}{c} u\left(y - \phi(y) - \underline{c}\right) + (1-\theta)\beta \mathbb{E}_{y'|y}\left[\mathcal{V}^d(y')\right] \\ + \theta\beta \mathbb{E}_{y',\kappa'|y}\left[\mathcal{V}^e(0, d_{y'}, y', \kappa')\right] \end{array} \right\}. \tag{24}$$

This model version with endogenous promise-keeping contains the baseline model from Section 2.3 as a special case. To see this, suppose that the support of κ is given as $\mathbb{K} \in \{\underline{\kappa}, \overline{\kappa}\}$, with $0 \leq \underline{\kappa} < \overline{\kappa}$, and that $\lambda \in [0,1]$ is the probability of $\overline{\kappa}$.

For $\underline{\kappa} = 0$ and $\bar{\kappa} = \min_\kappa \{\kappa \in \mathbb{R} : \mathcal{V}^c(b,d,y) \geq \mathcal{V}^o(b,y) - \kappa,\ \forall\ (b,d,y) \in \mathbb{B} \times \{0,1\} \times \mathbb{Y}\}$, the promise-keeping probability (23) reduces to the constant λ and the model collapses to the model of Section 2.3.

A.2 Policy Problem with Recursive Preferences

In this section, I show how the Bellman equations and the policy-trade offs change relative to Sections 2.3 and 2.5 if one allows for recursive preferences à la Epstein and Zin (1989, 1991) and Weil (1990).

Bellman Equations With Epstein-Zin-Weil preferences, the value of commitment $\mathcal{V}^c(\cdot)$, the option value of default $\mathcal{V}^o(\cdot)$, and the bond price schedule $q(\cdot)$ are still determined by (6), (10), and (12), respectively. Only the Bellman equations for $\mathcal{V}^r(\cdot)$ and $\mathcal{V}^d(\cdot)$ change. More specifically, the value of repayment now is given by

$$\mathcal{V}^r(b,y) = \max_{b' \in \mathbb{B},\, \mathbf{d}_{y'} \in \{0,1\}^Y} \left\{ \begin{array}{c} u(y + q\left(b', \mathbf{d}_{y'}, y\right) b' - b - \underline{c}) \\ +\beta H^{-1} \left[\begin{array}{c} \lambda \mathbb{E}_{y'|y}\left[H\left(\mathcal{V}^c(b', d_{y'}, y')\right)\right] \\ +(1-\lambda)\mathbb{E}_{y'|y}\left[H\left(\mathcal{V}^o(b', y')\right)\right] \end{array} \right] \end{array} \right\} \tag{25}$$

instead of (11) and the value of default by

$$\mathcal{V}^d(y) = \max_{\mathbf{d}_{y'} \in \{0,1\}^Y} \left\{ \begin{array}{c} u\left(y - \phi(y) - \underline{c}\right) \\ +\beta H^{-1} \left[\begin{array}{c} (1-\theta)\mathbb{E}_{y'|y}\left[H\left(\mathcal{V}^d(y')\right)\right] \\ +\theta\lambda \mathbb{E}_{y'|y}\left[H\left(\mathcal{V}^c(0, d_{y'}, y')\right)\right] \\ +\theta(1-\lambda)\mathbb{E}_{y'|y}\left[H\left(\mathcal{V}^o(0, y')\right)\right] \end{array} \right] \end{array} \right\}, \tag{26}$$

instead of (13), where

$$H(x) \equiv \frac{(1 + (1-\beta)(1-\gamma)x)^{(1-\alpha)/(1-\gamma)} - 1}{(1-\beta)(1-\alpha)},$$

as in Karantounias (2018).

First-Order Conditions The government's Euler equation now is given by

$$\begin{aligned}(1-\lambda)u_c(c)\frac{\partial \mathbb{E}_{y'|y}\left[\mathcal{D}(b', y')\right]}{\partial b'} b' - \mu_{\underline{b}} + \mu_{\overline{b}} \quad &= \quad \lambda \mathbb{E}_{y'|y}\left[(1-d_{y'})\left(u_c(c) - \tilde{\beta}\varphi_c^{\frac{\gamma-\alpha}{1-\alpha}} u_c(c')\right)\right] \\ &\quad + (1-\lambda)\mathbb{E}_{y'|y}\left[(1-d')\left(u_c(c) - \tilde{\beta}\varphi_o^{\frac{\gamma-\alpha}{1-\alpha}} u_c(c')\right)\right],\end{aligned} \tag{27}$$

where

$$\varphi_i \equiv \frac{\tilde{\mathcal{V}}^i(b', d_{y'}, y')^{1-\alpha}}{\lambda \mathbb{E}_{y'|y}\left[\tilde{\mathcal{V}}^c(b', d_{y'}, y')^{1-\alpha}\right] + (1-\lambda)\mathbb{E}_{y'|y}\left[\tilde{\mathcal{V}}^o(b', y')^{1-\alpha}\right]}, \; i \in \{c, o\},$$

with

$$\tilde{\mathcal{V}}^i(\cdot) \equiv \left(1 + (1-\beta)(1-\gamma)\mathcal{V}^i(\cdot)\right)^{1/(1-\gamma)}, \;\; i \in \{c, d, o, r\}.$$

For $\alpha = \gamma$, this Euler equation reduces to (14), whereas for $\alpha \neq \gamma$, it features a wedge that distorts the government's incentive to borrow.

Now consider the optimality condition for default,

$$\begin{aligned} u_c(c)b' &= \tilde{\beta}\frac{\tilde{\mathcal{V}}^d(\hat{y})^{1-\alpha}-\tilde{\mathcal{V}}^r(b',\hat{y})^{1-\alpha}}{(1-\beta)(1-\alpha)} \\ &\quad\times\left(\lambda\mathbb{E}_{y'|y}\left[\tilde{\mathcal{V}}^c(b',d_{y'},y')^{1-\alpha}\right]+(1-\lambda)\mathbb{E}_{y'|y}\left[\tilde{\mathcal{V}}^o(b',y')^{1-\alpha}\right]\right)^{-\frac{\gamma-\alpha}{1-\alpha}} \\ &\quad-\mu_y+\mu_{\overline{y}}, \end{aligned} \tag{28}$$

which reduces to first-order condition (17) for $\alpha=\gamma$. Similar to Euler equation (27), optimality condition (28) features a wedge for $\alpha\neq\gamma$, given by the second expression on the RHS, that distorts the default decision $d_{y'}$.

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