An Elementary Theory of Directed Technical Change and Wage Inequality*

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This paper generalizes central results from the theory of (endogenously) directed technical change to settings where technology does not take a labor-augmenting form and with arbitrarily many levels of skill. Building on simple notions of complementarity, the results remain intuitive despite their generality. The developed theory allows to study the endogenous determination of labor-replacing, that is, automation technology through the lens of directed technical change theory. In an assignment model with a continuum of differentially skilled workers and capital, where capital perfectly substitutes for labor in the production of tasks, any increase in the relative supply of skilled workers stimulates investment into improving the productivity of capital, potentially leading skill premia to increase in relative skill supply. Relatedly, trade with a skill-scarce country discourages improvements in capital productivity, potentially reversing the standard Heckscher-Ohlin effects.

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1. Introduction

Since the 1980s, many advanced economies have witnessed substantial increases in wage inequality between groups of workers with different levels of educational attainment. A broad empirical literature attributes parts of this increase to skill-biased technical change.[1] Appealing to skill-biased technical change as an exogenous explanation for the observed changes in

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[1] See Bound and Johnson (1992), Katz and Murphy (1992), and Goldin and Katz (2008) on skill-biased technical change in general, and Graetz and Michaels (2018), Acemoglu and Restrepo (2020), and Dauth, Findeisen, Suedekum and Woessner (2019) on the effects of automation technology in particular.

the wage structure, however, is not entirely satisfactory. After all, the technologies that are used in an economy are eventually chosen by economic agents, about whose decisions economics should have something to say. This is the starting point for the theory of endogenously directed technical change (see Acemoglu, 1998; Kiley, 1999).[2] Central results of the theory predict how the skill bias of technical change depends on the supply of skills firms face on the labor market. In particular, they provide conditions under which (i) there is *weak relative equilibrium bias of technology* (weak relative bias, henceforth), meaning that any increase in the relative supply of skill induces skill-biased technical change, and (ii) there is *strong relative equilibrium bias of technology* (strong relative bias, henceforth), meaning that the positive effect of the induced technical change on the skill premium dominates the (typically negative) direct effect, such that the skill premium increases in relative skill supply (e.g. Acemoglu, 2002, 2007). With the notable exception of Acemoglu (2007) (discussed below), these conditions are limited to settings in which aggregate production takes the specific form $F(\theta_1 L_1, \theta_2 L_2)$, where L_1 and L_2 denote the supply of skilled and unskilled labor, and θ_1 and θ_2 represent the endogenous, differentially labor-augmenting technology.

At the same time, the most recent literature on the effects of technical change on wage inequality analyzes labor-replacing (that is, automation) technology, typically in assignment models with labor and capital where capital perfectly substitutes for labor in the production of tasks (e.g. Acemoglu and Autor, 2011; Autor and Dorn, 2013; Acemoglu and Restrepo, 2018a; Feng and Graetz, 2020; Aghion, Jones and Jones, 2019). In these models, the relevant technology variables can in general not be represented as labor-augmenting technology, such that they are outside the scope of the main results on directed technical change described above.

This paper generalizes the central results from directed technical change theory on weak and strong relative bias beyond the special case of differentially labor-augmenting technology and thereby makes them applicable to automation technology in Roy-like assignment models.[3] The first part of the paper derives general conditions for weak and strong relative bias that are independent of any functional form restriction, drawing on techniques from the theory of monotone comparative statics (Milgrom and Shannon, 1994). Besides making directed technical change theory applicable to automation technology, the results clarify the general mechanisms, based on simple notions of complementarity, that underlie weak and strong relative bias. The second part applies these results to obtain novel insights about the endogenous determination of automation technology in a Roy-like assignment model.

The first part starts from a reduced-form characterization of wages and equilibrium technology that is shown to arise from a range of different microfoundations of endogenous technical change, including standard approaches from endogenous growth theory. Building on this

[2]Henceforth, I use the terms "endogenously directed technical change", "directed technical change", and "endogenous technical change" equivalently.

[3]At first glance, it may seem that Uzawa's theorem provides a justification for the restriction to labor-augmenting technology. But Uzawa's theorem only applies to the component of technology that grows over time on a balanced growth path, whereas the literature on endogenously directed technical change has mainly been concerned with the component of technology that is stationary on a balanced growth path, inducing changes in the stationary long-run distribution of (relative) wages. Moreover, with the labor share and the (risk-free) real interest rate declining over several decades (e.g. Karabarbounis and Neiman, 2014; Caballero, Farhi and Gourinchas, 2017), the general desirability for a model to generate balanced growth is no longer obvious.

reduced-form characterization, conditions are identified under which there is weak relative bias, that is, any increase in relative skill supply induces skill-biased technical change. The only essential condition is that the skill bias of technology is scale invariant, in the sense that a proportional change in the supply of all skill levels does not induce biased technical change. This can be guaranteed by a restriction close to homotheticity of aggregate production in all labor inputs, which is remarkably weak compared to existing results (e.g. Acemoglu, 2007, Theorem 1). Most importantly, the restriction to differentially labor-augmenting technology from previous work can be deleted without replacement.[4]

While an increase in the relative supply of skill tends to induce skill-biased technical change, it also has a direct effect on the wage distribution, which typically depresses skill premia. The second set of results provides necessary and sufficient conditions for the occurrence of strong relative bias, meaning that the effect of the induced technical change dominates the direct effect such that skill premia increase with relative skill supply. It is shown that the induced technical change effect dominates everywhere if and only if the aggregate production function is quasiconvex. Reversely, if and only if aggregate production is quasiconcave, the direct effect dominates everywhere. These conditions provide an interesting analogy to endogenous growth theory, where convexity of aggregate production along rays through the origin (that is, increasing returns to scale) is required to generate persistent growth in a wide class of models (cf. Romer, 1986). As in these models, the aggregate (quasi-)convexity requirement discovered here has implications for the market structures needed in a model to analyze the case where skill premia increase in relative skill supply. In particular, either deviations from perfect competition or external effects between firms' technologies are needed.

While my baseline results apply to local changes (in the sense of differential calculus) in two different labor inputs, I show that they transfer in natural ways to global changes in labor supply and to settings with arbitrarily many different types of labor.

I demonstrate how to apply my directed technical change results by analyzing the endogenous determination of automation technology in the Roy-like assignment model proposed by Teulings (1995) (see Costinot and Vogel, 2010 for decisive progress in comparative statics for this model), augmented to incorporate capital as an additional production factor as in Acemoglu and Autor (2011) or Feng and Graetz (2020). In the model, a continuum of differentially skilled workers and capital, taking the form of machines that perfectly substitute for labor in the production of tasks, are assigned to a continuum of tasks, which in turn are combined to produce a single final good. In line with recent forecasts on the future automation potential for different tasks (e.g. Frey and Osborne, 2017; Arntz, Gregory and Zierahn, 2016), capital is assumed to have comparative advantage in less complex tasks than labor, such that any increase in the set of tasks performed by capital (automation) displaces low-skilled workers from some of their previous tasks.[5]

[4]The results in this part of the paper imply a LeChatelier Principle for relative demand curves, analogous to the conventional LeChatelier Principle that applies to absolute demand curves (e.g. Milgrom and Roberts, 1996). For an explicit formulation of the implied LeChatelier Principle for relative demand see Loebbing (2016), an earlier version of the present paper.

[5]This assumption is also broadly supported by recent estimates of the impact of industrial robots (Graetz and Michaels, 2018; Acemoglu and Restrepo, 2020) and a wider set of automation technologies in US manufacturing (Lewis, 2011) on the structure of employment and wages.

I endogenize the productivity of capital in the model by allowing firms to choose capital productivity and the productivity by which they transform tasks into final goods subject to a technology frontier. While this specification of technology does not have the labor-augmenting form required by previous results, it is covered by the generalized directed technical change results from the first part of the paper. Using the results on weak relative bias, I find that any increase in relative skill supply induces an improvement in capital productivity, which in turn raises skill premia.

In the baseline model, where firms' technology choices depend on each other only via (competitive) market prices, aggregate production is quasiconcave. Applying my directed technical change results, this immediately implies that there cannot be strong relative bias, that is, the induced improvement in capital productivity cannot be strong enough to make skill premia increase in relative skill supply. This result is overturned once I allow for external effects between firms' technology choices.[6]

Finally, adopting an insight from Costinot and Vogel (2010), I show that, in the assignment model, the introduction of trade between two countries can be analyzed by the same tools as a change in labor supply. My results can thus be readily applied to study the impact of trade on automation technology. I find that trade with a skill-scarce country reduces capital productivity in a skill-abundant country, leading to a fall in skill premia. This counteracts the usual Heckscher-Ohlin effects that would determine the impact of trade on skill premia absent endogenous technology. Under strong relative bias, the novel directed technical change effect dominates and trade causes skill premia in the skill-abundant country to fall after adjustment of technology.

The remainder of the paper is structured as follows. Section 2 introduces the reduced-form characterization of wages and equilibrium technology that provides the basis for the general results on directed technical change in the following sections. Section 3 presents these results for local labor supply changes and two different levels of skill. Sections 4 and 5 generalize them to global labor supply changes and arbitrarily many levels of skill. Section 6 applies the results to endogenous automation technology in assignment models, and Section 7 concludes.

Related Literature The paper is related to several strands of the existing literature. The first part of the paper extends the literature on directed technical change and wage inequality (e.g. Acemoglu, 1998, 2002 and Kiley, 1999), generalizing the key theoretical results of that literature. Most closely related is Acemoglu (2007), who provides a directed technical change analysis on a similar level of generality. In contrast to the present paper, Acemoglu (2007) analyzes the effects of technical change induced by changes in the supply of a given skill level on the absolute wage of that skill, rather than on relative wages between different skills. From a purely theoretical perspective, the first part of the present paper can thus be viewed as the completion of a general theory of the effects of skill supply on the direction of technical change, with the first part on absolute wages given by Acemoglu (2007) and the second part on relative wages presented here. The analysis of relative wages is indispensable when the

[6]The same effect can be achieved in a model where technology, embodied in intermediate inputs, is supplied by a monopolistically competitive R&D sector, following the monopolistic competition approach in endogenous growth theory (see Online Appendix C.3.2).

goal is to study implications of endogenous technical change for wage inequality.

The second part of the paper bridges the gap between the literature on directed technical change and the more recent strand of work on technical change and wage inequality in Roy-like assignment models. Costinot and Vogel (2010), Acemoglu and Autor (2011), and Feng and Graetz (2020) analyze the impact of exogenous technical change on wage inequality in such models. More closely related are Acemoglu and Restrepo (2018a), Hémous and Olsen (2020), and Acemoglu and Restrepo (2019). Acemoglu and Restrepo (2018a) and Hémous and Olsen (2020) study the endogenous evolution of automation technology, but focus on its response to exogenous technology shocks rather than to changes in the structure of labor supply and international trade. Acemoglu and Restrepo (2019) analyze the effects of the demographic structure on endogenous automation, with the focus on the effects of automation on productivity and the labor share rather than on wage inequality.

The analysis of trade and automation in Section 6.4 is related to existing work on the effects of trade and technology on wage inequality. Part of this literature employs the assignment framework used here as well, but considers settings with exogenous technology (see Costinot and Vogel, 2015 for a survey of the use of assignment models in international trade).[7] Analyses of trade and wage inequality with endogenous technology are presented by Acemoglu (2003), Yeaple (2005), and Sampson (2014), but none of them uses the assignment model with labor-replacing capital employed here. All three papers find that trade (with a skill-scarce country) induces skill-biased technical change, which is opposite to my result. I discuss the differences between their approaches and my analysis in more detail at the end of Section 6.4.

2. A Simple Framework for Directed Technical Change

I analyze directed technical change in general equilibrium models with endogenous technology, exogenous labor supply, and a single consumption good. Acemoglu (2007) shows that, in a large class of such models, wages and technology are characterized by the same set of equations in equilibrium. Hence, instead of providing a complete, but necessarily specific, microfoundation of technology choices, I directly work with this general set of equilibrium conditions. This allows me to treat a diverse set of models within a simple, unifying framework. For microfounded models fitting into this framework, see Acemoglu (2007) and Online Appendix C.2.[8]

Let L denote the exogenous labor supply in the economy. In the main text, there is a finite number N of different skill types, such that $L = (L_1, L_2, ..., L_N) \in \mathbb{R}^N_{++}$, where L_s is the supply

[7] Closely related is parallel work by Krenz, Prettner and Strulik (2018) who provide a joint analysis of offshoring and automation in an assignment model with capital that perfectly substitutes for labor in task production. Their model, however, does not feature endogenous capital productivity and hence cannot generate strong relative bias and the ensuing reversal of the Heckscher-Ohlin effects.

[8] All models in Acemoglu (2007) and Online Appendix C.2 are static, but there is an equivalence between the equilibria of the static models and the constant growth paths of corresponding dynamic model versions. In particular, the models in Online Appendix C.2 can be extended to dynamic versions, which generate constant growth paths with stationary relative wages between skill groups. These relative wages are identical to the relative wages that prevail in equilibrium of the static models. My comparative statics results thus also apply on the constant growth paths of appropriately specified dynamic models. See Loebbing (2016, Section 3.2 and Appendix B) for details.

level for a given skill type $s \in S = \{1, 2, ..., N\}$. All results go through with a continuum of skills, but some clarifications are needed regarding the meaning of differentiation and partial derivatives in this case. Hence, I treat the continuum case separately in Appendix A.

Throughout the paper, I will call a higher index s a higher level of skill, and a type with a higher s a more skilled type of labor. While this suggests that wages should increase in s, none of my results requires a particular ordering of wages or that the ordering of wages remains constant when labor supply changes.

Production is described by the real-valued aggregate production function $F(L, \theta)$, where $\theta \in \Theta$ represents the (endogenous) technology and Θ denotes the set of feasible technologies. I impose the following assumptions throughout the analysis.

Assumption 1. *The set of feasible technologies Θ is a compact topological space. The aggregate production function F is continuous in technology θ and continuously differentiable in labor supply L. The gradient $\nabla_L F(L, \theta)$ is strictly positive everywhere.*

Compactness of Θ and continuity of F in θ ensure that there always exists a technology that maximizes aggregate production. Existence and strict positiveness of the marginal products of labor ensure that wages and wage ratios are always determined in equilibrium. Finally, the restriction that marginal products of labor are continuous is used exclusively in the proofs of Theorems 3 and 3′, and thus irrelevant for many of my results.[9]

Assumption 1 imposes hardly any restrictions on the set of feasible technologies Θ, supporting a general notion of technology. A technology θ can, for example, represent the allocation of production factors to tasks in an economy (see the example in Online Appendix C.3.1); a particular production technique that can be adopted by firms at no cost (see the example in Section C.3); or the distribution of costly investment into the quality of a range of intermediate inputs, each embodying a particular technology (see the example in Online Appendix C.3.2).

Since my results contrast situations where technology adjusts to changes in labor supply with situations where technology is fixed, it is useful to distinguish between an exogenous-technology equilibrium and an endogenous-technology equilibrium. In an exogenous-technology equilibrium, wages are determined by

$$w(L, \theta) = \nabla_L F(L, \theta) , \tag{1}$$

while technology θ is exogenous.

In an endogenous-technology equilibrium, technology is determined according to

$$\theta^*(L) \in \operatorname*{argmax}_{\theta \in \Theta} F(L, \theta) , \tag{2}$$

while wages again satisfy (1), but with the equilibrium technology $\theta^*(L)$ in place of θ. It is useful to introduce the notation

$$w^*(L) := w(L, \theta^*(L))$$

[9] It guarantees that the endogenous-technology production function $F^*(L)$ (see below) is absolutely continuous, which in turn allows me to apply an envelope theorem by Milgrom and Segal (2002).

for wages in endogenous-technology equilibrium. Similarly, let

$$F^*(L) := F(L, \theta^*(L))$$

denote aggregate production in endogenous-technology equilibrium. As mentioned above, microfoundations that give rise to these equilibrium conditions are presented in Acemoglu (2007) and Online Appendix C.2.

The equilibrium technology $\theta^*(L)$ as given by (2) may not be unique. In this case, I assume that $\theta^*(L)$ is some point-valued selection from the set of maximizers in (2). Whenever possible (i.e., whenever an order is defined and the maximizer set has a supremum), the selection should be made from the supremum of the maximizer set. Otherwise, it can be arbitrary. The results could also be derived in terms of sets of relative wages induced by sets of equilibrium technologies, but this would complicate the exposition without much apparent gain.

Before delving into the analysis, it is instructive to briefly review the most general existing results on directed technical change and wage inequality in the present environment. These results are structured along two hypotheses – weak and strong relative equilibrium bias of technology (Acemoglu, 2002, 2007) – and provide conditions for each of the two hypotheses to be true.

The results are restricted to a setting with two skill levels, $N = 2$, and local changes in labor supply, dL. The weak relative bias hypothesis states that any increase in relative skill supply induces skill-biased technical change. Acemoglu (2007) shows that the hypothesis is true if technology takes a labor-augmenting form. My results will imply this as a corollary.

Corollary 1 (Local Weak Bias with Two Skills and Labor-Augmenting Technology, Acemoglu 2007, Theorem 1)**.** *Suppose $N = 2$, $\Theta = \mathbb{R}^2_{++}$ and F takes the form*

$$F(L, \theta) = G(\theta_1 L_1, \theta_2 L_2) - C(\theta) ,$$

with G twice continuously differentiable, concave, and homothetic, and C twice continuously differentiable, strictly convex, and homothetic.

Then, for any initial labor supply L and any change dL such that $dL_2/L_2 \geq dL_1/L_1$ (an increase in relative skill supply), the induced technical change $d\theta^* = \nabla_L \theta^*(L) dL$ raises the skill premium:

$$\nabla_\theta \frac{w_2(L, \theta^*(L))}{w_1(L, \theta^*(L))} d\theta^* \geq 0 .$$

The central restriction here is that technology must take the labor-augmenting form $G(\theta_1 L_1, \theta_2 L_2)$. In Section 3, I show that this restriction can be dispensed with as long as the homotheticity restrictions are maintained.

While an increase in the relative supply of skilled labor induces skill-biased technical change, it also has a direct effect on the skill premium. If aggregate production is concave in labor, as is typically the case at a point of equilibrium, this direct effect is negative. It is thus natural to ask whether the negative direct effect or the positive induced technical change effect dominates; that is, whether, after adjustment of technology, the skill premium decreases or increases in relative skill supply.

The strong relative bias hypothesis states that the directed technical change effect dominates and the skill premium increases in relative skill supply. For labor-augmenting technologies, Acemoglu (2007) shows that this is the case whenever the elasticity of substitution between the two skill types is large enough.

Corollary 2 (Local Strong Bias with Two Skills and Labor-Augmenting Technology, Acemoglu 2007, Theorem 1)**.** *Suppose $N = 2$, $\Theta = \mathbb{R}^2_{++}$ and F takes the form*

$$F(L, \theta) = G(\theta_1 L_1, \theta_2 L_2) - C(\theta) ,$$

with G twice continuously differentiable, concave, and homothetic, and C twice continuously differentiable, strictly convex, and homothetic.

Denote the local elasticity of substitution between the two labor inputs by

$$\sigma(L, \theta) := -\frac{\partial \log (L_2/L_1)}{\partial \log (F_{L_2}/F_{L_1})}$$

and the local elasticity of substitution between the two technology variables in the cost function C by

$$\delta(L, \theta) := \frac{\partial \log (\theta_2/\theta_1)}{\partial \log (C_{\theta_2}/C_{\theta_1})} .$$

Then, for any initial labor supply L and any change dL such that $dL_2/L_2 \geq dL_1/L_1$, the skill premium increases after adjustment of technology if and only if the elasticities of substitution are large enough, that is,

$$\nabla_L \frac{w_2^*(L)}{w_1^*(L)} dL \geq 0$$

if and only if $\sigma(L, \theta^*(L)) - 1/\delta(L, \theta^*(L)) \geq 2$.

In Section 3, I provide a generalized condition for strong relative bias that applies beyond the case of labor-augmenting technology. When applied to the case of labor-augmenting technology, the generalized condition is, of course, equivalent to the condition in Corollary 2.

3. Directed Technical Change with Local Labor Supply Changes and Two Skill Levels

To facilitate the comparison with existing results, I develop my results at first for two skill levels and local labor supply changes, dropping only the restriction to labor-augmenting technology.

3.1. Weak Relative Bias

My results require the following scale-invariance condition on the skill bias of the equilibrium technology.

Assumption 2. *For any labor supply L and any $\lambda \in \mathbb{R}_{++}$,*

$$\frac{F_{L_2}(L,\theta^*(L))}{F_{L_1}(L,\theta^*(L))} = \frac{F_{L_2}(L,\theta^*(\lambda L))}{F_{L_1}(L,\theta^*(\lambda L))} \,.$$

In words, Assumption 2 says that a proportional change in labor supply levels does not induce biased technical change (i.e., technical change that alters the skill premium). I discuss in detail below which restrictions Assumption 2 imposes on the form of aggregate production F. At first, however, I show that Assumption 2 is sufficient for weak relative bias.

Proposition 1 (Local Weak Bias with Two Skills)**.** *Suppose that $N = 2$, $\Theta \subset \mathbb{R}^M$ (for arbitrary $M \in \mathbb{N}$), F is twice continuously differentiable, and the equilibrium technology θ^* is differentiable in labor supply L. Moreover, suppose that Assumption 2 holds.*

Then, for any initial labor supply L and any change in labor supply dL with $dL_2/L_2 \geq dL_1/L_1$, the induced technical change $d\theta^* = \nabla_L \theta^*(L) dL$ raises the skill premium:

$$\nabla_\theta \frac{w_2(L,\theta^*(L))}{w_1(L,\theta^*(L))} d\theta^* \geq 0 \,.$$

Proof. The proof (i) shows that Assumption 2 allows to focus on labor supply changes in direction of the isoquant, and (ii) applies an envelope argument along the isoquant to derive the desired result.

For the first step, note that we can decompose the labor supply change dL into a proportional component and a component that points along the isoquant. In particular, we write $dL = \widetilde{dL} + \lambda L$ for some $\lambda \in \mathbb{R}$. Since output is strictly increasing in labor, we can always find a λ such that the remainder $\widetilde{dL}$ leaves output unchanged, that is, $w^T \widetilde{dL} = 0$. Since dL is an increase in relative skill supply, this implies

$$w_2(L,\theta^*(L))\widetilde{dL}_2 = -w_1(L,\theta^*(L))\widetilde{dL}_1 \geq 0 \,. \tag{3}$$

The proportional component of dL, λL, induces technical change that leaves the skill premium unchanged by Assumption 2. Thus, the induced technical change effect on the skill premium will be the same for the change dL as for its component $\widetilde{dL}$, so we can restrict attention to $\widetilde{dL}$ in the following.

Consider now the second-order effects of $\widetilde{dL}$ on output with and without technology adjustment. With technology adjustment, the first-order effect is, by the envelope theorem,

$$\frac{d}{d\mu} F^*(L + \mu \widetilde{dL}) \bigg|_{\mu=0} \equiv \nabla_L F(L,\theta^*(L))\widetilde{dL} \,.$$

Hence, we obtain the second-order effect as

$$\frac{d^2}{(d\mu)^2} F^*(L + \mu \widetilde{dL}) \bigg|_{\mu=0} = \widetilde{dL}^T \nabla^2_{LL} F(L,\theta^*(L))\widetilde{dL} + \widetilde{d\theta}^{*T} \nabla^2_{L\theta} F(L,\theta^*(L))\widetilde{dL} \,,$$

where $\widetilde{d\theta}^*$ is the technical change induced by $\widetilde{dL}$. Without technology adjustment, we obtain

$$\left.\frac{d^2}{(d\mu)^2}F(L+\mu\widetilde{dL},\theta^*(L))\right|_{\mu=0} = \widetilde{dL}^T\nabla^2_{LL}F(L,\theta^*(L))\widetilde{dL} \; .$$

The two functions $F^*(.)$ and $F(.,\theta^*(L))$ coincide at L but F^* is greater everywhere else. So, F^* must be less concave than F at L (e.g. Dixit, 1990, pp. 113–114) and, hence,

$$\widetilde{d\theta}^{*T}\nabla^2_{L\theta}F(L,\theta^*(L))\widetilde{dL} \geq 0 \; .$$

This situation is illustrated in Figure 1. Replacing marginal products with wages and rearranging, we obtain

$$\nabla_\theta\left(w^T(L,\theta^*(L))\widetilde{dL}\right)\widetilde{d\theta}^* \geq 0 \; . \tag{4}$$

Equation (4) is the key to my results on weak relative bias. It states that technology will always adjust in a way that increases the output gains from the initial change in labor supply. Put differently, the induced technical change will always be complementary to the labor supply change. Extensions of equation (4) will also appear in the generalizations in Sections 4 and 5.

It is now easy to see that, for an increase in relative skill supply, the complementarity logic of equation (4) implies that the induced technical change must raise the skill premium. In particular, rearrange equation (4),

$$\nabla_\theta w_2(L,\theta^*(L))\widetilde{dL}_2\widetilde{d\theta}^* \geq -\nabla_\theta w_1(L,\theta^*(L))\widetilde{dL}_1\widetilde{d\theta}^* \; ,$$

and divide this by equation (3), assuming $\widetilde{dL} \neq 0$ (for $\widetilde{dL} = 0$ the proposition is trivially true), to obtain

$$\frac{\nabla_\theta w_2(L,\theta^*(L))\widetilde{d\theta}^*}{w_2(L,\theta^*(L))} \geq \frac{\nabla_\theta w_1(L,\theta^*(L))\widetilde{d\theta}^*}{w_1(L,\theta^*(L))} \; ,$$

which implies

$$\nabla_\theta\frac{w_2(L,\theta^*(L))}{w_1(L,\theta^*(L))}\widetilde{d\theta}^* \geq 0 \; .$$

□

By establishing weak relative bias, Proposition 1 essentially puts forward a Le Chatelier principle for relative, instead of absolute, demand curves. The original Le Chatelier principle as proposed by Samuelson (1947) states that long-run demand (when all inputs can adjust) is more elastic than short-run demand (when some inputs are fixed). Transferred to a general equilibrium environment where the supply of labor is inelastic, this implies that an increase in some type of labor reduces its (absolute) wage by more in the short run than in the long run, that is, the long-run adjustment of inputs raises the wage (see the absolute bias results in Acemoglu, 2007). Proposition 1 now establishes this result for the relative supply and the relative wage between two different types of labor.

The proof uses the same type of envelope argument that can also be used to prove the original Le Chatelier principle (e.g. Dixit, 1990, pp. 115–116), but applies it along the isoquant instead of in direction of a single input factor.

Weak Relative Bias

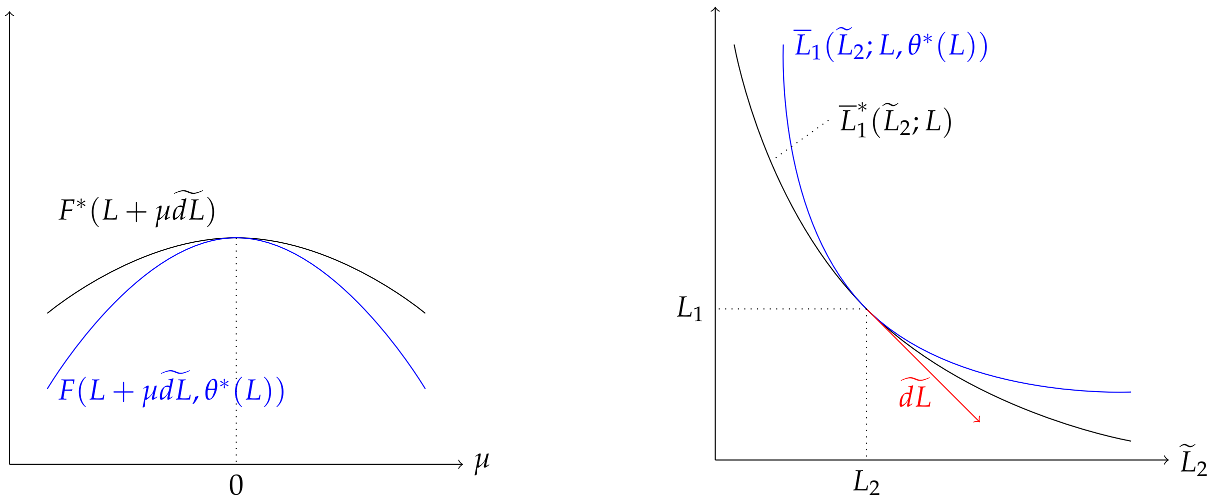

Figure 1. Illustration of the proof of Proposition 1. The left panel shows output along the line $L + \mu \widetilde{dL}$, which passes through the initial labor supply L (at $\mu = 0$) and is tangent to the isoquant at L. The marginal output gain when moving away from L declines more slowly if technology adjusts (the black curve) than if it is fixed (the blue curve), leading to weak relative bias. The right panel shows the isoquants through L when technology adjusts (black curve) and when it is fixed (blue curve). When moving in direction of $\widetilde{dL}$, the slope of the isoquant increases more slowly when technology adjusts, which provides an alternative way to prove weak relative bias.

An alternative perspective on the proof is provided in the right panel of Figure 1. If technology adjusts, the relative wage is given by the negative of the slope of the isoquant of $F^*(.)$ through L (the black curve); if technology is fixed, the relative wage is the negative of the slope of the isoquant of $F(., \theta^*(L))$ through L (the blue curve). The endogenous-technology isoquant (the one of F^*), however, is the lower envelope of a family of fixed-technology isoquants, including that of $F(., \theta^*(L))$. Hence, at the point of tangency L, the endogenous-technology isoquant is less convex than the fixed-technology isoquant. So, when moving from L in direction of L_2 along the isoquants, the skill premium falls by more when technology is fixed (i.e., along the fixed-technology isoquant) than when it is allowed to adjust (along the endogenous-technology isoquant). It follows that the adjustment of technology must raise the skill premium.

The intuition behind weak relative bias is readily derived from the proof and especially the key equation (4). An increase in relative skill supply raises the return to complementary changes in technology. Since complementarity is a symmetric relationship, such changes in technology in turn raise the marginal output gain from increasing relative skill supply, which is given by the skill premium.

While Proposition 1 establishes that Assumption 2 is sufficient for weak relative bias, the assumption is clearly also necessary if we look for a result applying to increases in relative skill supply in all directions. In particular, if there is a proportional change in labor supply inducing strictly skill-biased technical change (i.e., Assumption 2 is violated), we can always find both an increase in relative skill supply that induces skill-biased technical change (sufficiently close to the proportional change) and one that induces the opposite type of technical change (sufficiently close to the negative of the proportional change). Even in such cases, however, the second part of the proof of Proposition 1 remains valid and we still have weak relative bias for changes in labor supply along the isoquant.

What restrictions does Assumption 2 imply for the form of aggregate production F? To explore this, I present two alternative conditions on F each of which is sufficient for Assumption 2. The second of these conditions includes the case of labor-augmenting technology from the existing results in Corollaries 1 and 2 as a special case.

Condition 1. The aggregate production function $F(L, \theta)$ can be written as $g(f(L, \theta), L)$ for some function $g : \mathbb{R}_{++}^{N+1} \to \mathbb{R}$ that is strictly increasing in f, and a function $f : \mathbb{R}_{++}^{N} \times \Theta \to \mathbb{R}_{++}$, that is linear homogeneous in L.[10]

The nested structure of F ensures that the equilibrium technology θ^* can be found by maximizing the inner function f only. Since this inner function is linear homogeneous, the equilibrium technology is invariant to the scale of labor supply, that is, it does not respond to proportional changes in labor supply. This implies that Assumption 2 is satisfied.

Condition 1 is compatible with various different interpretations of technology θ: (i) it can describe the allocation of capital or some other scarce factor over a set of sectors or activities; (ii) it may represent a collection of productivity levels in different activities or sectors that can be adopted from an exogenous set of feasible productivity menus Θ; (iii) it may represent productivity levels that can be enhanced by innovation-specific inputs that are in fixed supply, such that technology is restricted by a constraint like $C(\theta) \leq \overline{C}$, where C is a strictly increasing cost function and $\overline{C}$ denotes the exogenous supply of innovation inputs. In all of these cases, Condition 1 essentially requires that the optimal factor allocation/technology menu/allocation of R&D resources is invariant to the scale of labor supply, which does not seem overly restrictive.

If, however, in case (iii) innovations are produced at least in parts from final good, F must include some cost of higher θ such that at some point (net) output ceases to increase in θ. If this cost is multiplicative with gross output, this case can still be accommodated by Condition 1.[11] If the cost is additive, however, there is a conflict with the homogeneity requirement in Condition 1 (the inner function f must be linear homogeneous in L). This is the case, for example, in the setting with labor-augmenting technology considered in Corollaries 1 and 2. To include such cases as well, I construct the following alternative condition, which again is sufficient for Assumption 2.

Condition 2. The aggregate production function $F(L, \theta)$ is strictly concave in θ and can be written as $G(L, \theta) - C(\theta)$ with $G : \mathbb{R}_{++}^{N} \times \Theta \to \mathbb{R}$ and $C : \Theta \to \mathbb{R}$ satisfying the following:

1. $G(L, \theta)$ can be written as $g(f(L, \theta))$ with $g : \mathbb{R} \to \mathbb{R}$ strictly increasing and $f : \mathbb{R}_{++}^{N} \times \Theta \to \mathbb{R}$ linear homogeneous in L.

2. $G(L, \theta)$ can be written as $\tilde{g}(\tilde{f}(L, \theta))$ with $\tilde{g} : \mathbb{R} \to \mathbb{R}$ strictly increasing and $\tilde{f} : \mathbb{R}_{++}^{N} \times \Theta \to \mathbb{R}$ linear homogeneous in θ.

[10] Note that this is slightly different from homotheticity of F in L because it allows the outer function to depend on L but not on θ (whereas homotheticity would allow dependence on θ but not on L).

[11] Suppose, for example, F can be written as $g(L, \theta)(1 - c(\theta))$ for some strictly increasing cost function c. If g is homogeneous in L, this example still satisfies Condition 1.

3. C is homothetic.

Strict concavity of F in θ implies that the equilibrium technology is uniquely characterized by first-order conditions. The three homotheticity-like conditions ensure that, when scaling labor supply by some factor and technology by a potentially different factor, then all marginal products of labor change by a common factor and all marginal products of technology also change by a common but potentially different factor. Therefore, after scaling labor supply up or down, we can always restore the first-order conditions by scaling technology up or down as well. As a result, scaling labor supply leads to a scaling of technology, which in turn leaves relative wages unaffected. So, Assumption 2 is satisfied.

Condition 2 includes the labor-augmenting technology case of Corollaries 1 and 2 as a special case. Proposition 1 thus provides a strict generalization of the existing weak relative bias result from Acemoglu (2007) presented in Corollary 1.

3.2. Strong Relative Bias

The strong bias condition from existing work (Corollary 2) is restricted to environments with labor-augmenting technology. Since weak relative bias holds much more generally, one might expect that also the strong bias condition has an insightful generalization. I provide such a generalization in the following.

The only restriction needed is that aggregate production is homothetic when accounting for the endogenous adjustment of technology.

Assumption 3. *The endogenous-technology production function F^* is homothetic.*

Just like Assumption 2 for weak relative bias, Assumption 3 is indispensable if we seek a condition for strong bias that pertains to all increases or decreases in relative skill supply instead of only to those pointing in a certain direction. If Assumption 3 is violated, we can always find both an increase in relative skill supply that raises the skill premium after adjustment of technology and one that lowers it.[12]

If Assumption 3 holds, however, all increases in relative skill supply have the same effect on the skill premium, irrespective of their direction in the L_1-L_2-plane. I show in the following that this effect is positive – that is, there is strong relative bias – if and only if the aggregate production function is locally quasiconvex.[13]

Definition 1. A function $g : \mathbb{R}^2 \to \mathbb{R}$, $(x, y) \mapsto g(x, y)$, is locally quasiconvex at (x, y) if and only if $d^2\bar{x}(\tilde{y}; x, y)/(d\tilde{y})^2|_{\tilde{y}=y} \leq 0$, where $\bar{x}(.; x, y) : Y \to \mathbb{R}$ is the isoquant given by $g(\bar{x}(\tilde{y}; x, y), \tilde{y}) = g(x, y)$ for all $\tilde{y} \in Y$ and some suitably defined domain Y.

In words, a function is locally quasiconvex if and only if its isoquant is locally concave in the sense of having a negative second derivative.

[12] Without Assumption 3, Proposition 2 below would still hold for changes in relative skill supply that point along the isoquant of F^*.

[13] The connection between strong relative bias and (quasi-)convexity is already partially anticipated in footnote 22 in Acemoglu (2007). There, Acemoglu (2007) notes that, in the case of labor-augmenting technology, strong relative bias arises if and only if a particular "modified production function" becomes convex. Here, I show that strong relative bias is indeed equivalent to quasiconvexity of the original (i.e., non-"modified") aggregate production function in much more general environments.

Proposition 2 (Local Strong Bias with Two Skills). *Suppose that $N = 2$, $\Theta \subset \mathbb{R}^M$ (for arbitrary $M \in \mathbb{N}$), F is twice continuously differentiable, and the equilibrium technology θ^* is differentiable in labor supply L. Moreover, suppose that Assumption 3 holds.*

Then, for any initial labor supply L and any change in labor supply dL with $dL_2/L_2 \geq dL_1/L_1$, the skill premium increases after adjustment of technology,

$$\nabla_L \frac{w_2^*(L)}{w_1^*(L)} dL \geq 0 \,,$$

if and only if the aggregate production function F^* is locally quasiconvex at L.

Proof. As in the proof of Proposition 1, we can decompose any increase in relative skill supply dL into a proportional component λL and a component $\widetilde{dL}$ that points along the isoquant of F^*, that is, $w^{*T}(L)\widetilde{dL} = 0$. By Assumption 3, the proportional component has no effect on the skill premium after adjustment of technology, so we can restrict attention to the isoquant component $\widetilde{dL}$.

Denote the isoquant of F^* through L by $\overline{L}_1^*(.;L)$. Its slope is equal to the negative of the skill premium:

$$\left.\frac{d\overline{L}_1^*(\widetilde{L}_2;L)}{d\widetilde{L}_2}\right|_{\widetilde{L}_2=L_2} = -\frac{F_{L_2}^*(\overline{L}_1^*(L_2;L),L_2)}{F_{L_1}^*(\overline{L}_1^*(L_2;L),L_2)} \,.$$

Consequently, the second derivative is the marginal change in the skill premium when moving along the isoquant:

$$\begin{aligned}
\left.\frac{d^2\overline{L}_1^*(\widetilde{L}_2;L)}{(d\widetilde{L}_2)^2}\right|_{\widetilde{L}_2=L_2} &= \left.\frac{d}{d\widetilde{L}_2}\left(-\frac{F_{L_2}^*(\overline{L}_1^*(\widetilde{L}_2;L),\widetilde{L}_2)}{F_{L_1}^*(\overline{L}_1^*(\widetilde{L}_2;L),\widetilde{L}_2)}\right)\right|_{\widetilde{L}_2=L_2} \\
&= -\left.\frac{d}{d\widetilde{L}_2}\frac{w_2^*(\overline{L}_1^*(\widetilde{L}_2;L),\widetilde{L}_2)}{w_1^*(\overline{L}_1^*(\widetilde{L}_2;L),\widetilde{L}_2)}\right|_{\widetilde{L}_2=L_2} \\
&= -\delta\nabla_L \frac{w_2^*(L)}{w_1^*(L)}\widetilde{dL}
\end{aligned}$$

for some $\delta \in \mathbb{R}_{++}$, where the last line follows from the fact that $\widetilde{dL}$ is proportional to $\left(d\overline{L}_1^*(\widetilde{L}_2;L)/d\widetilde{L}_2\Big|_{\widetilde{L}_2=L_2}, 1\right)$ by construction. This yields the equivalence between a locally concave isoquant of F^* and strong relative bias. □

Strong relative bias is illustrated in Figure 2. The figure basically shows the same graphs as Figure 1, but, in contrast to Figure 1, aggregate production F^* is convex along the isoquant (left panel) and, consequently, the endogenous-technology isoquant $\overline{L}_1^*$ is concave (right panel).

By establishing a link between convexity properties of aggregate production and strong relative bias, Proposition 2 reveals a parallel to endogenous growth theory. There, increasing returns to scale in aggregate production are necessary for persistent growth in a large class of models (cf. Romer, 1994; Acemoglu, 2009). While increasing returns to scale constitute a failure of concavity along lines through the origin, the failure of concavity required for strong relative bias concerns the isoquants of F^* and is in this sense orthogonal to returns to scale.

Strong Relative Bias

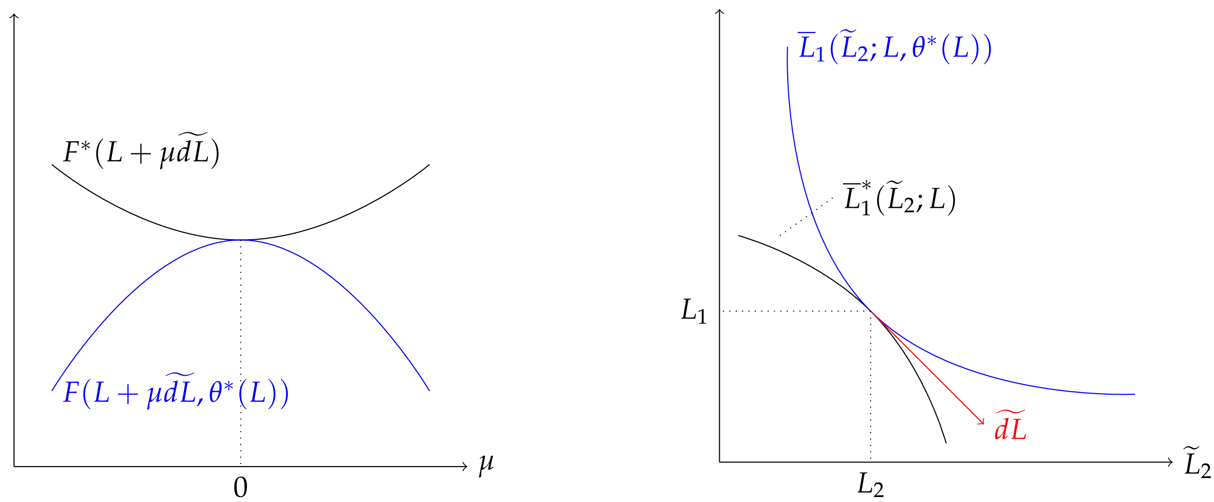

Figure 2. Illustration of strong relative bias. The panels reproduce the graphs from Figure 1, but now for the case of strong relative bias. In the left panel, output is convex if technology adjusts (the black curve). Hence, when moving from L in direction $\widetilde{dL}$, the output gain in this direction, and thereby the skill premium, increases. In the right panel, the isoquant is concave if technology adjusts (the black curve). Thus, when moving in direction $\widetilde{dL}$, the negative of the slope of the isoquant, and hence the skill premium, increases.

From a modeling perspective, Proposition 2 is informative about how (not) to set up a model that features strong relative bias. For example, in a setting where identical firms choose their technologies independently of each other, production functions must be concave in labor and technology around the equilibrium. By Proposition 2, strong relative bias cannot occur in such a model. In Online Appendix C.2, I discuss two ways to introduce strong relative bias, both of which create some form of interdependence between firms' technology choices. In the first approach, there are ad hoc spillovers between firms' technologies, reminiscent of learning-by-doing models of endogenous growth (e.g. Romer, 1986; Lucas, 1988). In the second approach, interdependence occurs via the market for technologies, where technology firms supply their innovations to final good firms, and non-rivalry of innovations implies that technology firms sell their ideas to all active final good firms at once. This specification follows monopolistic competition based models of endogenous growth such as Romer (1990) and Aghion and Howitt (1992). In both approaches, interdependence between firms' technologies breaks the requirement that production functions are jointly concave in labor and technology, enabling quasiconvexity and strong relative bias.

The baseline model in Online Appendix C.2.1, without strong relative bias, may also be interpreted as describing a process of pure technology adoption, whereas the other models incorporate some features of true innovation (such as spillovers from imperfect protection of an individual firms' knowledge, or imperfect competition from the partial protection of intellectual property). With this interpretation, Proposition 2 admits the conclusion that strong relative bias cannot arise from technology adoption alone but requires some portion of innovation.

4. Directed Technical Change with Global Labor Supply Changes

The above results move beyond labor-augmenting technology but still maintain the focus of previous results on local labor supply changes. In the following, I show how both results on weak and strong relative bias extend to global changes in labor supply.

4.1. Weak Relative Bias

With local labor supply changes, weak relative bias originates from the complementarity encapsulated in equation (4): the induced technical change is such that it raises the return to the underlying change in labor supply. This complementarity also arises with global labor supply changes: let L and L' denote the initial and the eventual labor supply, respectively; then, by definition of the equilibrium technology, the output gain of moving from L to L' must be greater under the new than under the old technology:

$$F(L', \theta^*(L')) - F(L, \theta^*(L')) \geq F(L', \theta^*(L)) - F(L, \theta^*(L)) \, . \tag{5}$$

The skill premium, however, is determined by the output gain from a local increase in relative skill supply, not from a global one. Thus, to relate the induced technical change to the skill premium, we need complementarity with local instead of global labor supply changes. While complementarity with the global change, as formalized in equation (5), implies complementarity with at least some local changes on the path from L to L', it does not necessarily do so for all of these local changes.[14] In particular, complementarity relationships may change on the way from L to L', such that the induced technical change is skill biased when evaluated at the initial labor supply but not at the new one.

Milgrom and Roberts (1996) face the same problem in their global extension of the Le Chatelier principle (for absolute instead of relative demand). They preclude reversals of complementarity by imposing quasisupermodularity and a single-crossing property on the production function.

I follow the same path here, with two differences. First, I order technologies according to their skill bias. In doing so, it appears that two different technologies may lead to the same skill premium and hence have the same skill bias. Thus, the resulting order is not antisymmetric, giving rise to a preorder instead of a partial order. Consequently, the definitions of a lattice and of quasisupermodularity must be adapted to the preorder environment as follows.

Definition 2 (Skill Bias)**.** For any two technologies $\theta, \theta' \in \Theta$, θ is skill biased relative to θ' if and only if

$$\frac{F_{L_2}(L, \theta)}{F_{L_1}(L, \theta)} \geq \frac{F_{L_2}(L, \theta')}{F_{L_1}(L, \theta')}$$

for all L. We write $\theta \succeq^{sb} \theta'$.

[14] Formally, with $dL := L' - L$, equation (5) implies that

$$\int_0^1 \left(w^T(L + \mu dL, \theta^*(L')) - w^T(L + \mu dL, \theta^*(L)) \right) dL \, d\mu \geq 0 \, .$$

The integrand must be positive at some but does not have to be positive at all μ. Thereby, we obtain a close counterpart to equation (4) at some but not necessarily all points between L and L'.

Definition 3 (Prelattice)**.** The preordered set $(\Theta, \succeq^{sb})$ is a prelattice if and only if any two technologies $\theta, \theta' \in \Theta$ have a supremum and an infimum in Θ.

Definition 4 (Prequasisupermodularity)**.** The function $F(L, \theta)$ is prequasisupermodular in θ under the preorder $\succeq^{sb}$ if, for any L and $\theta, \theta' \in \Theta$,

$$F(L, \underline{\theta}) \leq F(L, \theta) \text{ for all } \underline{\theta} \in \inf(\theta, \theta') \Rightarrow F(L, \theta') \leq F(L, \overline{\theta}) \text{ for some } \overline{\theta} \in \sup(\theta, \theta') ,$$

where $\inf(\theta, \theta')$ denotes the set of infima of θ and θ', and $\sup(\theta, \theta')$ denotes the set of suprema.

The second and more substantial difference to Milgrom and Roberts (1996) is that I can dispense with a single-crossing assumption between technology and labor. By ordering technologies according to their skill bias, I essentially get the effect of single crossing 'for free'.[15] Indeed, the following proposition shows that prequasisupermodularity alone is sufficient to extend weak relative bias to global labor supply changes.

Proposition 3 (Global Weak Bias with Two Skills)**.** *Suppose that $N = 2$, $(\Theta, \succeq^{sb})$ is a prelattice, and F is prequasisupermodular in θ under $\succeq^{sb}$. Moreover, suppose that Assumption 2 holds.*

Then, for any two labor supply vectors L and L' with $L_2'/L_1' \geq L_2/L_1$, technology is more skill biased under L' than under L, that is, $\theta^*(L') \succeq^{sb} \theta^*(L)$.

Proof. As for local weak relative bias, we first use Assumption 2 to focus on labor supply changes along the isoquant. In particular, by Assumption 2, we can scale the new labor supply L' up or down without changing the skill bias of the corresponding equilibrium technology. Hence, without loss of generality, we can assume that L' is on the same exogenous-technology isoquant as L, that is, $F(L, \theta^*(L)) = F(L', \theta^*(L))$.

The second step is to use prequasisupermodularity and the 'natural' complementarity between increases in relative skill supply and skill-biased technical change to show that the new equilibrium technology must be a supremum of the new and the old technology. For that, let $\overline{L}_1(\widetilde{L}_2; L, \theta^*(L))$ denote the isoquant through L at the exogenous technology $\theta^*(L)$, such that $F(\overline{L}_1(\widetilde{L}_2; L, \theta^*(L)), \widetilde{L}_2, \theta^*(L)) = F(L, \theta^*(L))$ for all $\widetilde{L}_2$. Note that this isoquant passes through both L and L', with $L_2' \geq L_2$.

Now, take any infimum $\underline{\theta} \in \inf(\theta^*(L), \theta^*(L'))$. Since $\theta^*(L)$ is skill biased relative to $\underline{\theta}$, we have

$$\frac{w_2(\overline{L}_1(\widetilde{L}_2; L, \theta^*(L)), \widetilde{L}_2, \theta^*(L))}{w_1(\overline{L}_1(\widetilde{L}_2; L, \theta^*(L)), \widetilde{L}_2, \theta^*(L))} \geq \frac{w_2(\overline{L}_1(\widetilde{L}_2; L, \theta^*(L)), \widetilde{L}_2, \underline{\theta})}{w_1(\overline{L}_1(\widetilde{L}_2; L, \theta^*(L)), \widetilde{L}_2, \underline{\theta})}$$

for all $\widetilde{L}_2$ in the support of the isoquant. Rearranging the inequality, we obtain:

$$\begin{aligned} 0 &\geq w_2(\overline{L}_1(\widetilde{L}_2; L, \theta^*(L)), \widetilde{L}_2, \underline{\theta}) - w_1(\overline{L}_1(\widetilde{L}_2; L, \theta^*(L)), \widetilde{L}_2, \underline{\theta}) \frac{w_2(\overline{L}_1(\widetilde{L}_2; L, \theta^*(L)), \widetilde{L}_2, \theta^*(L))}{w_1(\overline{L}_1(\widetilde{L}_2; L, \theta^*(L)), \widetilde{L}_2, \theta^*(L))} \\ &= \frac{d}{d\widetilde{L}_2} F(\overline{L}_1(\widetilde{L}_2; L, \theta^*(L)), \widetilde{L}_2, \underline{\theta}) . \end{aligned}$$

[15] Yet, the skill bias order, together with prequasisupermodularity, does not imply single crossing between relative skill supply and technology. See Online Appendix C.1 for a more detailed comparison with standard results from monotone comparative statics.

Hence, when moving along the isoquant from L to L', output under technology $\underline{\theta}$ declines. We thus obtain the following order of output levels,

$$F(L', \underline{\theta}) \leq F(L, \underline{\theta}) \leq F(L, \theta^*(L)) = F(L', \theta^*(L)) ,$$

where the second inequality stems from the equilibrium technology condition (2). Since these inequalities apply to all infima of $\theta^*(L)$ and $\theta^*(L')$, the outer (in)equalities and prequasisupermodularity imply that there must be a supremum $\overline{\theta} \in \sup(\theta^*(L), \theta^*(L'))$ with

$$F(L', \overline{\theta}) \geq F(L', \theta^*(L')) .$$

So, $\overline{\theta}$ is an element of the maximizer set in the equilibrium technology condition (2). Since F is continuous and prequasisupermodular in θ, the maximizer set in (2) is a complete subprelattice of Θ.[16] For this case, we assumed that the equilibrium technology $\theta^*(L')$ is selected from the supremum of the maximizer set. By transitivity, $\theta^*(L')$ must then also be contained in the supremum of $\theta^*(L)$ and $\theta^*(L')$, which completes the proof. □

4.2. Strong Relative Bias

When extending strong relative bias to global labor supply changes, there is no need to prevent changes in complementarity relationships via quasisupermodularity. Instead, if the endogenous-technology wage function w^* is differentiable almost everywhere, we can simply integrate over all marginal changes in the skill premium when moving from L to L' and apply the local results from Proposition 2 along the path.

We thus obtain that any (global) increase in relative skill supply raises the skill premium after adjustment of technology (i.e., there is global strong bias everywhere) if and only if the aggregate production function F^* is quasiconvex. Analogously, any increase in relative skill supply reduces the skill premium after adjustment of technology (i.e., there is global strong bias nowhere) if and only if F^* is quasiconcave. Given that conditions for the existence of strong bias, rather than for its absence, are of particular interest, the contrapositive of the latter statement seems more interesting: there exists an increase in relative skill supply that raises the skill premium after adjustment of technology if and only if F^* is not quasiconcave.

Proposition 4 establishes that these insights hold even if wages are not differentiable.

Proposition 4 (Global Strong Bias with Two Skills)**.** *Suppose that $N = 2$ and Assumption 3 is satisfied. Then, the following holds.*

1. *There exists an increase in relative skill supply that strictly raises the skill premium after adjustment of technology if and only if F^* is not quasiconcave.*

2. *Any increase in relative skill supply raises the skill premium after adjustment of technology if and only if F^* is quasiconvex.*

Proof. As described above, for the case where the wage function w^* is differentiable almost everywhere, the proposition follows immediately from path integration of relative wage changes

[16] See, for example, Corollary 2 and the subsequent discussion in Milgrom and Shannon (1994).

and application of Proposition 2. The general case is covered by the even more general Theorem 3 presented in the next section. □

Proposition 4 basically provides the same insights as the local strong bias condition in Proposition 2. It corroborates that a failure of quasiconcavity of aggregate production is key for strong relative bias, with all the implications discussed above.

5. Directed Technical Change with Many Skill Levels

In environments with more than two different types of labor, the results for two skill levels can in principle be applied to any pair of labor types, holding all other types' supply levels constant. When applied to each pair from a potentially large set of labor types, however, Assumptions 2 and 3 become increasingly restrictive. So, in the following, I present extensions of the two-skill results without imposing Assumptions 2 and 3 to hold pairwise. The main insights from the two-skill case generalize in fairly natural ways.

5.1. Weak Relative Bias

I present two results on weak relative bias for many skills. The first applies to local labor supply changes in arbitrary direction, imposing only minimal restrictions on the production function. The drawback is that it is only informative about the ratio of (weighted) average wages within two potentially coarse groups of labor types. The second result makes predictions about wage ratios for every pair of labor types and global changes in labor supply, but requires substantially stronger restrictions on the production function and the direction of labor supply changes.

For the first result, I impose a many-skill equivalent of Assumption 2, saying that a proportional change in all labor types shall not induce technical change that affects relative wages.

Assumption 4. *For any labor supply L, any $\lambda \in \mathbb{R}_{++}$, and any two skill levels s and $\widetilde{s}$,*

$$\frac{F_{L_s}(L,\theta^*(L))}{F_{L_{\widetilde{s}}}(L,\theta^*(L))} = \frac{F_{L_s}(L,\theta^*(\lambda L))}{F_{L_{\widetilde{s}}}(L,\theta^*(\lambda L))} .$$

Under this assumption, we can sort skill types into two exhaustive but otherwise arbitrary groups and prove that a version of weak relative bias holds between these groups.

Theorem 1 (Local Weak Bias with Many Skills)**.** *Suppose that $\Theta \subset \mathbb{R}^M$ (for arbitrary $M \in \mathbb{N}$), F is twice continuously differentiable, and the equilibrium technology θ^* is differentiable in labor supply L. Moreover, suppose that Assumption 4 holds.*

Consider a partition of the skill set $\{S^{(1)}, S^{(2)}\}$, an initial labor supply L, and a change in labor supply dL. Let $\widetilde{dL}$ denote the labor supply change that leads to equivalent changes in relative labor supply but leaves output unchanged, that is, $\widetilde{dL} = dL + \lambda L$ for $\lambda \in \mathbb{R}$ such that $w^T(L,\theta^*(L))\widetilde{dL} = 0$. Suppose that $\sum_{s\in S^{(i)}} \widetilde{dL}_s \neq 0$ for $i = 1, 2$.

Then, if $\sum_{s\in S^{(2)}} w_s(L,\theta^*(L))\widetilde{dL}_s > 0$, the induced technical change $d\theta^* = \nabla_L\theta^*(L)dL$ raises the weighted average wage in group $S^{(2)}$ relative to the weighted average wage in group $S^{(1)}$,

$$\nabla_\theta \frac{\overline{w}_{S^{(2)},\widetilde{dL}}(L,\theta^*(L))}{\overline{w}_{S^{(1)},\widetilde{dL}}(L,\theta^*(L))} d\theta^* \geq 0 \,,$$

where the weighted average wages are given by, for $i = 1, 2$,

$$\overline{w}_{S^{(i)},\widetilde{dL}}(L,\theta^*(L)) = \left| \sum_{s\in S^{(i)}} w_s(L,\theta^*(L)) \frac{\widetilde{dL}_s}{\sum_{\tilde{s}\in S^{(i)}} \widetilde{dL}_{\tilde{s}}} \right| \,.$$

Proof. The proof follows closely the proof of the local weak bias result for two skills (Proposition 1), replacing individual wages by weighted average wages within the two groups.

First, by the same arguments as for Proposition 1, we can focus on the technical change $\widetilde{d\theta}^*$ induced by the re-scaled labor supply change $\widetilde{dL}$ instead of the technical change $d\theta^*$ induced by dL. (The relative wage effects of $\widetilde{d\theta}^*$ and $d\theta^*$ are the same by Assumption 4.)

Next, the envelope arguments leading to equation (4) in the proof of Proposition 1 are independent of the dimensionality of labor supply. So, equation (4), showcasing the complementarity between labor supply change and induced technical change, holds here as well:

$$\nabla_\theta \left(w^T(L,\theta^*(L))\widetilde{dL} \right) \widetilde{d\theta}^* \geq 0 \,.$$

Separating the wages of the two skill groups $S^{(1)}$ and $S^{(2)}$, we obtain:

$$\nabla_\theta \left(w^T_{S^{(2)}}(L,\theta^*(L))\widetilde{dL}_{S^{(2)}} \right) \widetilde{d\theta}^* \geq \nabla_\theta \left(-w^T_{S^{(1)}}(L,\theta^*(L))\widetilde{dL}_{S^{(1)}} \right) \widetilde{d\theta}^* \,, \tag{6}$$

where the subscript $S^{(i)}$, for $i = 1, 2$, indicates that the corresponding vector contains the entries for all skills in $S^{(i)}$, and only those. Moreover, the labor supply change $\widetilde{dL}$ satisfies, by construction and by hypothesis,

$$w^T_{S^{(2)}}(L,\theta^*(L))\widetilde{dL}_{S^{(2)}} = -w^T_{S^{(1)}}(L,\theta^*(L))\widetilde{dL}_{S^{(1)}} > 0 \,. \tag{7}$$

Dividing (7) by (6) yields

$$\frac{\nabla_\theta \left(w^T_{S^{(2)}}(L,\theta^*(L))\widetilde{dL}_{S^{(2)}} \right) \widetilde{d\theta}^*}{w^T_{S^{(2)}}(L,\theta^*(L))\widetilde{dL}_{S^{(2)}}} \geq \frac{\nabla_\theta \left(-w^T_{S^{(1)}}(L,\theta^*(L))\widetilde{dL}_{S^{(1)}} \right) \widetilde{d\theta}^*}{-w^T_{S^{(1)}}(L,\theta^*(L))\widetilde{dL}_{S^{(1)}}} \,,$$

which implies that

$$\nabla_\theta \frac{w^T_{S^{(2)}}(L,\theta^*(L))\widetilde{dL}_{S^{(2)}}}{-w^T_{S^{(1)}}(L,\theta^*(L))\widetilde{dL}_{S^{(1)}}} \widetilde{d\theta}^* \geq 0 \,.$$

Multiplying by $|\sum_{s\in S^{(1)}}(-\widetilde{dL}_{S^{(1)}})| / |\sum_{s\in S^{(2)}}(-\widetilde{dL}_{S^{(2)}})|$ yields the desired result. □

Theorem 1 provides a group-level version of weak relative bias: a change dL that raises the effective relative labor supply of some group of skills $S^{(2)}$ – in the sense that the transformed labor supply change $\widetilde{dL}_{S^{(2)}}$ of this group has a positive effect on output – induces technical change that raises the weighted average wage of this group relative to the remaining skills. The weights in the average wages are determined by the transformed labor supply change $\widetilde{dL}$.

Due to the central role of the transformed labor supply change, Theorem 1 is somewhat difficult to interpret in full generality.[17] I therefore discuss two more transparent special cases in the following. First, suppose that labor supply changes are proportional within each group, with the common rate of change being larger in group $S^{(2)}$ than in group $S^{(1)}$. Then, the transformed labor supply change also takes the simple form $\widetilde{dL}_s = \lambda_i L_s$ for all $s \in S^{(i)}$, with $\lambda_2 > 0 > \lambda_1$. Consequently, the weights simplify to the usual quantity-based weights $L_s / \sum_{\tilde{s} \in S^{(i)}} L_{\tilde{s}}$. So, in this case, an increase in the relative supply of skills in group $S^{(2)}$ induces technical change that raises average earnings per unit of labor in group $S^{(2)}$ relative to average earnings per unit of labor in the remaining group of skills; equivalently, the induced technical change raises group $S^{(2)}$'s share of total earnings.

A second important special case is obtained when the supply of only one skill type $\tilde{s}$ increases while all other supply levels are constant. Choosing $S^{(2)} = \{\tilde{s}\}$, the transformed labor supply change has the same form as in the proportional case above – indeed, this is again a special case of the proportional case. Moreover, the weighted average wage in $S^{(2)}$ is just type $\tilde{s}$'s wage and the weighted average wage in $S^{(1)}$ is earnings per unit of labor in this group. As a result, an increase in $\tilde{s}$'s labor supply induces technical change that raises $\tilde{s}$'s wage relative to earnings per unit of labor among all other skill types, or, equivalently, $\tilde{s}$'s share of total earnings.

Corollary 3 (Local Weak Bias with Many Skills)**.** *Suppose that $\Theta \subset \mathbb{R}^M$ (for arbitrary $M \in \mathbb{N}$), F is twice continuously differentiable, and the equilibrium technology θ^* is differentiable in labor supply L. Moreover, suppose that Assumption 4 holds.*

Then, for any initial labor supply L, an increase in some type $\tilde{s}$'s labor supply induces technical change $d\theta^*$ that raises this type's wage relative to average earnings per labor unit among all other types,

$$\nabla_\theta \frac{w_{\tilde{s}}(L, \theta^*(L))}{\overline{w}_{-\tilde{s}}(L, \theta^*(L))} d\theta^* \geq 0 \,,$$

where

$$\overline{w}_{-\tilde{s}}(L, \theta^*(L)) = \sum_{s \in S \setminus \{\tilde{s}\}} w_s(L, \theta^*(L)) \frac{L_s}{\sum_{s' \in S \setminus \{\tilde{s}\}} L_{s'}} \,.$$

To move beyond changes in average wages and provide a more detailed account of how the induced technical change affects the wage distribution, further restrictions are needed. I present now a set of conditions under which any pervasive increase in relative skill supply induces technical change that leads to a pervasive increase in skill premia.

A pervasive increase in relative skill supply is defined, following Costinot and Vogel (2010), as a labor supply change such that the supply of more skilled workers increases relative to the supply of less skilled workers for any pair of skill levels. Similarly, a pervasive increase in skill premia is a change in skill premia such that the wage of more skilled workers increases relative

[17] If the initial labor supply change dL does not vary systematically between the two groups, or if the two groups have very unequal sizes in terms of initial income shares, the transformed change $\widetilde{dL}_s$ may even switch its sign within a group and, consequently, some weights in the average wages may be negative.

to the wage of less skilled workers for any pair of skills. Accordingly, I define pervasively skill-biased technical change as follows.

Definition 5 (Pervasive Skill Bias)**.** For any two technologies $\theta, \theta' \in \Theta$, θ is pervasively skill biased relative to θ' if and only if

$$\frac{F_{L_s}(L,\theta)}{F_{L_{s'}}(L,\theta)} \geq \frac{F_{L_s}(L,\theta')}{F_{L_{s'}}(L,\theta')}$$

for all $s \geq s'$ and for all L. We write $\theta \succeq^{psb} \theta'$.

With this definition of skill bias, we obtain an extension of the global weak bias result of Proposition 3.

Theorem 2 (Global Weak Bias with Many Skills)**.** *Suppose that* $(\Theta, \succeq^{psb})$ *is a prelattice,* F *is prequasisupermodular in* θ *under* $\succeq^{psb}$*, and Assumption 4 holds.*

Then, for any two labor supply vectors L *and* L' *with* $L'_s/L'_{s'} \geq L_s/L_{s'}$ *for all* $s \geq s'$*, the technology under* L' *is pervasively skill biased relative to the technology under* L*, that is,* $\theta^*(L') \succeq^{psb} \theta^*(L)$.

Sketch of proof. The proof uses basically the same reasoning as the proof of the two-skill case (i.e., Proposition 3). All details are presented in Appendix B.1. The basic steps are as follows.

First, by the usual arguments, Assumption 4 allows to focus on L' on the exogenous-technology isoquant that passes through L.

Next, we take an infimum $\underline{\theta}$ of $\theta^*(L)$ and $\theta^*(L')$ and note that $F(L,\underline{\theta}) \leq F(L,\theta^*(L))$. We then show that, since $\theta^*(L)$ is pervasively skill biased relative to $\underline{\theta}$, the output gain when moving from L to L' is greater under $\theta^*(L)$ than under $\underline{\theta}$. This yields $F(L',\underline{\theta}) \leq F(L',\theta^*(L))$.

By prequasisupermodularity, it follows that there must be a supremum $\overline{\theta}$ of $\theta^*(L)$ and $\theta^*(L')$ such that $F(L',\overline{\theta}) \geq F(L',\theta^*(L'))$. This implies, by the same arguments as in the proof of Proposition 3, that $\theta^*(L')$ itself must be a supremum and, hence, $\theta^*(L') \succeq^{psb} \theta^*(L)$. □

Theorem 2 provides a direct extension of the result for two skills in Proposition 3. It is important, however, to note that the requirement of prequasisupermodularity becomes substantially more restrictive when applied to environments with many skills. To see this, note that prequasisupermodularity is automatically satisfied in situations where any two technologies can be ordered according to their skill bias. So, prequasisupermodularity becomes restrictive only when the skill-bias order is incomplete.

With two skills, the only reason why the skill-bias order can be incomplete is that relative wage effects are reversed when moving through the labor supply space: a given technical change raises the skill premium at some labor input but lowers it at another input. Thus, prequasisupermodularity is satisfied as soon as the relative wage effects of technical change are stable (in terms of sign, not magnitude) over the labor supply space.

With many skills, this is not enough: there may be changes in technology that do not affect skill premia uniformly but, instead, raise them in some part and lower them in another part of the wage distribution. Such technologies cannot be ranked by skill bias, even if technical change effects are stable in the above sense. In this case, prequasisupermodularity requires that technical changes that raise skill premia in different parts of the wage distribution must not be substitutes. For illustration, imagine two technical changes, one that raises skill premia in the upper part and one that raises skill premia in the lower part of the wage distribution. Prequasisupermodularity requires that, if one of these changes raises output absent the other, it must still raise output when the other change has been implemented.

While these restrictions are substantial, Section 6 and Online Appendix C.3 provide natural examples where they are satisfied.

As a final extension, I show that Theorem 2 also holds when the skill set is not finite but a continuum. The arguments from the finite case are essentially unchanged with a continuum of skills, but their transfer requires some mathematical clarification. I provide this transfer in Appendix A.

5.2. Strong Relative Bias

For strong relative bias, the transfer from local to global labor supply changes does not require any additional restrictions (see the two-skills case above). Therefore, I directly extend the global strong bias result for two skills, Proposition 4, to the case with an arbitrary number of skills. Indeed, Theorem 3 provides a straightforward generalization of Proposition 4.

Theorem 3 (Global Strong Bias with Many Skills)**.** *Suppose Assumption 3 holds. Then, the following is true.*

1. *If there exists a pervasive increase in relative skill supply that strictly raises all skill premia after adjustment of technology, then F^* is not quasiconcave.*

   Moreover, if F^* is not quasiconcave on some line along which relative skill supply increases pervasively, then there exists a pervasive increase in relative skill supply that does not lower all skill premia after adjustment of technology.

2. *If every pervasive increase in relative skill supply raises all skill premia after adjustment of technology, then F^* is quasiconvex on all lines along which relative skill supply increases pervasively.*

   Moreover, if F^* is quasiconvex, then no pervasive increase in relative skill supply strictly lowers all skill premia after adjustment of technology.

Proof. See Appendix B.2. □

Following Proposition 4, the first part of Theorem 3 connects strong relative bias – here in the sense that a pervasive increase in relative skill supply leads to a pervasive increase in skill premia – to a failure of quasiconcavity of the aggregate production function F^*. As in the two-skills case, a failure of quasiconcavity is necessary for strong relative bias. Unlike with two skills, however, it is not generally sufficient. This is for two reasons. First, with many skill levels, a pervasive increase in relative skill supply may increase some and decrease other skill premia. Second, quasiconcavity may fail only in directions, which are not considered in the theorem, that is, directions along which some skill supply ratios rise while others fall. In consequence, there is only a partial converse: without quasiconcavity on a line along which relative skill supply rises, some pervasive increase in relative skill supply does not lower all skill premia. As soon as there are only two skill levels, this becomes a full converse: with two skills there is only one skill premium and if this does not fall, it must rise; moreover, any direction in the L_1-L_2-plane is one along which relative skill supply either (weakly) falls or rises.

Essentially the same adjustments are necessary in the second part of Theorem 3, connecting strong relative bias to quasiconvexity of F^*. Again, if there are only two skill levels, the statement collapses to the second part of Proposition 4.

6. Endogenous Automation Technology

I now demonstrate how to apply the results of the previous sections by analyzing directed technical change in assignment models where different types of labor and capital are assigned endogenously to a range of tasks. Such models provide a natural formalization of automation, whereby labor is replaced by capital in the production of certain tasks.

Important technology variables in these models, such as the productivity of capital, do not have the labor-augmenting form required by existing results on directed technical change. Hence, the generality of my results is indispensable in the following analysis.

Relative to settings with labor-augmenting technology, a directed technical change analysis in assignment models with automation technology has a number of benefits.[18] First, the results align well with intuitive notions of technical change. Second, they can be tested directly in empirical work, as labor-replacing technology variables can be identified with empirical measures of concrete automation technologies.[19] Third, they make statements about a form of technical change that is widely perceived to be among the most important determinants of future changes in the employment and wage structure. Finally, the literature on assignment models of the type analyzed here is growing rapidly, with applications in labor (e.g. Acemoglu and Autor, 2011), trade (e.g. Costinot and Vogel, 2010), growth (e.g. Acemoglu and Restrepo, 2018a), and public economics (e.g. Rothschild and Scheuer, 2013). Bringing results on directed technical change to the assignment environment keeps them connected to the newest strand of the theoretical literature on wage and income inequality.

In the main text, I focus on the endogenous determination of capital productivity in a setting where firms choose their technologies from a predetermined set. Online Appendix C.3 provides a more extensive treatment of the assignment framework and shows that the results also hold in a setting where technologies are embodied in intermediate inputs produced by an R&D sector that controls the quality of the intermediate inputs. Moreover, in Online Appendix C.3.1, I show that my directed technical change results can also be used to study the role of the assignment of factors to tasks itself, demonstrating how general the notion of technology used in the previous sections is.

[18]See Acemoglu and Restrepo (2018b) for a complementary list of advantages of the approach with labor-replacing technology.

[19]See for example the use of counts of industrial robots as a measure for automation technology by Graetz and Michaels (2018); Acemoglu and Restrepo (2020); Dauth et al. (2019); Abeliansky and Prettner (2017); Acemoglu and Restrepo (2019), the use of survey data on the adoption of various automation technologies in manufacturing by Lewis (2011), and the use of data on harvesting machines in agriculture by Clemens, Lewis and Postel (2018).

6.1. Setup

The analysis builds on the assignment model by Teulings (1995), augmented to incorporate capital as an additional production factor. There is a continuum of tasks (or intermediate goods), indexed by $x \in X = [0,1]$, and a single final good. A continuum of competitive firms produces the final good out of tasks according to

$$Y = \beta \left(\int_0^1 Y_x^{\frac{\epsilon-1}{\epsilon}} \, dx \right)^{\frac{\epsilon}{\epsilon-1}}$$

with $\beta > 0$, and $\epsilon > 0$ being the elasticity of substitution across tasks. Tasks in turn are produced linearly from capital and labor,

$$Y_x = \alpha K_x + \int_0^1 \gamma(s,x) L_{s,x} \, ds \, ,$$

where K_x denotes the amount (or density) of capital assigned to task x, $L_{s,x}$ is the amount of labor of skill s assigned to task x (or the joint density of labor over skills and tasks), and $\alpha > 0$ and $\gamma(s,x) > 0$ are (task-specific) productivities of capital and the differentially skilled types of labor. The wage of one unit of labor of skill s is denoted by w_s.

There is a continuum of skills, indexed by $s \in S = [0,1]$, and labor supply $L = \{L_s\}_{s \in S}$ (or the marginal density of labor over skills) is exogenous. The total amount of capital is denoted by $K = \int_0^1 K_x \, dx$. Capital is produced at marginal cost r from final good. This mimics the steady state of dynamic models in which capital is accumulated over time and the long-run interest rate is fixed by preferences and depreciation.

The productivities of capital, α, and final good production, β, are endogenous. In particular, firms choose their productivity levels subject to the frontier

$$g(\alpha, \beta) \leq \overline{g} \, , \tag{8}$$

where g is continuous, strictly increasing, and strictly convex. For later reference, denote the combination of productivities that maximizes their product $\alpha\beta$ by

$$(\overline{\alpha}, \overline{\beta}) \in \underset{(\alpha,\beta) \in \mathbb{R}^2_{++}}{\operatorname{argmax}} \, \{\alpha\beta \mid g(\alpha,\beta) \leq \overline{g}\} \, . \tag{9}$$

I show below that we can neglect all productivity pairs with $\alpha \geq \overline{\alpha}$, as those are never chosen by firms.[20]

An exogenous-technology equilibrium consists of wages for each skill type, a joint distribution of labor over tasks and skills, and a distribution of capital over tasks, such that labor markets clear and firms maximize profits, given exogenous productivities α and β.[21] An endogenous-technology equilibrium is an exogenous-technology equilibrium for productivi-

[20] A further restriction needed is that the cost of capital respects the lower bound $r > \overline{\alpha}\overline{\beta}$. This is because final good and task production are linear in capital while capital production is linear in final good. Such linearity in circular production may enable infinite output, analogously to unbounded growth of the AK-type in a dynamic model, if the marginal cost of capital is too low.

[21] I focus on exogenous-technology equilibria in which the productivities α and β are symmetric across firms.

ties α and β that maximize firm profits subject to the productivity frontier (8).

In equilibrium, factors are assigned to tasks according to their comparative advantage, which is determined by the labor productivity schedule $\gamma(s,x)$. I assume that γ is twice differentiable and satisfies

$$\frac{\partial^2 \log \gamma(s,x)}{\partial s \partial x} > 0 \quad \text{and} \quad \frac{\partial \log \gamma(s,x)}{\partial x} > 0 \tag{10}$$

for all s, x. The first part ensures that more skilled workers have comparative advantage in tasks with a higher index x (more complex tasks, henceforth). Since the productivity of capital α is constant across tasks (a normalization without material impact on the results), the second part implies that every type of labor has comparative advantage versus capital in more complex tasks.[22]

The pattern of comparative advantage induces a simple assignment rule for factors to tasks. In particular, for given labor inputs L and productivities α, β, the profit-maximizing assignment is characterized by (i) a threshold task $\tilde{x}$ such that all tasks below $\tilde{x}$ are produced by capital and all tasks above by labor; (ii) a strictly increasing and onto matching function $m : S \to [\tilde{x}, 1]$ such that $m(s)$ denotes the task assigned to workers of skill s. This characterization follows from a direct extension of the arguments in Costinot and Vogel (2010).[23]

Now, let $F(L, \alpha, \beta)$ denote net output $Y - r\int_0^1 K_x \, dx$ under the profit-maximizing assignment rules $\tilde{x}$, $\{K_x\}_{x \in [0,\tilde{x}]}$, and m, given labor inputs L and productivities α, β. In any symmetric exogenous-technology equilibrium, wages must satisfy, for all s,

$$w_s(L, \alpha, \beta) = F_s(L, \alpha, \beta) \ , \tag{11}$$

where $F_s(L, \alpha, \beta)$ denotes the Gateaux derivative of F in direction of L_s. This derivative corresponds to the notion of the marginal product of labor of skill s in the case with a continuum of skills.[24]

In a symmetric endogenous-technology equilibrium, where firms choose productivities to maximize profits, we must have

$$(\alpha^*(L), \beta^*(L)) \in \underset{\alpha,\beta}{\operatorname{argmax}} \{F(L, \alpha, \beta) \mid g(\alpha, \beta) \leq \bar{g}\} \tag{12}$$

and, for all s,

$$w_s^*(L) = w_s(L, \alpha^*(L), \beta^*(L)) = F_s(L, \alpha^*(L), \beta^*(L)) \ .$$

Symmetric exogenous- and endogenous-technology equilibria exist for any labor input L if and only if the endogenous-technology production function $F^*(L)$ is concave, which I assume henceforth.[25]

Assumption 5. *The endogenous-technology production function $F^*(L) := F(L, \alpha^*(L), \beta^*(L))$ is con-*

[22] See Online Appendix C.3.1 for a discussion of these assumptions.

[23] See Lemma 5 in Online Appendix C.3.1 for details.

[24] Appendix A provides a detailed explanation of derivative concepts in the continuum case.

[25] The proof of this statement uses standard arguments and is provided in Online Appendix C.2 (Observation 1) for a general class of directed technical change models.

cave.

At exogenous productivities, F is always concave in L in the present model. For concavity of the envelope F^*, we need that the productivity frontier g is sufficiently convex. Moreover, if g is sufficiently convex, the equilibrium technology $(\alpha^*(L), \beta^*(L))$ will be unique, which I also assume to be the case from here on.[26]

6.2. Weak Relative Bias of Automation Technology

By equations (11) and (12), the model fits into the general framework analyzed in the previous sections.[27] To apply the results on weak relative bias, it remains to verify that the particular conditions of Theorem $2'$ (the continuum counterpart of Theorem 2 for weak bias with many skills) are satisfied. We can establish the following.

Lemma 1.

1. *The aggregate production function $F(L, \alpha, \beta)$ is linear homogeneous in L at all feasible α, β.*

2. *Technologies are ordered according to their pervasive skill bias as follows:*

$$(\alpha', \beta') \succeq^{psb} (\alpha, \beta) \quad \Leftrightarrow \quad \alpha' \geq \alpha$$

for all (α', β') and (α, β) in the set

$$\Theta = \left\{ (\alpha, \beta) \in \mathbb{R}^2_+ \mid g(\alpha, \beta) = \overline{g},\, \alpha \leq \overline{\alpha} \right\} ,$$

where $\overline{\alpha}$ is uniquely determined by equation (9).

3. *For all L, $(\alpha^*(L), \beta^*(L)) \in \Theta$.*

Proof. See Appendix B.3. □

Linear homogeneity of aggregate production immediately implies that the equilibrium technology $(\alpha^*(L), \beta^*(L))$ is unaffected by the scale of labor supply and, thus, Assumption 4 (scale invariance of skill bias) is satisfied. The remaining parts of Lemma 1 show that we can order any two technologies according to their pervasive skill bias once we restrict attention to technologies in the set Θ. This set in turn is large enough to contain all possible equilibrium technologies. Hence, restricting attention to Θ is without loss of generality. This has two important consequences: (i) the ordered set $(\Theta, \succeq^{psb})$ is a chain and hence a prelattice, (ii) aggregate production F is prequasisupermodular in (α, β) on $(\Theta, \succeq^{psb})$, because any function on a chain is prequasisupermodular.

Thus, all conditions of Theorem $2'$ are satisfied and we obtain the following application of weak relative bias.

Corollary 4 (Weak Bias of Automation)**.** *Any pervasive increase in relative skill supply from L to L', where $L'_{s'}/L'_s \geq L_{s'}/L_s$ for all $s' \geq s$, induces improvements in capital productivity: $\alpha^*(L') \geq \alpha^*(L)$.*

[26] Alternatively, we could always select the equilibrium technology with the highest $\alpha^*(L)$, in line with the selection rule imposed in Section 2.

[27] More precisely, it fits into the extension to a continuum skill space in Appendix A.

Proof. By Theorem 2′, we have $(\alpha^*(L'), \beta^*(L')) \succeq^{psb} (\alpha^*(L), \beta^*(L))$. By Lemma 1, this implies $\alpha^*(L') \geq \alpha^*(L)$. □

The increase in capital productivity must come at the cost of decreased final good productivity. Moreover, the proof of Corollary 4 shows that the shift from final good to capital productivity is pervasively skill biased, raising all skill premia. Finally, Lemma 3 (provided in Appendix B.3) reveals that the shift towards capital productivity is accompanied by a reallocation of tasks from labor to capital, the epitome of automation.

It is worth to note at this point that the present assignment model is a challenging environment for comparative statics. Labor supply has infinite dimension and there is generally no closed-form solution for the aggregate production function. Direct computations based on the implicit function theorem would seem cumbersome at best. Yet, after establishing two simple properties in Lemma 1, my generalized directed technical change theorems deliver clear-cut results.

6.3. Strong Relative Bias of Automation Technology

By Theorem 3′ (the extension of Theorem 3 to a continuum of skills), strong relative bias requires that the aggregate production function fails to be quasiconcave. In the present model, this is ruled out by Assumption 5, which is necessary for a symmetric equilibrium to generally exist. Hence, we can never have a situation where the induced improvement in capital productivity is so strong that a pervasive increase in relative skill supply leads to a pervasive increase in skill premia.

Corollary 5 (No Strong Bias of Automation)**.** *There is no pervasive increase in relative skill supply that strictly raises all skill premia.*

The impossibility of strong relative bias is due to the fact that, in the present model, firms choose their technologies independently of each other. This precludes (quasi-)convexity in the aggregate endogenous-technology production function, because individual and aggregate production functions are identical.

I now introduce spillover effects between firms' technology choices into the model. This creates a separation between individual and aggregate production functions, enabling convexity in the aggregate and, thereby, strong relative bias. In Online Appendix C.3.2, I show that the same effects can be achieved with a setup in which an R&D sector supplies technology-embodying intermediate inputs to the aggregate of all final good firms. This separates the choice of labor demand from technology choices, allowing for convexity in labor and technology jointly while maintaining concavity in each factor individually.

I introduce spillover effects via the productivity frontier. In particular, let an individual firm i's productivity frontier be given by

$$\widetilde{g}(\alpha_i, \beta_i, \alpha_{-i}, \beta_{-i}) \leq \overline{g} \, ,$$

where α_{-i} and β_{-i} collect the productivities of all other firms. Denote by $\widehat{g}(\alpha, \beta)$ the symmetric technology frontier, that is,

$$\widehat{g}(\alpha,\beta) \equiv \widetilde{g}(\alpha_i,\beta_i,\alpha_{-i},\beta_{-i}) \quad \text{if} \quad \alpha_j = \alpha \text{ and } \beta_j = \beta \text{ for all } j \ .$$

I assume that there are positive knowledge spillovers in the sense that an individual firm's 'technology cost' is minimized when all other firms choose the same technology:

$$\widehat{g}(\alpha,\beta) \leq \widetilde{g}(\alpha,\beta,\alpha_{-i},\beta_{-i}) \quad \text{for all feasible} \quad \alpha,\beta,\alpha_{-i},\beta_{-i} \ .$$

Clearly, such spillover effects do not change the structure of symmetric exogenous-technology equilibria. With endogenous technology, however, they create a distinction between individual and aggregate production functions. In particular, for a given labor input L, an individual firm chooses its technology to achieve

$$\widetilde{F}^*(L,\alpha_{-i},\beta_{-i}) := \max_{\alpha,\beta} \left\{F(L,\alpha,\beta) \mid \widetilde{g}(\alpha,\beta,\alpha_{-i},\beta_{-i}) \leq \overline{g}\right\} \ . \tag{13}$$

The symmetric technology $(\alpha^*(L),\beta^*(L))$ that maximizes aggregate output, however, yields

$$\widehat{F}^*(L) := \max_{\alpha,\beta} \left\{F(L,\alpha,\beta) \mid \widehat{g}(\alpha,\beta) \leq \overline{g}\right\} \ . \tag{14}$$

It can be verified that the technology $(\alpha^*(L),\beta^*(L))$, when chosen by all firms, also solves the individual technology choice problem (13).

For $(\alpha^*(L),\beta^*(L))$ to be an equilibrium technology, we additionally need that the wages given by $w_s^*(L) = F_s(L,\alpha^*(L),\beta^*(L))$ support the symmetric labor input choices L by individual firms, that is,

$$L \in \operatorname*{argmax}_{\widetilde{L}} \widetilde{F}^*(\widetilde{L},\alpha^*_{-i}(L),\beta^*_{-i}(L)) - \int_0^1 w_s^*(L)\widetilde{L}_s \, ds \ ,$$

where $\alpha^*_{-i}(L)$ and $\beta^*_{-i}(L)$ denote the collections with $\alpha_j = \alpha^*(L)$ and $\beta_j = \beta^*(L)$ for all firms $j \neq i$. This is guaranteed if the individual endogenous-technology production function $\widetilde{F}^*(L,\alpha_{-i},\beta_{-i})$ is concave in L.[28]

Assumption 6. *The individual endogenous-technology production function $\widetilde{F}^*(L,\alpha_{-i},\beta_{-i})$ is concave in L for all α_{-i},β_{-i}.*

So, under Assumption 6, the technology $(\alpha^*(L),\beta^*(L))$ is part of an endogenous-technology equilibrium for any feasible labor supply L.[29]

The key distinction to the case without spillovers is that we only have to restrict the curvature of the individual endogenous-technology production function $\widetilde{F}^*$, not the aggregate function $\widehat{F}^*$. Indeed, it is easy to see that the aggregate endogenous-technology function is the

[28]The formal argument is provided in Online Appendix C.2 (Observation 2) for a general class of models with spillover effects.

[29]Other equilibria may exist but are inefficient, as technology would not maximize output. I focus on the efficient equilibrium technologies $(\alpha^*(L),\beta^*(L))$ here.

upper envelope of the individual functions:

$$\widehat{F}^*(L) = \max_{\alpha,\beta} \left\{ \widetilde{F}^*(L, \alpha_{-i}, \beta_{-i}) \mid \alpha_j = \alpha \,\forall j,\, \beta_j = \beta \,\forall j,\, \widehat{g}(\alpha, \beta) \leq \overline{g} \right\} .$$

Thus, on any line $\tau \mapsto l(\tau) = L + \tau(L' - L)$, aggregate production must be less concave than individual production:

$$\frac{d^2}{(d\tau)^2} \widehat{F}^*(l(\tau)) \geq \frac{d^2}{(d\tau)^2} \widetilde{F}^*(l(\tau), \alpha^*_{-i}(L), \beta^*_{-i}(L)) .$$

Assumption 6 now commands that the individual production function $\widetilde{F}^*$ is concave on the line, but this leaves the possibility that

$$\frac{d^2}{(d\tau)^2} \widehat{F}^*(l(\tau)) > 0 \geq \frac{d^2}{(d\tau)^2} \widetilde{F}^*(l(\tau), \alpha^*_{-i}(L), \beta^*_{-i}(L)) . \tag{15}$$

If this happens on a line along which relative skill supply increases pervasively, we have strong relative bias according to Theorem $3'$.

Corollary 6 (Strong Bias of Automation with Spillovers)**.** *Consider a line $\tau \mapsto l(\tau) = L + \tau(L' - L)$ with $L'_{s'}/L'_s \geq L_{s'}/L_s$ for all $s' \geq s$. Suppose (15) holds everywhere on this line. Then,*

$$\frac{w^*_{s'}(L')}{w^*_s(L')} > \frac{w^*_{s'}(L)}{w^*_s(L)}$$

for a strictly positive mass of (and potentially all) skill pairs with $s' > s$.

Proof. First, construct the auxiliary line $\widetilde{l}(\tau) = L + \tau(\widetilde{L}' - L)$ with $\widetilde{L}' = \lambda L'$ and λ such that

$$\widehat{F}^*(L) = \widehat{F}^*(\widetilde{L}') .$$

This line can also be written as $\widetilde{l}(\tau) = \lambda l(\tau) + (1 - \lambda)(1 - \tau)L$. Together with linear homogeneity of $\widehat{F}^*$, this implies that, at all $\tau \in [0, 1]$,

$$\frac{d^2}{(d\tau)^2} \widehat{F}^*(\widetilde{l}(\tau)) = \lambda^2 \frac{d^2}{(d\tau)^2} \widehat{F}^*(l(\tau)) > 0 .$$

Thus, the restriction of $\widehat{F}^*$ to $\widetilde{l}(\tau)$ must have a local minimum at some $\tau \in (0, 1)$, so it is not quasiconcave. Moreover, by construction, relative skill supply increases along $\widetilde{l}(\tau)$. So, part 1 of Theorem $3'$ implies that some skill premia must increase strictly when moving from L to L'.[30] Since wages are continuous in skill, this must hold for a strictly positive mass of skill pairs. □

The conditions of Corollary 6 ensure that a pervasive increase in relative skill supply strictly raises a non-zero mass of skill premia. This includes the case where all skill premia increase

[30] Part 1 of Theorem $3'$ implies that there will be some pervasive increase in relative skill supply that does not generate a pervasive fall in skill premia. The proof of the theorem clarifies that this pervasive increase in relative skill supply is located on the line along which quasiconcavity fails.

in response to a pervasive increase in relative skill supply. Online Appendix C.3.2 provides such an example.

To summarize, spillover effects between firms' technology choices allow for a distinction between individual and aggregate endogenous-technology production functions, which creates the potential for convexity in the aggregate production function and, thus, strong relative bias.

6.4. Automation and International Trade

The previous sections have analyzed how automation technology responds to changes in the structure of labor supply. Such changes can occur for institutional, demographic, or cultural reasons but also because of international trade, which makes foreign labor accessible to domestic firms. I now demonstrate that the previous results are easily applicable to study the impact of international trade on automation. The key insight is that, in the present model, international trade affects the economy as if it were a change in labor supply.

I consider two countries, North and South. The Northern economy is modeled by the spillover model of the previous subsection. The South differs from the North in three aspects. First, the South has pervasively smaller relative skill supply, that is, $L^S_{s'}/L^S_s \leq L^N_{s'}/L^N_s$ for all skill pairs $s' \geq s$, where L^S denotes labor supply in the South and L^N is labor supply in the North. Second, Southern firms do not choose their productivities themselves but copy them from Northern firms at some loss. In particular, Southern firms use productivities $\delta\alpha^N$ and $\delta\beta^N$ with $\delta \in (0,1)$, where α^N and β^N are the equilibrium technologies in the North. Finally, labor productivity can be lower in the South, $\gamma^S(s,x) = \Delta\gamma^N(s,x)$ with $\Delta \in (0,1]$ and γ^S and γ^N denoting the labor productivity schedules in the South and North, respectively. The difference in labor productivities is largely irrelevant for the following analysis but realistically allows for differences in wage levels between the two countries.

I compare autarky with a free-trade situation, where countries can freely trade tasks and final goods, but workers are immobile.[31] Capital and final good productivities are higher in the North, so, under free trade, both will only be produced in the North. Thereby, we obtain the following pattern of trade: the North imports tasks performed by labor from the South, combines them with the tasks performed by Northern labor and capital to produce final goods, and ships part of the final output back to the South in exchange for the Southern labor tasks.

It follows that free-trade equilibria are equivalent to autarky equilibria in the North where Northern labor supply is extended by the effective units of Southern labor to $L^N + \Delta L^S$.[32] Since the South has pervasively smaller relative skill supply, the effective world labor supply $L^N + \Delta L^S$ also features pervasively smaller relative skill supply than the Northern labor supply L^N.

We thus obtain that trade opening has the same effects in the North as a pervasive fall in relative skill supply from L^N to $L^N + \Delta L^S$. Corollary 4 then immediately implies the following weak relative bias result.

[31] Intermediate cases with partial trade opening or trade costs can be treated in a similar way.

[32] For a formal derivation of this result, see Online Appendix C.3.3.

Corollary 7 (Weak Bias of Automation from Trade). *Relative to autarky, free trade leads to a fall in Northern capital productivity: $\alpha^{N*} \geq \alpha^{NT*}$, where α^{N*} and α^{NT*} are Northern capital productivities in autarky and with free trade, respectively.*

The fall in Northern capital productivity (and simultaneous rise in final good productivity) induced by trade causes a pervasive fall in skill premia in the North, according to Lemma 1. This counteracts the usual Heckscher-Ohlin effects, which cause an increase in skill premia in the skill-abundant North. If there is strong relative bias, the effects of the induced technical change are strong enough to outweigh the Heckscher-Ohlin effects, leading to a situation where trade opening causes a fall in Northern skill premia after adjustment of technology.

In particular, let $\widehat{F}^{N*}$ denote the endogenous-technology aggregate production function in the North under autarky, equivalent to the endogenous-technology production function $\widehat{F}^{*}$ in Section 6.3. Suppose that $\widehat{F}^{N*}$ is convex along the line between L^N and $L^N + \Delta L^S$. Then, by Corollary 6, directed technical change effects dominate and trade leads to a fall in at least some skill premia in the North.

Corollary 8 (Strong Bias of Automation from Trade). *Suppose that $\widehat{F}^{N*}$ is strictly convex on the line between L^N and $L^N + \Delta L^S$. Then, relative to autarky, free trade leads to a fall in Northern skill premia in some parts of the wage distribution:*

$$\frac{w_{s'}^{N*}}{w_{s}^{N*}} \geq \frac{w_{s'}^{NT*}}{w_{s}^{NT*}}$$

for a strictly positive mass of (and potentially all) skill pairs with $s' > s$, where w^{N*} and w^{NT*} denote Northern wages in autarky and with free trade, respectively.

So, if aggregate production fails to be quasiconcave, directed technical change overwrites the usual Heckscher-Ohlin effects and trade with a skill-scarce country reduces wage inequality in some parts of the skill-abundant country's wage distribution.[33]

The results are in contrast to those from Acemoglu (2003), who studies the interaction between trade and directed technical change in a setting with labor-augmenting technology. The main difference is that, in Acemoglu (2003), the tasks produced by Southern labor cannot be traded directly but only after they have been combined with technology-embodying intermediate inputs in the South. Northern technology firms thus do not direct their R&D activities towards world labor supply but exclusively towards labor supply in the North even when there is trade. In such a setting, trade opening is not equivalent to a decrease in relative skill supply but operates solely through an increase in the relative price of skill-intensive goods in the North, which leads to skill-biased technical change.[34]

[33] In the South, Heckscher-Ohlin effects call for a reduction in inequality, but again, there is a countervailing force: trade integrates Southern labor into the Northern final good production chain and exposes it to competition from Northern capital, which disproportionately affects low-skilled workers. If the productivity difference between Northern and Southern capital under autarky is large enough (i.e., δ is small enough), this latter effect dominates and trade leads to an increase in wage inequality in the South, again counter to the usual Heckscher-Ohlin effects. These results, however, do not depend on the endogeneity of technology and are thus not discussed in further detail.

[34] This result also depends on the assumption that intellectual property rights are enforced only in the North and this does not change with trade opening. If property rights were enforced in the South starting with the opening of trade, results would likely be more similar to those obtained in my analysis.

Relatedly, Sampson (2014) and Yeaple (2005) study the effect of trade on wage inequality with endogenous technology in models of intra-industry trade. In their models, trade does not stem from differences in endowments and technology but from product differentiation and economies of scale as in Krugman (1980). To export, firms must incur a fixed cost, which incentivizes them to invest in more productive technologies. Complementarity between firm productivity and worker skill then gives rise to a positive effect of trade on skill premia.[35] Hence, as in Acemoglu (2003), these models predict that trade induces skill-biased technical change, opposite to the model presented above.

7. Conclusion

The first part of the paper develops general results, based on simple concepts, about the effects of the supply of skills on the skill bias of technical change. The results are independent of the functional form of aggregate production, hold for a variety of different microfoundations of endogenous technology choices, for settings with more than two and potentially infinitely many different levels of skill, and apply to both discrete and infinitesimal changes in the supply of skills. They show that, under a scale-invariance restriction on the skill bias of technology, any increase in the relative supply of skills induces skill-biased technical change. Moreover, the total effect of an increase in relative skill supply on skill premia, accounting both for the induced technical change effect and the direct effect, can be positive only if aggregate production fails to be quasiconcave. This generalizes upon existing results on weak and strong relative equilibrium bias of technology, which are limited to the special case of differentially labor-augmenting technology, two skill levels, and infinitesimal changes in the supply of skills.

The second part uses the developed theory to derive novel predictions on endogenous automation technology in assignment models of the type proposed by Teulings (1995). In the model investigated, a continuum of differentially skilled workers and capital, taking the form of machines that perfectly substitute for labor in the production of tasks, are assigned to a continuum of tasks, which in turn are combined to produce a single final good. Three results stand out. First, any increase in relative skill supply induces an improvement in capital productivity, which in turn leads to a rise in skill premia. Second, when there are external effects between firms' technologies, the effects of the induced improvement in capital productivity can be strong enough to outweigh the negative direct effect on skill premia, such that skill premia increase in relative skill supply. Third, in a two-country version of the model, trade with a skill-scarce country reduces capital productivity and, thereby, lowers skill premia in a skill-abundant country. This effect may overturn the usual Heckscher-Ohlin effects, such that trade causes a fall in inequality in the skill-abundant country after adjustment of technology.

There are several starting points for future research. First, the results of the first part and the results on the effects of skill supply on automation may serve as a starting point for future explorations of the implications of endogenous technical change in general and endogenous

[35] A similar effect is present in Burstein and Vogel (2017). While individual firms' technologies are exogenous in their model, trade induces a reallocation of market shares towards more productive firms, making technology more skill biased in the aggregate.

automation in particular for the design of redistributive policies, such as redistributive labor income taxation. The results on the interaction of international trade and automation may as well be the starting point for an analysis of optimal trade policy along the lines of Costinot, Donaldson, Vogel and Werning (2015). Second, the predictions on determinants of automation technology from the second part should be of interest for empirical work. Especially the predictions on the effects of trade on automation are testable once a suitable source of exogenous variation across observational units in the exposure to trade is found. Finally, moving beyond the analysis of low-skill automation by relaxing the assumption that machines always have comparative advantage versus workers in less complex tasks seems an important goal for future theory.

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A. Global Weak and Strong Bias With a Continuum of Skills

Here, I consider the case where the skill set S is given by the continuum $[0,1]$. I show that the results from Section 5 (for an arbitrary but finite number of skills) are essentially unchanged with a continuum of skills.

Labor supply is given by $L \in \mathcal{L}^2_{++}([0,1])$, the space of strictly positive, square-(Lebesgue)-integrable functions on the unit interval. Thus, for each $\theta \in \Theta$, the aggregate production function $F(\,.\,,\theta)$ is a functional on $\mathcal{L}^2_{++}([0,1])$.

Assumption 1 on Θ and F remains valid: Θ is compact, F is continuous in θ, continuously differentiable in L, and features strictly positive marginal products of labor. The differentiability assumptions, however, require some clarification. In particular, I assume that F is continuously Fréchet differentiable in L and – to distinguish it from the finite-dimensional gradient in the main text – denote the Fréchet derivative of F in L at (L,θ) by $D_L F(L,\theta)$.

For every (L,θ), the Fréchet derivative is a bounded linear functional from $\mathcal{L}^2([0,1])$ to $\mathbb{R}$. By the Riesz representation theorem, it can be represented by a square-integrable function $s \mapsto F_s(L,\theta)$, such that

$$D_L F(L,\theta)[dL] = \int_0^1 F_s(L,\theta) dL_s \, ds \,,$$

for every $dL \in \mathcal{L}^2([0,1])$. The function $F_s(L,\theta)$ is determined only up to a subset of skills of measure zero. To avoid this indeterminacy, I assume that, at every (L,θ), there exists a continuous representing function $F_s(L,\theta)$.[36] This continuous function is then necessarily unique. I call the value of the unique continuous function $F_s(L,\theta)$ at s the marginal product of labor type s. The assumption of strictly positive marginal products of labor now translates into $F_s(L,\theta)$ being strictly positive for all s and at all feasible (L,θ).

I summarize the continuum version of Assumption 1 as follows.

Assumption 1′. *The set of feasible technologies Θ is a compact topological space. The aggregate production function F is continuous in technology θ and continuously Fréchet differentiable in labor supply L. At every (L,θ), the Fréchet derivative of F with respect to L can be represented (as described above) by a continuous function $s \mapsto F_s(L,\theta) \in \mathbb{R}_{++}$.*

[36] Instead, we could accept the economically irrelevant degree of indeterminacy in marginal products of labor and formulate the results below for almost all, instead of all, relative wages. This qualification would be the only adjustment needed, the remainder of the results would be unchanged.

With these clarifications, the equilibrium conditions for technology and wages are the same as in the finite-skills case. Technology satisfies

$$\theta^*(L) \in \max_{\theta \in \Theta} F(L, \theta)$$

with the same qualifications regarding uniqueness and selection as with finite skills. Wages equal marginal products of labor,

$$w_s(L, \theta) = F_s(L, \theta) \quad \text{for all } s \, ,$$

where $F_s(L, \theta)$, here and henceforth always, denotes the unique representing function mentioned in Assumption 1′.

We can now extend the weak and strong relative bias results for many skills to the continuum setting. I focus on extensions of the global results, Theorem 2 for weak relative bias and Theorem 3 for strong relative bias. Both theorems are unchanged, except that the skill set is a continuum and, for clarity, we replace Assumption 1 by Assumption 1′.

Theorem 2′ (Global Weak Bias with a Continuum of Skills). *Let $S = [0, 1]$ and impose Assumption 1′. Suppose that $(\Theta, \succeq^{psb})$ is a prelattice, F is prequasisupermodular in θ under $\succeq^{psb}$, and Assumption 4 holds.*

Then, for any two labor supply vectors L and L' with $L'_s / L'_{s'} \geq L_s / L_{s'}$ for all $s \geq s'$, the technology under L' is pervasively skill biased relative to the technology under L, that is, $\theta^*(L') \succeq^{psb} \theta^*(L)$.

Proof. The proof of Theorem 2 goes through, with the following adjustments.

First, the path $\widetilde{L}(\tau)$ in Lemma 2 now is a function from $[0, 1]$ to $\mathcal{L}^2_{++}([0, 1])$ instead of a function from $[0, 1]$ to $\mathbb{R}^N_{++}$. In the proof of Lemma 2, the differential equation that yields $g(\tau)$ must be adjusted to

$$\int_{S^{up}} w_s(\widetilde{L}(\tau), \theta^*(L))(L'_s - L_s)\, ds + \int_{S^{down}} w_{s'}(\widetilde{L}(\tau), \theta^*(L))(L'_{s'} - L_{s'})\, ds' \frac{dg(\tau)}{d\tau} = 0 \, .$$

The remainder of the proof of Lemma 2 is unchanged.

Next, in the proof of Theorem 2, equation (20) for the change of $F(., \underline{\theta})$ along the path $\widetilde{L}(\tau)$ now becomes

$$\frac{d}{d\tau} F(\widetilde{L}(\tau), \underline{\theta}) = \int_0^1 \left[w_s(\widetilde{L}(\tau), \underline{\theta}) - \frac{w_{\tilde{s}}(\widetilde{L}(\tau), \underline{\theta})}{w_{\tilde{s}}(\widetilde{L}(\tau), \theta^*(L))} w_s(\widetilde{L}(\tau), \theta^*(L)) \right] \frac{d\widetilde{L}_s(\tau)}{d\tau}\, ds \, .$$

All the remaining parts of the proof are unchanged. □

Theorem 3′ (Global Strong Bias with a Continuum of Skills). *Let $S = [0, 1]$ and impose Assumption 1′. Suppose Assumption 3 holds. Then, the following is true.*

1. *If there exists a pervasive increase in relative skill supply that strictly raises all skill premia after adjustment of technology, then F^* is not quasiconcave.*

   Moreover, if F^* is not quasiconcave on some line along which relative skill supply increases pervasively, then there exists a pervasive increase in relative skill supply that does not lower all skill premia after adjustment of technology.

2. *If every pervasive increase in relative skill supply raises all skill premia after adjustment of technology, then F^* is quasiconvex on all lines along which relative skill supply increases pervasively. Moreover, if F^* is quasiconvex, then no pervasive increase in relative skill supply strictly lowers all skill premia after adjustment of technology.*

Proof. The structure of the proof is the same as for Theorem 3, but the adjustments needed to accommodate a continuum of skills are numerous enough to warrant a separate exposition here. I only prove part 1 of the theorem, however. As for Theorem 3, part 2 is proven analogously to part 1. So, from part 1 and the proof of Theorem 3, the proof of part 2 can be grasped immediately.

Step 1. I start by showing that, if there are labor supply vectors L and L' with $L'_{s'}/L'_s \geq L_{s'}/L_s$ for all $s' \geq s$ (a pervasive increase in relative skill supply) such that $w^*_{s'}(L')/w^*_s(L') > w^*_{s'}(L)/w^*_s(L)$ for all $s' \geq s$ (all skill premia increase), then F^* cannot be quasiconcave.

Let $H(L) = \{\widetilde{L} \in \mathcal{L}^2_{++}([0,1]) \mid D_L F(L, \theta^*(L))[\widetilde{L} - L] = 0\}$ be the hyperplane tangent to the isoquant of F at L, holding technology fixed at $\theta^*(L)$. By Assumption 3, we can restrict attention to cases where $L' \in H(L)$. Let $\widetilde{L}(\tau)$ parameterize the line through L and L', such that $\widetilde{L}(0) = L$ and $\widetilde{L}(1) = L'$.

Now, to derive a contradiction, suppose that F^* is quasiconcave. Then, $H(L)$ must be tangent to the (convex) upper contour set of F^* at L. Hence, the restriction of F^* to the line $\widetilde{L}(\tau)$ must attain its maximum at $\tau = 0$ (i.e., at L). For univariate functions, quasiconcavity is equivalent to unimodality. So, $F^*(\widetilde{L}(\tau))$ must decrease in τ for $\tau \geq 0$.

Next, consider the derivative of $F^*(\widetilde{L}(\tau))$ with respect to τ at $\tau = 1$, which, by the envelope theorem, is given by

$$\frac{d}{d\tau} F^*(\widetilde{L}(1)) = \int_0^1 w^*_s(L') \frac{d\widetilde{L}_s(1)}{d\tau} \, ds \, ,$$

where we used that $\widetilde{L}(1) = L'$. Since both $F(\widetilde{L}(\tau), \theta^*(L))$ and the derivative $d\widetilde{L}(\tau)/d\tau$ are constant in τ (the latter because $\widetilde{L}(\tau)$ is a line), we have

$$\frac{d}{d\tau} F(\widetilde{L}(0), \theta^*(L)) = \int_0^1 w_s(L, \theta^*(L)) \frac{d\widetilde{L}_s(0)}{d\tau} \, ds = \int_0^1 w^*_s(L) \frac{d\widetilde{L}_s(1)}{\tau} \, ds = 0 \, .$$

Combining the previous two equations, we can write

$$\frac{d}{d\tau} F^*(\widetilde{L}(1)) = \int_0^1 \left[w^*_s(L') - \lambda w^*_s(L) \right] \frac{d\widetilde{L}(1)}{d\tau} \, ds \tag{16}$$

for any scalar λ.

Let $\widetilde{s} \in (0,1)$ denote a skill level such that $d\widetilde{L}_s(1)/d\tau \leq 0$ for all $s < \widetilde{s}$ and $d\widetilde{L}_s(1)/d\tau > 0$ for all $s > \widetilde{s}$. Such a skill exists for two reasons: first, we consider a pervasive increase in relative skill supply; second, the vectors L and L' must disagree on a subset of skills with positive measure. The latter is true because otherwise, L and L' were the same element in $\mathcal{L}^2([0,1])$ (elements in this space are only identified up to a subset of skills of measure zero). This in turn would imply that the Fréchet derivative and, thus, wages were the same at L and L', contradicting the fact that all skill premia increase strictly when moving from L to L'.

Given such a skill $\tilde{s}$, set

$$\lambda = \frac{w^*_{\tilde{s}}(L')}{w^*_{\tilde{s}}(L)} .$$

Inserting this into equation (16), we obtain

$$\frac{d}{d\tau} F^*(\tilde{L}(1)) = \int_0^1 \left[w^*_s(L') - \frac{w^*_{\tilde{s}}(L')}{w^*_{\tilde{s}}(L)} w^*_s(L) \right] \frac{d\tilde{L}(1)}{d\tau}\, ds . \tag{17}$$

By hypothesis, we have

$$\frac{w^*_s(L')}{w^*_{\tilde{s}}(L')} > (<) \frac{w^*_s(L)}{w^*_{\tilde{s}}(L)}$$

for all $s > (<)\tilde{s}$. Rearranging yields

$$w^*_s(L') - \frac{w^*_{\tilde{s}}(L')}{w^*_{\tilde{s}}(L)} w^*_s(L) > (<) 0$$

for all $s > (<)\tilde{s}$. By construction, the derivative $d\tilde{L}_s(1)/d\tau$ has exactly the same sign, that is, $d\tilde{L}_s(1)/d\tau > (\leq) 0$ for all $s > (<)\tilde{s}$. Thus, from (17) we obtain that $dF^*(\tilde{L}(1))/d\tau > 0$, which contradicts our previous result that $F^*(\tilde{L}(\tau))$ is decreasing in τ for $\tau \geq 0$.

So, the initial claim that F^* is quasiconcave must be false, which completes the first part of the proof.

Step 2. In the second part, I show that, if F^* is not quasiconcave along some line $\tilde{L}(\tau)$ with $d \log \tilde{L}_s(\tau)/d\tau$ increasing in s, then there exist L and L' with $L'_{s'}/L'_s \geq L_{s'}/L_s$ for all $s' \geq s$ (a pervasive increase in relative skill supply) such that $w^*_{s'}(L')/w^*_s(L') > w^*_{s'}(L)/w^*_s(L)$ for at least one pair $s' > s$ (at least one skill premium increases strictly).

First, if F^* is not quasiconcave on $\tilde{L}(\tau)$, it must be possible to parameterize the line such that

$$F^*(\tilde{L}(0)) = F^*(\tilde{L}(1)) > F^*(\tilde{L}(\tilde{\tau}))$$

for some $\tilde{\tau} \in (0, 1)$. (This again follows from the fact that, for a univariate function, quasiconcavity is equivalent to unimodality.)

By the envelope theorem in Corollary 4 of Milgrom and Segal (2002), this implies

$$F^*(\tilde{L}(\tilde{\tau})) - F^*(\tilde{L}(0)) = \int_0^{\tilde{\tau}} \int_0^1 w^*_s(\tilde{L}(\tau)) \frac{d\tilde{L}_s(\tau)}{d\tau}\, ds\, d\tau < 0$$
$$F^*(\tilde{L}(1)) - F^*(\tilde{L}(\tilde{\tau})) = \int_{\tilde{\tau}}^1 \int_0^1 w^*_s(\tilde{L}(\tau)) \frac{d\tilde{L}_s(\tau)}{d\tau}\, ds\, d\tau > 0 .$$

It follows that there exist $\tau_1 < \tau_2$ such that

$$\int_0^1 w^*_s(\tilde{L}(\tau_1)) \frac{d\tilde{L}_s(\tau_1)}{d\tau}\, ds < 0 < \lambda \int_0^1 w^*_s(\tilde{L}(\tau_2)) \frac{d\tilde{L}_s(\tau_2)}{d\tau}\, ds ,$$

for any scalar $\lambda > 0$. Since $\tilde{L}(\tau)$ is a line, $d\tilde{L}(\tau_1)/d\tau$ and $d\tilde{L}(\tau_2)/d\tau$ are equal. Thus, the two inequalities imply

$$\int_0^1 \left[w^*_s(\tilde{L}(\tau_1)) - \lambda w^*_s(\tilde{L}(\tau_2)) \right] \frac{d\tilde{L}_s(\tau_1)}{d\tau}\, ds < 0 .$$

Let $\widetilde{s} \in (0,1)$ denote a skill such that $d\widetilde{L}_s(\tau_1)/d\tau \geq 0$ if $s > \widetilde{s}$ and $d\widetilde{L}_s(\tau_1)/d\tau \leq 0$ if $s < \widetilde{s}$. Such a skill level must exist because we consider an increase in relative skill supply. Then, replace the constant λ as in Step 1 to obtain

$$\int_0^1 \left[w_s^*(\widetilde{L}(\tau_1)) - \frac{w_{\widetilde{s}}^*(\widetilde{L}(\tau_1))}{w_{\widetilde{s}}^*(\widetilde{L}(\tau_2))} w_s^*(\widetilde{L}(\tau_2)) \right] \frac{d\widetilde{L}_s(\tau_1)}{d\tau} \, ds < 0 \, . \tag{18}$$

Next suppose, to derive a contradiction, that all skill premia decrease weakly from τ_1 to τ_2, that is,

$$\frac{w_s^*(\widetilde{L}(\tau_1))}{w_{\widetilde{s}}^*(\widetilde{L}(\tau_1))} \gtrless \frac{w_s^*(\widetilde{L}(\tau_2))}{w_{\widetilde{s}}^*(\widetilde{L}(\tau_2))} \quad \text{if} \quad s \gtrless \widetilde{s} \, .$$

It follows that the first vector in the inner product on the left-hand side of inequality (18) has positive (negative) entries for s above (below) $\widetilde{s}$. But by construction of $\widetilde{s}$, the same holds for the second vector $d\widetilde{L}(\tau_1)/d\tau$. Their product must hence be positive, in contradiction to inequality (18).

Thus, when moving from $\widetilde{L}(\tau_1)$ to $\widetilde{L}(\tau_2)$ (a pervasive increase in relative skill supply), at least one skill premium must increase strictly. □

B. Omitted Proofs

This section contains all proofs omitted from the main text.

B.1. Proof of Theorem 2

The proof of Theorem 2 uses the following lemma.

Lemma 2. *Consider any L and L' such that $F(L, \theta^*(L)) = F(L', \theta^*(L))$. Then, there exists a differentiable path $\widetilde{L} : [0,1] \to \mathbb{R}^N_{++}$, $\tau \mapsto \widetilde{L}(\tau)$, such that*

(i) $\widetilde{L}(0) = L$ *and* $\widetilde{L}(1) = L'$,

(ii) every component $\widetilde{L}_s(\tau)$, *for* $s \in S$, *is monotonic,*

(iii) $F(\widetilde{L}(\tau), \theta^*(L)) = F(L, \theta^*(L))$ *for every* $\tau \in [0,1]$.

Proof. We construct a path with the desired properties. Let S^{up} denote the set of skills with $L'_s > L_s$ and $S^{down} = S \setminus S^{up}$. Then, for all $s \in S^{up}$, let

$$\widetilde{L}_s(\tau) = L_s + \tau(L'_s - L_s) \, .$$

For all $s \in S^{down}$, let

$$\widetilde{L}_s(\tau) = L_s + g(\tau)(L'_s - L_s)$$

for a real-valued, differentiable function g.

Finally, choose g such that it solves the differential equation

$$\sum_{s \in S^{up}} w_s(\widetilde{L}(\tau), \theta^*(L))(L'_s - L_s) + \sum_{s' \in S^{down}} w_{s'}(\widetilde{L}(\tau), \theta^*(L))(L'_{s'} - L_{s'}) \frac{dg(\tau)}{d\tau} = 0$$

with the initial condition $g(0) = 0$. Such a solution exists by the Peano existence theorem. Moreover, by construction, we have that $dg(\tau)/d\tau \geq 0$ for all $\tau \in (0,1)$ and $g(1) = 1$. Thereby, the constructed path $\widetilde{L}$ satisfies all properties required by the lemma. □

With Lemma 2, we can prove Theorem 2. First, by Assumption 4, we can scale L' up or down without inducing technical change that affects relative wages. So, we can restrict attention to L' such that $F(L, \theta^*(L)) = F(L', \theta^*(L))$.

Next, take some infimum $\underline{\theta} \in \inf(\theta^*(L), \theta^*(L'))$ and consider a monotonic and differentiable path $\widetilde{L}(\tau)$ from L to L' as described by Lemma 2. How does $F(\widetilde{L}(\tau), \underline{\theta})$ vary along this path? As $F(\widetilde{L}(\tau), \theta^*(L))$ is constant in τ, we can write, at any point $\tau \in (0,1)$:

$$\frac{d}{d\tau}F(\widetilde{L}(\tau), \underline{\theta}) = \frac{d}{d\tau}F(\widetilde{L}(\tau), \underline{\theta}) - \lambda \frac{d}{d\tau}F(\widetilde{L}(\tau), \theta^*(L)) \tag{19}$$

for any scalar λ.

Now, fix an arbitrary τ and let $\tilde{s}$ denote a skill level such that $d\widetilde{L}_s(\tau)/d\tau \leq 0$ for all $s \leq \tilde{s}$ and $d\widetilde{L}_s(\tau)/d\tau \geq 0$ for all $s > \tilde{s}$. Such a skill exists because we consider a pervasive increase in relative skill supply. Then, set

$$\lambda = \frac{w_{\tilde{s}}(\widetilde{L}(\tau), \underline{\theta})}{w_{\tilde{s}}(\widetilde{L}(\tau), \theta^*(L))} \,.$$

Inserting this into equation (19) and writing the right-hand side more extensively, we obtain

$$\frac{d}{d\tau}F(\widetilde{L}(\tau), \underline{\theta}) = \left[w^T(\widetilde{L}(\tau), \underline{\theta}) - \frac{w_{\tilde{s}}(\widetilde{L}(\tau), \underline{\theta})}{w_{\tilde{s}}(\widetilde{L}(\tau), \theta^*(L))} w^T(\widetilde{L}(\tau), \theta^*(L)) \right] \frac{d\widetilde{L}(\tau)}{d\tau} \,. \tag{20}$$

Since $\theta^*(L)$ is pervasively skill biased relative to $\underline{\theta}$, we have

$$\frac{w_s(\widetilde{L}(\tau), \theta^*(L))}{w_{\tilde{s}}(\widetilde{L}(\tau), \theta^*(L))} \geq (\leq) \frac{w_s(\widetilde{L}(\tau), \underline{\theta})}{w_{\tilde{s}}(\widetilde{L}(\tau), \underline{\theta})}$$

for all $s \geq (\leq) \tilde{s}$. Rearranging yields

$$w_s(\widetilde{L}(\tau), \underline{\theta}) - \frac{w_{\tilde{s}}(\widetilde{L}(\tau), \underline{\theta})}{w_{\tilde{s}}(\widetilde{L}(\tau), \theta^*(L))} w_s(\widetilde{L}(\tau), \theta^*(L)) \leq (\geq) 0$$

for all $s \geq (\leq) \tilde{s}$. By construction, the entries of the vector $d\widetilde{L}(\tau)/d\tau$ have exactly the opposite signs. Thus, from (20) we obtain that $dF(\widetilde{L}(\tau), \underline{\theta})/d\tau \leq 0$.

Since this holds for all $\tau \in (0,1)$, it must hold that

$$F(L', \underline{\theta}) \leq F(L, \underline{\theta}) \leq F(L, \theta^*(L)) = F(L', \theta^*(L)) \,,$$

where the second inequality stems from the equilibrium technology condition (2). This holds for any infimum $\underline{\theta}$. Thus, by prequasisupermodularity, there must exist a supremum $\overline{\theta}$ of $\theta^*(L)$ and $\theta^*(L')$ such that

$$F(L', \overline{\theta}) \geq F(L', \theta^*(L')) \,.$$

This, by the same arguments as in the proof of Proposition 3, implies that $\theta^*(L')$ itself must be a supremum of $\theta^*(L)$ and $\theta^*(L')$. Hence, $\theta^*(L') \succeq^{psb} \theta^*(L)$.

B.2. Proof of Theorem 3

Part 1 of Theorem 3 I start by proving part 1 of Theorem 3. The proof is divided into two steps.

Step 1. I first show that, if there are labor supply vectors L and L' with $L'_{s'}/L'_s \geq L_{s'}/L_s$ for all $s' \geq s$ (a pervasive increase in relative skill supply) such that $w^*_{s'}(L')/w^*_s(L') > w^*_{s'}(L)/w^*_s(L)$ for all $s' \geq s$ (all skill premia increase), then F^* cannot be quasiconcave.

Let $H(L) = \{\widetilde{L} \in \mathbb{R}^N_{++} \mid \nabla_L F(L, \theta^*(L))(\widetilde{L} - L) = 0\}$ be the hyperplane tangent to the isoquant of F at L, holding technology fixed at $\theta^*(L)$. By Assumption 3, we can restrict attention to cases where $L' \in H(L)$. Let $\widetilde{L}(\tau)$ parameterize the line through L and L', such that $\widetilde{L}(0) = L$ and $\widetilde{L}(1) = L'$.

Now, to derive a contradiction, suppose that F^* is quasiconcave. Then, $H(L)$ must be tangent to the (convex) upper contour set of F^* at L. Hence, the restriction of F^* to the line $\widetilde{L}(\tau)$ must attain its maximum at $\tau = 0$ (i.e., at L). For univariate functions, quasiconcavity is equivalent to unimodality. So, $F^*(\widetilde{L}(\tau))$ must decrease in τ for $\tau \geq 0$.

Next, consider the derivative of $F^*(\widetilde{L}(\tau))$ with respect to τ at $\tau = 1$, which, by the envelope theorem, is given by

$$\frac{d}{d\tau} F^*(\widetilde{L}(1)) = w^{*T}(L') \frac{d\widetilde{L}(1)}{d\tau} \,,$$

where we used that $\widetilde{L}(1) = L'$. Since both $F(\widetilde{L}(\tau), \theta^*(L))$ and the derivative $d\widetilde{L}(\tau)/d\tau$ are constant in τ (the latter because $\widetilde{L}(\tau)$ is a line), we have

$$\frac{d}{d\tau} F(\widetilde{L}(0), \theta^*(L)) = w^T(L, \theta^*(L)) \frac{d\widetilde{L}(0)}{d\tau} = w^{*T}(L) \frac{d\widetilde{L}(1)}{\tau} = 0 \,.$$

Combining the previous two equations, we can write

$$\frac{d}{d\tau} F^*(\widetilde{L}(1)) = w^{*T}(L') \frac{d\widetilde{L}(1)}{d\tau} - \lambda w^{*T}(L) \frac{d\widetilde{L}(1)}{d\tau} \tag{21}$$

for any scalar λ.

Let $\tilde{s}$ denote a skill level such that $d\widetilde{L}_s(1)/d\tau \leq 0$ for all $s \leq \tilde{s}$ and $d\widetilde{L}_s(1)/d\tau \geq 0$ for all $s > \tilde{s}$. Such a skill exists because we consider a pervasive increase in relative skill supply. Then, set

$$\lambda = \frac{w^*_{\tilde{s}}(L')}{w^*_{\tilde{s}}(L)} \,.$$

Inserting this into equation (21), we obtain

$$\frac{d}{d\tau} F^*(\widetilde{L}(1)) = \left[w^{*T}(L') - \frac{w^*_{\tilde{s}}(L')}{w^*_{\tilde{s}}(L)} w^{*T}(L) \right] \frac{d\widetilde{L}(1)}{d\tau} \,. \tag{22}$$

By hypothesis, we have

$$\frac{w^*_s(L')}{w^*_{\tilde{s}}(L')} > (<) \frac{w^*_s(L)}{w^*_{\tilde{s}}(L)}$$

for all $s > (<)\tilde{s}$. Rearranging yields

$$w_s^*(L') - \frac{w_{\tilde{s}}^*(L')}{w_{\tilde{s}}^*(L)} w_s^*(L) > (<) 0$$

for all $s > (<)\tilde{s}$. By construction, the entries of the vector $d\tilde{L}(\tau)/d\tau$ have exactly the same signs, that is, $d\tilde{L}_s(1)/d\tau \geq (\leq) 0$ for all $s > (<)\tilde{s}$, with strict inequalities at least for $s = 1$ and $s = N$ (otherwise, L would be proportional to L', which, by Assumption 3, is incompatible with a strict increase in all skill premia). Thus, from (22) we obtain that $dF^*(\tilde{L}(1))/d\tau > 0$, which contradicts our previous result that $F^*(\tilde{L}(\tau))$ is decreasing in τ for $\tau \geq 0$.

So, the initial claim that F^* is quasiconcave must be false, which completes the first step of the proof.

Step 2. In the second step, I show that, if F^* is not quasiconcave along some line $\tilde{L}(\tau)$ with $d \log \tilde{L}_s(\tau)/d\tau$ increasing in s, then there exist L and L' with $L'_{s'}/L'_s \geq L_{s'}/L_s$ for all $s' \geq s$ (a pervasive increase in relative skill supply) such that $w_{s'}^*(L')/w_s^*(L') > w_{s'}^*(L)/w_s^*(L)$ for at least one pair $s' > s$ (at least one skill premium increases strictly).

First, if F^* is not quasiconcave on $\tilde{L}(\tau)$, it must be possible to parameterize the line such that

$$F^*(\tilde{L}(0)) = F^*(\tilde{L}(1)) > F^*(\tilde{L}(\tilde{\tau}))$$

for some $\tilde{\tau} \in (0, 1)$. (This again follows from the fact that, for a univariate function, quasiconcavity is equivalent to unimodality.)

By the envelope theorem in Corollary 4 of Milgrom and Segal (2002), this implies

$$\begin{aligned} F^*(\tilde{L}(\tilde{\tau})) - F^*(\tilde{L}(0)) &= \int_0^{\tilde{\tau}} \nabla_L F^*(\tilde{L}(\tau)) \frac{d\tilde{L}(\tau)}{d\tau} \, d\tau < 0 \\ F^*(\tilde{L}(1)) - F^*(\tilde{L}(\tilde{\tau})) &= \int_{\tilde{\tau}}^1 \nabla_L F^*(\tilde{L}(\tau)) \frac{d\tilde{L}(\tau)}{d\tau} \, d\tau > 0 \, . \end{aligned}$$

It follows that there exist $\tau_1 < \tau_2$ such that

$$\nabla_L F^*(\tilde{L}(\tau_1)) \frac{d\tilde{L}(\tau_1)}{d\tau} < 0 < \lambda \nabla_L F^*(\tilde{L}(\tau_2)) \frac{d\tilde{L}(\tau_2)}{d\tau} \, ,$$

for any scalar $\lambda > 0$. Since $\tilde{L}(\tau)$ is a line, $d\tilde{L}(\tau_1)/d\tau$ and $d\tilde{L}(\tau_2)/d\tau$ are equal. Thus, the two inequalities imply

$$\left[w^{*T}(\tilde{L}(\tau_1)) - \lambda w^{*T}(\tilde{L}(\tau_2)) \right] \frac{d\tilde{L}(\tau_1)}{d\tau} < 0 \, .$$

As in Step 1, let $\tilde{s}$ denote a skill such that $d\tilde{L}_s(\tau_1)/d\tau$ is greater (smaller) zero if s is greater (smaller) $\tilde{s}$. Then replace the constant λ to obtain

$$\left[w^{*T}(\tilde{L}(\tau_1)) - \frac{w_{\tilde{s}}^*(\tilde{L}(\tau_1))}{w_{\tilde{s}}^*(\tilde{L}(\tau_2))} w^{*T}(\tilde{L}(\tau_2)) \right] \frac{d\tilde{L}(\tau_1)}{d\tau} < 0 \, . \tag{23}$$

Next suppose, to derive a contradiction, that all skill premia decrease from τ_1 to τ_2, that is,

$$\frac{w_s^*(\widetilde{L}(\tau_1))}{w_{\widetilde{s}}^*(\widetilde{L}(\tau_1))} \gtreqless \frac{w_s^*(\widetilde{L}(\tau_2))}{w_{\widetilde{s}}^*(\widetilde{L}(\tau_2))} \quad \text{if} \quad s \gtreqless \widetilde{s} \, .$$

It follows that the first vector in the inner product on the left-hand side of inequality (23) has positive (negative) entries for s above (below) $\widetilde{s}$. But by construction of $\widetilde{s}$, the same holds for the second vector $d\widetilde{L}(\tau_1)/d\tau$. Their product must hence be positive, in contradiction to inequality (23).

Thus, when moving from $\widetilde{L}(\tau_1)$ to $\widetilde{L}(\tau_2)$ (a pervasive increase in relative skill supply), at least one skill premium must increase strictly.

Part 2 of Theorem 3 The proof of the second part of Theorem 3 is again divided into two steps. Step 1(2) follows closely step 2(1) in the proof of part 1 above.

Step 1. I first show that, if every pervasive increase in relative skill supply raises all skill premia, then F^* must be quasiconvex on all lines along which relative skill supply increases pervasively. The proof is by contradiction.

For that purpose, suppose that F^* is not quasiconvex on some line $\widetilde{L}(\tau)$ with $d \log \widetilde{L}_s(\tau)/d\tau$ increasing in s. Then, it must be possible to parameterize the line such that

$$F^*(\widetilde{L}(0)) = F^*(\widetilde{L}(1)) < F^*(\widetilde{L}(\widetilde{\tau}))$$

for some $\widetilde{\tau} \in (0, 1)$.

By the envelope theorem in Corollary 4 of Milgrom and Segal (2002), this implies

$$F^*(\widetilde{L}(\widetilde{\tau})) - F^*(\widetilde{L}(0)) = \int_0^{\widetilde{\tau}} \nabla_L F^*(\widetilde{L}(\tau)) \frac{d\widetilde{L}(\tau)}{d\tau} \, d\tau > 0$$

$$F^*(\widetilde{L}(1)) - F^*(\widetilde{L}(\widetilde{\tau})) = \int_{\widetilde{\tau}}^1 \nabla_L F^*(\widetilde{L}(\tau)) \frac{d\widetilde{L}(\tau)}{d\tau} \, d\tau < 0 \, .$$

It follows that there exist $\tau_1 < \tau_2$ such that

$$\nabla_L F^*(\widetilde{L}(\tau_1)) \frac{d\widetilde{L}(\tau_1)}{d\tau} > 0 > \lambda \nabla_L F^*(\widetilde{L}(\tau_2)) \frac{d\widetilde{L}(\tau_2)}{d\tau} \, ,$$

for any scalar $\lambda > 0$. Since $\widetilde{L}(\tau)$ is a line, $d\widetilde{L}(\tau_1)/d\tau$ and $d\widetilde{L}(\tau_2)/d\tau$ are equal. Thus, the two inequalities imply

$$\left[w^{*T}(\widetilde{L}(\tau_1)) - \lambda w^{*T}(\widetilde{L}(\tau_2)) \right] \frac{d\widetilde{L}(\tau_1)}{d\tau} > 0 \, .$$

Denote again by $\widetilde{s}$ a skill such that $d\widetilde{L}_s(\tau_1)/d\tau$ is greater (smaller) zero if s is greater (smaller) $\widetilde{s}$. Then replace the constant λ to obtain

$$\left[w^{*T}(\widetilde{L}(\tau_1)) - \frac{w_{\widetilde{s}}^*(\widetilde{L}(\tau_1))}{w_{\widetilde{s}}^*(\widetilde{L}(\tau_2))} w^{*T}(\widetilde{L}(\tau_2)) \right] \frac{d\widetilde{L}(\tau_1)}{d\tau} > 0 \, . \tag{24}$$

We know by hypothesis that all skill premia increase from τ_1 to τ_2, that is,

$$\frac{w_s^*(\widetilde{L}(\tau_1))}{w_{\tilde{s}}^*(\widetilde{L}(\tau_1))} \lesseqgtr \frac{w_s^*(\widetilde{L}(\tau_2))}{w_{\tilde{s}}^*(\widetilde{L}(\tau_2))} \quad \text{if} \quad s \gtreqless \tilde{s}\,.$$

It follows that the first vector in the inner product on the left-hand side of inequality (24) has negative (positive) entries for s above (below) $\tilde{s}$. But by construction of $\tilde{s}$, the entries of the second vector $d\widetilde{L}(\tau_1)/d\tau$ have the opposite signs. Their product must hence be negative, in contradiction to inequality (24).

Thus, the initial claim that F^* is not quasiconvex on $\widetilde{L}(\tau)$ must be false. Since this reasoning holds for any line along which relative skill supply increases pervasively, F^* must be quasiconvex on all such lines.

Step 2. Here, I show that, if F^* is quasiconvex, then no pervasive increase in relative skill supply can strictly reduce all skill premia. The proof is again by contradiction.

To this end, suppose that there exist L and L' with $L'_{s'}/L'_s \geq L_{s'}/L_s$ for all $s' \geq s$ such that $w^*_{s'}(L')/w^*_s(L') < w^*_{s'}(L)/w^*_s(L)$ for all $s' \geq s$.

Let $H(L) = \{\widetilde{L} \in \mathbb{R}^N_{++} \mid \nabla_L F(L, \theta^*(L))(\widetilde{L} - L) = 0\}$ be the hyperplane tangent to the isoquant of F at L, holding technology fixed at $\theta^*(L)$. By Assumption 3, we can restrict attention to cases where $L' \in H(L)$. Let $\widetilde{L}(\tau)$ parameterize the line through L and L', such that $\widetilde{L}(0) = L$ and $\widetilde{L}(1) = L'$.

Since F^* is quasiconvex, $H(L)$ must be tangent to the (convex) lower contour set of F^* at L. Hence, the restriction of F^* to the line $\widetilde{L}(\tau)$ must attain its minimum at $\tau = 0$ (i.e., at L). Moreover, the equivalence between quasiconcavity and unimodality for univariate functions (see step 1 of part 1 above) implies that $F^*(\widetilde{L}(\tau))$ must increase in τ for $\tau \geq 0$.

Next, consider the derivative of $F^*(\widetilde{L}(\tau))$ with respect to τ at $\tau = 1$, which, by the envelope theorem, is given by

$$\frac{d}{d\tau} F^*(\widetilde{L}(1)) = w^{*T}(L') \frac{d\widetilde{L}(1)}{d\tau}\,,$$

where we used that $\widetilde{L}(1) = L'$. Since both $F(\widetilde{L}(\tau), \theta^*(L))$ and the derivative $d\widetilde{L}(\tau)/d\tau$ are constant in τ (the latter because $\widetilde{L}(\tau)$ is a line), we have

$$\frac{d}{d\tau} F(\widetilde{L}(0), \theta^*(L)) = w^T(L, \theta^*(L)) \frac{d\widetilde{L}(0)}{d\tau} = w^{*T}(L) \frac{d\widetilde{L}(1)}{\tau} = 0\,.$$

Combining the previous two equations, we can write

$$\frac{d}{d\tau} F^*(\widetilde{L}(1)) = w^{*T}(L') \frac{d\widetilde{L}(1)}{d\tau} - \lambda w^{*T}(L) \frac{d\widetilde{L}(1)}{d\tau} \tag{25}$$

for any scalar λ.

Let $\tilde{s}$ denote a skill level such that $d\widetilde{L}_s(1)/d\tau \leq 0$ for all $s \leq \tilde{s}$ and $\widetilde{L}_s(1)/d\tau \geq 0$ for all $s > \tilde{s}$. Such a skill exists because we consider a pervasive increase in relative skill supply. Then, set

$$\lambda = \frac{w^*_{\tilde{s}}(L')}{w^*_{\tilde{s}}(L)}\,.$$

Inserting this into equation (25), we obtain

$$\frac{d}{d\tau}F^*(\widetilde{L}(1)) = \left[w^{*T}(L') - \frac{w^*_{\widetilde{s}}(L')}{w^*_{\widetilde{s}}(L)}w^{*T}(L)\right]\frac{d\widetilde{L}(1)}{d\tau} \,. \tag{26}$$

By hypothesis, we have

$$\frac{w^*_s(L')}{w^*_{\widetilde{s}}(L')} < (>) \frac{w^*_s(L)}{w^*_{\widetilde{s}}(L)}$$

for all $s > (<)\widetilde{s}$. Rearranging yields

$$w^*_s(L') - \frac{w^*_{\widetilde{s}}(L')}{w^*_{\widetilde{s}}(L)}w^*_s(L) < (>)0$$

for all $s > (<)\widetilde{s}$. By construction, the entries of the vector $d\widetilde{L}(\tau)/d\tau$ have exactly the opposite signs, that is, $d\widetilde{L}_s(1)/d\tau \geq (\leq)0$ for all $s > (<)\widetilde{s}$, with strict inequalities at least for $s = 1$ and $s = N$ (otherwise, L and L' would be proportional, which, by Assumption 3 is incompatible with strictly decreasing skill premia). Thus, from (26) we obtain that $dF^*(\widetilde{L}(1))/d\tau < 0$, which contradicts our previous result that $F^*(\widetilde{L}(\tau))$ is increasing in τ for $\tau \geq 0$.

So, the initial claim must be false: if F^* is quasiconvex, there cannot exist a pervasive increase in relative skill supply that strictly lowers all skill premia.

B.3. Proof of Lemma 1

It is useful to first establish the following lemma.

Lemma 3. *For any feasible technology (α, β) and labor supply L, let $\widetilde{x}(L, \alpha, \beta)$ denote the assignment threshold for capital in the exogenous-technology equilibrium, that is, let $\widetilde{x}(L, \alpha, \beta)$ be such that all tasks below it are produced by capital and all tasks above it by labor.*

Then, for any feasible (α, β) and (α', β'), if $\widetilde{x}(L, \alpha, \beta) \leq \widetilde{x}(L, \alpha', \beta')$,

$$\frac{w_{s'}(L, \alpha, \beta)}{w_s(L, \alpha, \beta)} \leq \frac{w_{s'}(L, \alpha', \beta')}{w_s(L, \alpha', \beta')} \quad \textit{for all } s \leq s' \,.$$

If $\widetilde{x}(L, \alpha, \beta) < \widetilde{x}(L, \alpha', \beta')$, the inequality for skill premia holds strictly for at least a set of skills of strictly positive measure.

Proof. Given $\widetilde{x}(L, \alpha, \beta)$, a profit-maximizing assignment of workers to tasks requires that

$$w_s(L, \alpha, \beta) = \max_{x \in [\widetilde{x}(L, \alpha, \beta), 1]} \frac{\partial Y}{\partial Y_x}\gamma(s, x) \,,$$

where $\partial Y/\partial Y_x$ denotes the Gateaux derivative of the final good production function in direction of Y_x. Taking logs and differentiating with respect to s yields, by an envelope argument,

$$\frac{d \log w_s(L, \alpha, \beta)}{ds} = \frac{\partial \log \gamma(s, m(s))}{\partial s} \,,$$

where $m(s)$ is the matching function that returns the task assigned to each skill s.

Now, consider an increase in the threshold task from $\widetilde{x}(L,\alpha,\beta)$ to $\widetilde{x}(L,\alpha',\beta')$. The same arguments as in Lemma 5 in Costinot and Vogel (2010) imply that this shifts the matching function upwards everywhere: $m'(s) \geq m(s)$ for all s, where $m'(s)$ denotes the matching function under (L,α',β'). By log supermodularity of $\gamma(s,x)$, as assumed in equation (10), this implies

$$\frac{d \log w_s(L,\alpha,\beta)}{ds} = \frac{\partial \log \gamma(s,m(s))}{\partial s} \leq \frac{\partial \log \gamma(s,m'(s))}{\partial s} = \frac{d \log w_s(L,\alpha',\beta')}{ds} .$$

For the strict version, note that the matching function must shift upwards strictly at least for those skills matched to tasks on $(\widetilde{x}(L,\alpha,\beta),\widetilde{x}(L,\alpha',\beta'))$, which, by the above reasoning, implies that skill premia for this segment of skills increase strictly. □

I proceed with the proof of Lemma 1.

Part 1. For linear homogeneity of the aggregate production function, note that firm profits are linear homogeneous in labor and capital. Thus, the optimal capital input scales with labor supply, the optimal assignment is independent of the scale of labor supply, and aggregate output in equilibrium is linear homogeneous in labor (see Lemma 5 in Online Appendix C.3.1 for more details).

Part 3. For the restriction of the set of feasible technologies, note that we can write net output as

$$\beta\left[\left(\int_0^{\widetilde{x}} \widetilde{K}_x^{\frac{\epsilon-1}{\epsilon}}\,dx + \int_{\widetilde{x}}^1 \left(\gamma(m^{-1}(x),x)L_{m^{-1}(x)}\frac{dm^{-1}(x)}{dx}\right)^{\frac{\epsilon-1}{\epsilon}} dx\right)^{\frac{\epsilon}{\epsilon-1}} - \frac{r}{\alpha\beta}\int_0^{\widetilde{x}} \widetilde{K}_x\,dx\right], \tag{27}$$

where $\widetilde{K}_x := \alpha K_x$ for all x.

Consider now a feasible technology (α,β) (i.e., one that satisfies the productivity frontier) with $\alpha > \overline{\alpha}$. By definition of $\overline{\alpha}$, we have that $\alpha\beta \leq \overline{\alpha}\overline{\beta}$ and $\beta < \overline{\beta}$. With the previous representation of net output, it is clear that (α,β) must lead to strictly lower net output than $(\overline{\alpha},\overline{\beta})$, irrespective of labor supply L, and will hence never be chosen in equilibrium.

Finally, it is obvious that firms will never choose a technology that doesn't satisfy the productivity frontier with equality. Thus, the only potential equilibrium technologies are those in Θ.

Part 2. To establish the ordering of technologies, consider any $(\alpha,\beta),(\alpha',\beta') \in \Theta$ with $\alpha' > \alpha$. At given technology and labor input, the assignment rules $\widetilde{x}$, m, and $\{K_x\}_{x\in[0,\widetilde{x}]}$ are chosen such as to maximize net output (27), which is equivalent to the maximization

$$\widetilde{Y}(L,\alpha,\beta) := \max_{\widetilde{x},m,\{\widetilde{K}_x\}_{x\in[0,\widetilde{x}]}} \left(\int_0^{\widetilde{x}} \widetilde{K}_x^{\frac{\epsilon-1}{\epsilon}}\,dx + \int_{\widetilde{x}}^1 \left(\gamma(m^{-1}(x),x)L_{m^{-1}(x)}\frac{dm^{-1}(x)}{dx}\right)^{\frac{\epsilon-1}{\epsilon}} dx\right)^{\frac{\epsilon}{\epsilon-1}} - \frac{r}{\alpha\beta}\int_0^{\widetilde{x}} \widetilde{K}_x\,dx .$$

Technology enters $\widetilde{Y}$ only via the product $\alpha\beta$. The product $\alpha g^{-1}(\alpha,\overline{g})$, where $g^{-1}(\alpha,\overline{g})$ is the inverse of the productivity frontier g with respect to β, is strictly concave in α. Thus, $\overline{\alpha} \geq \alpha' > \alpha$ implies that $\alpha'\beta' > \alpha\beta$. It follows that technology (α',β') leads to a strictly higher 'pseudo net output' $\widetilde{Y}$ than technology (α,β). Moreover, since $\widetilde{Y}(L,\alpha,\beta) \equiv F(L,\alpha,\beta)/\beta$, $\widetilde{Y}$ is linear homogeneous in L. We thus obtain:

$$\widetilde{Y}(L,\alpha',\beta') = \int_0^1 \widetilde{w}_s(L,\alpha',\beta')L_s\,ds > \int_0^1 \widetilde{w}_s(L,\alpha,\beta)L_s\,ds = \widetilde{Y}(L,\alpha,\beta)\,, \tag{28}$$

where $\widetilde{w}_s(L,\alpha,\beta) := w_s(L,\alpha,\beta)/\beta$ (for all s) is the marginal product of labor type s in the pseudo net output function $\widetilde{Y}$.

Next, the optimal assignment threshold $\widetilde{x}$ must satisfy the condition

$$\widetilde{w}_{\underline{s}}(L,\alpha,\beta) = \frac{\gamma(\underline{s},\widetilde{x}(L,\alpha,\beta))r}{\alpha\beta}\,.$$

Suppose now, to derive a contradiction, that $\widetilde{x}(L,\alpha',\beta') \leq \widetilde{x}(L,\alpha,\beta)$. Then, the threshold condition implies that $\widetilde{w}_{\underline{s}}(L,\alpha',\beta') < \widetilde{w}_{\underline{s}}(L,\alpha,\beta)$. Moreover, by Lemma 3, skill premia are lower under (α',β') than under (α,β). Thus, we obtain $\widetilde{w}_s(L,\alpha',\beta') < \widetilde{w}_s(L,\alpha,\beta)$ for all s, a contradiction to (28).

So, we must have $\widetilde{x}(L,\alpha',\beta') > \widetilde{x}(L,\alpha,\beta)$. By Lemma 3, this implies that all skill premia are greater under (α',β') than under (α,β), and strictly greater on a skill set of non-zero measure. Thus, $(\alpha',\beta') \succeq^{psb} (\alpha,\beta)$ and $(\alpha,\beta) \not\succeq^{psb} (\alpha',\beta')$.

This establishes the equivalence between increases in α and pervasive skill bias given by part 2 of Lemma 1.

C. Supplementary Material (Online Appendix)

This appendix collects further results that complement the material from the main text. Section C.1 provides a brief discussion of the relationship between global weak relative bias and standard results from monotone comparative statics. Section C.2 presents three models of directed technical change that fit into the general framework analyzed in the main text. It provides conditions for existence and uniqueness of exogenous- and endogenous-technology equilibria, with an emphasis on their implications for strong relative bias. Finally, Section C.3 contains complementary results on the assignment model presented in Section 6 of the main text.

C.1. Weak Relative Bias and Monotone Comparative Statics

In this section, I briefly discuss the relationship between my weak relative bias results and existing results in monotone comparative statics. In particular, I explain that Theorem 2 on global weak relative bias is not an application of the main theorem of monotone comparative statics, Theorem 4 in Milgrom and Shannon (1994), although the two are closely related.

The relevant part of Theorem 4 from Milgrom and Shannon (1994) says the following.

Theorem 4 (cf. Milgrom and Shannon, 1994)**.** *Let $(X, \succeq^a)$ be a lattice and $(P, \succeq^b)$ a partially ordered set. Consider a family of functions $\{f(\cdot; p)\}_{p \in P}$ with $f : X \times P \to \mathbb{R}$. Let $f(x; p)$ be quasisupermodular in x and have the single-crossing property in $(x; p)$. Then,*

$$p \succeq^b p' \Rightarrow \sup \operatorname*{argmax}_{x \in X} f(x; p) \succeq^a \sup \operatorname*{argmax}_{x \in X} f(x; p').$$

It can be shown that the theorem still holds when $\succeq^a$ and $\succeq^b$ are preorders and F is prequasisupermodular in x. The important difference between Theorem 4 and Theorem 2 is that the former imposes the single crossing-property in $(x; p)$ on F.[37] The latter instead uses specifically designed (pre)order relations to order technology and labor supply: technologies are ordered by the pervasive skill-bias order $\succeq^{psb}$ while labor supply is (implicitly) ordered according to a "pervasive relative skill supply" order $\succeq^{prs}$, such that $L \succeq^{prs} L'$ if and only if $L_s / L_{s'} \geq L'_s / L_{s'}$ for all $s \geq s'$. Indeed, these specific orderings already introduce a complementarity between changes along $\succeq^{prs}$ (pervasive increases in relative skill supply) and changes along $\succeq^{psb}$ (pervasively skill-biased technical change). In Theorem 4, such a complementarity is assumed via the single-crossing property. One can show, however, that the conditions of Theorem 2 do not imply the single-crossing property in $(\theta; L)$ for F. Therefore, given the specific environment introduced in the main text (the preorder relations and the structure of the labor supply space), Theorem 2 cannot be obtained as a corollary to Theorem 4.

C.2. Models of Directed Technical Change

This section presents three microfounded models of directed technical change fitting into the general framework analyzed in the main text. In the baseline model presented first, a contin-

[37] The single-crossing property in $(x; p)$ means that $F(x', p) - F(x, p) \geq (>) 0$ implies $F(x', p') - F(x, p') \geq (>) 0$ for any $x' \succeq^a x$ and $p' \succeq^b p$.

uum of perfectly competitive firms chooses labor inputs and technologies simultaneously. The only interaction between firms comes from market prices. This setting requires concavity in aggregate production and thus precludes strong relative bias. The next model enables strong relative bias by introducing externalities between firms' technology choices. Finally, in the last model presented, labor demand and technology choices are made by different sets of firms, again enabling convexity of the aggregate production function and strong relative bias.

C.2.1. Baseline Model

In the baseline model, there is a continuum of firms and a continuum of workers. Workers inelastically supply labor L and consume a single final good. They make no meaningful decisions. Firms are identical and produce the final good from labor according to a production function $F(L_i, \theta_i)$, where L_i is firm i's labor input and θ_i firm i's technology. The mass of firms is one.

Labor supply is differentiated according to skill levels s, that is, $L = \{L_s\}_{s \in S}$. Every L_s is a positive real number. The skill set can be of arbitrary size, that is, $S \subset \mathbb{R}$ is either a finite set or an interval. The technology variables θ_i are restricted to the set Θ. F and Θ satisfy Assumption 1 or 1′ (in the continuum case).

In an exogenous-technology equilibrium, all firms' technologies are fixed at some $\theta \in \Theta$. Firms choose their labor inputs L_i to maximize profits, taking wages $w = \{w_s\}_{s \in S}$ and technologies $\theta_i = \theta$ as given. The labor market clears. In a symmetric exogenous-technology equilibrium, wages must satisfy

$$w(L, \theta) = \nabla_L F(L, \theta) , \tag{29}$$

where the final good is used as the numéraire.[38]

An endogenous-technology equilibrium is an exogenous-technology equilibrium with technologies chosen such that firms maximize profits taking wages as given. In a symmetric endogenous-technology equilibrium, the symmetric technology choice of firms, denoted θ^*, satisfies

$$\theta^*(L) \in \underset{\theta \in \Theta}{\operatorname{argmax}}\, F(L, \theta) . \tag{30}$$

Moreover, wages are given by $\nabla_L F(L, \theta^*(L))$.

Observation 1. *In the baseline model, there exists a symmetric exogenous-technology equilibrium at any pair (L, θ) if and only if $F(L, \theta)$ is concave in L at any θ. If $F(L, \theta)$ is strictly concave in L, the symmetric equilibrium is the unique exogenous-technology equilibrium.*

Moreover, there exists a symmetric endogenous-technology equilibrium at any L if and only if the endogenous-technology production function $F^*(L)$ is concave. If $F^*(L)$ is strictly concave and $\operatorname{argmax}_{\theta \in \Theta} F(L, \theta)$ *is a singleton for all L, the symmetric equilibrium is the unique endogenous-technology equilibrium at any L.*

[38] In the case of a continuum of skill levels, the gradient ∇_L is replaced by the Fréchet derivative D_L, see Appendix A.

Proof. For a given labor supply $\overline{L}$ and technology θ, a symmetric exogenous-technology equilibrium exists if and only if we can find wages w such that

$$F(L,\theta) - wL$$

is maximized with respect to L at $\overline{L}$. Let $F_\theta^+(L) := F(L,\theta)$ be the production function at fixed technology θ. Then, the problem is equivalent to finding a hyperplane that is tangent to the graph of F_θ^+ at $\overline{L}$ and lies above $F_\theta^+(L)$ at all L. Such hyperplanes exist for all $\overline{L}$ if and only if $F_\theta^+(L)$ is concave.

If $F(L,\theta)$ is strictly concave in L, the profit maximization problem has a unique solution for any wage vector for which a solution exists. Hence, all firms must have the same labor input, and the symmetric equilibrium is the only one that can exist.

A symmetric endogenous-technology equilibrium at a given labor supply $\overline{L}$ exists if and only if we can find wages w such that

$$F^*(L) - wL$$

is maximized with respect to L at $\overline{L}$. This is because $\operatorname{argmax}_{\theta\in\Theta} F(\overline{L},\theta)$ is always non-empty by compactness of Θ and continuity of F (such that $F^*(L)$ is well defined). The existence proof then proceeds as for the exogenous-technology equilibrium but with F^* in the place of F_θ^+.

If $F^*(L)$ is strictly concave and $\operatorname{argmax}_{\theta\in\Theta} F(\overline{L},\theta)$ is a singleton, the profit maximization problem of firms has a unique solution for any wage vector for which a solution exists. Hence, all firms choose the same labor input and technology, and the symmetric equilibrium is the only one that can exist.

□

The important insight is that a symmetric endogenous-technology equilibrium exists at all L only if the endogenous-technology production function F^* is concave. According to Theorems 3 and $3'$, this prevents pervasive strong relative bias, in the sense that a pervasive increase in relative skill supply raises all skill premia after adjustment of technology.

C.2.2. Spillover Model

The spillover model is identical to the baseline model except for that it includes externalities between firms' technologies. In particular, the production function of firm i is now given by $\widetilde{F}(L_i,\theta_i,\overline{\theta})$, where $\overline{\theta}$ is the average of all firms' technology choices, $\overline{\theta} = \int_0^1 \theta_i \, di$. For the average to be well defined, let Θ be a convex subset of $\mathbb{R}^N$. Instead of the average, we could use any other function of all firms' technologies that is insensitive to any single firm's θ_j. Denote by $F(L,\theta) := \widetilde{F}(L,\theta,\theta)$ the symmetric-technology production function, which gives output as a function of labor inputs and a common technology for all firms.

The equilibrium definitions are as in the baseline model. At fixed technology $\theta_i = \theta$ for all i, an exogenous-technology equilibrium is given by wages and labor inputs for each firm, such that firms choose their labor inputs to maximize profits given wages. As in the baseline model, wages have to satisfy

$$w(L,\theta) = \nabla_L \widetilde{F}(L,\theta,\theta) = \nabla_L F(L,\theta) \tag{31}$$

in any symmetric exogenous-technology equilibrium.

An endogenous-technology equilibrium is given by wages, labor inputs, and technologies for all firms, such that firms choose their labor inputs and technologies to maximize profits, taking wages and the technologies of other firms as given.

Let

$$\theta^*(L) \in \operatorname*{argmax}_{\theta \in \Theta} F(L,\theta) \tag{32}$$

be a common technology across firms that maximizes output at symmetric labor inputs. Moreover, suppose that the spillovers across firms' technologies are such that each firm benefits from other firms choosing similar technologies to its own.

Assumption 7. *For each firm i and any labor input L_i,*

$$\widetilde{F}(L_i,\theta_i,\theta_i) \geq \widetilde{F}(L_i,\theta_i,\overline{\theta})$$

for all feasible $\overline{\theta}$.

In words, for any individual technology θ_i, firm i's productivity is maximized when the other firms, on average, choose θ_i as well. This captures the notion that part of the knowledge about how to work with a given technology is non-excludable, such that firms' productivity increases when other firms operate the same technology and much useful knowledge spills over. A perhaps more stringent formalization would have any firm's productivity decrease in the average distance between its own and other firms' technologies. Such a modification is straightforward and thus omitted.

Under Assumption (7) and appropriate conditions on $\widetilde{F}$, any technology $\theta^*(L)$ as described above forms a symmetric endogenous-technology equilibrium when combined with wages $w(L,\theta^*(L)) = \nabla_L F(L,\theta^*(L))$ and symmetric labor inputs L for all firms. More comprehensively, the following results hold.

Observation 2. *In the spillover model, there exists a symmetric exogenous-technology equilibrium at any pair (L,θ) if and only if the symmetric technology function $F(L,\theta)$ is concave in L at any θ. If F is strictly concave in L, the symmetric exogenous-technology equilibrium is the unique exogenous-technology equilibrium. Wages in any symmetric exogenous-technology equilibrium are given by equation (31).*

Moreover, suppose Assumption 7 holds and the endogenous-technology function $\widetilde{F}^*(L,\theta) := \max_{\theta_i} \widetilde{F}(L,\theta_i,\theta)$ is concave in L. Then, for any labor supply L, any technology $\theta^*(L)$ as given by equation (32) forms a symmetric endogenous-technology equilibrium in combination with wages $w(L,\theta^*(L)) = \nabla_L F(L,\theta^*(L))$ and symmetric labor inputs L for each firm.

Proof. The exogenous-technology equilibrium is equivalent to the exogenous-technology equilibrium of the baseline model, so the first part follows directly from Observation 1.

For the second part, we have to show that there are wages w such that the pair $(L, \theta^*(L))$ maximizes firm profits

$$\widetilde{F}(L_i, \theta_i, \theta^*(L)) - wL_i$$

with respect to (L_i, θ_i). First, for any $\theta' \in \Theta$ we have

$$\widetilde{F}(L, \theta^*(L), \theta^*(L)) \geq \widetilde{F}(L, \theta', \theta') \geq \widetilde{F}(L, \theta', \theta^*(L)) \, .$$

It follows that $\widetilde{F}^*(L, \theta^*(L)) = \widetilde{F}(L, \theta^*(L), \theta^*(L))$. Now let $w^* = \nabla_L \widetilde{F}(L, \theta^*(L), \theta^*(L))$ and note that by the envelope theorem,

$$w^* = \nabla_L \widetilde{F}^*(L, \theta^*(L)).$$

Concavity of $\widetilde{F}^*$ in L then implies that profits are indeed maximized at $(L, \theta^*(L))$ when wages are given by w^*. □

Uniqueness of the symmetric endogenous-technology equilibrium can easily be ensured by restricting spillovers to be sufficiently weak in an appropriate sense. The more interesting result, however, is that existence of a symmetric endogenous-technology equilibrium as characterized by equations (31) and (32) only requires concavity of $\widetilde{F}^*(L, \theta)$ in L, and not in L and θ jointly. In consequence, also the symmetric technology function $F(L, \theta)$ does not have to be jointly concave in L and θ. The reason is that existence of a symmetric equilibrium only requires concavity in the choice variables of an individual firm, whereas the function $F(L, \theta)$ combines an individual firm's technology and the average technology across firms in the variable θ (by restricting the two to be the same). Therefore, in a symmetric endogenous-technology equilibrium of the spillover model, there may be strong relative bias.

C.2.3. Monopolistic Competition Model

The distinction between concavity of individual decision problems and the aggregate production function becomes even more transparent in the monopolistic competition model. There are now two types of firms, a continuum of final good firms and a continuum of technology firms. Final good firms produce the single consumption good (the numéraire), using labor and technology-embodying intermediate goods as inputs. Their production function is $\widetilde{F}(L_i, Q_i)$, where L_i is firm i's labor input and $Q_i = (Q_{i,k})_{k=1,2,\ldots,K}$ is a vector of aggregates of technology-embodying intermediate goods. In particular, for each k,

$$Q_{i,k} = \int_0^1 \theta_{k,x} q_{i,k,x}^{\kappa} \, dx \; ,$$

where (k, x) indexes technology firms, $q_{i,k,x}$ is the quantity of firm (k, x)'s intermediate good used by final good firm i, and $\theta_{k,x}$ is the intermediate's quality. Technology firms are monopolistically competitive with substitution parameter $\kappa \in (0, 1)$. They produce their intermediate goods at constant marginal cost η_k from final good, facing inverse demand

$$p_{k,x} = \frac{\partial \widetilde{F}(L_i, Q_i)}{\partial Q_{i,k}} \kappa \theta_{k,x} q_{i,k,x}^{\kappa - 1}$$

from final good firm i. Since inverse demand is iso-elastic, all technology firms charge a price of $p_{k,x} = \eta_k/\kappa$. The symmetric price is denoted by p_k henceforth. Moreover, denote the total output of firm (k,x) by $q_{k,x}$. Then, profits of firm (k,x) are given by

$$\pi_{k,x}(\theta_{k,x}) = \max_{(q_i)_{i\in[0,1]}} \left\{ \kappa\theta_{k,x} \int_0^1 \frac{\partial \widetilde{F}(L_i, Q_i)}{\partial Q_{i,k}} q_i^\kappa \, di - \eta_k \int_0^1 q_i \, di - C_k(\theta_{k,x}) \right\}.$$

The first order condition for the firm's quality choice is

$$\kappa \int_0^1 \frac{\partial \widetilde{F}(L_i, Q_i)}{\partial Q_{i,k}} q_{i,k,x}^\kappa \, di = -\frac{dC_k(\theta_{k,x})}{d\theta_{k,x}}.$$

It can be verified that the elasticity of the optimal $q_{i,k,x}$ in $\theta_{k,x}$ is $1/(1-\kappa)$. Then, assuming that the elasticity of $dC_k/d\theta$ is always greater than $\kappa/(1-\kappa)$, the first-order condition has a unique solution, which is necessary and sufficient for a maximum. In summary, technology firms' problem of choosing price and quality of their output has a unique solution which is necessarily symmetric across firms. The symmetric quantities and qualities are denoted by $q_{i,k}$ and $\theta = (\theta_k)_{k=1,2,\ldots,K}$ henceforth.

Equilibrium conditions can now directly be stated in terms of the symmetric choices of technology firms. In particular, an exogenous-technology equilibrium is a collection of labor inputs L_i, intermediate inputs $q_{i,k}$, intermediate prices p_k, and wages w, such that final good firms choose their labor and intermediate inputs to maximize profits taking prices and wages as given, technology firms choose their prices to maximize profits taking inverse demand curves from final good firms and the quality levels of their output as given, and the labor market clears. An endogenous-technology equilibrium additionally consists of quality levels θ_k for technology firms, and requires technology firms to choose both their prices and quality levels to maximize profits, taking inverse demand curves from final good firms as given.

To characterize symmetric equilibria in the form of equations (1) and (2), define the following "modified production function":

$$F(L_i, \theta) := \max_{(q_k)_{k=1,2,\ldots,K}} \left\{ \widetilde{F}(L_i, (\theta_k q_k^\kappa)_{k=1,2,\ldots,K}) - \frac{\eta_k}{\kappa} q_k \right\} - \frac{1}{\kappa} \sum_{k=1}^K C_k(\theta_k).$$

Then, with technology firms' decisions given by $p_k = \eta_k/\kappa$ and θ, final good firms' objective is equivalent to maximizing

$$F(L_i, \theta) - wL_i$$

with respect to L_i.[39] Therefore, by the same arguments as in the previous models, a symmetric exogenous-technology equilibrium exists at all L and θ if and only if $F(L_i, \theta)$ is concave in L_i at all θ. Moreover, in such a symmetric equilibrium, wages are given by

$$w = \nabla_L F(L, \theta) \,. \tag{33}$$

If $F(L_i, \theta)$ is also strictly concave in L_i at all θ, any exogenous-technology equilibrium will

[39] Note that final good firms take θ as given, such that the presence of the term $\sum C_k(\theta_k)$ in $F(L_i, \theta)$ does not change the maximization problem.

feature symmetric labor inputs and wages given by equation (33).

For a symmetric endogenous-technology equilibrium, take any technology

$$\theta^*(L) \in \operatorname*{argmax}_{\theta \in \mathbb{R}_+^K} F(L, \theta). \tag{34}$$

Such a technology must satisfy the first-order conditions

$$\frac{\partial \widetilde{F}(L, Q^*(L))}{\partial Q_k} \kappa q_k^*(L)^\kappa = \frac{dC_k(\theta^*(L)_k)}{d\theta_k}$$

for all k, where $Q^*(L) = \theta^*(L) q^*(L)^\kappa$ and $q^*(L)$ is a solution to

$$\max_{(q_k)_{k=1,2,\ldots,K}} \left\{ \widetilde{F}(L, (\theta_k^*(L) q_k^\kappa)_{k=1,2,\ldots,K}) - \eta_k / \kappa q_k \right\}.$$

Thereby, $\theta^*(L)$ and $q_k^*(L)$ jointly satisfy technology firms' first-order conditions and final goods firms' inverse demand for intermediates, when labor inputs are symmetric. For a symmetric endogenous-technology equilibrium, it remains to find wages w such that symmetric labor inputs maximize

$$F(L_i, \theta^*(L)) - wL \, .$$

Such wages, again, exist at all L if and only if F is concave in L_i at $\theta^*(L)$. Moreover, they will clearly satisfy $w = \nabla_L F(L, \theta^*(L))$. We have therefore established that a symmetric endogenous-technology equilibrium with equilibrium technology given by (34) and wages by (33) exists whenever F is concave in L_i.

Observation 3. *In the monopolistic competition model, there exists a symmetric exogenous-technology equilibrium at any pair (L, θ) if and only if $F(L, \theta)$ is concave in L at any θ. If F is strictly concave in L, labor inputs are symmetric in any exogenous-technology equilibrium. Whenever labor inputs are symmetric, wages are given by equation (33).*

Moreover, if $F(L, \theta)$ is concave in L, there exists a symmetric endogenous-technology equilibrium with equilibrium technology satisfying equation (34) and wages given by (33).

Uniqueness of the endogenous-technology equilibrium can be ensured by imposing that F is strictly pseudoconcave in θ – such that a unique technology satisfies technology firms' first order conditions at symmetric final good firm choices – and $\widetilde{F}$ is strictly concave in the $q_{i,k,x}$ – such that all final good firms indeed choose the same intermediate quantities. The more important insight from Observation 3 is, however, that existence of symmetric endogenous and exogenous-technology equilibria can be guaranteed without any restriction on the curvature of $F(L, \theta)$ in L and θ jointly. Only restrictions on the curvature of F in L (for existence) and in θ (for uniqueness) individually are needed. In particular, the endogenous-technology function $F^*(L) = F(L, \theta^*(L))$ can be quasiconvex, as required for strong relative bias.

Finally, note that the monopolistic competition model embeds static versions of well-known models from previous work as special cases. First, when

$$\widetilde{F}(L, Q) = QL^{1-\kappa} \, ,$$

with L denoting labor supply of a single skill level, we obtain a static version of the standard monopolistic competition based growth models developed by Romer (1990) and Aghion and Howitt (1992). Since this model neither features wage inequality nor biased technical change, its static version is not very interesting. A more interesting case is obtained when

$$\widetilde{F}(L,Q) = \left[\left(Q_1 L_1^{1-\kappa} \right)^{\rho} + \left(Q_2 L_2^{1-\kappa} \right)^{\rho} \right]^{(1/\rho)} .$$

This is a static version of the seminal directed technical change model by Acemoglu (1998).

C.3. Endogenous Automation Technology: Further Results

This section complements the analysis of the assignment model presented in Section 6 of the main text. First, I present additional comparative statics results, especially on the extent of automation as measured by the set of tasks performed by capital. Second, I propose a monopolistic competition version of the assignment model that allows for strong relative bias, providing an alternative to the model with ad-hoc spillover effects from the main text. Finally, I formally derive the equivalence between the free-trade and the integrated equilibrium in the two-country model used in the main text.

Throughout the analysis, I provide some standard results on the assignment model that are omitted from the main text to streamline the exposition there.

C.3.1. The Extent of Automation

Here, I analyze how the extent of automation, as measured by the set of tasks performed by capital, responds to changes in relative skill supply. On the way, I provide a formal proof of the particular pattern of the equilibrium assignment of factors to tasks claimed in the main text.

I use a version of the assignment model from the main text where the productivities of capital and final good aggregation, α and β, are exogenous. So, final good is produced from tasks according to

$$Y = \beta \left(\int_0^1 Y_x^{\frac{\epsilon-1}{\epsilon}} \, dx \right)^{\frac{\epsilon}{\epsilon-1}} ;$$

tasks are produced from labor and capital according to

$$Y_x = \alpha K_x + \int_{\underline{s}}^{\overline{s}} \gamma(s,x) L_{s,x} \, ds ;$$

and capital is produced from final good at marginal cost r. Labor supply $L = \{L_s\}_{s \in S}$ is exogenous.

Markets for final good, tasks, capital, and labor are perfectly competitive. The final good is the numéraire, task prices are denoted by $\{p_x\}_{x \in X}$, the price for capital by p_c, and wages by $w = \{w_s\}_{s \in S}$. An equilibrium consists of wages, task prices, a price for capital, a joint distribution of labor over tasks and skills, and distributions of capital and task output levels over tasks such that all markets clear given profit maximizing behavior by firms.

The equilibrium assignment of factors to tasks is governed by the comparative advantage assumption (see equation (10) in the main text)

$$\frac{\partial^2 \log \gamma(s,x)}{\partial s \partial x} > 0 \quad \text{and} \quad \frac{\partial \log \gamma(s,x)}{\partial x} > 0 \quad \text{for all } s, x. \tag{35}$$

As mentioned in the main text, I briefly discuss this assumption in the following.

The first part of the assumption determines comparative advantage between different skills and gives a meaning to the task index x. A higher x now indicates a task in which more skilled workers have comparative advantage. In this sense, x can be viewed as a measure of a task's complexity.

The second part concerns the comparative advantage between capital and workers and is more restrictive. It implies that capital will always perform a set of least complex tasks while workers sort into tasks of higher complexity. Low-skilled workers will thus always be the first to lose their tasks to capital when automation technology advances. Though restrictive, there are good reasons for this assumption in the present context. First, empirical studies suggest that the use of industrial robots, an important form of automation technology in the manufacturing sector, has negative effects on low-skilled workers' wages and employment shares, while results for medium-skilled workers are ambiguous and high-skilled workers may gain somewhat on both margins (see Graetz and Michaels, 2018; Acemoglu and Restrepo, 2020).[40] Second, recent forecasts of the future potential for automation across occupations predict invariably that the risk of automation decreases almost monotonically with average education levels of workers in a given occupation (Frey and Osborne, 2017; Arntz et al., 2016; Nedelkoska and Quintini, 2018). Third, Lewis (2011) shows empirically that investment into various automation technologies in US manufacturing in the 1980s and 1990s was a substitute for the least-skilled but a complement to medium-skilled workers.

As claimed in the main text, the comparative advantage assumption implies a particular pattern for the assignment of factors to tasks. Here, I formalize and prove this statement.

Lemma 4. *In any equilibrium, there exists an automation threshold $\tilde{x} \in X$ and a strictly increasing and continuous matching function $m : S \to [\tilde{x}, 1]$ such that*

$$\begin{aligned} L_{s,x} > 0 &\quad \textit{if and only if} \quad x = m(s) \\ K_x > 0 &\quad \textit{if and only if} \quad x < \tilde{x}. \end{aligned}$$

Proof. First, suppose there exist $x < x'$ such that $L_{s,x} > 0$ and $K_{x'} > 0$. This requires that the cost per efficiency unit of capital is greater (smaller) than that of labor type s in task x (x'), that is,

$$\frac{w_s}{\gamma(s,x)} \leq \frac{r}{\alpha} \quad \text{and} \quad \frac{w_s}{\gamma(s,x')} \geq \frac{r}{\alpha} \,.$$

[40] In more detail, Graetz and Michaels (2018) analyze a panel of industrialized countries and find negative (positive) effects of robot use on the share of hours worked and the wage bill share of low-skilled (high-skilled) workers, whereas results for medium-skilled workers are insignificant. Acemoglu and Restrepo (2020) find that across US commuting zones the effects of robots on wages and employment to population ratios are monotonic over five education groups, with the largest negative effects for the least educated group. Observation periods in both studies start in 1993 and end in 2005 (Graetz and Michaels) and 2007 (Acemoglu and Restrepo).

But this implies

$$\frac{\gamma(s,x')}{\gamma(s,x)} \leq 1\,,$$

which contradicts the assumed pattern of comparative advantage between labor and capital. Therefore, there exists $\tilde{x} \in X$ such that (i) $K_x > 0$ only if $x \leq \tilde{x}$ and (ii) $L_{s,x} > 0$ only if $x \geq \tilde{x}$ for all s. Moreover, it is obvious that $K_x > 0$ for all $x < \tilde{x}$, as otherwise a task would not be produced at all, increasing its relative price arbitrarily and hence violating task firms' profit maximization conditions.

Second, conditional on $\tilde{x}$, the assignment of labor to tasks in $[\tilde{x}, \bar{x}]$ is the same as in a model without capital and with a task set of $[\tilde{x}, 1]$. Such a model is analyzed by Costinot and Vogel (2010), whose Lemma 1 establishes existence of a continuous and strictly increasing matching function m as proposed in Lemma 4.

It remains to argue why the threshold task $\tilde{x}$ is performed by labor and not by capital. But this question turns out to be irrelevant, as we have defined an equilibrium of the model in terms of distributions of labor and capital over tasks, and a density corresponding to a distribution is only unique up to a set of measure zero. This means that whether we let $\tilde{x}$ be performed by labor or by capital, the distributions of labor and capital over tasks, and hence the equilibrium itself, do not change. So, we can always represent the equilibrium distributions by capital and labor densities such that $K_{\tilde{x}} = 0$ and $L_{s,\tilde{x}} > 0$. □

With the assignment summarized by $\tilde{x}$ and m, we can give a concise formal definition of equilibrium, which is helpful when rigorously establishing some standard properties of the equilibrium allocation below. In particular, an equilibrium consists of

- an automation threshold $\tilde{x}$, a matching function $m : S \to [\tilde{x}, 1]$, an assignment of capital to tasks $\{K_x\}_{x \in X}$, and task output $\{Y_x\}_{x \in X}$;
- task prices $\{p_x\}_{x \in X}$, wages $\{w_s\}_{s \in S}$, and a capital price p_c;

such that

(E1)	$Y_x = \alpha K_x$ if $x < \tilde{x}$ and $Y_x = \gamma(m^{-1}(x), x) L_{m^{-1}(x)} \frac{dm^{-1}(x)}{dx}$ if $x \geq \tilde{x}$;	(market clearing)
(E2)	$p_x = \frac{\partial Y}{\partial Y_x}$ for all x;	(final good firms)
(E3)	$m(s) \in \text{argmax}_{x \in X}\, \gamma(s,x) p_x$ for all s;	(task producers)
(E4)	$w_s = \gamma(s, m(s)) p_{m(s)}$ for all s;	
(E5)	$p_c = r = \alpha p_x$ for all $x < \tilde{x}$;	
(E6)	$\frac{w_{\underline{s}}}{\gamma(\underline{s},\tilde{x})} = \frac{r}{\alpha}$.	

Condition (E1) establishes that the markets for tasks, capital, and labor clear. It derives the amount of labor used in a given task x (the marginal density of labor at x) via a change of variable from the exogenous supply of skills L_s (the marginal density of labor at s), using the assignment of skills to tasks $m(s)$ and labor market clearing. (E2) follows from final good firms' profit maximization. Task producers' profit maximization is reflected in the remaining conditions: each skill is assigned to the task where its marginal product is greatest (E3); this marginal product determines the wage (E4); capital is assigned where its marginal product is greatest and this marginal product determines the price of capital, which in turn must be equal to capital's marginal cost (E5); and the threshold task $\widetilde{x}$ is determined such that task producers are indifferent between using capital and skill $\underline{s}$ in this task (E6).

Now, let the threshold task $\widetilde{x}$ take the role of technology θ in the general results from the main text and define an exogenous- and an endogenous-technology equilibrium accordingly. In particular, in an exogenous-technology equilibrium, $\widetilde{x}$ is fixed while all other equilibrium objects are determined according to conditions (E1) to (E5). An endogenous-technology equilibrium corresponds exactly to the equilibrium definition above, based on the full set of conditions (E1) to (E6).

The following lemma establishes that the general results from the main text are applicable.

Lemma 5. *For any $\widetilde{x} \in (0,1)$ there exists a unique exogenous-technology equilibrium. Let $F(L,\widetilde{x})$ denote aggregate net production, that is, $Y - rK$, and $w(L,\widetilde{x})$ denote wages in this equilibrium. Then:*

1. *$F(L,\widetilde{x})$ is linear homogeneous in L.*

2. *Wages correspond to marginal products in F, that is, $w_s(L,\widetilde{x}) = F_s(L,\widetilde{x})$ for all s.*

3. *Any increase in $\widetilde{x}$ raises all skill premia, that is, $\widetilde{x} \geq \widetilde{x}' \Leftrightarrow \widetilde{x} \succeq^{psb} \widetilde{x}'$.*

Moreover, for any labor supply L there exists a unique endogenous-technology equilibrium with automation threshold $\widetilde{x}^*(L)$ such that

4. *$\widetilde{x}^*(L) \in \operatorname{argmax}_{\widetilde{x} \in X} F(L,\widetilde{x})$.*

Proof. Consider first existence and uniqueness of the exogenous-technology equilibrium. For any $\widetilde{x} \in (0,1)$, the assignment of labor to the task set $[\widetilde{x},1]$ is equivalent to the labor assignment in Costinot and Vogel (2010), whose Lemma 1 establishes existence of a unique assignment function m as required by the equilibrium definition. Moreover, given m, the assignment of capital to $[0,\widetilde{x})$ is clearly uniquely determined by the requirement that all marginal products $\alpha \partial Y / \partial Y_x$ must equal r. Then, capital and labor assignment together uniquely determine task quantities, task prices, and wages via conditions (E1), (E2), and (E4).

Consider now the three properties of the exogenous-technology equilibrium proposed by the lemma.

1. Consider $L' = \lambda L$. Let K_x denote the equilibrium capital density under L, let $K'_x = \lambda K_x$, and analogously for $Y'_x = \lambda Y_x$. It is then easy to check that K'_x and Y'_x, with all other equilibrium objects unchanged, form an equilibrium under the new labor supply L'. This is because final good and task production are linear homogeneous, such that scaling all inputs by a common factor does not change prices. Linear homogeneity in production also implies that final good production Y changes by the factor λ in the new equilibrium. Since aggregate capital K changes by λ as well, this must also hold for aggregate net production $Y - rK$.

2. Since all markets are competitive, the equality of wages and marginal products of labor follows from standard Walrasian equilibrium arguments.

3. The ordering of automation thresholds according to their pervasive skill bias is already established in Lemma 3 in the main text.

Finally, consider the endogenous-technology equilibrium. Again since all markets are competitive, standard reasoning along the lines of the first welfare theorem implies that the automation threshold $\widetilde{x}^*$ satisfies

$$\widetilde{x}^*(L) \in \operatorname*{argmax}_{\widetilde{x} \in X} F(L, \widetilde{x})$$

in any endogenous-technology equilibrium (otherwise task producers could choose a different $\widetilde{x}$ and earn positive profits thereby). Moreover, any such $\widetilde{x}$ must satisfy condition (E6) and hence forms an endogenous-technology equilibrium (when combined with the corresponding exogenous-technology equilibrium). Existence then follows from the fact that $F(L, \widetilde{x})$ is continuous in $\widetilde{x}$ and X is compact.[41]

For uniqueness, suppose that there are two equilibrium technologies $\widetilde{x}_1^* < \widetilde{x}_2^*$ at some labor supply L. Then, using the assumption about comparative advantage between capital and labor (Assumption 1), (E6) implies that the least-skilled worker earns less under $\widetilde{x}_1^*$ than under $\widetilde{x}_2^*$, that is,

$$w_{\underline{s}}(L, \widetilde{x}_1^*) < w_{\underline{s}}(L, \widetilde{x}_2^*) \ . \tag{36}$$

On the other hand, we know from point 3 of Lemma 5 that skill premia must also be smaller under $\widetilde{x}_1^*$. Finally, since $F(L, \widetilde{x}_1^*) = F(L, \widetilde{x}_2^*)$ and F is linear homogeneous in L, we have

$$\int_S w_s(L, \widetilde{x}_1^*) L_s \, ds = \int_S w_s(L, \widetilde{x}_2^*) L_s \, ds \ .$$

In combination with the fact that skill premia are greater under $\widetilde{x}_2^*$, this requires that the least-skilled worker earns less under $\widetilde{x}_2^*$, a contradiction to inequality (36). □

Given Lemma 5, Theorem 2′ on global weak relative bias is immediately applicable and we obtain the following result.

Corollary 9. *Any pervasive increase in relative skill supply from L to L', where $L'_{s'}/L'_s \geq L_{s'}/L_s$ for all $s' \geq s$, induces automation: $\widetilde{x}^*(L') \geq \widetilde{x}^*(L)$.*

Proof. By Theorem 2′, we have $\widetilde{x}^*(L') \succeq^{psb} \widetilde{x}^*(L)$. By Lemma 5, this implies $\widetilde{x}^*(L') \geq \widetilde{x}^*(L)$. □

In words, any pervasive increase in relative skill supply induces automation in the sense that the set of tasks performed by capital expands.

It is also straightforward to verify that the aggregate production function is quasiconcave in the present version of the model. Thus, adjustments of the extent of automation $\widetilde{x}$ cannot create strong relative bias.

Corollary 10. *There is no pervasive increase in relative skill supply that strictly raises all skill premia after adjustment of the degree of automation $\widetilde{x}^*$.*

[41] To see that $F(L, \widetilde{x})$ is continuous in $\widetilde{x}$, note that (E1) and the final good production function imply existence of an aggregate production function $\widetilde{F}(L, m, \{K_x\}_x, \widetilde{x})$ that is continuous in $\widetilde{x}$. Since in the exogenous-technology equilibrium $\{K_x\}_x$ and m are such that they maximize aggregate production, the reduced form production function $F(L, \widetilde{x})$ is the upper envelope of $\left\{\widetilde{F}(L, m, \{K_x\}_x, \widetilde{x})\right\}_{m, K_x}$. As the upper envelope of a family of continuous (in $\widetilde{x}$) functions, F is itself continuous.

Moreover, after adjustment of $\widetilde{x}^*$, any pervasive increase in relative skill supply from L to L', where $L'_{s'}/L'_s \geq L_{s'}/L_s$ for all $s' \geq s$, raises the least skilled worker's wage: $w_0(L', \widetilde{x}^*(L')) \geq w_0(L, \widetilde{x}^*(L))$.

Proof. The first part follows directly from quasiconcavity of the aggregate production function and Theorem 3′.

For the second part, note that the ratio $\gamma(0,x)/\alpha$ increases in x by the comparative advantage assumption (10). So, using condition (E6) and the fact that $\widetilde{x}^*(L) \leq \widetilde{x}^*(L')$ (by Corollary 9), we obtain

$$w_0(L, \widetilde{x}^*(L)) = \frac{\gamma(0, \widetilde{x}^*(L))}{\alpha} r \leq \frac{\gamma(0, \widetilde{x}^*(L'))}{\alpha} r = w_0(L', \widetilde{x}^*(L')) \ .$$

□

C.3.2. Monopolistic Competition Model

This section provides an alternative to the spillover assignment model presented in Section 6.3 of the main text. In the model, technology choices are made by monopolistically competitive technology firms supplying technology-embodying intermediate inputs to final good firms. The ensuing separation between technology choices and labor demand allows for convexity in the aggregate production function and strong relative bias, just as the spillover effects in the model from the main text.

After establishing aggregate equivalence to the model from the main text, I also provide a specific example of a pervasive increase in relative skill supply that raises all skill premia, proving a claim from the main text.

In the monopolistic competition model, final good production takes the form

$$Y = \int_0^1 \beta_i q_{\beta,i}^{\kappa} \, di \left(\int_{\underline{x}}^{\overline{x}} Y_x^{\frac{\epsilon-1}{\epsilon}} \, dx \right)^{\frac{(1-\kappa)\epsilon}{\epsilon-1}} , \tag{37}$$

where the $q_{\beta,i}$ are technology-embodying intermediate goods to be described further below. Tasks are produced according to

$$Y_x = \int_0^1 \alpha_i q_{\alpha,i}^{\kappa} \, di \, K_x^{1-\kappa} + \int_{\underline{s}}^{\overline{s}} \gamma(s,x) L_{s,x} \, ds \ ,$$

where, again, the $q_{\alpha,i}$ are technology-embodying intermediate goods, which are required to produce tasks using capital. The comparative advantage assumption in (10) is maintained: $\log \gamma(s,x)$ is strictly increasing in x and supermodular. Capital is produced at marginal cost r from final good. The markets for final good, capital, and tasks are perfectly competitive.

The technology-embodying intermediates, in contrast, are supplied under monopolistic competition. In particular, there is a continuum of α-monopolists, indexed by $i \in [0,1]$, who produce $q_{\alpha,i}$ at marginal cost η_α from final good. Analogously, there is a continuum of β-monopolists who produce $q_{\beta,i}$ at marginal cost η_β from final good.[42] The inverse demand for

[42]In a slight abuse of notation, I use the same index to denote α- and β-monopolists. This shall not implicate that a given monopolist produces both $q_{\alpha,i}$ and $q_{\beta,i}$, although this would not change the results.

$q_{\alpha,i}$, derived from task producers' optimization, is given by

$$p_{\alpha,i} = \kappa \alpha_i q_{\alpha,i}^{\kappa-1} \int_0^{\widetilde{x}} p_x K_x^{1-\kappa}\, dx\;, \tag{38}$$

which makes use of the result from Lemma 4 (which carries over to the present setting) that capital is used in a subset of tasks $[0, \widetilde{x})$. Analogously, the inverse demand for $q_{\beta,i}$ is

$$p_{\beta,i} = \kappa \beta_i q_{\beta,i}^{\kappa-1} \left(\int_{\underline{x}}^{\overline{x}} Y_x^{\frac{\epsilon-1}{\epsilon}}\, dx \right)^{\frac{(1-\kappa)\epsilon}{\epsilon-1}} . \tag{39}$$

Since both inverse demand functions are iso-elastic, monopolists will charge a constant markup over marginal cost. Intermediate good prices will thus be given by $p_{\alpha,i} = \eta_\alpha/\kappa$ and $p_{\beta,i} = \eta_\beta/\kappa$ in equilibrium.

Besides supplying intermediate goods, α- and β-monopolists can also invest into the quality of their products. For a quality level α_i, an α-monopolist must employ R&D resources of $\alpha_i^{1/\rho}$, with $\rho \in (0, 1-\kappa)$. Analogously, a β-monopolist must employ $\beta_i^{1/\rho}$ units of R&D resources to obtain a quality level β_i. In order to isolate effects on the direction of technical change from effects on the aggregate amount of resources spent on R&D activities, fix the total amount of R&D resources at D.[43] This implies an R&D resource constraint of $\int_0^1 \left(\alpha_i^{1/\rho} + \beta_i^{1/\rho} \right) di = D$.

Denote the unit price of R&D resources by p_D. Each α-monopolist then chooses α_i to maximize profits

$$\pi_{\alpha,i}(\alpha_i) = \max_q \left\{ \kappa \alpha_i q^\kappa \int_{\underline{x}}^{\widetilde{x}} p_x K_x^{1-\kappa}\, dx - \eta_\alpha q - p_D \alpha_i^{1/\rho} \right\} .$$

With $\rho \in (0, 1-\kappa)$, it can be verified that profits are pseudoconcave in α_i, so the first-order condition for the choice of α_i is necessary and sufficient for an optimum:

$$\rho \kappa q_{\alpha,i}^\kappa \int_{\underline{x}}^{\widetilde{x}} p_x K_x^{1-\kappa}\, dx = p_D \alpha_i^{\frac{1-\rho}{\rho}} . \tag{40}$$

Analogously, β-monopolists' profit maximization leads to the following first-order condition for the choice of β_i:

$$\rho \kappa q_{\beta,i}^\kappa \left(\int_{\underline{x}}^{\overline{x}} Y_x^{\frac{\epsilon-1}{\epsilon}}\, dx \right)^{\frac{(1-\kappa)\epsilon}{\epsilon-1}} = p_D \beta_i^{\frac{1-\rho}{\rho}} . \tag{41}$$

With this characterization of technology choices, the equilibrium definition from the previous section can be extended appropriately. Since all α-monopolists and all β-monopolists choose the same $q_{\alpha,i}$ and α_i, or, respectively, the same $q_{\beta,i}$ and β_i, it is convenient to define an equilibrium in terms of their symmetric choices q_α, α, q_β, and β.

An equilibrium consists of

- an automation threshold $\widetilde{x}$, a matching function $m : S \to [\widetilde{x}, 1]$, an assignment of capital to tasks $\{K_x\}_{x \in X}$, task output $\{Y_x\}_{x \in X}$, technology intermediate quantities q_α and q_β, and productivity levels α and β;

[43] This is equivalent to the assumption of a fixed amount of "research labor" often made in dynamic models with directed technical change; see, for example, Acemoglu and Restrepo (2018a).

- task prices $\{p_x\}_{x\in X}$, wages $\{w_s\}_{s\in S}$, a capital price p_c, technology intermediate prices p_α and p_β, and a price of R&D resources p_D;

such that

(E1)′	$Y_x = \alpha q_\alpha^\kappa K_x^{1-\kappa}$ if $x < \widetilde{x}$ and $Y_x = \gamma(m^{-1}(x), x) L_{m^{-1}(x)} \frac{dm^{-1}(x)}{dx}$ if $x \geq \widetilde{x}$;	(market clearing)
(E2)′	$p_x = \frac{\partial Y}{\partial Y_x}$ for all x;	(final good firms)
(E3)′	q_β satisfies equation (39);	(final good firms)
(E4)′	$m(s) \in \text{argmax}_{x\in X}\, \gamma(s,x) p_x$ for all s;	(task producers)
(E5)′	$w_s = \gamma(s, m(s)) p_{m(s)}$ for all s;	(task producers)
(E6)′	$\left(\frac{p_\alpha}{\kappa\alpha}\right)^\kappa \left(\frac{p_c}{(1-\kappa)\alpha}\right)^{1-\kappa} = p_x$ for all $x < \widetilde{x}$ and $p_c = r$;	(task producers)
(E7)′	$\frac{w_0}{\gamma(0,\widetilde{x})} = \left(\frac{p_\alpha}{\kappa\alpha}\right)^\kappa \left(\frac{r}{(1-\kappa)\alpha}\right)^{1-\kappa}$;	(task producers)
(E8)′	q_α satisfies equation (38);	(task producers)
(E9)′	$p_\alpha = \frac{\eta_\alpha}{\kappa}$ and $p_\beta = \frac{\eta_\beta}{\kappa}$;	(technology firms)
(E10)′	α, β, and p_D satisfy equations (40), (41), and $\alpha^{\frac{1}{\rho}} + \beta^{\frac{1}{\rho}} = D$.	(technology firms)

The full list of conditions (E1)′ to (E10)′ defines an endogenous-technology equilibrium. An exogenous-technology equilibrium, in contrast, is characterized by conditions (E1)′ to (E9)′, given an exogenously fixed pair (α, β).

The following lemma verifies that the model is covered by the directed technical change theorems from the main text.

Lemma 6. *For any (α, β) such that $\alpha^{1/\rho} + \beta^{1/\rho} = D$, there exists a unique exogenous-technology equilibrium. Define $F(L, \alpha, \beta)$ as a "modified aggregate production function",*

$$F(L, \alpha, \beta) := Y - rK - \frac{\eta_\alpha}{\kappa} q_\alpha - \frac{\eta_\beta}{\kappa},$$

with Y, K, q_α, and q_β being quantities in the exogenous-technology equilibrium, and let $w(L, \alpha, \beta)$ denote wages in the exogenous-technology equilibrium. Then:

1. *$F(L, \alpha, \beta)$ is linear homogeneous in L.*

2. *Wages equal marginal products, that is, $w_s(L, \alpha, \beta) = F_s(L, \alpha, \beta)$ for all s.*

3. *Technologies are ordered according to their pervasive skill bias as follows:*

$$(\alpha', \beta') \succeq^{psb} (\alpha, \beta) \quad \Leftrightarrow \quad \alpha' \geq \alpha$$

for any (α', β') and (α, β) in the set

$$\Theta = \left\{ (\alpha, \beta) \in \mathbb{R}_+^2 \mid \alpha^{1/\rho} + \beta^{1/\rho} = D,\, \alpha \leq \overline{\alpha} \right\},$$

where $\overline{\alpha}$ is uniquely determined by (cf. equation (9) in the main text)

$$(\overline{\alpha}, \overline{\beta}) \in \underset{(\alpha,\beta)\in\mathbb{R}_{++}^2}{\text{argmax}} \left\{ \alpha\beta \mid \alpha^{1/\rho} + \beta^{1/\rho} = D \right\}.$$

Moreover, for all L and all

$$(\alpha^*(L), \beta^*(L)) \in \underset{(\alpha,\beta)\in\mathcal{D}}{\operatorname{argmax}} F(L, \alpha, \beta) \,,$$

where $\mathcal{D} = \left\{(\alpha, \beta) \in \mathbb{R}^2_+ \mid \alpha^{1/\rho} + \beta^{1/\rho} = D\right\}$ *is the innovation possibilities frontier, there exists an endogenous-technology equilibrium with equilibrium productivity levels* $\alpha^*(L)$ *and* $\beta^*(L)$.

Finally, for all L, $(\alpha^*(L), \beta^*(L)) \in \Theta$.

Proof. We can establish existence and uniqueness of the exogenous-technology equilibrium analogously to existence and uniqueness of the endogenous-technology equilibrium in Lemma 5 in the previous section. First, note that the exogenous-technology equilibrium is equivalent to the equilibrium of an otherwise identical model where the intermediate goods q_α and q_β are produced at marginal costs η_α/κ and η_β/κ and supplied under perfect competition. Call the equilibrium of this perfectly competitive model the "auxiliary equilibrium". We prove existence and uniqueness of the auxiliary equilibrium.

Suppose at first that $\tilde{x}$ is fixed and consider conditions (E1)′ to (E6)′, (E8)′, and (E9)′. By the same arguments as in the proof of Lemma 5, the matching function m is uniquely determined by the equilibrium conditions and $\tilde{x}$. Similarly, the relative assignment of capital over tasks $[0, \tilde{x})$ is uniquely determined by the requirement that p_x is constant over these tasks (E6)′. The intermediate quantity q_β is determined by (E3)′ conditional on task quantities Y_x. Solving equation (39) for q_β and substituting the resulting expression into final good production leads to a final good production function of the same form as in the setting without technology-embodying intermediate goods in Section C.3.1. Analogously, solving equation (38) for q_α and plugging the result into task production (E1)′ yields a reduced form task production function that, for $x < \tilde{x}$, only depends on capital. We can then use the derived final good production function and the reduced form task production function to uniquely determine the scale of $\{K_x\}_{x\in[\underline{x},\tilde{x})}$ (note that the relative assignment of capital to tasks was already determined before, via condition E6′). Via the capital assignment, q_α and q_β are then determined uniquely via (E3)′ and (E8)′.

Considering the determination of $\tilde{x}$, the same arguments as in the proof of Lemma 5 imply that $\tilde{x}$ must maximize net aggregate production in the auxiliary equilibrium, and that there is exactly one $\tilde{x}$ consistent with the equilibrium conditions. This establishes existence and uniqueness of the auxiliary equilibrium and, by equivalence, of the exogenous-technology equilibrium.

Consider now the properties of the modified aggregate production function $F(L, \alpha, \beta)$ and wages $w(L, \alpha, \beta)$ proposed in the lemma.

1. For linear homogeneity of F, suppose L is scaled by $\lambda > 0$. It is then easily verified that, when scaling K_x, q_α, and q_β by λ, while keeping all prices, $\tilde{x}$, and the matching function unchanged, all equilibrium conditions are still satisfied. In this new equilibrium, all quantities are scaled by λ, so the value of F will also be scaled by λ, establishing linear homogeneity of F in L.

2. For equality of wages and the marginal product of labor in F, note that $F(L, \alpha, \beta)$ mea-

sures aggregate production in the auxiliary equilibrium. Since the auxiliary equilibrium is perfectly competitive, standard Walrasian equilibrium arguments imply equality of wages and the marginal product of labor. Then, equivalence between auxiliary and exogenous-technology equilibrium allows to transfer this result to the exogenous-technology equilibrium.

3. The proof for the ordering of technologies (α, β) is analogous to the proof of part 2 of Lemma 1 in the main text.

Consider now the endogenous-technology equilibrium, where α and β are determined via (E10)′. Take any

$$(\alpha^*(L), \beta^*(L)) \in \underset{(\alpha,\beta)\in\mathcal{D}}{\operatorname{argmax}}\, F(L, \alpha, \beta)\,,$$

and recall that

$$F(L, \alpha, \beta) = Y - rK - \frac{\eta_\alpha}{\kappa} q_\alpha - \frac{\eta_\beta}{\kappa}\,,$$

where the quantities Y, K, q_α, and q_β take their exogenous-technology equilibrium values. Moreover, let

$$F^+(L, \tilde{x}, \{K_x\}, m, q_\alpha, q_\beta, \alpha, \beta) = Y^+ - rK - \frac{\eta_\alpha}{\kappa} q_\alpha - \frac{\eta_\beta}{\kappa}$$

be net output at quantities $(L, \tilde{x}, \{K_x\}, m, q_\alpha, q_\beta, \alpha, \beta)$, that is, Y^+ is gross output at these given quantities as derived from equation (37) and condition (E1)′. Since the exogenous-technology equilibrium values of $(\tilde{x}, \{K_x\}, m, q_\alpha, q_\beta)$ maximize F^+, F is the upper envelope of the functions $F^+(L, ., \alpha, \beta)$ (where the dot shall signify that the upper envelope is taken with respect to the variables $(\tilde{x}, \{K_x\}, m, q_\alpha, q_\beta)$). Envelope arguments then imply that the technology pair $(\alpha^*(L), \beta^*(L))$ must satisfy the following Lagrange conditions:

$$\begin{aligned}
\frac{\partial F(L, \alpha^*(L), \beta^*(L))}{\partial \alpha} - \frac{\lambda}{\rho}\alpha^{\frac{1-\rho}{\rho}} &= \frac{\partial F^+(L, \tilde{x}^*, \{K_x^*\}, m^*, q_\alpha^*, q_\beta^*, \alpha^*(L), \beta^*(L))}{\partial \alpha} - \frac{\lambda}{\rho}\alpha^{\frac{1-\rho}{\rho}} \\
&= q_\alpha^{*\kappa} \int_0^{\tilde{x}^*} p_x K_x^{*1-\kappa}\, dx - \frac{\lambda}{\rho}\alpha^{\frac{1-\rho}{\rho}} = 0 \\
\frac{\partial F(L, \alpha^*(L), \beta^*(L))}{\partial \beta} - \frac{\lambda}{\rho}\beta^{\frac{1-\rho}{\rho}} &= \frac{\partial F^+(L, \tilde{x}^*, \{K_x^*\}, m^*, q_\alpha^*, q_\beta^*, \alpha^*(L), \beta^*(L))}{\partial \beta} - \frac{\lambda}{\rho}\beta^{\frac{1-\rho}{\rho}} \\
&= q_\beta^{*\kappa} \left(\int_X Y_x^{*\frac{\epsilon-1}{\epsilon}}\, dx \right)^{\frac{(1-\kappa)\epsilon}{\epsilon-1}} - \frac{\lambda}{\rho}\beta^{\frac{1-\rho}{\rho}} = 0\,,
\end{aligned}$$

where the $(\tilde{x}^*, \{K_x^*\}, m^*, q_\alpha^*, q_\beta^*)$ denote the exogenous-technology equilibrium quantities of the corresponding variables. Comparison of the conditions now reveals that, with $p_D = \lambda\kappa$, any (α, β) that satisfies the Lagrange conditions also satisfies the equilibrium condition (E10)′ and hence forms an endogenous-technology equilibrium.

Note also that the Lagrange conditions require

$$\frac{\partial F(L, \alpha, \beta)/\partial \alpha}{\partial F(L, \alpha, \beta)/\partial \beta} = \left(\frac{\alpha}{\beta}\right)^{\frac{1-\rho}{\rho}}\,.$$

When $\rho \to 0$, the right-hand side of the equation converges to a step function that is 0 for

$\alpha/\beta < 1$ and jumps to infinity at $\alpha/\beta = 1$. Thus, the equation will have a unique solution when ρ is sufficiently small. In the following, I assume that either ρ is sufficiently small to guarantee uniqueness of the endogenous-technology equilibrium or the equilibrium with the highest α^* is selected, in line with the selection rule imposed in the main text.

The proof for $(\alpha^*(L), \beta^*(L)) \in \Theta$ is again analogous to the corresponding part in the proof of Lemma 1 in the main text. □

Lemma 6 establishes that technology and wages are determined by the same conditions as in the assignment model analyzed in Section 6 of the main text. Thus, the weak relative bias result of Theorem 2′ is applicable and Corollary 4 transfers to the environment with monopolistic competition: any pervasive increase in relative skill supply induces an improvement of capital productivity α, which in turn causes a pervasive rise in skill premia.

More importantly, like the spillover model in the main text, the monopolistic competition version presented here allows for strong relative bias. In particular, no restriction on the curvature properties of the endogenous-technology production function $F^*(L) := F(L, \alpha^*(L), \beta^*(L))$ is required for equilibrium existence in Lemma 6. The reason is that technology choices and labor demand decisions are separated. For an interior solution of final good firms' profit maximization problems, concavity of $F(L, \alpha, \beta)$ in L is needed, which is always satisfied in the assignment model. For interior solutions to technology firms' maximization problems, (pseudo-)concavity in (α, β) is needed, which is also satisfied given the parameter restrictions imposed in the description of the model. No restriction, however, is needed on the curvature of F in L and (α, β) jointly and, thus, no restriction on the curvature of F^*.

As a consequence, Corollary 6 on strong relative bias in the spillover model from the main text transfers to the monopolistic competition framework: it is possible that a pervasive increase in relative skill supply induces some, or even all, skill premia to increase.

Complementing Corollary 6, I now describe an example of a pervasive increase in relative skill supply that indeed raises all skill premia. For this, consider the limit case with a subset of low-skilled workers $[0, \tilde{s}]$ who have no discernible comparative advantage over capital. Formally, for $s \in [0, \tilde{s}]$, $\gamma(s, x)$ is constant in x and hence proportional to capital productivity α. This case itself does not satisfy Assumption (10), but it is the limit of a sequence of cases all covered by the assumption. Since the equilibrium is continuous in the relevant parameters, we can still analyze the limit case on the basis of Lemma 6. The only complication is that the absence of strict comparative advantage between capital and workers with skill below $\tilde{s}$ means that the assignment of these factors to tasks is no longer uniquely determined. This indeterminacy, however, neither affects prices, nor task, nor final good quantities. So we can safely ignore it when analyzing the response of wages to changes in labor supply conditional on curvature properties of aggregate production.

In this limit case, consider now a proportional increase in the supply of all skill levels above $\tilde{s}$ by a factor of $\lambda > 1$. Holding capital productivity constant at its initial level, it is easy to see that all wages remain unchanged. In particular, let $\tilde{x}'$ be the threshold task such that skills above (below) $\tilde{s}$ sort into tasks above (below) $\tilde{x}'$ before the labor supply change, and suppose that the task assignment for skills above $\tilde{s}$ remains unchanged when labor supply changes. Then, capital adjusts in a way that raises all task quantities below $\tilde{x}'$ by the factor λ.

This holds all task ratios and hence all task prices unchanged, such that, given constant labor assignment, wages will be unchanged as well. Constancy of wages in turn confirms the initial assumption of an unchanged labor assignment. So, at constant capital productivity, skill premia do not change in response to the specific increase in relative skill supply described above. But by Corollary 4, capital productivity will increase in response to the increase in relative skill supply. This raises all skill premia above their initial level, because the increase in capital productivity is pervasively skill biased.

In addition, low-skilled workers' wages will fall in response to any increase in relative skill supply. This is because capital is a perfect substitute for low-skilled workers in all tasks, due to the absence of comparative advantage between these factors. Therefore, when the productivity of capital rises while its price stays constant, low-skilled workers' wages must fall.

Observation 4. *Consider the limit case where $\gamma(s,x)$ is constant in x for all $s \leq \widetilde{s}$ for some $\widetilde{s} \in (0,1)$ and a pervasive increase in relative skill supply from L to L' such that $L'_s = \lambda_1 L_s$ for all $s \leq \widetilde{s}$ and $L'_s = \lambda_2 L_s$ for all $s > \widetilde{s}$ with $\lambda_2 > \lambda_1$. This increase in relative skill supply raises all skill premia,*

$$\frac{w^*_{s'}(L')}{w^*_s(L')} \geq \frac{w^*_{s'}(L)}{w^*_s(L)}$$

for all $s' \geq s$.

Moreover, in the limit case described above, low-skilled workers' wages fall in response to any pervasive increase in relative skill supply.

Proof. The first part is proven in the text for $\lambda_1 = 1$. It holds for arbitrary $\lambda_1 < \lambda_2$ by zero-homogeneity of wages and technology in L.

The second part follows from the fact that, by the equilibrium condition (E7)',

$$w_0 = \left(\frac{p_\alpha}{\kappa\alpha}\right)^{\kappa} \left(\frac{r}{(1-\kappa)\alpha}\right)^{1-\kappa} \gamma(0,\widetilde{x}) \,, \tag{42}$$

observing that $\gamma(0,x)$ is constant in x in the limit case and that α increases in response to any increase in relative skill supply by Corollary 4. The result extends to all skills $s \leq \widetilde{s}$ by noting that the ratios w_s/w_0 are fixed for all $s \leq \widetilde{s}$, due to the absence of strict comparative advantage between these skills in the limit case. □

While Observation 4 is restricted to the limit case described above, continuity arguments imply that its results hold more broadly. In particular, a pervasive rise in skill premia and the drop in low-skilled workers' wages are generally likely whenever there are no tasks in which low-skilled workers maintain a strong comparative advantage over capital.

C.3.3. Equivalence Between Free Trade and Integrated Equilibrium

In the analysis of the two-country assignment model in the main text, I use the result that the equilibrium under free trade is equivalent to a hypothetical integrated equilibrium, where Southern labor supply moves to the North. Here, I derive this equivalence formally.

The formal derivation constructs full lists of equilibrium conditions for each of the two equilibria and transforms one list into the other. Such a full list of equilibrium conditions is already provided in Section C.3.2 for the monopolistic competition version of the assignment model. I therefore provide the equivalence proof for the monopolistic competition version as well; the proof for the version with spillover effects from the main text proceeds in close analogy.

In the two-country model, the Northern economy has the same structure as the closed economy of Section C.3.2. In particular, the final good is produced according to

$$Y^N = \int_0^1 \left(\beta_i^{N\frac{1}{\kappa}} q_{\beta,i}^{NN} + \beta_i^{S\frac{1}{\kappa}} q_{\beta,i}^{SN} \right)^\kappa di \left[\int_0^1 \left(Y_x^{NN} + Y_x^{SN} \right)^{\frac{\epsilon-1}{\epsilon}} dx \right]^{\frac{\epsilon(1-\kappa)}{\epsilon-1}} ,$$

where Y^N denotes final good production in the North, β_i^N and β_i^S are quality levels of the technology-embodying intermediate good (β, i) in North and South, respectively, $q_{\beta,i}^{NN}$ is the quantity of this intermediate good produced and utilized in the North, $q_{\beta,i}^{SN}$ is the quantity produced in the South and utilized in the North, and analogously Y_x^{NN} (Y_x^{SN}) is the quantity of task x produced and utilized in the North (produced in the South and utilized in the North). Tasks are produced according to

$$Y_x^N = \int_0^1 \left(\alpha_i^{N\frac{1}{\kappa}} q_{\alpha,i}^{NN} + \alpha_i^{S\frac{1}{\kappa}} q_{\alpha,i}^{SN} \right)^\kappa di \, K_x^{N1-\kappa} + \gamma^N(m^{N-1}(x), x) L_{m^{N-1}(x)}^N \frac{dm^{N-1}(x)}{dx} .$$

Here, Y_x^N denotes production of task x in the North, α_i^N and α_i^S are quality levels of the technology-embodying intermediate good (α, i) in the North and South, respectively, $q_{\alpha,i}^{NN}$ ($q_{\alpha,i}^{SN}$) is the quantity of this good produced and utilized in the North (produced in the South and utilized in the North), K_x^N is the amount of capital employed in task x in the North, $m^N(x)$ is the skill level assigned to task x in the North, and $\gamma^N(s, x)$ is the Northern labor productivity schedule. Tasks, labor, and the final good are supplied competitively. Tasks prices in the North are denoted by p_x^N, wages by w_s^N, and the Northern final good is the numéraire.

The technology-embodying intermediate goods are produced by monopolists at marginal cost η_α and η_β, respectively, from final good. The total quantity of good (α, i) produced in the North is denoted $q_{\alpha,i}^N$, and analogously for (β, i). Prices are $p_{\alpha,i}^N$ and $p_{\beta,i}^N$. The monopolists obtain quality levels α_i^N or β_i^N at costs $p_D \alpha_i^{N\frac{1}{\rho}}$ or $p_D \beta_i^{N\frac{1}{\rho}}$, respectively, where p_D denotes the price for R&D resources. R&D resources are in fixed supply D in the North.

The South is symmetric to the North, with two exceptions. First, there is no R&D sector in the South. Instead, quality levels α_i^S and β_i^S are copied with some loss δ from Northern monopolists, such that $\alpha_i^S = \delta \alpha_i^N$ and $\beta_i^S = \delta \beta_i^N$ for $\delta \in (0, \kappa)$. Second, since there are no fixed R&D expenditures required to produce them, the technology-embodying intermediates $q_{\alpha,i}^S$ and $q_{\beta,i}^S$ are supplied competitively.[44]

Final good market clearing now requires that $Y^N = Y^{NN} + Y^{NS}$, where Y^{NN} (Y^{NS}) is the amount of final good produced and consumed in the North (produced in the North and

[44]In addition, as in the main text, the South has pervasively smaller relative skill supply than the North and Southern labor productivity may be reduced relative to the North, $\gamma^S(s, x) = \Delta \gamma^N(s, x)$ for all (s, x) and some $\Delta \in (0, 1]$.

consumed in the South), and $Y^S = Y^{SS} + Y^{SN}$, where Y^{SS} and Y^{SN} are the Southern analogues of Y^{NN} and Y^{NS}. Task market clearing requires $Y_x^N = Y_x^{NN} + Y_x^{NS}$ and $Y_x^S = Y_x^{SS} + Y_x^{SN}$ for all x. The markets for technology-embodying intermediates clear if $q_{\alpha,i}^N = q_{\alpha,i}^{NN} + q_{\alpha,i}^{NS}$, $q_{\beta,i}^N = q_{\beta,i}^{NN} + q_{\beta,i}^{NS}$, and analogously for Southern intermediate good production. Finally, trade between the two countries is balanced if

$$
\begin{aligned}
Y^{NS} + \int_0^1 p_x^N Y_x^{NS}\,dx + \int_0^1 p_{\alpha,i}^N q_{\alpha,i}^{NS}\,di + \int_0^1 p_{\beta,i}^N q_{\beta,i}^{NS}\,di& \\
= Y^{SN} + \int_0^1 p_x^S Y_x^{SN}\,dx + \int_0^1 p_{\alpha,i}^S q_{\alpha,i}^{SN}\,di + \int_0^1 p_{\beta,i}^S q_{\beta,i}^{SN}\,di\,.&
\end{aligned}
$$

A trade equilibrium now consists of automation thresholds $\widetilde{x}^N$ and $\widetilde{x}^S$, matching functions $m^N(s)$ and $m^S(s)$, capital assignments $\{K_x^N\}_{x\in X}$ and $\{K_x^S\}_{x\in X}$, task production $\{Y_x^N\}_{x\in X}$ and $\{Y_x^S\}_{x\in X}$, task utilization $\{Y_x^{NN}\}_{x\in X}$, $\{Y_x^{NS}\}_{x\in X}$, $\{Y_x^{SS}\}_{x\in X}$, and $\{Y_x^{SN}\}_{x\in X}$, technology intermediate production q_α^N, q_α^S, q_β^N, and q_β^S, technology intermediate utilization q_α^{NN}, q_α^{NS}, q_α^{SS}, q_α^{SN} (and analogously for β), final good quantities Y^N and Y^S, and final good consumption Y^{NN}, Y^{NS}, Y^{SS}, Y^{SN}; task prices $\{p_x^N\}_{x\in X}$ and $\{p_x^S\}_{x\in X}$, wages $\{w_s^N\}_{s\in S}$ and $\{w_s^S\}_{s\in S}$, capital prices p_c^N and p_c^S, technology intermediate prices p_α^N, p_α^S, p_β^N, and p_β^S, a price for R&D resources p_D in the North, and a price for final good in the South p^S; such that all firms maximize profits, all markets clear, and trade is balanced. Note that the definition already uses symmetry across technology intermediate producers within each country.

The remainder of the section provides a characterization of a trade equilibrium in terms of the equilibrium of an integrated economy with labor supply $L^N + \Delta L^S$, where $\Delta \in (0,1]$ measures the difference in labor productivity between the two countries, that is, $\gamma^S(s,x) = \Delta\gamma^N(s,x)$.

We start from the observation that free trade in tasks and final good implies that the corresponding prices must be equal across countries, that is, $p_x^N = p_x^S$ for all x and $p^S = 1$ (the Northern final good is the numéraire). Since skills are assigned to those tasks in which their marginal product is greatest, and because the labor productivity difference Δ does not depend on tasks, equality of task prices implies equality of matching functions across countries. Denote the common matching function by $m^T(x)$. It follows immediately that there is also a common automation threshold $\widetilde{x}^T$ for both countries. Moreover, wages correspond to marginal products of skills in their respective tasks, so we must have $w_s^S = \Delta w_s^N$ for all skills. Finally, because final good prices are equal and the marginal cost of capital in terms of final good is r in both countries, there will be a common price of capital $p_c^T = r$.

Consider now the supply of technology-embodying intermediate goods for task production. If only Northern monopolists supplied the goods, they would again face iso-elastic demand, such that they would charge prices $p_\alpha^N = \eta_\alpha/\kappa$. This implies a price per efficiency unit of $\eta_\alpha/(\kappa\alpha^N)$. The price at which Southern producers just break even is $p_\alpha^S = \eta_\beta$, which implies a price per efficiency unit of η_α/α^S or, with $\alpha^S = \delta\alpha^N$, $\eta_\alpha/(\delta\alpha^N)$. Since we assumed that $\delta < \kappa$, Southern producers would incur losses when producing at Northern producers' monopoly prices. Hence, in equilibrium only Northern producers produce. Moreover, they charge monopoly prices $p_\alpha^N = \eta_\alpha/\kappa$. To obtain a condition for the quantity of these goods, consider inverse demand in the North (using symmetry across α-intermediates),

$$p_\alpha^N = \kappa \alpha^N q_\alpha^{NN\kappa-1} \int_0^{\tilde{x}^T} p_x K_x^{N1-\kappa}\, dx\ ,$$

and in the South,

$$p_\alpha^N = \kappa \alpha^N q_\alpha^{NS\kappa-1} \int_0^{\tilde{x}^T} p_x K_x^{S1-\kappa}\, dx\ .$$

Now let $q_\alpha := q_\alpha^N + q_\alpha^S = q_\alpha^N$ denote world production, and let $s_\alpha := q_\alpha^{NN}/q_\alpha$ be the share of world production utilized in the North. Then, linear homogeneity of task production in q_α^{NC} and K_x^C (for both countries $C = N, S$) for all tasks $x < \tilde{x}^T$ implies that the marginal product of capital in task x will be equal in both countries if and only if $K_x^N = s_\alpha K_x$ for all $x < \tilde{x}^T$, where $K_x := K_x^N + K_x^S$ is the world capital stock. Note that marginal products of capital must be equal because the price of capital is the same in both countries. With this result, we can rewrite the inverse demand for the α-intermediate in the North (or, equivalently, in the South) as

$$\begin{aligned}
p_\alpha^N &= \kappa \alpha^N s_\alpha^{\kappa-1} q_\alpha^{\kappa-1} \int_0^{\tilde{x}^T} p_x K_x^{N1-\kappa}\, dx \\
&= \kappa \alpha^N q_\alpha^{\kappa-1} \int_0^{\tilde{x}^T} p_x \left(\frac{K_x^N}{s_\alpha}\right)^{1-\kappa} dx \\
&= \kappa \alpha^N q_\alpha^{\kappa-1} \int_0^{\tilde{x}^T} p_x K_x^{1-\kappa}\, dx\ ,
\end{aligned} \tag{43}$$

which has the same form as the corresponding inverse demand in the closed economy (see equation (38)). Profits of α-monopolists at a given α are then given by

$$\pi_{\alpha,i}(\alpha_i^N) = \max_q \left\{ \kappa \alpha_i^N q^\kappa \int_0^{\tilde{x}^T} p_x K_x^{1-\kappa}\, dx - \eta_\alpha q - p_D \alpha_i^{N1/\rho} \right\}\ .$$

It follows that the first-order condition for a profit maximum in α_i^N also takes the same form as in the closed economy (using symmetry to drop the i):

$$\rho \kappa q_\alpha^\kappa \int_0^{\tilde{x}^T} p_x K_x^{1-\kappa}\, dx = p_D \alpha^{N\frac{1-\rho}{\rho}}\ . \tag{44}$$

Next, consider the production of tasks. Let $Y_x := Y_x^N + Y_x^S$ denote world production of tasks. The previous results now imply that

$$\begin{aligned}
Y_x &= \alpha^N s_\alpha q_\alpha^\kappa K_x^{1-\kappa} + \alpha^N (1 - s_\alpha) q_\alpha^\kappa K_x^{1-\kappa} \\
&= \alpha^N q_\alpha^\kappa K_x^{1-\kappa}
\end{aligned}$$

for all $x < \tilde{x}^T$, and

$$Y_x = \gamma^N(m^{T^{-1}}(x),x)L^N_{m^{T^{-1}}(x)}\frac{dm^{T^{-1}}(x)}{dx} + \gamma^S(m^{T^{-1}}(x),x)L^S_{m^{T^{-1}}(x)}\frac{dm^{T^{-1}}(x)}{dx}$$
$$= \gamma^N(m^{T^{-1}}(x),x)\left(L^N_{m^{T^{-1}}(x)} + \Delta L^S_{m^{T^{-1}}(x)}\right)\frac{dm^{T^{-1}}(x)}{dx}$$

for $x \geq \tilde{x}^T$. Again, both equations, written in terms of world quantities, take the same form as in the closed economy.

Considering the supply of intermediate goods for final good production (β-intermediates), for the same reason as in the case of α-intermediates only Northern monopolists will produce β-intermediates. They will therefore also face iso-elastic demand and charge constant markups, $p^N_{\beta,i} = \eta_\beta/\kappa$. To derive an inverse demand equation in terms of world quantities, define $s_\beta := (Y^{NN}_x + Y^{SN}_x)/Y_x$ as the share of world task output that is utilized in the North. Note that this share is constant across tasks, since otherwise the marginal products of tasks, and hence task prices, would differ across countries. Using s_β, inverse demand for β-intermediates in the North can be written as:

$$p^N_{\beta,i} = \kappa\beta^N_i q^{NN^{\kappa-1}}_{\beta,i}\left[\int_0^1 \left(Y^{NN}_x + Y^{SN}_x\right)^{\frac{\epsilon-1}{\epsilon}} dx\right]^{\frac{\epsilon(1-\kappa)}{\epsilon}}$$
$$= \kappa\beta^N_i q^{NN^{\kappa-1}}_{\beta,i} s_\beta^{1-\kappa}\left[\int_0^1 Y_x^{\frac{\epsilon-1}{\epsilon}} dx\right]^{\frac{\epsilon(1-\kappa)}{\epsilon}},$$

and inverse demand in the South:

$$p^N_{\beta,i} = \kappa\beta^N_i q^{NS^{\kappa-1}}_{\beta,i}\left[\int_0^1 \left(Y^{SS}_x + Y^{NS}_x\right)^{\frac{\epsilon-1}{\epsilon}} dx\right]^{\frac{\epsilon(1-\kappa)}{\epsilon}}$$
$$= \kappa\beta^N_i q^{NS^{\kappa-1}}_{\beta,i}(1-s_\beta)^{1-\kappa}\left(\int_0^1 Y_x^{\frac{\epsilon-1}{\epsilon}} dx\right)^{\frac{\epsilon(1-\kappa)}{\epsilon}}.$$

The two inverse demand equations imply that the share s_β also equals the share of world output of β-intermediates that is utilized in the North: $q^{NN}_{\beta,i} = s_\beta q_{\beta,i}$, or, using symmetry across intermediate varieties, $q^{NN}_\beta = s_\beta q_\beta$, where $q_{\beta,i}$ and q_β denote world output. With this observation, and again using symmetry across i, the inverse demand equations imply

$$p_\beta = \kappa\beta^N q_\beta^{\kappa-1}\left(\int_0^1 Y_x^{\frac{\epsilon-1}{\epsilon}} dx\right)^{\frac{\epsilon(1-\kappa)}{\epsilon}}, \tag{45}$$

which is the desired inverse demand equation in terms of world quantities. Profits of β-monopolists are then given by

$$\pi_{\beta,i}(\beta^N_i) = \max_q\left\{\kappa\beta^N_i q^\kappa\left(\int_0^1 Y_x^{\frac{\epsilon-1}{\epsilon}} dx\right)^{\frac{\epsilon(1-\kappa)}{\epsilon}} - \eta_\beta q\right\} - p_D\beta^{N^{\frac{1}{\rho}}}_i,$$

such that the first-order condition for a profit maximum in β^N_i becomes (using symmetry to drop the i):

$$\rho\kappa q_\beta^\kappa \left(\int_0^1 Y_x^{\frac{\epsilon-1}{\epsilon}}\, dx\right)^{\frac{(1-\kappa)\epsilon}{\epsilon-1}} = p_D {\beta^N}^{\frac{1-\rho}{\rho}} . \tag{46}$$

Finally, let $Y = Y^N + Y^S$ be world (gross) production of the final good. Using the share s_β, we can write

$$\begin{aligned} Y &= \beta^N (s_\beta q_\beta)^\kappa \left(\int_0^1 (s_\beta Y_x)^{\frac{\epsilon-1}{\epsilon}}\, dx\right)^{\frac{(1-\kappa)\epsilon}{\epsilon-1}} + \beta^N ((1-s_\beta) q_\beta)^\kappa \left(\int_0^1 ((1-s_\beta) Y_x)^{\frac{\epsilon-1}{\epsilon}}\, dx\right)^{\frac{(1-\kappa)\epsilon}{\epsilon-1}} \\ &= \beta^N q_\beta^\kappa \left(\int_0^1 Y_x^{\frac{\epsilon-1}{\epsilon}}\, dx\right)^{\frac{(1-\kappa)\epsilon}{\epsilon-1}} . \end{aligned} \tag{47}$$

This gives world final good production in terms of world quantities of tasks and β-intermediates.

Collecting the derived equations yields equilibrium conditions for world quantities and prices that are common across countries. These conditions exactly replicate the equilibrium conditions for the closed economy in Section C.3.2. In particular, in any trade equilibrium the common matching function $m^T(x)$, the common automation threshold $\tilde{x}^T$, world quantities $\{K_x\}_{x\in X}$, $\{Y_x\}_{x\in X}$, q_α, and q_β, and productivity levels α^N and β^N; the common prices $\{p_x\}_{x\in X}$, p_c^T, p_α^N, p_β^N, the Northern R&D price p_D, and Northern wages $\{w_s^N\}_{s\in S}$ satisfy the following conditions:

- (E1)″ $Y_x = \alpha^N q_\alpha^\kappa K_x^{1-\kappa}$ if $x < \tilde{x}^T$ and $Y_x = \gamma^N(m^{T^{-1}}(x), x) \left(L^N_{m^{T^{-1}}(x)} + \Delta L^S_{m^{T^{-1}}(x)}\right) \frac{dm^{T^{-1}}(x)}{dx}$ if $x \geq \tilde{x}$;
- (E2)″ $p_x = \frac{\partial Y}{\partial Y_x}$ for all x;
- (E3)″ q_β satisfies equation (45);
- (E4)″ $m^T(s) \in \operatorname{argmax}_{x\in X} \gamma^N(s,x) p_x$ for all s;
- (E5)″ $w_s^N = \gamma^N(s, m^T(s)) p_{m^T(s)}$ for all s;
- (E6)″ $\left(\frac{p_\alpha^N}{\kappa\alpha^N}\right)^\kappa \left(\frac{p_c^T}{(1-\kappa)\alpha^N}\right)^{1-\kappa} = p_x$ for all $x < \tilde{x}^T$ and $p_c^T = r$;
- (E7)″ $\frac{w_0^N}{\gamma^N(0,\tilde{x}^T)} = \left(\frac{p_\alpha^N}{\kappa\alpha^N}\right)^\kappa \left(\frac{r}{(1-\kappa)\alpha}\right)^{1-\kappa}$;
- (E8)″ q_α satisfies equation (43);
- (E9)″ $p_\alpha^N = \frac{\eta_\alpha}{\kappa}$ and $p_\beta^N = \frac{\eta_\beta}{\kappa}$;
- (E10)″ α^N, β^N, and p_D satisfy equations (44), (46), and ${\alpha^N}^{\frac{1}{\rho}} + {\beta^N}^{\frac{1}{\rho}} = D$.

These conditions are identical to conditions (E1)′ to (E10)′ for the closed economy. Therefore, the collection of world quantities and prices described in (E1)″ to (E10)″ is identical to the equilibrium of a closed economy with labor supply $L^N + \Delta L^S$ and parameter values of the North. The difference between world quantities and prices under trade and the Northern autarky equilibrium is thus identical to the difference in Northern autarky variables that arises from a hypothetical change in labor supply from L^N to $L^N + \Delta L^S$. This verifies the claim in Section 6.4 of the main text that the effects of trade integration on technology α^N and wages w^N are the same as the effects of a change in labor supply from L^N to $L^N + \Delta L^S$.