Political Competition with Endogenous Party Formation and Citizen Activists

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Abstract

This paper studies the effects of endogenous party formation on political platforms. It develops a model in which parties allow like-minded citizens to, first, share the cost of running in a public election and, second, coordinate on a policy platform. The paper characterizes the set of political equilibria with two competing parties and with one uncontested party. In two-party equilibria, the distance between both platforms is always positive but limited, in contrast to the median voter model and the citizen candidate model. In one-party equilibria, the median voter can be worse off than in all equilibria with two competing parties.

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1 Introduction

This paper studies electoral competition between political parties that are endogenously formed by policy-interested citizens. The analysis explicitly accounts for two central aspects of the empirically evident group-character of political parties: First, the foundation and the composition of parties result from the strategic interaction of heterogeneous citizens. Second, the nomination of party candidates and the selection of policy platforms result from the strategic interaction of heterogeneous party members. The paper thereby contributes to the economic theory of electoral competition, initiated by Downs (1957). Most previous papers in this field have studied elections between independent candidates or parties that act as unitary agents. In particular, there is no party formation in the commonly used workhorse models of political economy, including the classical median voter model (Downs 1957) and the citizen candidate model (Osborne & Slivinski 1996, Besley & Coate 1997). The main goal of this paper is to investigate the effects of endogenous party formation on the policies implemented in political equilibria. More precisely, I study which policy platforms can be offered by political parties with stable membership structures, i.e., with sets of party members such that no citizen has an incentive to change his party affiliation.

For this purpose, I develop a formal model of political competition with endogenous party formation. This model has four central features. First, the political process is democratic in every respect: there is a (large) set of citizens with heterogeneous policy preferences each of whom is entitled to join a party, to become the party's candidate for a public office and to vote in a public election for this office. Both the number and the composition of the competing parties are determined endogenously within the political process. Second, political activity is costly: Parties have to pay an exogenous cost of running to enter political competition, and citizens have to pay an exogenous membership fee to join a party. The members' payments are used to finance the cost of running mentioned above. Third, party members coordinate their behavior in primary elections. The members of each party select one candidate from their ranks to run for the public office. This allows the party members to commit to the candidate's ideal policy, thereby choosing a policy platform. Fourth, there is electoral risk. In particular, the citizens perceive the median voter's ideal policy as the realization of a random variable with a probability distribution that satisfies a set of regularity conditions.

The model builds on the citizen candidate framework by Osborne & Slivinski (1996) and Besley & Coate (1997). In particular, the two first features are similar in spirit to the citizen candidate model, but are adapted to suit a model with endogenous party formation. In contrast, the modeling of endogenous party formation and within-party coordination (third feature) and the assumption of electoral risk (fourth feature) differ from the previous literature. The latter modeling decision helps to make crucial trade-offs and mechanisms at work more visible and less degenerate.

The paper provides three contributions to the theoretical literature on electoral competition. First, I develop a novel theoretical framework that allows to study party formation and political competition simultaneously. From a conceptual perspective, the identification of policy platforms that can be supported by stable party membership structures is novel to the literature on electoral competition. In particular, a party can be stable if and only if (i) no party member can benefit from leaving the party and saving the membership cost, and (ii) no independent agent can benefit from joining a party and potentially becoming its candidate. Otherwise, some citizen has an interest to change his party affiliation, anticipating the effect on the parties' policy platforms and the implemented policy.

The second contribution is given by a complete characterization of the policy platforms that can be offered in political equilibria with two competing parties and with one uncontested party. For the main result of this paper, I revisit the classical question whether the equilibrium platforms of two competing parties are fully convergent as in Downs (1957), or divergent as in Osborne & Slivinski (1996) and Besley & Coate (1997). In contrast to these models, the platform distance in political equilibria of my model is always strictly positive but limited. Intuitively, endogenous party formation gives rise to both a centrifugal and a centripetal force. On the one hand, parties can only be stable if their policy platforms are sufficiently different – otherwise, no citizen would be willing to support a party and bear the membership costs. On the other hand, parties can only be stable if their policy platforms are sufficiently close to each other – otherwise, a moderate independent citizen would enter and be nominated as candidate to increase the party's electoral prospects. In the benchmark case of full electoral certainty, the policy platforms in two-party equilibria are uniquely determined and exhibit a strictly positive distance that depends only on the cost of party membership. This limit case, hence, makes the difference to the results of both the median voter model and the citizen candidate model most obvious. As acknowledged by Besley & Coate (1997), a dissatisfactory feature of the citizen candidate model is given by the multiplicity of political equilibria with two candidates: the distance platform may be small or very large.[1] Endogenous party formation substantially reduces this multiplicity of equilibria.

[1]See also Dhillon & Lockwood (2002), De Sinopoli & Turrini (2002) and Roemer (2003).

Finally, I derive a novel result on the difference between one-party and two-party equilibria, which sheds light on the desirability of democratic (multi-party) competition. In particular, I show that the platform of an uncontested party can deviate more strongly from the median voter position than the platforms of two competing parties. Intuitively, the members of competing parties need to balance their own policy preferences and those of the electorate at large, while the members of an uncontested party can focus on the preferences within their own group. As a result, the median voter is ex post worse off in some one-party equilibria than in every two-party equilibrium. This result is in line with the conventional view that democratic competition (between multiple parties) is beneficial because it provides incentives for each party to respect the voters' interests. By contrast, the citizen candidate models by Osborne & Slivinski (1996) and Besley & Coate (1997) give rise to the opposite conclusion.[2]

In the following sections, I focus on a basic model in which the agents have linear and symmetric policy preferences, and the parties' electoral prospects depend on their policy platforms only. The qualitative results do not change, however, if policy preferences are concave or asymmetric. They also remain unchanged if I assume that the winning probability is continuously increasing in the campaign contributions one party collects relative to the other one. In particular, the platform distance in political equilibria with two active parties is bounded both from below and from above in all these model versions.

The paper proceeds as follows. Section 2 discusses the related literature, and Section 3 presents the model. Section 4 solves the electoral game and derives the set of political equilibria with two competing parties and with one uncontested party. Section 5 provides comparative statics and results for the limit cases of electoral certainty and zero cost of party membership. Section 6 briefly discusses some extensions of the basic model. Section 7 concludes. All formal proofs are relegated to the appendix.

2 Related literature

The model builds on the citizen candidate framework introduced by Besley & Coate (1997) and Osborne & Slivinski (1996). In both versions of this model, the set of candidates is determined endogenously from the set of citizens who are not only entitled to vote in a democratic election, but can also decide to run as (individual)

[2]In both models, the median voter is strictly better off in each equilibrium with one uncontested candidate than in any equilibrium with two competing candidates.

candidates, facing an exogenous cost of candidacy. There are no parties, and citizens cannot coordinate their political behavior. In both model versions, there is a multiplicity of political equilibria with either one or two candidates. Their main insight is that the endogeneity of the candidate set eliminates the possibility of fully convergent platforms in two-candidate equilibria. This impossibility result is in sharp contrast to the classical predictions of the median voter model by Downs (1957) and the probabilistic voting model by Lindbeck & Weibull (1987), but in line with empirical observations. In both versions of the citizen candidate model, there exist equilibria with strongly polarized candidates. In Besley & Coate (1997), in particular, the platform distance in two-candidate equilibria is only bounded by the extremes of the policy space.[3]

My paper contributes to a strand in the literature that extends the basic citizen candidate framework to accommodate political parties. Most closely related, Levy (2004) studies endogenous party formation under the assumption that parties can credibly commit to any policy in the Pareto set of their members, while individual candidates can only commit to their own ideal policy. Hence, parties allow to convexify the set of credible policy proposals. She shows that party formation *increases* the set of potential equilibrium policies, but only if the policy space is multi-dimensional. Building on this work, Dotti (2021) studies the parties that can form in an elected assembly and shows that their effect on the equilibrium policy depends on the status quo policy, see also Pech (2012).[4] Complementing these results, I show that endogenous party formation *reduces* the set of potential equilibrium policies by eliminating the equilibria with the largest platform distance (which may be considered as political failures). Importantly, this effect of party formation does not depend on whether the policy space is one-dimensional or multi-dimensional.

A few additional papers extend the citizen candidate to accommodate party formation, but do not study the effects of party formation on political polarization. For example, Rivière (1999) and Osborne & Tourky (2008) provide game-theoretical explanations for Duverger's law, i.e., the prevalence of two-party systems under the plurality rule. Morelli (2004) studies the implications of alternative electoral systems for the formation of parties by agents with heterogeneous policy preferences. Snyder & Ting (2002) and Poutvaara & Takalo (2007) show that parties may serve as

[3] In Osborne & Slivinski (1996), there also exist multiple equilibria with different levels of polarization. The platform distance is only limited by the assumption of sincere instead of strategic voting; it is not related to the candidates' behavior or coordination.

[4] Both Pech (2012) and Dotti (2021) find that the equilibrium policy cannot make the median legislator worse off than the status quo policy. Hence, if the median legislator prefers to maintain the status quo, party formation cannot affect the implemented policy. None of these papers studies the effects of parties on the platform distance in two-party equilibria.

brand names or screening devices, which provide superior information about the candidates' preferences or quality, respectively.

These previous papers do neither examine the effects of party formation on political polarization nor on the multiplicity of equilibria in the citizen candidate model. Furthermore, they either consider only the case of electoral uncertainty or strongly restrict the type space. By contrast, I study the implications of party formation on platform choice in a general setting, allowing for different degrees of electoral uncertainty and a large set of agents with heterogeneous policy preferences.[5]

To my knowledge, only two previous papers investigate the effect of political parties on platform choice within the citizen candidate framework. Cadigan & Janeba (2002) study the competition between two exogenously given parties in a US-style presidential election with primary elections and identify a strong connection between membership structures and party platforms. As they do not endogenize membership structures, however, the model only delivers limited insights into the effects of party formation. Jackson et al. (2007) use a similar, one-dimensional model to compare the effects of different nominations processes on the chosen platforms and the equilibrium policy. Their focus is on political competition between parties with exogenous membership structures as well. In both models, any combination of platforms can represent a political equilibrium for some given membership structures.[6]

In addition, there is a small number of papers on the formation of political parties outside the citizen candidate framework. Most closely related, Roemer (2006) studies the effects of endogenous party formation and campaign contributions by policy-motivated citizens. He identifies a unique equilibrium with positive but limited policy convergence. The result is driven by (i) his notion of a cooperative "Kantian equilibrium", in which the agents consider joint deviations by all party members, and (ii) the assumption that the policy platforms are chosen through a bargaining process in which an agent's influence depends on her contribution. In contrast, I apply a non-cooperative equilibrium notion and assume that each party member has exactly one vote in the primary elections. In the papers by Herrera et al. (2008), Campante (2011) and Ortuño-Ortin & Schultz (2005), there is no endogenous party formation. However, citizen can make contributions to exogenously given parties and, thereby, affect the parties' platform choices indirectly. Poutvaara (2003) studies a model where the citizens form political parties based on expressive

[5]Dhillon (2005) surveys the existing theoretical models with pre-election as well as post-election party formation, with a particular focus on papers that extend the citizen candidate model.

[6]In an extension, Jackson et al. (2007) also consider a case with endogenous parties where voters can join parties at zero cost and the median voter is known ex ante. They find that the median voter's preferred policy always prevails if nomination is by voting of the party members, thereby confirming Levy (2004).

(non-strategic) objectives only. He predicts a positive but limited platform distance, as in my model with strategic membership choices by policy-orientied citizens.[7]

Finally, this paper also relates to a literature on the effects of electoral uncertainty on political competition. Wittman (1983) and Calvert (1985) extend the model by Downs (1957) to study elections between two policy-oriented candidates with exogenous ideal policies. In this model, electoral uncertainty leads to the adoption of divergent policy platforms. The level of divergence depends on the candidates' ideal policies, which are exogenously given and not determined within the political process. Based on this framework, Bernhardt et al. (2009*a*) show that a moderate degree of platform divergence increases voter welfare if there is electoral uncertainty and voter preferences are positively correlated, see also Bernhardt et al. (2009*b*). This is related to my result that the median voter can be better off in equilibria with two parties than with one uncontested party.[8] Eguia (2007) studies the effect of electoral uncertainty in the citizen candidate model. Without party formation, electoral uncertainty mainly eliminates equilibria with a single candidate and leads to the existence of asymmetric equilibria with two candidates. Per se, it does not limit political polarization. All these papers focus on the behavior of individual candidates and do not examine the effects of party formation.

3 The model

This section introduces the basic model, including the set of agents, their preferences and the policy space. Subsequently, I explain the political process and define the notion of a political equilibrium.

3.1 The environment

The population consists of a large finite set $\mathcal{N}$ of agents. The utility of each agent i depends on some implemented policy $x \in \mathbb{R}$ and a pair (α_i^L, α_i^R) of payments she makes. I explain the role of these payments below. Specifically, the preferences of i can be captured by the utility function

$$u\left(x, \omega_i, \alpha_i^L, \alpha_i^R\right) = v(x - \omega_i) - \alpha_i^L - \alpha_i^R \,, \tag{1}$$

[7]Besides, a few papers study models with endogenous party formation under proportional electoral systems in which the implemented policy is given as a weighted sum of the party platforms (e.g., Gomberg et al. 2004, Gerber & Ortuño-Ortin 1998). In these models, all political equilibria involve an extremely high level of political polarization.

[8]In my model, the platform of an uncontested party can differ more strongly from the expected median voter position than the platforms of two competing parties. In contrast to Bernhardt et al. (2009*a*,*b*), this result also holds if there is no electoral uncertainty.

where ω_i denotes agent i's ideal point in the policy space $\mathbb{R}$ and $v(x-\omega_i)$ captures her policy payoff. For most of this paper, I concentrate on the case of linear Euclidean preferences with $v(x-\omega_i) = -|x-\omega_i|$.[9]

Policy x emerges from a political process that I describe in detail below. At the beginning of this process, only the ideal points of a subset $\mathcal{A} \subset \mathcal{N}$ of agents are publicly observable. I refer to these agents as *political activists*, and to the collection $\Omega = \{\omega_i\}_{i\in\mathcal{A}}$ of their ideal points as the *activist population.* The ideal points of the remaining agents in $\mathcal{N} \setminus \mathcal{A}$ are only revealed at a later stage of the political process. This assumption creates uncertainty about the ideal point m of the median voter. Specifically, I assume that the median position m is commonly perceived to be the realization of a random variable with *distribution function* Φ and density ϕ, satisfying the following assumptions.

Assumption 1. *The distribution function Φ is twice continuously differentiable, log-concave, and symmetric with $\Phi(x) = 1 - \Phi(-x) > 0$ for all $x \in \mathbb{R}$, and satisfies* $\lim_{x\to-\infty} \Phi(x) = 0$.

Assumption 1 imposes a set of regularity conditions on Φ. They imply that the expected value of m is normalized to 0, the density ϕ has full support on $\mathbb{R}$, and the ratio $\phi(x)/\Phi(x)$ is monotonically decreasing.[10]

3.2 The political process

Policy x is determined in a game with four stages: (i) party formation, (ii) candidate selection, (iii) general election, and (iv) policy implementation. The political process is structured by 2 parties, labeled L and R.[11]

Party formation. At the first stage, the ideal policies of all activists become public information. Each activist $i \in \mathcal{A}$ chooses her payments $\alpha_i^L \geq 0$ and $\alpha_i^R \geq 0$, interpreted as contributions to the parties L and R, respectively. For simplicity, I assume that each agent can only make a positive contribution to one party.[12] Activist i enters party $J \in \{L, R\}$ if α_i^J exceeds an exogenous threshold $c > 0$. Hence, the *member set* of party J is given by $\Omega_J = \left\{\omega_i \mid i \in \mathcal{A} : \alpha_i^J \geq c\right\}$. The threshold c can be interpreted as a membership cost. It can be thought of as a monetary payment,

[9] I consider cases with non-linear or asymmetric preferences in Section 6.

[10] These conditions are standard in the literature and satisfied by many commonly used distribution functions, including the normal, the logistic and the Laplace distributions.

[11] The restriction to two parties has no effect on the my main results: If a pair of platform is a robust equilibrium outcome given that at most two parties can form, then it is also a robust equilibrium outcome given that $K > 2$ parties can form.

[12] In political equilibria, this restriction is not binding.

but also as hours worked or effort spent for the party's electoral campaign and party meetings. Agents in $\mathcal{N} \setminus \mathcal{A}$ do not make contributions and cannot join a party.

I impose the following assumption on the threshold c and the distribution Φ.

Assumption 2. *The membership cost c and the median voter distribution Φ satisfy the condition $c\,\phi(0) \geq 1$.*

Assumption 2 can either be interpreted as a lower bound on c or as an upper bound on the electoral risk implied by the median voter distribution Φ.[13] I prefer the latter interpretation. The assumption allows to avoid case distinctions in the formal analysis below. In Section 6, I explain how the results may change when it is violated.

Party J becomes *active* if the sum of its contributions $\alpha^J \sum_{i \in \mathcal{A}} \alpha_i^J$ exceeds another threshold $C \geq 3c$. The threshold C can be thought of as an exogenous cost of running in the general election (as in the citizen candidate models by Besley & Coate 1997 and Osborne & Slivinski 1996). The assumption $C \geq 3c$ rules out the prevalence of parties with less than three members in political equilibria.

Candidate selection. At the second stage, the members of each active party J observe the member set Ω_J, but not the member set of the competing party Ω_{-J} (see further details below). They nominate a *presidential candidate* from their ranks through primary elections. Specifically, if party J is active, a set of simultaneous pairwise elections between all agents in the member set Ω_J takes place. The candidate is drawn with equal probability from the agents in the top cycle, i.e., from the agents with the lowest number of pairwise defeats.[14] If a member of J is a Condorcet winner, then she is nominated with certainty. The candidate cannot make binding policy commitments. As will become clear, her ideal point can be interpreted as party J's policy platform, denoted by ℓ for party L and by r for party R. An inactive party does not have a leader; its platform is denoted by $\emptyset$.

General election. At the third stage, the ideal points of both candidates become public information. Moreover, the ideal points of all non-activists in set $\mathcal{N} \setminus \mathcal{A}$ and, thereby, the ex post location of the median voter are realized. Then, the candidates of all active parties run in a general election. They cannot make binding policy commitments. Each agent in $\mathcal{N}$ casts her vote for one of the candidates. If there

[13]In the limit case where the electoral risk vanishes, $\phi(0)$ goes to ∞. In this case, hence, Assumption 2 is satisfied whenever c is strictly positive.

[14]The same results arise under the following assumption: If there is a median party member, she can nominate a candidate from Ω_L. If there is no median member, then the candidate is randomly drawn from the party members nominated by the two midmost members.

are two active parties, the candidate with the highest share of votes becomes the president. If both candidates receive the same vote share, the president is determined by tossing a fair coin. If there is only one active party, the candidate of this party directly becomes president. If there is no active party, the presidential position remains empty.

Policy implementation stage. At the final stage, the president independently chooses the implemented policy $x \in \mathbb{R}$. If there is no president, no policy is implemented and the utility of each agent in $\mathcal{N}$ equals $-\infty$.

3.3 Party structures, information structure and political equilibria

A party structure is given by a partition of the activist population Ω into the member sets Ω_L and Ω_R, and the set of independent activists $\Omega \setminus (\Omega_L \cup \Omega_R)$. An outcome is given by a pair of platforms (ℓ, r). I assume the following information structure, illustrated in Figure 2 in Appendix B. At the party formation stage, the set of activists Ω_A and the median voter distribution Φ are public information. At the primary election stage, the members of party J can observe their own member set Ω_J, but not the member set Ω_{-J} of the other party. This implies that, for the members of J, all party structures with the same member set Ω_J are contained in one information set. Hence, the members of party L choose their candidate based on a set of beliefs about the platform r of the competing party R (and vice versa). At the general election stage, the platforms ℓ and r – the ideal points of both parties' candidates – are publicly observable.

A strategy α_i of agent i with ideal point ω_i specifies (i) her contributions α_i^L and α_i^R, (ii) her primary voting choices for each possible member set Ω_J such that i is a member of party J, (iii) her voting choice at the general election stage for each potential pair of platforms (ℓ, r), and (iv) her policy choice in case of becoming president. A belief of agent i specifies, for each member set Ω_J such that i is a member of party J, an expectation about the policy platform of the competing party $-J$. A political equilibrium is given by a Perfect Bayesian equilibrium of the game defined above, consisting of a strategy profile $\alpha = (\alpha_i)_{i \in A}$ and a belief system such that, first, the strategies of all agents are sequentially rational given the belief system and, second, the belief system is derived from the strategy profile α using Bayes' rule whenever possible. I focus on political equilibria in pure strategies in which the agents do neither choose weakly dominated actions at the candidate selection stage nor at the general election stage.

I am interested in the set of equilibrium outcomes of the game defined above. A complication is that this set depends crucially on the activist population $\Omega = \{\omega_i\}_{i \in \mathcal{A}}$. Hence, one option would be to study the set of equilibrium outcomes for one specific example of Ω. Another option is to solve for outcomes that arise in equilibria for "many different" activist populations. I follow the second path because my main interest is in identifying the equilibrium outcomes in large populations with a rich heterogeneity in policy preferences (i.e., with many different bliss points distributed over the entire set $\mathbb{R}$).

To formalize this idea, I introduce the following notion of robustness with respect to the activist population. I denote by $\mathcal{P}(\Omega)$ the set of equilibrium outcomes for a specific set Ω. I say that outcome (ℓ, r) is a *potential equilibrium outcome* if there is some finite population Ω such that $(\ell, r) \in \mathcal{P}(\Omega)$. It is a *robust equilibrium outcome* if there is some finite population Ω such that (ℓ, r) is an equilibrium outcome for Ω and any perturbation of Ω, i.e., if $(\ell, r) \in \mathcal{P}(\Omega)$ and $(\ell, r) \in \mathcal{P}(\Omega \cup \{\omega_k\})$ for any $\omega_k \in \mathbb{R}$. Hence, this notion of robustness eliminates equilibrium outcomes that can only arise under certain restrictions on the preference heterogeneity in Ω. The analysis below shows that the set of robust equilibrium outcomes is non-empty for any combination of the membership fee c, the cost of running C, and the median distribution Φ.

A simple example helps to explain the notion of population-robustness. Consider an allocation such that party R runs with platform $r = \tilde{\omega}$ and party L is inactive, $\ell = \emptyset$. Trivially, $(\tilde{\omega}, \emptyset)$ is an equilibrium outcome if all activists in the population Ω have the same ideal point $\tilde{\omega} \in \mathbb{R}$. But to what extent does this result depend on the obviously strong assumption that all activists have identical policy preferences? Put differently, is it also an equilibrium outcome if the population contains one additional activist with some other ideal point $\omega_i \in \mathbb{R}$? Proposition 3 below shows that, if the electoral risk implied by Φ is small, the pair $(\tilde{\omega}, \emptyset)$ is a *robust equilibrium outcome* if and only if the policy $\tilde{\omega}$ is located close enough to the expected location of the median voter, $E[m] = 0$. I consider the latter result as providing deeper insights about electoral competition in large electorates with heterogeneous policy preferences than the former.[15]

[15] Instead of using this notion of robustness, I could assume that the agents' ideal points are independent random draws from some probability distribution with full support on $\mathbb{R}$, and study the set of equilibria in the limit case where the number of activists goes to infinity. Asymptotically, this set of equilibrium outcomes equals the set of robust equilibrium outcomes identified below.

4 Equilibrium analysis

In the following, I solve the game backwards, starting with the policy implementation and the general election subgames. Both subgames can be solved straightforwardly.

Policy implementation. At the final stage, let candidate j be the president. To maximize her individual utility, she implements her own ideal policy ω_j. Hence, the nomination of agent j as presidential candidate of party J implies a credible commitment to policy w_j. Correspondingly, I henceforth refer to the candidates' ideal points as the parties' policy platforms ℓ and r, respectively.

General election. At the general election stage, each citizen observes the platforms of all active parties, and casts her vote for one of them. If there are two active parties, the citizens anticipate the winner's policy choice. Hence, agent i's weakly dominant action is to vote for the party whose platform is closer to her own ideal point ω_i. As preferences are single-peaked, party L wins the election if and only if the median voter prefers it to party R. As a convention, let party L be the party with the smaller (more leftist) platform, $\ell \leq r$.[16] Then, party L wins the election if and only if the median voter's ideal point m is below $(\ell + r)/2$.

Ex ante, the agents only know the probability distribution Φ of the median voter position $m \in \mathbb{R}$. Hence, there is electoral risk: for any pair (ℓ, r), both parties win the election with positive probabilities. Specifically, the winning probability $p(\ell, r)$ of party L is given by

$$p(\ell, r) = \Phi\left(\frac{\ell + r}{2}\right) \in (0, 1) \ . \tag{2}$$

Candidate selection. At the candidate selection stage, the members of each active party J nominate a candidate for the general election. To simplify the exposition, I henceforth focus on candidate selection in the leftist party L, and on cases where party L has an odd number of members $\#\Omega_L$.

At this stage, each member of party L can observe the member set Ω_L, i.e., the bliss points of all her party fellows. I denote by m_L the ideal point of party L's median member. The members of party L select a candidate from their ranks and, thereby, choose the platform ℓ from the set Ω_L through a series of pairwise elections. They cannot observe the member set Ω_R of the competing party, however. Hence, the voting behavior at this stage is based on a set of beliefs with respect to the

[16] Otherwise, both parties are re-labeled.

platform of party R.[17] Let the members of L have a common point belief about this platform, denoted by $\hat{r} \in \mathbb{R} \cup \emptyset$ and referred to as the expected platform.[18] In the following, I distinguish between two cases.

I start with the case where the members of L expect party R to be inactive, $\hat{r} = \emptyset$. In this case, pairwise voting gives rise to a straightforward outcome.

Lemma 1. *Let $\hat{r} = \emptyset$ and $\#\Omega_L$ be odd. Then, the platform of party L is given by the party median m_L.*

If party L runs in an uncontested election, its candidate will become president and implement her ideal policy with certainty. Hence, the members of L effectively choose the implemented policy x when voting on platform ℓ. Their preferences over the platform ℓ coincide with their single-peaked policy preferences $v(x - \omega_i)$. By standard arguments, if there is a party median, its ideal policy is a Condorcet winner in the set Ω_L. Hence, the party median wins a pairwise election against any other option.[19]

Next, let the members of L expect party R to be active and its platform to be $\hat{r} > m_L$. In this case, the members need to choose their preferred platform ℓ in a general election against party R. This requires to decide between more extreme platforms close to the party median m_L and more moderate platforms closer to $\hat{r}$. The former platforms provide a larger policy payoff in case of winning for the majority of members; the latter platforms offer a higher winning probability. The *policy effect function*

$$\Gamma(\ell, \hat{r}) := p(\ell, \hat{r})(\hat{r} - \ell) \ . \tag{3}$$

combines both aspects to measure the effect of platform ℓ on the expected policy $E[x] = p(\ell, \hat{r}) \ \ell + [1 - p(\ell, \hat{r})] \ \hat{r}$. Denote by $\gamma(\hat{r}, \Omega_L)$ the platform that maximizes the policy effect $\Gamma(\cdot, \hat{r})$ over the elements of Ω_L. The following lemma identifies the optimal candidate selection in terms of this function.

Lemma 2. *Let $\hat{r} \geq m_L$ and $\#\Omega_L$ be odd. Then, the platform of party L is given by the maximum of m_L and $\gamma(\hat{r}, \Omega_L)$.*

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[17] Alternatively, the members of party L can hold beliefs about the composition of member set Ω_R, which then map into a more compact belief about platform r. As only the policy platform is payoff-relevant, I only consider the latter.

[18] In a pure-strategy equilibrium with platforms ℓ and r, the beliefs must be consistent with equilibrium strategies so that all members of party L must hold the correct point belief $\hat{r} = r$.

[19] Lemma 1 naturally extends to the case with an even number of party members. In this case, the two midmost members with ideal points m_{L-} and $m_{L+} \geq m_{L-}$ are nominated with equal probability.

Lemma 2 follows from two insights that generalize beyond the details of my model. First, consider some party member i with ideal point $\omega_i < \hat{r}$. When choosing platform ℓ, member i strictly prefers ω_i to any policy right of $\hat{r}$ and to any policy left of ω_i. Among the remaining platforms in the interval $[\omega_i, \hat{r})$, moreover, i prefers the platform that maximizes $\Gamma(\ell, \hat{r})$, i.e., that shifts the expected policy most strongly towards her own ideal point ω_i. Under Assumption 1, in particular, function $\Gamma(\ell, \hat{r})$ is strictly quasi-concave and has a unique maximizer in $(-\infty, \hat{r})$. Figure 3 in Appendix B depicts $\Gamma(\ell, \hat{r})$ for an example with a normally distributed population median m.

Second, the agents' preferences on platform ℓ are in general not single-peaked. But they satisfy a single-crossing property à la Gans & Smart (1996) for any distribution function Φ, as I show in the formal proof of Lemma 2. Consequently, voting is monotonic in any pairwise election and, if there is a median member, then her preferred platform represents a Condorcet winner among the elements in the member set Ω_L. With pairwise elections, this Condorcet winner prevails.[20]

Lemma 2 follows from the combination of these two insights: A majority of the members of party L cast their votes for the platform that maximizes $\Gamma(\ell, \hat{r})$ over the set of available platforms in the interval $[m_L, \hat{r})$. In this model, the set of available platforms is given by Ω_L, the ideal points of party L's members. Hence, platform ℓ is either given by the constrained maximizer $\gamma(\hat{r}, \Omega_L)$ or by the ideal point m_L of the party median, whatever is larger.[21]

Party formation. Lemmas 1 and 2 identify platform ℓ for any combination of the member set Ω_L and the platform belief $\hat{r}$. Correspondingly, platform r is pinned down for any combination of Ω_R and the platform belief $\hat{\ell}$. In a political equilibrium, the beliefs on both platforms must be consistent with the ideal points of the presidential candidates, $\hat{r} = r$ and $\hat{\ell} = \ell$. With an exogenous party structure (Ω_L, Ω_R), these conditions would already pin down the equilibrium platforms (ℓ, r).

In my model, however, the party structure emerges endogenously from the activists' actions at the first stage. In an equilibrium, the agents anticipate the effects of their actions on the platforms (ℓ, r) and, ultimately, on the implemented policy x. In a political equilibrium, party structures must therefore be stable in the sense that

(I) no member of party $J \in \{L, R\}$ can profitably leave her party, and

[20] The same outcome arises if the median party member is entitled to nominate his preferred membe, see Jackson et al. (2007) and Poutvaara (2003).

[21] Again, this result can easily be generalized to the case with an even number of party members. Then, the two midmost members with ideal points m_{L-} and m_{L+} are nominated with equal probability if $m_{L+} > \gamma(\hat{r}, \Omega_L)$. Otherwise, the member with ideal point $\gamma(\hat{r}, \Omega_L)$ is nominated.

(II) no independent citizen can profitably join party L or party R.

Conditions (I) and (II) are necessary and jointly sufficient conditions for a political equilibrium.[22] Henceforth, I refer to a party structure as *exit-stable* if it satisfies (I), and as *entry-stable* if it satisfies (II). In the following, I use these conditions to derive the set of robust equilibrium outcomes.

4.1 Political equilibria with two active parties

I now study the set of robust political equilibria with two active parties. For this purpose, I first establish a property on the party structures in any robust equilibrium. Then, I subsequently characterize the symmetric and asymmetric pairs of platforms that constitute robust equilibrium outcomes.

Efficient parties in two-party equilibria. Consider a party structure with two active parties that gives rise to a robust political outcome (ℓ, r). Recall that party J is *active* if the sum of the contributions it receives is larger than the exogenous cost of running C. It is said to be *efficient* if

$$\sum_{i \in \mathcal{A}} \alpha_{iJ} \in [C, C + c) \ .$$

In this case, there is no over-contribution in the sense that, if any of its members would leave party J, it would become inactive.

Lemma 3. *Let Assumptions 1 and 2 hold. In any robust political equilibrium with two active parties, each party with winning probability* $1/2$ *or lower is efficient.*

Consider a two-party equilibrium in which party L has lower electoral prospects, $p(\ell, r) \leq 1/2$. Lemma 3 implies that each member of party L is pivotal: If one member would leave L, she would cause party L's inactivity and the certain implementation of platform r. Intuitively, party L can be seen as a local public good from which all citizens with similar policy preferences benefit. As common with public goods, there is free-riding in equilibrium: although each citizen with an ideal point close to or left of platform ℓ benefits from the activity of party L, she prefers to bear as little of the provision cost as possible herself.

Lemma 3 also implies that, in equilibrium, citizens do not join a party L to merely affect the choice of its platform ℓ. If there were an equilibrium with a non-efficient party, then its members would only prefer to stay active to avoid changes

[22] In principle, there is a third necessary condition for a political equilibrium: Given a stable party structure, no member of party L can profitably join party R and vice versa. It turns out, however, that this third condition does not further restrict the set of equilibrium platforms.

in platform ℓ. Lemma 3 clarifies that this motive is never the (only) incentive for party membership in a two-party equilibrium.

The proof of Lemma 3 is by contradiction. For the basic idea, consider a potential equilibrium in which party L is not efficient and its platform is given by ℓ_0. Then, the exit of the member with the most leftist ideal point would not cause party L's inactivity, but may shift its platform to a more rightist position $\ell_1 > \ell_0$ with probability 1/2. The member is only willing to maintain her membership in L if her policy loss would be large enough to exceed the saved membership cost c. But if this were true, the entry of an independent citizen with a more rightist ideal point close to ℓ_1 would lead to the same shift in the platform of party L. Moreover, the policy gain of this entrant would unambiguously exceed the membership cost c, as shown in the proof of Lemma 3. Hence, an allocation where the weaker party is non-efficient cannot be *exit-stable* and *entry-stable* at the same time.

Symmetric outcomes with two active parties. The following paragraphs characterize the set of robust political outcomes with two active parties. A particular focus is on the platform distance $\Delta = r - \ell$ that is implied by a robust political outcome. I start with the set of symmetric outcomes such that $r = \Delta/2$ and $\ell = -r = -\Delta/2$. Hence, symmetric outcomes differ only in the platform distance $\Delta \geq 0$. In any equilibrium with a symmetric outcome, both parties L and R have a winning probability of 1/2 in the general election.

Proposition 1. *A symmetric pair of platforms* $(-\Delta/2, \Delta/2)$ *is a robust political outcome if and only if the platform distance* Δ *is between* $\underline{\Delta} = 2c$ *and*

$$\bar{\Delta} = c \, \frac{\Phi(c/2)}{\Phi(c/2) - 1/2} > 2c \, . \tag{4}$$

Proposition 1 establishes the existence of symmetric equilibria with two active parties. Moreover, it shows that the platform distance in these equilibria is bounded from below and from above, and provides closed-form solutions for both bounds. This implies that there is a limited policy divergence in each robust two-party equilibrium, in contrast to the workhorse models by Downs (1957) and Besley & Coate (1997). I continue by explaining the intuition behind both bounds.

First, the lower bound $\underline{\Delta} = 2c$ follows from the requirement that party L must be *exit-stable*. If both parties' platforms are closer to each other, then no agent is willing to bear the cost of party membership. To see this, consider a party structure (Ω_L, Ω_R) such that each party member contributes exactly c to her party and the party platforms are symmetric with $r > 0$ and $\ell = -r < 0$. By Lemma 3, both parties must be efficient. Hence, if any member of L reduces her contribution from c to 0, party L becomes inactive and the competing party's platform r is surely implemented. For a leftist party member i with $\omega_i \leq \ell$, leaving the party thus implies a direct utility gain from saving the membership fee c and a policy loss of $1/2(r - \ell) = \Delta/2$. If the platform distance Δ is smaller than the lower bound $\underline{\Delta} = 2c$, then the direct gain from saving c exceeds the policy loss. Hence, the deviation is profitable and party structure (Ω_L, Ω_R) is not *exit-stable.*

If the platform distance is larger than $\underline{\Delta}$, by contrast, then the deviation yields a policy loss that exceeds the membership fee for members of L with ideal points at or below the platform ℓ. Consequently, party structure (Ω_L, Ω_R) is *exit-stable* if all members of L have ideal points weakly below $-2c$ and, correspondingly, all members of R have ideal points weakly above $2c$.

Second, the upper bound $\bar{\Delta}$ follows from the requirement of *entry-stability.* If the platform distance is larger than $\bar{\Delta}$, then an independent agent with a moderate ideal point in (ℓ, r) benefits from joining a party. To see this, consider again a party structure with two active parties and symmetric platforms, $\ell = -r < 0$. Assume that an independent agent j with ideal point $\omega_j \in (\ell, r)$ deviates by contributing $\alpha_i^L = c$ and joining party L. This deviation is profitable if two conditions are met. First, the entrant is nominated as L's new candidate. By Lemma 2, this condition is met if her ideal point ω_j yields a larger policy effect than the initial platform ℓ. Second, for the entrant j, the policy gain from switching from the initial platform ℓ to her ideal point ω_i must be large enough to exceed the membership cost c. Intuitively, this condition is met if the distance between ω_j and ℓ is large enough.

For any symmetric outcome (ℓ, r) with a platform distance above $\bar{\Delta}$, both conditions are satisfied for some ideal points ω_j in some subset of $(\ell, 0)$. This implies that (ℓ, r) can only be an equilibrium outcome if the activist population Ω does not contain an agent with an ideal point in this subset. As soon as one agent has such an ideal point, the party structure is not *entry-robust.* Put differently, no platform pair with $\Delta > \bar{\Delta}$ is a robust equilibrium outcome. For a symmetric platforms with $r - \ell \in (0, \bar{\Delta})$, by contrast, no agent with any ideal point in (ℓ, r) can profitably join a party: For any agent who becomes presidential candidate if joining, the policy gain is dominated by the membership cost c.

Asymmetric outcomes with two active parties. In the next step, I consider asymmetric outcomes (ℓ, r) such that $\ell \neq -r$. For outcomes with $\ell + r > 0$, party L has a larger winning probability $p(\ell, r) > 1/2$ in the general election. For outcomes with $\ell + r < 0$, party L has a winning chance below $1/2$. It proves useful to rewrite the party platforms as $\ell = -\Delta/2 + \varepsilon$ and $r = \Delta/2 + \varepsilon$, where $\Delta = r - \ell$ is the platform distance and $\varepsilon = (r + \ell)/2$ is a measure of party L's electoral advantage. The following proposition fully characterizes the set of robust equilibrium outcomes in terms of Δ and ε. It makes use of the auxiliary functions $x_a(\varepsilon) := c\,[4\,\Phi(-\varepsilon)]^{-1} - \varepsilon$ and $Z(\varepsilon) := c\Phi(x_a(\varepsilon))\,[2\Phi(-\varepsilon)\ \ (\Phi(x_a(\varepsilon)) - \Phi(-\varepsilon))]^{-1}$.

Proposition 2. *There is a threshold $\bar{\varepsilon} > 0$ such that a pair $(\ell, r) = (-\Delta/2 + \varepsilon, \Delta/2 + \varepsilon)$ is a robust equilibrium outcome if and only if the electoral advantage satisfies $\varepsilon \in [-\bar{\varepsilon}, \bar{\varepsilon}]$ and the platform distance Δ is between*

$$\Delta_{low}(\varepsilon) := \frac{c}{\Phi(-|\varepsilon|)} \tag{5}$$

and

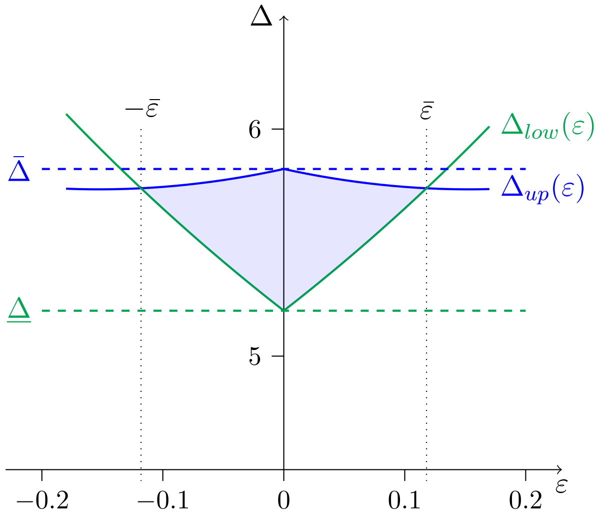
$$\Delta_{up}(\varepsilon) := \min\{Z(-\varepsilon), Z(\varepsilon)\}\ . \tag{6}$$

Proposition 2 generalizes the previous insights from symmetric outcomes. It shows that, first, there exist robust equilibrium outcomes with asymmetric platforms and, second, the distance between these platforms is bounded both from below and from above. It also provides closed-form expressions for both bounds, conditional on the electoral advantage ε. The intuition for this result is the same as in the case of symmetric outcomes. If the lower bound is violated, then party members benefit from leaving their party: The policy loss from this deviation is smaller than the membership cost c. If the upper bound is violated, then a moderate independent with some ideal point in (ℓ, r) benefits from joining a party and becoming its candidate: The policy gain from this deviation exceeds the membership cost.

Figure 1 illustrates these insights for a numerical example where the median voter distribution Φ is a standard normal distribution and c equals 2.6. Specifically, the solid green line depicts the lower bound $\Delta_{low}(\varepsilon)$ on the platform distance as a function of the electoral advantage ε. The solid blue line shows how the upper bound $\Delta_{up}(\varepsilon)$ varies with ε. The shaded area between both lines gives the combinations of Δ and ε for which the pair of platforms $\ell = -\Delta/2 - \varepsilon$ and $r = \Delta/2 - \varepsilon$ is a robust equilibrium outcome. Figure 4a in Appendix B depicts the same set of outcomes in a diagram with platform r on the horizontal axis and platform ℓ on the vertical axis.

Proposition 2 and Figure 1 provide several further insights. First, they clarify that only a limited degree of asymmetry is possible. There are no robust equilibrium outcomes in which the electoral advantage of one party is above the threshold $\bar{\varepsilon}$, i.e., in which the electoral prospects of both parties differ too much. At $\varepsilon = \bar{\varepsilon}$, the lower bound $\Delta_{low}(\varepsilon)$ and the upper bound $\Delta_{up}(\varepsilon)$ coincide, see Figure 1. Hence, any

Figure 1: Platform distance in robust equilibrium outcomes.



Notes: Figure 1 depicts the set of robust equilibrium outcomes for a numerical example in which Φ is a standard normal distribution and c equals 2.6. The solid green line shows the lower bound $\Delta_{low}(\varepsilon)$ on the platform distance as a function of the electoral advantage ε; the solid blue line shows the upper bound $\Delta_{up}(\varepsilon)$. The shaded area shows the combinations of Δ and ε in robust equilibrium outcomes.

party structure with an electoral advantage above $\bar{\varepsilon}$ is either not exit-stable or not entry-stable.[23]

Second, the weaker party is less stable in the following sense. Consider an outcome with $\varepsilon > 0$ so that party R has a winning probability below one half. If the platform distance is slightly below the lower bound $\Delta_{low}(\varepsilon)$, then the members of party R profit from leaving their party, while the members of the dominant party L prefer to maintain their membership. Hence, policy convergence is limited by the requirement of exit-stability for the weaker party.

With respect to the upper bound on Δ, the picture is less clear. Joining party R is profitable for some independent agent if $\Delta > Z(\varepsilon)$, while joining party L is profitable if $\Delta > Z(-\varepsilon)$. In general, it is unclear which of these conditions is more restrictive. In the numerical example illustrated in Figure 1, specifically, the former condition is more restrictive with $Z(\varepsilon) < Z(-\varepsilon)$ for any $\varepsilon > 0$. Hence, policy divergence is limited by the requirement of entry-stability for the weaker party as well.[24]

Third, in Figure 1, both bounds on the platform distance are more restrictive for asymmetric outcomes than for symmetric outcomes with $\varepsilon = 0$. The following corollary shows that this insight generalizes beyond the numerical examples consid-

[23] Specifically, the upper bound $\bar{\varepsilon}$ is below c if $\Phi(c) > 3/4$ (i.e., if the electoral risk is small).

[24] Further numerical simulations indicate that this is the typical pattern.

ered in Figure 1. Under Assumptions 1 and 2, $\underline{\Delta}$ and $\bar{\Delta}$ bound the platform distance $\Delta = r - \ell$ in any robust equilibrium outcome – whether symmetric or asymmetric.

Corollary 1. *Every robust equilibrium outcome with two active parties involves a platform distance* $\Delta \in [\underline{\Delta}, \bar{\Delta}]$.

4.2 Political equilibria with one uncontested party

In the following, I derive two results on the set of political equilibria in which only one party is active. In such an allocation, the incentives for party membership differ substantially from those in an allocation with two active parties. With two active parties, potential party members are interested in improving the electoral prospects of one party relative to the other one. With one active party, the implemented policy is equal to this party's platform. Thus, potential party members are mainly interested in affecting decision-making within their party. Without loss of generality, I henceforth assume that party R is active and has an odd number of members.

First, I derive a condition on the member set Ω_R of the uncontested party. Specifically, let $m_{R>}$ denote the ideal point $\omega_i \in \Omega_R$ that is adjacent to and weakly larger than the party median m_R, and $m_{R<}$ denote the ideal point that is adjacent to and weakly smaller than m_R.

Lemma 4. *Let* $\#\Omega_R$ *be odd. In any robust political equilibrium in which only party* R *is active,* Ω_R *satisfies* $\max\{m_{R>} - m_R, m_R - m_{R<}\} \leq 2c$.

Lemma 4 implies that, in any robust one-party equilibrium, the uncontested party has to be sufficiently coherent. Specifically, there must only be a limited distance between the ideal points of the median party member and the most similar party members. Otherwise, an independent agent could profit from joining party L; the policy gain from shifting platform r would be large enough to compensate the membership cost c. Hence, the party must be sufficiently coherent to be *entry-stable.*

Second, I derive the set of robust equilibrium outcomes with one active party (i.e., with $r \in \mathbb{R}$ and $\ell = \emptyset$). It follows from another aspect of *entry-stability.* Assume that only party R is active and no activist makes any contributions to party L. Then, an independent agent i could still choose to enter L and enable it to run in the general election by contributing the entire cost of running, $\alpha_i^L = C$. In a robust one-party equilibrium, this deviation must not be profitable for an agent with any ideal point ω_i.

Proposition 3. *If* $\omega_i \Phi(-\omega_i/2) \leq C$ *for all* $\omega_i \in \mathbb{R}$*, then there is a threshold* $\bar{x} \in [0, C)$ *such that* $(\emptyset, r)$ *is a robust equilibrium outcome if and only if* $r \in [-\bar{x}, \bar{x}]$. *Otherwise, there is no robust equilibrium outcome with* $r \in \mathcal{R}$ *and* $\ell = \emptyset$.

By Proposition 3, equilibria with an uncontested active party exist if and only if the electoral risk implied by Φ is small enough given the cost of running C or, vice versa, the cost C is large enough given distribution Φ. If such equilibria exist, the platform of the uncontested party has to be sufficiently close to the expected median voter position 0. Intuitively, the cost of running C works as a barrier to market entry, i.e., to the formation of a competing party. An independent citizen i can only benefit from starting party L if the expected effect on the implemented policy, $p(\omega_i, r)\,|r - \omega_i|$ is large enough to exceed C. A more moderate platform r and a smaller level of electoral risk imply that the electoral prospects of potential entrants are limited, deterring independent citizens from challenging party R.

For the logic behind the condition in Proposition 3, consider an allocation in which platform r equals the expected median voter position 0. If an independent agent i with some ideal $\omega_i < -2C$ starts a new party, she realizes a policy gain of $\omega_i\ p(\omega_i, 0) = \omega_i\ \Phi(-\omega_i/2) < \omega_i/2$. If the electoral risk is small, the winning probability $p(\omega_i, 0)$ is too low to compensate the cost of running C. If the same is true for any ideal point below $-2C$, then there is a robust one-party equilibrium in which the party's platform equals the median position, $r = 0$. By continuity, there are also robust equilibria in which platform r differs sufficiently little from 0. If the electoral risk is high, by contrast, then there is an independent agent who would even profit from running against a party with the expected median position. In this case, there is no robust political equilibrium with an uncontested party.

5 Comparative statics and limit results

In the previous sections, I have identified the set of policy platforms that can arise in political equilibria with one and two active parties, given some fixed median voter distribution Φ, membership cost c and cost of running C. This section investigates the effects of changes in the cost parameter c and in the degree of electoral risk implied by Φ. Additionally, it studies the set of equilibrium platforms for the limit cases of costless party membership and electoral certainty.

The first set of results focuses on the comparative static effects on the lower and upper bounds on the platform distance $\Delta = r - \ell$. I start by considering variations in the membership cost c.

Proposition 4. *Both bounds* $\underline{\Delta}$ *and* $\bar{\Delta}$ *on the platform distance are strictly increasing in the membership cost* c. *For* $c \to 0$, *the platform distance in robust equilibrium outcomes is between* $\lim_{c\to 0} \underline{\Delta} = 0$ *and* $\lim_{c\to 0} \bar{\Delta} = 1/\phi(0) > 0$.

Consider first the lower bound on the platform distance, $\Delta \geq 2c$. In equilibrium, party members are only willing to maintain their activity if each party's activity has a sufficiently large effect on expected policy, i.e., if the platform distance is large enough. As the cost of political activity c becomes larger, the party members require a larger policy effect and platform distance to maintain their political engagement. If the membership cost approaches zero, however, the members get willing to accept more closer platforms. In the limit, party membership is costless and even consistent with full policy convergence.

With respect to the most divergent equilibria, an increase in the membership cost c tightens the combined coordination and free-riding problem faced by potential activists. Consider an independent agent i whose ideal policy $\omega_i \in (\ell, r)$ would increase the policy effect of party L, relative to platform ℓ. As long as the difference between the policy effects of ω_i and ℓ is not large enough to exceed c, the independent agent prefers to free-ride on the current members of party L. With an increasing membership cost c, an even larger difference between ω_i and ℓ is required to make joining party L profitable. Hence, more extreme platforms by both parties can be supported in a two-party equilibrium.

If the membership cost converges to zero, in contrast, this coordination problem vanishes: an independent agent is willing to join party L whenever his ideal point ω_i allows to achieve a larger policy effect than ℓ. Put differently, the median party member is always able to recruit his preferred candidate and to choose his preferred policy platform. Consider a case where both median party members have extreme policy preferences, $m_L \to -\infty$ and $m_R \to \infty$. Then, each party selects the platform that maximizes the policy effect $\Gamma(q, -q)$, given its opponent's platform $-q$. These mutually best responses are given by $\ell = -\left[2\phi(0)\right]^{-1}$ and $r = \left[2\phi(0)\right]^{-1}$, respectively, as I show in the appendix. The difference between the implied platforms is given by $1/\phi(0)$.

Next, I study the effect of variations in the electoral risk as implied by the distribution Φ. For this purpose, I restrict my attention to distributions within some family of distributions that satisfy the following assumption.

Assumption 3. *The probability distribution Φ belongs to a family of distribution functions $(\Phi_\sigma)_{\sigma \in \mathbb{R}_+}$ such that, for any fixed $x < 0$,*

(i) $\frac{d\Phi_\sigma(x)}{d\sigma} > 0$, *and*

(ii) $\lim_{\sigma \to 0} \Phi_\sigma(x) = 0$.

If Assumption 3 is satisfied, all distributions within the family $(\Phi_\sigma)_{\sigma \in \mathbb{R}_+}$ can be ordered with respect to the implied electoral risk. In particular, Φ_σ is a meanpreserving spread of $\Phi_{\sigma'}$ if and only if $\sigma > \sigma'$. For example, this assumption is satisfied by the families of normal distributions with mean 0, of logistic distributions and of Laplace distributions. Treating σ as a parameter that measures the degree of electoral risk, I can derive the following result.

Proposition 5. *Let Assumption 3 hold. The upper bound $\bar{\Delta}$ on the platform distance is strictly increasing in the electoral risk σ, while the lower bound $\underline{\Delta}$ is not affected. In the limit case $\sigma \to 0$, the pair of platforms $(-c, c)$ is the unique robust equilibrium outcome with two active parties.*

By Proposition 5, the upper bound of the platform distance $\Delta = r - \ell$ is increasing in the degree of electoral risk, while the lower bound remains constant. Intuitively, the more uncertain the outcome of an election is, the more attractive a party median finds his own ideal point m_L compared to the ideal point $\omega_i \in (\ell, r)$ of any more moderate citizen. Put differently, an extreme party median becomes less interested in recruiting a moderate citizen and selecting him as the party's candidate. Hence, an increase in electoral risk implies that more divergent platforms can be supported in two-party equilibria.

In the limit case of full electoral certainty, $\sigma \to 0$, the location of the population median at policy 0 is perfectly known. Thus, the members of each party prefer a platform that is slightly closer to the median voter than the one of the competing party. In this case, the upper bound $\bar{\Delta}$ coincides with the lower bound $\underline{\Delta} = 2c$, so that all symmetric equilibria with a platform distance above $2c$ are eliminated. Moreover, there exist no asymmetric equilibria because the weaker party has a zero chance of winning the election. As a result, only the platforms $\ell = -c$ and $r = c$ can be supported by stable parties in a two-party equilibrium. Given these platforms, the membership cost c is (i) just large enough to deter moderate citizen with ideal point $\omega_i \in (-c, c)$ from entering one of the parties and (ii) just small enough to deter party members from leaving their party and causing the inactivity of their party.

In this case, the difference between the citizen candidate models by Osborne & Slivinski (1996) and Besley & Coate (1997) and my model with endogenous party formation becomes most obvious.[25] In both types of models, the costs of political activity give rise to a lower bound on the platform distance in two-candidate equilibria. Without party formation, there exist also political equilibria with more divergent parties. Hence, there is a large multiplicity of two-party equilibria with different platform distances. This well-known weakness of the citizen candidate model contrasts sharply with the unique pair of equilibrium platforms established in Proposition 5.

[25] Osborne & Slivinski (1996) and Besley & Coate (1997) focus on the case of electoral certainty.

A result of special interest can be derived for the twofold limit case, where party membership is costless and there is full electoral certainty. This is the only case for which every two-party equilibrium involves full convergence of both party platforms at the median voter position.

Corollary 2. *Let Assumption 3 hold. If and only if both* $\sigma \to 0$ *and* $c \to 0$*, both party platforms* ℓ *and* r *equal the median voters' ideal policy in every two-party equilibrium.*

This result confirms the famous Downs (1957) result – full policy convergence – for a special case only. Arguably, both conditions (zero membership costs, no electoral risk) appear quite restrictive from an applied perspective. The basic message of Corollary 2 and the previous results is thus the following: In any political competition between endogenously formed parties, there is a centripetal force that pushes the competing parties' platforms to converge towards the median voter. Full convergence is not a robust prediction, however, but only a natural limit case that results if all kinds of frictions (costs of activity, limited information) vanish. By contrast, this convergence result does not occur in the citizen candidate model: In this model, there continues to be a multiplicity of equilibria with two candidates even in the corresponding limit case with zero cost of running and full electoral certainty. This includes equilibria with full convergence but also equilibria with strongly diverging platforms (see Besley & Coate 1997, Osborne & Slivinski 1996).

The last result compares the implemented policies in two-party and one-party equilibria in the limit case of full electoral certainty, but assuming strictly positive levels of the membership cost c and the cost of running C, but focusing on the case of full electoral certainty. It seems natural to ask whether two competing parties cater more or less to the preferences of the (decisive) median voter than a single, uncontested party.[26] In the citizen candidate model by Besley & Coate (1997), the implemented policy in any one-candidate equilibrium is ex post closer to median voter's ideal point than in the unique two-candidate equilibrium. With endogenous party formation, I come to a different conclusion.

Corollary 3. *Let Assumption 3 hold. In the limit case* $\sigma \to 0$*, there are one-party equilibria in which the median voter is ex post strictly worse off than in every two-party equilibrium.*

In the limit case with full electoral certainty, $\sigma \to 0$, the unique robust outcome with two active parties involves the platforms $\ell = -c/2$ and $r = c/2$ by Proposition

[26]For a symmetric distribution of ideal points in the population, this is equivalent to asking whether social welfare (i.e., the integral over all citizens' policy payoffs) is larger or smaller in two-party equilibria than in one-party equilibria.

5. With respect to one-party equilibria, outcome $(r, \emptyset)$ is a robust equilibrium outcome if and only if platform r is located between $-C/2$ and $C/2$. As $C \geq 3c$, the implemented policy r in some one-party equilibria deviates further from the median voter position at zero than the potential policies in a two-party equilibrium. Corollary 3 thus conflicts with the results of the citizen candidate model, but confirms the conventional view that political competition is beneficial, ensuring that politicians respect the voters' interest.[27]

For the intuition behind this insight, consider a party whose members have very coherent, but extreme policy preferences (in the sense of differing strongly from the median voter's preferences). If this is the only active party, it is relatively complicated for moderate independent citizens to stand up against this party and affect political decisions. If there are two competing parties, then moderate independents can more easily become active by joining or supporting the less extreme party. Loosely speaking, political competition reduces the barriers to political activity, working as a safeguard against political extremism. In the model, this idea is represented by the difference between the cost of party membership c and the cost of setting up a competitive party $C > 3c$.

6 Extensions

All previous results have been derived under the assumptions that (i) there is only limited electoral risk by Assumption 2 and (ii) the agents have Euclidean (i.e., linear and symmetric) policy preferences. The main insights from my analysis generalize beyond these simplifying assumptions, however. In the following, I sketch some natural extensions and their effects on the results. More detailed discussions and formal results on symmetric equilibria are provided in Online Appendix C.

Higher electoral risk. First, assume that the median voter distribution Φ implies a higher level of electoral risk, so that Assumption 2 is violated. I have to distinguish two cases. For an intermediate level of electoral risk, Propositions 1 to 5 continue to hold. Only Corollary 1 may become invalid: In two-party equilibria, the upper bound $\Delta_{up}(\varepsilon)$ on the platform distance in asymmetric equilibria might be larger for some $\varepsilon \neq 0$ than the upper bound $\bar{\Delta}$ in symmetric equilibria. Numerical simulations show, though, that $\bar{\Delta}$ in many cases remains an upper bound on the platform

[27]This result extends to cases with a low level of electoral risk as measured by σ. For higher levels of σ, the set of supportable platforms in one-party equilibria is closer to the expected median voter position. For even higher levels of electoral risk, there exist no equilibria with one uncontested party.

distance in all robust equilibrium outcomes.

For an even higher level of electoral risk, by contrast, there is an additional class of two-party equilibria with a platform distance below the lower bound $\underline{\Delta} = 2c$. In particular, even full convergence to the expected median voter, $\ell = r = 0$, becomes a robust equilibrium outcome. In the equilibria of this class, both parties are inefficient, party L has members with ideal points right of the platform of party R, and party R has members with ideal points left of the platform of L. Hence, both parties are ideologically fragmented. Crucially, the members of party L are not active to avoid the implementation of platform r, but to avoid that their own party runs with an even worse platform than r. Proposition C.1 in Online Appendix C provides a necessary and sufficient condition for the existence of these convergent equilibria.[28]

Concave preferences. Second, assume that the agents have concave policy preferences so that agent i considers each additional deviation of policy x from his ideal point ω_i more harmful. Intuitively, this variation has no effect on the mechanisms underlying the bounds on the platform distance in political equilibria. First, if the platform distance is too small, no agent is willing to support a party. Second, if the platform distance is too large, a moderate agent can profitably join a party and become its presidential candidate because she offers superior electoral prospects. In particular, with concave preferences, the party members become even more interested in increasing the electoral prospects of their party.

In Online Appendix C, I confirm this intuition for a case where the agents' policy preferences can be represented by a CARA payoff function,

$$v(x - \omega_i) = \frac{1}{a}\left(1 - e^{a|x-\omega_i|}\right) \ .$$

I focus on cases with a strictly positive curvature parameter $a > 0$, which implies concave policy preferences. I find that the qualitative insights from the benchmark model remain valid. In particular, I can provide closed-form expressions for the upper and lower bounds on the platform distance given any value of $a > 0$.

Asymmetric policy preferences. Third, assume that an agent i does not only care for how much policy x differs from her ideal point ω_i, but also for whether policy x is located to the left or to the right of ω_i. I consider such asymmetric preferences

[28] Under Assumption 3, there exist two thresholds $\sigma_1 > 0$ and $\sigma_2 > \sigma_1$ such that (i) Assumption 2 holds if and only if $\sigma \leq \sigma_1$ and (ii) political equilibria with platform distance below $\underline{\Delta}$ exist if and only if $\sigma \geq \sigma_2$. If Φ is a normal distribution with mean zero and standard deviation σ, for example, threshold σ_1 equals $c/\sqrt{2\pi} \approx 0.4c$ and threshold σ_2 is close to $5.88c$.

by using a payoff function of the form

$$v_i(x-\omega_i) = \begin{cases} -b_{Li}(\omega_i - x) & \text{for} \quad x \le \omega_i \ , \\ -b_{Ri}(x-\omega_i) & \text{for} \quad x > \omega_i \ , \end{cases} \tag{7}$$

where the taste parameters $b_{Li} > 0$ and $b_{Ri} > 0$ capture the agent's sensitivity with respect to leftward and rightward deviations of policy x from ω_i, respectively. Again, this variation does not affect the intuition underlying the lower and upper bounds on the platform distance in two-party equilibria.

In Online Appendix C, specifically, I focus on a case where each leftist agent with ideal point $\omega_i < 0$ considers deviations to the right of her ideal point more harmful than deviations to the left, while the opposite is true for each rightist agent with $\omega_i > 0$. Again, I can generalize my results from the basic model and provide closed-form expressions for the lower and upper bounds on the platform distance.

No exogenous costs of running. In the basic model, I have assumed that party J can run in the general election if and only if its contributions suffice to finance the exogenous cost of running, $\sum_{i\in\mathcal{A}} \alpha_i^J \ge C$, while the contributions do not affect the party's electoral prospects otherwise. As a result, each party member is pivotal in keeping her party active in political equilibria. I can relax this assumption by assuming that there is no exogenous cost of running and that the winning probability of party L is increasing in the contributions it receives relative to party R. As long as the winning probabilities also depend on the party platforms ℓ and r, the basic intuition extends to this model version: On the one hand, if there is too little policy divergence, then no agents is willing to support a party. On the other hand, if there is too much policy divergence, then a moderate agent can profitable enter one party and become its presidential candidate because he can offer much better electoral prospects. Intuitively, joining a party even becomes more profitable in this model version because the entrant's contribution further increases the electoral prospects of her party.

In Online Appendix C, in particular, I show that my qualitative results extend to a model version in which the winning probability of party L equals

$$\tilde{p}(\ell, r, C_L, C_R) = \tilde{\Phi}\left(\frac{\ell + r}{2} + \beta \, \frac{C_L - C_R}{C_L + C_R}\right) \ , \tag{8}$$

where $\tilde{\Phi}$ is a probability distribution satisfying Assumptions 1 and 2, the fraction $(C_L - C_R)/(C_L + C_R)$ measures the relative contributions to party L, and β is a parameter that governs the sensitivity of the winning probability with respect to contributions relative to policy platforms.[29]

7 Conclusion

This paper has investigated electoral competition between endogenously formed parties in a new model that arguably brings theory closer to real-world politics. The analysis has focused on the policy platforms that can be offered by stable political parties in robust political equilibria. In particular, I have derived the implications of *entry-stability* and *exit-stability* for equilibria with two competing parties and with one uncontested party. I have provided two main results. First, the platform distance in two-party equilibria is always strictly positive, but limited. This result is in contrast to the classical Downs (1957) result of full policy convergence, which fails to comply with empirical observations. It is also in contrast to the results of the citizen candidate models by Osborne & Slivinski (1996) and Besley & Coate (1997), in which there is a multiplicity of two-party equilibria with indeterminate platform distance. The difference can be seen most obviously in the benchmark case of full electoral certainty, where both parties' platforms are uniquely determined in the party formation model only. Second, the implemented policy can differ more strongly from the median voter's position in equilibria with one uncontested party than in equilibria with two competing parties. Hence, multi-party competition can be seen as a safeguard against political extremism. Again, this result is in contrast to the findings of the citizen candidate model.

For the sake of clarity, the analysis of this paper has focused on a simple environment, including an abstract one-dimensional policy space. The model is tractable enough to study more complex policy decisions, however, especially in the commonly studied benchmark case of full electoral certainty. For example, it could be used to investigate political competition over linear income taxation as in Dixit & Londregan (1998) or non-linear income taxation as in Brett & Weymark (2017). A richer model could also allow for, e.g., a larger number of potential parties, alternative rules for intra-party decision-making, or a different modeling of electoral uncertainty.

[29]One interpretation of (8) is that, in the general election, a share s of the population vote based on the policy platforms ℓ and r, while the remaining share $1 - s$ of the electorate vote based on idiosyncratic party preferences and the parties' relative campaign expenditures financed by their contributions.

References

Bernhardt, D., Duggan, J. & Squintani, F. (2009*a*), 'The case for responsible parties', *American Political Science Review* **103**(4), 570–587.

Bernhardt, D., Duggan, J. & Squintani, F. (2009*b*), 'Private polling in elections and voter welfare', *Journal of Economic Theory* **144**, 2021–2056.

Besley, T. & Coate, S. (1997), 'An economic model of representative democracy', *Quarterly Journal of Economics* **112**(1), 85–114.

Brett, C. & Weymark, J. (2017), 'Voting over selfishly optimal nonlinear income tax schedules', *Games and Economic Behavior* **101**, 172–188.

Cadigan, J. & Janeba, E. (2002), 'A citizen-candidate model with sequential elections', *Journal of Theoretical Politics* **14**(4), 387–407.

Calvert, R. (1985), 'Robustness of the multidimensional voting model, candidate motivations, uncertainty, and convergence', *American Journal of Political Science* **39**, 69–95.

Campante, F. R. (2011), 'Redistribution in a Model of Voting and Campaign Contributions', *Journal of Public Economics* **95**(7), 646–656.

De Sinopoli, F. & Turrini, A. (2002), 'A remark on voters' rationality in a model of representative democracy', *Journal of Public Economic Theory* **4**(2), 163–170.

Dhillon, A. (2005), Political parties and coalition formation, *in* G. Demange & M. Wooders, eds, 'Group Formation in Economics: Networks, Clubs, and Coalitions', Cambridge University Press, Cambridge, pp. 289–311.

Dhillon, A. & Lockwood, B. (2002), 'Multiple equilibria in the citizen-candidate model of representative democracy', *Journal of Public Economic Theory* **4**(2), 171–184.

Dixit, A. & Londregan, J. (1998), 'Ideology, tactics, and efficiency in redistributive politics', *Quarterly Journal of Economics* **113**(2), 497–529.

Dotti, V. (2021), 'Reaching across the aisle to block reforms', *Economic Theory* **72**, 533–578.

Downs, A. (1957), *An Economic Theory of Democracy*, Harper and Row, New York.

Eguia, J. (2007), 'Citizen candidates under uncertainty', *Social Choice and Welfare* **29**(2), 317–331.

Gans, J. & Smart, M. (1996), 'Majority voting with single-crossing preferences', *Journal of Public Economics* **59**(2), 219–237.

Gerber, A. & Ortuño-Ortin, I. (1998), 'Political compromise and endogenous formation of coalitions', *Social Choice and Welfare* **15**(3), 445–454.

Gomberg, A., Marhuenda, F. & Ortuño-Ortin, I. (2004), 'A model of endogenous political party platforms', *Economic Theory* **24**(2), 373–394.

Herrera, H., Levine, D. K. & Martinelli, C. (2008), 'Policy platforms, campaign spending and voter participation', *Journal of Public Economics* **92**(3-4), 501 – 513.

Jackson, M. O., Mathevet, L. & Mattes, K. (2007), 'Nomination processes and policy outcomes', *Quarterly Journal of Political Science* **2**(1), 67–92.

Levy, G. (2004), 'A model of political parties', *Journal of Economic Theory* **115**(2), 250–277.

Lindbeck, A. & Weibull, J. W. (1987), 'Balanced-budget redistribution as the outcome of political competition', *Public Choice* **52**, 273–297.

Morelli, M. (2004), 'Party formation and policy outcomes under different electoral systems', *Review of Economic Studies* **71**(3), 829–853.

Ortuño-Ortin, I. & Schultz, C. (2005), 'Public funding of political parties', *Journal of Public Economic Theory* **7**(5), 781–791.

Osborne, M. J. & Slivinski, A. (1996), 'A model of political competition with citizen-candidates', *Quarterly Journal of Economics* **111**(1), 65–96.

Osborne, M. J. & Tourky, R. (2008), 'Party formation in single-issue politics', *Journal of the European Economic Association* **6**(5), 974–1005.

Pech, G. (2012), 'Intra-party decision making, party formation, and moderation in multiparty systems', *Mathematical Social Sciences* **63**, 14–22.

Poutvaara, P. (2003), 'Party platforms with endogenous party membership', *Public Choice* **117**(1-2), 79–98.

Poutvaara, P. & Takalo, T. (2007), 'Candidate quality', *International Tax and Public Finance* **14**(1), 7–27.

Rivière, A. (1999), Citizen candidacy, party formation and Duverger's law, Royal Holloway Discussion Paper No. dpe00/1.
URL: *http://idei.fr/doc/conf/eco/riviere.pdf*

Roemer, J. E. (2003), Indeterminacy of citizen candidate equilibrium, Cowles foundation discussion paper no. 1410, yale university.
URL: *https://cowles.yale.edu/sites/default/files/files/pub/d14/d1410.pdf*

Roemer, J. E. (2006), 'Party competition under private and public financing: A comparison of institutions', *Advances in Theoretical Economics* **6**(1), 1229.

Snyder, J. M. & Ting, M. M. (2002), 'An informational rationale for political parties', *American Journal of Political Science* **46**(1), 90–110.

Wittman, D. (1983), 'Candidate motivation: A synthesis of alternative theories', *American Political Science Review* **77**, 142–157.

Appendix

A Proofs

Proof of Lemma 1

Proof. Given $\hat{r} = \emptyset$, the members of party L expect their candidate to become president and implement his ideal point w_j with certainty. Thus, nominating a party member is equivalent to selecting a policy from the set Ω_L. The Euclidean policy preferences $v(x-\omega_i) = -|x - \omega_i|$ are single-peaked. By standard arguments, single-peakedness implies that voting is monotonic in any election between two elements of Ω_L. With an odd number of party members, the party median m_L is a Condorcet winner in Ω_L. Hence, the median party member prevails in party L's pairwise elections. □

With an even number of party members, there are two midmost members with ideal points m_{L-} and $m_{L+} \geq m_{L-}$. As voting is monotonic, both members do not loose a pairwise election against any other element in Ω_L. Hence, the two midmost members are nominated as candidates with equal probability. In the special case were both have the same ideal point, $m_{L+} = m_{L-}$, party platform ℓ is pinned down uniquely.

Proof of Lemma 2

Lemma 2 identifies the optimal choice of party platform ℓ for any member set Ω_L and belief $\hat{r}$. To prove it, I first show that the agents' platform preferences satisfy a single-crossing property and depend on the policy effect function $\Gamma(\ell, r)$. Then, I prove that $\Gamma(\ell, r)$ is strictly quasi-concave under Assumption 1. Lemma 2 follows from the combination of these two results.

Lemma A.1. *Given any platform belief $\hat{r}$, the platform preferences of party members over the set of potential platforms satisfy a version of the single-crossing property by Gans & Smart (1996).*

Proof. Given platform ℓ and belief $\hat{r}$, the expected policy payoff of agent i with ideal point ω_i is $E\left[v(x - \omega_i) \mid \ell, \hat{r}\right] = p(\ell, \hat{r})\ v(\ell - \omega_i) + [1 - p(\ell, \hat{r})]\ v(\hat{r} - \omega_i)$. Consider two potential platforms ℓ_1 and ℓ_2 such that $\ell_1 < \ell_2 < \hat{r}$. Agent i prefers ℓ_1 to ℓ_2 if and only if

$$F(\ell_1, \ell_2, \hat{r}, \omega_i) := p(\ell_1, \hat{r})\left[v(\ell_1 - \omega_i) - v(\hat{r} - \omega_i)\right] - p(\ell_2, \hat{r})\left[v(\ell_2 - \omega_i) - v(\hat{r} - \omega_i)\right] \quad \text{(A.1)}$$

is positive. With $v(x - \omega_i) = -|x - \omega_i|$, $F(\ell_1, \ell_2, \hat{r}, \omega_i)$ is equal to $\Gamma(\ell_1, \hat{r}) - \Gamma(\ell_2, \hat{r})$ for all $\omega_i \leq \ell_1$, and equal to $\Gamma(\ell_2, \hat{r}) - \Gamma(\ell_1, \hat{r})$ for all $\omega_i \geq \hat{r}$. The derivative with respect to

ω_i is given by

$$\frac{dF(\ell_1,\ell_2,\hat{r},\omega_i)}{d\omega_i} = \begin{cases} 0 & \text{for} \quad \omega_i \leq \ell_1 \ , \\ -2\ p(\ell_1,\hat{r}) < 0 & \text{for} \quad \omega_i \in (\ell_1,\ell_2] \ , \\ 2\ \left[p(\ell_2,\hat{r}) - p(\ell_1,\hat{r})\right] > 0 & \text{for} \quad \omega_i \in (\ell_2,\hat{r}] \ , \\ 0 & \text{for} \quad \omega_i \geq \hat{r} \ . \end{cases}$$

Hence, F has a unique root in ω_i if $\Gamma(\ell_1,\hat{r}) \neq \Gamma(\ell_2,\hat{r})$. For $\Gamma(\ell_1,\hat{r}) > \Gamma(\ell_2,\hat{r})$, all agent with ω_i below this root prefer ℓ_1 and we get the following single-crossing property

$$\begin{aligned} F(\ell_1,\ell_2,\hat{r},\omega_i) \leq 0 \quad &\Rightarrow \quad F(\ell_1,\ell_2,\hat{r},w_j) < 0 \ \forall \ w_j > \omega_i \ , \ \text{and} \\ F(\ell_1,\ell_2,\hat{r},\omega_i) \geq 0 \quad &\Rightarrow \quad F(\ell_1,\ell_2,\hat{r},w_k) > 0 \ \forall \ w_k < \omega_i \ . \end{aligned}$$

For $\Gamma(\ell_1,\hat{r}) < \Gamma(\ell_2,\hat{r})$, all agents with ω_i below the root prefer platform ℓ_2 and we get

$$\begin{aligned} F(\ell_1,\ell_2,\hat{r},\omega_i) \leq 0 \quad &\Rightarrow \quad F(\ell_1,\ell_2,\hat{r},w_j) < 0 \ \forall \ w_j < \omega_i \ , \ \text{and,} \\ F(\ell_1,\ell_2,\hat{r},\omega_i) \geq 0 \quad &\Rightarrow \quad F(\ell_1,\ell_2,\hat{r},w_k) > 0 \ \forall \ w_k > \omega_i \ . \end{aligned}$$

For $\Gamma(\ell_1,\hat{r}) = \Gamma(\ell_2,\hat{r})$, all agents with $\omega_i \in (\ell_1,\hat{r})$ strictly prefer the moderate platform ℓ_2, while all other agents are indifferent between both platforms. Trivially, the preferences satisfy

$$F(\ell_1,\ell_2,\hat{r},\omega_i) \leq 0 \quad \Rightarrow \quad F(\ell_1,\ell_2,\hat{r},w_j) \leq 0 \ \forall \ w_j > \omega_i \ .$$

Finally, if $\ell_1 \leq \hat{r} \leq \ell_2$ with $\ell_1 < \ell_2$, then $F(\ell_1,\ell_2,\hat{r},\omega_i)$ has exactly one root in (ℓ_1,ℓ_2) and the implied platform preferences again satisfy a single-crossing property. □

Lemma A.2. *For any $r \in \mathbb{R}$, function $\Gamma(\ell,r) = p(\ell,r)(r-\ell)$ has a unique maximizer $\gamma^*(r) \in (-\infty,r)$ and is strictly quasi-concave in its first argument for $\ell \in (-\infty,r)$. For $r \lesseqgtr [2\ \phi(0)]^{-1}$, the maximizer satisfies $\gamma^*(r) \lesseqgtr -r$.*

Proof. Fix some $r \in \mathbb{R}$. Both Φ and Γ are continuously differentiable by Assumption 1. For any $\ell < r$, the first and second derivatives of $\Gamma(\ell,r)$ with respect to ℓ are given by

$$\begin{aligned} \Gamma_\ell(\ell,r) &= \frac{\phi(z)}{2}(r-\ell) - \Phi(z) \ \text{and} \\ \Gamma_{\ell\ell}(\ell,r) &= \frac{\phi'(z)}{4}(r-\ell) - \phi(z) \ , \end{aligned}$$

where I write $z = (r+\ell)/2$ for a more concise notation. At any extreme value of Γ in ℓ, the second derivative is given by

$$\Gamma_{\ell\ell}(\ell,r) = \frac{\phi'(z)}{2}\frac{\Phi(z)}{\phi(z)} - \phi(z) = \Phi(z)\left[\frac{1}{2}\ \frac{\phi'(z)}{\phi(z)} - \frac{\phi(z)}{\Phi(z)}\right] \ .$$

By the log-concavity imposed in Assumption 1, the term in brackets is strictly negative for any z. Consequently, Γ is strictly quasi-concave in its first argument: It has at most one local maximum and no local minimum in (∞, r).

To show the existence of a local maximum, I consider the limit of $\Gamma(\ell, r)$ for ℓ converging to $-\infty$. For any $r \in \mathbb{R}$, this limit is given by

$$\lim_{\ell \to -\infty} \Gamma(\ell, r) = \lim_{\ell \to -\infty} \frac{r - \ell}{\frac{1}{\Phi(z)}} = \lim_{\ell \to -\infty} \frac{1}{\frac{\phi(z)}{2\Phi(z)^2}} = \lim_{\ell \to -\infty} 2\frac{\Phi(z)^2}{\phi(z)} = 0 \, .$$

The last equality sign follows because, first, $\lim_{l \to -\infty} \Phi(z) = 0$ and, second, $\Phi(z)/\phi(z) < \Phi(0)/\phi(0) < \infty$ for any $z < 0$ by the log-concavity of Φ imposed in Assumption 1. Hence, $\Gamma(\ell, r)$ converges to zero for $\ell \to -\infty$. Moreover, $\Gamma(\ell, r)$ is strictly positive for all $\ell \in (-\infty, r)$, and weakly negative for all $\ell \geq r$. Consequently, Γ has a unique maximizer $\gamma^*(r) \in (-\infty, r]$ for any $r \in \mathbb{R}$.

Finally, for $\ell = -r < 0$, the first derivative is given by $\Gamma_\ell(-r, r) = r \ \phi(0) - \frac{1}{2}$. If $r > [2 \ \phi(0)]^{-1}$, then this derivative is positive, implying that $\gamma^*(r) > -r$. If instead $r < [2 \ \phi(0)]^{-1}$, then $\Gamma_\ell(-r, r) < 0$, which implies that $\gamma^*(r) < -r$. □

Lemma A.3. *Fix a belief $\hat{r} > \omega_i$, and any set $\mathcal{B} \cup \mathbb{R}$ of potential platforms of party L. For an agent with ideal point $\omega_i \in \mathcal{B}$, the expected policy payoff is maximized by the maximum of ω_i and $\gamma(\hat{r}, \mathcal{B}) = \arg\max_{\ell \in \mathcal{B}} \Gamma(\ell, \hat{r})$.*

Proof. First, agent i prefers her ideal point ω_i to any platform $x \geq \hat{r}$ because $E[v(x - \omega_i) \mid \ell = \omega_i, \hat{r}] > v(\hat{r} - \omega_i) \geq E[v(x - \omega_i) \mid \ell > \hat{r}, \hat{r}]$.

Second, agent i prefers ω_i to any more leftist platform $\ell < \omega_i$ because $F(\ell, \omega_i, \hat{r}, \omega_i) = p(\ell, \hat{r}) v(\ell - \omega_i) + [p(\omega_i, \hat{r}) - p(\ell, \hat{r})] v(\hat{r} - \omega_i) < 0$. Specifically, ω_i yields a larger winning probability, $p(\omega_i, \hat{r}) > p(\ell, \hat{r})$, and a larger payoff in case of winning, $v(\omega_i - \omega_i) > v(\ell - \omega_i)$.

Third, for any platform ℓ in $[\omega_i, \hat{r})$, agent i's expected policy payoff is given by $E[v(x - \omega_i) \mid \ell, \hat{r}] = \Gamma(\ell, \hat{r}) + v(\hat{r} - \omega_i)$. Hence, it is maximized by platform $\gamma(\hat{r}, \mathcal{B})$ if $\omega_i \leq \gamma(\hat{r}, \mathcal{B})$. If instead $\omega_i > \gamma(\hat{r}, \mathcal{B})$, then the quasi-concavity of Γ ensures that agent i prefers ω_i to any platform in $(\omega_i, \hat{r})$. □

Lemma 2 follows from combining Lemmas A.1 and A.3 if party L has an odd number of members. By Lemma A.3, the median member with ideal point m_L prefers platform $\ell = \max\{m_L, \gamma(\hat{r}, \Omega_L)\}$ to any other element in Ω_L. By Lemma A.1, a majority of party members prefer this platform to any other available platform: Either the party median and all more leftist members, or the party median and all more rightist members.

If party L has an even number of members and the ideal points of the two midmost members are given by $m_{L-} < \hat{r}$ and $m_{L+} \in (m_{L-}, \hat{r})$, we have to distinguish two cases. First, if $m_{L+} \leq \gamma(\hat{r}, \Omega_L)$, both midmost members and all more leftist members strictly prefer $\gamma(\hat{r}, \Omega_L)$ to any other element in Ω_L. Second, if $m_{L+} > \gamma(\hat{r}, \Omega_L)$, then both m_{L-} and m_{L+} receive at least half of the votes in any pairwise election by Lemma A.1. As all other elements of Ω_L lose at least one pairwise election, m_{L-} and m_{L+} are in the top cycle and become party L's platform with equal probability.

Proof of Lemma 3

Proof. To prove Lemma 3, I distinguish three cases. First, I show that there is no two-party equilibrium with an inefficient party that has only one member. Second, I show that there is no robust two-party equilibrium with an inefficient party in which at least one party member has an ideal point between m_L and m_R. Second, I show that, under Assumption 2, there is no such equilibrium in which all members of the weaker party have ideal points outside of (m_L, m_R). The proofs for all three cases are by contradiction, and concentrates on cases in which both parties have an odd number of members.

No inefficient party with a single member. Consider a potential two-party equilibrium with outcome (ℓ, r) and membership structure (Ω_L, Ω_R) such that party L is not efficient, $\sum_{i \in \mathcal{A}} \alpha_i^L \geq C + c$. Assume that party L has a unique member i. This implies that either i contributes $\alpha_i^L \geq C + c$, or some independent agent j contributes $\alpha_j^L \in (0, c)$. In the former case, member i can profitable deviate by reducing α_i^L without affecting platform $\ell = \omega_i$. In the latter case, the independent j can profitable deviate by reducing α_j^L without affecting platforms ℓ or r.

No inefficient party with members in (ℓ, r). Consider an equilibrium in which party L is inefficient and has at least three members. In this case, each member must contribute exactly c; otherwise, she could reduce her contribution without affecting platform ℓ. Denote by $\ell_+ = \min\{z \in \Omega_L : \ z > m_L\}$ the ideal point of the party member that is closest to the right of m_L. Assume that $\ell_+ \in [\ell, r)$. I show that this is inconsistent with party L being inefficient in equilibrium. For this purpose, recall that ℓ equals the maximum of m_L and $\gamma(r, \Omega_L)$ by Lemma 2.

First, assume that L's platform is given by $\ell = \gamma(r, \Omega_L) > m_L$. In this case, the party structure cannot be exit-stable. In particular, if the leftist member i with ideal point $\omega_i \leq m_L$ leaves the party, the two then-midmost members continue to prefer policy $\gamma(r, \Omega_L)$ to any other element of Ω_L. Hence, member i can profitably leave party L to save c without affecting platform ℓ and the expected policy payoff.

Second, assume that L's platform is $\ell = m_L \geq \gamma(r, \Omega_L)$. In this case, the party structure cannot be exit-stable and entry-stable at the same time. On the one hand, if the most leftist member i of L leaves L, then there are two midmost members with ideal points ℓ and $\ell_+ \in [\ell, r)$. The new platform of L is drawn from ℓ and ℓ_+ with equal probability, see proof of Lemma 2. Thus, the deviation implies that agent i saves at least the membership cost c but bears a policy loss of $-F(\ell, \ell_+, r, \ell)/2$. Exit-stability requires that this deviation is not profitable,

$$F(\ell, \ell_+, r, \ell) \geq 2c \ , \tag{A.2}$$

i.e., that the distance between ℓ and the adjacent member to the right ℓ_+ is sufficiently large. On the other hand, if an independent agent k with ideal point $\omega_k \in (\ell, \ell_+)$ joins L, she becomes one of the two midmost member (together with the initial party median). Then, the new platform is drawn with equal probability from ℓ and ω_k. Entry-stability requires that the policy gain from joining L, $F(\omega_k, \ell, r, \omega_k)/2$ is below c for any such independent agent in the activist population Ω. For $\omega_k = \ell_+$, however, the policy gain of the entrant k is larger than the policy loss of a leaving member j with $\omega_j < \ell$,

$$F(\ell_+, \ell, r, \ell_+) = F(\ell, \ell_+, r, \ell) + 2\ [p(\ell_+, r) - p(\ell, r)]\ (r - \ell_+) > F(\ell, \ell_+, r, \ell) \ . \tag{A.3}$$

By continuity of F, the same is true for the policy gain of an entrant with ideal point slightly below ℓ_+. Hence, if party structure (Ω_L, Ω_R) is exit-stable with an inefficient party, it cannot be entry-stable once the activist population Ω contains an independent agent with ideal point in some interval between ℓ and ℓ_+, who benefits from joining L and becoming its presidential candidate with positive probability. Put differently, the party structure is not entry-stable given a perturbation $\Omega \cup \{\omega_k\}$ with ω_k close to ℓ_+. I conclude that there is no robust equilibrium in which one party is inefficient and has at least one member with ideal point in (ℓ, r).

No inefficient party without members in (ℓ, r). It remains to show that there is no robust equilibrium in which an inefficient party has a winning probability of 1/2 or lower, and all its members have ideal points to the left of ℓ or to the right of r. If Assumption 2 holds, such a party cannot be exit-stable and entry-stable at the same time.

Assume that both platforms satisfy $\ell + r \leq 0$ so that party L has a winning probability $p(\ell, r) \leq 1/2$. Denote by $\ell_- = \max\{\omega_i \in \Omega_L : \omega_i < m_L\}$ the member whose ideal point is closest to the left of m_L. As shown above, L cannot be inefficient in equilibrium if $\ell_+ < r$. Consider the remaining case where $\ell_+ \geq r$. On the one hand, if this member leaves L, the new platform is given by ℓ_- and ℓ with equal chance. This deviation is profitable for j if the membership cost c exceeds the policy loss $-F(\ell, \ell_-, r, \omega_j)/2 = \Gamma(\ell_-, r) - \Gamma(\ell, r)$. On the other hand, if some independent agent with ideal point $\omega_k \in [\ell_-, \ell)$ joins L, the new platform is drawn from ω_k and ℓ. For this agent, joining L is profitable if $F(\omega_k, \ell, r, \omega_k)/2 = \Gamma(\omega_k, r) - \Gamma(\ell, r)$ exceeds c. Hence, party L can only be exit-stable and entry-stable at the same time if, first, $\Gamma(\ell_-, r) - \Gamma(\ell, r) = 2c$ and, second, $\Gamma(\omega_k, r) \leq \Gamma(\ell_-, r)$ for all independent agents with ideal points in (ℓ_-, ℓ). By the quasi-concavity of Γ, the second condition holds if and only if $\Gamma_\ell(\ell_-, r) \leq 0$.

To complete the proof, I now show that $\Gamma_\ell(\ell_-, r) > 0$ if $\ell + r \leq 0$ and Assumption 2 holds. Note that $\ell_- < \ell \leq r$ and $r + \ell \leq 0$ jointly imply that $p(\ell_-, r) < 1/2$. Hence, the condition $\Gamma(\ell_-, r) - \Gamma(\ell, r) = 2c$ requires that

$$\begin{aligned}
\frac{r-\ell_-}{2} &= \frac{c}{p(\ell_-, r)} + \frac{p(\ell, r)}{2p(\ell_-, r)}(r-\ell) > 2c\,, \text{and} \\
\frac{\ell_- - \ell}{2} &= -\frac{c}{p(\ell_-, r)} - \frac{p(\ell, r) - p(\ell_-, r)}{p(\ell_-, r)}(r-\ell) < -2c\,.
\end{aligned}$$

With $r + \ell \leq 0$, the second inequality also ensures that $(r + \ell_-)/2 < -2c$ and, by the log-concavity of Φ, $\Phi\left(\frac{r+\ell_-}{2}\right)/\phi\left(\frac{r+\ell_-}{2}\right) < \Phi(-2c)/\phi(-2c)$. This finally yields the conclusion that the derivative of Γ is given by

$$\Gamma_\ell(\ell_-, r) = \phi\left(\frac{r+\ell_-}{2}\right)\left[\frac{r-\ell_-}{2} - \frac{\Phi\left(\frac{r+\ell_-}{2}\right)}{\phi\left(\frac{r+\ell_-}{2}\right)}\right] > \phi\left(\frac{r+\ell_-}{2}\right)\left[2c - \frac{\Phi(-2c)}{\phi(-2c)}\right]\,.$$

By Assumption 2, c is larger than $1/\phi(0) = 2\Phi(0)/\phi(0)$. By the quasi-concavity of Φ, moreover, $\Phi(0)/\phi(0) > \Phi(-2c)/\phi(-2c)$ for any $c > 0$. Hence, the derivative $\Gamma_\ell(\ell_-, r)$ is strictly positive. Consequently, whenever party L is exit-stable, an agent with some ideal point between the party median ℓ and the next-leftist member ℓ_- can profitably deviate by joining and becoming L's candidate with positive probability. I conclude that there is no robust equilibrium in which the weaker party is inefficient. □

Proof of Proposition 1

Proof. Consider a symmetric outcome with $\ell = -\Delta/2$ and $r = \Delta/2$ for some $\Delta \geq 0$. The platform distance is given by $r - l = \Delta$, and the winning probability of each party is given by $p(\ell, r) = \Phi(0) = 1/2$. Hence, the policy effect of each party is given by $\Gamma(\ell, r) = \Delta/2 \geq 0$. In the following, I first show that the pair $(-\Delta/2, \Delta/2)$ is not a robust equilibrium outcome if $\Delta < \underline{\Delta}$ or $\Delta > \bar{\Delta}$. Then, I prove that it is a robust equilibria outcome if $\Delta \in [\underline{\Delta}, \bar{\Delta}]$.

Lower bound on platform distance. I start by showing that there is no robust equilibrium outcome with platform distance below $\underline{\Delta} = 2c$. By Lemma 2, the party median m_L is weakly smaller than ℓ. Assume that a party member with $\omega_i \leq m_L$ deviates by reducing her contribution from $\alpha_i^L \geq c$ to zero and leaving the party. By Lemma 3, each party is efficient. Hence, the deviation makes party L inactive and ensures the implementation of policy r. The change in i's expected utility is given by

$$v(r-\omega_i) - \left[\frac{1}{2}v(\ell-\omega_i) + \frac{1}{2}v(r-\omega_i) - \alpha_i^L\right] = \alpha_i^L - \Gamma(\ell, r) \geq c - \frac{\Delta}{2}\,.$$

If $\Delta < 2c$, the deviation is profitable and the initial party structure is not exit-stable. We can conclude that there is no symmetric equilibrium with platform distance below $\underline{\Delta} = 2c$.

Upper bound on platform distance. Next, I show that there is no robust political equilibrium with platform distance above the upper bound $\bar{\Delta}$. By Lemma 2, the platform of party L is only $\ell = -\Delta/2$ if no agent with ideal point $\omega_i \in (\ell, r)$ such that $\Gamma(\omega_i, r) > \Gamma(\ell, r) = \Delta/2$ is member of party L. If such an agent would join L, he would by nominated as candidate by Lemma 2. Moreover, joining L would be profitable for such an agent if and only if the implied policy gain

$$\begin{aligned} F(\omega_i, \ell, r, \omega_i) &= p(\omega_i, r)\ \ [v(0) - v(r - \omega_i)] - \frac{1}{2}\,[v(\omega_i - \ell) - v(r - \omega_i)] \\ &= p(\omega_i, r)(r - \omega_i) - \frac{1}{2}\,[-\Delta/2 + \Delta/2 - 2\omega_i] \\ &= \Gamma(\omega_i, \Delta/2) + \omega_i \end{aligned}$$

is strictly larger than c. Hence, $(-\Delta/2, \Delta/2)$ can only be an equilibrium outcome if there is no independent agent with ideal point ω_i such that (i) $\Gamma(\omega_i, \Delta/2) > \Delta/2$ and (ii) $\Gamma(\omega_i, \Delta/2) + \omega_i > c$. I now show that, if and only if $\Delta > \bar{\Delta}$, there is a policy $\omega_i \in (\ell, r)$ that satisfies both (i) and (ii).

For the *if* part, let $\hat{\omega}(\Delta) = \frac{\Delta}{2}\ \frac{\Phi(c/2)-1}{\Phi(c/2)} \in (-\Delta/2, 0)$. The policy effect is

$$\Gamma\left(\hat{\omega}(\Delta), \Delta/2\right) = [\Delta/2 - \hat{\omega}(\Delta)]\ \ \Phi\left(\frac{\hat{\omega}(\Delta) + \Delta/2}{2}\right) = \frac{\Delta}{2}\ \frac{\Phi\left(\frac{\Delta}{2}\ \frac{\Phi(c/2)-1/2}{\Phi(c/2)}\right)}{\Phi(c/2)}\ .$$

Thus, condition (i) is equivalent to

$$\Phi\left(\frac{\Delta}{2}\ \frac{\Phi(c/2) - 1/2}{\Phi(c/2)}\right) > \Phi(c/2)\ ,$$

and condition (ii) can be rewritten as

$$\Gamma\left(\hat{\omega}(\Delta), \Delta/2\right) + \hat{\omega}(\Delta) = \frac{\Delta}{2}\ \frac{1}{\Phi(c/2)}\ \left[\Phi\left(\frac{\Delta}{2}\ \frac{\Phi(c/2) - 1/2}{\Phi(c/2)}\right) + \Phi(c/2) - 1\right] > c\ .$$

Both conditions are satisfied if and only if Δ exceeds $\bar{\Delta} = c\ \frac{\Phi(c/2)}{\Phi(c/2)-1/2}$. In this case, joining party L is a profitable deviation for an independent agent with ideal point ω_i close enough to $\hat{\omega}(\Delta)$. As a result, the symmetric pair of platform $(\Delta/2, \Delta/2)$ is no robust equilibrium outcome if $\Delta > \bar{\Delta}$.

For the *only-if* part, the quasi-concavity of Γ implies that condition (i) is satisfied for some ideal point $\omega_i \in (\ell, r)$ if and only if $\Delta > 1/\phi(0)$, see last sentence of Lemma A.2. In this case, there is a unique cutoff $\theta(\Delta) \in (\ell, r)$ such that $\Gamma(\omega_i, r) > \Delta/2$ if and only if $\omega_i \in (-\Delta/2, \theta(\Delta))$. Hence, agents with ideal points in $(-\Delta/2, \theta(\Delta))$ can affect L's platform if joining for any member set Ω_L. Moreover, the benefit of joining L is strictly increasing in the entrant's ideal point ω_i,

$$\frac{dF(\omega_i, -\Delta/2, \Delta/2, \omega_i)}{d\omega_i} = \frac{1}{2}\,\phi\left(\frac{\omega_i + \Delta/2}{2}\right)\ \ (\Delta/2 - \omega_i) - \Phi\left(\frac{\omega_i + \Delta/2}{2}\right) + 1 > 0\ .$$

This implies that if an agent with ideal point $\theta(\Delta)$ cannot profitably deviate by joining L and becoming its candidate, then no other agent can as well. We have shown above that $\Gamma(\hat{\omega}(\Delta), \Delta/2)$ is weakly smaller than both $\Delta/2$ and $\hat{\omega}(\Delta) + c$ if $\Delta \leq \bar{\Delta}$. This implies that, first, agents with ideal point above $\hat{\omega}(\Delta)$ cannot become candidates if joining and, second, joining party L is not profitable for any agent with ideal point below $\hat{\omega}(\Delta)$. I conclude that, if $\Delta \leq \bar{\Delta}$, there is no ideal point $\omega_i \in (\ell, r)$ for whom the conditions (i) and (ii) are jointly satisfied.

Robust equilibrium outcomes. Finally, I show that the platform pair $(-\Delta/2, \Delta/2)$ is a robust equilibrium outcome if Δ is between $\underline{\Delta} = 2c$ and $\bar{\Delta}$. Consider an activist set Ω such that there are $z \geq C/c$ agents with each of the ideal points $-\Delta/2$ and $\Delta/2$, and a party structure (Ω_L, Ω_R) such that both parties are efficient and all their members contribute exactly c. Moreover, let all members of L have ideal point $-\Delta/2$, and all members of R have ideal point $\Delta/2$. Then, first, $-\Delta/2$ and $\Delta/2$ are trivially the equilibrium platforms of L and R, respectively. Second, with $\Delta \geq \underline{\Delta} = 2c$, no member would benefit from leaving; her policy loss would exceed the membership cost c. Third, if any independent agent joins L, the party median m_L would remain equal to $-\Delta/2$. Hence, an entrant i can affect L's platform only if his ideal point ω_i if preferred to $-\Delta/2$ by the initial members. As shown above, if $\Delta \leq \bar{\Delta}$, then joining party L is not profitable for any agent satisfying this condition. Hence, party L is both exit-stable and entry-stable for the activist set Ω and any $\Omega \cup \{\omega_i\}$. By symmetry, the same is true for party R. Finally, no party member would benefit from changing his party affiliation. For example, if a member of L would join R, this would not affect R's platform but induce L's inactivity. This would ensure the implementation of policy $r = \Delta/2$ and make the agent strictly worse off. Hence, $(-\Delta/2, \Delta/2)$ is a robust equilibrium outcome if and only if $\Delta \in [\underline{\Delta}, \bar{\Delta}]$. □

Proof of Proposition 2

Proof. I proceed in the same steps as in the proof of Proposition 1. Consider an asymmetric pair of platforms with platforms $\ell = -\Delta/2 + \varepsilon$ and $r = \Delta/2 + \varepsilon$ for some platform distance $\Delta > 0$ and $\varepsilon \in \mathbb{R}$. The winning probability of party L is $p(\ell, r) = \Phi(\varepsilon)$; it is smaller than $1/2$ if and only if $\varepsilon > 0$. The winning probability of party R is $\Phi(-\varepsilon)$. Hence, ε is a measure of party L's electoral advantage. The policy effect of party L is $\Gamma(-\Delta/2 - \varepsilon, \Delta/2 + \varepsilon) = \Delta\ \Phi(-\varepsilon)$. In the following, I fix a $\varepsilon < 0$ so that $p(\ell, r) < 1/2$. The results for $\varepsilon > 0$ follow by symmetry.

Lower bound on platform distance. Party L is efficient by Lemma 3. Thus, if one of its members leaves L, policy $r = \Delta/2 + \varepsilon$ is implemented with certainty. This deviation is profitable to any member with $\omega_i \leq \ell$ if the implied policy loss is smaller than the membership cost, $p(\ell, r)(r - \ell) = \Delta\ \Phi(\varepsilon) < c$. Hence, exit-stability of party L requires that $\Delta \geq c/\Phi(\varepsilon)$. Correspondingly, exit-stability of party R requires that $\Delta \geq c/\Phi(-\varepsilon)$. Combining both conditions, the platform distance for any robust equilibrium outcome must be above $\Delta_{low}(\varepsilon) = c/\Phi(-|\varepsilon|) = \max\{c/\Phi(\varepsilon), c/\Phi(-\varepsilon)\}$.

Upper bound on platform distance Δ. As explained in the proof of Proposition 1, a pair (ℓ, r) cannot be an equilibrium outcome if there is an agent $i \in \mathcal{A}$ with ideal point $\omega_i \in (\ell, r)$ such that (i) $\Gamma(\omega_i, r) > \Gamma(\ell, r)$ so that i becomes the presidential candidate if she joins L and (ii) $F(\omega_i, \ell, r, \omega_i) > c$ so that her utility goes up if becoming presidential candidate. For an asymmetric pair $(r, \ell) = (-\Delta/2 + \varepsilon, \Delta/2 + \varepsilon)$, we have $\Gamma(\ell, r) = \Delta\ \Phi(\varepsilon)$ and

$$\begin{aligned} F(\omega_i, \ell, r, \omega_i) &= [1 - p(\omega_i, r)]\, v(r - \omega_i) - [\Phi(\varepsilon) v(\omega_i - \ell) + [1 - \Phi(\varepsilon)]\, v(r - \omega_i)] \\ &= p(\omega_i, r)(r - \omega_i) - \Phi(\varepsilon)\, [r + \ell - 2\omega_i] \\ &= \Gamma(\omega_i, r) + 2\Phi(\varepsilon)\, [\omega_i - \varepsilon]\ . \end{aligned}$$

I now show that, if $\Delta > Z(\varepsilon)$, there is a position $\omega_i \in (\ell, r)$ that satisfies both (i) and (ii). For this purpose, define the auxiliary function

$$\tilde{\omega}(\Delta, \varepsilon) \coloneqq \frac{\Delta}{2}\ \frac{\Phi(x_a(\varepsilon)) - 2\Phi(\varepsilon)}{\Phi(x_a(\varepsilon))} + \varepsilon \in (\ell, r)\ ,$$

with $x_a(\varepsilon) = c\, [4\Phi(\varepsilon)]^{-1} + \varepsilon > \varepsilon$ as defined in the main text. The policy effect of platform $\tilde{\omega}(\Delta, \varepsilon)$ against $r = \Delta/2 + \varepsilon$ is given by

$$\begin{aligned} \Gamma(\tilde{\omega}(\Delta, \varepsilon), r) &= [\Delta/2 + \varepsilon - \tilde{\omega}(\Delta, \varepsilon)]\ \Phi\left(\frac{\tilde{\omega}(\Delta, \varepsilon) + \Delta/2 + \varepsilon}{2}\right) \\ &= \Delta\ \frac{\Phi(\varepsilon)}{\Phi(x_a(\varepsilon))}\ \Phi\left(\Delta/2 \frac{\Phi(x_a(\varepsilon)) - \Phi(\varepsilon)}{\Phi(x_a(\varepsilon))} + \varepsilon\right)\ . \end{aligned}$$

For $\omega_i = \tilde{\omega}(\Delta, \varepsilon)$, condition (i) is thus equivalent to

$$\Phi\left(\Delta/2 \frac{\Phi(x_a(\varepsilon)) - \Phi(\varepsilon)}{\Phi(x_a(\varepsilon))} + \varepsilon\right) > \Phi(x_a(\varepsilon)) = \Phi\left(\frac{c}{4\Phi(\varepsilon)} + \varepsilon\right)\ .$$

Moreover, condition (ii) is equivalent to

$$\Delta\ \frac{\Phi(\varepsilon)}{\Phi(x_a(\varepsilon))}\ \left[\Phi\left(\frac{\Delta}{2}\ \frac{\Phi(x_a(\varepsilon)) - \Phi(\varepsilon)}{\Phi(x_a(\varepsilon))} + \varepsilon\right) + \Phi(x_a(\varepsilon)) - 2\Phi(\varepsilon)\right] > c\ .$$

Both conditions are satisfied if and only if $\Delta > Z(\varepsilon) = \frac{c}{2\Phi(\varepsilon)}\ \frac{\Phi(x_a(\varepsilon))}{\Phi(x_a(\varepsilon)) - \Phi(\varepsilon)}$. In this case, hence, joining L is a profitable deviation for an agent with ideal point close enough to $\tilde{\omega}(\Delta, \varepsilon)$.

By corresponding arguments, if and only if $\Delta > Z(-\varepsilon)$, joining part R is a profitable deviation for an agent with ideal point close enough to $-\tilde{\omega}(\Delta, -\varepsilon)$. As a result, the pair

$(-\Delta/2+\varepsilon, \Delta/2+\varepsilon)$ is no robust equilibrium outcome if $\Delta > \Delta_{up}(\varepsilon) = \min\{Z(\varepsilon), Z(-\varepsilon)\}$.

Upper bound on electoral advantage ε. Let again $\varepsilon < 0$. I have shown that $(-\Delta/2+\varepsilon, \Delta/2+\varepsilon)$ can only be a robust equilibrium outcome if Δ is between $\Delta_{low}(\varepsilon)$ and $\Delta_{up}(\varepsilon) \leq Z(\varepsilon)$, which requires that

$$\begin{aligned} Z(\varepsilon) = \frac{c}{2\Phi(\varepsilon)} \frac{\Phi(x_a(\varepsilon))}{\Phi(x_a(\varepsilon)) - \Phi(\varepsilon)} &\geq \Delta_{low}(\varepsilon) = \frac{c}{\Phi(\varepsilon)} \\ \Leftrightarrow 2\Phi(\varepsilon) &\geq \Phi(x_a(\varepsilon)) \ . \end{aligned}$$

This inequality is satisfied for $\varepsilon = 0$ because $2\Phi(0) - \Phi(x_a(0)) = 1 - \Phi(c/2) > 0$. It is violated for $\varepsilon \to -\infty$ because $x_a(\varepsilon)$ converges to 0 so that $2\Phi(\varepsilon) - \Phi(x_a(\varepsilon))$ approaches $-1/2$. Moreover, the derivative of $2\Phi(\varepsilon) - \Phi(x_a(\varepsilon))$ is given by

$$\begin{aligned} \frac{d\left[2\Phi(\varepsilon) - \Phi(x_a(\varepsilon))\right]}{d\varepsilon} &= 2\phi(\varepsilon) + \phi(x_a(\varepsilon))\left[\frac{c}{4\Phi(\varepsilon)}\frac{\phi(\varepsilon)}{\Phi(\varepsilon)} - 1\right] \\ &> 2\phi(x_a(\varepsilon))\left[\frac{\phi(\varepsilon)}{\phi(x_a(\varepsilon))} - \frac{1}{2}\right] \\ &= 2\phi(x_a(-\varepsilon))\frac{\Phi(\varepsilon)}{\Phi(x_a(\varepsilon))}\left[\frac{\Phi(x_a(\varepsilon))}{\phi(x_a(\varepsilon))}\frac{\phi(\varepsilon)}{\Phi(\varepsilon)} - \frac{\Phi(x_a(\varepsilon))}{2\Phi(\varepsilon)}\right] \ . \end{aligned}$$

As $x_a(\varepsilon) > \varepsilon$, the first term in the bracket is strictly larger than 1 by the log-concavity of Φ. For any ε such that $2\Phi(\varepsilon) \geq \Phi(x_a(\varepsilon))$, the second term in the bracket is smaller than 1, implying that the derivative is strictly positive for $\varepsilon < 0$. Hence, $2\Phi(\varepsilon) - \Phi(x_a(\varepsilon))$ has a unique root $-\bar{\varepsilon} \in (-\infty, 0)$. I conclude that, for an electoral advantage below this root $-\bar{\varepsilon}$, the requirement $\Delta_{up}(\varepsilon) \geq \Delta_{low}(\varepsilon)$ is violated: Any party structure giving rise to platforms with $\varepsilon < -\bar{\varepsilon}$ is either not exit-stable or not entry-stable. By symmetry, the same is true for ε above $\bar{\varepsilon}$.

Existence of robust equilibrium outcomes. Finally, consider a pair of platforms $\ell = \Delta/2 + \varepsilon$ and $r = \Delta/2 + \varepsilon$ such that $-\bar{\varepsilon} \leq \varepsilon < 0$ and $\Delta \in [\Delta_{low}(\varepsilon), \Delta_{up}(\varepsilon)]$. Assume that both parties are efficient and have $n \geq C/c \geq 3$ members, each contributing exactly c. Further, assume that all members of party L have the same ideal point ℓ, and all members of R have the same ideal point r. Hence, no agent can affect any of the party medians m_L or m_R by changing her party affiliation. With $\Delta \geq \Delta_{low}(up)$, no member of L can benefit from leaving her party because the policy loss $\Delta\Phi(\varepsilon)$ exceeds the membership cost c. As party R has a winning probability of $\Phi(-\varepsilon) > 1/2$, the policy loss from leaving is even higher for members of R. Hence, party R is exit-stable as well.

It remains to show that no agent with ideal point $\omega_i \in (\ell, r)$ can benefit from joining a party and becoming its candidate. With a constant party median m_L, joining party L is only profitable for i if (i) $\Gamma(\omega_i, r) > \Delta\ \Phi(-\varepsilon)$ and (ii) $F(\omega_i, \ell, r, \omega_i) > c$. As shown above, if $\Delta \leq Z(\varepsilon)$, neither condition is satisfied for $\omega_i = \tilde{\omega}(\Delta, \varepsilon)$ and, moreover, $(\tilde{\omega}(\Delta, \varepsilon) + r)/2$

is weakly smaller than $x_a(\varepsilon)$. This implies that $p(\tilde{\omega}(\Delta, \varepsilon), r) \leq \Phi(x_a(\varepsilon))$. The derivative of the entry benefit with respect to ω_i is given by

$$\frac{dF(\omega_i, \ell, r, \omega_i)}{d\omega_i} = \frac{1}{2}\,\phi\left(\frac{\omega_i + r}{2}\right)\,(r - \omega_i) - p(\omega_i, r) + 2\Phi(\varepsilon)\ .$$

For any $\omega_i \leq \tilde{\omega}(\Delta, \varepsilon)$, this derivative is larger than $2\Phi(\varepsilon) - \Phi(x_a(\varepsilon))$ and thus, positive for any $\varepsilon \geq -\bar{\varepsilon}$, see last step above. We can conclude that condition (ii) is not satisfied for any $\omega_i \leq \tilde{\omega}(\Delta, \varepsilon)$. By the quasi-concavity of Γ, condition (i) is not satisfied for any $\omega_i \geq \tilde{\omega}(\Delta, \varepsilon)$. Hence, agent i cannot profitably join party L for any ideal point in (ℓ, r): Party L is entry-stable. Corresponding arguments imply that joining party R is only profitable for some independent agent if $\Delta > Z(-\varepsilon)$. Thus, if $\Delta \leq \Delta_{up}(\varepsilon) = \min\{Z(\varepsilon), Z(-\varepsilon)\}$, then no independent agent with some ideal point $\omega_i \in (\ell, r)$ can profitably join any party. As a last point, note that changing the party affiliation is not profitable for any party member either: It does not provide a direct gain from saving c, but has the same negative policy effect as becoming independent. □

Proof of Corollary 1

Proof. First, the lower bound $\Delta_{low}(\varepsilon) = c/\Phi(-|\varepsilon|)$ is strictly decreasing for $\varepsilon < 0$ and strictly increasing for $\varepsilon > 0$. Hence, it has a unique minimum at $\varepsilon = 0$, where $\Delta_{low}(0) = \underline{\Delta} = c/2$.

Second, the upper bound is given by $\Delta_{up}(\varepsilon) =\in \{Z(\varepsilon), Z(-\varepsilon)\}$ and satisfies $\Delta_{up}(0) = \bar{\Delta}$. I now show that $Z(\varepsilon)$ is increasing in ε for all $\varepsilon \in (-\bar{\varepsilon}, 0)$. For this purpose, define

$$\tilde{Z}(\varepsilon) = \frac{c}{2Z(\varepsilon)} = \Phi(\varepsilon)\left[1 - \frac{\Phi(\varepsilon)}{\Phi(x_a(\varepsilon))}\right]\ .$$

The derivative of $\tilde{Z}(\varepsilon)$ is given by

$$\frac{d\tilde{Z}(\varepsilon)}{d\varepsilon} = \phi(\varepsilon)\left[1 - 2\frac{\Phi(\varepsilon)}{\Phi(x_a(\varepsilon))}\right] + \phi(x_a(\varepsilon))\ \frac{\Phi(\varepsilon)^2}{\Phi(x_a(\varepsilon))^2}\ x_a'(\varepsilon)\ ,$$

where the term in brackets is negative for any $\varepsilon \in [0, \bar{\varepsilon}]$, see proof of Proposition 2. The first and second derivatives of $x_a(\varepsilon)$ are

$$\begin{aligned} x_a'(\varepsilon) &= -\frac{c}{4\Phi(\varepsilon)}\,\frac{\phi(\varepsilon)}{\Phi(\varepsilon)} + 1\ , \text{ and} \\ x_a''(\varepsilon) &= \frac{c}{4\Phi(\varepsilon)}\left[\frac{\phi(\varepsilon)^2}{\Phi(\varepsilon)^2} - \frac{d\left(\phi(\varepsilon)/\Phi(\varepsilon)\right)}{d\varepsilon}\right]\ . \end{aligned}$$

The second derivative is strictly positive because $\phi(\varepsilon)/\Phi(\varepsilon)$ is decreasing in ε by the log-concavity of Φ imposed in Assumption 1. At $\varepsilon = 0$, the first derivative equals $-c\,\phi(0) + 1$, which is negative under Assumption 2. Thus, $x_a'(\varepsilon) < 0$ for all $\varepsilon < 0$. This implies that

$\tilde{Z}(\varepsilon)$ is strictly decreasing, $Z(\varepsilon)$ is strictly increasing, and $\Delta_{up}(\varepsilon) \leq Z(\varepsilon) < \Delta_{up}(0) = \bar{\Delta}$ for all $\varepsilon \in [-\bar{\varepsilon}, 0)$. By symmetry, $Z(-\varepsilon)$ is strictly decreasing in ε and $\Delta_{up}(\varepsilon) \leq Z(-\varepsilon) < \bar{\Delta}$ for all $\varepsilon \in (0, \bar{\varepsilon}]$. □

Proof of Lemma 4

Proof. Consider a potential equilibrium in which only party R is active. Let the member set Ω_R be such that $m_R - m_{R<} > 2c$, and consider an independent agent i with ideal point $\omega_i \in (m_{R<}, m_R)$. Assume i joins party R by contributing $\alpha_{Ri} = c$. After this deviation, the initial median member and the entrant are the two midmost members. Hence, the party platform r is given by ω_i and m_R with equal probability by Lemma 2. As R is the only active party, the implemented policy equals r. The entrant's utility increases by $1/2(m_R - \omega_i) - c$. For ω_i close enough to $m_{R<}$, joining party R is a profitable deviation for i. Correspondingly, if $m_{R>} - m_R > 2c$, joining R is profitable for an independent agent with some ideal point $\omega_i \in (m_R, m_{R>})$. Hence, there is no robust political equilibrium with $\max\{m_R - m_{R<}, m_{R>} - m_R\} > 2c$.

□

Proof of Proposition 3

Proof. Consider a potential political equilibrium in which party R is active, efficient and satisfies $\max\{m_R - m_{R<}, m_{R>} - m_R\} \leq 2C$. By Lemma 4, the last condition implies that no independent agent profits from joining R. The efficiency of R implies that no party member profits from leaving R.

It remains to check whether an independent agent can profit from joining party L. Let $\alpha_i^L = 0$ for all $i \in \mathcal{A}$ (i.e., party L has no members). Assume now that some independent agent i with ideal point $\omega_i < r$ deviates by contributing $\alpha_i^L = C$. Then, party L runs with platform $\ell = \omega_i$. The policy gain to agent i equals $\Gamma(\omega_i, r) = p(\omega_i, r)\ (r - \omega_i)$. Contributing more than C yields the same policy gain, but is more costly. Contributing less than C has no policy effect. Hence, there is a profitable deviation for agent i if and only if $\Gamma(\omega_i, r) > C$. In particular, the policy effect $\Gamma(\omega_i, r)$ has a unique maximizer $\omega_i = \gamma^*(r) < r$, see Lemma A.2. Hence, the pair $(r, \emptyset)$ is a robust equilibrium outcome if and only if

$$\Gamma(\gamma^*(r), r) = (r - \gamma^*(r))\ \Phi\left(\frac{r + \gamma^*(r)}{2}\right) \leq C\,. \tag{A.4}$$

For any $r > C$, this condition is violated because $\Gamma(\gamma^*(r), r) \geq \Gamma(-r, r) = r$. Besides, the left-hand side is strictly increasing in r because

$$\frac{d\Gamma(\gamma^*(r), r)}{dr} = \underbrace{\Gamma_\ell(\gamma^*(r), r)}_{=0}\ \frac{d\gamma^*(r)}{dr} + \Gamma_r(\gamma^*(r), r)$$

$$= \quad \frac{r-\gamma^*(r)}{2}\phi\left(\frac{r+\gamma^*(r)}{2}\right)+\Phi\left(\frac{r+\gamma^*(r)}{2}\right)>0\,.$$

Hence, there are two possible cases. First, condition (A.4) is satisfied for $r = 0$ if and only if $-\omega_i\ \Phi(\omega_i/2) \leq C$ for all $\omega_i \in \mathbb{R}$. In this case, there is a unique threshold $\bar{x} \in [0, C)$ such that $\Gamma(\gamma^*(\bar{x}), \bar{x}) = C$. There is a robust one-party equilibrium with $\ell = \emptyset$ and any $r \in [0, \bar{x}]$. By symmetry, there is also a one-party equilibrium with any $r \in [-\bar{x}, 0)$.

Second, if condition (A.4) is violated for $r = 0$, then it is also violated for any $r > 0$. Hence, there is no robust one-party equilibrium with any platform $r \geq 0$, because an independent agent with some ideal point ω_i could profitably start party L on her own. By symmetry, there is neither a robust one-party equilibrium with $r < 0$.

□

Proof of Proposition 4

Proof. First, the result for the lower bound $\underline{\Delta} = 2c$ follows immediately. Second, the upper bound is given by $\bar{\Delta} = c\ \Phi(c/2)\ [\Phi(c/2) - 1/2]^{-1}$. Its derivative with respect to c is given by

$$\frac{d\bar{\Delta}}{dc} = \frac{\Phi(c/2)}{[\Phi(c/2)-1/2]^2}\ K(c)\ ,\text{where } K(c) = \Phi(c/2) - \frac{1}{2} - \frac{c}{4}\ \frac{\phi(c/2)}{\Phi(c/2)}\ .$$

For $c = 0$, the term $K(c)$ equals zero. For all $c \geq 0$, its derivative is strictly positive,

$$K'(c) = \frac{\phi(c/2)}{2\Phi(c/2)}\underbrace{[\Phi(c/2)-1/2)]}_{\geq 0} - \frac{c}{4}\ \underbrace{\frac{d\,(\phi(c/2)/\Phi(c/2))}{dc}}_{<0} > 0,$$

because $\phi(x)/\Phi(x)$ is decreasing in x by the log-concavity of Φ. Hence, $\bar{\Delta}$ is strictly increasing in c for any $c > 0$.

For the limit case $c = 0$, the lower bound is trivially given by $\underline{\Delta} = 0$. Using l'Hôpital's rule, the limit of the upper bound $\bar{\Delta}$ follows as

$$\lim_{c\to 0}\bar{\Delta} = \lim_{c\to 0} c\ \frac{\Phi(c/2)}{\Phi(c/2)-1/2} = \lim_{c\to 0}\frac{\Phi(c/2)+c/2\ \phi(c/2)}{1/2\ \phi(c/2)} = \frac{1}{\phi(0)}\ ,$$

where I exploit that $\Phi(0) = 1/2$. □

Proof of Proposition 5

Proof. The derivative of the lower bound $\underline{\Delta} = 2c$ with respect to σ is trivially zero. This also implies that $\underline{\Delta}$ remains equal to $2c$ for $\sigma \to 0$. The derivative of $\bar{\Delta}$ with respect to σ is given by

$$\frac{d\bar{\Delta}}{dc} = -\frac{c}{2}\ \frac{d\Phi_\sigma(c/2)}{d\sigma}\ \frac{1}{[\Phi(c/2)-1/2]^2} > 0\,,$$

where the positive sign follows because $d\Phi(c/2)/d\sigma < 0$ for any $c > 0$ by Assumption 3. For $\sigma \to 0$, $\Phi(c/2)$ goes to 1. Hence, the upper bound $\bar{\Delta}$ converges to $2c$.

For the upper bound $\bar{\varepsilon} > 0$ on the electoral advantage, recall that $x_a(\varepsilon) = \varepsilon + c\,[4\Phi(\varepsilon)]^{-1}$ and that $\Phi(x_a(\varepsilon))/\Phi(\varepsilon) \leq 2$ for all $\varepsilon \in [-\bar{\varepsilon}, 0)$. Fix some $\varepsilon < 0$. For $\sigma \to 0$, $\Phi(\varepsilon)$ converges to 0, while $x_a(\varepsilon)$ goes to $+\infty$. Hence, I find that $\Phi(x_a(\varepsilon))$ converges to 1 so that $\Phi(x_a(\varepsilon))/\Phi(\varepsilon) > 2$ for any $\varepsilon < 0$. This implies that $\lim_{\sigma\to 0} \bar{\varepsilon} = 0$. As a result, all asymmetric equilibria vanish in the limit case without any electoral risk. □

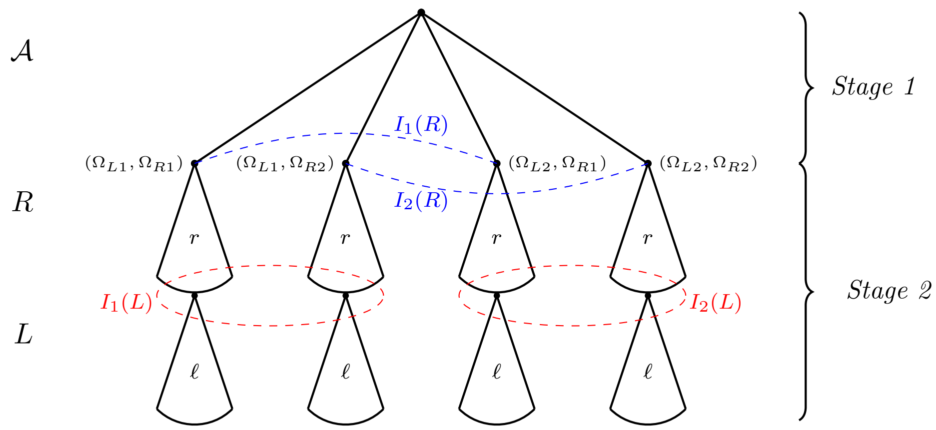
Proof of Corollary 3

Proof. In the limit case of electoral certainty, $\sigma \to 0$, the party platforms are given by $(\ell, r) = (-c, c)$ in every two-party-equilibrium by Proposition 5. The median voter with ideal point $\omega_i = 0$ is indifferent between both parties and does not make any contributions. Her utility equals $E[v(x)] = -c$.

By Proposition 3, there is a robust one-party equilibria with any platform $r \in [-\bar{x}, \bar{x}]$, where $\bar{x}$ satisfies $\Gamma(\gamma^*(\bar{x}), \bar{x}) = C$. For $\sigma \to 0$, $p(-r + \mu, r) = 1$ and $\Gamma(-r + \mu, r) = 2r - \mu$ for any $\mu > 0$. This implies that $\lim_{\sigma\to 0} \gamma^*(r) = -r$ and $\lim_{\sigma\to 0} \bar{x} = C/2$. As a result, there is a robust one-party equilibrium with platform r if and only if $r \in [-C/2, C/2]$. In such an equilibrium, the utility of the median voter is given by $v(r) = -|r|$ (or lower if she makes contributions to party R). Thus, in each equilibrium with platform r in the intervals $[-C/2, -c)$ or $(c, C/2]$, the median voter's utility is strictly below $-c$, her utility in every two-party equilibrium. □

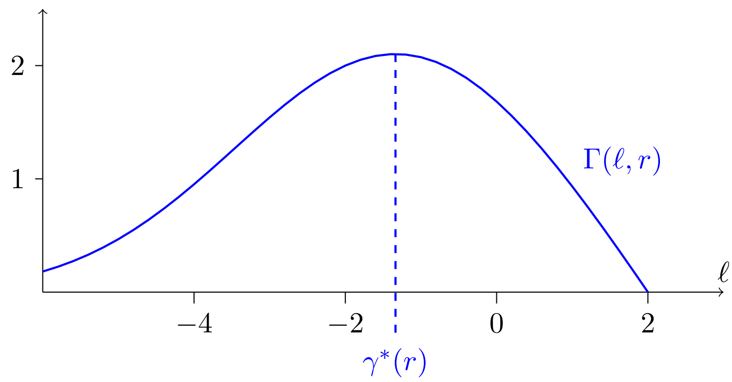
B Figures

Figure 2: The party formation and candidate selection stages



Notes: Figure 2 illustrates the information setting at the party formation and candidate selection stages. At the party formation stage, all activists in $\mathcal{A}$ simultaneously make their membership decisions, thereby determining the member sets Ω_L and Ω_R. For illustration, the figure only depicts two possible members sets Ω_{L1} and Ω_{L2} for party L and two possible member sets Ω_{R1} and Ω_{R2} for party R. At the candidate selection stage, the members of party L observe their own member set, but cannot observe whether the member set of party R is Ω_{R1} or Ω_{R2}. Hence, when selecting platform ℓ, the members of party L only know whether they are in information set $I_1(L)$ or in information set $I_2(L)$. Correspondingly, the members of party R can only observe their own member set, i.e., know whether they are in information set $I_1(R)$ or in information set $I_2(R)$.

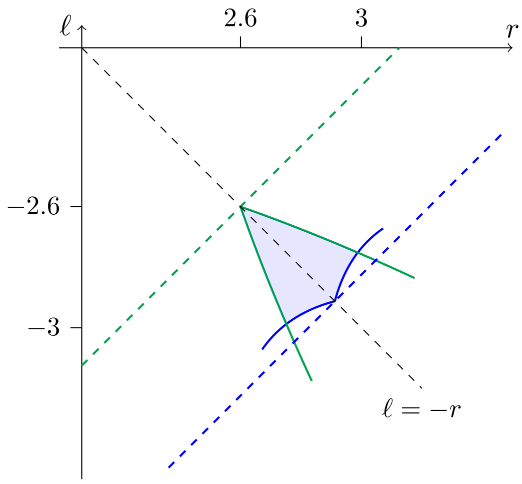
Figure 3: The policy effect function $\Gamma(\ell, r)$



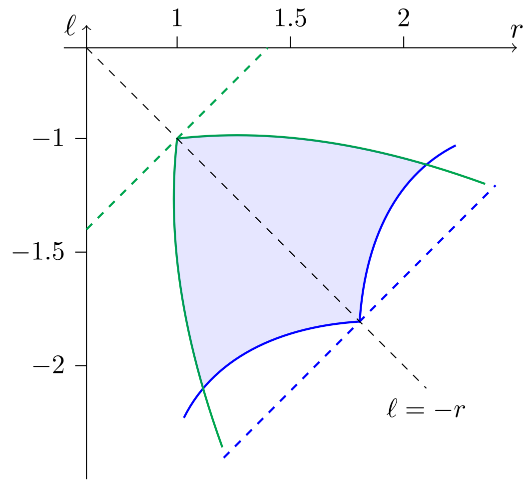
Notes: Figure 3 depict the policy effect $\Gamma(\ell, r)$ as a function of platform ℓ for a numerical example in which Φ is a standard normal distribution and the policy platform of party R is given by $r = 2$. The policy effect function is maximized by policy $\gamma^*(r) \approx -1.34$.

Figure 4: Robust equilibrium outcomes.

(a) Example 1: $c = 2.6$.



(b) Example 2: $c = 1$.



Notes: Figure 4 depicts the sets of robust equilibrium outcomes for two numerical examples in which Φ is a standard normal distribution. Subfigure 4a considers a case with $c = 2.6$ for which Assumption 2 is satisfied. Subfigure 4b considers a case with $c = 1$ for which Assumption 2 is violated. In both subfigures, the solid green lines follow from the lower bound $\Delta_{low}(\varepsilon)$ on the platform distance; the solid blue lines follow from the upper bound $\Delta_{up}(\varepsilon)$. For comparison, see Figure 1.

Online Appendix

C Extensions

C.1 Higher electoral risk and convergent equilibria

Under Assumption 2, there are no robust equilibrium outcomes with platform distance below $2c$ by Proposition 1 and Corollary 1. If Assumption 2 is violated, by contrast, there may exist robust equilibria with more convergent platforms. The following proposition focuses on symmetric platforms with platform distance below $2c$. It shows that such equilibria exist if and only if the level of electoral risk as implied by the median voter distribution Φ is large enough.

For the sake of concreteness, consider a party structure such that, in each party $J \in \{L, R\}$, the median member has ideal point $m_J = 0$, the next-leftist member has an ideal point $-\hat{\omega} < 0$, and the next-rightist member has an ideal point $\hat{\omega}$. Moreover, both parties are not efficient so that they would remain active if one member leaves her party. If the electoral risk is large enough, then there is a robust political equilibrium with such a party structure and fully convergent platforms $r = \ell = 0$. In this equilibrium, any member of party L prefers to remain in her party not to avoid that the competing party R wins, but to avoid that her own party L runs with a platform that makes her (much) worse off than the platform of R. Any equilibrium of this type is fragile in the following two ways: First, any party member except the party medians and any independent agent with ideal point below $-\hat{\omega}$ or above $\hat{\omega}$ is exactly indifferent between joining a party and staying independent. Second, the equilibrium vanishes if policy preferences are (slightly) concave or (slightly) asymmetric.

Proposition C.1. *If $\Gamma(\gamma^*(0), 0) \geq 2c$, there is a threshold $\hat{\beta} \in [0, 1]$ such that $(-\beta c, \beta c)$ is a robust equilibrium outcome for any $\beta \in [0, \hat{\beta}]$. If $\Gamma(\gamma^*(0), 0) < 2c$, the pair $(-\beta c, \beta c)$ is no robust equilibrium outcome for any $\beta \in [0, 1)$.*

Proof. Consider a potential equilibrium with platforms $\ell = m_L = -\beta c$ and $r = m_R = \beta c$, and a party structure (Ω_L, Ω_R) such that the next-midmost members of party L have ideal points $\ell_- < -\beta c$ and $\ell_+ > -\beta c$, respectively. A member of party L with $\omega_i \geq \ell_+$ can profitably leave L unless $\Gamma(\ell_-, r) \geq \Gamma(\ell, r) + 2c = (2 + \beta)c$, and an independent agent with $\omega_i \leq \ell_-$ can profitably join party L unless $\Gamma(\ell_-, r) \leq \Gamma(\ell, r) + 2c = (2 + \beta)c$. Both conditions are only compatible if the ideal point ℓ_- of the next-leftist member of L satisfies

$$\Gamma(\ell_-, \beta c) = p(\ell_-, \beta c)(\beta c - \ell_-) = (2 + \beta)c \; . \tag{C.1}$$

Moreover, joining party L is profitable for some agent with ideal point $\omega_i \in (\ell_-, \ell)$ unless $\Gamma_\ell(\ell_-, r) \leq 0$. By the quasi-concavity of Γ, if $G(\beta) = \Gamma(\gamma^*(\beta c), \beta c) - (2 + \beta)c \geq 0$, both conditions are satisfied for some $\omega_i \in [\gamma^*(\beta c), -\beta c)$. Otherwise, the conditions are not satisfied for any $\omega_i < -\beta c$.

By corresponding arguments, exit-stability for members with $\omega_i \leq \ell_-$ and entry-stability for members with $\omega_i \geq \ell_+$ jointly require that

$$p(\ell_+, \beta c)(\ell_+ - \beta c) = \Gamma(-\ell_+, -\beta c) = (2-\beta)c$$

and that $\Gamma_\ell(-\ell_+, -\beta c) \leq 0$. If and only if $G(-\beta) = \Gamma(\gamma^*(-\beta c), -\beta c) - (2-\beta)c \geq 0$, both conditions are satisfied for some $\ell_+ \in (\gamma^*(-\beta c), -\beta c)$.

The derivative of $G(\beta)$ is strictly negative,

$$G'(\beta) \;\; = \;\; c\; [\Gamma_r(\gamma^*(\beta c), \beta c) - 1] = c\; [2\, p(\gamma^*(\beta c), \beta c) - 1] < 0\;,$$

because $p(\gamma^*(\beta c), \beta c) < 1/2$ due to $\gamma^*(\beta c) < -\beta c$. Hence, $G(\beta) \geq 0$ implies that $G(\beta') > 0$ for all $\beta' < \beta$. As a result, the condition on ℓ_+ is satisfied whenever the condition on ℓ_- is satisfied. Moreover, if $G(0) = \Gamma(\gamma^*(0), 0) - 2c \geq 0$, there exists a threshold $\hat{\beta} \geq 0$ such that $G(\hat{\beta}) = 0$. In this case, $G(\beta) \geq 0$ and $(-\beta c, \beta c)$ is a robust equilibrium outcome if and only if $\beta \in [0, \hat{\beta}]$. If instead $G(0) < 0$, then $(-\beta c, \beta c)$ is no robust equilibrium outcome for any $\beta \in [0, 1]$. □

As a final remark, note that Assumption 2 is equivalent to $c/2 \geq \Phi(0)/\phi(0)$. By contrast, the condition $Z(0) \geq 0$ requires that $\gamma^*(0) < -4c$ or, equivalently, that

$$\Gamma_\ell(-4c, 0) = 2c\phi(-2c) - \Phi(-2c) < 0 \;\Leftrightarrow\; 2c < \frac{\Phi(-2c)}{\phi(-2c)} < \frac{\Phi(0)}{\phi(0)}\;,$$

where the last inequality holds by the log-concavity of Φ. Hence, Assumption 2 ensures that $Z(0) < 0$, thereby ruling out the existence of robust equilibrium outcomes with platform distance below $2c$.

C.2 Concave policy preferences

In the basic model, I have assumed that an agent's utility is linearly decreasing in the distance between the implemented policy x and her ideal point ω_i. In the following, I show that my main results generalize to a model with non-linear policy preferences. Specifically, I assume that policy preferences are captured by the CARA utility function

$$v(x - \omega_i) = \frac{1}{a}\left(1 - e^{a|x-\omega_i|}\right) \tag{C.2}$$

with a strictly positive curvature parameter $a > 0$. This implies that utility is concave and decreasing in the distance between policy x and ideal point ω_i. The limit case $a \to 0$ coincides with the linear preferences studied in the main text.

In the following, I first show that the agent's implied preferences on the platform of party L satisfy a single-crossing property. Second, I derive the equilibrium outcome at the candidate selection stage in party L given any member set Ω_L and any belief $\hat{r}$ about the platform of the competing party R. Based on these preliminary results, I then show that the platform distance in robust equilibrium outcomes is bounded both from below and from above. For the last result, I focus on symmetric outcomes such that $\ell = -r$.

Lemma C.1. *Let the policy preferences be given by* (C.2)*. Given any $a > 0$ and any platform belief $\hat{r}$, the platform preferences of party L's members satisfy a version of the single-crossing property by Gans & Smart (1996).*

Proof. Consider two potential platforms $\ell_1 < r$ and $\ell_2 \in (\ell_1, r)$, so that $0 < p(\ell_1, r) < p(\ell_2, r) < 1$. Agent i with ideal point ω_i prefers ℓ_1 to ℓ_2 if $F(\ell_1, \ell_2, r, \omega_i) > 0$, where F is defined by (A.1). As in the benchmark case, $F(\ell_1, \ell_2, r, \ell_2) < 0$. The derivative of F with respect to ω_i is given by

$$\frac{dF(\ell_1, \ell_2, r, \omega_i)}{d\omega_i} = \begin{cases} -aF(\ell_1, \ell_2, r, \omega_i) & \text{for} \quad \omega_i < \ell_1 \ , \\ -aF(\ell_1, \ell_2, r, \omega_i) - 2p(\ell_1, r) & \text{for} \quad \omega_i \in (\ell_1, \ell_2) \ , \\ aF(\ell_1, \ell_2, r, \omega_i) + 2e^{a(r-\omega_i)}\left[p(\ell_2, r) - p(\ell_1, r)\right] & \text{for} \quad \omega_i \in (\ell_2, r) \ , \\ aF(\ell_1, \ell_2, r, \omega_i) & \text{for} \quad \omega_i > r \ . \end{cases}$$

There are three possible cases. First, $F(\ell_1, \ell_2, r, \ell_1) \geq 0$ implies that

$$F(\ell_1, \ell_2, r, r) = -F(\ell_1, \ell_2, r, \ell_1) + p(\ell_2, r)\left[v(r - \ell_1) + v(0) - v(r - \ell_2) - v(\ell_2 - \ell_1)\right] < 0 \ ,$$

where the term in brackets is strictly negative for any $a > 0$ by Karamata's inequality. In this case, there is a unique threshold $\omega' \in (\ell_1, \ell_2)$ such that $F(\ell_1, \ell_2, r, \omega)$ is strictly positive for all $\omega < \omega'$ and strictly negative for all $\omega > \omega'$. Second, $F(\ell_1, \ell_2, r, r) \geq 0$ implies that $F(\ell_1, \ell_2, r, \ell_1) < 0$. In this case, there is a unique $\omega' \in (\ell_2, r)$ such that $F(\ell_1, \ell_2, r, \omega)$ is strictly negative for all $\omega < \omega'$ and strictly positive for all $\omega > \omega'$. Finally, it is possible that $F(\ell_1, \ell_2, r, \omega)$ is weakly negative for all $\omega \in \mathbb{R}$. In all three cases, a version of the single-crossing property holds. □

This implies that, for any member set Ω_L with an odd number of elements and any platform belief $\hat{r}$, the preferred platform of the party median is a Condorcet winner in the primary elections of party L. In particular, a member with ideal point ω_i prefers the platform that maximizes $E[v(x - \omega_i) \mid \ell, r] = e^{-a\omega_i}\ \Gamma^a(\ell, r) + v(r - \omega_i)$ over the set of available platforms in $[\omega_i, r]$, where

$$\Gamma^a(\ell, r) = \frac{p(\ell, r)}{a}\left[e^{ar} - e^{a\ell}\right]$$

generalizes the policy effect function $\Gamma(\ell, r)$ for non-linear preferences.

Lemma C.2. *Let the policy preferences be given by* (C.2) *with* $a > 0$*, let* $\hat{r} > m_L$ *and* $\#\Omega_L$ *be odd. Function* $\Gamma^a(\ell, r, m_L)$ *is strictly quasi-concave in* ℓ *for* $\ell \in [m_L, r)$*. The platform of party L is given by the maximum of* m_L *and* $\gamma_a(\hat{r}, \Omega_L) \coloneqq \arg\max_{\ell \in \Omega_L} \Gamma^a(\ell, r)$*.*

Proof. The derivative of $\Gamma^a(\ell, r)$ in ℓ is given by

$$\Gamma^a_\ell(\ell, r) = \frac{1}{2a}\,\phi\left(\frac{\ell + r}{2}\right)\,\left[e^{ar} - e^{a\ell}\right] - \Phi\left(\frac{\ell + r}{2}\right)\,e^{a\ell}\,.$$

It equals zero if

$$2\frac{\Phi\left(\frac{\ell+r}{2}\right)}{\phi\left(\frac{\ell+r}{2}\right)} = \frac{1}{a}\left[e^{a(r-\ell)} - 1\right]\,, \tag{C.3}$$

where the left-hand side is increasing in ℓ by the log-concavity of Φ, and the right-hand side is decreasing in ℓ for any $a > 0$. Hence, function Γ^a is strictly quasi-concave in ℓ. Denote by $\gamma_a(\hat{r}, \Omega_L)$ the platform that maximizes $\Gamma^a(\ell, r)$ over $\ell \in \Omega_L$. As in the linear case, an agent prefers his own ideal point ω_i to all lower platforms below ω_i or above r. With an odd number of party members, the preferred platform of the party median prevails in any pairwise vote by Lemma C.2. Hence, the platform is given by m_L if $m_L \geq \gamma_a(\hat{r}, \Omega_L)$, and by $\gamma_a(\hat{r}, \Omega_L)$ otherwise. □

Lemma C.3. *Let the policy preferences be given by* (C.2) *with* $a > 0$*. In every robust political equilibrium with two active parties and symmetric platforms, both parties are efficient.*

Proof. The proof follows the same steps as the proof of Lemma 3. I now provide a sketch of these steps, more details are available on request. Assume that there is a symmetric equilibrium with $\ell = -r$ in which party L is inefficient such that $\sum_{i \in \mathcal{A}} \alpha_i^L \geq C + c$.

First, in such an equilibrium, each member of L must contribute exactly c. Otherwise, she could reduce her contribution without any policy loss. Second, the party platform ℓ must equal the median member's ideal point m_L. Otherwise, a member with $\omega_i < m_L$ could leave party L without incuring a policy loss. Third, exit-robustness requires that party members with ideal points below m_L cannot profitably leave party L. Formally, we must have $F(m_L, m_{L>}, r, m_{L<}) \geq 2c$, where $m_{L>}$ and $m_{L<}$ are the party members with ideal points closest above and below, respectively, the party median m_L. Fourth, entry-robustness requires that no independent agent with ideal point $\omega_j \geq m_{L>}$ can profitably join party L. Formally, this implies that $-F(m_L, m_{L>}, r, \omega_j) \leq 2c$ must be satisfied. For an agent with $\omega_j = r + m_L - m_{L-}$, however, Karamata's inequality implies that $-F(m_L, m_{L>}, r, \omega_j) > F(m_L, m_{L>}, r, m_{L<})$. Hence, if the activist population Ω contains an agent with this ideal point and party L is inefficient, the party structure cannot be exit-robust and entry-robust at the same time. As a result, there is no robust political equilibrium with an inefficient party. □

Proposition C.2. *Let the policy preferences be given by* (C.2) *with some fixed parameter* $a > 0$. *A pair of platforms* $(-\Delta/2, \Delta/2)$ *is a robust equilibrium outcome if* Δ *is between*

$$\underline{\Delta}_a := \ln[2ac + 1]/a$$

and a threshold

$$\tilde{\Delta}_a \leq \bar{\Delta}_a := \frac{1}{a} \ln \left[\frac{e^{a x_a}\ \Phi(x_a/2) - 1/2}{\Phi(x_a/2) - 1/2} \right]$$

with $x_a := \ln \left[ac + \sqrt{a^2 c^2 + 1} \right] /a$. *It is no robust equilibrium outcome if* $\Delta > \bar{\Delta}_a$.

Proof. The proof follows the same steps as the one for Proposition 1 in the main text. Consider a symmetric equilibrium with $\ell = -\Delta/2$ and $r = \Delta/2$ for some $\Delta > 0$. The platform distance is given by Δ, and the winning probability of each party is $\Phi(0) = 1/2$.

Lower bound on platform distance. Consider a pair of platforms $\ell = -\Delta/2 < 0$ and $r = \Delta/2 > 0$. The presidential candidate of party L contributes $\alpha_i^L \geq c$. Lemma C.3 implies that, if she reduces her contribution to zero, party L becomes inactive and policy $r = \Delta/2$ is implemented for sure. The agent's utility changes by

$$\alpha_i^L - e^{a\Delta/2}\ \Gamma^a(-\Delta/2, \Delta/2) \geq c - \frac{e^{a\Delta} - 1}{2a} \ .$$

Hence, the deviation is strictly profitable if $\Delta < \underline{\Delta}^a = \ln[2ac + 1]/a$.

Upper bound on platform distance. A symmetric pair of platforms cannot be a robust equilibrium outcome if there is an independent agent i with an ideal point $\omega_i \in (-\Delta/2, \Delta/2)$ such that (i) $\Gamma^a(\omega_i, \Delta/2) > \Gamma^a(-\Delta/2, \Delta/2) = (e^{a\Delta/2} - e^{-a\Delta/2})/(2a)$, and (ii) $F(\omega_i, -\Delta/2, \Delta/2, \omega_i) > c$. Condition (i) ensures that, if i deviates by joining party L, she becomes presidential candidate. Condition (ii) implies that i profits from this deviation. The policy gain in condition (ii) equals

$$F(\omega_i, -\Delta/2, \Delta/2, \omega_i) \quad = \quad e^{-a\omega_i}\ \Gamma^a(\omega_i, \Delta/2) + \frac{1}{2a} e^{a\Delta/2} \left(e^{a\omega_i} - e^{-a\omega_i} \right) \ .$$

Consider an agent with ideal point $\hat{\omega}_a(\Delta) = x_a - \Delta/2$, where $x_a = \ln[ac + \sqrt{a^2c^2 + 1}]/a$. For this agent, both conditions (i) and (ii) are satisfied if and only if Δ exceeds $\bar{\Delta}_a = \frac{1}{a} \ln \left[\frac{e^{a x_a}\ \Phi(x_a/2) - 1/2}{\Phi(x_a/2) - 1/2} \right]$. This implies that the pair $(-\Delta/2, \Delta/2)$ is no robust equilibrium outcome if $\Delta > \bar{\Delta}_a$.

Existence of robust equilibrium outcomes. In the final step, I show that there exists a threshold $\tilde{\Delta}_a \in \left(\underline{\Delta}_a, \bar{\Delta}_a \right]$ such that the pair of platforms $(-\Delta/2, \Delta/2)$ is a robust equilibrium outcome if $\Delta \in \left[\underline{\Delta}_a, \bar{\Delta}_a \right]$. For this purpose, assume that party L is efficient and has three or more members, each of whom contributes exactly c and has ideal point $\ell = -\underline{\Delta}_a/2 = -\ln[2ac + 1]/(2a)$. Correspondingly, party R is efficient with the same number of members, each of whom contributes c and has ideal point $r = \underline{\Delta}_a/2$. Then, as shown above, no member can profitably leave her party. Moreover, if an independent agent with ideal point below ℓ or above r joins a party, she cannot affect the party's platform. Finally, assume that an moderate independent agent with any ideal point $\omega_i \in (\ell, r)$ joins party L and becomes the party's candidate. Then, her net utility change is negative because

$$\begin{aligned} F(\omega_i, \ell, r, \omega_i) &= \frac{p(\omega_i, r)}{a}\left[e^{a(\Delta/2-\omega_i)} - 1\right] + \frac{1}{2a}\left[e^{a(\Delta/2+\omega_i)} - e^{a(\Delta/2-\omega_i)}\right] \\ &< \frac{1}{2a}\left[e^{a(\Delta/2-\omega_i)} + e^{a(\Delta/2+\omega_i)} - 1 - (2ac+1)\right] < c \\ \Leftrightarrow e^0 + a^{a\Delta} &> e^{a(\Delta/2-\omega_i)} + e^{a(\Delta/2+\omega_i)} , \end{aligned}$$

where the inequality in the last line is true by Karamata's inequality. By continuity, there is a threshold $\tilde{\Delta}_a > \underline{\Delta}_a$ such that, if $\Delta \in [\underline{\Delta}_a, \tilde{\Delta}_a]$, joining a party is not profitable for an agent with any ideal point in $(-\Delta/2, \Delta/2)$, while if Δ is slightly above $\tilde{\Delta}_a$, joining a party is profitable for some agent. As shown above, $\tilde{\Delta}_a$ must be weakly below $\bar{\Delta}_a$. □

C.3 Asymmetric preferences

In the basic model, I have assumed that the agent's policy preferences are symmetric so that agent i is indifferent between any pair of policies (x_1, x_2) that are equally distant from her ideal point ω_i, $x_2 - \omega_i = \omega_i - x_1 > 0$. In the following, I show that my results do not change qualitatively if policy preferences are asymmetric. Specifically, I assume that, for a leftist agent with ideal point ω_i below the expected median of 0, the policy payoff is given by

$$v_i(x - \omega_i) = \begin{cases} -(\omega_i - x) & \text{for} \quad x \le \omega_i \ , \\ -b(x - \omega_i) & \text{for} \quad x > \omega_i \ . \end{cases} \tag{C.4}$$

For a rightist agent with ideal point $\omega_i > 0$, by contrast, the policy payoff is given by the form

$$v_i(x - \omega_i) = \begin{cases} -b(\omega_i - x) & \text{for} \quad x \le \omega_i \ , \\ -(x - \omega_i) & \text{for} \quad x > \omega_i \ . \end{cases} \tag{C.5}$$

The basic model with symmetric preferences is nested with $b = 1$. For the case where parameter b is above 1, leftist agents are better off with a policy x_1 below their ideal point ω_i than with an equally distant policy $x_2 = 2\omega_i - x_1$. Intuitively, this implies that they are more sensitive to rightward deviations than to leftward deviations from their ideal point. The opposite is true for rightist agents.[30]

Lemma C.4. *Let the policy preferences be given by* (C.4) *and* (C.5) *with* $b > 0$. *Given any platform belief* $\hat{r} \in \mathbb{R}$, *the platform preferences of party* L*'s members satisfy a version*

[30] For completeness, I assume that an agent with ideal point $\omega_i = 0$ has the same (symmetric) policy preferences as in the basic model.

of the single-crossing property by Gans & Smart (1996).

Proof. Consider two alternative platforms ℓ_1 and ℓ_2 such that $\ell_1 < \ell_2 < r$, and $0 < p(\ell_1, r) < p(\ell_2, r) < 1$. Agent i with ideal point ω_i prefers ℓ_1 to ℓ_2 if and only if $F(\ell_1, \ell_2, r, \omega_i) > 0$, where F is defined by (A.1). First, if an agent with ideal point $\omega_i \leq \ell_1$ strictly prefers one platform, then each agent with ideal point $\omega_i \geq r$ strictly prefers the other platform because $F(\ell_1, \ell_2, r, r) = -F(\ell_1, \ell_2, r, \ell_1)/b$. Second, the derivative of F with respect to ideal point ω_i is given by

$$\frac{dF(\ell_1, \ell_2, r, \omega_i)}{d\omega_i} = \begin{cases} 0 & \text{for} \quad \omega_i < \ell_1 \ , \\ -(1+b)p(\ell_1, r) < 0 & \text{for} \quad \omega_i \in (\ell_1, \ell_2) \ , \\ (1+b)\left[p(\ell_2, r) - p(\ell_1, r)\right] > 0 & \text{for} \quad \omega_i \in (\ell_2, r) \ , \\ 0 & \text{for} \quad \omega_i > r \ . \end{cases}$$

These properties jointly imply that, if $F(\ell_1, \ell_2, r, \ell_1) > 0$, there is a unique root $\omega' \in (\ell_1, \ell_2)$ such that agent i prefers ℓ_1 if and only if her ideal point satisfies $\omega_i < \omega'$. If instead $F(\ell_1, \ell_2, r, r) > 0$, there is a unique $\hat{\omega}' \in (\ell_2, r)$ such that agent i prefers ℓ_1 if and only if $\omega_i > \omega'$. Finally, it is possible that $F(\ell_1, \ell_2, r, \omega) = 0$ for all $\omega \leq \ell_1$ and all $\omega_i \geq r$. In this case, all agent with ideal points in (ℓ_1, r) strictly prefer platform ℓ_2, while the other agents are indifferent. In all three cases, a version of the single-crossing property holds. □

This implies that, for any member set Ω_L and platform belief $\hat{r}$, if there is a unique party median, her preferred platform is a Condorcet winner in the primary election of party L. In particular, any member with ideal point $\omega_i < \hat{r}$ prefers the platform that maximizes $b\Gamma(\ell, r)$ over the set of available platforms in $[\omega_i, r)$. As a result, Lemma 2 continues to hold: For any $\hat{r} > m_L$ and $\#\Omega_L$ odd, the chosen platform ℓ is the maximum of m_L and $\gamma(\hat{r}, \Omega_L)$. Similarly, Lemma 3 extends to asymmetric parties: Both parties are efficient in every robust equilibrium with symmetric platforms $\ell = -\Delta/2 \leq 0$ and $r = \Delta/2 \geq 0$. Based on these intermediate results, I can now identify the set of symmetric platforms that represent robust equilibrium outcomes.

Proposition C.3. *Let the policy preferences be given by* (C.4) *and* (C.5) *with* $b > 0$. *A pair of platforms* $\ell = -\Delta/2$ *and* $r = \Delta/2$ *is a robust equilibrium outcome if the platform distance* $r - \ell = \Delta$ *is between*

$$\underline{\Delta}_b \coloneqq \frac{2c}{b}$$

and a threshold

$$\tilde{\Delta}_b \leq \bar{\Delta}_b \coloneqq \tilde{c}\,\frac{\Phi(\tilde{c}/2)}{\Phi(\tilde{c}/2) - 1/2}$$

with $\tilde{c} = 2c/(1+b)$. *It is no robust equilibrium outcome if* $\Delta > \bar{\Delta}_b$. *If* $b \in (0,1]$, *the pair* $(-\Delta/2, \Delta/2)$ *is a robust equilibrium outcome if and only if* $\Delta \in \left[\underline{\Delta}_b, \bar{\Delta}_b\right]$.

Proof. The proof follows the same steps as the one for Proposition 1 in the main text.

Consider a pair of platforms $\ell = -\Delta/2$ and $r = \Delta/2$ for some platform distance $\Delta > 0$. The winning probability of each party is given by 1/2.

Lower bound on platform distance. Assume that a member of party L with ideal point $\omega_i \leq \ell$ reduces her contribution to L from $\alpha_i^L \geq c$ to zero. Then, L becomes inactive and policy r is implemented for sure. For agent i, this deviation yields a policy loss of $b\Gamma(\ell, r) = b\Delta/2$. It is profitable if this policy loss is below c, i.e., if Δ is below $\underline{\Delta}_b = 2c/b$.

Upper bound on platform distance. A symmetric pair of platforms cannot be an equilibrium outcome if there is an independent agent i with an ideal point $\omega_i \in (-\Delta/2, \Delta/2)$ that satisfies the conditions (i) $b\Gamma(\omega_i, \Delta/2) > b\Gamma(-\Delta/2, \Delta/2) = b\Delta/2$ and (ii) $F(\omega_i, -\Delta/2, \Delta/2, \omega_i) = b\,\Gamma(\omega_i, \Delta/2) + \Delta(1-b)/4 + \omega_i(1+b)/2 > c$. If both conditions hold, then agent i can profitably join party L and become its presidential candidate.

Consider an agent with ideal point $\hat{\omega}_b(\Delta) = \Delta[\Phi(\tilde{c}/2) - 1]/[2\,\Phi(\tilde{c}/2)] \in (-\Delta/2, 0)$, where $\tilde{c} = 2c/(1+b)$. For this agent, both conditions (i) and (ii) are satisfied if and only if Δ exceeds $\bar{\Delta}_b = \tilde{c}\,\Phi(\tilde{c}/2)/[\Phi(\tilde{c}/2) - 1/2]$. Hence, the platforms $[-\Delta/2, \Delta/2]$ are no robust equilibrium outcome for any $\Delta > \bar{\Delta}_b$.

Existence of robust equilibrium outcomes. In the last step, I show that there exists a threshold $\tilde{\Delta}_b \in (\underline{\Delta}_b, \bar{\Delta}_b]$ such that a pair $(-\Delta/2, \Delta/2)$ is a robust equilibrium outcome for any $\Delta \in [\underline{\Delta}_b, \tilde{\Delta}_b]$. For this purpose, assume that each party has three or more members, each of whom contribute exactly c. Moreover, each member of party L has ideal point $\ell = -\Delta/2$, and each member of party R has ideal point $r = \Delta/2$. For $\Delta \geq \underline{\Delta}_b$, both parties are exit-stable; no member can profitably leave her party. Moreover, for an independent agent with ideal point below ℓ or above r, joining a party is not profitable as she cannot affect the party platforms.

Finally, consider an independent agent with ideal point $\omega_i \in (\ell, r)$. I have to distinguish two cases. First, if $b \in (0,1)$, $F(\omega_i, -\Delta/2, \Delta/2, \omega_i)$ is strictly increasing in ω_i for all $\omega_i < 0$. For $\Delta < \bar{\Delta}_b$, both conditions (i) and (ii) are not satisfied for $\omega_i = \hat{\omega}_a(\Delta) < 0$. Hence, condition (ii) can neither be satisfied for any $\omega_i \leq \hat{\omega}_a(\Delta)$. For any $\omega_i \geq \hat{\omega}_a(\Delta)$, on the other hand, condition (i) is not satisfied by the quasi-concavity of $\Gamma(\omega_i, \Delta/2)$ in ω_i. Hence, any pair $(-\Delta/2, \Delta/2)$ with $\Delta < \tilde{\Delta}_b = \bar{\Delta}_b$ is entry-robust.

Second, if $b > 1$, $F(\omega_i, -\Delta/2, \Delta/2, \omega_i)$ may be non-monotonic in ω_i. Assume that $\Delta = \underline{\Delta}_b$ so that $\ell = -r = -c/b$. For an agent with any ideal point $\omega_i \geq 0$, on the one hand, condition (i) cannot be satisfied. For an agent with ideal point $\omega_i \in (-c/b, 0)$, on the other hand, condition (ii) cannot be satisfied because

$$
\begin{aligned}
F(\omega_i, -\underline{\Delta}_b/2, \underline{\Delta}_b/2, \omega_i) - c &= bp\left(\omega_i, \frac{c}{b}\right)\left(\frac{c}{b} - \omega_i\right) + \frac{1-b}{2}\,\frac{c}{b} + \frac{1+b}{2}\,\omega_i \\
&< c + \frac{1-b}{2}\left(\frac{c}{b} + \omega_i\right) < c\,.
\end{aligned}
$$

Hence, party L is entry-stable, and the pair $(-\underline{\Delta}_b/2, \underline{\Delta}_b/2)$ is a robust equilibrium outcome. By continuity, there is a threshold $\tilde{\Delta}_b > \underline{\Delta}_b$ such that $(-\Delta/2, \Delta/2)$ is also a robust equilibrium outcome for any $\Delta \in [\underline{\Delta}_b, \tilde{\Delta}_b]$, but not for Δ slightly above $\tilde{\Delta}_b$. As shown above, $\tilde{\Delta}_b$ is weakly below $\bar{\Delta}_b$. □

C.4 No exogenous costs of running

In the basic model, I assume that the campaign contributions a party collects have no effect on their winning probability, once they exceed the cost of running C. I now consider a version of the model in which each party can enter the general election whenever it has a member and a presidential candidate. Hence, there is no exogenous cost of running. Instead, I assume that, if both parties L and R compete in the general election, the winning probability of party L is increasing in their campaign expenses $C_L = \sum_{i \in \mathcal{A}} \alpha_i^L$ and decreasing in the expenses $C_R = \sum_{i \in \mathcal{A}} \alpha_i^R$ of party R. Specifically, I solve an extended model under the assumption that the winning probability of party L equals

$$\tilde{p}(\ell, r, C_L, C_R) = \tilde{\Phi}\left(\frac{\ell + r}{2} + \beta\, \frac{C_L - C_R}{C_L + C_R}\right) , \tag{C.6}$$

where $\tilde{\Phi}$ is a distribution function satisfying Assumptions 1 and 2. The fraction $(C_L - C_R)/(C_L + C_R)$ captures party L's relative campaign expenses, and β is a measure of how sensitive the electoral prospects are with respect to campaign expenses. Equation (C.6) can be micro-founded by assuming, e.g., that only a share $s \in (0, 1)$ of voters behave strategically based on policy preferences as specified in (1). The remaining share $1 - s$ of voters is *impressionable*: They cast their votes based on the relative campaign expenses $(C_L - C_R)/(C_L + C_R)$, an idiosyncratic party preference ν_i, and a common preference shock μ with distribution function $\tilde{\Phi}$, in the spirit of the probabilistic voting model by Lindbeck & Weibull (1987). To ensure the existence of political equilibria with two competing parties, I impose the following assumption on the set of parameters.

Assumption 4. *The exogenous parameters c and β and the distribution $\tilde{\Phi}$ satisfy the condition*

$$c\, \frac{\tilde{\phi}(\beta/3)}{\tilde{\Phi}(\beta/3)} < \beta\, \tilde{\phi}(0) < 1 \, .$$

Otherwise, I maintain the assumptions of the basic model: All activists are policy-oriented with linear policy preferences; agent i enters party $J \in \{L, R\}$ if $\alpha_i^J \geq c$ with some $c > 0$; the members of each party nominate a presidential candidate from their ranks; and the winning candidate in the general election implements her ideal policy.

At the candidate selection stage, all insights from the basic model remain valid. Conditional on any level of the relative campaign expenses $(C_L - C_R)/(C_L + C_R)$, the agent's implied policy preferences satisfy a single-crossing condition. Each member of party L with ideal point $\omega_i < \hat{r}$ prefers the platform ℓ that maximizes $\tilde{\Gamma}(\ell, r, C_L, C_R) =$

$\tilde{p}(\ell, r, C_L, C_R)(r-\ell)$ over the elements in Ω_L that are located in $[\omega_i, r)$. Hence, the equilibrium platform of L equals the maximum of the party median m_L and $\tilde{\gamma}(r, C_L, C_R, \Omega_L) = \arg\max_{\ell\in\Omega_L} \tilde{\Gamma}(\ell, r, C_L, C_R)$.

Henceforth, I focus on symmetric platform pairs with $\ell = -\Delta/2$ and $r = \Delta/2$. By the following proposition, the platform distance in these equilibria is bounded from above and from below as well, as in the basic model. A crucial difference is that, in the equilibria of this extended model, the parties are not efficient in the sense that any party member is pivotal for the activity of party R. However, any party member i can increase the winning probability of her party J by raising the contribution α_i^J. In an equilibrium, the sum of campaign contributions a party collects satisfies the first-order condition

$$\frac{d\tilde{p}(\ell, r, C_L, C_R)}{dC_L}(r-\ell) - 1 = \beta\ \Delta\ \tilde{\phi}(0)\ \frac{2C_R}{(C_L+C_R)^2} - 1 \leq 0 \tag{C.7}$$

for party L, and a corresponding first-order condition for party R.[31] In the following, I focus on equilibria in which condition (C.7) is satisfied with a strictly equality and both parties collect identical contributions, $C_L = C_R$.

Proposition C.4. *Assume that the winning probability is given by* (C.6) *and that Assumption 4 holds. Let $C_L = C_R$ and condition C.7 be satisfied with equality. The pair of platforms $\ell = -\Delta/2 \leq 0$ and $r = \Delta/2$ is a robust equilibrium outcome if Δ is between*

$$\tilde{\Delta}_{low}(\beta) := \frac{2c}{\beta\tilde{\phi}(0)} > 2c\ ,$$

and a threshold $\tilde{\Delta} \in \left(\tilde{\Delta}_{low}(\beta), \tilde{\Delta}_{up}(\beta)\right)$, with

$$\tilde{\Delta}_{up}(\beta) := c\ \frac{\tilde{\Phi}\,(c/2+\beta/3)}{\tilde{\Phi}\,(c/2+\beta/3) - \tilde{\Phi}\,(\beta/3)}\ .$$

It is no robust equilibrium outcome if $\Delta \geq \Delta_{up}(\beta)$.

Proof. Again, the proof follows the same steps as the one for Proposition 1. Fix some $\beta > 0$ and consider a symmetric pair of platforms $\ell = -\Delta/2$ and $r = \Delta/2$ with some platform distance $\Delta \geq 0$.

Lower bound on platform distance. If $C_L = C_R$ and condition (C.7) is satisfied with equality, then this condition requires that $C_R = \beta\tilde{\phi}(0)\Delta/2$. Both parties can only run if they have at least one member, i.e., if $C_R \geq c$. This implies that Δ must be weakly larger than $\tilde{\Delta}_{low}(\beta) = \frac{2c}{\beta\,\tilde{\phi}(0)}$. Under Assumption 4, $\beta\phi(0)$ is below 1, ensuring that $\tilde{\Delta}_{low}(\beta) > 2c$.

[31] Condition (C.7) can be satisfied with a strict inequality if (i) each member i of L has an ideal point $\omega_i \leq \ell$ and contributes exactly $\alpha_i^L = c$, and (ii) none other activists makes any contribution to L. In all other cases, condition (C.7) is satisfied with a strict equality.

Upper bound on platform distance. Consider an allocation such that $\ell = -\Delta/2$, $r = \Delta/2$, and $C_L = C_R = \beta\tilde{\phi}(0)\Delta/2$, see above. The relative contribution $(C_L - C_R)/(C_L + C_R)$ equals 0, and each party has a winning probability of $\tilde{\Phi}(0) = 1/2$. Assume now that an independent agent i deviates by contributing $\alpha_i^L = c$ and joining party L. This deviation raises the relative contribution $(C_L - C_R)/(C_L + C_R)$ to $c/[\beta\tilde{\phi}(0)\Delta + c] > 0$ and the winning probability of party L to $\tilde{\Phi}(\tilde{\beta}) > 1/2$, where I write $\tilde{\beta} = \beta c/[\beta\tilde{\phi}(0)\Delta + c]$ for a concise notation.

The pair $(-\Delta/2, \Delta/2)$ cannot be an equilibrium outcome if there is an agent $i \in \mathcal{A}$ with ideal point ω_i such that (i) $\tilde{\Gamma}(\omega_i, \Delta/2, C_L + c, C_R) = \tilde{\Phi}\left(\frac{\omega_i + \Delta/2}{2} + \tilde{\beta}\right)(\Delta/2 - \omega_i) > \Delta\,\tilde{\Phi}(\tilde{\beta})$ and (ii) $\Delta\,\tilde{\Phi}(\tilde{\beta}) + \omega_i > c$. If both conditions are satisfied, then agent i can profitably join party L and become its presidential candidate.

For $\omega_i = \tilde{\omega}(\Delta, \tilde{\beta}) := \Delta/2\left[1 - 2\frac{\tilde{\Phi}(\tilde{\beta})}{\tilde{\Phi}(c/2+\tilde{\beta})}\right]$, we have

$$\tilde{\Phi}\left(\frac{\omega_i + \Delta/2}{2} + \tilde{\beta}\right)\left(\frac{\Delta}{2} - \omega_i\right) = \tilde{\Phi}\left(\frac{\Delta}{2}\left[1 - \frac{\tilde{\Phi}(\tilde{\beta})}{\tilde{\Phi}(c/2 + \tilde{\beta})}\right] + \tilde{\beta}\right)\Delta\frac{\tilde{\Phi}(\tilde{\beta})}{\tilde{\Phi}(c/2 + \tilde{\beta})},$$

so that condition (i) is satisfied if and only if

$$\Delta > c\,\frac{\tilde{\Phi}(c/2 + \tilde{\beta})}{\tilde{\Phi}(c/2 + \tilde{\beta}) - \tilde{\Phi}(\tilde{\beta})}. \tag{C.8}$$

If (C.8) holds, condition (ii) is satisfied as well because

$$\Delta\,\tilde{\Phi}(\tilde{\beta}) + \tilde{\omega}(\Delta, \tilde{\beta}) = \Delta\left[\tilde{\Phi}(\tilde{\beta}) + \frac{1}{2} - \frac{\tilde{\Phi}(\tilde{\beta})}{\tilde{\Phi}(c/2 + \tilde{\beta})}\right] > c\,\frac{\tilde{\Phi}(\tilde{\beta}) + \frac{1}{2} - \frac{\tilde{\Phi}(\tilde{\beta})}{\tilde{\Phi}(c/2+\tilde{\beta})}}{1 - \frac{\tilde{\Phi}(\tilde{\beta})}{\tilde{\Phi}(c/2+\tilde{\beta})}},$$

which is strictly larger than c because $\tilde{\Phi}(\tilde{\beta}) > 1/2$ for any $\beta > 0$.

To complete this step, recall that $\tilde{\beta} = \beta c/[\beta\tilde{\phi}(0)\Delta + c]$ is an endogenous object. However, for any $\Delta \geq \tilde{\Delta}_{low}(\beta)$, we have $\beta\tilde{\phi}(0)\Delta \geq 2c$ and, hence, $\tilde{\beta} \leq \beta/3$. By the log-concavity of $\tilde{\Phi}$, the right-hand side of (C.8) is strictly increasing in $\tilde{\beta}$. Thus, if the platform distance Δ is equal to or larger than the bound $\tilde{\Delta}_{up}(\beta) = c\,\tilde{\Phi}(c/2 + \beta/3)\,[\tilde{\Phi}(c/2 + \beta/3) - \tilde{\Phi}(\beta/3)]^{-1}$, there unambiguously exists an $\omega_i \in (-\Delta/2, \Delta/2)$ such that both conditions (i) and (ii) are satisfied. Put differently, the pair $(-\Delta/2, \Delta/2)$ is no robust equilibrium outcome if Δ is weakly larger than the bound $\tilde{\Delta}_{up}(\beta)$, which is expressed in exogenous variables only.

Existence of robust equilibrium outcomes. Consider a party structure such that all members of party L have ideal point $\ell = -\Delta/2$, and all members of party R have ideal point $r = \Delta/2$ with $\Delta \geq \tilde{\Delta}_{low}(\beta)$. Assume that the first-order condition (C.7) holds with equality for both parties. Hence, no member can profitably leave her party, and no agent can profitably change her party contribution.

I now show that there is a threshold $\tilde{\Delta} > \tilde{\Delta}_{low}(\beta)$ such that both parties are also entry-stable if $\Delta \in [\tilde{\Delta}_{low}(\beta), \tilde{\Delta}]$. By an adaption of the arguments in Lemma A.2, $\tilde{\Gamma}(\omega_i, \Delta/2, C_L+c, C_R)$ is strictly quasi-concave and has a unique maximizer $\tilde{\gamma}(\Delta/2) < \Delta/2$ in its first argument, where I suppress the dependence of $\tilde{\gamma}$ on C_L, C_R and c. Moreover, there is a unique $\Delta' > 0$ such that $\tilde{\gamma}(\Delta'/2) = -\Delta'/2$ or, equivalently,

$$\Delta' = 2\frac{\tilde{\Phi}\left(\frac{\beta c}{\beta\phi(0)\Delta'+c}\right)}{\tilde{\phi}\left(\frac{\beta c}{\beta\phi(0)\Delta'+c}\right)} .$$

For all $\Delta \in [0, \Delta')$, $\tilde{\gamma}(\Delta/2) < -\Delta/2$ and $\tilde{\Gamma}_\ell(-\Delta/2, \Delta/2, C_L, C_R) < 0$. This implies that, for $\Delta \in [0, \Delta')$, there is no $\omega_i \in (-\Delta/2, \Delta/2)$ such that condition (i) in the previous step is satisfied. Hence, Δ' is strictly smaller than $\tilde{\Delta}_{up}(\beta)$.

I now show that $\Delta' > \tilde{\Delta}_{low}(\beta)$ under Assumption 4. For $\Delta = \tilde{\Delta}_{low}(\beta)$, we have $C_R = C_L = c$, $\tilde{\beta} = \beta/3$, and

$$\tilde{\Gamma}_\ell(-\Delta/2, \Delta/2, 2c, c) \quad = \quad \frac{\tilde{\Delta}_{low}(\beta)}{2}\,\tilde{\phi}(\beta/3) - \tilde{\Phi}(\beta/3) = \left[\frac{c}{\beta\,\tilde{\phi}(0)} - \frac{\tilde{\Phi}(\beta/3)}{\tilde{\phi}(\beta/3)}\right]\tilde{\phi}(\beta/3) < 0 \,,$$

where the inequality follows from Assumption 4. Thus, Δ' is strictly larger than $\tilde{\Delta}_{low}(\beta)$. As a result, the pair of platforms $(-\Delta/2, \Delta/2)$ is a robust equilibrium outcome for any $\Delta \in [\tilde{\Delta}_{low}(\beta), \Delta']$: Given any such platforms, no independent agent with ideal point $\omega_i \in (\ell, r)$ becomes presidential candidate if she joins a party. □