Supplementary Material for "Pareto-Improving Tax Reforms and the Earned Income Tax Credit"

C Empirical analysis

In this section, we first provide additional background information on the 1974 US tax-and-transfer system. Second, we conduct various robustness checks on the result that there was scope for a Pareto improvement by means of a two bracket tax cut. Third, we refine our analysis by looking separately at single parents who differ in the number of children and discuss the desirability of tagging in this dimension. Finally, we analyze the US tax-and-transfer system for single parents in the post-reform years 1975 to 1978 and analyze the scope for further improvements of the EITC.

C.1 Data description and benchmark calibration

This section provides a description of our data and explains the choices for our benchmark calibration. We start with a description of the 1974 US tax-and-transfer system. Subsequently, we describe how we obtained estimates of the relevant income distributions and our benchmark assumptions on behavioral responses to tax reforms.

Status quo tax function: the US tax and transfer system in 1974. We take account of the federal income tax and the two largest welfare programs, Aid for Families with Dependent Children (AFDC) and Supplementary Nutrition Assistance Programs (SNAP, also called Food Stamps). ADFC was available only for single parents and varied to some extent across US states. We focus on the AFDC rules in California. SNAP was a federal program that was available both for single parents and childless singles, but more generous for single parents. Programs for single parents also depended on the number of children. In our benchmark analysis, we focus on the tax-transfer schedule for single parents with two children.

Table 1: Sources for US tax-transfer system, 1974-1978.

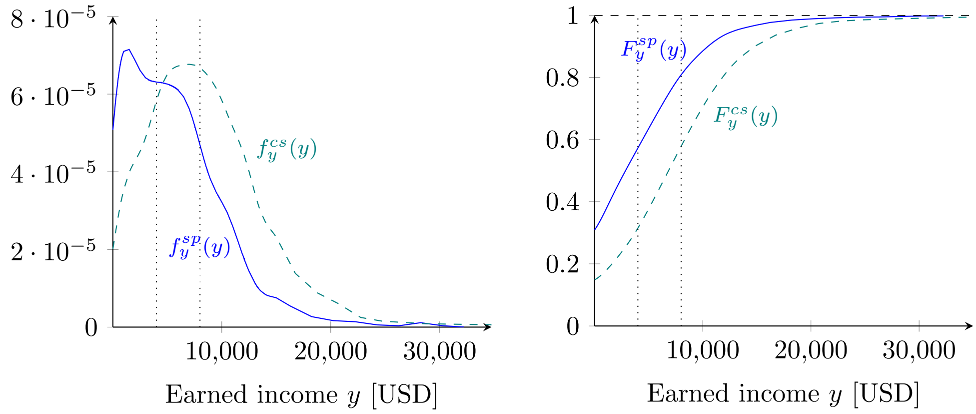
Information	Years	Sources
Income tax	1974-1975	Internal Revenue Service, "Instructions for Form 1040", years 1974, 1975. URLs: https://www.irs.gov/pub/irs_prior/i1040--1974.pdf; www.irs.gov/pub/irs_prior/i1040--1975.pdf.
	1974-1978	Internal Revenue Service, "Statistics of Income, Individual income tax returns", years 1974, 1975, 1978. URLs: https://www.irs.gov/pub/irs-soi/74inar.pdf; https://www.irs.gov/pub/irs-soi/75inar.pdf; www.irs.gov/pub/irs-soi/78inar.pdf.
EITC	1975, 1978	Tax Policy Center, "Earned Income Tax Credit Parameters, 1975-2021". URL: https://www.taxpolicycenter.org/statistics/eitc-parameters.
AFDC	1974-1978	Office of the Assistant Secretary for Planning and Evaluation, "Aid to Families with Dependent Children. The Baseline", 1998. URL: https://aspe.hhs.gov/basic-report/aid-families-dependent-children-baseline
	1974	US Department of Health, Education and Welfare, "Aid to Families with Dependent Children: Standards for Basic Needs, July 1974", 1974. URL: https://hdl.handle.net/2027/mdp.39015088906634.
	1975, 1978	TRIM3 project, "TRIM3 AFCD Rules". Accessible at trim3.urban.org.
SNAP	1974-1975	US Bureau of the Census, "Characteristics of Households Purchasing Food Stamps. Current Population Reports", Series 9-23, No. 61, 1976. URL: https://www2.census.gov/library/publications/1976/demographics/p23-061.pdf.
	1978	Federal Register Vol. 43, No. 95, May 16, 1978. Accessible at https://www.govinfo.gov/app/collection/fr.
SNAP, Payroll tax	1974-1978	Social Security Administration, "Annual Statistical Supplement. Section 2: Program Provisions and SSA Administrative Data", 2010. URL: https://www.ssa.gov/policy/docs/statcomps/supplement/2010/2a1-2a7.html.

In our data, the median number of children in single-parent households was two, and the arithmetic mean was about 2.2. Figure 4 in the main text depicts the effective tax rates for this subgroup of single parents and for childless singles. For the years 1975 and later, we also account for the Earned Income Tax Credit. Table 1 below depicts the sources we use for computing the US tax and transfer systems for single parents and childless singles in the years 1974 to 1978.

Income distribution. We estimate the 1974 income distributions based on Current Population Survey (CPS) data, using the Annual Social and Economics Supplement from the March 1975 wave. We proceed in the same way for the years 1975 and 1978. Specifically, we consider for each year the sample of non-married individuals aged 25 to 60 who do neither co-habit with an unmarried spouse nor with another adult family member. We partition this sample into childless singles and single parents. For the benchmark analysis reported in the main text, we estimate the earned income distribution based on the set of all single parents (i.e., with any number of children). In the subsequent analysis, we also look into the income distributions in the subsets of single parents with one, two, and three children.

In line with the EITC rules, we consider as earned income the sum of (self-reported) wage income and self-employment income. In this sample, 30.9% of single parents and 14.8% of childless singles have zero or negative incomes, while 49.9% of single parents and 43% of childless singles had strictly positive incomes below $8,000$ USD, the eligibility threshold of the EITC. For our benchmark analysis, we estimate the distributions of earned income for both groups using a non-parametric kernel density estimation with a Gaussian kernel. In the benchmark, we use bandwidths of 997 USD for single parents and $1,200$ USD for childless singles, following Silverman's rule. Figure 9 shows the estimated pdf and the cdf of both income distributions.

Behavioral responses to taxation. We draw on a rich literature providing estimates of labor supply responses at the intensive and the extensive margin, see



(a) Probability density functions. (b) Cumulative distribution functions.

Figure 9: Income distributions of single parents and childless singles, US 1974.

Notes: Figure 9 shows the kernel estimates of the US income distributions among single parents (solid blue lines) and childless singles (dashed teal lines) in 1974. Panel (a) depicts the probability density functions; panel (b) depicts the cumulative distribution functions of both income distributions. The dotted vertical lines mark the endpoints of the phase-in range at $4,000$ USD and the phase-out range at $8,000$ USD of the 1975 EITC.

Source: Authors' calculations (see Table 1 for details).

the discussions in Saez, Slemrod and Giertz (2012) or Chetty et al. (2013). Based on a meta-study and focusing on population-wide averages, Chetty et al. (2013) suggest an intensive-margin elasticity of labor supply with respect to the net-of-tax rate of 0.33, and an extensive-margin elasticity with respect to net labor income of 0.25. Bargain, Dolls, Neumann, Peichl and Siegloch (2014) provide similar estimates for a sample of childless singles. For single parents, various studies find larger responses at the extensive margin. Specifically, Bastian (2020) estimates labor supply responses of single mothers to the 1975 EITC introduction, and finds an average participation elasticity of 0.58. Most earlier studies find similar or even larger participation responses by single mothers, e.g., Meyer and Rosenbaum (2001). In contrast, Kleven (2019) recently estimated a participation elasticity close to zero based on EITC reforms in the 1990s. Besides, several studies find that persons with little formal education and low incomes respond more strongly at the extensive margin than persons with higher education and higher incomes – see, e.g., Juhn, Murphy and Topel (1991), Juhn, Murphy and Topel (2002), Meghir and Phillips (2010). There is only limited empirical evidence on the relevance of income effects for labor supply. Recent evidence by Cesarini, Lindqvist, Notowidigdo and Östling (2017) based on Swedish lottery winners suggests (i) a marginal propensity to earn out of unearned income (MPE) of -0.08, (ii) with about two thirds of income effects arising at the intensive margin, one third at the extensive margin, and (iii) with little heterogeneity in income effects along the income distribution. Imbens, Rubin and Sacerdote (2001) report similar estimates with an MPE of -0.11 for the US, see also Holtz-Eakin, Joulfaian and Rosen (1993).

In our benchmark calibration, we assume an average participation elasticity of 0.58 for single parents and 0.25 for childless singles, and an intensive-margin elasticity of 0.33 for both subgroups, following Bastian (2020) and Chetty et al. (2013). Moreover, we assume that participation elasticities are decreasing with income in both groups, according to the function $\pi_0(y) = \pi_a - \pi_b \ (y/\tilde{y})^{1/2}$, where $\tilde{y}$ equals 50,000 USD. Similar assumptions are employed by Jacquet et al. (2013) and Hansen (2021). For single parents, we assume that the participation elasticity falls from 0.67 at very low incomes to 0.4 at incomes above 50,000 USD (i.e., $\pi_a = 0.67$, $\pi_b = 0.27$), giving rise to an average value of 0.58. For childless singles, we assume π to fall from 0.4 to 0.1 (i.e., $\pi_a = 0.4$, $\pi_b = 0.3$), giving rise to an average value around 0.25. In the benchmark calibration, we leave out income effects. In the sensitivity analysis below, we consider a large range of alternative assumptions on labor supply elasticities.

C.2 Sensitivity analysis

In the following, we provide an extensive sensitivity analysis. We start by considering alternative assumptions on the behavioral responses to tax reforms. Then, we repeat our analysis using alternative estimates of the income distributions among single parents. Finally, we consider alternative representations of the US tax-transfer system. We find that our main results are robust to variations in all these dimensions: The 1974 US tax-transfer system for single parents was Pareto-inefficient, and there existed Pareto-improving tax reforms with two brackets similar to the 1975 EITC introduction.

Labor supply elasticities. Figure 10 plots the revenue function $y \mapsto \mathcal{R}_{sp}(y)$ under different assumptions on the extensive-margin elasticity π_0. This aspect deserves particular attention because the optimal tax literature has established the result that an EITC with negative marginal taxes can only be optimal with labor supply responses at the extensive margin, see Saez (2002) and Hansen (2021). Moreover, Kleven (2019) recently challenged the conventional view that the participation responses of single parents are particularly large. In Figure 10, we therefore show the revenue function $y \mapsto \mathcal{R}_{sp}(y)$ for a range of different assumptions on participation elasticities. The blue line is our benchmark case in which the participation elasticity is falling from 0.67 at very low incomes to 0.4 at high incomes, giving an average elasticity of $\bar{\pi} = 0.58$. Additionally, the black line is a case where the elasticity is zero at all income levels; the brown line depicts a case where the elasticity is falling from 0.3 to 0.1 ($\bar{\pi} = 0.23$); the teal line shows a case

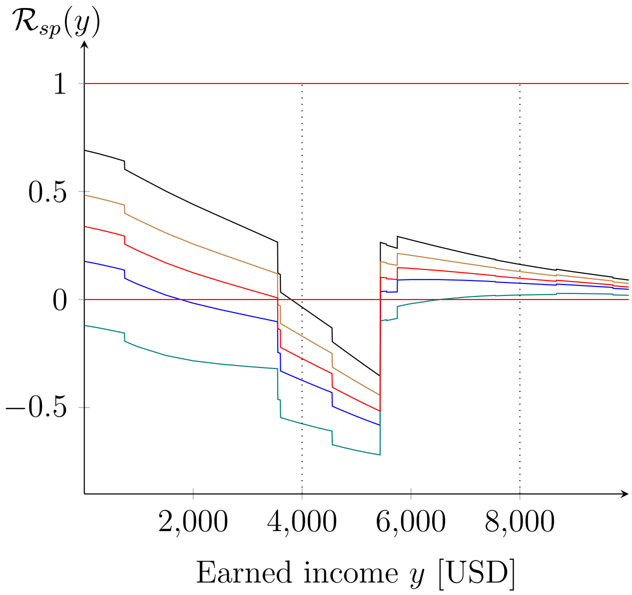


Figure 10: Pareto test of the 1974 US tax-transfer system, different participation elasticities.

Notes: Figure 10 shows the function $y \mapsto \mathcal{R}_{sp}(y)$ for single parents in 1974, assuming different participation elasticities: constant at 0 (black line), falling with income from 0.3 to 0.1 (brown line), falling from 0.67 to 0.4 (blue line), falling from 1 to 0.75 (teal line). The dotted vertical lines mark the endpoints of the phase-in range at $4,000$ USD and the phase-out range at $8,000$ USD of the 1975 EITC.

Source: Authors' calculations (see Table 1 for details).

where it is falling from 1 to 0.75 ($\bar{\pi} = 0.92$). Finally, the red line depicts a case with an elasticity of 0.4 at all income levels. In all these cases, the intensive-margin elasticity is held constant at $\varepsilon = 0.33$, as in our benchmark calibration.

The qualitative result is the same in all cases: for single parents, the 1974 US tax system was not Pareto-efficient. There existed Pareto-improving tax cuts with one and two brackets. This is even true in the limit case of vanishing labor supply responses at the extensive margin. Hence, the finding that an EITC-like two-bracket tax cut for low-income earners was Pareto-improving holds irrespective of what we assume about the strength of behavioral responses to taxation at the extensive margin. Quantitatively, they play a role, though. Higher participation elasticities imply larger revenue gains from a two-bracket tax cut; i.e., higher participation elasticities make an EITC-like reform more attractive (see Figure 10).

Figure 11 plots the revenue function $y \mapsto \mathcal{R}_{sp}(y)$ for different levels of the intensive-margin elasticity ε between 0 and 0.5, while holding the average participation elasticity at the benchmark level of 0.58. We find that the 1974 US tax system for single parents violated the monotonicity condition for Pareto efficiency for any level of the intensive-margin elasticity in this range. Again, this even remains true in the limit case where labor supply does not respond at the intensive margin, i.e., for $\varepsilon = 0$. More specifically, we find the inefficiencies in the 1974 tax-transfer system to be quantitatively more pronounced with higher levels of the intensive-margin elasticity.

In Figure 12, we plot the revenue function $y \mapsto \mathcal{R}_{sp}(y)$ under the assumption that the intensive-margin elasticity ε varies with income. The left panel compares a case where ε is equal to 0.3 at all income levels (blue line) with cases where ε is linearly increasing from 0.1 at the bottom to 0.5 at incomes above $15,000$ USD (teal line), or decreasing from 0.5 at the bottom to 0.1 at incomes above $15,000$ USD (brown line). As can easily be seen, in all cases, the revenue function violates the monotonicity condition from Theorem 1 so that there is room for a Pareto improvement by means of a symmetric two-bracket tax cut. In an attempt

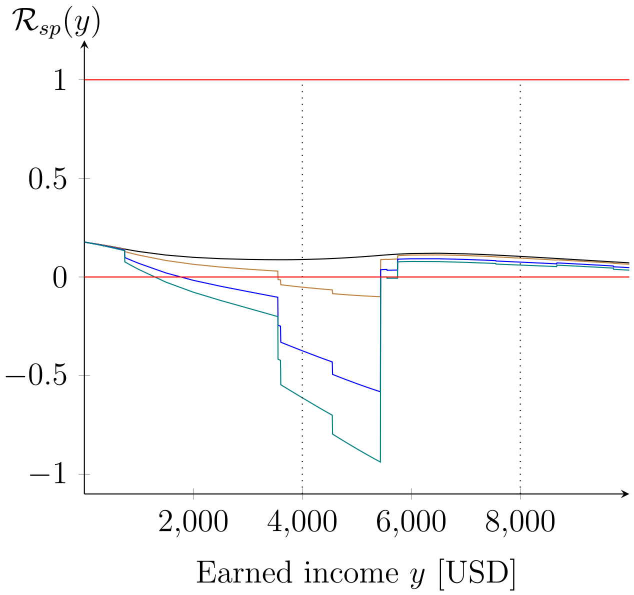
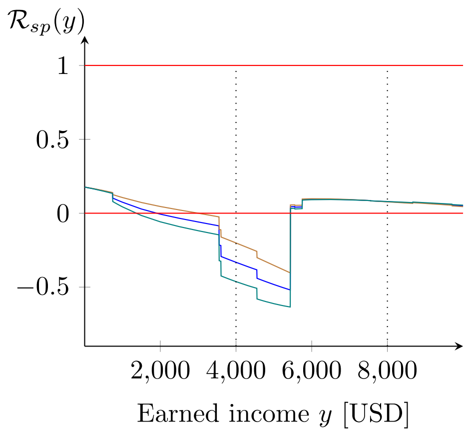


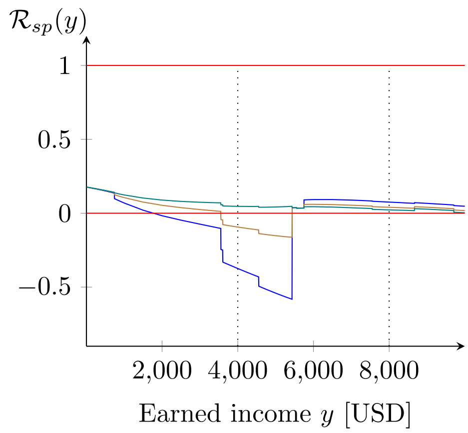
Figure 11: Pareto test of the 1974 US tax-transfer system, different intensive-margin elasticities.

Notes: Figure 11 shows the marginal revenue function $\mathcal{R}_{sp}(y)$ for single parents in 1974, assuming an intensive-margin elasticity of 0 (brown line), 0.1 (black line), 0.33 (blue line, benchmark) and 0.5 (teal line). The dotted vertical lines mark the endpoints of the phase-in range at $4,000$ USD and the phase-out range at $8,000$ USD of the 1975 EITC.

Source: Authors' calculations (see Table 1 for details).



(a) Continuous variations in ε.



(b) Discontinuous drops in ε.

Figure 12: Pareto test of the 1974 US tax-transfer system, heterogeneous intensive-margin elasticities.

Notes: Figure 12a shows the marginal revenue function $\mathcal{R}_{sp}(y)$ for single parents in 1974, assuming that the intensive-margin elasticity ε varies with income: falling from 0.5 to 0.1 (brown line), or increasing from 0.1 to 0.5 (teal line), or equal to 0.3 at all income levels. Figure 12b shows function $\mathcal{R}_{sp}(y)$ under the assumption that the intensive-margin elasticity ε jumps in two steps from 0.13 over 0.33 to 0.73 (brown line), or from 0.03 over 0.33 to 0.93 (teal line), or equals 0.33 at all income levels (blue line, benchmark). In all cases, the average participation elasticity is held at the benchmark level of 0.58. The dotted vertical lines mark the endpoints of the phase-in range at $4,000$ USD and the phase-out range at $8,000$ USD of the 1975 EITC.

Source: Authors' calculations (see Table 1 for details).

to identify the limits of this robustness analysis, we also explore some extreme cases in which the elasticity ε varies with income in a discontinuous way. Specifically, the right panel compares the benchmark case where ε is equal to 0.33 at all income levels (blue line) with two extreme cases. The brown line shows a case with an elasticity of 0.13 at the bottom, an upward jump to 0.33 at income level $5,433$ USD, and a further jump to 0.73 at $5,478$ USD. Our qualitative results remain unchanged in this case. The teal line shows a case where both upward jumps are even more pronounced, from 0.03 at the bottom over 0.33 to 0.93 at the top. In this second case, the revenue function $y \mapsto \mathcal{R}_{sp}(y)$ gets close to satisfying the monotonicity condition for a Pareto-efficient tax schedule (see Figure 12b).

Income effects. Finally, we consider a model in which income effects in labor supply arise due to concave consumption utility

$$u(c, y, \omega, \gamma) = \psi(c) - k(y, \gamma) - \gamma \ \mathbb{1}_{y>0} \ ,$$

with $\psi'(c) > 0$, $\psi''(c) < 0$ for all $c > 0$. With income effects, our sufficient-statistics formula for the revenue function $y \mapsto \mathcal{R}(y)$ becomes

$$\begin{aligned} \mathcal{R}(y) \quad &= \quad (1 - \mathcal{M}_0)^{-1} \left\{ 1 - F_y(y) - f_y(y) \ y \ \varepsilon(y) \ \frac{T_0'(y)}{1 - T_0'(y)} \right. \\ & \left. - \int_y^{\bar{y}} f_y(y') \pi(y') \frac{T_0(y')}{y' - T_0(y')} dy' - \int_y^{\bar{y}} f_y(y') \eta(y') T_0'(y') dy' \right\} \ , \end{aligned} \tag{26}$$

where the multiplier term $\mathcal{M}_0$ is given by

$$\mathcal{M}_0 = \int_0^{\bar{y}} f_y(y) \eta(y)) T_0'(y) dy + \int_0^{\bar{y}} f_y(y) \nu(y) T_0(y) dy \ . \tag{27}$$

In this formula, $\eta(y)$ captures income effects at the intensive margin (i.e., the effect of an increase of the base transfer c_0 on the earnings of workers with pre-reform income y); $\nu(y)$ captures income effects at the extensive margin (i.e., the effect of an increase of the base transfer c_0 on the employment rate of agents who choose to earn y when becoming active on the labor market). With concave consumption utility and additively separable preferences, $\eta(y)$ and $\nu(y)$ are both negative. Our benchmark case with quasi-linear utility is nested with $\eta(y) = \nu(y) = 0$. We formally derive a version of formula (26) in part F of the Supplementary Material below. Specifically, Proposition 4 gives the revenue function $y \mapsto \mathcal{R}(y)$ in terms of the model's primitives based on a general framework that allows for complementaries between consumption and leisure.

By (26), income effects matter for the revenue effect of a one-bracket tax increase at income y in two ways. First, they increase labor supply at the intensive margin for all agents with income above y, for whom the reform has reduced net income directly. This effect is captured by the last term in (26). With positive marginal taxes, this implies an upward shift of the revenue function. Second, when the revenue gain is rebated lump sum, this spurs further income effects at both margins all across the income distribution. This is captured by the term $\mathcal{M}_0$ in (27). With positive marginal taxes and positive participation tax rates, income effects make $\mathcal{M}_0$ negative and decrease the multiplier $(1 - \mathcal{M}_0)^{-1}$. Consequently, relative to the benchmark without income effects, the revenue function $y \mapsto \mathcal{R}(y)$ shifts towards the horizontal axis, i.e., its absolute value is decreased.

Figure 13 compares the revenue function $\mathcal{R}_{sp}(y)$ for our benchmark calibration to two scenarios with income effects. The teal line shows the revenue function $y \mapsto \mathcal{R}_{sp}(y)$ for a case with $\eta(y) = -0.08$ and $\nu(y) = -8 \times 10^{-6}$, corresponding to an MPE of -0.1 close to the estimates by Cesarini et al. (2017) and Imbens et al. (2001). The brown line shows $y \mapsto \mathcal{R}_{sp}(y)$ for a larger MPE of -0.4, corresponding to $\eta(y) = -0.24$ and $\nu(y) = -2.4 \times 10^{-5}$. The blue line is the benchmark calibration without income effects. In all cases, we maintain our benchmark assumptions about substitution effects at both margins, i.e., $\varepsilon = 0.33$ and $\bar{\pi} = 0.58$. Figure 13 demonstrates that our conclusion on the desirability of an EITC is robust to the consideration for income effects.

Summing up, our sensitivity analysis demonstrates that our main result does not hinge on the specifics of our benchmark calibration. The 1974 US tax and transfer system for single parents was not Pareto-efficient and there existed Pareto-improving reforms akin to the introduction of the EITC. This conclusion is obtained for a wide range of empirically plausible assumptions about labor supply responses at the intensive and extensive margin.

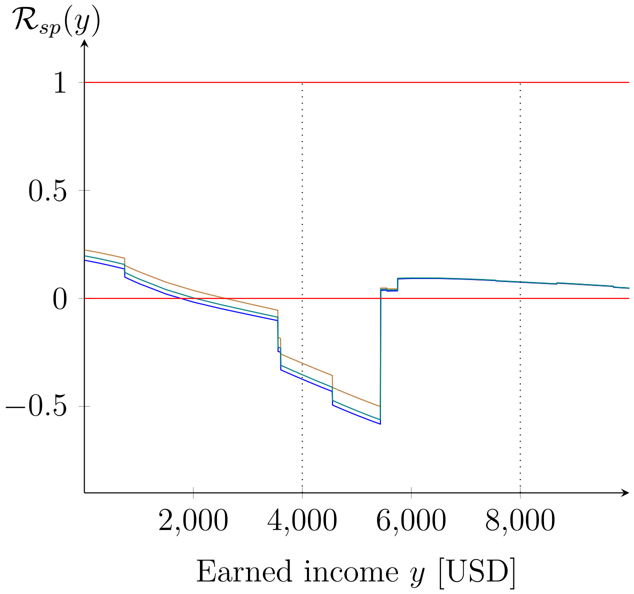
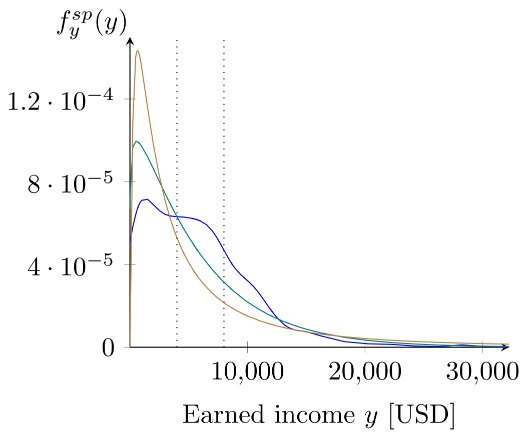


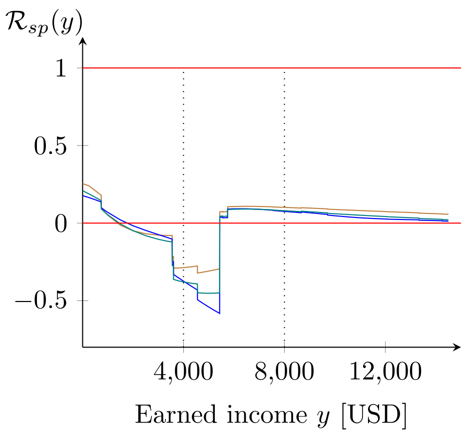
Figure 13: Pareto test of the 1974 US tax-transfer system, accounting for income effects.

Notes: Figure 10 shows the revenue function for single parents $y \mapsto \mathcal{R}_{sp}(y)$ in 1974 as a function of earned income, assuming income effects at both margins with an MPE of -0.08 (teal line) or -0.24 (brown line). For comparison, the blue line shows the benchmark case without income effects. The dotted vertical lines mark the endpoints of the phase-in range at $4,000$ USD and the phase-out range at $8,000$ USD of the 1975 EITC.

Source: Authors' calculations (see Table 1 for details).



(a) Income distributions.



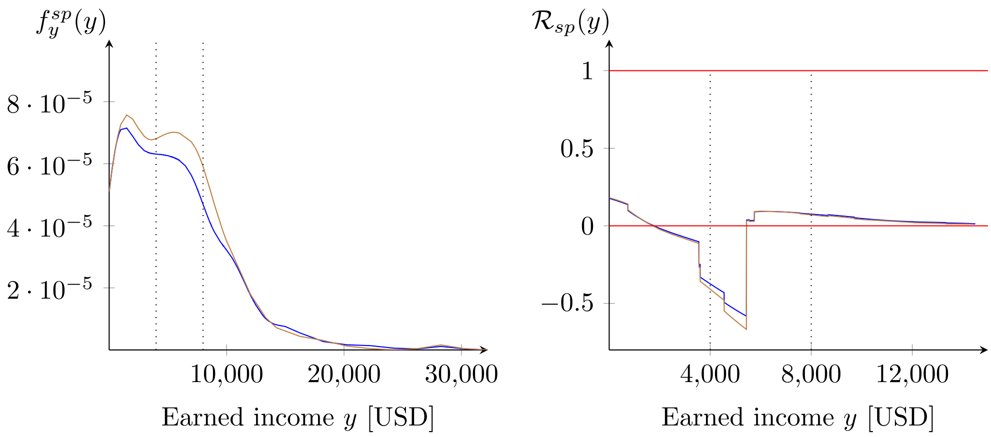
(b) Marginal revenue functions.

Figure 14: Pareto test of 1974 US tax system for single parents, parametric income distributions.

Notes: Figure 14a shows alternative estimates of the income distribution for single parents in 1974: kernel estimation (blue, benchmark), gamma distribution (teal), log-normal distribution (brown). Figure 14b shows the revenue function $y \mapsto \mathcal{R}_{sp}(y)$ for single parents in 1974, based on the kernel estimation (blue), gamma distribution (teal), and lognormal distribution (brown), using benchmark values for the elasticities at both margins. The dotted vertical lines mark the endpoints of the phase-in range at $4,000$ USD and the phase-out range at $8,000$ USD of the 1975 EITC.

Source: Authors' calculations (see Table 1 for details).

Alternative estimates of the income distribution. As alternatives to our benchmark calibration, we first verified that variations in the bandwidth choice for our kernel estimates do not substantially affect our results. We also fitted gamma and lognormal distributions to the earned income data in the March 1975 CPS. Figure 14 shows versions of the revenue function that result with these parametric distributions. The differences to the benchmark case are small.



(a) Income distributions.

(b) Marginal revenue functions.

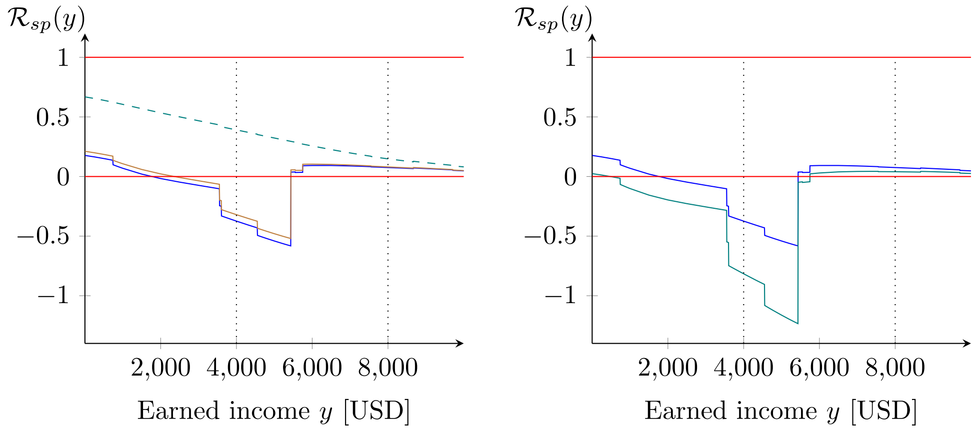
Figure 15: Pareto test of 1974 US tax system for single parents, different samples.
Notes: Figure 15a shows the pdfs of the distributions of earned income based on the samples of all single parents (blue, benchmark) versus single parents with exactly two children (brown line) on the basis of the March 1975 CPS. Figure 15b gives the revenue function $y \mapsto \mathcal{R}_{sp}(y)$ for single parents in 1974, based on the estimated income distributions for all single parents (blue) versus single parents with exactly two children (brown), using benchmark values for the elasticities at both margins. The dotted vertical lines mark the endpoints of the phase-in range at $4,000$ USD and the phase-out range at $8,000$ USD of the 1975 EITC.
Source: Authors' calculations (see Table 1 for details).

For the benchmark analysis reported in the main text, we estimate the earned income distribution based on the set of all single parents ($N = 1,494$), with no conditioning on the number of children. As an alternative, we also considered non-parametric kernel estimates of the income distribution for the smaller sample of single parents with exactly two children ($N = 453$). Figure 15 shows that this does not make much of a difference.

Alternative representations of the tax and transfer system. As final robustness checks, we consider three alternative representations of the 1974 US tax and transfer system for single parents. In all three cases, our results are virtually unchanged. First, we take into account the wealth and assets tests for welfare recipients. In 1974, only families with liquid assets below $1,500$ USD were eligible for welfare transfers (AFDC, SNAP). Using the approximation of liquid assets suggested by Giannarelli (1992), we find that about 8% of single parents with earned incomes below the relevant thresholds failed the programs' assets test. Moreover, single parents were only eligible for the EITC if the sum of their earned income and capital income was below $8,000$ USD. In the March 1975 CPS data, only 0.9% of the single parents with incomes in the EITC range lost eligibility due to high capital incomes. Hence, we find that about 7% of single parents were eligible for the EITC, but not for welfare programs.

Our benchmark analysis in the main text ignores the assets test. Thereby, we provide an answer to the question whether, in 1974, introducing an EITC with the same assets tests as AFDC and SNAP would have been Pareto-improving. Alternatively, one can ask whether the tax-transfer system would have been Pareto-improved by a tax reform without an assets test, conditional on the assets test for welfare recipients being in place. To answer this question, the brown line in Figure 16a shows a version of the revenue function $y \mapsto \mathcal{R}_{sp}(y)$ under the assumption that 7% of single parents were not eligible for AFDC and SNAP, while all other single parents were eligible. For comparison, the blue line shows $y \mapsto \mathcal{R}_{sp}(y)$ for the benchmark case where all single parents are eligible for welfare, and the dashed teal line shows $y \mapsto \mathcal{R}_{sp}(y)$ for the counterfactual case where no single parent was eligible for welfare. As Figure 16a shows, taking into account the wealth test leaves our results qualitatively unchanged: There was scope for Pareto improvements by means of both one-bracket and two-bracket reforms.

Second, we take into account social security contributions (payroll taxes) to compute the effective tax rates for single parents. By contrast, our benchmark



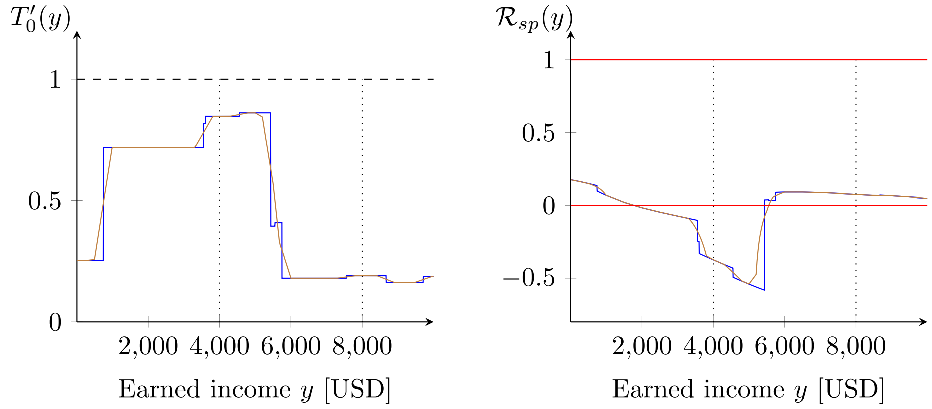
(a) Accounting for wealth test. (b) Accounting for payroll tax.

Figure 16: Pareto test of 1974 US tax system, accounting for wealth test or payroll taxes.

Notes: Figure 16a shows the revenue function for single parents $y \mapsto \mathcal{R}_{sp}(y)$ taking into account the wealth test for AFDC and SNAP eligibility (brown line) versus ignoring it as in the benchmark (blue line). The dashed teal line shows $y \mapsto \mathcal{R}_{sp}(y)$ ignoring AFDC and SNAP, i.e., based on the statutory income tax schedule alone. Figure 16b plots $y \mapsto \mathcal{R}_{sp}(y)$ taking into account the employee share of social security contributions (teal line) versus ignoring them as in the benchmark (blue line). The dotted vertical lines mark the endpoints of the phase-in range at $4,000$ USD and the phase-out range at $8,000$ USD of the 1975 EITC.

Source: Authors' calculations (see Table 1 for details).

analysis takes into account the statutory income tax and welfare transfers, but ignores payroll taxes. Specifically, Figure 16b illustrates the effect of taking into account the employee share of payroll taxes, which was 5.85% at the time. The teal line gives the revenue function $y \mapsto \mathcal{R}_{sp}(y)$ for this case, while the blue line shows the benchmark case without payroll taxes. With payroll taxes, we find even larger inefficiencies in the 1974 US tax-transfer system for single parents, and two-bracket tax cuts for low-income earners become even more attractive. Taking into account both the employee share and the employer share of payroll taxes would reinforce this effect.



(a) Marginal tax rates. (b) Marginal revenue functions.

Figure 17: Pareto test of 1974 US tax system, using smoothed tax schedule.

Notes: Figure 17a shows a smoothed version of the marginal tax rates for single parents (teal line) versus the statutory marginal tax rates (blue line). Figure 17b shows the revenue function $y \mapsto \mathcal{R}_{sp}(y)$ based on the smoothed tax schedule (teal line) versus the statutory tax schedule as in the benchmark (blue line). The dotted vertical lines mark the endpoints of the phase-in range at $4,000$ USD and the phase-out range at $8,000$ USD of the 1975 EITC.

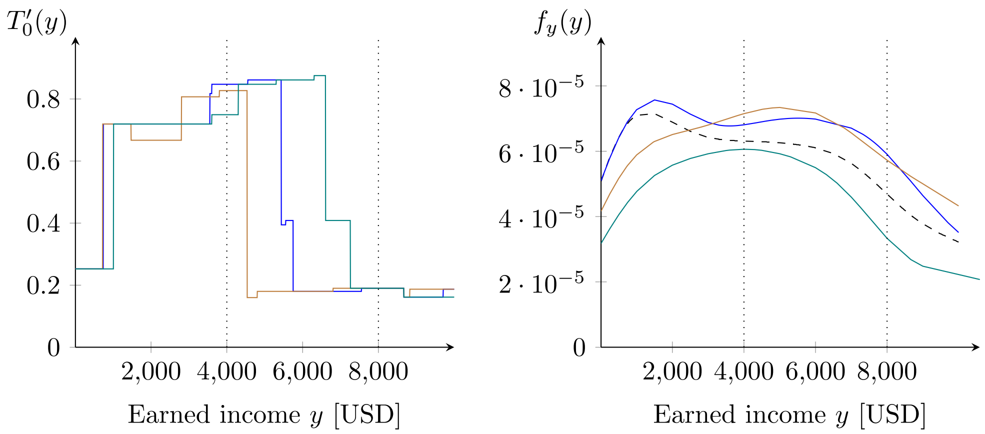
Source: Authors' calculations (see Table 1 for details).

Third, we consider a smoothed version of the 1974 tax-transfer schedule for single parents that eliminates kinks. Such smoothed tax schedules are commonly used in the literature on implicit marginal welfare weights. Specifically, Figure 17a compares the statutory marginal tax $T_0'(y)$ (blue line) with a smooth alternative that is given by the average marginal tax in intervals of length 500 USD, $[T_0(y+250) - T_0(y-250)]/500$ (brown line). In Figure 17b, the brown line shows the revenue function $y \mapsto \mathcal{R}_{sp}(y)$ based on the smoothed schedule and the corresponding participation taxes, while the blue line shows $y \mapsto \mathcal{R}_{sp}(y)$ based on the statutory (benchmark) tax schedule. Again, the differences between these curves are small.

C.3 Heterogeneity with respect to household size

In this section, we take a closer look at single parents who differ in the number of children. In 1975, a single EITC schedule was introduced for all single parents, irrespective of the household size. Decades later, the US authorities introduced additional, more generous EITC schedules for single parents with two or more children (in 1991), and for single parents with three or more children (in 2009). Our analysis provides two insights. First, the 1974 US tax-transfer system was not Pareto-efficient, irrespective of what subgroup of single parents we consider. Second, the size of the inefficiencies differed across these subgroups: Using the inefficiency measure introduced in Section 4, the 1974 tax-transfer system was more inefficient for single parents with three children than for those with two children, and the least inefficient for single parents with one child. Correspondingly, the best available tax reforms differed across subgroups, and the introduction of differentiated EITC schedules would have made sense already in the mid-1970s.

Figure 18 demonstrates pronounced heterogeneity among single parents in two dimensions. First, Figure 18a shows that the 1974 US tax-transfer schedules differed substantially depending on the number of children: For single parents with more children, effective marginal taxes were much higher and these high marginal tax rates were concentrated on higher incomes. Specifically, marginal taxes above 70% applied for single parents with one child up to annual incomes around $4,500$ USD; for those with two children up to annual incomes around $5,400$ USD; for those with three children up to annual incomes around $6,600$ USD. The main reason for these differences is that the maximum amount of welfare



(a) Marginal tax schedules.

(b) Income distributions.

Figure 18: Tax rates and income distributions for single parents with 1, 2, and 3 children.

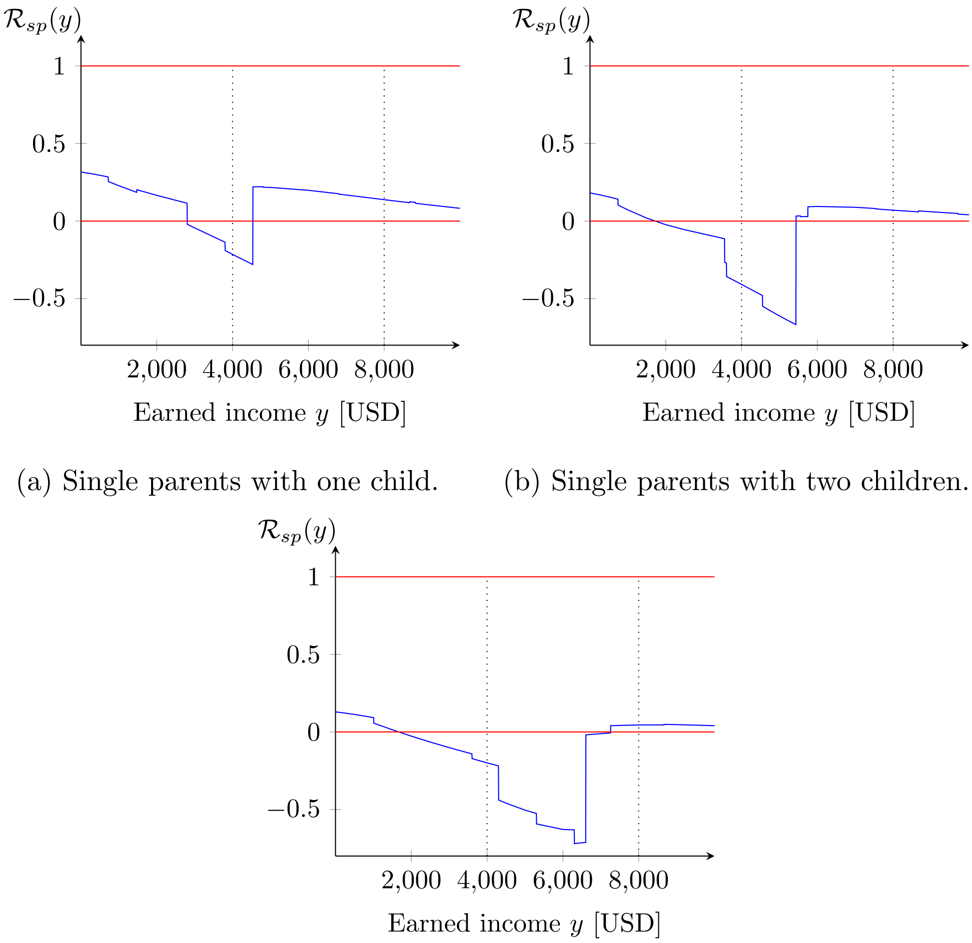
Notes: Figure 18 shows the tax and transfer schedules and the income distributions among single parents with one child (brown lines), two children (blue lines), and three children (teal lines). Specifically, Figure 18a shows the marginal taxes $y \mapsto T_0'(y)$ in 1974. Figure 18b shows the kernel estimates of the 1974 pdf $y \mapsto f_y(y)$ of earned incomes; the dashed black line plots the estimated pdf among all single parents. In both panels, the dotted vertical lines mark the endpoints of the phase-in range at $4,000$ USD and the phase-out range at $8,000$ USD of the 1975 EITC.

Source: Authors' calculations (see Table 1 for details).

transfers (AFDC, SNAP) was increasing in family size. As the transfers were phased out at the same rates, the phasing-out of welfare transfers occurred at higher incomes for families with more children. Second, Figure 18b shows that there were large differences in the distributions of earned income across single parents: Parents with more children had on average lower incomes. Specifically, it shows the kernel estimates of the pdfs of earned income based on CPS March 1975 data, using a Gaussian kernel with a bandwidth of 1200 for single parents with one and two children and a bandwidth of 1400 for single parents with three children. For comparison, the dashed black line shows the estimated pdf of the income distribution among all single parents. Among single parents with positive incomes, the average incomes were $5,037$ USD for those with one child; $4,165$ USD for those with two children; and $3,550$ USD for those with three children. The shares of single parents without any earned income were 20.9% (one child); 24.9% (two children); and 43.5% (three children).

To perform separate tests for Pareto efficiency of the 1974 US tax system in these subgroups, Figure 19 shows the revenue function $y \mapsto \mathcal{R}_{sp}(y)$ for single parents with one child in the upper left panel, for those with two children in the upper right panel, and for those with three children in the lower panel. For all cases, we assume an intensive-margin elasticity of 0.33 and participation elasticities that decrease with income from 0.67 to 0.4 as in the benchmark calibration. First, Figure 19 clarifies that the 1974 US tax-transfer schedules were not Pareto-efficient for any subgroup of single parents. All panels show that there were Pareto-improving tax reforms with both one and two brackets. Second, the relevant income ranges differed: With more children, the inefficiencies arise at higher income levels and cover a wider range of incomes.

The inefficiency measure developed in Section 4 confirms the visual impression from Figure 19. For single parents with one child, the revenue-maximizing tax cut was a two-bracket tax cut that reduced marginal taxes in the income range $2,003$ to $4,533$ USD and increased them in the income range $4,533$ to $7,063$ USD, accompanied by two further two-bracket tax cuts in the income ranges $1,365$



(a) Single parents with one child.

(b) Single parents with two children.

(c) Single parents with three children.

Figure 19: Pareto test of 1974 US tax system, single parents with 1, 2, and 3 children.

Notes: Figure 19 shows the revenue functions $y \mapsto \mathcal{R}_{sp}(y)$ in 1974 for single parents with one child (upper left panel), single parents with two children (upper right panel), and single parents with three children (lower panel). In all panels, the dotted vertical lines mark the endpoints of the phase-in range at $4,000$ USD and the phase-out range at $8,000$ USD of the 1975 EITC.

Source: Authors' calculations (see Table 1 for details).

to $1,575$ USD and $8,559$ to $8,775$ USD. This reform allowed to raise the base transfer to each agent in this group by 6.45 USD per percentage point change in the marginal tax rates, with more than 99% of that amount coming from the income range $2,003$ to $7,063$ USD.

For single parents with two children, the revenue-maximizing Pareto-improvement was a two-bracket tax cut that reduced marginal taxes in the income range $1,305$ to $5,748$ USD and increased marginal taxes in the income range $5,748$ to $10,191$ USD. This reform allowed to raise the base transfer to each household in this group by 13.64 USD per percentage point change in marginal taxes.

For single parents with three children, finally, the revenue-maximizing tax cut was a two-bracket tax cut that reduced marginal taxes in the income range $1,339$ to $7,257$ USD and increased them in the income range $7,257$ to $13,175$ USD. It allowed to raise the base transfer to single parents with three children by 18.22 USD per percentage point change in marginal taxes. For completeness, we note that this heterogeneity is mainly driven by the differences in the tax-transfer schedules shown in Figure 18a. If we compute the revenue functions $y \mapsto \mathcal{R}_{sp}(y)$ using the same income distribution for all subgroups of single parents, the inefficiencies differ even more in size than if we use subgroup-specific income distributions as in Figure 19.

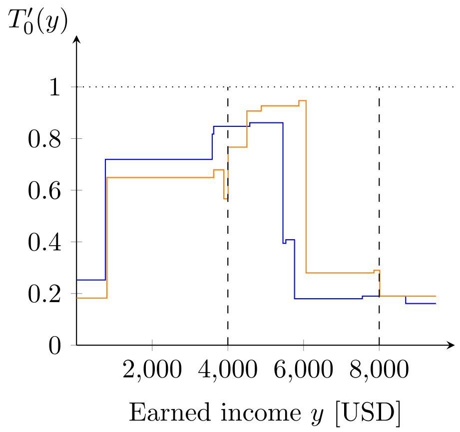
Summing up, we find that the inefficiencies in the 1974 US tax-transfer system were substantially larger for single parents with more children. Moreover, the best available reforms differed across the subgroups: For lone parents with more children, EITC-like reforms on much wider and higher income ranges were Pareto-improving than for those with less children. We conclude that, already in 1974, the introduction of differentiated EITC schedules – as implemented decades later – would have been desirable.

C.4 Results for further post-reform years

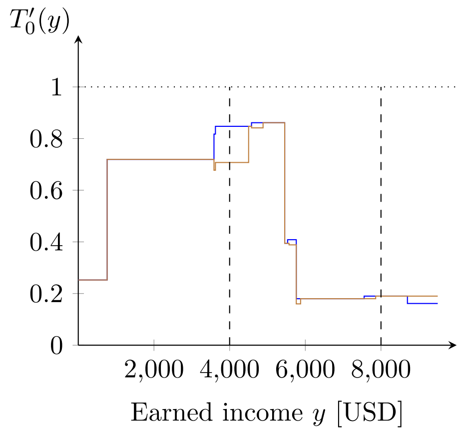
We now extend our analysis to the US tax-transfer systems after the 1975 EITC introduction. We start by explaining how the effective marginal tax-transfer schedules for single parents with two children changed from 1974 to 1975. We also provide a graphical decomposition of the overall change due to adjustments of the statutory income tax, changes of welfare transfers, and changes due to the introduction of the EITC. Second, we demonstrate that the 1975 tax-transfer schedule was not Pareto-efficient. We then show that these inefficiencies remained and even increased until 1978, partly due to adjustments of the tax schedule and the welfare transfers that were independent of the EITC. Third, we take a closer look at the tax-transfer system for single parents in 1978, when the first expansion of the EITC as of 1979 was decided. We find that the 1979 EITC reform went into a Pareto-improving direction.

Figure 20 illustrates the 1975 changes in the effective tax-transfer schedule for single parents. The upper left panel compares the marginal taxes for low-income earners in 1974 (blue line) and 1975 (orange line). The other three panels associate these changes to the elements of the tax-transfer system: the statutory income tax, the EITC, and the welfare transfers AFDC and SNAP. In the statutory income tax, there were minor adjustments in the standard deduction and a disregard for personal exemptions. This implied a reduction of marginal taxes in the lowest tax bracket (see Figure 20b in the upper right panel). The introduction of the EITC reduced the marginal tax by 10 percentage points between 0 and $4,000$ USD, and increased the marginal tax by 10 percentage points between $4,000$ and $8,000$ USD (see Figure 20c in the lower left panel). Finally, adjustments in the AFDC and SNAP parameters implied increases in the maximum transfer amounts and in the eligibility thresholds. As a result, both transfers were phased out over wider income ranges, and marginal tax rates increased substantially between $5,400$ and $6,100$ USD (see Figure 20d in the lower right panel.) We note that the EITC introduction implied the most substantial tax changes for single parents, but the AFDC and SNAP adjustments also played a role.

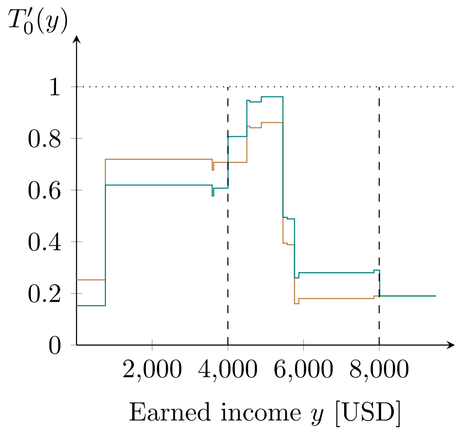
In Section 5 in the main text, we study whether the 1975 EITC introduction went into a Pareto-improving direction, holding all other elements of the tax-transfer system fixed. This is the reform we seek to evaluate, and therefore it



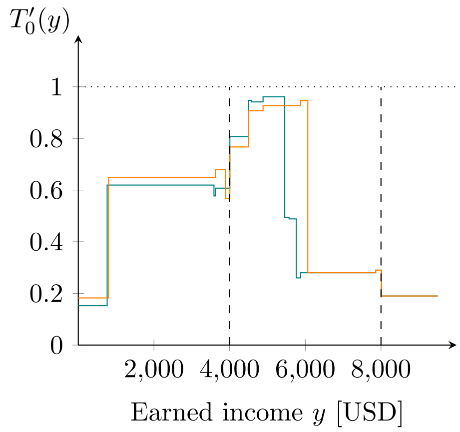
(a) Marginal taxes 1974 versus 1975.



(b) Changes in statutory income tax.



(c) Changes due to EITC.



(d) Changes in AFDC and SNAP.

Figure 20: Changes in effective marginal taxes for single parents, 1974 to 1975.

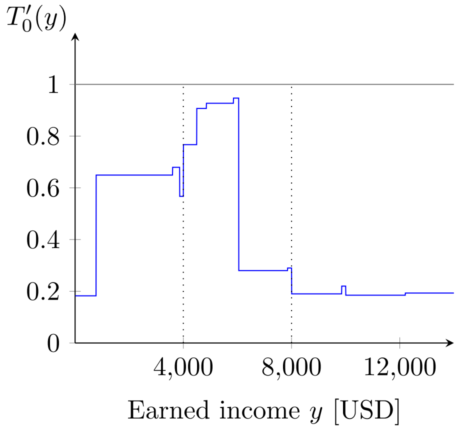
Notes: Figure 20a compares the effective marginal taxes for single parents with two children in 1974 (blue line) and 1975 (orange line). Figures 20b to 20d decompose these differences into three elements of the tax-transfer systems. Figure 20b shows how the marginal tax in 1974 was changed by adjustments of the statutory income tax (brown line vs. blue line); Figure 20c shows the effect of the EITC introduction (teal line vs. brown line); Figure 20d accounts for changes in AFDC and SNAP parameters (orange line vs. teal line). The dotted vertical lines mark the endpoints of the phase-in range at $4,000$ USD and the phase-out range at $8,000$ USD of the 1975 EITC.

Source: Authors' calculations (see Table 1 for details).

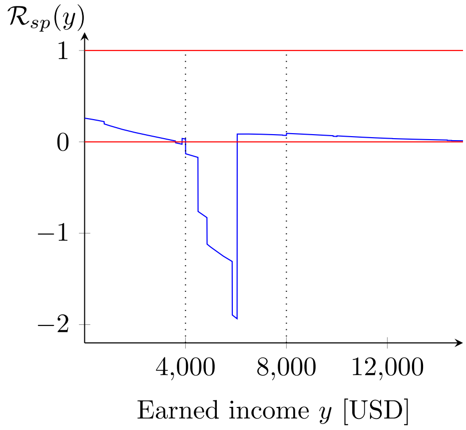
would not be appropriate to evaluate several changes in the tax and transfer system simultaneously. We find that this reform was not Pareto-improving according to our benchmark calibration, while a similar two-bracket tax cut in an alternative income range would have been. In particular, the 1975 EITC further increased the excessively high marginal taxes between 4,000 and 5,750 USD, while a reduction would have been Pareto-improving. As can be seen from Figure 20a, the same pattern emerges if we take account of all changes in the tax-transfer schedule between 1974 and 1975. This suggests that, due to the suboptimal design of the EITC, the 1975 US tax schedule neither was Pareto-efficient.

We confirm this hypothesis using the conditions for Pareto efficiency in Theorems 1 and 2 applied to the tax and transfer system for single parents prevailing after the introduction of the EITC. Figure 21 provides a graphical illustration of this test for 1975, right after the EITC introduction, and for 1978, before the first EITC expansion. It shows that the US tax-transfer system remained inefficient throughout this time. More specifically, the upper right panel shows the revenue function $y \mapsto \mathcal{R}_{sp}(y)$ for the 1975 tax system. On the one hand, it shows that the conditions for Pareto efficiency were now violated on a smaller income range than in 1974. In 1975, the revenue-maximizing tax cut was a two-bracket reform that reduced marginal taxes between 2,525 and 6,045 USD, and increased them between 6,045 and 9,565 USD (i.e., on a smaller interval than in 1974). On the other hand, function $y \mapsto \mathcal{R}_{sp}(y)$ attains even more negative values for some incomes. As a result, we find that the post-EITC tax system was even more inefficient than the pre-EITC tax system: In 1975, a two-bracket tax cut that changed marginal taxes by one percentage point allowed to increase the base transfer to single parents 21.9 USD. In 1974, only an increase in the base transfer by 12.6 USD was feasible.

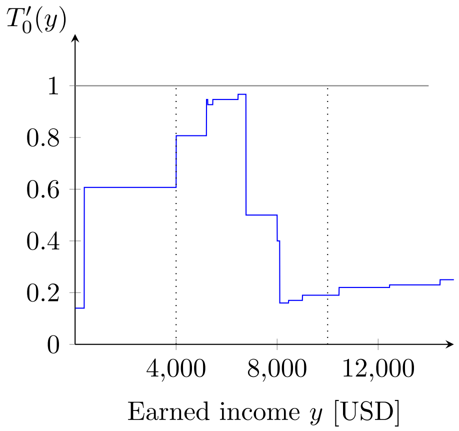
From 1975 to 1978, the income range with inefficiencies widened further, eventually covering the majority of the phase-out range of the EITC between 4,000 and 8,000 USD (see Figure 21). This increase was driven partly by secular changes such as wage growth, and partly by a series of minor adjustments in deductions,



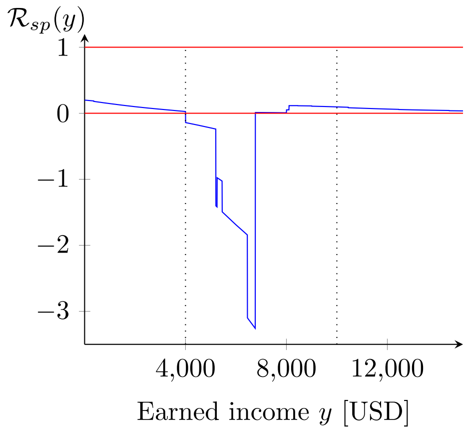
(a) Marginal tax 1975.



(b) Revenue function 1975.



(c) Marginal tax 1978.



(d) Revenue function 1978.

Figure 21: Pareto test of US tax-transfer system, 1975 and 1978.

Notes: Figure 21 shows the marginal tax rates (left panels) and the revenue function $y \mapsto \mathcal{R}_{sp}(y)$ (right panels) for single parents in the years 1975 and 1978. In the upper panels, the vertical dotted lines mark the endpoints of the phase-in range at $4,000$ USD and the phase-out range at $8,000$ USD of the 1975 EITC. In the lower panels, the dashed vertical lines at $4,000$ and $10,000$ USD mark the income range that was affected by the 1979 EITC expansion.

Source: Authors' calculations (see Table 1 for details).

income disregards and AFDC/SNAP parameters. In 1978, finally, the tax-transfer system involved larger inefficiencies than both in 1974 and 1975. It was possible to increase the base transfer to each single parent by up to 35.4 USD by a two-bracket tax cut that changed marginal taxes by only one percentage point. Specifically, this revenue-maximizing tax cut required to reduce marginal taxes between $3,256$ and $8,100$ USD and to increase them between $8,100$ and $12,944$ USD.

Interestingly, the 1979 EITC reform was fairly similar to this revenue-maximizing tax cut. The US government strongly expanded the EITC in the tax year 1979, using a two-bracket tax cut with the following properties: The phase-in range was expanded from $4,000$ to $5,000$ USD; a plateau range between $5,000$ and $6,000$ USD was introduced and the phase-out range was adjusted and now went from $6,000$ to $10,000$ USD. While the phase-in rate was kept at 10%, the phase-out rate was raised from 10% to 12.5%. Altogether, the 1979 EITC reform was a tax cut in direction

$$\tilde{h}_{79}(y) = \begin{cases} -0.2 & \text{for} \quad y \in [4.000; 5.000) \ , \\ -0.1 & \text{for} \quad y \in [5.000; 6.000) \ , \\ 0.025 & \text{for} \quad y \in [6.000; 8.000) \ , \\ 0.125 & \text{for} \quad y \in [8.000; 10.000) \ , \\ 0 & \text{for} \quad y \notin [4.000, 10.000) \ . \end{cases}$$

To check whether this reform went into a Pareto-improving direction, we evaluate condition (9) using the reform direction $y \mapsto \tilde{h}_{79}(y)$ and the 1978 revenue function $y \mapsto \mathcal{R}_{sp}(y)$. Based on our benchmark calibration, we find that the 1979 EITC reform direction was indeed Pareto-improving: It raised tax revenue and allowed to increase the base transfer to all single parents, while directly reducing the tax liabilities for all taxpayers with earned incomes between $4,000$ and $10,000$ USD.

To sum up, we have demonstrated that the 1974 EITC reform did neither go in a Pareto-improving direction nor did it restore a Pareto-efficient tax system; mainly because the 1974 EITC was too limited in the income range that it covered. By contrast, the 1979 reform expanded the EITC to incomes up to $10,000$ USD, thereby correcting the initial weaknesses and achieving a Pareto improvement.

D Pareto efficiency when earnings are bounded away from zero and bounded from above

Suppose that $y \mapsto \mathcal{R}(y)$ is non-increasing over $\mathcal{Y}$, so that there is no scope for a Pareto improvement by a two-bracket reform. Then, by Theorem 2 and Corollary 1, the status quo tax system is Pareto-efficient if $\mathcal{R}$ is bounded from above by 1 for the lowest incomes, and bounded from below by 0 for the highest incomes. Hence, our Pareto test only requires to verify one-bracket-efficiency at the very top and the very bottom. Under more restrictive assumptions, even the conditions for one-bracket-efficiency at the extremes become dispensable as we show in the following corollary. For this purpose, we denote by $y_{min} := \inf y_0(\Theta)$ the infimum of the income levels, and by $y_{max} := \sup y_0(\Theta)$ the supremum of the income levels chosen under the status quo tax policy.

Corollary 2 *If $y_{min} > 0$, $y_{max} < \bar{y}$, and $y \mapsto \mathcal{R}(y)$ is non-increasing, then there is no Pareto-improving direction in the class of continuous functions.*

Under the conditions of Corollary 2, the monotonicity of $y \mapsto \mathcal{R}(y)$ is sufficient for Pareto efficiency, i.e., there is no need to invoke the requirements that this function must be bounded from below by 0 and from above by 1. The conditions are that all types choose their incomes in some interior subset of $\mathcal{Y}$. In this case, as we show formally below, we have $\mathcal{R}(y) = 1$ for all $y \in [0, y_{min})$, and $\mathcal{R}(y) = 0$ for all $y \in (y_{max}, \bar{y}]$. Consequently, all three sufficient conditions in Theorem 2 are satisfied if $y \mapsto \mathcal{R}(y)$ is non-increasing.[44]

The conditions in Corollary 2 hold, for instance, in a Mirrleesian model of income taxation with only intensive-margin responses when Inada conditions ensure positive and bounded incomes for everybody.[45] By contrast, there is typically a

[44]Note that Corollary 2 provides a more compact expression of our sufficient conditions, but it should not be interpreted as showing that two-bracket reforms can achieve strictly more than one-bracket reforms. In particular, a two-bracket reform with one bracket below y_{min} or above y_{max} is economically equivalent to a one-bracket reform.

[45]Technically, this also requires that the parameter $\bar{y}$ in $\mathcal{Y} = [0, \bar{y}]$ is chosen so large that this upper bound does not interfere with individual choices.

positive mass of taxpayers with zero income in models with an intensive and an extensive margin. In this case, the requirement of boundedness does not follow from the requirement of monotonicity. Put differently, with a mass of non-working people, there can exist Pareto-improving reforms with one bracket even if there is no Pareto-improving reform with two brackets.

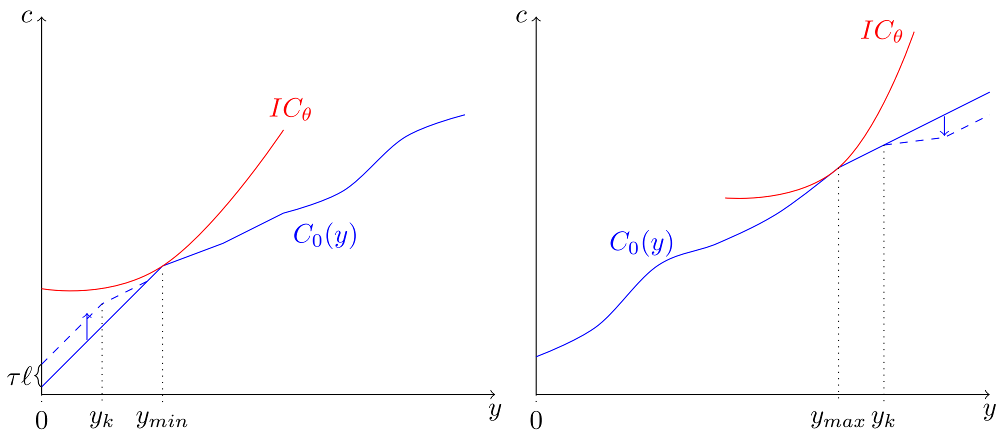
D.1 Proof of Corollary 2

Recall that $y_0(\Theta)$ is the image function of y_0, i.e., the set of income levels that are individually optimal for some type in Θ given the status quo tax system. The infimum of this set is denoted by y_{min} and the supremum by y_{max}.

Lemma 4 *If $y_{min} > 0$, then $\mathcal{R}(\hat{y}) = 1$ for any $\hat{y} \in [0, y_{min})$.*

Proof Fix a one-bracket reform $(\tau, \ell, \hat{y})$ such that $\hat{y} \geq 0$, $\hat{y} + \ell < y_{min}$ and assume that $\tau\ \ell$ is close to zero. This implies that, at any income level $y \geq y_{min}$, the tax burden increases by $\tau\ \ell$. We now argue that there are no behavioral responses to such a reform. More specifically, we show that "no behavioral responses" is consistent with both utility-maximizing behavior and the government budget constraint. When there are no behavioral responses and $y_0(\theta) > \hat{y} + \ell$ for all θ, this implies that the change of aggregate tax revenue equals $R^s(\tau, \ell, \hat{y}) = \tau\ \ell$. Since this additional tax revenue is rebated lump-sum, this also implies that all taxpayers receive additional transfers of $\tau\ \ell$. Hence, for any income in $y^*(\Theta)$, the additional tax payment and the additional transfer cancel each other out, implying that taxpayers face the same budget set before and after this reform, $C_1(y) = C_0(y)$. Moreover, for $\tau\ \ell$ sufficiently small, incomes smaller than y_{min} remain dominated by $y_0(\theta) \geq y_{min}$ for each θ. From $R^s(\tau, \ell, \hat{y}) = \tau\ \ell$, we obtain $R^s_{\tau\ell}(0, 0, \hat{y}) = 1$, which completes the proof. $\square$

Figure 22a illustrates these arguments: The solid blue line depicts the status quo budget set $C_0(y)$, the dashed blue line shows the upward shift by $\tau\ \ell$ in the post-reform budget set $C_1(y)$ for incomes below y_k. The red line, finally, shows an indifference curve of the lowest-earning type θ such that $y_0(\theta) = y_{min}$ before the reform. As apparent from Figure 22a, the type continues to prefer y_{min} to any lower income and does not change her behavior as long as $\tau\ \ell$ is small enough.



(a) A tax increase below y_{min}.

(b) A tax increase above y_{max}.

Figure 22: Illustration of small one-bracket reforms below y_{min} and above y_{max}.

Lemma 5 *If $y_{max} < \bar{y}$, then $\mathcal{R}(y) = 0$ for any $y \in (y_{max}, \bar{y}]$.*

Proof Fix a one-bracket reform $(\tau, \ell, \hat{y})$ such that $\hat{y} \in (y_{max}, \bar{y})$ and $\tau\ \ell > 0$. This reform does not change the tax burden at any income $y \leq y_{max}$. It increases the tax burden for incomes above $\hat{y}$ that were already dominated by some income below y_{max} for each type θ prior to the reform. We now argue that there are no behavioral responses to such a reform. More specifically, we show that "no behavioral responses" is consistent with both utility-maximizing behavior and the government budget constraint. When there are no behavioral responses and $y_0(\theta) < \hat{y}$ for each θ, this implies that the aggregate tax revenue does not change, $R^s(\tau_k, \ell_k, y_k) = 0$. This also implies that the base transfer is not changed, $c_1 = c_0$. Hence, for any income $y \leq \hat{y}$, taxpayers face the same budget set before and after this reform, $C_1(y) = C_0(y)$. Moreover, because of the tax increase for incomes larger than $\hat{y}$, these incomes remain dominated by some income below y_{max} for each type θ. From $R^s(\tau, \ell, \hat{y}) = 0$, we obtain $\mathcal{R}(\hat{y}) = R^s_{\tau\ell}(0, 0, \hat{y}) = 0$, which completes the proof. $\square$

Figure 22b illustrates these arguments. Again, the solid blue line depicts the status quo budget set $C_0(y)$, and the dashed blue line depicts the downward shift in the budget set due to the reform. The red depicts an indifference curve of a type θ who chooses the highest income y_{max} before the reform. As apparent from Figure 22b, the type continues to prefer y_{max} to any higher income and does not change her behavior after the reform.

Corollary 2 follows from combining Theorem 2 with Lemmas 4 and 5. Thus, if $y_{min} > 0$ and $y_{max} < \bar{y}$, monotonicity of $\mathcal{R}$ is a sufficient condition for the non-existence of Pareto-improving tax reforms. The lower and upper bounds on $\mathcal{R}$ become dispensable. Put differently, whenever there is a Pareto-improving reform with one bracket, there is also a Pareto-improving two-bracket tax cut.

Now suppose that the conditions in Corollary 2 are violated, so that either $y_{min} = 0$ or $y_{max} = \bar{y}$. In particular, the case with a mass of people with zero incomes is both empirically relevant and a typical outcome in the model with labor supply responses at the intensive and the extensive margin that we focus on in Section 5. In this case, it is possible that $\mathcal{R}(y)$ is monotonically decreasing over $(0, \bar{y})$ even though $\mathcal{R}(y) > 1$ for positive incomes close to 0. Then, there exists a Pareto-improving one-bracket reform, but no such reforms with two brackets. Hence, one-bracket efficiency at the bottom remains a substantive constraint.

In contrast, in the case $y_{max} = \bar{y}$, we can simply raise the upper threshold $\bar{y}$ of the set of feasible incomes to make sure that all people choose incomes below $\bar{y}$. After this adjustment, $\mathcal{R}(y) = 0$ for all incomes between y_{max} and the adjusted level of $\bar{y}$ by Lemma 5. Then, $\mathcal{R}(y) < 0$ for some income level close to $\bar{y}$ implies a violation of the condition that $y \mapsto \mathcal{R}(y)$ must be non-increasing over $[0, \bar{y}]$. Hence, whenever there is a Pareto-improving reform that reduces marginal taxes in one bracket of incomes below y_{max}, there is also a Pareto-improving two-bracket reform. In particular, any reform that combines the previously mentioned one-bracket tax cut with a tax increase in some bracket of incomes above y_{max} will do the job. It should be noted, however, that such a two-bracket reform is economically equivalent to a one-bracket reform by the arguments given in the proof of

Lemma 5: The second bracket has no effect on behavior as no type will choose an income in or above the second bracket.

E Pareto-improving tax reforms imply negative weights in the "inverse tax problem"

The literature on the "inverse tax problem" looks at observed tax policies to infer a government's redistributive concerns, using a revealed-preferences logic.[46] Specifically, it inverts the necessary (first-order) conditions for a welfare maximum to identify the government's "implicit marginal welfare weights" for people with different income levels, i.e., the marginal welfare weights that make the observed tax system optimal conditional on a given set of labor supply elasticities and an income distribution. How is this exercise related to Pareto-efficient taxation? It is a necessary condition for the Pareto efficiency of a tax system that it maximizes a Paretian social welfare function, i.e., an objective that is increasing in the individual utility of each type. Thus, if there is an observed tax system for which the "implicit welfare weights" at some incomes are negative, then this tax system cannot be Pareto-efficient. This suggests that necessary conditions for the non-negativity of implicit welfare weights are at the same time necessary conditions for Pareto efficiency. In the following, confirm this conjecture: we show that, whenever our approach detects a Pareto inefficiency, the implicit weights for people in certain income ranges are indeed negative.

Welfare implications. Let there be a social welfare function,

$$\mathcal{W}(\tau, h) := \mathbf{E}\left[g(\theta)\; v(\tau, h, \theta)\right] \;,$$

where $g : \theta \mapsto g(\theta)$ specifies weights for different types of individuals. Upon using (3), the welfare implications of a refomr in direction h away from a given the status

[46]See, e.g., Christiansen and Jansen (1978), Blundell et al. (2009), Bourguignon and Spadaro (2012), Bargain et al. (2011), Jacobs et al. (2017), Lockwood and Weinzierl (2016), or Hendren (2020).

quo can be written as

$$\mathcal{W}_\tau(0,h) = \lambda_0 \left\{ R_\tau(0,h) - \frac{1}{\lambda_0} \mathbf{E}_y \left[\tilde{g}(y)\ h(y) \right] \right\} \ , \tag{28}$$

where $\lambda_0 = \mathbf{E}\left[g(\theta)\ u_c(\cdot)\right]$ is a welfare-weighted population average of the marginal utility of consumption, evaluated at the status quo policy with $\tau = 0$. The expression $\mathbf{E}_y\left[\tilde{g}(y)\ h(y)\right]$, by contrast, looks only at people whose tax burden is affected by the reform, i.e., people with $h[y_0(\theta)] \neq 0$. Specifically, $\tilde{g}(y) = \mathbf{E}\left[g(\theta)\ u_c(\cdot) \mid y_0(\theta) = y\right]$ is commonly referred to as the *marginal social welfare weight* of people with income y. Under appropriate regularity assumptions, the status quo tax system is welfare-maximizing if and only if $\mathcal{W}_\tau(0,h) = 0$ for any admissible direction h.

Pareto-improving reforms and the inverse tax problem. The canonical optimal tax problem is to find the tax policy that maximizes a given social welfare function. The literature on the *inverse tax problem*, by contrast, takes the tax policy (T_0, c_0) as given and tries to find the function $\tilde{g} : y \mapsto \tilde{g}(y)$ such that the first-order conditions are satisfied, i.e., $\mathcal{W}_\tau(0,h) = 0$ for any direction h. For example, Jacobs et al. (2017) show how, upon smoothing the tax schedule and income distribution, $\tilde{g}(y)$ can be recovered pointwise for each income level y. The solution $y \mapsto \tilde{g}(y)$ to this inverted problem can be interpreted as giving *implicit welfare weights* if the first-order conditions characterize a global welfare maximum, see Bourguignon and Spadaro (2012) for a detailed analysis.

As we show now, the existence of Pareto-improving reforms according to Theorem 1 implies that the implicit welfare weights are negative at certain income levels, and hence contradicts the maximization of a Paretian social welfare function.

Suppose that $\mathcal{R}(y_1) < 0$ so that a small one-bracket tax cut at income level y_1 is Pareto-improving. The marginal welfare implications of a single-bracket reform h^s at income y_1 are given by

$$\mathcal{W}_{\tau\ell}(0,h^s) = \lambda_0 \left\{ \mathcal{R}(y_1) - \frac{1}{\lambda_0} \left[1 - F_y(y_1)\right]\ \mathbf{E}_y\left[\tilde{g}(y) \mid y > y_1\right] \right\} .$$

If the first-order condition for welfare-maximization is satisfied, $\mathcal{W}_{\tau\ell}(0, h^s) = 0$, then we have

$$\mathbf{E}_y \left[\tilde{g}(y) \mid y > y_1\right] < 0 \ ,$$

i.e., on average, the implicit welfare weights of people with incomes above y_1 are negative.

Analogously, suppose that $\mathcal{R}(y_1) > 1$ so that a small one-bracket tax increase at income level y_1 is Pareto-improving. Then, the first-order condition requires that

$$\mathbf{E}_y \left[\tilde{g}(y) \mid y < y_1\right] < 0 \ ,$$

i.e., on average, the implicit welfare weights of people with incomes below y_1 are negative.

Finally, assume that a small two-bracket tax cut between incomes y_1 and $y_2 > y_1$ is Pareto-improving, $\mathcal{R}(y_2) > \mathcal{R}(y_1)$. The marginal welfare implications of such a two-bracket reform h_2 with $\tau_1 < 0$, $\tau_2 > 0$ and $\tau_1 \, \ell_1 + \tau_2 \, \ell_2 = 0$ are

$$\begin{aligned}
\mathcal{W}_{\tau\ell}(0, h_2) &= \lambda_0 \Big\{ \tau_1 \, \ell_1 \, \mathcal{R}(y_1) + \tau_2 \, \ell_2 \, \mathcal{R}(y_2) \\
&\qquad - \frac{1}{\lambda_0} \tau_1 \, \ell_1 \left[1 - F_y(y_1)\right] \ \mathbf{E}_y \left[\tilde{g}(y) \mid y > y_1\right] \\
&\qquad - \frac{1}{\lambda_0} \tau_2 \, \ell_2 \left[1 - F_y(y_2)\right] \ \mathbf{E}_y \left[\tilde{g}(y) \mid y < y_2\right] \Big\} \\
&= \lambda_0 \, \tau_2 \, \ell_2 \Big\{ \mathcal{R}(y_2) - \mathcal{R}(y_1) \\
&\qquad + \frac{1}{\lambda_0} \left[F_y(y_2) - F_y(y_1)\right] \ \mathbf{E}_y \left[\tilde{g}(y) \mid y \in (y_1, y_2)\right] \Big\} \ .
\end{aligned}$$

If such a reform satisfies the first-order condition, $\mathcal{W}_{\tau\ell}(0, h_2) = 0$, then

$$\mathbf{E}_y \left[\tilde{g}(y) \mid y \in (y_1, y_2)\right] < 0 \ ,$$

i.e., the implicit welfare weights of people with incomes between y_1 and y_2 are negative.

To sum up, whenever one of the necessary Pareto conditions from Theorem 1 is violated, the implicit welfare weights of people in some part of the income distribution are negative. This implies a contradiction to the maximization of a Paretian social welfare function.

F Sufficient statistics with behavioral responses at the intensive and the extensive margin

We consider a setting with two-dimensional heterogeneity. Individuals differ both in fixed and variable costs associated with the generation of income. Such a framework has been suggested by the literature that analyzes earnings subsidies from an optimal-tax perspective (see, e.g., Saez (2002), Jacquet et al. (2013), or Hansen (2021)). The analysis in this part of the Supplementary Material yields a characterization of $y \mapsto R^s_{\tau\ell}(0,0,y)$ that depends on labor supply elasticities at the intensive and the extensive margin.

Henceforth, fixed costs are captured by a parameter γ, variable costs by a parameter ω. Thus, we write $\theta = (\omega, \gamma)$ for an individual's type. Variable cost types and fixed cost types belong, respectively, to subsets of the positive reals that we denote by $\Omega = [\underline{\omega}, \overline{\omega}]$ and $\Gamma = [\underline{\gamma}, \overline{\gamma}]$. The joint distribution is denoted by F. The utility that an individual with type (ω, γ) derives from a (c, y)-pair that involves positive earnings is denoted by $u(c, y, \omega, \gamma)$. We denote by $u^{no}(c_0)$ the utility of individuals with no earnings. The function u^{no} is assumed to be increasing and weakly concave.

Variable costs. To capture variable costs, we assume that preferences satisfy the Spence-Mirrlees single-crossing property: Consider two individuals with the same fixed cost type γ, and an arbitrary point in the (c, y)-space with $y > 0$. At any such point, an individual with a higher ω-type has a flatter indifference curve. The interpretation is that, due to her lower variable costs, she needs less compensation for a marginal increase of her earnings. Formally, for any given γ, $\omega' > \omega$ implies that

$$-\frac{u_y(c, y, \omega', \gamma)}{u_c(c, y, \omega', \gamma)} < -\frac{u_y(c, y, \omega, \gamma)}{u_c(c, y, \omega, \gamma)} \ .$$

for any pair (c, y) with $y > 0$.

Let $C : y \mapsto C(y)$ be a non-decreasing function, interpreted as the boundary of a budget set that individuals face. An implication of the Spence-Mirrlees single crossing property is as follows: Consider two individuals who differ only in the variable cost type. If type ω weakly prefers an earnings level y' over an earnings level $y < y'$, then any type $\omega' > \omega$ strictly prefers y' over y. More formally, for any γ, any pair ω', ω with $\omega' > \omega$, and any pair y', y with $y' > y$,

$$u(C(y'), y', \omega, \gamma) \geq u(C(y), y, \omega, \gamma) \quad \text{implies} \quad u(C(y'), y', \omega', \gamma) > u(C(y), y, \omega', \gamma) .$$

Fixed costs. Fixed costs affect the compensation that individuals demand for positive earnings. Let $\pi(c, y, \omega, \gamma)$ be such that

$$u(c + \pi(c, y, \omega, \gamma), y, \omega, \gamma) = u^{no}(c_0) .$$

We assume that π is an increasing function of γ.[47]

An implication is as follows: Consider two individuals who differ only in their fixed-cost type γ. Given a non-decreasing consumption schedule $C : y \mapsto C(y)$, if type γ prefers an earnings level of 0 over an earnings level of $y > 0$, then any individual with a fixed-cost type $\gamma' > \gamma$, will also prefer 0 over y; for any ω, any pair γ', γ with $\gamma' > \gamma$, and any $y > 0$,

$$u^{no}(c_0) \geq u(C(y), y, \omega, \gamma) \quad \text{implies} \quad u^{no}(c_0) > u(C(y), y, \omega, \gamma') .$$

The Spence-Mirrlees single-crossing property preserves monotonicity of choices in variable costs. For a given continuous consumption schedule $C_0 : y \mapsto C_0(y)$, $y_0(\omega, \gamma)$ is the utility-maximizing choice of type (ω, γ). By the Spence-Mirrlees single-crossing property, $\omega' > \omega$ implies $y_0(\omega', \gamma) \geq y_0(\omega, \gamma)$. In particular, $y_0(\omega', \gamma) = 0$ implies $y_0(\omega, \gamma) = 0$. Thus, for any given γ, there is a cutoff type $\hat{\omega}_0(\gamma)$ so that $\omega < \hat{\omega}_0(\gamma)$ implies $y_0(\omega, \gamma) = 0$, whereas $\omega \geq \hat{\omega}_0(\gamma)$ implies $y_0(\omega, \gamma) > 0$.

The earnings function will generally exhibit an upward jump at $\hat{\omega}(\gamma)$. With C continuous, raising y slightly above 0 comes only with a small gain in consumption

[47] This property holds for various preference specifications that have been explored in the literature. In particular, it holds for separable utility functions of the form $u(c, y, \omega, \gamma) = \tilde{u}(c, y, \omega) - \gamma \ \mathbf{1}_{y>0}$, where $\mathbf{1}$ is the indicator function. It also holds for specifications with monetary fixed costs $u(c, y, \omega, \gamma) = \tilde{u}(c - \gamma \ \mathbf{1}_{y>0}, y, \omega)$. The two classes coincide if the function $\tilde{u}$ is quasi-linear in c.

utility, but an upward jump of effort costs. Thus, a significant increase of earnings is needed to have a gain in consumption utility that offsets these effort costs. Moreover, by our assumption on fixed costs, $\gamma' > \gamma$ implies that $\hat{\omega}(\gamma') \geq \hat{\omega}(\gamma)$.

The earnings function y_0 is bounded away from zero for all (ω, γ) with $\omega > \hat{\omega}(\gamma)$. Over this domain, we take y_0 to be a non-decreasing function of γ. Thus, $y_0(\omega, \gamma) > 0$, $y_0(\omega, \gamma') > 0$ and $\gamma' > \gamma$ imply that $y_0(\omega, \gamma') \geq y_0(\omega, \gamma)$.[48]

For a given reform direction h, we denote by $y^*(e, \tau, h, \omega, \gamma)$ the solution to the problem

$$\max_{y \geq 0} \; u(c_0 + e + y - T_0(y) - \tau \; h(y), y, \omega, \gamma)$$

and the corresponding indirect utility by $v(e, \tau, h, \omega, \gamma)$. The parameter e stands for a source of income that is exogenous from an individuals' perspective. In the subsequent analysis, e will be equal to the change in tax revenues, $e = R(\tau, h)$. If $y^*(e, \tau, h, \omega, \gamma) = 0$, then $v(e, \tau, h, \omega, \gamma) = u^{no}(c_0 + e)$.

The earnings function y^* exhibits a discontinuity at $\hat{\omega}(\cdot)$. Earnings are zero for types below $\hat{\omega}(\cdot)$ and bounded away from zero for types above. Individuals with type $\hat{\omega}(\cdot)$ are indifferent between earnings of zero and a strictly positive earnings level. It is convenient to assume that these individuals have positive earnings. Thus, we assume that

$$y^*(R(\cdot), \tau, \hat{\omega}(\cdot), \gamma) > 0 \; .$$

Intensive-margin responses. For one-bracket reforms, the derivative of the function y^*_τ with respect to τ gives how earnings respond to small changes in marginal tax rates for incomes that lie in that bracket. These are the behavioral responses at the intensive margin.

Extensive-margin responses. For a given reform direction h, we view the cutoff type $\hat{\omega}$ not only as a function of γ, but also as a function of the size of the

[48] For separable utility functions of the form $u(c, y, \omega, \gamma) = \tilde{u}(c, y, \omega) - \gamma \; \mathbf{1}_{y>0}$, $y_0(\omega, \gamma) > 0$, $y_0(\omega, \gamma') > 0$ and $\gamma' > \gamma$ imply that $y_0(\omega, \gamma') = y_0(\omega, \gamma)$. For specifications $u(c, y, \omega, \gamma) = \tilde{u}(c - \gamma \; \mathbf{1}_{y>0}, y, \omega)$ with concave consumption utility, $y_0(\omega, \gamma) > 0$, $y_0(\omega, \gamma') > 0$ and $\gamma' > \gamma$ imply that $y_0(\omega, \gamma') > y_0(\omega, \gamma)$.

reform as measured by τ. Formally, for given γ, the cutoff type $\hat{\omega}(\tau,\gamma)$ is defined as the value of ω that solves

$$u^{no}(c_0 + R(\tau,h)) = v(\tau,\omega,\gamma) \ .$$

The effect of a small change of the reform intensity τ on the cutoff type $\hat{\omega}$ is obtained by computing a total differential of this equation. This yields, invoking again the envelope theorem,

$$u_c^{no}(\cdot)\ R_\tau(\tau,h) = u_c(\cdot)\,(R_\tau(\tau,h) - h(\cdot)) + u_\omega(\cdot)\ \hat{\omega}_\tau(\tau,\gamma)\ ,$$

where the functions u_c, u_ω and h are evaluated at $y = y^*(R(\cdot),\tau,\hat{\omega}(\cdot),\gamma)$. Equivalently,

$$\hat{\omega}_\tau(\tau,\gamma) = \frac{u_c^{no}(\cdot)}{u_\omega(\cdot)} R_\tau(\tau,h) - \frac{u_c(\cdot)}{u_\omega(\cdot)}\,(R_\tau(\tau,h) - h(\cdot))\ . \tag{29}$$

To interpret these expressions, consider the following thought experiment: a fraction $F_\Omega(\hat{\omega}(\tau,\gamma) \mid \gamma)$ of individuals with fixed cost type γ has zero earnings, where $F_\Omega(\cdot \mid \gamma)$ is the distribution of variable cost types ω conditional on the fixed cost type being γ. Consider a small increase of transfers only for the unemployed, the marginal effect on $F_\Omega(\cdot \mid \gamma)$ is given by

$$P^{no}(\tau,\gamma) := f_\Omega(\hat{\omega}(\tau,\gamma) \mid \gamma)\ \frac{u_c^{no}(\cdot)}{u_\omega(\cdot)}\ , \tag{30}$$

where the letter P is chosen to indicate a marginal effect on Participation. Alternatively, the effect of a transfer only to those with positive earnings is given by

$$P(\tau,\gamma) = f_\Omega(\hat{\omega}(\tau,\gamma) \mid \gamma)\ \frac{u_c(\cdot)}{u_\omega(\cdot)}\ . \tag{31}$$

Thus, assuming that the distribution F and these marginal effects are known is equivalent to assuming that the extensive-margin elasticities and semi-elasticities that are ubiquitous in the related literature are known.

Notation. It is convenient to use a shorthand for endogenous variables at the status quo. For instance, we will occasionally write $\hat{\omega}_0(\gamma) := \hat{\omega}(0,\gamma)$ for the type at the participation margin, among those with fixed cost type γ. We write $y_0(\omega, \gamma) := y^*(0, 0, h, \omega, \gamma)$ for income in the status quo, and similarly for other variables. Given a fixed-cost type γ, we denote by $\omega_0(y, \gamma)$ the variable cost type who chooses earnings of y in the status quo. If we evaluate partial derivatives at the status quo, we occasionally write $\hat{\omega}_{0\tau}(\gamma)$ or $y_{0e}(\omega, \gamma)$ and so on. We, moreover, write $\hat{y}_0(\gamma) := y_0(\hat{\omega}_0(\gamma), \gamma)$ for the status quo income of the cutoff type among those with fixed costs of γ. Finally,

$$\mathcal{K}(\omega_0(y, \gamma) \mid \gamma) = E_\Omega \left[T_0'(y_0(s, \gamma))\; y_{0e}(s, \gamma) \mid s \geq \omega_0(y, \gamma), \gamma\right] \ ,$$

is a measure of the size of income effects among those individuals with fixed cost type γ who have earnings exceeding y.

Revenue implications of reforms with one bracket. By our analysis in the previous section, to understand whether a given tax system can be reformed in a Pareto-improving way, we need to check whether the function $y \mapsto R^s_{\tau\ell}(0, 0, y)$ is bounded from below by 0, bounded from above by 1, and non-increasing. The following proposition provides a characterization of this function for the given setup with variable and fixed costs of productive effort. A version of this function in terms of empirically estimable objects can be found in (26) in part C of the Supplementary Material.

Proposition 4 *Suppose that, for any given γ, $\omega_0(y, \gamma)$ is strictly increasing in y, whenever $y > 0$. Also suppose that, for all (ω, γ), income in the status quo satisfies the first-order conditions of utility-maximization whenever $y_0(\omega, \gamma) > 0$. Then,*

$$R^s_{\tau\ell}(0, 0, y) = (1 - \mathcal{M}_0)^{-1} \Big(\mathcal{I}(y) - \mathcal{X}(y)\Big) \ ,$$

where

$$\mathcal{M}_0 = E_\Gamma\Big[(1 - F(\hat{\omega}_0(\gamma) \mid \gamma))\; \mathcal{K}(\hat{\omega}_0(\gamma) \mid \gamma) - T_0(\hat{y}_0^*(\gamma))\; \left(P_0^{no}(\gamma) - P_0(\gamma)\right)\Big] \ ,$$

$$\begin{aligned}\mathcal{I}(y) \quad &= \quad T_0'(y)\; E_\Gamma \left[f(\omega_0(y, \gamma) \mid \gamma) \tfrac{y_{0\tau}(\omega_0(y,\gamma),\gamma)}{y_{0\omega}(\omega_0(y,\gamma),\gamma)}\right] \\ &\quad + E_\Gamma \left[(1 - \mathcal{K}(\omega_0(y, \gamma) \mid \gamma))(1 - F(\omega_0(y, \gamma) \mid \gamma))\right] \ ,\end{aligned}$$

and

$$\mathcal{X}(y) = \int_{\underline{\gamma}}^{\overline{\gamma}} T_0(\hat{y}_0(\gamma))\ P_0(\gamma)\ \mathbf{1}(\hat{y}_0(\gamma) \geq y)\ f_\Gamma(\gamma) d\gamma\ .$$

The proof of Proposition 4 can be found below. Proposition 4 shows that the revenue effect of a small single-bracket reform at income level y, $R^s_{\tau\ell}(0,0,y)$, can be decomposed into a term $\mathcal{I}(y)$ that includes a mechanical effect and an intensive-margin effect, an extensive-margin effect $\mathcal{X}(y)$, and a multiplier $\mathcal{M}_0$. The assumptions that $\omega_0(y,\gamma)$ is strictly increasing in y and that individual behavior can be described by first-order conditions are made for ease of exposition. They avoid complications due to bunching.

The term $\mathcal{I}(y)$ consists of a mechanical effect, a behavioral response from an income effect and a behavioral response from a substitution effect. The mechanical effect is that individuals with an income larger than y now pay additional taxes. This yields a revenue gain that is proportional to the mass of these people, $E_\Gamma\left[1 - F(\omega_0(y,\gamma) \mid \gamma)\right]$. With income effects, these people also seek to make up for the fact that the tax reform makes them poorer and they respond with an increase of their earnings. Together the mechanical and the income effect amount to $E_\Gamma\left[(1 - \mathcal{K}(\omega_0(y,\gamma) \mid \gamma))(1 - F(\omega_0(y,\gamma) \mid \gamma))\right]$. The substitution effect is relevant for people with an income of y. They have less of an incentive to exert productive effort when the marginal tax rate for incomes close to y is increased. This is captured by the expression

$$T_0'(y)\ E_\Gamma\left[f(\omega_0(y,\gamma) \mid \gamma)\frac{y_{0\tau}(\omega_0(y,\gamma),\gamma)}{y_{0\omega}(\omega_0(y,\gamma),\gamma)}\right]\ .$$

Alternatively, this term can be written as

$$\frac{T_0'(y)}{1 - T_0'(y)}\left(1 + \frac{1}{\varepsilon_0(y)}\right) E_\Gamma\left[f(\omega_0(y,\gamma) \mid \gamma)\right]\ ,$$

where $\varepsilon_0(y)$ is the elasticity of earnings with respect to the net-of-tax rate among people with pre-reform earnings of y.

The extensive-margin effect is shaped by the employment response of those individuals who are close to indifferent between staying out of the labor market and entering. More specifically, $\mathcal{X}(y)$ gives an average for all types who choose earnings of at least y when entering the labor force. A change of the marginal rates in a bracket that begins at y has no effect on individuals who only consider incomes lower than y. For those who consider an income of y or above, there is a negative effect on participation and this tends to lower the revenue that is raised by such a reform. Naturally, the effect of this employment response on tax revenue depends on the tax payment that these individuals pay when entering the labor force and which is lost when they stay out.

The multiplier $\mathcal{M}_0$ mitigates all previously discussed effects. It reflects income effects at both margins, which appear if utility is non-linear in consumption. In this case, a tax reform that yields a revenue gain and hence increased lump-sum transfers has a negative effect on the earnings of market participants at the intensive margin. This effect is captured by the first component of $\mathcal{M}_0$,

$$E_\Gamma\Big[(1 - F(\hat{\omega}_0(\gamma) \mid \gamma))\,\mathcal{K}(\hat{\omega}_0(\gamma) \mid \gamma)\Big] .$$

In addition, there are also income effects at the extensive margin. Specifically, the increased lump-sum transfers makes non-participation more attractive with concave consumption utility such that $(P_0^{no}(\gamma) - P_0(\gamma)) > 0$. This effect is captured by the second component in $\mathcal{M}_0$. It implies a loss of tax revenue if $T_0(y) > 0$.

F.1 Proof of Proposition 4

The change in tax revenue associated with a one-bracket reform (τ, ℓ, y_a) can be written as

$$R^s(\tau, \ell, y_a) = \mathbb{E}_\Gamma\left[\mathcal{R}^s(\tau, \ell, y_a \mid \gamma)\right] ,$$

where $\mathcal{R}^s(\tau, \ell, y_a \mid \gamma)$ is the change in tax revenue due to individuals with a given fixed cost type γ, and $\mathbb{E}_\Gamma$ is an expectations operator that indicates the computation of a population average using the marginal distribution F_Γ of fixed costs. Also, note that

$$\begin{aligned}\mathcal{R}^s(\tau, \ell, y_a \mid \gamma) &= \mathbb{E}_\Omega\left[T_1(y^*(R^s(\cdot), \tau, h, \omega, \gamma) - T_0(y_0(\omega, \gamma)) \mid \gamma\right] \\ &= \textstyle\int_{\underline{\omega}}^{\overline{\omega}} \{T_1(y^*(R^s(\cdot), \tau, h, \omega, \gamma) - T_0(y_0(\omega, \gamma))\} f_\Omega(\omega \mid \gamma)\, d\omega ,\end{aligned}$$

where $f_\Omega(\cdot \mid \gamma)$ is the density of the conditional distribution of ω for given γ. The change in revenue associated with a marginal change of τ can be written as

$$R^s_\tau(\tau, \ell, y_a) = \mathbb{E}_\Gamma \left[\mathcal{R}^s_\tau(\tau, \ell, y_a \mid \gamma) \right] \ , \tag{32}$$

where

$$\begin{aligned} \mathcal{R}^s_\tau(\tau, \ell, y_a \mid \gamma) &= \frac{d}{d\,\tau} \int_{\underline{\omega}}^{\overline{\omega}} T_1(y^*(R^s(\cdot), \tau, h, \omega, \gamma) f_\Omega(\omega \mid \gamma)\, d\omega \\ &= \frac{d}{d\,\tau} \int_{\hat{\omega}(\tau,\gamma)}^{\overline{\omega}} T_1(y^*(R^s(\cdot), \tau, h, \omega, \gamma) f_\Omega(\omega \mid \gamma)\, d\omega \ . \end{aligned}$$

By standard arguments,

$$\begin{aligned} \mathcal{R}^s_\tau(\tau, \ell, y_a \mid \gamma) &= \frac{d}{d\,\tau} \int_{\hat{\omega}(\tau,\gamma)}^{\overline{\omega}} T_1(y^*(R^s(\cdot), \tau, h, \omega, \delta) f_\Omega(\omega \mid \gamma)\, d\omega \\ &= -T_1(y^*(R^s(\cdot), \tau, h, \hat{\omega}(\cdot), \gamma)) f(\hat{\omega}(\cdot) \mid \gamma)\, \hat{\omega}_\tau(\cdot) \\ &\quad + \int_{\hat{\omega}(\tau,\gamma)}^{\overline{\omega}} \frac{d}{d\tau}\, T_1(y^*(R^s(\cdot), \tau, h, \omega, \gamma) f_\Omega(\omega \mid \gamma)\, d\omega \ , \end{aligned} \tag{33}$$

i.e., the change of tax revenues can be decomposed into a change that comes from extensive-margin responses and a change that comes from intensive-margin responses to the tax reform.

A small change of the marginal tax rate. Equations (32), (33) and (29) imply that

$$R^s_\tau(0, \ell, y_a) = \mathbb{E}_\Gamma \left[\mathcal{R}^s_\tau(0, \ell, y_a \mid \gamma) \right] \ , \tag{34}$$

where

$$\begin{aligned} \mathcal{R}^s_\tau(0, \ell, y_a \mid \gamma) &= -T_0(y_0(\hat{\omega}_0(\gamma), \gamma)) f(\hat{\omega}_0(\gamma) \mid \gamma)\, \hat{\omega}_{0\tau}(\cdot) \\ &\quad + \int_{\hat{\omega}_0(\gamma)}^{\overline{\omega}} \left(\frac{d}{d\tau}\, T_1(y^*(R^s(\cdot), \tau, h, \omega, \gamma) \right)_{|\tau=0} f_\Omega(\omega \mid \gamma)\, d\omega \ , \end{aligned} \tag{35}$$

and

$$\hat{\omega}_{0\tau} = \frac{u^{no}_{0c}(\cdot)}{u_{0\omega}(\cdot)} R^s_\tau(0, \ell, y_a) - \frac{u_{0c}(\cdot)}{u_{0\omega}(\cdot)} \left(R^s_\tau(0, \ell, y_a) - h(y_0(\hat{\omega}_0(\gamma), \gamma)) \right) \ . \tag{36}$$

By the arguments in Bierbrauer and Boyer (2018),

$$\int_{\hat{\omega}_0(\gamma)}^{\overline{\omega}} \left(\frac{d}{d\tau}\, T_1(y^*(R^s(\cdot), \tau, \omega, \delta) \right)_{|\tau=0} f_\Omega(\omega \mid \gamma)\, d\omega$$

$$= R^s_\tau(0, \ell, y_a)\, (1 - F_\Omega(\hat{\omega}_0(\gamma) \mid \gamma))\, \mathcal{K}(\hat{\omega}_0(\gamma) \mid \gamma) + \mathcal{J}(\ell, y_a \mid \gamma) \tag{37}$$

where

$$\mathcal{K}(\hat{\omega}_0(\gamma) \mid \gamma) = E_\Omega \left[T_0'(y_0(s, \gamma))\, y_{0e}(s, \gamma) \mid s \geq \hat{\omega}_0(\gamma), \gamma \right] ,$$

as defined in the body of the text, and

$$\begin{aligned} \mathcal{J}(\ell, y_a \mid \gamma) \;\; = \;\; & \int_{\omega_0(y_a|\gamma)}^{\omega_0(y_a+\ell\,|\gamma)} T_0'(y_0(\omega, \gamma))\, y_{0\tau}(\omega, \gamma)\, f(\omega \mid \gamma) d\omega \\ & + \int_{\omega_0(y_a|\gamma)}^{\omega_0(y_a+\ell\,|\gamma)} (y_0(\omega, \gamma) - y_a) \left[1 - T_0'(y_0(\omega, \gamma))\, y_{0e}(\omega, \gamma) \right] f(\omega \mid \gamma) d\omega \\ & + \ell \Big(1 - F(\omega_0(y_a + \ell \mid \gamma)) \\ & - \int_{\omega_0(y_a+\ell\,|\gamma)}^{\overline{\omega}} T_0'(y_0(\omega, \gamma))\, y_{0e}(\omega, \gamma) f(\omega \mid \gamma) d\omega \Big) \end{aligned}$$

and $\omega_0(y_a \mid \gamma)$ and $\omega_0(y_a + \ell \;\mid \gamma)$ are, respectively, the ω-types who choose income levels of y_a and $y_a + \ell$ in the status quo. Equations (30), (31), (35), (36) and (37) imply that

$$\begin{aligned} R^s_\tau(0, \ell, y_a \mid \gamma) \;\; = \;\; & R^s_\tau(0, \ell, y_a)\, M_0(\gamma) \\ & - T_0(y_0(\hat{\omega}_0(\gamma), \gamma))\, P_0(\gamma)\, h(y_0(\hat{\omega}_0(\gamma), \gamma)) \\ & + \mathcal{J}(\ell, y_a \mid \gamma) \end{aligned} \tag{38}$$

where the multiplier $M_0(\gamma)$ is given by

$$M_0(\gamma) := (1 - F(\hat{\omega}_0(\gamma)) \mid \gamma)\, \mathcal{K}(\hat{\omega}_0(\gamma) \mid \gamma) - T_0(y_0(\hat{\omega}^0(\gamma), \gamma))\, \left(P_0^{no}(\gamma) - P_0(\gamma) \right) .$$

We let $\mathcal{M}_0 = E_\Gamma[M_0(\gamma)]$. Then, equations (34) and (38) imply that

$$R^s_\tau(0, \ell, y_a) = (1 - \mathcal{M}_0)^{-1} \Big(E_\Gamma[\mathcal{J}(\ell, y_a \mid \gamma)] - E_\Gamma[\mathcal{Z}(\ell, y_a \mid \gamma)] \Big) , \tag{39}$$

where

$$\mathcal{Z}(\ell, y_a \mid \gamma) = T_0(y_0(\hat{\omega}_0(\gamma), \gamma))\;\; P_0(\gamma)\, h(y_0(\hat{\omega}_0(\gamma), \gamma)) .$$

If y_a is a very high, the tax reform affects only high incomes. Plausibly, y_a is then also above the income level that individuals at the extensive margin would consider, i.e. $y_a > \max_\gamma \; y_0(\hat{\omega}_0(\gamma), \gamma)$. In this case, $h(y_0(\hat{\omega}_0(\gamma), \gamma)) = 0$ and hence $\mathcal{Z}(\ell, y_a \mid \gamma) = 0$.

A small change of the marginal tax rate for a narrow bracket. We are interested in determining how much additional tax revenue a reform generates that involves a small change of marginal tax rates for incomes in a narrow bracket. To this end, we provide a characterization of $R^s_{\tau\ell}(0,0,y_a)$, i.e., of the cross-derivative of tax revenue with respect to the change of the marginal tax rate τ and the length of the bracket ℓ at the status quo. To understand the logic of this exercise, note that our previous derivations imply that $R^s_\tau(0,0,y_a)=0$, i.e., if the length of the bracket is zero, the marginal tax rate remains unchanged at each income level. Consequently, there is no effect on tax revenue. If, however, $R^s_{\tau\ell}(0,0,y_a)>0$, then moving from $\ell=0$ to some $\ell=\varepsilon$ for $\varepsilon>0$ but small, implies that $R^s_\tau(0,\varepsilon,y_a)>0$, so that a small change of the marginal tax rate then has a positive effect on revenue. More formally, if $R^s_{\tau\ell}(0,0,y_a)>0$ there exist $\delta>0$ and $\varepsilon>0$ so that $R^s(\delta,\varepsilon,y_a)>0$.

It follows from equation (39) that

$$R^s_{\tau\ell}(0,0,y_a)=(1-\mathcal{M}_0)^{-1}\Big(E_\Gamma[\mathcal{J}_\ell(0,y_a\mid\gamma)]-E_\Gamma[\mathcal{Z}_\ell(0,y_a\mid\gamma)]\Big)\ , \tag{40}$$

where, by the arguments in Bierbrauer and Boyer (2018),

$$\begin{aligned} E_\Gamma[\mathcal{J}_\ell(0,y_a\mid\gamma)] \;&=\; T_0'(y_a)E_\Gamma\left[f(\omega_0(y_a,\gamma)\mid\gamma)\frac{y_{0\tau}(\omega_0(y_a,\gamma),\gamma)}{y_{0\omega}(\omega_0(y_a,\gamma),\gamma)}\right] \\ &\quad +E_\Gamma\left[(1-\mathcal{K}(\omega_0(y_a,\gamma)\mid\gamma))(1-F(\omega_0(y_a,\gamma)\mid\gamma))\right] . \end{aligned} \tag{41}$$

We now work towards a characterization of $\mathcal{Z}_\ell(0,y_a):=E_\Gamma[\mathcal{Z}_\ell(0,y_a\mid\gamma)]$. Suppose that $y_a\le\max_\gamma\ y_0(\hat\omega_0(\gamma),\gamma)$ and denote by $\gamma_0(y_a)$ and $\gamma_0(y_a+\ell)$, respectively, the types at the extensive margin who earn, respectively, incomes of y_a and $y_b=y_a+\ell$, i.e.

$$y_0(\hat\omega_0(\gamma_0(y_a)),\gamma_0(y_a))=y_a\ ,\quad\text{and}\quad y_0(\hat\omega_0(\gamma_0(y_a+\ell)),\gamma_0(y_a+\ell))=y_a+\ell\ .$$

Armed with this notation, we can write

$$\begin{aligned} E_\Gamma[\mathcal{Z}_\ell(\ell,y_a\mid\gamma)] \;&=\; \int_{\gamma_0(y_a)}^{\gamma_0(y_a+\ell)} T_0(\hat y_0(\gamma))\ P_0(\gamma)\ \ (\hat y_0(\gamma)-y_a)\ \ f_\Gamma(\gamma)\,d\gamma \\ &\quad +\ell\int_{\gamma_0(y_a+\ell)}^{\overline{\gamma}} T_0(\hat y_0(\gamma))\ P_0(\gamma)\ \ f_\Gamma(\gamma)\,d\gamma\ , \end{aligned}$$

where $\hat{y}_0(\gamma)$ is a shorthand for $y_0(\hat{\omega}_0(\gamma),\gamma)$. Straightforward computations yield

$$\mathcal{Z}_\ell(0,y_a) = \int_{\gamma_0(y_a)}^{\overline{\gamma}} T_0(\hat{y}_0(\gamma))\ P_0(\gamma) f_\Gamma(\gamma) d\gamma\ .$$

Now, to accommodate both the case $y_a \leq \max_\gamma\ \hat{y}_0(\gamma)$ and the case $y_a > \max_\gamma\ \hat{y}_0(\gamma)$, we will write henceforth

$$\mathcal{Z}_\ell(0,y_a) = \int_{\underline{\gamma}}^{\overline{\gamma}} T_0(\hat{y}_0(\gamma))\ P_0(\gamma)\ \mathbf{1}(\hat{y}_0(\gamma) \geq y_a)\ f_\Gamma(\gamma) d\gamma\ .$$

Proposition 4 as stated in the text now follows upon adopting the more concise notation

$$\mathcal{I}(y) := \mathcal{J}_\ell(0,y) \quad \text{and} \quad \mathcal{X}(y) := \mathcal{Z}_\ell(0,y)\ ,$$

for any earnings level y.

F.2 Implications of Proposition 4

F.2.1 Diamond's model

With quasi-linear in consumption preferences and without fixed costs of labor market participation, we have

$$\mathcal{M}_0 = 0\ , \quad \text{and} \quad \mathcal{X}(y) = 0\ ,$$

and

$$\mathcal{I}(y) = T_0'(y)\ \ f(\omega_0(y))\ \ \frac{y_{0\tau}(\omega_0(y))}{y_{0\omega}(\omega_0(y))} + 1 - F(\omega_0(y))\ . \tag{42}$$

This can be rewritten as

$$\mathcal{I}(y) = -\frac{T_0'(y)}{1-T_0'(y)}\ \ f(\omega_0(y))\ \ \left(1+\frac{1}{\varepsilon_0(y)}\right)^{-1} \omega_0(y) + 1 - F(\omega_0(y))\ . \tag{43}$$

Getting from equation (42) to equation (43) requires to invoke the first-order condition characterizing $y_0(\omega)$ for the purpose of deriving comparative statics results that yield a characterization of $y_{0\tau}(\omega_0(y))$ and $y_{0\omega}(\omega_0(y))$. Thus,

$$R^s_{\tau\ell}(0,0,y) = -\frac{T_0'(y)}{1-T_0'(y)}\ \ f(\omega_0(y))\ \ \left(1+\frac{1}{\varepsilon_0(y)}\right)^{-1} \omega_0(y) + 1 - F(\omega_0(y))\ ,$$

which is equation (7) in the main text using that

$$F_y(y) = F(\omega_0(y)) \quad \text{and} \quad f_y(y) = f(\omega_0(y))\, \omega_0'(y) = f(\omega_0(y))\ \frac{1}{y_{0\omega}(\omega_0(y))}\ .$$

and that, for any y in $y_0(\Omega)$, $\varepsilon_0(y) = -\frac{y}{1-T_0'(y)}\ y_{0\tau}(\cdot)$.

F.2.2 Fixed costs as an extension of Diamond's model

We now consider an extension of Diamond (1998) that includes fixed costs of labor market participation. Preferences are now given by

$$u(c, y, \omega, \gamma) = c - \frac{1}{1 + \frac{1}{\epsilon}} \left(\frac{y}{\omega}\right)^{1+\frac{1}{\epsilon}} - \gamma \; \mathbb{1}_{y>0} \; ,$$

where ϵ is a parameter. Again, the absence of income effect implies that $\mathcal{M}_0 = 0$. We can, once more, rewrite $\mathcal{I}(y)$ using the distribution of incomes F_y so that

$$F_y(y) = E_\Gamma \left[F_\Omega(\omega_0(y, \gamma) \mid \gamma)\right] \; .$$

This yields

$$\mathcal{I}(y) = -\frac{T_0'(y)}{1 - T_0'(y)} \; \varepsilon_0(y) \; y f_y(y) \; + 1 - F_y(y) \; .$$

While this expression looks exactly as in Diamond's model, the distribution of incomes F_y is now shaped by the joint distribution of fixed and variable costs. We also rewrite

$$\mathcal{X}(y) = \int_{\underline{\gamma}}^{\overline{\gamma}} T_0(\hat{y}_0(\gamma)) \; P_0(\gamma) \; \mathbf{1}(\hat{y}_0(\gamma) \geq y) \; f_\Gamma(\gamma) \; d\gamma \; .$$

in a way that is more handy in the context of our application: We first note that, with quasi-linear in consumption preferences, $P_0(\gamma) = f_\Omega(\hat{\omega}_0(\gamma) \mid \gamma)$, and therefore

$$\mathcal{X}(y) = \int_{\underline{\gamma}}^{\overline{\gamma}} T_0(\hat{y}_0(\gamma)) \; \mathbf{1}(\hat{y}_0(\gamma) \geq y) \; f_\Omega(\hat{\omega}_0(\gamma) \mid \gamma) \; f_\Gamma(\gamma) \; d\gamma \; .$$

We then note that $\hat{y}_0$ is an increasing function.[49] Thus, if we denote by $\gamma_0(y)$ be value of γ for which $\hat{y}_0(\gamma) = y$, then we can write $\mathcal{X}(y)$ as

$$\mathcal{X}(y) = \int_{\gamma_0(y)}^{\overline{\gamma}} T_0(\hat{y}_0(\gamma)) \; f_\Omega(\hat{\omega}_0(\gamma) \mid \gamma) \;\; f_\Gamma(\gamma) d\gamma \; .$$

[49]This follows from the following observations. First, recall that, by definition, $\hat{y}_0(\gamma) = y_0(\hat{\omega}_0(\gamma), \gamma)$. Second, with quasi-linear in consumption preferences, the function $y_0 : (\omega, \gamma) \mapsto y_0(\omega, \gamma)$ is increasing in the first argument and constant in the second argument. Third, $\hat{\omega}_0 : \gamma \mapsto \hat{\omega}_0(\gamma)$ is increasing, as higher rents from labor market participation are needed to offset larger fixed costs.

We seek an interpretation of $\mathcal{X}(y)$ in terms of extensive-margin elasticities. Therefore, consider the following thought experiment: For pre-tax incomes in an interval $[y_0, y]$, after-tax incomes are marginally decreased. Overall employment is then reduced by

$$L_E(y_0, y) = \int_{\gamma_0(y_0)}^{\gamma_0(y)} \frac{f_\Omega(\hat{\omega}_0(\gamma) \mid \gamma)}{u_\omega(\ )} f_\Gamma(\gamma)\, d\gamma \ .$$

Denote the derivative of $L_E(y_0, y)$ with respect to y by $l_E(y)$ and apply Leibnitz rule to derive

$$l_E(y) = f_\Omega(\hat{\omega}_0(\gamma_0(y)) \mid \gamma_0(y))\, u_\omega(\ )^{-1}\, f_\Gamma(\gamma_0(y))\, \gamma_0'(y) \ .$$

Using the fundamental theorem of calculus, we can now also write

$$L_E(y_0, y) = \int_{y_0}^{y} l_E(y')\, dy' \ .$$

Thus, $l_E(y)$ is a marginal effect. One that measures the mass of people flowing out of the pool of people who earn y and into the pool of people who earn zero, when consumption for people earning y is slightly reduced. We will now rewrite $\mathcal{X}(y)$ in a way that highlights the significance of this employment response. We perform a substitution using

$$y' = \hat{y}_0(\gamma) \quad \text{and} \quad dy' = \frac{\partial\, \hat{y}_0^*(\gamma)}{\partial\, \gamma}\, d\gamma \ .$$

Also, note that $\gamma_0 : y' \mapsto \gamma_0(y')$ is the inverse of the function $\hat{y}_0$, so that

$$\gamma_0'(y')\, dy' = d\gamma$$

Finally, note that $\hat{y}_0(\gamma_0(y')) = y'$. Thus,

$$\begin{aligned}
\mathcal{X}(y) &= \int_{y}^{\overline{y}_0} T_0(y')\, \frac{f_\Omega(\hat{\omega}_0(\gamma_0(y')) \mid \gamma_0(y'))}{u_\omega(\ )}\, f_\Gamma(\gamma_0(y'))\, \gamma_0'(y')\, dy' \\
&= \int_{y}^{\overline{y}_0} T_0(y')\, l_E(y')\, dy' \\
&= \int_{y}^{\overline{y}_0} \frac{T_0(y')}{y' - T_0(y')}\, \frac{l_E(y')}{f_y(y')}\, (y' - T_0(y'))\ f_y(y')\, dy' \\
&= \int_{y}^{\overline{y}_0} \frac{T_0(y')}{y' - T_0(y')}\, \pi_0(y')\, f_y(y')\, dy' \ ,
\end{aligned}$$

where $\overline{y}_0$ is the highest level of y in the support of F_y. We interpret

$$\pi_0(y') := \frac{l_E(y')}{f_y(y')}\ (y' - T_0(y'))$$

as an extensive-margin elasticity, it relates a percentage change in the mass of people at pre-tax income y due to extensive-margin responses to a percentage change in their after-tax labor income $y - T_0(y)$. The literature typically refers to $\mathbb{E}_y[\pi_0(y')]$ as the participation elasticity.

Upon collecting terms, and upon assuming an unbounded distribution of income in the status quo, we obtain

$$\begin{aligned} R^s_{\tau\ell}(0,0,y) &= 1 - F_y(y) - \varepsilon_0(y)\, y\, f_y(y)\, \frac{T_0'(y)}{1-T_0'(y)} \\ &\quad - \int_y^\infty f_y(y')\, \pi_0(y')\, \frac{T_0(y')}{y' - T_0(y')}\, dy' \, . \end{aligned} \tag{44}$$

Note that (44) coincides with equation (8) in the body of the text.

G Analysis of the 2018 US tax-transfer system

There were many reforms of the US tax-and-transfer system since the mid-1970s. To get a sense of whether there was progress in the US tax policy over time, this section studies the 2018 tax-and-transfer system for single parents. We find that it was still not Pareto-efficient. But, based on the inefficiency measure introduced in Section 4, the inefficiencies in the current tax system are quantitatively an order of magnitude smaller than those in the mid-1970s.

Calibration. Again, our analysis focuses on the group of single parents with two children in California. We parametrize the revenue function given in (8) for the 2018 tax system that applied to this group.

We take account of the federal income tax and various transfer and welfare programs: the federal Earned Income Tax Credit, the Child Tax Credit (CTC, a partly refundable tax credit introduced in 1998), SNAP and Temporary Assistance for Needy Families (TANF, the successor of AFDC). Table 2 below presents a summary of the sources that we use for this purpose. As of 2018, the maximum

Table 2: Sources for US tax-transfer system, 2018.

Information	Sources
Income tax	Internal Revenue Service, "1040 Instructions, Tax Year 2018", 2019. Accessible at `https://www.irs.gov/pub/irs-dft/i1040gi--dft.pdf`.
EITC	Tax Policy Center, "Earned Income Tax Credit Parameters, 1975-2018", 2018. Accessible at `https://www.taxpolicycenter.org/file/178859`.
AFDC	CCWRO, "CalWORKs (also known as AFDC/TANF)", 2018. Accessible at `https://www.ccwro.org/advocateresources/public-assistance-table`.
	City and County of San Francisco Human Services Agency, "CalWORKs Eligibility Handbook", 2018. URL: `https://www.sfhsa.org/file/6406`.
SNAP	New Mexico Department of Human Services, "Income Eligibility Guidelines for SNAP and Financial Assistance", 2018. URL: `https://www.hsd.state.nm.us/uploads/FileLinks/26463f122f47474487faee4922e09ce8/ISD_017_Income_Eligibility_Guidelines_for_SNAP_and_Financial_Assistance__FFY18_1.pdf`.

EITC amounts to 5,716 USD for single taxpayers with two children. It is phased in at a rate of 0.4 for annual incomes below 14,290 USD and phased out at a rate of 0.2106 for incomes between 18,660 and 45,802 USD.

Figure 23 shows the effective marginal tax, $T_0'(y)$, and the participation tax rate, $T_0(y)/y$, for single parents that result from the interplay of these programs. In 2018, effective marginal tax rates for low incomes are much lower than in the 1970s. Most notably, they are negative due to the EITC phase-in for incomes below 2,700 USD (only), and close to 60% in an income range between 14,300 and 18, 500 USD. In contrast to the 1974 tax system, the end of the EITC phase-in range in now closely aligned to the full phasing out of transfers (TANF, SNAP) around 18, 000 USD.

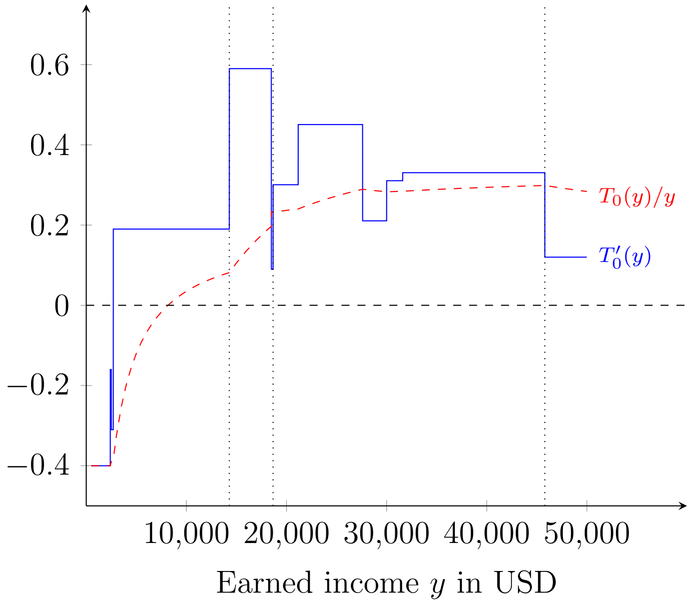


Figure 23: US income tax and transfer schedule in 2018 for single parents.

Notes: Figure 23 shows the 2018 effective marginal tax $T_0'(y)$ (solid blue line) and the participation tax rate $T_0(y)/y$ (dashed red line) for single parents with two children as functions of earned income in 2018 USD. The dotted vertical lines mark the endpoint of the phase-in range at 14, 290 USD, the starting point of the phase-out range at 18, 660 USD, and the endpoint of the phase-out range at 45, 802 USD.

Source: Authors' calculations (see Table 2 for details).

We estimate the income distribution among single parents based on data from the March 2019 wave of the CPS. As before, we use a non-parametric kernel density estimation. The share of single parents with no income has gone down considerably since the mid-1970s. In 2018, it amounted to 15.2%, about half of the share in the 1970s. Around 54.1% of single parents had strictly positive incomes below 45, 802 USD and were, therefore, eligible for the EITC. In our data, 81.4% of single parents were female.

For the behavioral responses to taxation, we stick to the benchmark case with an intensive-margin elasticity of 0.33 and participation elasticities that are decreasing from 0.67 for very low income levels to 0.4 for incomes above 50.000 USD.

Empirical results. Figure 24 plots the revenue function $y \mapsto \mathcal{R}_{sp}(y)$ for the US tax-and-transfer system for single parents in 2018. Remember that, for the 1974 US tax system, this revenue function violated two necessary conditions for Pareto efficiency: It was negative at incomes between $2,000$ and $5,500$ USD, and increasing in the range between $5,000$ and $6,000$ USD. Hence, there existed both Pareto-improving one-bracket reforms and Pareto-improving two-bracket reforms. By contrast, the 2018 version of $y \mapsto \mathcal{R}_{sp}(y)$ is bounded from below by 0 and from above by 1. Hence, we no longer find Pareto improvements by means of one-bracket reforms. But there are still several income ranges where the function is increasing. This indicates that, even today, there exist Pareto-improving tax reforms with two brackets.

The visual impression from Figure 24 is that, despite these non-monotonicities, the inefficiencies in the 2018 US tax system are smaller in magnitude than those in the 1974 tax system. To verify this conjecture, we apply the method introduced in Section 4 to find the revenue-maximizing tax cut – the *best* Pareto-improving reform at the time – and use the net revenue gain from this reform as a measure of the current tax system's inefficiency. We find that, in 2018, the revenue-maximizing tax cut was given by a combination of the following two-bracket tax cuts: (i) one between $15,990$ and $20,980$ USD that includes the starting point of the EITC phase-in range, (ii) one between $25,200$ and $30,000$ USD, and (iii) one between $42,359$ and $49,245$ USD that includes the EITC eligibility threshold. Based on our calibration, this reform would have allowed to raise the base transfer to each single-parent household by about three USD (in 2018 units) for each percentage-point change in marginal tax rates. By contrast, the revenue-maximizing tax cut in 1974 would have allowed to raise the base transfer to single parents by about 12.7 USD in 1974 units, or about 64 USD in 2018 units, per percentage-point change in marginal taxes. We conclude that the set of US tax reforms since the 1970s did not fully eliminate the inefficiencies in the tax-transfer system, but reduced them

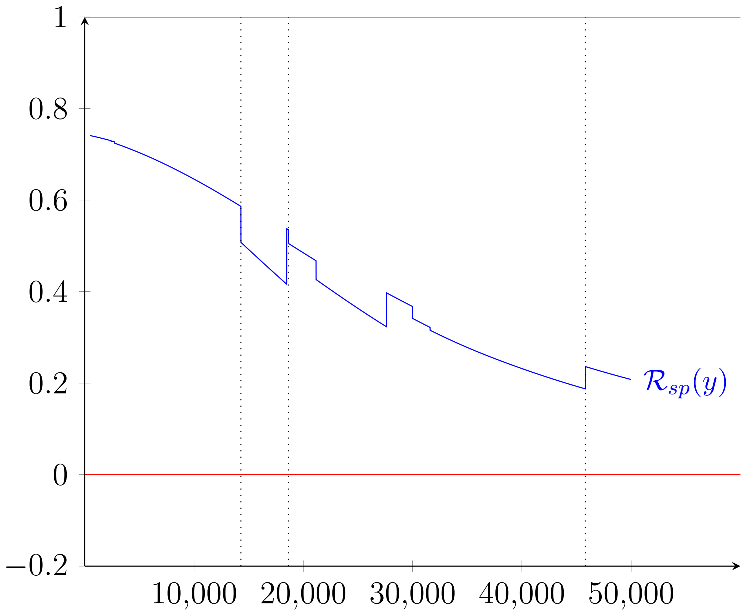


Figure 24: Pareto test of the 2018 tax system for single parents.

Notes: Figure 24 shows the revenue function $y \mapsto \mathcal{R}_{sp}(y)$ for single parents in 2018, assuming an intensive-margin elasticity of 0.33 and an participation elasticity that is decreasing from 0.67 at very low incomes to 0.4 at high incomes. The dotted vertical lines mark the endpoint of the phase-in range at $14,290$ USD, the starting point of the phase-out range at $18,660$ USD, and the endpoint of the phase-out range at $45,802$ USD.

Source: Authors' calculations (see Table 2 for details).

substantially.

Finally, note that these results for the 2018 US tax system are robust in several dimensions. Most importantly, the recent empirical evidence suggests that labor supply elasticities have decreased in magnitude over time, especially those related to the extensive margin. We therefore also considered alternative calibrations with smaller labor supply elasticities at both margins. Qualitatively, our results remain unchanged: the marginal revenue function satisfies the lower and upper boundary conditions, but violates the monotonicity condition. Hence, we conclude that the 2018 US tax-and-transfer system for single parents continues to be Pareto-inefficient, but these inefficiencies are an order of magnitude smaller than those in the mid-1970s.