Endogenous Growth, Green Innovation and GDP Deceleration in a World with Polluting Production Inputs*

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Abstract

We study economic growth and pollution control in a model with endogenous rate and direction of technical change. Economic growth results from growth in the quantity and productivity of polluting intermediates. Growth of GDP and pollution can be decoupled by reducing the pollution intensity of a given quantity through costly research (green innovation) and by containing rebound effects from productivity growth on polluting intermediate quantity. The latter implies that also the rate of GDP growth remains below productivity growth (deceleration). While neither green innovation nor deceleration is chosen under laissez-faire, both contribute to long-run optimal pollution control for reasonable parameter values.

Keywords: Endogenous Growth, Direction of Technical Change, Pollution, Green Innovation, Rebound Effect

JEL Codes: O30, O41, O44, Q55Ker

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1 Introduction

When it comes to the question of whether and how economic growth and environmental conservation should be reconciled, it is widely argued that technical change will play a crucial role (see for instance the Stern Review, Stern (2007), IPCC AR5 Synthesis Report (2014)). Technological development can help to decouple economic growth from pollution, be this by "green innovations", reducing the pollution intensity of production inputs or by productivity enhancing innovations, raising input productivity. However both types of innovations are not undisputed: Boosting green innovation diverts resources from other research activities and may thus reduce GDP growth. Productivity enhancing innovations in turn are feared by others for their so-called rebound effects, which raise economic activity and pollution. Rebound effects of technical progress are one reason for environmental activists like Greenpeace to believe that the world economy should give up economic growth and converge towards stationary levels of consumption and production.

Consider a transport company carrying $Y = F(L_Y, X, Q_t)$ ton-miles of freight with L_Y driver-hours and X truck-hours (truck weight times running hours). The variable Q_t measures the efficiency of the current truck generation. Truck-hours involve the combustion of fuel, which in turn causes emissions X/B_t, where $1/B_t$ measures the emissions caused by running a truck of the current generation for an hour. A **green innovation** raises B_t. An example of such an innovation is the introduction or the improvement of catalytic converters. Given total factor employment, it always reduces pollution without affecting output. Any green innovation is thus unequivocally environment friendly. A **productivity enhancing innovation** raises Q_t. This may be the introduction of a new truck version built with lighter materials like aluminium, carbon or titanium. Such an innovation is not dirty or clean per se. Whether it is dirty or clean hinges on its effect on polluting input use. The latter in turn depends on how the innovation affects factor prices and output demand: Any productivity enhancing innovation *can* be used to reduce polluting input use X while keeping output Y constant. In the example, transport companies could deliver the same Y ton-miles as before the innovation, running the same number but lighter trucks. This would reduce X, fuel consumption, and emissions X/B_t. Instead, as delivery becomes cheaper, the transport company may prefer business expansion over fuel saving, shifting to larger and heavier trucks. The increase in output triggered by the innovation 'rebounds' on the amount by which polluting inputs are actually reduced (compared to the benchmark of constant output). This effect is usually referred to as the **rebound effect** of productivity enhancing innovations.[1] In the first example, with constant Y, there is zero rebound. In contrast, if X is kept constant, there is said to be 'full rebound' (rebound effect of 100%). Pollution is not affected. The concern about productivity enhancing innovation arises because the rebound effect may even be larger, what is sometimes referred to as 'backfire': rebounds which induce a net increase in input use. The production of trucks itself involves the transportation of factors and intermediate goods. Cheaper transportation will thus also raise the efficiency in truck production. Keeping constant the total amount of resources that flows into the production of trucks and transportation as before the innovation, demand and

[1] Following Berkhout et al. (2000) we can define the rebound effect as the percentage $\frac{\Delta_X^{\mathrm{p}} - (X_0 - X_1)}{\Delta^{\mathrm{p}}} \cdot 100$ of potential input saving $\Delta_X^{\mathrm{p}} = X_0 - X^{\mathrm{p}}$ that is lost due to increased input use, where X^{p} is defined by $F(L_{Y0}, X_0, Q_0) = F(L_{Y0}, X^{\mathrm{p}}, Q_1)$. In the first example with constant Y and L_Y, the rebound effect is 0% (since $X_1 = X^{\mathrm{p}}$ and hence $\frac{\Delta_X^{\mathrm{p}} - (X_0 - X^{\mathrm{p}})}{\Delta_X^{\mathrm{p}}} = 0$). In the second example with constant X, the rebound effect is 100% (since $X_1 = X_0$ and $\frac{\Delta_X^{\mathrm{p}} - (X_0 - X_1)}{\Delta_X^{\mathrm{p}}} = 1$). While Berkhout et al. (2010) refer to energy inputs, we apply the definition more generally to polluting inputs.

output will therefore grow by more than just the efficiency gain in transportation: X rises and, as a by-product, emissions X/B_t rise as well. Such strong rebounds were already discussed by William Stanley Jevons (1865), who describes how the invention of the steam engine – by reducing the amount of coal needed to produce a given amount of Joules – led to increased coal consumption.[2]

A minimal demand by environmental activists is that rebound effects of productivity innovations should be kept below their maximal level. Since this reduces not only polluting input growth, but also GDP and consumption growth below their potential growth rates, we speak of **deceleration** in this case. Environmental activists often advocate reducing the rebound effect to zero, which can be achieved either by strong deceleration given productivity growth or by giving up productivity growth altogether.[3]

We address the question of whether persistent growth of output and consumption is socially optimal and if so, how its environmental effects should best be controlled. Our main finding is that for empirically plausible parameter values, it is not sufficient to partly direct research toward green innovation to reduce the environmental impact of productivity growth. The rebound effect of productivity enhancing innovation should be controlled as well: The rate of polluting input growth $\widehat{X}$ should be held below the rate of productivity growth $\widehat{Q}$. This involves deceleration: The rate of GDP growth $\widehat{Y}$ remains below its potential growth rate, also determined by the rate of productivity growth $\widehat{Q}$. Our results imply that policy may have to stimulate technical development and green innovation in particular while at the same time setting incentives to control the rebound effect which offsets potential efficiency gains in the laissez-faire equilibrium.

Our paper offers a comprehensive analysis of the market equilibrium and the social optimum in a model with endogenous technical change. Research can be directed to productivity enhancing innovation raising input efficiency and/or green innovation decreasing the pollution intensity of inputs. The long-run laissez-faire equilibrium does not internalize pollution externalities. Consequently, no research is directed toward green innovation, nor is there deceleration. Productivity growth comes with maximal rebound and GDP growth. We set up the model so as to make sure that growth would be optimal under standard conditions if pollution were not taken into account. Further, our setup ensures that optimal growth should always be accompanied by green innovation, once pollution is accounted for.[4] We show that if production is relatively inelastic with respect to polluting intermediate quantity, there must be control of the rebound effect as well. We argue in the main text that this is the empirically plausible case.[5] The intuition behind our main result is that with inelastic production, restricting the rebound effect of productivity growth comes relatively cheap, without incurring a large loss in potential consumption growth: Deceleration does not have to be too strong. Further, the social return to green innovation rises in the production elasticity of the polluting input. It is therefore comparatively

[2] An overview over the different channels through which technical progress may trigger rebound effects, of which those mentioned in the text are two examples, is for example given in Gillingham, Rapson and Wagner (2015).

[3] The demand often even goes beyond avoiding rebound effects by giving up growth: The so-called 'degrowth' movement shares the belief that the world economy has surpassed sustainable levels of economic activity, so that downsizing -'degrowth'- is unavoidable (see Ariès (2005) and Latouche (2004) for example).

[4] This result is driven by the existence of fixed costs in each individual research unit. Once a research unit is opened up and the fixed costs are paid, making intermediates at least marginally cleaner while making them more productive generates almost no additional cost.

[5] We can, however, fully characterize the long-run optimum also in the opposite case. For a full characterization of all cases, we refer the interested reader to an extended appendix to this paper, available upon request from the authors.

small if this elasticity is low. A socially optimal path thus features all three elements of the title: (Strictly positive) endogenous growth, green innovation and GDP deceleration.

While the terms "controlling the rebound effect" for $\widehat{X} < \widehat{Q}$ and "deceleration" for $\widehat{Y} < \widehat{Q}$ may suggest that polluting input growth and output growth are determined *given* productivity growth, it should be clear that the planner chooses innovation and input allocation *simultaneously.* It goes without saying that a planner who chooses to "control the rebound effect" will not first plan strong GDP growth and than decelerate to control pollution. Remember that a productivity innovation is neither clean nor dirty per se. It acts clean if it is used to reduce polluting inputs (given output). Accordingly, total productivity growth $\widehat{Q}$ can be partitioned into clean productivity growth $\widehat{Q} - \widehat{X}$ (productivity innovations used to reduce polluting input) and GDP enhancing dirty productivity growth $\widehat{X}$ (productivity innovations used to raise output rebounding on polluting input). Instead of saying that the planner controls the rebound effect and accepts the concomitant decelaration, we may equivalently say that clean productivity growth is part of optimal policy. And an equivalent way to phrase our main result is that if the elasticity of output with respect to the polluting input X is low, then long-run optimum features persistent GDP growth (causing rebound), clean productivity innovation, and green innovation. When we continue to use the terms "deceleration" and "controling the rebound effect" also in context of a planner optimum, this is to remain consistent with the above terminology.

Even if the optimal path for our model economy features deceleration, this does not imply a decline of GDP and consumption in absolute terms. In our model, persistent GDP degrowth towards stationary GDP and consumption levels is preferable to a path with unconstrained pollution growth as it is chosen in the entirely unregulated economy. However, giving up consumption growth altogether is never optimal (for a sufficiently patient household).

Although we restrict our analysis to the characterization of the long-run equilibrium and the social optimum, by the very nature of our undertaking, we cannot confine the analysis to balanced growth paths. Along a balanced growth path, the growth rate of intermediate input quantity equals the rates of productivity and of output growth. Controlling the rebound effect requires to keep the growth rate of polluting intermediate inputs persistently below the rate of productivity growth. To cover this possibility, we extend the analysis beyond balanced growth to solutions characterized by growth rates which converge towards constant values only asymptotically. We call such solutions 'asymptotically-balanced growth solutions'.[6]

While a number of papers on technological change, economic growth and pollution have studied the optimal direction of technical change, the optimality of deceleration has so far not been explicitly addressed. Closest to our model are two contributions by Hart (2004) and Ricci (2007). These authors also consider the choice between a lower pollution intensity and greater productivity. However, they neglect the possibility to lower pollution growth by reducing the rebound effect of productivity growth so that deceleration is not part of the optimal solution of their models. Ricci (2007) concentrates on the analysis of balanced growth paths. Along those, by definition, deceleration cannot occur. In Hart (2004), not only the quantity component of output but output itself has a negative effect on the environment. There is thus no clean productivity growth. In both

[6] Asymptotically-balanced growth paths in environmental economic models have been described, e.g., in an Ak-model by Withagen (1995) and in a general one-sector growth model with non-renewable resources (but without pollution) by Groth and Schou (2002).

Hart (2004) and Ricci (2007) therefore, green innovation remains the only way to decouple economic growth and pollution growth.

While the understanding of green innovation in Hart (2004) and Ricci (2007) is similar to ours, a different definition is given in Grimaud and Rouge (2008) and Acemoglu, Aghion, Bursztyn and Hemous (2012). In contrast to the present paper, these articles assume the existence of completely clean substitutes for polluting production inputs. Building on Acemoglu (2002) both papers assume separate research sectors, one for a polluting and one for a non-polluting production input.[7] Green innovation increases the productivity of the clean good while leaving the pollution intensity of the dirty input unchanged. Pollution in these papers is optimally controlled by shifting the composition of GDP towards the clean sector. Grimaud, Lafforgue and Magné (2011) consider three forms of R&D (raising the productivity of non-polluting factors, raising the productivity of polluting factors, reducing the polluting impact of polluting factors), but without detailed microfoundation of the R&D sectors. They numerically derive the optimal growth path and study different policy scenarios but do not discuss rebound effects. None of these papers studies persistent deceleration.

The Cobb-Douglas specification of our model limits the extent to which clean inputs (labor) can substitute for polluting inputs (intermediates). Reality often allows for more elastic substitution. Nevertheless, no currently known production technology is completely clean. Transportation and storage of renewable energy, manufacturing and disposal of batteries, solar cells or wind turbines generate emissions and other forms of pollution. Even within relatively clean sectors, technical progress can be directed to productivity gains and/or to towards reductions in pollution intensity and control of rebound effects matters, so that decelerations remains an issue.[8]

The outline of our paper is as follows: Section 2 presents the model setup. We then determine the laissez-faire equilibrium in section 3. In section 4, we characterize the unique long-run optimum. Our main result, theorem 1, shows that for empirically reasonable parameter constellations, the optimal solution includes both green innovation and deceleration to decouple output- and pollution growth. Section 5 extends the baseline model to include a polluting non-renewable resource. Section 6 concludes.

2 The model

2.1 Setup

In each period, a representative household receives utility $v(c_t) = \frac{\sigma_c}{\sigma_c - 1} c_t^{\frac{\sigma_c - 1}{\sigma_c}}$ from per-capita-consumption $c_t = \frac{C_t}{L}$ and utility $\phi^E(E_t) = \psi \frac{\sigma_E}{\sigma_E - 1} E_t^{\frac{\sigma_E - 1}{\sigma_E}}$ from environmental quality E_t. We assume as, for example, Stokey (1998) as well as Aghion and Howitt (1998, chapter 5), that utility is additively separable. Discounted

[7] Several authors, among them Smulders and de Nooij (2003) and Hassler, Krusell and Olovsson (2012) use the framework by Acemoglu (2002) to analyze energy-saving technical change in a setup without pollution externality. As we are mainly concerned with the pollution externality, we do not refer to these contributions in detail here.

[8] Furthermore, any transition to sectors with relatively 'clean' technology is bound to come slowly: More than 80% of today's energy consumption is not produced from renewable energy but from oil, gas and coal and total energy consumption is growing for all forms of energy, particularly in non-OECD countries (International Energy Outlook (EIA (2016))) Even though the rapid growth in the consumption of coal is projected to decline in the coming decades, growth in other fossil fuel continues and the share of oil, gas and also coal in world energy consumption is expected to remain close to 80%.

intertemporal utility is given by

$$U = \int_0^\infty e^{-\rho t} \left(\frac{\sigma_c}{\sigma_c - 1} c_t^{\frac{\sigma_c - 1}{\sigma_c}} + \psi \frac{\sigma_E}{\sigma_E - 1} E_t^{\frac{\sigma_E - 1}{\sigma_E}} \right) L dt \tag{1}$$

where ρ is the rate of time preference, L total household labor-supply and $\sigma_c, \sigma_E > 0$, $\sigma_c, \sigma_E \neq 1$ are the intertemporal substitution elasticities of consumption and environmental quality respectively. ψ measures the weight of environmental quality in instantaneous utility. Utility is increasing and strictly concave in both arguments.

Environmental quality is inversely related to the stock of pollution originating from the intermediate sector:

$$E_t = \frac{1}{S_t} \tag{2}$$

While utility is concave in E_t, the relation between environmental quality and pollution is convex. Depending on σ_E, the disutility $\phi^S(S_t) = -\phi^E(E_t) = \frac{\sigma_E}{1-\sigma_E} S_t^{\frac{1-\sigma_E}{\sigma_E}}$ of pollution can be concave or convex in S_t. We assume that it is convex by restricting σ_E to

$$\sigma_E < \frac{1}{2} \tag{3}$$

so that the marginal disutility of pollution increases in the pollution stock. The assumption of convex disutility also rules out parameter constellations for which the utility impact of pollution asymptotically becomes negligible relative to that of consumption in a growing economy. This is not an interesting case for the long-run analysis of the trade-off between economic growth and a clean environment. Not only the long-run laissez-faire equilibrium but also the long-run optimal solution would be similar to those in non-environmental models of growth through creative destruction.

The representative household allocates an amount L_{Yt} of its labor supply L to final-good production, L_{Xt} to intermediate production and an amount L_{Dt} to research:

$$L = L_{Yt} + L_{Xt} + L_{Dt} \tag{4}$$

Final output Y_t is produced from labor L_{Yt} and intermediate goods X_{it} of various productivity levels Q_{it}, $i \in [0, 1]$ with the production function

$$Y_t = L_{Yt}^{1-\alpha} \int_0^1 X_{it}^{\alpha} Q_{it}^{1-\alpha} di \tag{5}$$

where $0 < \alpha < 1$. In our example Y_t would be tons of freight times miles transported, L_{Yt} driving hours, intermediate goods the different components of a truck (engine, body, tires ...) and Q_{it} the state of technology associated with the component.[9] Y_t is used for consumption only.

$$Y_t = c_t L. \tag{6}$$

Intermediate goods are produced with the production function

$$X_{it} = \varphi L_{Xit} Q_t \tag{7}$$

[9] To adapt the truck example from the introduction to the Dixit-Stiglitz aggregator in (5), assume that before transporting goods, the carrier chooses a truck type by combining the desired quantities X_{it} of a number of components $i \in [0, 1]$.

where $\varphi > 0$ is a parameter and $Q_t = \int_0^1 Q_{it} di$ measures aggregate productivity. An increase in overall productivity thus has a positive spillover on the production of truck components, which needs less labor.[10] Truck use requires a certain amount of fuel, which is proportional to truck weight times running hours, given by the sum $X_t = \int_0^1 X_{it} di$. Breaking fuel consumption down on the various components, X_{it} units of intermediate good i are thus associated with a flow R_{it} of fossil resources used.[11] In the baseline specification of our model, R_{it} does not have to be accounted for explicitly. Formally, this amounts to the assumption that there is an infinite supply of these resources in each period, such that their price is zero. We show in Section 5 that for parameter constellations well in line with empirical evidence, the alternative assumption of a finite initial resource stock does not affect the long-run social optimum so that our main results still hold.

The combustion of fuel causes emissions, which are ceteris paribus also proportional to truck weight times running hours, i. e. X_t. However, it is possible to reduce the emissions intensity/increase the cleanliness of the various components through R&D. Let B_{it} be the cleanliness of component i. To keep the optimization problem tractable, we assume that aggregate cleanliness simply is the average of individual intermediates' cleanliness at t: $B_t = \int_0^1 B_{it} di$ and that the pollution flow at time t is given by X_t/B_t.

If, in addition, a fraction δ of the pollution stock is cleaned up by natural regeneration processes in every period, the pollution stock evolves according to the equation of motion:[12]

$$\dot{S_t} = \frac{X_t}{B_t} - \delta S_t \tag{8}$$

Note that due to natural regeneration, pollution growth eventually ceases if there is no growth in intermediates and therefore fossil resource use. However, if X_t grows, S_t asymptotically grows at the same rate for a given technology level B_t. Still, even in this case, pollution growth remains below its potential as long as the rebound effect of productivity growth is restricted. Along a balanced growth path in a standard growth model, productivity growth is used to expand output and more of the polluting input is used, so that $\widehat{X}_t = \widehat{Q}_t$. Remember that productivity growth is said to have a rebound effect if it is not fully used to reduce polluting input use (such that $\widehat{X}_t < -\widehat{Q}_t$) and that there is backfire if $\widehat{X}_t > 0$. For ease of exposition, we do not differentiate between partially controlled backfire and partially controlled rebound unless necessary, saying that there is partial control of the rebound effect on any growth path with $\widehat{X}_t < \widehat{Q}_t$. As has been explained, controlling the rebound effect comes at the cost of a slowdown in output growth below potential: There is deceleration.

Definition 1 *There is (partial) control of the rebound effect whenever $\widehat{X}_t < \widehat{Q}_t$. Controlling the rebound effect goes along with deceleration, $\widehat{Y}_t < \widehat{Q}_t$.*

[10] Our results would not change if we assumed that instead of labor, a part Y_X of final output had to be spent on the production of intermediates and $X_{it} = \varphi L_{Xit} Q_t$ were replaced by $X_{it} = \varphi Y_{Xit}$. THe externality which is explicitely assumed in the present $L_{Xit} Q_t$ specification would occure implicitly as a pecunary externality in the Y_X specification since final goods would become cheaper with rising Q_t. In both cases the dependence of X_{it} on aggregate productivity Q_t ensures that the allocation of labor supply and thus growth rates of the aggregate variables in our model can be constant in the long run.

[11] Fossil resources can of course also be inputs in the production of intermediate goods. While the emissions from these fossil fuels accrue before the goods are actually used, we do not separate them from emissions arising from the use of intermediate goods.

[12] In general, we use a dot above a variable to indicate its derivative with respect to time, while we mark growth rates with a cicumflex.

The two sources of slow pollution accumulation (besides natural regeneration) become apparent when rewriting (8) as $\dot{S}_t = \frac{X_t}{Q_t}\frac{Q_t}{B_t} - \delta S_t$: First, $\dot{S}_t$ is small whenever Q_t/B_t is small, which means a sufficiently large share of research must have been oriented towards green innovation in the past. Second, pollution accumulates more slowly if the rebound effect of productivity growth has been controlled such that X_t/Q_t is smaller. If the rebound effect remains uncontrolled ($\widehat{X}_t = \widehat{Q}_t$), the stock of pollution remains constant ($\widehat{S}_t = \frac{d\widehat{S}_t}{dt} = 0$) if and only if $\widehat{B}_t = \widehat{Q}_t$.[13] This suggests the definition of a natural benchmark for the direction of technical change:

Definition 2 *The* ***direction of technical change*** *is ecologically* ***neutral*** *if and only if* $\widehat{B}_t = \widehat{Q}_t$, ***productivity-oriented*** *if and only if* $\widehat{B}_t < \widehat{Q}_t$, *and* ***green*** *if and only if* $\widehat{B}_t > \widehat{Q}_t$.

Both productivity Q and cleanliness B change over time due to innovations from a continuum of R&D-sectors. Entry to the research sector for any intermediate X_{it} is not restricted. For research unit $j \in [0, \infty]$, improving Q_{it} by a rate q_{ijt} and B_{it} by a rate b_{ijt} requires

$$l_{Dijt}(q_{ijt}, b_{ijt}, Q_{it}, B_{it}, Q_t, B_t) = q_{ijt}^2 \frac{Q_{it}}{Q_t} + b_{ijt}^2 \frac{B_{it}}{B_t} + d\frac{Q_{it}}{Q_t} \tag{9}$$

units of labor. We call q_{ijt} and b_{ijt} the step-size of an innovation with respect to productivity and cleanliness, respectively. The wage rate is denoted by w_{Dt}. Then $w_{Dt}d\frac{Q_{it}}{Q_t} > 0$ are fixed entry costs for unit j in sector i. Variable costs for each dimension of technology improvement are quadratic in the step-size.[14] Total costs $w_{Dt}l_{Dijt}$ rise with the level of sectoral relative to aggregate productivity Q_{it}/Q_t and cleanliness B_{it}/B_t respectively. The underlying assumption is that technology improvements in a given sector are increasingly difficult the more advanced the technology in that sector is already while there are positive spillovers from the other sectors.[15] Given l_{Dijt}, a trade-off exists between making an intermediate more productive and making it cleaner. On the other hand, there is also an indirect positive relation between research orientations: Once fixed costs have been paid to innovate in one direction, a comparatively small additional labor-investment is needed to increase the other technology stock as well.

If a researcher j enters into the research sector for intermediate X_i at time t, he hires labor l_{Dijt} and chooses a step-size q_{ijt} and b_{ijt} for the improvement in productivity and cleanliness respectively. The wage rate w_{Dt} is taken as given. Innovations occur at the exogenous, constant Poisson arrival-rate μ per unit of time for the individual researcher j. An innovation changes the sectoral productivity level by $q_{ijt}Q_{it}$ and the cleanliness of production by $b_{ijt}B_{it}$. The innovator obtains a patent for the production of the improved intermediate good. He then receives a profit flow from selling the intermediate. This flow eventually ceases when a new innovation arrives and the incumbent is replaced by another firm. If n_{it} units decide to enter research sector i

[13] $\widehat{S}_t = 0$ if and only if $X_t = \delta S_t B_t$ and $\frac{d\widehat{S}_t}{dt} = 0$ if, in addition, $\widehat{X}_t = \widehat{B}_t$. Since $\widehat{X}_t = \widehat{Q}_t$, this requires $\widehat{B}_t = \widehat{Q}_t$.

[14] While fixed costs are needed to guarantee a finite number of research units, assuming costs to be quadratic in the step-size ensures the existence of an efficient choice of the latter. As we explain in section 4.3, the presence of fixed costs creates a certain complementarity between pollution-reducing and productivity-enhancing innovation, which we believe to be realistic.

[15] Like the intermediate production function, labor required in R&D (equation (9)) must depend on the aggregate and additionally on the sectoral levels of technology to ensure asymptotically constant growth of the aggregate variables.

in t, innovations arrive at rate μn_{it} in this sector. The expected development of Q_{it} and B_{it} is given by

$$E[\Delta Q_{it}] = \int_0^{n_{it}} \mu q_{ijt} Q_{it} dj \tag{10}$$

$$E[\Delta B_{it}] = \int_0^{n_{it}} \mu b_{ijt} B_{it} dj. \tag{11}$$

While the sectoral technology level faces discontinuous jumps, aggregate technology evolves continuously because there is a continuum of sectors carrying out research. According to the law of large numbers, the average rates of change $\dot{Q}_t$ and $\dot{B}_t$ of Q and B approximately equal the respective expected rates of change, which are derived by aggregating over sectors in (10) and (11):

$$\dot{Q}_t = \int_0^1 \int_0^{n_{it}} \mu q_{ijt} Q_{it} dj di \tag{12}$$

$$\dot{B}_t = \int_0^1 \int_0^{n_{it}} \mu b_{ijt} B_{it} dj di. \tag{13}$$

2.2 Balanced and asymptotically-balanced growth

The subsequent analysis of our model in this and the following sections extends beyond balanced growth paths to 'asymptotically-balanced growth paths'. The definition below serves to clarify the terminology. Here and in the following, z_∞ refers to the limit $\lim_{t\to\infty} z_t$ of a variable z.

Definition 3 *Assume that for some initial state (Q_0, B_0, S_0), there exists a (market or planner) solution such that the sequence $\left(\widehat{Q}_t, \widehat{B}_{t,} \widehat{S}_t\right)_{t=0}^{\infty}$ converges towards the vector $\left(\widehat{Q}_\infty, \widehat{B}_\infty, \widehat{S}_\infty\right)$ for $t \to \infty$. We call such a solution an asymptotically-balanced growth (ABG) solution. We say that the model has an asymptotically unique ABG-solution if all ABG-solutions have the same limit vector $\left(\widehat{Q}_\infty, \widehat{B}_\infty, \widehat{S}_\infty\right)$.*
If there exist initial states (Q_0, B_0, S_0) such that the corresponding solution paths are characterized by $\left(\widehat{Q}_t, \widehat{B}_{t,} \widehat{S}_t\right) = \left(\widehat{Q}_\infty, \widehat{B}_\infty, \widehat{S}_\infty\right)$ for every t, we call the path defined by $\left(\widehat{Q}_t, \widehat{B}_{t,} \widehat{S}_t\right)_{t=0}^{\infty} = \left(\widehat{Q}_\infty, \widehat{B}_\infty, \widehat{S}_\infty\right)$ the unique balanced growth (BG)-path.

In abuse of terminology, we sometimes refer to the unique limit of all ABG-solutions for $t \to \infty$, characterized by the unique vector $\left(\widehat{Q}_\infty, \widehat{B}_\infty, \widehat{S}_\infty\right)$, as *the* ABG-solution.
Note that a BG-solution, defined by constant growth rates of Q, B and S for all t, is also an ABG-solution. The reverse is not true because there may not exist an initial state (Q_0, B_0, S_0) such that $\widehat{Q}_t$, $\widehat{B}_t$ and $\widehat{S}_t$ are constant for all t. We will see that while the economy has a unique BG-equilibrium and a unique ABG-optimum for any set of parameters, it need not have a BG-optimum. In particular, a BG-optimum does not exist in theorem 1.

3 The laissez-faire equilibrium

The laissez-faire equilibrium is given by sequences of plans for per-capita consumption $\{c_t\}_0^\infty$, assets $\{A_t\}_0^\infty$, labor supply in production $\{L_{Xit}, L_{Yt}\}_0^\infty$ and research $\{L_{Dt}\}_0^\infty$, demand for intermediates $\{X_{it}^d\}_0^\infty$, demand for labor in production $\{L_{Xit}^d, L_{Yt}^d\}_0^\infty$ and research labor demand $\{l_{Dijt}\}_0^\infty$, plans for the step-size $\{q_{ijt}\}_0^\infty$ and $\{b_{ijt}\}_0^\infty$ of innovation in productivity and cleanliness, as well as sequences of intermediate prices $\{p_{it}\}_0^\infty$ and wages $\{w_{Xit}, w_{Yt}, w_{Dt}\}_0^\infty$ in intermediate production, final good production and research and a path $\{r_t\}_0^\infty$ for the interest rate such that in every period t, (i) the representative household maximizes utility taking into account the budget constraint and the labor market constraint (4), (ii) profits from final and intermediate goods production as well as research profits are maximized, (iii) aggregate expected profits in each research sector i are zero (iv) the markets for intermediate goods, the three types of labor and assets clear (v) all variables with the possible exception of q_{ij} and b_{ij} are non-negative.

The solution of the model under laissez-faire follows closely that in standard endogenous growth models. Define an upper bound $\overline{\rho}^{\text{LF}}$ for the rate of time preference such that $\widehat{Q}^{\text{LF}} > 0$ if and only if $\rho < \overline{\rho}^{\text{LF}}$. Further, define a lower bound $\underline{\rho}^{\text{LF}}$ such that the transversality condition for assets is satisfied if and only if $\rho > \underline{\rho}^{\text{LF}}$.[16] The following proposition describes the balanced-growth equilibrium:

Proposition 1 ***BG laissez-faire equilibrium***
There exists a $\underline{\rho}^{LF}$ such that the transversality condition for assets is satisfied if and only if $\rho > \underline{\rho}^{LF}$.
For $\rho > \underline{\rho}^{LF}$, the model has a unique BG-laissez-faire equilibrium. Further, an upper bound $\overline{\rho}^{LF}$ for the rate of time preference exists such that economic growth is strictly positive if and only if $\rho < \overline{\rho}^{LF}$. Productivity growth leads to equally fast expansion of polluting quantity ($\widehat{X}_\infty^{LF} = \widehat{Q}_\infty^{LF}$). The rebound effect of productivity growth is not controlled and there is no green innovation. Pollution grows at the same rate as consumption, production and productivity. Given (3), i.e. $\sigma_E < 1/2$, a solution without long-run growth is socially preferable.

Proof. See appendix A.1. ■

From the previous section it is obvious that in our model, there are no incentives for producers to invest in cleaner intermediates or counteract the rebound effect of productivity growth. In a growing economy, there is unconstrained pollution growth. This is clearly suboptimal if the disutility of pollution is convex ($\sigma_E < 1/2$) but utility is concave in consumption: The marginal utility gain from an additional unit of consumption becomes negligible relative to the marginal utility loss generated by a unit increase in the pollution stock as consumption and pollution rise. Utility declines persistently without lower bound. If consumption growth is given up in the long run, the pollution stock and utility converge to constant values. Stationary long-run levels of consumption and production as called for by environmental activists are therefore welfare-improving over the laissez-faire equilibrium.

[16] The boundary values are $\overline{\rho}^{\text{LF}} = \frac{1}{2}\mu L \left(\left(\frac{1}{\alpha} + \frac{\alpha}{1-\alpha}\right)\left(\sqrt{1+d} - 1\right)\right)^{-1}$ and $\underline{\rho}^{\text{LF}} = \frac{1}{2}\alpha(1-\alpha)\left(1 - \frac{1}{\sigma_c}\right)(1+d)^{-1/2}\mu L$.

4 The Social Planner's solution

The social planner chooses the time paths of Q, B and S as well as consumption c, production Y, X, X_i, x_{ij}, the allocation of labor L_{Yt}, L_{Xt},L_{Xit},L_{Dt}, l_{Dijt}, the number of research units[17] $n_{it} = n_t$ and a step-size q_{ijt} and b_{ijt} for technology improvements in every period t so as to maximize utility (equation (1)). She takes into account the labor market constraint (4), the aggregate resource constraint (6), the effect of pollution on environmental quality (2), the equation of motion for pollution (8), the expected change in Q_i and B_i as given by (10) and (11) as well as the aggregate equations of motion for Q (12) and B (13).

Because all research units j are ex ante symmetric and research costs are convex in q_{ij} and b_{ij}, the social planner chooses the same q_{ijt}, b_{ijt} and therefore l_{Dijt} for every j in sector i. Further, the planner allocates intermediate production in every sector i to the latest innovator because he is the most productive and cleanest while marginal costs are the same for all j. We therefore omit the index j from now on. In fact, it is optimal to choose the same $q_{it} = q_t$ and $b_{it} = b_t$ in every sector, as we explain in appendix B.1. We also show there that given the allocation of resources over firms and sectors just described, the dynamic social planner's problem involves the sector-independent variables Q, B, S, c, X, L_Y, n, q and b only and we derive the first-order conditions.

The long-run optimal solution differs dependent on the parameter constellation considered. To simplify the analysis, we focus on the empirically most relevant case by making the following assumptions[18]:

$$\alpha/(1-\alpha) < 1 - \frac{(\sigma_c - 1)/\sigma_c}{(1-\sigma_E)/\sigma_E} \tag{14}$$

$$\rho > \rho^{\text{delta}} := \frac{1}{2}\left(1 + \left(\frac{\alpha}{1-\alpha}\right)^2\right)^{1/2} d^{-1/2}\mu L - \kappa \frac{(1-\sigma_E)/\sigma_E}{(1-\sigma_c)/\sigma_c}\delta \text{ for } \sigma_c < 1 \tag{15}$$

$$\text{where } \kappa = \left(\frac{1}{\sigma_c} + \frac{\alpha}{1-\alpha}\left(1 - \frac{(\sigma_c - 1)/\sigma_c}{(1-\sigma_E)/\sigma_E}\right)\right)$$

The second restriction excludes a boundary case[19] which does not lead to qualitatively different conclusions. The first restriction is crucial for the characteristics of the optimal path, as we explain in section 4.3. To see that condition (14) indeed describes the most likely parameter constellation, consider the relevant parameters, α, σ_c and σ_E: While there are little reliable empirical results on σ_E, we believe that disutility is convex in the pollution stock ($\sigma_E < 1/2$) so that the marginal disutility of pollution is the larger, the more polluted the environment is. As for the IES in consumption σ_c, the range $\sigma_c \in (0, 1)$ is suggested by a large body of empirical literature (e.g. Hall (1988), Ogaki and Reinhart (1998)). Defining a reasonable range for α is less straightforward. Setting α to the capital share would imply $\alpha \approx 1/3$. Interpreting X_t as energy, α would be

[17] To allow for an analytical solution to the planner problem we consider the constrained maximization problem with $n_{it} = n_t$ for all i.

[18] For a full characterization of all cases, we refer the interested reader to an extended appendix to this paper, available upon request from the authors.

[19] We show in lemma 2 that the pollution stock S decreases whenever $\sigma_c < 1$. S can at most decrease at the rate of natural regeneration ($\widehat{S}_\infty \geq -\delta$). To actually reach this rate of decrease, flow pollution would have to become zero and all economic activity would have to be given up. This can clearly not be optimal in finite time as a positive consumption level has to be maintained. Still, it can be optimal to approach $\widehat{S}_\infty = -\delta$ asymptotically by decreasing the pollution flow particularly fast. This case is more difficult to handle analytically and does not offer new insights. Condition (15) ensures that $\widehat{S}_\infty > -\delta$. Note that no such restriction is needed for $\sigma_c > 1$ as S increases in the long run in this case (see lemma 2).

substantially smaller than the capital share: Energy expenditures as a share of GDP amounted to 6.2% in the U.S. in 2015 (EIA (2017)). On the other hand, α is also the inverse of the mark-up in the intermediate production sector. Estimates for the manufacturing sector in the U.S. (Roeger (1995)) suggest values of α of at least 0.3. We consider values which do not exceed 0.5 as plausible. With $\sigma_E < 0.5$, $\sigma_c \in (0, 1)$ and $0 < \alpha \leq 0.5$, condition (14) is always satisfied.
If we choose a smaller range for α, so that α does not exceed the capital share of 1/3, condition (14) holds for $\sigma_c < 2$ which covers most empirical estimates of the IES in consumption. Setting α to the energy share in real GDP, even extremely high values of σ_c up to 4.4 as found by Fuse (2004) for Japan do not violate the condition.

Before analyzing optimal pollution control in a growing economy (see section 4.3), we shortly describe the conditions for positive growth and the development of the pollution stock along the optimal path.

4.1 Optimality of persistent economic growth

In standard models of endogenous growth, long-run growth is optimal for sufficiently patient households. This result carries over to our model with negative environmental externalities.

Lemma 1 ***Positive long-run consumption growth***
There exists a $\overline{\rho}$, such that for any rate of time preference $\rho < \overline{\rho}$, long-run optimal consumption growth is positive.

Proof. The upper bound $\overline{\rho} = \frac{1}{2}\left(1 + \left(\frac{\alpha}{1-\alpha}\right)^2\right)^{1/2} d^{-1/2}\mu L$ is derived in the extended appendix. The proof follows from the solution of the model, similar to the proof in standard endogenous growth models. ■

The result is not surprising as pollution accumulation can be restricted without giving up consumption growth altogether. Persistent economic growth must however be accompanied by continuous pollution control.

4.2 The optimal relation between economic growth and pollution accumulation

We show in this subsection that optimal growth does not automatically require constant or decreasing pollution levels. More precisely, we find that for our assumption of convex disutility of pollution ($\sigma_E < 1/2$) whether the pollution stock de- or increases in the long-run optimum depends on the intertemporal elasticity of substitution in consumption:

Lemma 2 ***Development of the pollution stock***
Long-run growth must be accompanied by a persistent restriction of pollution growth. In a growing economy, the optimal pollution stock S_t increases (decreases) for $\sigma_c > 1$ ($\sigma_c < 1$).

Proof. The first statement follows as a corollary of proposition 1. As to the second, we show in appendix B.2 that in a solution with asymptotically-balanced growth, under restriction (15), the following condition must hold:

$$\frac{\sigma_c - 1}{\sigma_c}\widehat{c}_\infty = \frac{1 - \sigma_E}{\sigma_E}\widehat{S}_\infty \tag{16}$$

Given $\widehat{c}_\infty > 0$, the left-hand side of (16) is positive whenever $\sigma_c > 1$ and negative for $\sigma_c < 1$. Under assumption (3) that the disutility of pollution is convex, $\frac{1-\sigma_E}{\sigma_E}$ on the right-hand side is positive. Therefore the right-hand side of equation (16) is negative if and only if $\widehat{S}_\infty < 0$ and positive if and only if $\widehat{S}_\infty > 0$. It follows that the pollution stock must increase whenever $\sigma_c > 1$ and decrease whenever $\sigma_c < 1$.[20] ∎

Equation (16) is the balanced-growth condition described in Gradus and Smulders (1996) which has become standard in models of the environment and endogenous growth: It requires the ratio of instantaneous marginal utility from consumption to instantaneous marginal disutility from pollution to develop proportionally to S/c. The elasticity of substitution between c and S is unity then.

4.3 Pollution control and the direction of technical change

As shown in lemma 1, long-run growth in the optimal solution must go along with a persistent restriction of pollution growth. It is intuitive that green innovation is always part of optimal pollution control: Once research units are opened up and the fixed costs (e.g., for equipment and fixed labor costs) have been paid, it is almost costless to make intermediates a little cleaner while making them more productive.

Unlike green innovation, restricting the rebound effect of productivity growth is not always optimal in a growing economy. Recall from the introduction that by the term 'restricting the rebound effect', we mean that the social planner uses productivity growth only partly to raise output (dirty productivity growth) and partly to reduce pollution (clean productivity growth). But the social cost from forgone GDP growth associated with choosing partly clean over dirty productivity growth may be too large compared to its social benefit from the reduction in pollution growth. Under the empirically likely parameter constellation given in condition (14), however, it is socially desirable to restrict the rebound effect and incur deceleration, or, equivalently, to choose clean productivity growth.

In the following theorem, we characterize the long-run optimal solution given conditions (3), (14) and (15). We define a lower bound ρ^{TVC} for the rate of time preference so that the transversality conditions are satisfied if and only if $\rho > \rho^{\text{TVC}}$[21].

Theorem 1 ***ABG optimum***

There exists a lower bound ρ^{TVC} for ρ such that the transversality conditions are satisfied if and only if $\rho > \rho^{TVC}$.

For $\rho^{TVC} < \rho < \overline{\rho}$, there exists an asymptotically unique ABG-optimum with the following properties: Pollution growth $\widehat{S}_\infty$ equals the growth rate of flow pollution, $\widehat{X}_\infty - \widehat{B}_\infty$. $\widehat{S}_\infty$ is reduced below the potential rate $\widehat{Q}_\infty$ both by green innovation ($\widehat{B}_\infty > 0$) and by restricting the rebound effect of productivity growth ($\widehat{X}_\infty < \widehat{Q}_\infty$). The latter goes along with deceleration ($\widehat{Y}_\infty < \widehat{Q}_\infty$). The ratio of green relative to productivity-improving innovation is $\widehat{B}_\infty/\widehat{Q}_\infty = \alpha/(1-\alpha)$. The direction of technical change is green (productivity-oriented), i.e., $\widehat{B}_\infty > \widehat{Q}_\infty$ ($\widehat{B}_\infty < \widehat{Q}_\infty$), if and only if $\alpha > 1/2$ ($\alpha < 1/2$).

[20] Note that (16) also suggests that under more general assumptions concerning the utility function, whether the pollution stock de- or increases depends on σ_E being smaller or larger than one as well.

[21] The formal expression for the critical value ρ^{TVC} is $\rho^{\text{TVC}} = \frac{1}{2}\frac{1-1/\sigma_c}{1+\frac{\alpha}{1-\alpha}\left(1-\frac{(\sigma_c-1)/\sigma_c}{(1-\sigma_E)/\sigma_E}\right)}\left(1+\left(\frac{\alpha}{1-\alpha}\right)^2\right)^{1/2} d^{-1/2}\mu L$ (see the extended appendix). Note that the condition $\rho > \rho^{\text{TVC}}$ is satisfied for any non-negative ρ if $\sigma_c < 1$. In this case, a positive lower bound for ρ is given by ρ^{delta} in condition (15).

Proof. See appendix B.3. ■

Given condition (14), i.e. $\alpha/(1-\alpha) < 1 - \frac{(\sigma_c-1)/\sigma_c}{(1-\sigma_E)/\sigma_E}$, reductions in pollution intensity are optimally combined with clean productivity growth. In other words, unlike in the laissez-faire solution, the rebound effect of productivity growth is restricted ($\widehat{Q}_\infty - \widehat{X}_\infty > 0$) and there is deceleration ($\widehat{Q}_\infty - \widehat{Y}_\infty > 0$).

In this case, the elasticity α of final good production $Y_t = X_t^\alpha (Q_t L_{Yt})^{1-\alpha}$ with respect to the polluting input X_t is rather small. As it follows from the production function that $\widehat{Q}_\infty - \widehat{Y}_\infty = \alpha\left(\widehat{Q}_\infty - \widehat{X}_\infty\right)$, a small elasticity α implies that restricting the rebound effect does not require strong deceleration. Controlling the rebound effect is therefore an attractive way for the social planner to control pollution growth. In our truck example, a small elasticity means that the use of lighter materials for the truck body allows to achieve a given amount of fuel saving without giving up much of the increase in tonne-miles transported which could be gained by increasing truck size and weight and thus fuel use.

Further, because the attractiveness of clean as opposed to dirty productivity growth increases when α decreases, it becomes less important to reduce the pollution intensity of intermediate goods. The smaller α, the lower therefore the social return to green as opposed to productivity-improving research.

The expression $1 - \frac{(\sigma_c-1)/\sigma_c}{(1-\sigma_E)/\sigma_E}$ is the ratio of green relative to productivity-improving innovation which yields the pollution growth rate reconcilable with asymptotically-balanced growth (according to equation (16)) when productivity growth is dirty, the rebound effect of productivity growth remains uncontrolled ($\widehat{Q}_\infty - \widehat{X}_\infty = 0$) and there is no deceleration. If the elasticity α is so small that $\alpha/(1-\alpha) < 1 - \frac{(\sigma_c-1)/\sigma_c}{(1-\sigma_E)/\sigma_E}$, the relative return to green research compared to the net benefit from clean productivity growth is too low to support this research orientation: It is not optimal to bring about the asymptotically-balanced pollution growth rate by green innovation alone. Research then remains rather productivity-oriented but productivity growth is relatively clean ($\widehat{Q}_\infty - \widehat{X}_\infty > 0$). Deceleration lowers the rebound effect of productivity growth and thereby helps to restrict pollution growth.

For larger values of α not reconcilable with (14), a balanced-growth optimum without deceleration exists. In the truck example, it is too costly in terms of tonne-miles transported to forgo an increase in truck size. While lighter materials are used in truck production, truck weight and fuel use are not reduced. Pollution control is achieved through green innovation only. This case is described in the extended appendix.

A solution without deceleration becomes less likely as σ_E increases if and only if $\sigma_c < 1$ and more likely if and only if $\sigma_c > 1$. As σ_E increases from close to zero to 1/2, the intertemporal elasticity of substitution in pollution ($\sigma_E/(1-2\sigma_E)$) increases and the optimal pollution path becomes steeper. If $\sigma_c < 1$, this means that the pollution stock must fall faster, so that stronger pollution control is required. If $\sigma_c > 1$, a larger positive pollution growth rate is accepted by the social planner, so that less pollution control is needed.

We have characterized the social optimum in the long run only. The set of necessary conditions generates a complex dynamic system which does not allow to determine the transition path analytically. Numerical analysis suggests, however, that for any initial state of the economy, there exists a path leading towards the long-run optimal solution.

We have pointed out that even with some control of the rebound effect, intermediate quantity may still rise: There may still be backfire. A restriction of the rebound effect below 100% occurs if intermediate quantity falls in absolute terms, not only per labor efficiency unit. There is then degrowth in intermediate quantity (but not in GDP). Because quantity degrowth requires extreme deceleration, it is optimal only if the ratio $\alpha/(1-\alpha)$ of production elasticities is particularly small. This result follows as a corollary from theorem 1:

Corollary 1 ***Quantity degrowth***
X_t converges to zero as Q_t grows in the ABG-solution of the social planner's problem described in theorem 1, i.e. there is quantity degrowth ($\widehat{X}_\infty < 0$), if and only if $\frac{\alpha}{1-\alpha} < (1-\alpha)\frac{(1-\sigma_c)/\sigma_c}{(1-\sigma_E)/\sigma_E}$.

Proof. Proof follows directly from setting $\widehat{X}_\infty < 0$ in equation (B.25) in the appendix. ■

Note that quantity degrowth can only be optimal if the pollution stock is required to decline in the long-run optimum (for $\sigma_c < 1$). Further, α should be substantially below the capital share. Quantity degrowth is likely to be optimal if α is interpreted as the energy share in GDP: Setting $\alpha \approx 0.09$ and $1/3 \leq \sigma_E < 1/2$, the optimal solution is characterized by quantity degrowth for almost all values of σ_c from the interval $(0, 1)$.

4.4 Environmental care and the pace of economic growth

In our model, a stronger research orientation towards green innovation implies slower productivity growth for given total research effort. Further, deceleration needed to control the rebound effect of productivity growth requires to give up potential consumption growth. Intuitively, one might therefore expect environmental care to slow down economic growth relative to the case where the negative environmental externality of intermediate goods is not taken into account.

We find, however, that the above intuition is not necessarily correct. Comparing the optimal solution of our baseline model to the optimum in a modified setting where the weight of pollution in utility is zero ($\psi = 0$), we observe the following: First, economic growth is positive for larger rates of time preference in our framework. Second, depending on parameters, growth rates of consumption, production and productivity may in fact be higher than in the model without a negative external effect from pollution.[22]

Moreover, the degree of the household's preference for a clean environment and therefore the strength of the negative pollution externality, as reflected in the size of ψ, does not influence long-run growth rates at all (given $\psi > 0$). The reason is that a stronger environmental preference does not alter the social return to productivity-oriented research, which is the driver of economic growth. The long-run relation between productivity growth and growth in intermediate quantity, consumption and output is fixed independently of the environmental preference on an ABG-path.[23]

Corollary 2 ***Environmental care and the pace of economic growth***
In the solution of theorem 1 compared to the optimal solution in a modified setting without negative external effect from pollution on utility ($\psi = 0$), (i) the condition on ρ for long-run growth in per capita consumption to

[22] A similar result can be obtained if the optimal solution with $\psi > 0$ is compared not to the optimum with $\psi = 0$ but to the laissez-faire equilibrium. It is, however, not possible to attribute faster growth to the environmental externality in particular in this case because equilibrium growth may be suboptimally slow as a result of several other externalities ('standing-on-shoulders' of previous innovators, firms cannot appropriate the whole consumer surplus).

[23] A similar result was found by Gradus and Smulders (1993) in a Lucas–Uzawa-model. While stronger environmental preference has no influence on long-run growth rates, it can be expected to affect the levels of the model variables along the long-run path. These effects can however not be analyzed without studying transitional dynamics.

be positive is less strict and (ii) long-run optimal growth in per capita consumption is faster if and only if the rate of time preference is sufficiently large.
Given $\psi > 0$, the strength of the representative household's preference for a clean environment, as reflected in the size of ψ, has no influence on long-run optimal growth rates.

Proof. See appendix B.4. ■

The driving force behind the result is a positive link between green and productivity-oriented research. Green innovation can lead to an increase in the optimal amount of labor devoted to research. It thereby fosters also productivity growth and therefore consumption growth. A similar effect has before been described by Ricci (2007).

5 The model with a non-renewable resource

Pollution in the baseline model arises as a by product of intermediate good usage. As an example, we have suggested that fossil fuels are used in proportion to intermediate goods in the production of the final good and that pollution is due to emissions of greenhouse gases contained in these fossil fuels. So far we have assumed that there is no restriction on the total amount of fossil fuel used over time. Indeed, it has been argued e.g. by Nordhaus (1974) and more recently Hart(2016) that technical change in extraction technology will help to overcome resource scarcity. Empirical support has amongst others been provided by Krautkrämer (1998) and Hart and Spiro (2011). Nevertheless, in this section, we discuss the robustness of our main results with respect to the consideration of a polluting non-renewable (fossil) resource stock. More precisely, intermediate production is assumed to explicitly use an exhaustible resource as production input. For simplification, other production inputs such as labor are ignored.[24] We also do not allow for technical progress in input production, so that resource scarcity cannot be overcome by improvements in resource efficiency. There thus ultimately has to be a decline in intermediate production over time, in other words: there must be quantity degrowth. We have argued before that judging by empirical estimates for the model parameters, a solution with quantity degrowth is reasonable. We prove that the results of our baseline model for the long-run social optimum still apply if the optimal solution of the baseline model is characterized by quantity degrowth and the initial resource stock is large enough.[25] The negative pollution externality of intermediate production then reduces optimal resource use in a way that a sufficiently large resource stock is never exhausted.

5.1 Setup

We denote the resource stock in period t by F_t. Starting from a finite positive initial level F_0, the resource stock is depleted proportionally to resource use:

$$\dot{F}_t = -R_t. \tag{17}$$

We assume that the resource is owned by the representative household and, for simplification, that it can be extracted at zero cost (see also Barbier (1999), Schou (2000) and Groth and Schou (2002)).

[24] Again, an alternative interpretation is that the resource is needed to use intermediates, i. e. fossil fuel is needed to operate a truck.

[25] The introduction of a scarce resource slows growth in the laissez-faire equilibrium, as is shown in the extended appendix.

The resource stock F_t must be non-negative for any t. Therefore total extraction must not exceed the initial stock F_0, a requirement which is formally represented in the condition

$$\int_0^\infty R_t dt \leq F_0. \tag{18}$$

Suppose that one unit of the intermediate good is produced by one unit of the non-renewable resource so that

$$X_{it} = R_{it} \tag{19}$$

is resource input in sector i and $X_t = \int_{i=0}^1 X_{it} di = R_t$ aggregate resource use in period t. With a finite resource stock, it is obvious that resource use and therefore intermediate production must ultimately decline to zero in the long run, both in the socially optimal solution and the laissez-faire equilibrium. There has to be quantity degrowth.

Lemma 3 *If intermediate goods are produced with a non-renewable resource according to equation (19), the growth rate $\widehat{X}$ of intermediate quantity is negative in the long run. Any solution path is characterized by quantity degrowth for $t \to \infty$.*

Proof. It follows from (19), that aggregate resource use is $R_t = X_t$. Substitution into equation (18) yields $\int_0^\infty X_t dt \leq F_0$. To satisfy the condition, the integral must converge, which requires $\lim_{t\to\infty} \widehat{X}_t = \widehat{X}_\infty < 0$ as a necessary condition. ■

We now consider the optimal outcome in more detail.[26]

5.2 Resource scarcity in the long-run social optimum

We first characterize the long-run social optimum in case of a binding natural resource constraint. This case is commonly studied in related literature (Schou (2000, 2002), Grimaud and Rouge (2008)). The Lagrange-multiplier λ_{Rt} for the natural resource constraint reflects the social costs of producing one unit of intermediates, i.e., the social price of the non-renewable resource. λ_{Rt} increases over time according to the modified Hotelling rule

$$\widehat{\lambda}_R = \rho. \tag{20}$$

While the social price λ_{Rt} of the non-renewable resource increases with progressing resource scarcity, the shadow price v_{St} of pollution moves along with the marginal disutility of pollution on an asymptotically-balanced growth path[27]. The shadow price therefore falls towards $v_{S\infty}^{\mathrm{R}} = 0$ as the stock of the polluting resource gets exhausted and the pollution stock declines. It is shown in the appendix that in this case, green innovation is no longer optimal in the long run, i.e.,

$$b_\infty^{\mathrm{R}} = \widehat{B}_\infty^{\mathrm{R}} = 0.$$

However, we have suggested earlier that the natural resource constraint need not be binding in the social planner's solution. We know from corollary 1 in subsection 4.3, that the social planner may choose to let the quantity of intermediates decrease in the long run even if there is no constraint imposed on intermediate

[26] As before, we focus on balanced and asymptotically-balanced growth solutions.
[27] Recall the derivation of equation (16) in the appendix.

production by resource scarcity. More precisely, this is the case if preferences are such that a declining pollution stock is desired and the factor elasticity of intermediates is particularly small so that quantity degrowth is not too costly in terms of foregone potential consumption growth. Whenever there is quantity degrowth in the long run, the integral $\int_0^\infty X_t dt$ converges to a finite value. In the modified setting where intermediates are produced from a non-renewable resource, the resource constraint is then not binding given that the initial resource stock is not too small. We prove in the appendix that the long-run optimal solution of the resource model is the same as in our baseline model without resources.

Proposition 2 ***ABG-optimum with an exhaustible resource***
Assume that intermediates are produced with a non-renewable resource according to equation (19). Assume further that the path $\{X_t\}_0^\infty$ for intermediate quantity is continuous.

There is always quantity degrowth in the long-run optimal solution. Further, the following holds:
(a) Binding resource constraint *If the resource constraint is binding, all labor in research and development is shifted to productivity improvements asymptotically and green innovation comes to a halt ($\widehat{B}_\infty^R = 0$).*
(b) Non-binding resource constraint *Given that the conditions for quantity degrowth in the baseline model (see corollary 1) are satisfied and given a sufficiently large (but finite) initial resource stock F_0, the natural resource constraint is not binding in the social planner's problem. There exists an asymptotically unique ABG-solution which for $t \to \infty$ is identical to the ABG-solution with quantity degrowth described in section 4.3. More precisely, growth in output and consumption is positive, given a sufficiently small rate of time preference ρ, and entirely driven by productivity growth. The pollution stock S declines both due to quantity degrowth and because the pollution intensity of intermediate goods is reduced by green innovation. The orientation of research and technical change is given by $\widehat{B}_\infty^R / \widehat{Q}_\infty^R = \alpha/(1-\alpha)$.*
Proof. *See appendix C.2.* ■

In case of a binding resource constraint, resource scarcity forces the social planner to save on polluting inputs to such an extent that investing in green innovation to bring about an even faster decline in pollution is not optimal in the long run. On the other hand, the depletion of the non-renewable resource poses an increasing threat to economic growth over time. Therefore, asymptotically, green innovation comes to a halt. All labor in the research sector is shifted towards productivity improvements. Productivity growth raises the productivity of intermediate goods and thereby dampens the adverse effects from resource scarcity on output and consumption growth.

With a binding resource constraint, the need to save scarce resources solves the pollution problem. Proposition 2, however, also suggests that under realistic conditions it may be vice versa: The preference for a clean environment may make it optimal to restrict resource use in a way that the resource stock is never exhausted. This also means that the inevitable deceleration induced by resource scarcity will not solve the pollution problem. We have pointed out that the parameter constellations for which there is quantity degrowth in the long-run optimal solution are well in line with empirical evidence. In particular, quantity degrowth has been shown to be a likely outcome of the social planner's optimization problem if the intermediate good is interpreted as energy input and its production elasticity α as the energy share in GDP. Further, although fossil resources are effectively bounded, the large stocks particularly of coal still in the ground suggests that the assumption of a finite but large initial resource stock is also realistic. We conclude that without too strong restrictions on the parameter range, the long-run results from the socially optimal solution of the baseline model extend to a model with a non-renewable resource.

6 Conclusion

Pollution accumulation in our endogenous growth model can be controlled by green innovation and by reducing rebound effects from productivity growth on input quantity. The latter goes along with a cost in terms of foregone potential growth in consumption and GDP which we referred to as 'deceleration'.

On a BGP, deceleration cannot occur since output, polluting intermediate inputs, consumption and productivity all grow at the same rate. This means that rebound effects are not restricted. A channel of pollution control is thus neglected in otherwise related literature focussing on balanced growth. The first contribution of this paper is to extend the analysis beyond balanced growth paths. This enables us to address the question of whether and when a deliberate reduction of consumption growth below productivity growth to decrease the growth of polluting inputs may be socially desirable.

By construction of our model, no growth would generally be socially preferable to the laissez-faire equilibrium (which exhibits neither green innovation nor deceleration). At the same time, given the possibility of pollution control, long-run economic growth is a desirable aim from a social planner's perspective. The second contribution of this paper is to show that for empirically reasonable parameter values, optimal pollution control involves green innovation and persistent reduction of rebound effects which requires persistent deceleration. Fostering productivity growth while investing in green innovation to decrease the pollution intensity of production does not achieve the optimal balance between consumption and pollution growth. It has to be ensured that productivity growth does not merely lead to a faster expansion of production: The rebound effect of productivity growth must be restricted. The model also shows that we cannot rely on resource scarcity to induce sufficient deceleration to solve the pollution problem.

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A Appendix to section 3 (Laissez-faire)

The derivation of the laissez-faire equilibrium can be found in an extended appendix to this paper, available upon request.

A.1 Proof of proposition 1

1. **Existence and Uniqueness:** Proof of existence and uniqueness follows the proof in the standard Schumpeterian growth model and is contained in the extended appendix.

2. **Welfare comparison:** To prove that for convex disutility of pollution, a path without long-run growth would be welfare-improving, consider the utility function as function of the pollution stock S which is obtained using (2):

$$U = \int_0^\infty e^{-\rho t} \left(\frac{\sigma_c}{\sigma_c - 1} c_t^{\frac{\sigma_c - 1}{\sigma_c}} - \psi \frac{\sigma_E}{1 - \sigma_E} S_t^{\frac{1-\sigma_E}{\sigma_E}} \right) L dt \tag{A.1}$$

For convex disutility of pollution ($\sigma_E < 1/2$), $\frac{1-\sigma_E}{\sigma_E}$ is at least one while $\frac{\sigma_c - 1}{\sigma_c}$ is smaller than one. Along the balanced-growth path, $\widehat{S}^{\text{LF}} = \widehat{S}_\infty^{\text{LF}} = \widehat{c}^{\text{LF}}$. Instantaneous utility $u_t = \frac{\sigma_c}{\sigma_c - 1} c_t^{\frac{\sigma_c-1}{\sigma_c}} - \psi \frac{\sigma_E}{1-\sigma_E} S_t^{\frac{1-\sigma_E}{\sigma_E}}$ converges to $-\phi^S(S_t) = -\psi \frac{\sigma_E}{1-\sigma_E} S_t^{\frac{1-\sigma_E}{\sigma_E}}$ and declines persistently towards $(-\infty)$. The long-run growth rate is $\frac{1-\sigma_E}{\sigma_E} \widehat{S}_\infty^{\text{LF}}$. Now assume instead that economic growth is given up in a period s: Consumption growth drops to zero instantly while pollution growth converges to zero over time. Initially, there is a loss in per-period-utility compared to the laissez-faire equilibrium. This loss is only transitory: In the long-run, the pollution stock is constant and so is utility, while utility decreases in the laissez-faire equilibrium. Therefore, from a certain time onwards, not growing yields a utility-gain in each period which increases as $t \to \infty$. Because of the concavity of the utility from consumption and convexity of the disutility from pollution, the transitional welfare-loss is smaller, the later in time the regime-switch occurs and converges to zero as $s \to \infty$. Giving up economic growth in the long-run therefore yields an increase in intertemporal welfare.

B Appendix to section 4 (Social Planner)

B.1 Maximization problem

To see that the optimal q_{it} and b_{it} are the same for all sectors i, i.e. $q_{it} = q_t$ and $b_{it} = b_t$, note that the social planner chooses the step-size in every sector i so as to reach a given rate of change $\dot{Q}_t$ and $\dot{B}_t$ in the respective aggregate technology level with a minimum labor investment. From the equations of motion (12) and (13) for Q and B together with the R&D-cost function (9) we can conclude that the marginal gain of an increase in b_i and q_i, in terms of faster technological progress, and the additional amount of labor required increase in the sectorial technology levels Q_{it} and B_{it} in the same way. Therefore sectorial differences are irrelevant for the optimal choice of q_i and b_i.

The dynamic optimization problem then depends on aggregate variables only: From (9), with $\int_0^1 Q_{it}di = Q_t$, $\int_0^1 B_{it}di = B_t$ and $n_{it} = n_t$, the amount of labor allocated to research in period t is $L_{Dt} = n_t(q_t^2 + b_t^2 + d)$. To produce X_t units of intermediates requires $L_{Xt} = \frac{1}{\varphi}\frac{X_t}{Q_t}$ units of labor. The labor market constraint can be written as

$$L = \frac{1}{\varphi}\frac{X_t}{Q_t} + L_{Yt} + n_t(q_t^2 + b_t^2 + d). \tag{B.1}$$

The equations of motion (12) for Q and (13) for B are:

$$\dot{Q}_t = \mu n q_t Q_t \tag{B.2}$$

$$\dot{B}_t = \mu n b_t B_t \tag{B.3}$$

Given aggregate intermediate production X_t the decision over X_{it} is static. The planner optimally allocates a higher share of aggregate intermediate production to the sectors with higher productivity level so as to maximize Y_t. The optimal X_{it} is:

$$X_{it} = X_t \frac{Q_{it}}{Q_t} \tag{B.4}$$

With (B.4), the aggregate resource constraint can be rewritten as:

$$L_{Yt}^{1-\alpha} X_t^{\alpha} Q_t^{1-\alpha} = c_t L \tag{B.5}$$

The dynamic maximization problem is solved by finding the optimal paths for Q, B, S, c, X, L_Y, n, q and b subject to (8), (B.1), (B.2), (B.3) and the resource constraint (B.5). The current-value Hamiltonian is given by:

$$\begin{aligned} H \quad = \quad & \left(\frac{\sigma_c}{\sigma_c - 1} c_t^{\frac{\sigma_c - 1}{\sigma_c}} - \psi \frac{\sigma_E}{1-\sigma_E} S_t^{\frac{1-\sigma_E}{\sigma_E}}\right) L \\ & + v_{St}\left(\frac{X_t}{B_t} - \delta S_t\right) \\ & + v_{Qt}\mu n_t q_t Q_t \\ & + v_{Bt}\mu n_t b_t B_t \\ & + \lambda_{Yt}\left(X_t^{\alpha} Q_t^{1-\alpha} L_{Yt}^{1-\alpha} - c_t L\right) \\ & + \lambda_{Lt}(L - \frac{1}{\varphi}\frac{X_t}{Q_t} - L_{Yt} - n_t(q_t^2 + b_t^2 + d)) \end{aligned}$$

where v_{St}, v_{Qt} and v_{Bt} are the shadow-prices of S_t, Q_t and B_t respectively and λ_{Yt} and λ_{Lt} are Lagrange-multipliers.

B.2 First-order conditions

The first-order conditions are:

$$\frac{\partial H}{\partial c_t} = 0 \Leftrightarrow \lambda_{Yt} = c_t^{-1/\sigma_c} \tag{B.6}$$

$$\frac{\partial H}{\partial X_t} = 0 \Leftrightarrow \frac{v_{St}}{B_t} + \lambda_{Yt}\alpha X_t^{\alpha-1} L_{Yt}^{1-\alpha} Q_t^{1-\alpha} - \lambda_{Lt}\frac{1}{\varphi Q_t} = 0 \tag{B.7}$$

$$\frac{\partial H}{\partial q_t} = 0 \Leftrightarrow v_{Qt}\mu n_t Q_t = 2\lambda_{Lt} n_t q_t \tag{B.8}$$

$$\frac{\partial H}{\partial b_t} = 0 \Leftrightarrow v_{Bt}\mu n_t B_t = 2\lambda_{Lt} n_t b_t \tag{B.9}$$

$$\frac{\partial H}{\partial n_t} = 0 \Leftrightarrow v_{Qt}\mu q_t Q_t + v_{Bt}\mu b_t B_t = \lambda_{Lt}\left(q_t^2 + b_t^2 + d\right) \tag{B.10}$$

$$\frac{\partial H}{\partial L_{Yt}} = 0 \Leftrightarrow \lambda_{Yt}(1-\alpha) X_t^{\alpha} Q_t^{1-\alpha} L_{Yt}^{-\alpha} = \lambda_{Lt} \tag{B.11}$$

$$\frac{\partial H}{\partial S_t} = \rho v_{St} - \dot{v}_{St} \Leftrightarrow -\psi S_t^{(1-2\sigma_E)/\sigma_E} L - \delta v_{St} = \rho v_{St} - \dot{v}_{St} \tag{B.12}$$

$$\frac{\partial H}{\partial Q_t} = \rho v_{Qt} - \dot{v}_{Qt}$$

$$\Leftrightarrow v_{Qt}\mu n_t q_t + \lambda_{Yt}(1-\alpha) X_t^{\alpha} Q_t^{-\alpha} L_{Yt}^{1-\alpha} + \lambda_{Lt}\frac{X_t}{\varphi}\frac{1}{Q_t^2} = \rho v_{Qt} - \dot{v}_{Qt} \tag{B.13}$$

$$\frac{\partial H}{\partial B_t} = \rho v_{Bt} - \dot{v}_{Bt} \Leftrightarrow -v_{St}\frac{X_t}{B_t^2} + v_{Bt}\mu n_t b_t = \rho v_{Bt} - \dot{v}_{Bt} \tag{B.14}$$

$$\frac{\partial H}{\partial v_{St}} = \dot{S}_t \Leftrightarrow \frac{X_t}{B_t} - \delta S_t = \dot{S}_t \tag{B.15}$$

$$\frac{\partial H}{\partial v_{Qt}} = \dot{Q}_t \Leftrightarrow \mu n_t q_t Q_t = \dot{Q}_t \tag{B.16}$$

$$\frac{\partial H}{\partial v_{Bt}} = \dot{B}_t \Leftrightarrow \mu n_t b_t B_t = \dot{B}_t \tag{B.17}$$

$$\frac{\partial H}{\partial \lambda_{Yt}} = 0 \Leftrightarrow X_t^{\alpha} Q_t^{1-\alpha} L_{Yt}^{1-\alpha} = c_t L \tag{B.18}$$

$$\frac{\partial H}{\partial \lambda_{Lt}} = 0 \Leftrightarrow L = \frac{1}{\varphi}\frac{X_t}{Q_t} + L_{Yt} + n_t(q_t^2 + b_t^2 + d) \tag{B.19}$$

Further, the transversality conditions $\lim_{t\to\infty}(e^{-\rho t} v_{Qt} Q_t) = \lim_{t\to\infty}(e^{-\rho t} v_{Bt} B_t) = \lim_{t\to\infty}(e^{-\rho t} v_{St} S_t) = 0$ as well as the non-negativity constraints $Q_t, B_t, S_t, c_t, X_t, L_{Yt}, n_t \geq 0$, $\forall t$ must hold.

From the first-order conditions, four key equations crucial for the determination of the long-run optimum are derived: The condition (16) for *asymptotically-balanced growth* in the text follows from the first-order conditions for X and S: The first-order condition (B.7) for X yields a relation $\widehat{v}_{S\infty} = (1 - 1/\sigma_c)\,\widehat{c}_\infty + \widehat{B}_\infty - \widehat{X}_\infty$ between the growth rates of the marginal utility c_t^{-1/σ_c} of consumption and the shadow price v_S of pollution for $t \to \infty$. From the first-order condition (B.12) for the pollution stock, it follows that along an ABG path, the ratio $S_t^{(1-2\sigma_E)/\sigma_E}/v_{St}$ must be constant for v_S to grow at a constant rate. In the long run, v_S must therefore grow at the same rate as the (instantaneous) marginal disutility $\psi S^{(1-2\sigma_E)/\sigma_E}$ of pollution, $\widehat{v}_{S\infty} = ((1-2\sigma_E)/\sigma_E)\,\widehat{S}_\infty$. Setting equal with the expression for $\widehat{v}_{S\infty}$ obtained from (B.7) and rearranging, taking into account that $\widehat{S}_\infty = \widehat{X}_\infty - \widehat{B}_\infty$ under condition (15), yields (16) in the proof of lemma 2.

We are interested in solution candidates with $n_\infty > 0$. Solving (B.8) and (B.9) for v_Q and v_B respectively, substituting in the first-order condition (B.10) for n and taking the limit for $t \to \infty$ yields

$$q_\infty^2 + b_\infty^2 = d \tag{B.20}$$

Condition (B.20) is an *indifference condition.* It guarantees that the social planner is indifferent between all possible values for n.

Dividing by v_{Qt}, setting $t = \infty$ and rearranging, (B.13) can be written as:

$$(1/\sigma_c)\,\widehat{c}_\infty + \rho = \frac{1}{2}\mu q_\infty^{-1}\left(L_{Y\infty} + \frac{1}{\varphi}\left(\frac{X}{Q}\right)_\infty\right) + \alpha \widehat{X}_\infty + (1-\alpha)\mu n_\infty q_\infty \tag{B.21}$$

Equation (B.21) is a version of the *consumption Euler-equation,* where we replaced the shadow-prices and Lagrange-multipliers as well as their growth rates using (B.8), (B.11) and (B.6).
Both research directions, that is, increasing Q and increasing B, must yield the same social net return. We manipulate the first-order condition (B.14) for B similarly to the one for Q, using (B.9) as well as the expression $v_{St} = \left(\lambda_{Lt}\frac{1}{\varphi Q_t} - \lambda_{Yt}\alpha X_t^{\alpha-1}L_{Yt}^{1-\alpha}Q_t^{1-\alpha}\right)B_t$ from (B.7), and equations (B.11) and (B.6). Setting equal the right-hand sides of (B.21) and the modified first-order condition for B, we obtain the *research-arbitrage condition*

$$\frac{1}{2}\mu q_\infty^{-1}\left(L_{Y\infty} + \frac{1}{\varphi}\left(\frac{X}{Q}\right)_\infty\right) = \frac{1}{2}\mu b_\infty^{-1}\left(\frac{\alpha}{1-\alpha}L_{Y\infty} - \frac{1}{\varphi}\left(\frac{X}{Q}\right)_\infty\right). \tag{B.22}$$

B.3 Proof of theorem 1

If growth rates are to be constant asymptotically, equation (B.22) requires intermediate quantity in efficiency units, more precisely the ratio $(X/Q)_\infty$, to be constant in the limit as well.

A balanced growth path, along which productivity and cleanliness grow at constant rates not only asymptotically, must be characterized by a strictly positive $(X/Q)_\infty$[28]. There must therefore be equal growth in intermediate quantity, productivity and (from the resource constraint) also consumption. Equation (16) then yields a ratio $\widehat{B}_\infty/\widehat{Q}_\infty$:

$$\widehat{B}_\infty/\widehat{Q}_\infty = 1 - \frac{(\sigma_c - 1)/\sigma_c}{(1-\sigma_E)/\sigma_E}. \tag{B.23}$$

If $\alpha/(1-\alpha) < 1 - \frac{(\sigma_c-1)/\sigma_c}{(1-\sigma_E)/\sigma_E}$ (see (14)), a balanced growth solution to the social planner's problem does not exist, because the ratio $\widehat{B}_\infty/\widehat{Q}_\infty$ in (B.23) is not reconcilable with equation (B.22) for any nonnegative $(X/Q)_\infty$. As $X/Q < 0$ has no sensible interpretation, the optimal solution is to let X/Q converge to zero asymptotically by choosing $\widehat{X}_\infty < \widehat{Q}_\infty$. According to (B.22), the optimal ratio $\widehat{B}_\infty/\widehat{Q}_\infty$ corresponds to

$$\widehat{B}_\infty/\widehat{Q}_\infty = \frac{\alpha}{1-\alpha}. \tag{B.24}$$

With the definition of the direction of technical change, it follows straightforwardly that technical change is green (productivity-oriented) if and only if $\alpha > 1/2$ ($\alpha < 1/2$).

To compute the relation between the growth rates $\widehat{X}_\infty$ and $\widehat{Q}_\infty$, we use (16), substituting $\widehat{X}_\infty - \widehat{B}_\infty = \widehat{X}_\infty - \frac{\alpha}{1-\alpha}\widehat{Q}_\infty$ for $\widehat{S}_\infty$ and $\alpha\widehat{X}_\infty + (1-\alpha)\,\widehat{Q}_\infty$ from the resource constraint for $\widehat{c}_\infty$. After some manipulation,

[28] On a balanced growth path, $(X/Q)_\infty = 0$ implies $X_t/Q_t = 0$ for all t. This is only possible if $X_t = c_t = 0$ for all t which cannot be an optimal path for X because the utility function satisfies the Inada-conditions for c_t.

we obtain:

$$\widehat{X}_\infty = \frac{1+\left(\frac{\alpha}{1-\alpha}\right)^2 - \left(1-\frac{(\sigma_c-1)/\sigma_c}{(1-\sigma_E)/\sigma_E}-\frac{\alpha}{1-\alpha}\right)}{1+\frac{\alpha}{1-\alpha}\left(1-\frac{(\sigma_c-1)/\sigma_c}{(1-\sigma_E)/\sigma_E}\right)}\widehat{Q}_\infty \tag{B.25}$$

For $\frac{\alpha}{1-\alpha} < 1 - \frac{(\sigma_c-1)/\sigma_c}{(1-\sigma_E)/\sigma_E}$, it is obvious that $\widehat{X}_\infty < \widehat{Q}_\infty$ given $\widehat{Q}_\infty > 0$.

The proof of uniqueness is contained in the extended appendix.

B.4 Proof of corollary 2

See the extended appendix.

C Appendix to section 5.2 (Optimum with a non-renewable resource)

C.1 First-order conditions

Three changes occur in the set of necessary first-order conditions compared to the baseline model: First, the shadow price λ_R of the non-renewable resource contributes to the marginal social cost of intermediate production instead of the marginal labor requirement, so that the first-order condition for X becomes

$$\frac{\partial H}{\partial X_t} = 0 \Leftrightarrow \frac{v_{St}}{B_t} + \lambda_{Yt}\alpha X_t^{\alpha-1}L_{Yt}^{1-\alpha}Q_t^{1-\alpha} - \lambda_{Rt} = 0. \tag{C.1}$$

In the first-order condition (B.13) for Q, the last term on the left-hand side $\left(\lambda_{Lt}\,(1/\varphi)\left(X_t/Q_t^2\right)\right.$ drops out because Q no longer affects the production of intermediate goods.

Second, the first-order conditions are complemented by a complementary slackness condition:

$$\frac{\partial H}{\partial \lambda_{Rt}} \leq 0 \Leftrightarrow F_0 - \int_0^\infty X_t dt \geq 0 \quad \lambda_{Rt} \geq 0 \quad \lambda_{Rt}\left(F_0 - \int_0^\infty X_t dt\right) = 0 \tag{C.2}$$

Third, labor is only allocated to research and output production. The first order condition for λ_{Lt} changes to:

$$\frac{\partial H}{\partial \lambda_{Lt}} = 0 \Leftrightarrow L = L_{Yt} + n_t(q_t^2 + b_t^2 + d) \tag{C.3}$$

The set of first-order conditions is otherwise unaffected by the modifications in the model setup.

C.2 Proof of proposition 2

C.2.1 (a) Binding constraint

(i) **Quantity degrowth:** If there is quantity degrowth, $S_\infty = 0$ so that $v_{S\infty} = 0$, while λ_R grows persistently. To satisfy the first-order condition (C.1) for X, the social marginal product of X in production must equal λ_R asymptotically:

$$c_\infty^{-1/\sigma_c}\alpha X_\infty^{\alpha-1}L_{Y\infty}^{1-\alpha}Q_\infty^{1-\alpha} = \lambda_{R\infty} \tag{C.4}$$

Note that we already substituted $\lambda_Y = c_\infty^{-1/\sigma_c}$ from the first-order condition for c. Condition (C.4) replaces condition (16) for asymptotically-balanced growth from the baseline model. Computing growth rates on both sides of (C.4) yields $(-1/\sigma_c \cdot \widehat{c}_\infty) - (1-\alpha)\left(\widehat{X}_\infty - \widehat{Q}_\infty\right) = \rho$. From this equation, using $\widehat{c}_\infty = \alpha\widehat{X}_\infty + (1-\alpha)\widehat{Q}_\infty$, we derive the growth rate $\widehat{X}^{\mathrm{R}}_\infty$ for any given $\widehat{Q}^{\mathrm{R}}_\infty$:

$$\widehat{X}^{\mathrm{R}}_\infty = \frac{1}{\frac{\alpha}{1-\alpha}\frac{1}{\sigma_c} + 1}\left(\left(1 - \frac{1}{\sigma_c}\right)\widehat{Q}^{\mathrm{R}}_\infty - \frac{1}{1-\alpha}\rho\right) \tag{C.5}$$

If $\sigma_c < 1$, it can be seen directly that $\widehat{X}^{\mathrm{R}}_\infty < 0$. For $\sigma_c > 1$ the transversality conditions, which require $\rho > \left(1 - \frac{1}{\sigma_c}\right)\widehat{Q}^{\mathrm{R}}_\infty$, together with $(1-\alpha) < 1$ guarantee that indeed $\widehat{X}^{\mathrm{R}}_\infty < 0$.

(ii) **Green Innovation:** The research-arbitrage equation is:

$$\frac{\mu}{2q_\infty} L_{Y\infty} = \frac{\mu}{2b_\infty} L_{Y\infty}\left(\frac{\alpha}{1-\alpha} - \frac{1}{1-\alpha}\left(\frac{\lambda_R}{\lambda_Y}\right)_\infty \left(\frac{X}{Q}\right)^{1-\alpha}_\infty L^{\alpha-1}_{Y\infty}\right) \tag{C.6}$$

Substituting (C.4) in (C.6) shows that investing in the cleanliness of technology is not optimal in the long run:

$$\begin{aligned} \frac{\mu}{2b_\infty} L_{Y\infty}\left(-\frac{\alpha}{1-\alpha} + \frac{\alpha}{1-\alpha}\right) &= (\rho - (1 - 1/\sigma_c)\,\widehat{c}_\infty) \\ \Leftrightarrow b^{\mathrm{R}}_\infty &= 0 \end{aligned}$$

From $q^2_\infty + b^2_\infty = d$ it follows that $q^{\mathrm{R}}_\infty = \sqrt{d}$ so that labor in the R&D-sector is entirely used for productivity-oriented innovation.

C.2.2 (b) Unbinding constraint

Given the assumption of continuity of the path for X, the integral $\int_0^\infty X_t dt$ converges if there is quantity degrowth in the long run (see the extended appendix). Therefore $\int_0^\infty X_t dt < F_0$ for a sufficiently large F_0. In this case, the natural resource constraint is not binding and it follows from (C.2) that $\lambda_{Rt} = 0$, $\forall t$. If $\lambda_R = 0$, differences in the first-order conditions compared to the baseline model only arise because labor is no longer used in intermediate production in the model of this section. But for parameter constellations such that there is quantity degrowth in the baseline model, labor use in intermediate production converges to zero in the baseline model as well, so that the first-order conditions and therefore the long-run solutions are identical for $t \to \infty$.