Separation of Credit Risk and Maturity Transformations*

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Abstract

This paper studies intermediation chains in which the transformation of credit risk and the transformation of maturities are performed in two separate steps – as in case of a securitization of assets and a financing of the senior tranches with short-term debt. The paper shows that the separation of the two transformations increases the default probability of the resulting short-term debt in comparison to a case in which the assets are directly financed with short-term debt. In spite of this increase of solvency risk, intermediation chains can emerge as optimal financing structure, because the securitization increases the liquidity. And the increase in liquidity allows to provide a larger amount of ‘money-like’ claims to investors. This relative advantage of a chain is lost, however, if the provision of money-like claims is supported by a public insurance of short-term debt. And if risk-shifting at the expense of such an insurance shall be prevented, intermediation chains have to be subject to higher capital requirements than a bank which directly finances the assets with short-term debt.

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1 Introduction

Financial intermediation involves the financing of risky long-term assets with relatively safe short-term debt, which means that it involves a transformation of maturities as well as a transformation of credit risk. The analysis of financial intermediaries usually assumes that the two transformations are performed by the same firm – for instance, by a bank that holds risky long-term assets and provides deposits to investors. I will refer to this mode of intermediation as ‘traditional bank’. During the recent decades, however, the form of financial intermediation has changed and it has often been split up into different steps, as illustrated in Pozsar et al. (2016) or GSBMR (2016). Take the following example: long-term, risky assets like mortgages, student loans, etc. are pooled and tranch by firm A, and A sells long-term senior tranches to firm B that finances these securities with short-term debt. This form of intermediation separates the maturity transformation (performed by firm B) from the transformation of credit risk (given by the tranching of firm A). I will refer to this form of intermediation as ‘intermediation chain’. The main contribution of this paper is to show that the separation of the two transformations decreases the stability of financial intermediation. This means: short-term debt issued by an intermediation chain has a larger default probability than short-term debt issued by a traditional bank, given the same underlying assets and the same amount of equity in the chain and in the bank. Put differently, a chain requires a larger amount of equity than a traditional bank in order to avoid a default of short-term debt.

Consider a set of assets whose value evolves stochastically and which is held by a traditional bank, which means that the assets are directly financed with short-term debt. If the equity of the bank has the initial value \( e \), the short-term debt defaults when the sum of shocks to the value of assets is larger than \( e \). Compare this to an intermediation chain in which firm A holds the same assets as the traditional bank and has an initial equity value \( e_A = e \), but it sells long-term debt to firm B, which finances this purchase by issuing short-term debt. If there are shocks to the assets, the equity value of firm A decreases. But such shocks also increase the conditional probability that the asset value will be smaller than the face value of the long-term debt when it matures. Consequently, the shocks decrease the value of the long-term debt. If firm B has not issued some equity \( e_B \) which can absorb this shock to the value of its portfolio, it has to default on its short-term debt.

This means that the initial value \( e_A + e_B \) of equity in the chain has to be larger than the equity value \( e \) of the traditional bank in order to achieve the same safety of the short-term debt, given the same underlying assets. This result is not particular to the case \( e_A = e \), but it holds for any distribution \((e_A, e_B)\) of equity in the chain with \( e_A > 0 \). The equity value \( e_A \) consists of the expected payoff of the equity claims. This includes the payoff of the equity claims after transitory shocks, from which the assets recover before the long-term debt becomes due. This part of the equity value \( e_A \) is not ‘available’ for absorbing shocks at intermediate dates. If there is a shock, the equity of firm A maintains some
value owing to the remaining possibility that the asset value will be larger than the debt liability at maturity, while the value of the long-term debt declines due to an increase of the conditional default probability. This observation might be rephrased as: there is no strict ‘seniority’ of long-term debt relative to equity with respect to shocks at intermediate dates. And while firm A maintains some equity in case of a shock at an intermediate date, firm B needs some additional equity in order to protect its short-term debt against the shock to the value of the long-term debt.

Given the relative increase of the default probability due to a separation of the transformations, one might wonder why intermediation chains have become an important form of financial intermediation. Do such chains allow for some efficiency gains or are they just due to regulatory arbitrage? And if the latter is true, one has to study how the regulation of chains and traditional banks has to be adjusted in order to avoid such regulatory arbitrage. This paper addresses these issues by providing two alternative explanations for the emergence of intermediation chains. The first explanation points out why a chain can be more efficient than a bank owing to an increase in liquidity. The second explanation illustrates that chains also emerge, if the short-term debt of both, banks and chains, is insured, while both forms of intermediation are subject to (prima facie) equally strict capital requirements. This explanation highlights that capital requirements for chains have to be higher than for banks in order to avoid regulatory arbitrage.

The two explanations are presented in a model, in which the incentive for issuing short-term debt is due to a premium for safe, ‘money-like’ claims, as it has been suggested by Gorton and Pennacchi (1990). A second feature of the model is that the selection of the assets that shall be financed is subject to an agency problem. This problem can be solved if the selecting agent retains a sufficient amount of junior claims to the assets. The retention of junior claims leads to a partial separation of credit risk and maturity transformations in an intermediation chain. The third feature of the model is that assets can be illiquid for two reasons: first, if the assets have to be liquidated, they have to be sold to arbitrageurs who demand a compensation for the opportunity costs of providing liquidity; second, the firm that initially holds the assets obtains special skills in operating these assets (like a commercial bank that establishes lending relationships, for instance) which are lost when the assets are liquidated and sold.

The traditional bank finances the illiquid assets by issuing short-term debt. This can lead to a coordination problem when the short-term debt has to be rolled over. The threat of a non-fundamental run and the corresponding liquidation loss constrain the level of safe short-term debt that the bank can issue and for which it can earn a premium. In intermediation chains, in contrast, the short-term debt claims do not refer directly to the underlying assets, but they refer to the long-term debt issued against the assets. If the

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1. As illustrated in Pozsar et al. (2016), GSBMR (2016) and many other papers.
2. This is in line with the literature on securitization and the related agency problems, see e.g. Gorton & Pennacchi (1995).
holders of the short-term debt withdraw their funding of the chain, the long-term debt has
to be liquidated, but not the underlying assets. The long-term debt is also illiquid to the
extent that the arbitrageurs have to be compensated for their opportunity costs of buying
the debt. The underlying assets, however, remain with the agents that have obtained
special skills in operating them. This means that the loss from liquidating long-term debt
is smaller than the loss from liquidating the underlying assets. As a consequence, an
intermediation chain can provide a higher level of short-term debt without facing non-
fundamental runs. If this reduction of liquidity risk is larger than the increase of solvency
risk due to the separation of the transformations, then the chain allows for issuing more
safe debt. Given a premium for safe debt, the chain is the optimal form of intermediation
in that case.

This explanation of intermediation chains applies to segments of financial markets that
have no access to an insurance of short-term debt. Since such an insurance can resolve
coordination problems that are due to illiquidity, it eliminates a constraint for the provi-
sion of safe claims. It thus increases the efficiency of both, bank and chain. But it also
negates the relative advantage of an intermediation chain that I have just pointed out.
An insurance of short-term debt has to be accompanied by appropriate insurance premi-
ums or capital requirements which ensure that the short-term debt is only insured against
liquidity risk, but not against solvency risk. Since an intermediation chain requires more
equity in order to remain solvent, the capital requirements or insurance premiums have to
be larger for a chain than for a bank. Larger capital requirements imply a lower level of
debt that can be issued. At the same time, the chain has no longer a relative advantage
from having less liquid assets. Consequently, the traditional bank is the privately optimal
form of intermediation in presence of an insurance of short-term debt and a regulation
which prevents that the insurance covers solvency risk.

The appropriate calibration of the capital requirements or insurance premiums, however,
requires an understanding of the increase in the default probability due to a separation
of the transformations. Imagine that the insurance provider is unaware of this problem,
but is guided by the idea of a ‘level playing field’, which means that it imposes the same
lower bound for equity on a chain as it imposes on a bank. Given the different default
probabilities, such a regulation implies a subsidization of the chain, because the insurance
covers insolvency risk of the chain that is not covered in case of a bank. Given this sub-
sidy, a chain can become the privately optimal form of intermediation, at the expense of
the insurance provider. The implication for regulatory debates is: if a public authority
provides an insurance of short-term debt that avoids inefficient non-fundamental runs,
while it wants to avoid a subsidization of intermediation chains, then it should require
that chains have more equity than traditional banks.

Additional related literature: Allen et al. (2015) and Gale & Gottardi (2017) analyze
the optimal distribution of equity between firms and banks that lend to these firms. The
difference to this paper is that they only address the transformation of credit risk, but
no maturity transformation. Maturity transformations and their division into different steps in an intermediation chain are studied in Flore (2018). But that paper focuses on maturity transformations and does not address the separation of credit risk and maturity transformation. Glode & Opp (2016) also analyze ‘intermediation chains’, but they study intermediation chains in which assets are traded along a chain of dealers without any maturity transformation or choice of capital structure. The literature on financial networks, following Allen & Gale (2000) and Freixas et al. (2000), describes a certain type of ‘intermediation chains’. The networks studied in that literature, however, consist of nodes that engage in the same type of maturity transformation and do not separate it from the credit risk transformation. Finally, there is also a literature that highlights the regulatory differences between traditional banks and intermediation chains. Examples for theoretical analyses are Hanson et al. (2015), Plantin (2015), Flore (2015), Luck & Schempp (2014) & (2016). None of these papers, however, has stressed the consequences of a separation of credit risk and maturity transformations for the regulation of financial intermediation.

The remainder of this paper is structured as follows: Section 2 illustrates with a simple example that a separation of credit risk and maturity transformations increases the default probability of short-term debt. Section 3 provides an explanation why intermediation chains with such a separation might yet be efficient. Section 4 points out that intermediation chains whose short-term debt is insured have to be subject to stronger regulation than insured banks. Section 5 shows that the key result of this paper is robust to a generalization of the risk structure.

2 Default Probabilities in Case of Separated Transformations of Maturity and Credit Risk

This section illustrates the key result of this paper with a simple example, before Section 5 shows that the result holds for very general cases. To highlight the purely mechanical fact that a separation of credit risk and maturity transformation increases the default probability of short-term debt, I abstract from any frictions and any description of agents and their behavior in this section.

Consider assets that yield either 1 or $1 - a$ at $t = 2$. At $t = 1$, there is a public signal about the probabilities of the two potential payoffs. The expected payoff of the assets, conditional on the information available at $t$, is denoted as $y_t$.

At $t = 1$, the uncertainty about the payoff at $t = 2$ is either resolved by a signal that the assets will yield 1 with certainty (I refer to this as a ‘good shock’ at $t = 1$), or the uncertainty remains until $t = 2$ (denoted as ‘bad shock’ at $t = 1$). The respective probabilities of the two cases are $1 - p_1$ and $p_1$. In the latter case, the remaining uncertainty about the payoff at $t = 2$ is resolved at $t = 2$: the assets either yield 1 (denoted as ‘good
shock' at $t = 2$) or just $1 - a$ (denoted as ‘bad shock’ at $t = 2$). The respective probabilities are $1 - p_2$ and $p_2$.

Assume that the assets are financed with equity and debt claims. I first consider the case that the debt is ‘short-term’, which means that is has to be rolled over at $t = 1$. Let us assume that the value of the equity and debt claims is determined in a market with complete information about the asset risk, risk-neutral pricing and a risk-free interest rate $r$ normalized to 0. In that case, debt with initial face value $D_S \in [0, 1]$ can be rolled over, if the new face value $D_{S,1}$ of the debt claim is such that the expected payoff of the claim at $t = 2$ equals $D_S$, which means: $D_S = E_{t=1}[\min\{D_{S,1}, y_2\}]$. If $D_S > E_{t=1}[y_2] = y_1$, no roll over is possible, but there is a debt default at $t = 1$. A debt default at $t = 2$ occurs for $D_{S,1} > y_2$. The probability of a default at $t = 1$ shall be denoted as $\phi_1$, and the probability that a debt default occurs either at $t = 1$ or at $t = 2$ shall be denoted as $\phi$. Both probabilities depend on the initial debt level $D_S$. Their form follows from the previous remarks and the fact that $D_S \leq 1 - p_2 a \Rightarrow D_{S,1} \leq 1$:

$$\phi_1(D_S) = \begin{cases} 0 & \text{for } D_S \in [0, 1 - p_2 a] \\ p_1 & \text{for } D_S \in (1 - p_2 a, 1] \end{cases}$$

$$\phi(D_S) = \begin{cases} 0 & \text{for } D_S \in [0, 1 - a] \\ p_1 p_2 & \text{for } D_S \in (1 - a, 1 - p_2 a] \\ p_1 & \text{for } D_S \in (1 - p_2 a, 1] \end{cases}$$

Let us now add an additional step to the financing of the assets. Assume that the assets described above are financed by issuing equity and ‘long-term’ debt, which means debt with face value $D_L \in [0, 1]$ that matures at $t = 2$ and does not have to rolled over at $t = 1$. Assume further that this long-term debt claim is held on a second balance sheet, where it is financed with equity and short-term debt. The short-term debt has an initial face value $M_S \in [0, D_L]$ and has to be rolled over at $t = 1$.

In order to refer to the different balance sheets, let us denote the financing of the assets with equity and long-term debt as ‘security issuer’, while the financing of the long-term debt with equity and short-term debt shall be denoted as ‘maturity transformer’. And for comparison, let us denote the

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3 Since a debt claim with $D_S > 1$ has the same payoff as one with $D_S = 1$, I focus on $D_S \in [0, 1]$.

4 Since the maximal possible payoff of the portfolio consisting of the long-term debt claim is just $D_L$, a debt claim with $M_S > D_L$ has the same payoff as for $M_S = D_L$, and I thus focus on $M_S \in [0, D_L]$. 

combination of both balance sheets as ‘intermediation chain’ and the financing structure described at the beginning of this section as ‘bank’. (The description focuses on the debt claims, but the values of the equity claims are implicitly given by the expected residual payoffs of the entities.)

![Figure 2: Schematic balance sheets of two different modes of financing.]

The expected payoff of the long-term debt at \( t = 2 \) conditional on the information at \( t \) is equal to \( E_t[\min\{D_L, y_2\}] \) and shall be denoted as \( y_1^D(D_L) \). If \( D_L \leq 1 - a \), then \( y_1^D(D_L) = D_L \) for \( t = 1, 2 \) independent of the shocks. If \( D_L \in (1 - a, 1] \), then \( y_1^D(D_L) \) evolves according to the event tree depicted in Fig. 3.

![Figure 3: Stochastic process of the value \( y_1^D(D_L) \) of the long-term debt.]

Since the portfolio of the maturity transformer consists of the long-term debt of the security issuer, the portfolio value at \( t \) is given by \( y_1^D(D_L) \). Analogous to the case of a bank, the short-term debt of the maturity transformer can be rolled over at \( t = 1 \), if the new face value \( M_{S,1} \) of debt claim is such that the expected payoff of the claim at \( t = 2 \) equals \( M_S \), which means: \( M_S = E_{t=1}[\min\{M_{S,1}, y_2^D(D_L)\}] \). If \( M_S > E_{t=1}[y_2^D(D_L)] = y_1^D(D_L) \), the maturity transformer defaults on its short-term debt at \( t = 1 \). The value \( y_1^D(D_L) = (1 - p_2)D_L + p_2(1 - a) \) of the long-term debt at \( t = 1 \) in case of a bad shock shall be denoted as \( y_-(D_L) \). The maturity transformer defaults on its short-term debt at \( t = 2 \) in case of \( M_{S,1} > y_2^D(D_L) \). The probability of a default at \( t = 1 \) shall be denoted as \( \phi_{M,1} \), and the probability that a default of the short-term debt occurs either at \( t = 1 \) or at \( t = 2 \) shall be denoted as \( \phi_M \). Both probabilities depend on \( M_S \) and on \( D_L \). Their form follows from the previous remarks and the fact that \( M_S \leq y_-(D_L) \Rightarrow M_{S,1} \leq D_L \):
Both ways of financing the assets entail a maturity transformation as well as a transformation of credit risk. In the bank, however, both transformations are performed on one balance sheet, while the transformations are partly separated in the intermediation chain: the maturity transformer performs the entire maturity transformation, while both balance sheets perform a part of the credit risk transformation in case of the maturity transformer performs the entire maturity transformation, while both balance sheets perform a part of the credit risk transformation in case of \( D_L < 1 \) and \( M_S < D_L \). The cases \( D_L = 1 \) and \( M_S = D_L \) are degenerate in the sense that they do no allow for a distinction between equity and debt. For \( D_L = 1 \) or \( M_S = D_L \), the security issuer or the maturity transformer only sell a single claim that receives the entire payoff of the respective portfolio. A comparison of the default probabilities of bank and intermediation chain leads to:

**Proposition 1**

Given the same underlying assets, short-term debt of an intermediation chain has a larger default probability than the same level of short-term debt of a bank:

\[
\phi_M(M_S, D_L) \geq \phi(M_S) \forall M_S \in [0, 1], \text{ and for } D_L < 1 : \ \phi_M(M_S, D_L) > \phi(M_S) \forall M_S \in (y_-(D_L), 1 - p_2 a] .
\]

The same statement holds for the default probabilities \( \phi_{M,1} \) and \( \phi_1 \) at \( t = 1 \).

**Proof:** The result follows from comparing \( \phi_{M,1} \) with \( \phi_1 \) and \( \phi_M \) with \( \phi \). The respective pairs are almost identical functions of the face value of the short-term debt, denoted as either \( D_S \) or \( M_S \). The only difference is that the interval boundary \( 1 - p_2 a \) in \( \phi_1 \) and \( \phi \) is replaced by \( y_-(D_L) \) in \( \phi_{M,1} \) and \( \phi_M \). The boundary is thus shifted downward, since \( y_-(D_L) \leq 1 - p_2 a \), with a strict inequality for \( D_L < 1 \). This implies that short-term debt of the chain with face value \( M_S \in (y_-(D_L), 1 - p_2 a] \) has the default probability \( p_1 \), while short-term debt of the bank with face value \( D_S \in (y_-(D_L), 1 - p_2 a] \) only has the default probability \( p_1 p_2 \).

Consider the example of short-term bank debt with face value \( D_S = 1 - p_2 a \). The debt can be rolled over in case of a bad shock at \( t = 1 \). The new face value is \( D_{S,1} = 1 \), which implies that there is also no default at \( t = 2 \), if the asset value recovers to \( y_2 = 1 \). The default probabilities are thus \( \phi_1 = 0 \) and \( \phi = p_1 p_2 \). For \( M_S = 1 - p_2 a \) and \( D_L < 1 \), in contrast, the maturity transformer has to default on the short-term debt in case of a bad
shock at \( t = 1 \). The maturing debt is larger than the value \( y_− = (1 - p_2)D_L + p_2(1 - a) \) of the maturity transformer’s portfolio in that state. Consequently, \( φ_1 = φ = p_1 \).

Whereas the short-term debt of the bank is a direct claim to the asset payoff, the short-term debt of the maturity transformer refers to the asset payoff via the long-term debt. And a long-term debt claim to the assets is a less valuable backing of the short-term debt than the assets themselves, even if the face value of the long-term debt is larger than the face value of the short-term debt. Consider the case \( M_S = 1 - p_2 a \) and \( D_L > 1 - p_2 a \). The value \( y_−(D_L) \) of the long-term debt in case of a bad shock at \( t = 1 \) is smaller than the asset value \( 1 - p_2 a \) in that state, although \( D_L > 1 - p_2 a \). The reason is that the value of the long-term debt is more sensitive to the remaining downside risk of the assets than to their upside risk. The values of both, assets and long-term debt, decrease in case of a bad shock at \( t = 1 \), because the conditional probability of a low payoff \( 1 - a \) increases from \( p_1 p_2 \) to \( p_2 \). But there is also the upside risk that the asset value recovers to \( 1 > D_L \) before the long-term debt becomes due at \( t = 2 \). The long-term debt, however, benefits only from a part of this upside risk. There is a residual payoff \( 1 - D_L \) in case of an asset recovery which accrues to the equity of the security issuer. This implies that the equity of the security issuer maintains a strictly positive value in case of a bad shock at \( t = 1 \). At the same time, the value of the long-term debt declines to \( y_−(D_L) < 1 - p_2 a \), so that a maturity transformer with short-term debt \( M_S = 1 - p_2 a \) becomes insolvent. Since the value of the long-term debt equals the asset value minus the equity value of the security issuer, the long-term debt is a less valuable backing of short-term debt in case of a bad shock than the assets themselves. This implies that an intermediation chain has a larger default probability than a bank.

To illustrate the result further, let me briefly rephrase it in terms of equity that has to be issued at \( t = 0 \) in order to avoid defaults. The equity values of the bank, the security issuer and the maturity transformer at \( t = 0 \) shall be denoted as \( e_B, e_I \) and \( e_M \), respectively. Owing to risk-neutrality and \( r = 0 \), the equity values at \( t = 0 \) are given as the expected payoff of the respective equity claims at \( t = 2 \). Appendix A determines the relation between these equity values and the face values of debt, and it derives the default probabilities \( φ, φ_1, φ_M, \) and \( φ_{M,1} \) of the short-term debt as functions of \( e_B \) and \( (e_I, e_M) \). Based on that, Appendix A obtains the following reformulation of Proposition 1:

**Proposition 1 (alternative formulation)**

Given the same underlying assets and the same amount of equity, an intermediation chain has a larger default probability than a bank:

\[
\begin{align*}
& φ_M(e_I, e_M) ≥ φ(e_B) \quad ∀ e_I + e_M = e_B \in [0, 1], \quad \text{and} \\
& φ_M(e_I, e_M) > φ(e_B) \quad \text{for} \quad e_I + e_M = e_B ≥ (1 - p_1)p_2 a \\
& \quad \land e_M + \left( 1 - \frac{1 - p_2}{1 - p_1 p_2} \right) e_I < (1 - p_1)p_2 a .
\end{align*}
\]
The same statement holds for the default probabilities $\phi_{M,1}$ and $\phi_1$ at $t = 1$.

Being just a reformulation of Proposition 1, the proposition holds for the same reason. A part of the equity value of the security issuer is due to the remaining upside risk in case of a bad shock at $t = 1$, which means the equity payoff that is possible if the assets recover until $t = 2$. This part of the equity value does not ‘protect’ the long-term debt in case of a bad shock at $t = 1$, but the long-term debt losess value in that state, since the conditional default probability increases. And the maturity transformer needs a sufficient amount of equity to absorb this loss in value of its portfolio and to ensure the safety of its short-term debt.

To sum up, this section has shown that a separation of credit risk and maturity transformations increases the default probability of the resulting short-term debt. This result is a purely mechanical one, which does not depend on any frictions or any other assumptions. For the sake of illustration, I used a simple example. But Section 5 shows that the result holds for very general cases.

3 A Rationale for Intermediation Chains with Separated Transformations

This section provides an explanation why financial firms form intermediation chains with separated credit risk and maturity transformation, although such chains have a larger default probability than banks (i.e., the financing of assets directly with short-term debt). The first subsection describes the basic problem of a financial firm that chooses its capital structure and it introduces the frictions that are relevant for the emergence of intermediation chains. The second subsection derives the optimal capital structure for the case that the financial firm operates as a bank. The third subsection shows why an intermediation chain can be a more efficient form of financing than a bank.

3.1 The Relevant Frictions for the Choice of Financing

Assume that there are three dates $t = 0, 1, 2$ and three types of agents: first, the initial owner of a firm, which has assets as described in Section 2 and which can obtain the necessary funding $I$ for these assets at $t = 0$ by issuing equity and debt claims; second, a continuum $S_I$ of investors whose wealth at $t = 0$ adds up to $W_I = 1$ and who can buy financial claims either from the firm or from each other; and third, a continuum $S_A$ of arbitrageurs who are present at $t = 1$ and whose wealth at $t = 1$ adds up to $W_A = 1$. Investors and arbitrageurs are modeled in a very simple way. The investors are risk-neutral and buy or roll over a claim as long as its expected return is weakly larger than the ‘market rate’ $r = 0$ that they could obtain from alternative investments. The arbitrageurs are risk-neutral, too, but have access to a more profitable investment opportunity at $t = 1$ that yields a return $r_A > 0$ at $t = 2$. Consequently, they are only willing to buy claims or
assets from the firm at an appropriately discounted price.

I will show that intermediation chains with a separation of credit risk and maturity transformation can be rationalized, if one accounts for three frictions. The first friction leads to a deviation from the Modigliani-Miller Theorem and explains the use of short-term debt financing. The second friction motivates the separation of credit risk and maturity transformation in an intermediation chain. And the third friction establishes the key advantage of a chain over a bank. Let me explain these frictions:

I. ‘Money Premium’: Based on Gorton & Pennacchi (1990), I assume that the investors have a demand for safe claims and are willing to pay a premium for them, because they can use these claims as means of payment. For the questions addressed in this paper, it is sufficient to represent the benefits of safe claims in a simple way: by assuming that the investors pay a fee $\lambda$ per unit of safe claim per unit of time (similar to a fee for a deposit account). Also for simplicity, let us assume, first, that the fees are paid at the very end, after the debt has been paid off at $t = 2$, and second, that fees are only paid for debt that is safe when it is issued at $t = 0$.

II. ‘Risk Retention’: I assume that the selection of the assets by the initial firm owner at $t = 0$ entails an agency problem: instead of the ‘good set’ characterized above, she can also select a ‘bad set’, which has the same characteristics as the good set apart from a higher probability $p_1 + \delta p$ of a bad shock at $t = 1$. Her choice of assets is private information and she has a private benefit $\mu$ from choosing the bad set (for instance, because a poorer screening of loans entails less costly effort). I assume that the bad set is an inefficient choice due to $\mu < \delta p \cdot p_2 \cdot a$, which is the expected relative loss from choosing the bad set.

The initial owner refrains from selecting the bad assets in spite of the private benefit $\mu$, if she retains a set of claims to the assets whose expected loss from choosing bad assets is weakly larger than $\mu$.

Let us focus on the retention of equity as a commitment device. Later, when I will discuss the intermediation chain, I will indicate in Footnote 16 that a retention of debt claims would be a less efficient commitment device, because it means that fewer debt claims could be provided to investors. The expected loss $L_A(D_d; d)$ of the firm equity from choosing the bad set of assets depends on the face value $D_d$ and duration $d \in \{S, L\}$ of the firm debt.

To determine $L_A(D_d; d)$, one has to remember the evolution of the asset value, which has weaknesses.

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5 A microfoundation of the premium following Gorton & Pennacchi (1990) could be based on transaction needs that investors have between the dates, when some agents already receive the shocks about the assets. Given such transaction needs in presence of asymmetric information, safe claims are beneficial as means of payment, because they avoid costs of adverse selection. Such a microfoundation, however, would not change any results of this paper.

6 This allows to ignore tedious, uninteresting effects of paid fees on the safety and repricing of the debt. The assumption implies: even if investors withdraw their debt or transfer it in a payment process, they do not pay the fee for holding the safe claim (up to the withdrawal date) before the very end.

7 I thus neglect the possibility that an initially risky claim, which becomes safe after an increase of the asset value, earns a fee from that point onward.

8 While the payoff is 1 in case of a good shock at $t = 1$, it is $1 - a$ with probability $p_2$ in case of a bad shock at $t = 1$, whose probability increases by $\delta p$ as consequence of choosing the bad set.
been depicted in Figure [1]. In case of long-term debt, choosing the bad set increases the probability that the equity payoff at \( t = 2 \) is \( \max\{0, 1 - a - D_L\} \) (for an asset payoff \( 1 - a \)) instead of \( 1 - D_L \) (for an asset payoff \( 1 \)) by \( \delta_p p_2 \). In case of short-term debt, choosing the bad set increases the probability that the equity value at \( t = 1 \) is \( \max\{0, 1 - p_2 a - D_S\} \) (for an asset value \( 1 - p_2 a \)) instead of \( 1 - D_S \) (for an asset value \( 1 \)) by \( \delta_p \). The expected losses are thus:

\[
L_A(D_L; L) = \delta_p \cdot p_2 \cdot (1 - \max\{D_L, 1 - a\}) ,
L_A(D_S; S) = \delta_p \cdot (1 - \max\{D_S, 1 - p_2 a\}) .
\]

If the initial owner has no other device for credibly committing to the choice of the good set, the initial owner will retain a fraction \( \gamma \) of the firm equity with \( \gamma L_A(D_d; d) \geq \mu \), where \( d \in \{S, L\} \) is the duration of the firm debt. Given this retention of equity, choosing the good set is optimal for the initial owner and the investors account for this fact when they buy claims of the firm. If the initial owner held a smaller fraction of the firm equity, it would be optimal for her to choose the bad set, independent of the price that investors pay for their claims. Taking the choice of the bad set into account, however, the investors would only buy claims at prices that reflect the expected loss. In this way the initial owner would incur the cost of the inefficient choice of a bad set. As a result, she has an incentive to retain the fraction \( \gamma \geq \frac{\mu}{L_A(D_{d, d})} \) of equity in order to commit to the good set.

III. ‘Improved Liquidity’: I assume that both, the firm assets and the securities issued by the firm, are illiquid – but to different degrees. If all investors withdraw their funding of the intermediation chain at \( t = 1 \), the long-term debt has to be sold to the arbitrageurs. The arbitrageurs face opportunity costs that are given by their alternative investment opportunity with interest rate \( r_a \). Consequently, they do not pay \( y_1^0(D_L) \) for the long-term debt (which is the value of its expected payoff discounted with \( r = 0 \)), but they only pay \( (1 - l_D) \cdot y_1^0(D_L) \) with \( l_D \in (0, 1) \) given by \( 1 - l_D = \frac{1}{1 + r_a} \). The situation is similar, if investors withdraw their funding of the firm at \( t = 1 \), so that the firm assets have to be sold to the arbitrageurs. They will only pay a discounted price due to their opportunity costs. I assume, however, that the transfer of the underlying assets entails an additional efficiency loss. This can be interpreted as the costs of interrupting the smooth operation of the assets by specialized managers of the firm. In case of a bank with a portfolio of loans, for instance, this can be the interruption of valuable lending relationships. I assume that the sale of the assets to a ‘second-best user’ entails a decline of the expected asset payoff from \( y_1 \) to \( (1 - l_A) y_1 \). Consequently, the arbitrageurs only pay \( (1 - l) y_1 \) for the assets at \( t = 1 \) with \( (1 - l) := (1 - l_D)(1 - l_A) < 1 - l_D \).

To sum up, I assume that a long-term debt claim to a set of assets is more liquid than the assets themselves. The long-term debt might be sold at a depressed price in case of a withdrawal of its short-term funding, but this depressed price does not impair the underlying assets. The efficient operation of the underlying assets is protected by the long
duration of the claim to their payoff. If the assets are directly financed by short-term debt, in contrast, a withdrawal of the debt can enforce a sale of the assets themselves, which implies an inefficient interruption of their operation.

3.2 Financing the Assets as a Bank

Having introduced the relevant frictions, let us now study the choice of capital structure by the firm. For that purpose, one has to examine the premium \( \Lambda(D_L; d) \) that the firm can earn from providing safe claims. It depends on the evolution of the asset value depicted in Figure 1. If the firm issues long-term debt, the case is very simple owing to the assumptions stated above. For \( D_L \leq 1 - a \), the debt is safe from \( t = 0 \) until \( t = 2 \) and leads to a premium \( 2 \lambda D_L \), given that the investors pay a fee \( \lambda \) per unit of safe claim per unit of time. ‘Unit of claim’ refers to a unit of expected payoff at \( t = 2 \), which is equal to the face value in case of safe debt. For \( D_L > 1 - a \), the debt is risky and yields no premium. Consequently,

\[
\Lambda(D_L; L) = \lambda \cdot \begin{cases} 
2 D_L & \text{for } D_L \in [0, 1 - a] \\
0 & \text{for } D_L \in (1 - a, 1] 
\end{cases}.
\]

If the firm issues short-term debt, the illiquidity of the assets can reduce the safety of the debt claim due to a coordination problem at the roll-over date \( t = 1 \). If the following conditions hold, a premature liquidation of the firm can occur for certain debt levels: first, each investor in the continuum \( S_I \) and each arbitrageur in the continuum \( S_A \) only has an infinitesimal wealth, so that each investor/arbitrageur can only hold/buy an infinitesimal fraction \( \alpha \) of the firm debt; second, the investors and arbitrageurs have to decide simultaneously about rolling over or buying short-term debt and they cannot coordinate this decision; third, maturing debt has priority to outstanding debt.\(^9\) These features are not rationalized as optimal contractual arrangement, but they are rather taken as given constraints of financial intermediaries with decentralized short-term debt holders.

Given these features and a level \( D_S \) of short-term debt with \( (1 - l)y_1 < D_S \leq y_1 \) at \( t = 1 \), there is a ‘run equilibrium’ in which the investors and arbitrageurs do not roll over or buy the short-term debt, because they believe that the others do the same. The consequence of this collective action is that the assets have to be sold to the arbitrageurs at the discounted price \( (1 - l)y_1 \), as explained above.\(^10\) The collective withdrawal/refusal to buy is an equilibrium: if a debt holder withdraws her fraction \( \alpha \) of the debt when all other

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\(^9\) This means: if maturing debt is withdrawn at \( t = 1 \), the firm assets are liquidated in order to pay off the withdrawing debt claim, even if the overall payoff from the liquidation is too small to also pay off the debt that matures later.

\(^10\) Owing to \( y_1 \leq 1 \), the arbitrageurs with wealth \( W_A = 1 \) have sufficient funds to buy the assets. I assume that the wealth of the investors who are liquid when the assets are sold (which means the subset of investors in \( S_I \) who have not bought equity or debt of the firm) is smaller than \( (1 - l)y_1 \), so that selling the assets to the investors at the higher price \( (1 - l_A)y_1 \) is not possible. If the firm could sell a part of the assets to the liquid investors, and had to sell only the remaining part to the arbitrageurs, the relative loss would become smaller. As this would only change the quantitative, but not the qualitative results, I abstract from this possibility and assume that the firm can sell the assets only to the arbitrageurs.
debt holders do the same, she receives \( \alpha (1 - l) y_1 \) on average; if she rolled over her debt claim in that situation, she would only receive \( y^D_\gamma := \max\{0, (1 - l) y_1 - (1 - \alpha) D_S \} \), which is smaller than \( \alpha (1 - l) y_1 \) due to \( (1 - l) y_1 < D_S \). And no other investor or arbitrageur would buy the fraction \( \alpha \) of debt at a price larger than \( y^D_\gamma \), given that the expected payoff of the rolled over claim would equal \( y^D_\gamma \). But the revenue \( y^D_\gamma \) of such a resale would be insufficient to pay out the withdrawing claim with face value \( \alpha D_d \), so that the firm would still be liquidated.

If the occurrence of such a ‘run’ and the resulting loss due to a premature liquidation cannot be excluded, short-term debt with \( D_S > (1 - l)(1 - p_2 a) \) is not safe, given that \( y_1 \) is only \( 1 - p_2 a \) in case of a bad shock at \( t = 1 \). The premium \( \Lambda(D_S; S) \) for providing safe claims in form of short-term debt is thus given as:

\[
\Lambda(D_S; S) = \lambda \cdot \begin{cases} 
2 D_S & \text{for } D_S \in [0, 1-a] \\
(2 - p_1) D_S & \text{for } D_S \in (1-a, (1-l)(1-p_2 a)] \\
0 & \text{for } D_S \in ((1-l)(1-p_2 a), 1]
\end{cases}
\]

for the case \( 1 - a \leq (1-l)(1-p_2 a) \), on which I will focus by imposing Assumption \([1.a]^{12}\)

For \( D_S \leq 1 - a \), the debt is safe between \( t = 0 \) and \( t = 2 \), since the short-term debt can be rolled over at \( t = 1 \) without a change of the face value, given \( r = 0 \). The same holds for \( D_S \in (1-a, (1-l)(1-p_2 a)] \) in case of a good shock at \( t = 1 \). In case of a bad shock at \( t = 1 \) (occurring with probability \( p_1 \)), a claim with such a face value can still be rolled over, since the expected payoff \( y_1 = 1 - p_2 a \) of the assets is larger than \( D_S \) and there is no run.\([13]\)

But the debt becomes risky and thus earns no fee between \( t = 1 \) and \( t = 2 \).

Having determined the form of \( \Lambda(D_d; d) \), let us now study the decision problem of the firm owner, assuming that she wants to maximize her expected wealth at \( t = 2 \). Selling claims to the investors, the initial firm owner has no incentive to deviate from the investors’ reservation price for a claim, which is equal to the expected payoff of the claim (given \( r = 0 \) and risk-neutrality). This implies that the expected payoff \( P(D_d, \gamma) \) of the claims that are sold to investors (i.e., the firm debt plus the fraction \( 1 - \gamma \) of the firm equity) has to be weakly larger than \( I \) in order to obtain the funding of the assets at \( t = 0 \). Assume that, if the initial firm owner sells claims worth more than \( I \), she can also store her wealth at a rate \( r = 0 \). Her expected payoff at \( t = 2 \) would thus be \( y_0 - I \) (remember that \( y_0 \) is the expected payoff of the assets), if there were no frictions. Accounting for the frictions,
the decision problem is given as:

$$\max_{D_d \in [0,1], d \in \{S,L\}, \gamma \in [0,1]} \quad y_0 - I + \Lambda(D_d; d) - L_L(D_d; d) - (\delta_p p_2 a - \mu) \cdot 1_{\{\mu > \gamma L_A(D_d; d)\}}$$

s.t. $P(D_d, \gamma) \geq I$,

where $L_L(D_d; d)$ denotes the loss from premature liquidations (which is discussed below), and $\delta_p p_2 a - \mu$ is the cost of choosing the bad set of assets.\[14\]

**Assumption 1**

a) $(2 - p_1) \cdot ((1 - p_2) a - l \cdot (1 - p_2 a)) > p_1 \cdot (1 - a)$

b) $1 - a \geq I$

Let us impose these assumptions in order to focus on interesting cases. Assumption b) ensures in a simple way that enough claims can be sold in order to obtain the funding $I$ at $t = 0$. Assumption a) states that the the shocks at $t = 1$ and $t = 2$ might be relatively large (i.e., $a$ is large), but that the probabilities $p_1$ and $p_2$ of such shocks are small. By imposing this assumption, I study the provision of safe claims in presence of ‘tail risk’.

**Lemma 1**

If Assumption[14] holds, the optimal capital structure of the firm consists of short-term debt with face value $D_S^a = (1 - l)(1 - p_2 a)$ and the retention of a fraction $\gamma^* \in \left[\frac{\mu}{\delta_p p_2 a}, 1\right]$ of the firm equity by the initial owner.

**Proof:** The premium $\Lambda$ has two relative maxima: short-term debt with $D_S = (1 - l)(1 - p_2 a)$ which implies $\Lambda((1 - l)(1 - p_2 a); S) = \lambda(2 - p_1)(1 - l)(1 - p_2 a)$; and the debt level $D_d = 1 - a$ which (independent of $d \in \{S, L\}$) implies $\Lambda(1 - a; S) = \Lambda(1 - a; L) = \lambda 2(1 - a)$. The former maximum is larger than the latter one, if Assumption[14] a) holds. In case of short-term debt with $D_S^a = (1 - l)(1 - p_2 a)$, there are no runs and premature liquidations.

This implies $L_L(D_d; d) = 0$, which is the minimal possible liquidation loss. The term $y_0 - I$ in the objective function is independent of the capital structure. The loss from choosing the set of assets is minimized and equal to zero, if $\gamma L_A(D_d; d) \geq \mu$. This is the case for $\gamma \in \left[\frac{\mu}{\delta_p p_2 a}, 1\right]$, since $L_A((1 - l)(1 - p_2 a); S) = \delta_p p_2 a$. And even if the initial owner chooses $\gamma = 1$ and only issues debt, she can still raise the necessary funding $I$ at $t = 0$, since the expected payoff of the debt is $P((1 - l)(1 - p_2 a), 1) = (1 - l)(1 - p_2 a)$, which is weakly larger than $I$ given Assumption[14] a). And the initial wealth $W_I = 1$ of the investors is large enough to buy the claims at the price $P((1 - l)(1 - p_2 a), 1) < 1$.

This section has determined the optimal capital structure of a single firm that faces the frictions introduced in the previous section. Besides the necessity for the initial owner to

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\[14\] As explained above: if the owner does not retain an equity position, which is sufficiently large to disincentivize the choice of the bad set (which means $\gamma \cdot L_a(D_d) \geq \mu$), the investors will price that in, so that the loss $\delta_p p_2 a - \mu$ is borne by the initial firm owner.

\[15\] Assumption[14] a) implies $(1 - p_2) a - l(1 - p_2 a) > 0 \iff a - 1 + (1 - l)(1 - p_2 a) > 0 \iff (1 - l)(1 - p_2 a) > 1 - a$. From this follows that $(1 - l)(1 - p_2 a) > I$ given Assumption[14] b).
retain some claims in order to ensure the selection of good assets, the key feature is that
the premium for safe claims is maximized by issuing short-term debt. Short-term debt
with \( D_S = (1 - l)(1 - p_2 a) \) leads to a larger expected premium for safe claims than a debt
level \( 1 - a \), which is safe in all possible states, if two conditions hold: first, the probability
\( p_1 \) that the higher debt level \( D_S = (1 - l)(1 - p_2 a) \) becomes risky after \( t = 1 \) is relatively
small; second, the reduction \( (1 - p_2 a - l \cdot (1 - p_2 a) \) of the debt face value, which would be
necessary to achieve safety in all possible states, is relatively large. Following the previous
convention, the financing of the assets by issuing short-term debt shall be denoted as
‘bank’. This form of financing will now be compared to an intermediation chain.

3.3 Financing the Assets by Means of an Intermediation Chain

Let us study how the optimal financing of the assets changes, if one accounts for the
possibility that investors who hold debt of the firm can sell their own debt, which is
backed by the firm debt. I first describe a specific financing structure, before I explain
under which conditions this structure is the optimal one.

Consider the case that the firm is a ‘security issuer’ according to the previous convention
and sells long-term debt with face value \( D^1_L = 1 - \frac{\mu}{\delta p p_2} \). Assume that the initial owner
maintains all the equity of the firm. Due to \( \delta p p_2 a > \mu \), it holds that \( L_A(D^1_L; L) = \delta p p_2 (1 - \max\{D^1_L, 1 - a\}) = \delta p p_2 \left( 1 - \left( 1 - \frac{\mu}{\delta p p_2} \right) \right) = \mu \), so that the initial owner has no
incentive to choose the bad set of assets. The value \( P(D^1_L, 1) \) of the issued securities (i.e.
the expected payoff of the long-term debt) is \( (1 - p_1 p_2) \left( 1 - \frac{\mu}{\delta p p_2} \right) + p_1 p_2 (1 - a) \). This
is larger than \( 1 - a \geq I \), so that the necessary funding at \( t = 0 \) can be obtained. Since
the long-term debt is risky, the firm does not earn any premium for safe debt. But an
investor who buys and holds the long-term debt, can issue debt with face value \( M_d \) and
duration \( d \in \{ S, L \} \) against this portfolio. And if the debt is safe, she can earn a premium.

Let us consider a representative investor who does this, and let us refer to her as ‘margin
investor’. Let us refer to the financing of the long-term debt with the issuance of debt by
the margin investor as ‘maturity transformer’ (in anticipation of the choice of \( d \)). In the
explanation of Proposition 2 I point out the incentive for an investor to become a margin
investor.

The state-contingent value of the long-term debt issued by the firm equals \( y_D(D^1_L) = E_t[\min\{D^1_L, y_2\}] \). Since the liquidation of the long-term debt leads to the relative loss \( l_D \),
there is a coordination problem if the maturity transformer issues short-term debt with
\( M_S \in \left( (1 - l_D) y_D(D^1_L), y_D(D^1_L) \right) \) at \( t = 1 \). The problem is completely analogous to the
one for a bank that has been described in the previous subsection. The expected loss of
the maturity transformer due to premature liquidations shall be denoted as \( L^M_L \). Given
the possibility of a run, short-term debt of the maturity issuer is only safe up to the value
\( (1 - l_D) y_-(D^1_L) \), where \( y_-(D^1_L) = \left( (1 - p_2) D^1_L + p_2 (1 - a) \right) \) is the value of the \( D^1_L \)-claim
at \( t = 1 \) in case of a bad shock. Given that investors pay a fee \( \lambda \) per unit of safe claim and
per unit of time, the maturity transformer can earn a premium \( \Lambda_M(M_d; d) \) for providing safe claims. Remember that the processes \( y_t \) and \( y_t^D(D_L) \), depicted in the Figures 1 and 3, are very similar. Consequently, \( \Lambda_M(M_d; d) \) depends on \( M_d \) and \( d \) in the same way as \( \Lambda \) depends on \( D_d \) and \( d \), apart from a change of the boundary between the second and third interval in case of \( d = S \) from \( (1 - l)(1 - p_2 a) \) to \( (1 - l_D)y_-(D_L^1) \):

\[
\Lambda_M(M_S; S) = \lambda \cdot \begin{cases} 
2M_S & \text{for } M_S \in [0, 1 - a] \\
(2 - p_1)M_S & \text{for } M_S \in \left(1 - a, (1 - l_D)y_-(D_L^1)\right) \\
0 & \text{for } M_S \in \left((1 - l_D)y_-(D_L^1), 1\right)
\end{cases}
\]

\[
\Lambda_M(M_L; L) = \lambda \cdot \begin{cases} 
2M_L & \text{for } M_L \in [0, 1 - a] \\
0 & \text{for } M_L \in (1 - a, 1]
\end{cases}
\]

**Assumption 2**

a) \( (1 - l_D)(1 - p_2) \frac{\mu}{\delta p_2} < (l - l_D)(1 - p_2 a) \)
b) \( \lambda \cdot (2 - p_1)(1 - p_2) \frac{\mu}{\delta p_2} < \delta_p p_2 a - \mu \)

The purpose of this assumption is to focus on a parameter range for which two conditions hold: a) the relative increase \( l - l_D \) in liquidity owing to a chain is relatively large compared to size of claims which the security issuer has to retain in order to align incentives in the choice of assets (which is represented by \( \frac{\mu}{\delta p_2} \)); b) the loss \( \delta_p p_2 a - \mu \) from choosing bad assets is relatively large compared to the premium that can be earned from safe claims (represented by \( \lambda \)).

**Proposition 2**

If Assumptions 2 and 3 apply, the following statements hold:

a) The premium \( \Lambda_M \) of the maturity transformer is maximized by short-term debt with face value \( M_2^S = (1 - l_D)y_-(D_L^1) = (1 - l_D)\left(1 - p_2\right)\left(1 - \frac{\mu}{\delta p_2}\right) + p_2 \cdot (1 - a)\).

b) The intermediation chain described here is more efficient than a bank, because

\[
\Lambda_M(M_2^S; S) > \Lambda(D_2^S; S),
\]

while the chain faces no liquidation losses and the security issuer chooses the good assets.

Forming the intermediation chain is a Pareto improvement, if the maturity transformer transfers a fraction \( \omega \in [0, 1] \) of \( \Lambda_M \) to the security issuer with \( \omega \Lambda_M(M_2^S; S) > \Lambda(D_2^S; S) \).

c) The intermediation chain described above is the most efficient intermediation chain, which means: no \( D_d \in \mathbb{R}^+ \) and \( d \in \{S, L\} \) chosen by the security issuer and no \( M_P \in [0, D_d] \) and \( d' \in \{S, L\} \) chosen by the maturity transformer lead to a higher surplus

\[
\Lambda(D_d; d) + \Lambda_M(M_d; d') - L_L(D_d; d) - L_L^M(M_d; d') - (\delta_p p_2 a - \mu) \cdot 1_{\{\mu > \gamma \Lambda(D_d; d)\}}.
\]
The proof is given in Appendix B. **Statement a)** is the analogue of Lemma 1. It states that issuing short-term debt with a relatively large face value $M^1_S$ allows for providing more safe claims than issuing debt with face value $1-a$, which remains safe in all possible states. The necessary condition (given by Assumption 2a) is that a relatively large reduction of the debt level would be necessary to achieve safety in all possible states, while the probability of a bad shock, by which the debt level $M^1_S$ becomes risky, is small.

Since the payoff of the $M^1_S$-claim at $t=1$ is safe, it can be sold at $t=0$ for the price $M^1_S$, given that $r = 0$. The price for the $D^1_L$ claim is $y^0_0(D^1_L)$. This implies that the margin investor has to invest the amount $y^0_0(D^1_L) - M^1_S$ of its own wealth at $t=0$. She is willing to do so, because the equity in the maturity transformer has the same expected return $r=0$ as alternative investments, but it allows to earn the premium $\Lambda_M$ in addition.

So far, I have abstracted from potential agency problems between the margin investor and buyers of its short-term debt. Let us assume that the maturity transformer can implement a rule which restricts its portfolio to debt securities issued by the security issuer described above. The remaining agency problem is then that the margin investor might collude with the security issuer and might tolerate a selection of bad assets. As described above, this would lead to a higher probability of a bad shock at $t=1$. But since the margin investor provides a sufficiently large amount of equity to ensure the safety of the short-term debt, the debt holders do not incur losses from an increase in the probability of a bad shock at $t=1$. The loss would be borne by the margin investor instead, which means that she has no incentive to tolerate the selection of bad assets by the security issuer.

**Statement b)** points out that an intermediation chain can issue more safe debt than a bank despite the relatively higher default probability identified in Section 2. This is possible, if the relative increase in solvency risk is smaller than the decrease in illiquidity risk that the chains achieves. The separation of credit risk and maturity transformation leads to a reduction in the level of safe debt (this reduction is given by the left hand side of Assumption 2a). But the portfolio that is financed with short-term debt is more liquid, so that the a higher level of short-term debt can be chosen without facing the risk of a run (the resulting increase in the level of safe debt is given by the right hand side of Assumption 2a). If the latter is larger than the former, the intermediation chain can provide more safe claims than a bank, given the same underlying assets.

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16 Let me briefly explain here how the situation would change, if the owner of the security issuer retained debt claims in order to credibly commit to selecting the good assets. The relative loss of a long-term debt claim with face value $D_L$ from choosing a bad set is: $L^0(t; D_L) = \delta_p p_2 \max\{D_L - (1-a), 0\}$. For $D_L = 1$ (the case for which the debt is most sensitive to the choice of assets), the initial owner had to retain a fraction $\frac{\mu}{\delta_p p_2 \pi}$ in order to incur an expected loss from choosing the bad set that is weakly larger than $\mu$.

Selling the other fraction $\left(1 - \frac{\mu}{\delta_p p_2 \pi}\right)$ of $D_L$ to a maturity transformer, the safe value of this claim at $t=1$ is $\left(1 - \frac{\mu}{\delta_p p_2 \pi}\right) \left(1 - l_D\right) y(1) = \left(1 - \frac{\mu}{\delta_p p_2 \pi}\right) \left(1 - l_D\right) (1 - p_2 a)$. Due to $a < 1$, this is smaller than $(1 - l_D) y(1 - \frac{\mu}{\delta_p p_2 \pi}) = (1 - l_D) \left(1 - p_2 a - (1 - p_2) \frac{\mu}{\delta_p p_2 \pi}\right)$, which is the safe value of the $D_L$ claim at $t=1$ that the financial firm can sell if the owner retains equity. This means retaining equity rather than debt in order to solve the agency problem is more efficient, as it allows for a larger premium $\Lambda_M$. 

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The gains that are available due to an intermediation chain can be shared between the maturity transformer and the security issuer, if the former transfers a fraction $\omega$ of the premium $\Lambda_M$ to the latter. In that case, the firm holding the assets is better off with becoming a security issuer in a chain than with operating as a bank. The fraction $\omega$ of $\Lambda_M$ that is transferred from fund to bank depends on the bargaining situation between security issuer and maturity transformer (or on the competition in the market), which will not be further discussed here.

**Statement c)** follows from considering alternative intermediation chains. If the security issuer sold more long-term debt to the maturity transformer than $D_L^*$, the latter could provide more safe claims (as the short-term debt is backed by a more valuable debt claim). Choosing $D_L > D_L^*$, however, would entail that the equity held by the owner of the security issuer is so small that she would choose the bad assets. If the loss from choosing bad assets is relatively large compared to the premium that can be earned from safe claims, as stated by Assumption 2b), a chain with $D_L > D_L^*$ is less efficient than the chain with $D_L^*$. Choosing $D_L < D_L^*$ is also less efficient, because the reduction in the long-term debt entails that the maturity transformer can issue less safe debt. The remaining alternative is that the security issuer sells short-term debt to the maturity transformer. But such a chain cannot be more efficient than a bank, because it also implies a direct reference of short-term debt to the underlying assets. Consequently, the provision of safe claims is constrained by the same coordination problem as in case of a bank. The coordination problem is the same, even if the maturity transformer holds all short-term debt of the security issuer, because: if the dispersed investors in short-term debt issued by the maturity transformer withdraw their funding at $t = 1$, the maturity transformer has to withdraw its claim to the security issuer, which has to liquidate its assets then. Chains that sell long-term debt to the investors, in contrast, are always less efficient than a bank, because long-term debt is only safe for face values smaller than $1 - a$. This implies a premium $\Lambda_M$ that is smaller than $\Lambda(D_S'; S)$, as shown in Lemma 1.

To sum up, this section has illustrated that the formation of an intermediation chain with separation of credit risk and maturity transformations can be efficient in spite of the increase of the default probability that this separation entails. The efficiency has been shown in a setting in which, first, the provision of safe claims yields a premium, and second, the separation of credit risk and maturity transformation in a chain is due to an agency problem that affects the sale of asset-backed securities. The advantage of an intermediation chain is the indirect reference of the short-term debt to the underlying assets via long-term debt, which is more liquid than the underlying assets. The increased liquidity of the portfolio, to which the short-term debt refers directly, allows for issuing more short-term debt without facing a non-fundamental run. This allows for providing more safe claims.
4 The Regulation of an Intermediation Chain

The previous section has pointed out that an increase of liquidity can be a key advantage of an intermediation chain, as it allows for issuing more short-term debt without facing a run. As highlighted in Diamond & Dybvig (1983), however, coordination problems of short-term debt can be resolved by an insurance of the debt claims. While such an insurance can improve the provision of safe claims by both, bank and intermediation chain, it eliminates the relative advantage of the intermediation chain. And an insurance of short-term debt entails moral hazard, if the insured intermediaries can shift risk to the insurance provider – for instance, by issuing too little equity. To prevent such risk-shifting, the insurance has to be accompanied by capital requirements or fairly priced insurance premiums. This section highlights that an ostensibly equivalent regulation of bank and chain implies an implicit subsidy for the chain. This result is a direct consequence of the higher default probability of a chain.

Let us assume that there is a public authority, which insures the short-term debt of a bank at \( t = 1 \). The insurance means: if there is a run on the bank at \( t = 1 \), the authority takes over the bank and fully pays off all investors (independent of their withdrawal decision). The funds for the insurance payments are taken from the proceeds of liquidating the assets and – to fill the remaining funding gap – from a lump-sum tax imposed on all investors. This insurance resolves the coordination problem that a bank with \( D_S \in ( (1 - l)(1 - p_2 a), 1 - p_2 a ] \) would face in case of a bad shock at \( t = 1 \), since the expected payoff of a short-term debt holder is independent of the withdrawal decision of the other investors. For \( D_S \leq 1 - p_2 a \), the bank is able to roll over the debt in case of a bad shock at \( t = 1 \), because it can offer a risk-adjusted face value \( D_{S,1} \leq 1 \) for which \( E_{t=1}[\min\{D_{S,1}, y_2\}] = D_S \) holds. This is no longer possible for \( D_S > 1 - p_2 a \). Independent of the face value \( D_{S,1} \) that is offered, the expected payoff of the rolled over debt is smaller than \( D_S \), because the expected payoff of the underlying assets is only \( 1 - p_2 a \), given a bad shock at \( t = 1 \). As a consequence, the short-term debt holders will withdraw at \( t = 1 \) in order to receive the insured payoff \( D_S \). Knowing that they have a safe payoff \( D_S \) at \( t = 1 \), the investors are willing to pay \( D_S \) at \( t = 0 \) for the debt claims. This implies a subsidy for the bank, since the insurance provider covers the difference between \( D_S \) and the asset value \( 1 - p_2 a \) in case of a bad shock. To avoid this subsidy, the authority can impose a capital requirement which enforces \( D_S \leq 1 - p_2 a \). (I focus on capital requirements, but explain at the end why the results are the same for insurance premiums.) Let us denote this capital requirement as ‘fair regulation’, because it is the weakest constraint which prevents a bank from shifting risk to the insurance provider by issuing too much debt. The regulation can be rephrased as a requirement to issue equity at \( t = 0 \) that has the value \( e_B \geq (1 - p_1)p_2 a \).\(^{17}\)

In spite of this regulation, the bank benefits from the insurance, because the absence of

\(^{17}\)The equity value at \( t = 0 \) equals its expected payoff \((1 - p_1)(1 - D_S) + p_1 \cdot \max\{0, 1 - p_2 a - D_S\}\), which is weakly larger than \((1 - p_1)p_2 a \) for \( D_S \leq 1 - p_2 a \).
the coordination problem allows for issuing a higher amount of safe debt. Being insured and subject to fair regulation, the bank can earn the following premium $\Lambda^I$:

$$\Lambda^I(D_S; S) = \begin{cases} 2DS & \text{for } D_S \in [0, 1 - a] \\ (2 - p_1)DS & \text{for } D_S \in (1 - a, 1 - p_2 a] \end{cases}$$

$$\Lambda^I(D_L; L) = \Lambda(D_L; L)$$

**Observation 1**

An insurance of the bank debt at $t = 1$ accompanied by the constraint $e_B \geq (1 - p_1)p_2 a$ increases the bank surplus, as it allows for a premium $\Lambda^I(1 - p_2 a; S) > \Lambda((1 - l)(1 - p_2 a); S)$.

The insurance increases the level of safe short-term debt from $(1 - l)(1 - p_2 a)$ to $1 - p_2 a$. If Assumption 1 holds (which implies that short-term debt with $D_S = (1 - l)(1 - p_2 a)$ maximizes $\Lambda(D_d; d)$), then short-term debt with face value $D_S = 1 - p_2 a$ maximizes $\Lambda^I$ and leads to a larger premium than in the uninsured case.\(^{18}\) As a result, the insurance of a bank combined with a fair regulation increases efficiency (as pointed out before by other papers). Let us now study the insurance and regulation of an intermediation chain.

Assuming that the public authority also insures the short-term debt of the intermediation chain at $t = 1$, I will discuss two alternative regulations. The first regulation (denoted as ‘level playing field’) is the capital requirement $e_I + e_M \geq (1 - p_1)p_2 a$, where $e_I$ is the equity value of the security issuer at $t = 0$ and $e_M$ is the equity value of the maturity transformer at $t = 0$. This requirement might seem to be equally strong as the capital requirement $e_B \geq (1 - p_1)p_2 a$ imposed on the bank, as it requires the same amount of equity. The second regulation (denoted as ‘fair regulation’) is the weakest possible capital requirement that prevents risk-shifting and a subsidization of the chain.

Assume that the public authority ensures the short-term debt with face value $M_S$ at $t = 1$, as long as the chain complies with the imposed regulation. If $M_S$ exceeds the value $y_-(D_L)$ of the maturity transformer’s portfolio in case of a bad shock, the insurance implies a subsidy $p_1 \cdot (M_S - y_-(D_L))$ to the chain, for the same reasons as stated above for the case of a bank. The dependence of this subsidy, denoted as $S$, on the equity level

\(^{18}\) And for $D_S = 1 - p_2 a$ and $\gamma \in \left[\frac{\mu}{\sigma_{p_2}}, 1\right]$, the risk retention $\gamma L_A(1 - p_2 a; \gamma) = \gamma \delta_p p_2 a$ by the initial firm owner is still large enough to ensure the selection of the good set. In addition, the issuance of safe debt with face value $D_S = 1 - p_2 a$ raises the sufficient funding $1 - p_2 a > I$ at $t = 0$.\)
(e_I, e_M) can be derived from the relationships between (e_I, e_M) and (D_L, D_S) as:

\[ S(e_I, e_M) = \max \left\{ 0, p_1 \cdot \left( p_2 \cdot \left( a - \frac{e_I}{1 - p_1 p_2} \right) - \frac{e_M}{1 - p_1} \right) \right\} \]

**Assumption 3**: \((\frac{(1 - p_1) p_2}{1 - p_1 p_2}) p_2 a > \mu \).

The purpose of this assumption is only to illustrate the relevant mechanism for a simple case. At the end, I will indicate how the results change for the inverse relation.

**Proposition 3**

If Assumptions 2 and 3 apply, the following statements hold.

\(a\) ‘Level playing field’: if the short-term debt of the chain is insured, while the capital requirement \(e_I + e_M \geq (1 - p_1) p_2 a\) is imposed on the chain, then

- the chain can obtain a positive subsidy from the insurance, which is maximized for \(e_I^+ = (1 - p_1) p_2 a\) and \(e_M^+ = 0\) with \(S(e_I^+, e_M^+) = p_1 p_2 \frac{1 - p_2}{1 - p_1 p_2} a\);
- the equity levels \((e_I^+, e_M^+)\) imply that the short-term debt of the chain has the face value \(M_S^+ = 1 - \frac{(1 - p_1) p_2}{1 - p_1 p_2} a > p_2 a\), which leads to a larger premium than a bank can obtain: \(\Lambda_M(M_S^+; S) > \Lambda(1 - p_2 a; S)\);
- if Assumption 3 holds, the retention of the equity \(e_I^+\) by the initial firm owner is sufficient to incentivize the choice of the good assets.

Given a sufficient transfer between maturity transformer and security issuer, the firm owner chooses to be a security issuer in a chain rather than to be a bank.

\(b\) ‘Fair regulation’: if the short-term debt of the chain is insured, while the equity choice is constrained by lower bounds for \(e_I\) and \(e_M\) that ensure \(S(e_I, e_M) = 0\), then

- \(\tilde{e}_I = (1 - p_1 p_2) \frac{\mu}{\mu_p p_2}\) and \(\tilde{e}_M = (1 - p_1) p_2 \left( a - \frac{\tilde{e}_I}{1 - p_1 p_2} \right)\) are the combination \((e_I, e_M)\) of equity values that allows for the largest surplus of the chain (given by \(\Lambda_M\) minus a potential loss from selecting bad assets), while fulfilling \(S(e_I, e_M) = 0\);

[19] Deriving the relationships between \((e_I, e_M)\) and \((D_L, D_S)\), one can focus on cases with \(M_S \geq y_-(D_L)\) and \(D_L \geq 1 - a\), since smaller face values do not allow for any subsidy. For \(D_L \geq 1 - a\), the expected payoff of the security issuer’s equity is \(e_I = (1 - p_1 p_2) (1 - D_L) \Leftrightarrow D_L = 1 - \frac{e_I}{1 - p_1 p_2}\). And for \(M_S \geq y_-(D_L)\), the expected payoff of the maturity transformer’s equity is \(e_M = (1 - p_1) (D_L - M_S) \Leftrightarrow M_S = D_L - \frac{e_M}{1 - p_1} = 1 - \frac{e_I}{1 - p_1 p_2} - \frac{e_M}{1 - p_1}\). The subsidy can thus be written as the following function \(S(e_I, e_M)\) of \(e_I\) and \(e_M:\)

\[ S(e_I, e_M) = \max \left\{ 0, p_1 \cdot (M_S(e_I, e_M) - y_-(D_L(e_I))) \right\} = \max \left\{ 0, p_1 \left( D_L - \frac{e_M}{1 - p_1} - (1 - p_2) D_L - p_2 (1 - a) \right) \right\} \]

\[ = \max \left\{ 0, p_1 \left( 1 - \frac{e_I}{1 - p_1 p_2} - \frac{e_M}{1 - p_1} - (1 - p_2) \left( 1 - \frac{e_I}{1 - p_1 p_2} - p_2 (1 - a) \right) \right) \right\} = \max \left\{ 0, p_1 \left( p_2 \left( a - \frac{e_I}{1 - p_1 p_2} \right) - \frac{e_M}{1 - p_1} \right) \right\} \]
• the equity levels ($\tilde{e}_I, \tilde{e}_M$) ensure the selection of good assets, and they correspond to a face value $M_S = (1-p_2) \left( 1 - \frac{\mu}{\delta+p_2} \right) + p_2 (1-a) < 1-p_2 a$ of the short-term debt, which means that the premium is smaller than for a bank: $\Lambda_M(M_S;S) < \Lambda(1-p_2 a;S)$.

The bank thus allows for a larger private benefit than a chain, and the firm owner has no incentive to join a chain.

The proof is given in Appendix C. Both parts of Proposition 3 are consequences of the fact that the separation of credit risk and maturity transformations increase the amount of equity that is necessary to prevent a default of the short-term debt. Part b) highlights that a fair regulation, which avoids that the insurance against illiquidity becomes an insurance against insolvency, has to impose higher capital requirements on a chain than on a bank (i.e., the chain must issue more equity at $t = 0$ than a bank: $e_I + e_M > e_B$). The higher capital requirements imply that the chain cannot issue as much short-term debt as a bank. Consequently, the chain cannot earn the same premium as a bank. And since there is no subsidy, the firm has no incentive to join a chain, but prefers to operate as a bank. While the insurance increases the efficiency of both, bank and chain, it eliminates the relative advantage of the chain.

The situation is different, if the regulation does not account for the increase of insolvency risk in a chain, but requires the same level of equity from both financing structures, as discussed in part a). In that case, the insurance covers a part of the insolvency risk of the chain and subsidizes the chain in this way. For the reason highlighted in Section 2, the equity value of the security issuer partly consists of the upside risk after a bad shock $t = 1$. This equity value remains with the equity holders of the security issuer and is not accessible to pay off the short-term debt of the maturity issuer. The short-term debt has to be paid off by the insurance instead. The implicit subsidy increases with the equity value that the security issuer maintains in case of a bad shock, which means that it increases with the initial equity value of the security issuer. The subsidy is thus maximized, if the equity value that the regulation imposes (which is $(1-p_1) p_2 a$) is provided by the security issuer, while the maturity transformer has no equity at all: $e_I^+ = (1-p_1) p_2 a$ and $e_M^- = 0$.

The equity levels ($e_I^+, e_M^+$) also maximize the face value $M_S$ of the short-term debt on condition that $e_I + e_M \geq (1-p_1) p_2 a$, since the face value is implicitly given by $M_S = 1 - \frac{e_I}{1-p_1 p_2} - \frac{e_M}{1-p_1}$. The corresponding face value $M_S^+ = 1 - \frac{(1-p_1) p_2 a}{1-p_1 p_2} a$ is larger than the face value $D_S^+ = 1 - p_2 a$ of the short-term debt that the bank can issue. And if Assumption 3 holds, the retention of $e_I^+$ by the initial firm owner is sufficient to incentivize the choice of the good assets, since $\Lambda_A(D_L(e_I^+)) > \mu$. Consequently, a chain with ($e_I^+, e_M^+$) provides a larger private benefit than a bank, because it allows for a subsidy and a higher premium than a bank.

If Assumption 3 does not hold, the equity level $e_I^+$ is too small to ensure a selection of the good assets. In this case, the chain faces a trade-off: increasing $e_I$ above $e_I^+ = (1-p_1) p_2 a$ reduces the subsidy and the premium, but choosing a sufficiently large $e_I$ avoids the loss.
δ_{p} p_{2} a - \mu \) from the selection of bad assets. The optimal trade-off in that case depends on a further specification of the parameter relations.

Let me conclude the section with a brief remark on a regulation by means of insurance premiums as an alternative to capital requirements. Fair insurance premiums have to be calibrated such that they are equal to the potential subsidy \( S(e_{I},e_{M}) \) from choosing too little equity. If this is the case, they perfectly offset the incentive to choose such capital structures. The fair premiums for the chain have to be larger than for a bank, since the subsidy for a chain is larger than for a bank, given the same level of debt. If this is the case, the firm has no incentive to join a chain instead of operating as a bank. But if the insurance premiums for the chain are the same as for a bank, the chain receives a subsidy and might be chosen by the firm for that reason.

5 Generalization of the Key Result

The key result of this paper is that the separation of credit risk and maturity transformations increases the default probability of the issued short-term debt. This section shows that this result is not specific to the particular asset risk I have studied, but that it holds for generic distributions of risk. The result does not dependent on the frictions introduced in the Section 3, and I thus neglect them here again.

Consider assets that are financed at \( t = 0 \) by issuing equity and debt claims and that have a stochastic payoff \( y_{2} \in \mathbb{R}^{+} \) at \( t = 2 \), with probability density function \( f \). At \( t = 1 \), there is a public signal \( \sigma \in \Sigma \subset \mathbb{R}^{n} \) about the portfolio, from which all agents can infer an updated probability density function \( f(y_{2};\sigma) \). The probability density function of \( \sigma \) shall be denoted as \( g \). Let us again assume that claims are priced in a market with risk-neutral investors and complete information about the asset risk. The risk-free interest rate between \( t = 1 \) and \( t = 2 \) is denoted as \( r_{2} \), and I initially abstract from interest rate risk. In the last paragraph of this section, I explain why the result is robust to interest rate risk. Let us start with determining the default probability of short-term debt of a bank with initial face value \( D_{S} \). Assuming that the financial market is liquid, the short-term debt can be rolled over at \( t = 1 \) in case of a signal \( \sigma \) as long as the discounted expected payoff \( y_{1}(\sigma) \) of the assets is larger than \( D_{S} \), where \( y_{1}(\sigma) = \frac{1}{1+r_{2}} E \left[ y_{2}|\sigma \right] = \frac{1}{1+r_{2}} \int y_{2} f(y_{2};\sigma) \, dy_{2} \). A default occurs at \( t = 1 \) for \( y_{1}(\sigma) < D_{S} \) and at \( t = 2 \) for \( y_{2} < D_{S,1}(D_{S};\sigma) \), where \( D_{S,1}(D_{S};\sigma) \) is the face value of the short-term debt after being rolled over in presence of a signal \( \sigma \). It is implicitly given by

\[
(1 + r_{2}) D_{S} = \int \min \left\{ D_{S,1}(D_{S};\sigma), y_{2} \right\} \, f(y_{2};\sigma) \, dy_{2},
\]

so that the discounted expected payoff of the new claim (given by the r.h.s. divided by \( (1 + r_{2}) \)) equals the value \( D_{S} \) of the maturing claim. The probability \( \phi_{1}(D_{S}) \) that the bank defaults on its short-term debt at \( t = 1 \) and the probability \( \phi(D_{S}) \) that the bank
defaults either at \( t = 1 \) or \( t = 2 \) are thus given as:

\[
\phi_1(D_S) = \int \mathbf{1}_{\{y_1(\sigma) < D_S\}} g(\sigma) d\sigma,
\]

\[
\phi(D_S) = \phi_1(D_S) + \int_0^{D_S;1(D_S;\sigma)} f(y_2; \sigma) dy_2 \mathbf{1}_{\{y_1(\sigma) \geq D_S\}} g(\sigma) d\sigma.
\]

These default probabilities can be compared to the ones of short-term debt issued by an intermediation chain. The discounted expected payoff \( y_t^D \) of the long-term debt at \( t = 2 \) given the information at date \( t \) is \( \min\{D_L, y_2\} \) for \( t = 2 \) and \( y_t^D(D_L; \sigma) = \frac{1}{1 + r_2} \int \min\{D_L, y_2\} f(y_2; \sigma) dy_2 \) for \( t = 1 \). Given liquid markets, the maturity transformer can roll over its short-term debt with face value \( M_S \) as long as \( M_S \leq y_t^D(D_L; \sigma) \), while it has to default for \( M_S > y_t^D(D_L; \sigma) \). A default at \( t = 2 \) occurs for \( y_t^D(D_L) < M_S;1(M_S; \sigma) \), where \( M_S;1(M_S; \sigma) \) denotes the face value of the short-term debt after a roll-over in presence of signal \( \sigma \). This value is implicitly given by

\[
(1 + r_2) M_S
\]

\[
= M_S;1(M_S; \sigma) \int_{D_L}^{\infty} f(y_2; \sigma) dy_2 + M_S;1(M_S; \sigma) \int_{M_S;1(M_S; \sigma)}^{D_L} f(y_2; \sigma) dy_2 + \int_0^{M_S;1(M_S; \sigma)} y_2 f(y_2; \sigma) dy_2
\]

\[
= \int \min\{M_S;1(M_S; \sigma) , y_2\} f(y_2; \sigma) dy_2.
\]

The probability \( \phi_{M;1}(D_S) \) that the maturity transformer defaults on its short-term debt at \( t = 1 \) and the probability \( \phi_{M}(D_S) \) that it defaults either at \( t = 1 \) or \( t = 2 \) are thus given as:

\[
\phi_{M;1}(M_S, D_L) = \int \mathbf{1}_{\{y_t^D(D_L; \sigma) < M_S\}} g(\sigma) d\sigma
\]

\[
\phi_M(M_S, D_L) = \phi_{M;1}(M_S, D_L) + \int_0^{M_S;1(M_S; \sigma)} f(y_2; \sigma) dy_2 \mathbf{1}_{\{y_t^D(D_L; \sigma) \geq M_S\}} g(\sigma) d\sigma.
\]

**Proposition 4**

The statements of Proposition\footnote{The ordering of the integration boundaries takes into account that a short-term debt claim with face value \( M_S;1 > D_L \) is equivalent to one with face value \( M_S;1 = D_L \), as they both have the same payoff, and it is thus sufficient to account for the later case.} holds independent of the risk of the assets: given the same underlying assets, short-term debt issued by an intermediation chain has a larger default probability than the same level of short-term debt issued by a bank:

\[
\phi_{M;1}(M_S, D_L) \geq \phi_1(M_S) \land \phi_M(M_S, D_L) \geq \phi(M_S) \forall M_S \in \mathbb{R}^+,
\]

and the inequalities are strict

\[
\text{for } M_S \in \bigcap_{\sigma \in \Sigma^*} \left( y_t^D(D_L; \sigma) , y_1(\sigma) \right) \text{ for any } \Sigma^* \subset \Sigma \text{ with } \int_{\Sigma^*} \int \mathbf{1}_{\{y_2 > D_L\}} f(y_2; \sigma) dy_2 g(\sigma) d\sigma > 0.
\]
The proof is given in Appendix D. Being a generalization of Proposition 1, the proposition holds for the same reason: a long-term debt claim to assets is a less valuable backing of short-term debt than the assets themselves, because some upside risk of the assets in case of a transitory shock is not included in the long-term debt. The long-term debt is a strictly less valuable backing (which implies that there are debt levels for which the chain has a strictly larger default probability), as long as the face value $D_L$ of the long-term debt is smaller than the highest possible payoff of the assets at $t = 2$. This means: as long as there is a subset $\Sigma^* \in \Sigma$ of signals at $t = 1$ (with strictly positive measure) for which $y_2 > D_L$ has strictly positive probability.

Let me conclude with indicating why the result is robust to introducing interest rate risk. The basic idea is that the interest rate risk can be reinterpreted as part of the signal about the conditional value of the assets at $t = 1$. Assume that the interest rate $r_2$ is drawn at $t = 1$ from a distribution with probability density function $h$. The interest rate $r_2$ appears in the analysis of $\phi_1$ and $\phi_{M,1}$ only in the factor $\frac{1}{1+r_2} f(y_2; \sigma)$, which can be rewritten as $f'(y_2; \sigma')$ with $\sigma' = (\sigma, r_2)$ being a generalized version of the signal at $t = 1$. Correspondingly, weighted integrals can be rewritten as $\int I \left( \frac{1}{1+r_2} f(y_2; \sigma) \right) g(\sigma) h(r_2) d\sigma dr_2 = \int I \left( f'(y_2; \sigma') \right) g'(\sigma') d\sigma'$ for any integrand $I$. Applying these reparametrizations, each step in the comparison of $\phi_{M,1}$ and $\phi_1$ presented above and in Appendix D remains the same. Consequently, the derived statements about their relation do not change. Based on this relation between $\phi_{M,1}$ and $\phi_1$, one can simply repeat the arguments stated above which have shown that the weak inequality $\phi_{M,1} \geq \phi_1$ extends to $\phi_M \geq \phi$ and that the strong inequality $\phi_{M,1} > \phi_1$ extends to $\phi_M > \phi$. Each of these arguments has been independent of the parameter $r_2$ and each of them thus holds true for an infinite sum over potential realizations of $r_2$. 

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Appendix

A Rephrasing the Result in Terms of Equity Values

As mentioned in the main text, the equity values at \( t = 0 \) are given as the expected payoff of the respective equity claim at \( t = 2 \), given risk-neutrality and \( r = 0 \). Taking into account the state-contingent asset values, the relations between equity values and face values of debt are as follows. The equity value \( e_B \) of the bank is equal to \( (1 - p_1)(1 - D_S) + p_1 \max \{ 0, 1 - p_2 a - D_S \} \). The equity value \( e_I \) of the security issuer is equal to \( y_0 - D_L = 1 - p_1 p_2 a - D_L \) for \( D_L \leq 1 - a \), and equal to \( (1 - p_1 p_2)(1 - D_L) \) for \( D_L \in (1 - a, 1] \). And the equity value \( e_M \) of the maturity transformer is equal to \( (1 - p_1)(D_L - M_S) + p_1 \max \{ 0, y_-(D_L) - M_S \} \). Solving these relations for the debt face values as functions of the equity values and plugging the results into \( \phi_M(M_S, D_L) \) and \( \phi(D_S) \) leads to the following expressions for \( \phi_M \) and \( \phi \) as functions of \((e_I, e_M)\) and \( e_B \):

\[
\phi(e_B) = \begin{cases} 
0 & \text{for } e_B \in [(1 - p_1 p_2)a, 1] \\
 p_1 p_2 & \text{for } e_B \in [(1 - p_1)p_2 a, (1 - p_1 p_2)a) \\
 p_1 & \text{for } e_B \in [0, (1 - p_1) p_2 a) 
\end{cases}
\]

\[
\phi_M(e_I, e_M) = \begin{cases} 
 p_1 p_2 & \text{for } e_M + e_I < (1 - p_1 p_2)a \land e_M + \left( 1 - \frac{1 - p_2}{1 - p_1 p_2} \right) e_I \geq (1 - p_1)p_2 a \\
 p_1 & \text{for } e_M + \left( 1 - \frac{1 - p_2}{1 - p_1 p_2} \right) e_I < (1 - p_1)p_2 a 
\end{cases}
\]

Analogously, \( \phi_1 \) and \( \phi_{M,1} \) can be rewritten as functions of \( e_B \) and \((e_I, e_M)\), too. As before, the boundaries between the second and third interval of \( \phi \) and \( \phi_M \), respectively, are also the boundaries that separate \( \phi_1 = 0 \) from \( \phi_1 = p_1 \) and \( \phi_{M,1} = 0 \) from \( \phi_{M,1} = p_1 \).

Proposition 1 (alternative formulation)

Given the same underlying asset and the same amount of equity, an intermediation chain has a larger default probability than a bank:

\[
\phi_M(e_I, e_M) \geq \phi(e_B) \forall e_I + e_M = e_B \in [0, 1], \text{ and } \\
\phi_M(e_I, e_M) > \phi(e_B) \text{ for } e_I + e_M = e_B \geq (1 - p_1)p_2 a \\
\land e_M + \left( 1 - \frac{1 - p_2}{1 - p_1 p_2} \right) e_I < (1 - p_1)p_2 a .
\]

The same statement holds for the default probabilities \( \phi_{M,1} \) and \( \phi_1 \) at \( t = 1 \).

Again, the proof consists of comparing the functional forms of the default probabilities.
B Proof of Proposition 2

Statement a) follows from the fact that the premium \( \Lambda_M \) has two relative optima: short- or long-term debt with face value \( 1 - a \) (the duration does not matter in this case), or short-term debt with face value \( M^\dagger_S = (1 - l_D) (1 - p_l) + (1 - p_2) (1 - l_D)^2 + p_2 (1 - a) \)

Statement b) holds, because \( M^\dagger_S > D^\ast_S \) if Assumption \( 2 \) holds. This implies \( \Lambda_M(M^\dagger_S; S) = \lambda(2 - p_1) M^\dagger_S > \lambda(2 - p_1) D^\ast_S = \Lambda(D^\ast_S; S) \). Short-term debt with face value \( M^\dagger_S \) is safe and there is no run at \( t = 1 \) and thus no corresponding liquidation loss: \( L^M_M = 0 \). And the initial owner of the security issuer chooses the good assets, because she retains all equity of the security issuer, which incurs the loss \( L_A(D^\dagger_L; L) = \mu \) from choosing the bad assets.

Statement c) follows from studying how the value of the surplus \( \Lambda + \Lambda_M - L_L - L^M_L - \delta_p p_2 a - \mu \cdot 1_{(\mu > \gamma L_A(D_d; d)} \) changes, if the structure of the chain deviates from \( (D^\dagger_L; L) \) and \( (M^\dagger_S; S) \), for which the surplus simply equals \( \Lambda_M(M^\dagger_S; S) \). Let us start with focusing on \( d = L \), which means focusing on all chains in which the security issuer sells long-term debt to the maturity transformer. The only change of the surplus from choosing \( D_L < D^\dagger_L \) is that the upper bound of the second interval of \( \Lambda_M \) decreases, which implies that the maximum of the premium \( \Lambda_M \) decreases. Choosing \( D_L > D^\dagger_L \) increases the upper bound of the second interval and thus the maximum of \( \Lambda_M \). The second consequence is, however, that \( \gamma L_A(D_L; L) \) falls below \( \mu \) even for \( \gamma = 1 \), so that the security issuer will no longer select the good assets. This leads to a loss \( \delta_p p_2 a - \mu \). If Assumption \( 2 \) b) holds, this loss is larger than the maximal increase of \( \Lambda_M \) which can be obtained by choosing \( D_L = 1 \) instead of \( D^\dagger_L = 1 - \frac{\mu}{\delta_p p_2} \). This maximal possible increase is equal to \( \lambda \left( (2 - p_1) (1 - l_D) (1 - p_2) + \frac{\mu}{\delta_p p_2} - \delta_p M^\dagger_S \right) \).

Liquidation losses play no role, since the maximization of \( \Lambda_M \) implies an exclusion of runs on the maturity transformer (so that \( L^M_M = 0 \), while the long-term debt of the security issuer cannot run (so that \( L_L = 0 \)).

Let us now consider \( d = S \), which means to consider all chains in which the security issuer sells short-term debt to the maturity transformer. The direct reference of short-term debt to the illiquid assets means that the provision of safe claims is constrained by the same coordination problem as in case of the bank. And statement b) has already shown that the provision of safe claims by a bank is less efficient than by a chain with \( (D^\dagger_L; L) \) and \( (M^\dagger_S; S) \). The coordination problem is present although the maturity transformer holds

\[ 21 \text{When } D_L \leq 1 - a, \text{ the long-term debt becomes safe. But if a premium is paid for it, it is paid by the maturity transformer, so that there is no increase of the overall surplus of the chain.} \]
all of the short-term debt, given that the maturity transformer issues short-term debt and each investors only holds a fraction \( \alpha < 1 \) of \( M_S \). If all investors withdraw their \( M_S \) claim at \( t = 1 \), the security issuer has to withdraw its \( D_S \) claim, which leads to the liquidation of the underlying assets. The maturity transformer has to sell short-term debt, if it wants to issue safe claims with a face value larger than \( 1 - a \), which is lowest possible payoff at \( t = 2 \). The fact that the issuance of debt with face value \( 1 - a \) leads to a lower premium than \( \Lambda_M(M_S^1, S) \) has already been shown statement a) of the proposition.

C Proof of Proposition 3

The first statement concerning the ‘level playing field’ follows from the functional form of \( S(e_I, e_M) \). Its derivative w.r.t. to both, \( e_I \) and \( e_M \) is negative, so that the minimal possible sum \( e_I + e_M = (1 - p_1)p_2 a \) maximizes \( S \). And since the derivative w.r.t. \( e_M \) is smaller than w.r.t. \( e_I \) due to \(-\frac{1}{1-p_1} < -\frac{p_2}{1-p_1 p_2}\), choosing \( e_M^+ = 0 \) and \( e_I^+ = (1 - p_1)p_2 a \) maximizes \( S \). Inserting \( e_I^+ \) and \( e_M^+ \) into \( M_S = 1 - \frac{e_I^+}{1-p_1 p_2} - \frac{e_M^+}{p_2} \) and accounting for \( \frac{1-p_1}{1-p_1 p_2} < 1 \) leads to the second statement. A retention of the equity \( e_I^+ \) by the firm owner leads to the loss \( L_A(D_S^1; L) = L_A \left( 1 - \frac{e_I^+}{1-p_1 p_2}; L \right) = L_A \left( 1 - \frac{u(p_1)p_2}{1-p_1 p_2}; a; L \right) = \delta_p p_2 \left( 1 - \max \left( 1 - \frac{1-(1-p_1)p_2}{p_2} a, 1 - a \right) \right) = \delta_p p_2 \left( \frac{1-p_1}{1-p_1 p_2} a \right) \) from choosing bad assets, which is larger than \( \mu \), if Assumption \( 3 \) holds. The consequence of these three statements is that the chain allows for a higher private benefit than a bank: is allows for a larger premium and an additional subsidy without having higher agency costs.

The result concerning the ‘fair regulation’ can be derived from the following observations. The premium \( \Lambda_M \) has two relative maxima, of which the one at \( M_S = 1-a \) leads a premium \( \lambda 2(1 - a) < \Lambda(D_S^1; S) \) (as shown in Lemma \( 1 \)). At the other relative maximum, the premium equals \( \lambda (2 - p_1)MS \) and thus increases in \( MS \). The derivative of \( M_S(e_I, e_M) = 1 - \frac{e_I}{1-p_1 p_2} - \frac{e_M}{p_2} \) w.r.t. \( e_M \) is negative. But \( S(e_I, e_M) = 0 \) only holds for \( (e_I, e_M) \) with \( e_M \geq (1 - p_1)p_2 \left( a - \frac{e_I}{1-p_1 p_2} \right) \). This implies that this constraint has to be binding in case that \( \Lambda_M \) is maximal for \( M_S > 1 - a \). For a binding constraint, one can define the function \( e_M(e_I) \), can insert the function into \( M_S(e_I, e_M) \) and can differentiate the expression w.r.t. \( e_I \). This leads to a negative term due to \( p_2 < 1 \), which implies that minimizing \( e_I \) maximizes \( M_S \) and \( \Lambda_M \). For \( e_I < (1-p_1)p_2 \frac{\mu}{\delta_p p_2} \), however, the owner of the security issuer will choose the bad set of assets, which leads to an efficiency loss \( \delta p p_2 a - \mu \). If Assumption \( 2b \) holds, this loss is larger than the maximal possible increase of \( \Lambda_M \) by choosing \( e_I = 0 \) instead of \( e_I = (1-p_1)p_2 \frac{\mu}{\delta_p p_2} \), which is \( \lambda \left( (2 - p_1)(1 - p_2) \frac{\mu}{\delta_p p_2} - \delta_p (1 - p_2 a) \right) \). (This has been shown in statement c of Proposition 2.) This means that \( \hat{e}_I = (1-p_1 p_2) \frac{\mu}{\delta_p p_2} \) is the optimal choice of equity. Given \( e_M = (1-p_1)p_2 \left( a - \frac{e_I^+}{1-p_1 p_2} \right) \), this choice implies the face value \( M_S = (1-p_2) \left( 1 - \frac{e_I^+}{\delta_p p_2} \right) + p_2 (1 - a) \) of short-term debt. This choice of short-term debt allows for a higher premium than \( M_S = 1 - a \) if Assumptions \( 1 \) and \( 2 \) hold, as shown in statement a of Proposition 2 (this statement shows that already \( (1 - l_D)\bar{M}_S \)
allows for a larger premium than \( M_S = 1 - a \). But due to \( M_S < D_S^* = 1 - p_2 a \), it follows that \( \Lambda_M(M_S; S) < \Lambda(D_S^*; S) \). As the chain obtains no subsidy and can only earn a smaller premium than a bank, the financial firm has no incentive to become a security issuer instead of a bank.

### D Proof of Proposition 4

The weak inequality \( \phi_{M_1}(M_S, D_L) \geq \phi_1(M_S) \) for arbitrary \( M_S \in \mathbb{R}^+ \) follows from

\[
y^0_1(D_L; \sigma) = \frac{1}{1 + r_2} \int \min\{D_L, y_2\} f(y_2; \sigma) dy_2 \leq \frac{1}{1 + r_2} \int y_2 f(y_2; \sigma) dy_2 = y_1(\sigma).
\]

The weak inequality \( \phi_M \geq \phi \) is a consequence of \( \phi_{M_1} \geq \phi_1 \) and the following observation concerning the second terms in \( \phi_M \) and \( \phi \). If \( y_1(\sigma) \geq D_S = M_S \), there are two possibilities. First, \( y^0_1(D_L; \sigma) < M_S \) and the signal \( \sigma \) contributes its full weight \( g(\sigma) d\sigma \) to the difference \( \phi_{M_1} - \phi_1 \). This contribution to \( \phi_{M_1} - \phi_1 \) larger than the signal’s contribution \( \int_0^{D_S} f(y_2; \sigma) dy_2 \cdot g(\sigma) d\sigma \) to the second term in \( \phi \). This implies that the signal’s contribution to the difference \( \phi_M - \phi \) is positive. Second, \( y^0_1(D_L; \sigma) \geq M_S \) and the signal \( \sigma \) has the same contribution to \( \phi_1 \) and \( \phi_{M_1} \) (which is zero) and the same contribution to the second terms in \( \phi \) and \( \phi_M \), because \( M_S(M_S; \sigma) = D_S(M_S; \sigma) \) for \( M_S = D_S \) owing to the identical form of the pricing relations for \( M_S \) and \( D_S \).

Strict inequalities apply when the face value \( D_L \) of the long-term debt is smaller than the highest possible payoff of the assets at \( t = 2 \), which means whenever there is subset \( \Sigma^* \in \Sigma \) of signals at \( t = 1 \) (with strictly positive measure) for which \( y_2 > D_L \) has strictly positive probability. This condition is expressed by \( \int_{\Sigma^*} \int \mathbb{1}_{\{y_2 > D_L\}} f(y_2; \sigma) dy_2 g(\sigma) d\sigma > 0 \).

For any signal \( \sigma \) in such a set \( \Sigma^* \), the value \( y^0_1(D_L; \sigma) = \frac{1}{1 + r_2} \int \min\{D_L, y_2\} f(y_2; \sigma) dy_2 \) of the long-term debt at \( t = 1 \) is strictly smaller than the value \( y_1(\sigma) = \frac{1}{1 + r_2} \int y_2 f(y_2; \sigma) dy_2 \) of the underlying assets at \( t = 1 \). Consequently, the default probability \( \phi_{M_1}(M_S) = \int \mathbb{1}_{\{y^0_1(D_L; \sigma) < M_S\}} g(\sigma) d\sigma \) of the chain is strictly larger than the probability \( \phi_1(D_S) = \int \mathbb{1}_{\{y_1(\sigma) < D_S\}} g(\sigma) d\sigma \) of the bank for all \( D_S = M_S \) that are in the interval between \( y^0_1(D_L; \sigma) \) and \( y_1(\sigma) \) for such \( \sigma \in \Sigma^* \).

The strict inequality \( \phi_{M_1} > \phi_1 \) for \( M_S = D_S \in \bigcap_{\sigma \in \Sigma^*} (y^0_1(D_L; \sigma), y_1(\sigma)) \) extends to \( \phi_M > \phi \) for two reasons. First, the contribution of all \( \sigma \notin \Sigma^* \) to \( \phi_M \) is weakly larger than their contribution to \( \phi \), as explained two paragraphs before. Second, for all \( \sigma \in \Sigma^* \) it holds that \( y_1(\sigma) > D_S \) and \( y^0_1(D_L; \sigma) < M_S \), so that these signals contribute their full weight \( g(\sigma) d\sigma \) to the difference \( \phi_{M_1} - \phi_1 \). This contribution to \( \phi_{M_1} - \phi_1 \) is not completely negated by the signals’ contribution to the second terms in \( \phi_M \) and \( \phi \), because the conditional probability of a bank default at \( t = 2 \) after a signal \( \sigma \in \Sigma^* \) is strictly smaller than one: \( \int_0^{D_S} f(y_2; \sigma) dy_2 < 1 \) for \( \sigma \in \Sigma^* \). This is true, because the pricing relation \( (1 + r_2) D_S = \int \min\{D_S, (D_S; \sigma)\} f(y_2; \sigma) dy_2 \) implies that \( \int_{D_S} f(y_2; \sigma) dy_2 > 0 \) for \( D_S < y_1(\sigma) = \frac{1}{1 + r_2} \int y_2 f(y_2; \sigma) dy_2 \).
References


