

# Politically feasible reforms of non-linear tax systems\*

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## Abstract

We develop a framework for a political economy analysis of tax reforms. We document that most tax reforms give rise to tax cuts or tax increases that are a monotonic function of income. We prove a median voter theorem for monotonic reforms of non-linear tax systems. We then focus on simple reforms that involve a change of the tax rates for incomes in a certain bracket. We characterize the conditions under which such reforms are politically feasible. We also develop a sufficient statistics approach that identifies whether a given tax system admits reforms that are politically feasible.

*Keywords:* Non-linear income taxation; Tax reforms; Political economy; Optimal taxation.

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# 1 Introduction

We present a new approach for a political economy analysis of non-linear tax systems. We focus on tax reforms. Our main results provide a characterization of tax reforms that are politically feasible in the sense that a majority of individuals prefers the reform over the status quo.

Previous literature has focussed on models of voting over tax schedules. The set of non-linear tax policies is multi-dimensional policy space. Thus, the median voter's preferred policy is not a Condorcet winner. This complicates any analysis of voting over non-linear tax schedules. One way of dealing with this complication is to restrict attention to a subset of tax systems for which a median voter theorem applies.<sup>1</sup> These restrictions limit the scope for a comprehensive political economy analysis of top tax rates, earned income tax credits, or taxes for the middle class.

Our main contributions are as follows: First, we prove a median voter theorem for reforms of non-linear tax systems. The theorem applies to reforms that satisfy a monotonicity condition. We document that this monotonicity condition is typically satisfied in practical tax policy. The analysis of politically feasible reforms, moreover, provides an explanation for the frequent observation that income tax schedules are more progressive for middle incomes than for low or high incomes. Second, we discuss the relation between reforms that are politically feasible and reforms that are attractive from a welfare perspective. Thereby we contribute to bridging the gap between political economy analyses of non-linear taxes and analyses of optimal welfare-maximizing taxes. Political economy approaches frequently limit attention to a subset of tax systems for tractability, whereas the normative literature seeks to avoid *a priori* restrictions on tax policies. A focus on monotonic reforms makes it possible to analyse whether non-linear tax systems can be reformed in way that is both politically feasible and welfare improving, or, whether the status quo is efficient in the sense that the scope for politically feasible welfare improvements has been exhausted. Finally, we develop a sufficient statistics approach that identifies politically feasible reforms empirically, given data on the actual tax policy, the behavioral responses to taxation and the distribution of incomes.

**A median voter theorem for monotonic tax reforms.** We focus on monotonic tax reforms, i.e. on reforms with the property that the difference between taxes due after the reform and taxes due before the reform is a monotonic function of income. We prove a median voter theorem according to which a monotonic reform is politically feasible if and only if it is supported by the taxpayer with median income (Theorem 1). We moreover

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<sup>1</sup>For instance, the well-known prediction due to Meltzer and Richard (1981) that tax rates are an increasing function of the difference between median and average income is obtained by focussing on linear income taxes.

document that, in practice, most tax reforms are monotonic.<sup>2</sup> For instance, the tax cuts under Bush (2001-2003) in the US and in the 80s under Thatcher in the UK involved tax cuts for all incomes, with larger cuts for larger incomes. By our Theorem 1, if the person with median income is among the beneficiaries of such a reform, a majority will be in favor. Another example of a monotonic reform is one that involves higher taxes, with increases that are larger for “the rich.” For instance, the tax reforms in the UK in 2010 and in France in 2013 were of this type. Again, if the median voter appreciates the reform, there is a majority in favor of it. In an extension of Theorem 1, we also provide conditions under which reforms that are monotonic only for incomes above or below the median income are politically feasible. This applies to reforms of the US income tax under Clinton and Obama. Both involved lower taxes for low incomes and higher taxes for high incomes.

**Political feasibility and welfare.** Monotonic reforms also play a prominent role in the theory of welfare-maximizing taxation. Characterizations of optimal tax systems via the perturbation method often look at the welfare implications of reforms that we refer to as *simple* in what follows.<sup>3</sup> A simple reform involves a small change of the marginal tax rates for incomes that lie in a certain bracket.<sup>4</sup> Welfare-maximizing approaches then strike a balance between the distortions due to the increase of tax rates in the bracket and the revenue due to the increased tax receipts from incomes above the bracket. A characterization of welfare-optimal taxes can be obtained from the condition that there must not exist a simple reform that yields welfare gains.

Thus, simple reforms have welfare implications that are well understood. Moreover, they are monotonic so that Theorem 1 applies. Hence, by focussing on simple reforms we can provide a more detailed analysis of how the set of politically feasible reforms relates to the set of welfare-maximizing reforms. To see where the tension between welfare considerations and political feasibility may come from, note that political feasibility requires that a reform makes a sufficiently large number of individuals better off. Welfare considerations, by contrast, tradeoff utility gains and losses of different individuals. A reform that yields large gains to a small group of individuals and comes with small losses for a large group can be welfare-improving, but might not be politically feasible. A reform that has small gains for many and large costs for few might be politically feasible without

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<sup>2</sup>We investigate a panel of 33 OECD countries and find that 78 percent of the tax reforms that took place since the year 2000 were monotonic reforms. For the United States, the United Kingdom, and France we also consider alternative data sources that allow us to cover a longer time horizon. This leads to similar conclusions: the share of monotonic reforms is 80% for the US (period 1981-2016), 84% for France (period 1916-2016), and 77% for the UK (period 1981-2016).

<sup>3</sup>See Piketty (1997), Saez (2001), Golosov, Tsivinski and Werquin (2014), or Jacquet and Lehmann (2016).

<sup>4</sup>Consequently, the tax payments of individuals with incomes in the bracket change linearly with their income; and, once the endpoint of the bracket is reached, the extra tax burden stays constant. Thus, a simple reform involves additional taxes that are a non-decreasing function of income.

being welfare-improving.

Theorem 2 provides a characterization of simple reforms that are politically feasible.<sup>5</sup> It considers a Pareto efficient tax system as the status quo. The theorem shows that tax cuts for below median incomes and tax increases for above median incomes are politically feasible. To see the logic, consider first a bracket of below median incomes. Lowering the marginal tax rates for incomes in this bracket leads to a tax cut for all individuals with higher income, including all individuals with above median income. It also involves a loss of tax revenue. If the status quo is efficient, then individuals with above median income are net beneficiaries, i.e. their gain from the tax cut outweighs their loss from the reduced tax revenue. Hence, there is a majority in favor of such a reform. Interestingly, the reduced rates apply to below median incomes, but the majority support relies on above median incomes. The mirror image is a reform that involves higher rates for above median incomes and which is supported by all individuals with below median income.

This characterization implies a marked discontinuity at the median level of income. Below, tax cuts are politically feasible. Above, higher tax rates are politically feasible. This marked discontinuity suggests an explanation for the observation that actual tax schedules often have a pronounced increase of marginal tax rates close to the median income:<sup>6</sup> if political economy forces push towards low tax rates below the median and towards high tax rates above the median, then there has to be an intermediate range that connects the low rates below the median with the high rates above the median.

As a corollary, Theorem 2 also provides an answer to the question whether a given tax system can be reformed in way that is both welfare-improving and politically feasible. For below median incomes, moving towards lower marginal tax rates is politically feasible. If, according to a given welfare measure, tax rates are too high in the status quo, then such a reform is also politically feasible. Otherwise, there is no simple reform that is both welfare-improving and politically feasible. Analogously, raising marginal tax rates is politically feasible for above median incomes. If tax rates are too low from a welfare perspective, then such a reform also yields a welfare gain. If not, there is no simple reform for above median incomes that is both welfare improving and politically feasible.

**Identifying politically feasible reforms.** We also show how to bring our analysis to the data. An implication of our analysis is that we only need Pareto bounds for marginal tax rates to identify simple reforms that are politically feasible. We build on sufficient statistics approaches to the analysis of non-linear tax systems. Specifically, we derive both upper and lower Pareto bounds for marginal tax rates. How tight these Pareto bounds are

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<sup>5</sup>A caveat that applies to any approach based on small reforms is that it can only identify directions for reform. While this is informative, it does not extend without further assumptions to large reforms, see Kleven (2018).

<sup>6</sup>In Section 6 we present evidence for the US and France. The Netherlands share this pattern, see Jacobs, Jongen and Zoutman (2017). In Germany it is referred to as the “Mittelstandsbauch” (middle class belly) of the income tax schedule.

depends on behavioral responses, in particular on the elasticity of taxable income (ETI). There is an inverse elasticity logic at play: the lower this elasticity, the more permissive are the Pareto bounds. Tax policy then has a larger room to maneuver in the sense that marginal tax rates can be lowered or increased without violating Pareto efficiency. We illustrate our approach using data for the United States, the United Kingdom, and France. For these empirical cases, the upper Pareto bound gets close to the status quo schedule for values of this elasticity that are discussed in the empirical literature. Thus, the discussion about the appropriate value of the ETI has implications for whether taxes on “the rich” can be increased in a politically feasible way. Low estimates suggest that the answer is “yes”, high estimates suggest that the answer is “no”. The lower bound does not give rise to such controversies. It is far away from the status quo for plausible values of the ETI. Thus, marginal taxes on “the poor” can be lowered in a politically feasible way.

**Outlook.** The remainder is organized as follows. The next section discusses related literature. Section 3 presents evidence on monotonic reforms. The formal framework is introduced in Section 4. Section 5 contains the median voter theorem for monotonic reforms. The characterization of politically feasible reforms can be found in Section 6. Section 7 discusses extensions of the median voter theorem for monotonic reforms to models of taxation that are richer than our basic setup, a generic Mirrleesian framework. Specifically, we consider the possibility to mix direct and indirect taxes as in Atkinson and Stiglitz (1976), the possibility to add sources of heterogeneity among individuals such as fixed costs of labor market participation or public goods preferences, and the possibility that taxpayers seek to mitigate income differences that are due to luck as opposed to effort, as in Alesina and Angeletos (2005). The last section contains concluding remarks. Unless stated otherwise, proofs are relegated to the Appendix.

## 2 Related literature

Any political economy approach to non-linear income taxation faces the difficulty that the set of non-linear tax schedules is a multi-dimensional policy space. With such a policy space, the existence of a Condorcet winner is not to be expected. Many political economy approaches to redistributive taxation deal with this complication by restricting attention to a subset of tax systems in which a Condorcet winner can be found. The advantage of this approach is that it allows for clear-cut political economy predictions. The disadvantage is that the set of tax systems may become too small for an analysis that is empirically appealing. We provide a more detailed discussion that relates our work to this literature in what follows. The main difference is that we look at preferences over *tax reforms* – as opposed to preferences over a subset of tax systems.

Well-known political economy approaches to redistributive income taxation use the

model of linear income taxation due to Sheshinski (1972). In this model, marginal tax rates are the same for all levels of income and the resulting tax revenue is paid out as a uniform lump-sum transfer. As has been shown by Roberts (1977), the median voter's preferred alternative is a Condorcet winner in the set of all linear income tax systems. Gans and Smart (1996) note that the median voter theorem for linear taxation reflects a more general result in social choice theory due to Rothstein (1990; 1991).<sup>7</sup> Our work is related in that we also draw on Rothstein's insight to prove a median voter theorem, albeit one that applies to tax reforms.

The analysis by Gans and Smart (1996) extends to a specific class of non-linear tax systems, namely those that can be ordered according to their degree of progressivity. Bénabou (2000) uses this framework for a dynamic political economy analysis of redistributive taxation. Specifically, he focusses on tax systems with a constant rate of progressivity, see also Heathcote, Storesletten and Violante (2017). We present median voter results that also apply to reforms of tax systems in this class. A limitation of it is that one cannot have varying degrees of progressivity for different income categories. Progressivity often times is particularly pronounced for middle incomes.

Median voter theorems for linear income taxation have been widely used on the assumption that voters are selfish. A prominent example is the analysis by Meltzer and Richard (1981). The explanatory power of this framework was found to be limited – see, for instance, the review in Acemoglu, Naidu, Restrepo and Robinson (2015) – and has led to analyses in which the preferences for redistributive tax policies are also shaped by prospects for upward mobility or a desire for a fair distribution of incomes.<sup>8</sup> In Section 7 we extend our basic analysis and prove a median voter theorem for reforms of non-linear tax systems that takes account of such demands for fairness.

Röell (2012), Bohn and Stuart (2013) and Brett and Weymark (2016; 2017) study non-linear taxes in the citizen-candidate framework due to Osborne and Slivinski (1996) and Besley and Coate (1997): citizens compete for office and lack powers of commitment. An elected candidate will therefore implement her own preferred policy. Voting over candidates is therefore equivalent to voting over the non-linear tax policies that the different candidates' would select if they could dictate tax policy. The median voter's preferred tax policy is a Condorcet winner in this set of selfishly optimal tax schedules.

The literature has also explored political economy approaches to non-linear taxation that do not give rise to median voter results. Non-linear taxation has, for instance, been squared with probabilistic voting, the political agency model of Barro (1973) and Ferejohn (1986), or pork-barrel spending.<sup>9</sup> Saez and Stantcheva (2016) study generalized

<sup>7</sup>An alternative condition for the validity of the median voter theorem is that preferences are single-peaked, see Black (1948).

<sup>8</sup>See, for instance, Piketty (1995), Bénabou and Ok (2001), Alesina and Angeletos (2005), Bénabou and Tirole (2006), or Alesina, Stantcheva and Teso (2018).

<sup>9</sup>See Farhi, Sleet, Werning and Yeltekin (2012), Scheuer and Wolitzky (2016), Acemoglu, Golosov and Tsyvinski (2008; 2010), or Bierbrauer and Boyer (2016).

welfare functions with weights that may reflect such political equilibrium outcomes. In this paper, we focus on tax reforms. Moreover, we do not analyze political competition as a strategic game that gives rise to equilibrium tax policies. Our framework is, however, consistent with such an approach. Game-theoretic models of political competition can be used to generate predictions of how the political process selects from the set of politically feasible reforms.

Pareto bounds for non-linear taxes play an important role in our characterization of politically feasible tax reforms. Upper bounds have previously been derived by Werning (2007) and Lorenz and Sachs (2016). Our work also contains the characterization of a lower Pareto bound.

Our characterization of politically feasible reforms uses perturbation arguments akin to those in the literature on welfare-maximizing tax systems. It complements both the heuristic approaches due to Piketty (1997) and Saez (2001) and approaches that use functional derivatives such as Golosov et al. (2014) and Jacquet and Lehmann (2016). Simple reforms – i.e. small changes of marginal tax rates for incomes in a particular bracket – play a prominent role for both. Such reforms induce discontinuities in the marginal tax schedule and hence give rise to discontinuous behavioral responses. Heuristic approaches proceed on the assumption that these discontinuities can be ignored provided that the reform is “sufficiently small.” The work on functional derivatives uses well-behaved approximations of simple reforms to avoid discontinuities. We neither use heuristics nor approximations. Instead, we explicitly compute functional derivatives in the presence of bunching.

The focus on reforms links our work to an older literature in public finance that seeks to complement the theory of optimal taxation – which characterizes welfare-maximizing tax systems and has no role for current tax policy – by a theory of incremental changes that apply to a given status quo, see Feldstein (1976). Weymark (1981), for instance, studies the scope for Pareto-improving reforms of a commodity tax system. Guesnerie (1995) provides a survey of this literature and contains an analysis of tax reforms that emphasizes political economy forces, formalized as a requirement of coalition-proofness. Our analysis goes beyond this earlier literature by combining results from social choice theory on the validity of median voter theorems with the perturbation approach to the analysis of non-linear tax systems.

### 3 Monotonic reforms

The focus on monotonic tax reforms enables a tractable political economy analysis of tax reforms in subsequent sections. Here, we define monotonic reforms more formally and argue that they play a prominent role in practical tax policy. Let  $T_0$  be the pre-reform and  $T_1$  be the post-reform tax schedule. The tax reform is said to be monotonic over a range of incomes  $\mathcal{Y} \subset \mathbb{R}_+$  if  $T_1 - T_0$  is a monotonic function for  $y \in \mathcal{Y}$ . Given a

cross-section distribution of income, we say that a reform is monotonic above (below) the median if  $T_1 - T_0$  is a monotonic function for incomes above (below) the median income. As will become clear, monotonicity at least above or below the median is key for our median voter results.

Tax reforms in OECD countries are typically monotonic over the whole range of incomes, but there are also notable deviations from this pattern. Many of these deviations are such, however, that monotonicity applies for incomes above or below the median. Other deviations are such that the non-monotonicity is negligible in magnitude. In the following, we explain how we arrive at these assertions. First, we analyze tax reforms in OECD countries from 2000 to 2016. Second, for the United States, the United Kingdom, and France we use additional sources to cover a longer time horizon and we look explicitly at specific tax reforms that took place in these countries.

The OECD provides annual data on key parameters of the statutory personal income tax systems of its member countries. In particular, it documents tax brackets and marginal tax rates. We use this information to construct the corresponding tax function.<sup>10</sup> A reform takes place if this tax function changes from one year to the next. The following table provides a summary statistic of how many reforms took place between the years 2000 and 2016 and of how many of those that took place were monotonic.

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Total number of possible reforms (#years * #countries):	528
Total number of reforms:	394
Number of monotonic reforms:	309 (78%)
Number of non-monotonic reforms:	85 (22%)

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Table 1: Summary statistics on the tax reforms for a panel of 33 OECD countries (2000-2016).

Table 1 is based on the OECD database (Table I.1. Central government personal income tax rates and thresholds: accessible on <http://stats.oecd.org/Index.aspx?DataSetCode=TABLEI1>). See Appendix D for a list of the countries that we cover.

Table 1 shows that most, but not all, reforms observed in OECD countries are monotonic. Specifically, 78% of the reforms are monotonic over the whole range of incomes. The complementary set includes reforms that are monotonic either above or below the median. It also includes reforms with non-monotonies that seem economically negli-

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<sup>10</sup>Constructing the tax functions with the OECD database does not do full justice to all types of individual heterogeneity that tax systems take account of. For instance, the OECD presents tax functions for singles without dependents. Neither does it distinguish the tax functions that are relevant for individuals with and without the possibility to deduct child care expenses. For the US we contrast the tax functions constructed with OECD database and the ones obtained by using the more detailed NBER TAXSIM micro-simulation model. We report on this robustness check in part D of the Appendix.

gible. We provide more specific examples of such reforms below. Not all countries have a fraction of monotonic reforms close to the average of 78%. For instance, the fraction of monotonic reforms is much smaller in Israel and Italy, and much larger in Belgium and Sweden. Summary statistics for all OECD countries can be found in the supplementary material for this paper. The Supplement also reports on findings obtained from additional sources for the US, the UK and France. The share of monotonic reforms is 80% for the US (period 1981-2016), 84% for France (period 1916-2016) and 77% for the UK (period 1981-2016). In the following, we illustrate these descriptive statistics by a discussion of specific examples.

An interesting reform took place in France in 1937 (first budget after the Front Populaire election in 1936). Figure 1 shows the reform induced change in the tax burden for different levels of income. The origin of the sawtooth pattern is a change in the description of the tax code. Tax law expressed tax liabilities by means of marginal tax rates before the reform and by means of average tax rates after the reform. This example illustrates that we tend to underestimate the prevalence of monotonic reforms by focussing on the instances where  $T_1 - T_0$  is a monotonic function: there are reforms, like the one in the Figure, that are essentially monotonic even though the function  $T_1 - T_0$  exhibits some small non-monotonicities.

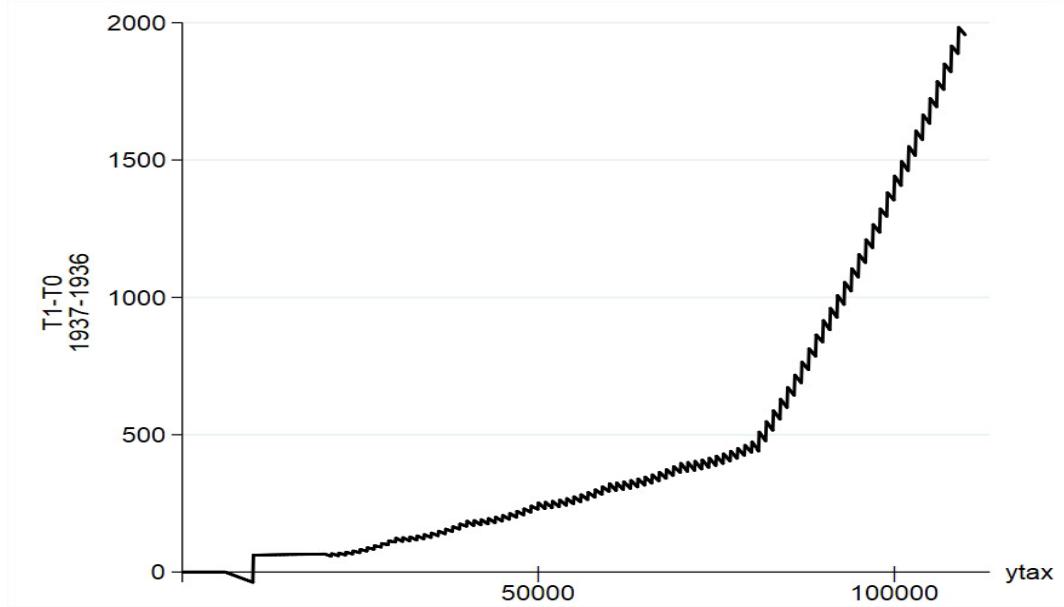


Figure 1: Reform of the French income tax in 1937

Examples of monotonic reforms are the tax cuts in the United Kingdom under the Thatcher government in 1988, the 2010 reform in the United Kingdom and 2013 reform in France that both involved the creation of a new bracket at the top, or the Bush tax cuts between 2001 and 2003 in the United States that involved tax cuts that were increasing in income (see Figure 2). The Reagan tax cuts are another example of a reform that

is essentially monotonic, also with larger tax cuts for larger incomes, but involves small non-monotonicities for low levels of income (see Figure 3).

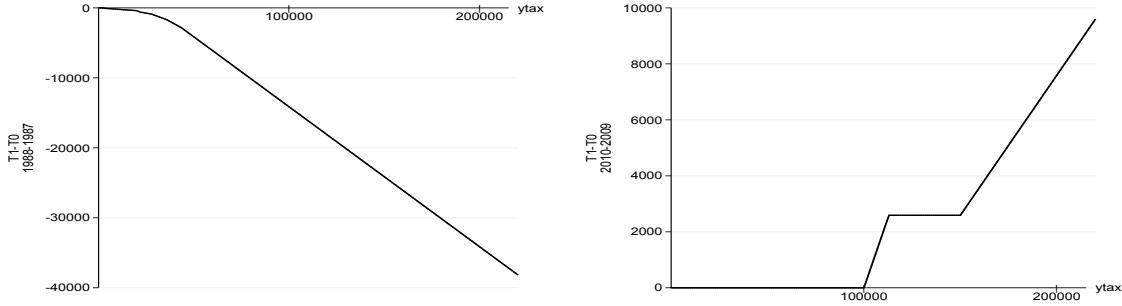


Figure 2: Reforms of the UK income tax

Figure 2 shows the reforms in the years 1988 (left panel) and 2010 (right panel).

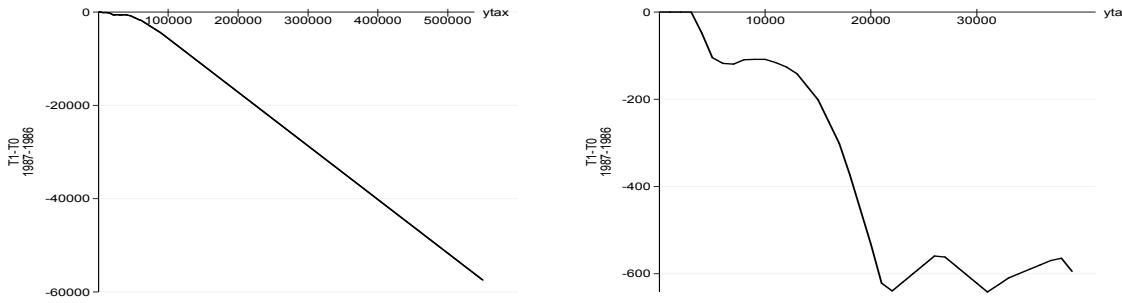


Figure 3: Reagan tax cuts

The figure on the left shows the whole range of incomes. The figure on the right focusses on low incomes where a non-monotonicity arises.

A frequent type of non-monotonicity is a reform that involves lower tax payments for the poor and higher tax payments for the rich. Our analysis shows that such reforms often involve tax cuts that increase in income until a threshold is reached. Above the threshold, the tax cuts decrease in income and eventually turn negative. For instance, the tax reforms in France in 1983 and 2011 (see Figure 4) or the US tax reforms in the years 1993 under Clinton, and 2013 under Obama are of this type (see Figure 5). Such reforms are monotonic either above or below the median.

## 4 The model

We study the political economy of tax reforms through the lens of a generic Mirrleesian model of income taxation: individuals value consumption and leisure, and maximize utility subject to a budget constraint that is shaped by a non-linear income tax system. We begin with a specification of preferences and then describe how individual choices as

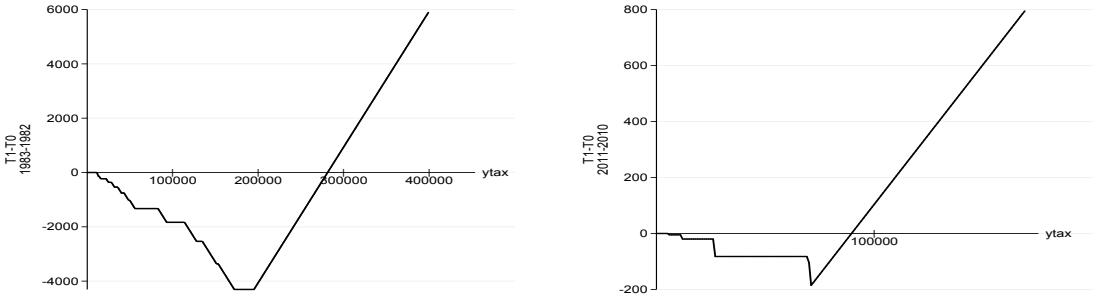


Figure 4: Reforms of the French income tax in years 1983 and 2011

Figure 4 shows the reforms in the years 1983 (left) and 2011 (right).

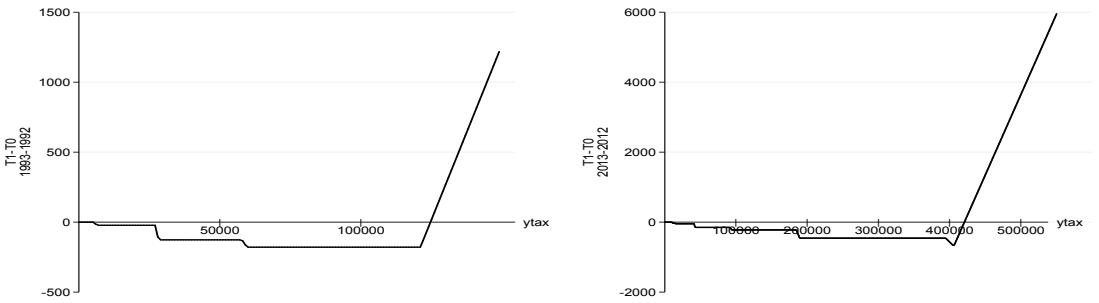


Figure 5: Reforms of the US income tax under Clinton and Obama

Figure 5 shows the reforms in the years 1993 (left) and 2013 (right).

well as measures of tax revenue, welfare and political support are affected by reforms of the tax system. We discuss extensions to richer models of taxation in Section 7.

## 4.1 Preferences

There is a continuum of individuals of measure 1. Individuals have a utility function  $u$  that is increasing in private goods consumption, or after-tax income,  $c$ , and decreasing in earnings or pre-tax income  $y$ . Individuals differ in their willingness to work harder in exchange for increased consumption. To formalize this we distinguish different types of individuals. The set of possible types is denoted by  $\Omega$  with generic entry  $\omega$ . The utility that an individual with type  $\omega$  derives from  $c$  and  $y$  is denoted by  $u(c, y, \omega)$ . The slope of an individual's indifference curve in a  $y$ - $c$ -diagram  $-\frac{u_y(c, y, \omega)}{u_c(c, y, \omega)}$  measures how much extra consumption an individual requires as a compensation for a marginally increased level of pre-tax income. We assume that this quantity is decreasing in the individual's type, i.e. for any pair  $(c, y)$ , and any pair  $(\omega, \omega')$  with  $\omega' > \omega$ ,

$$-\frac{u_y(c, y, \omega')}{u_c(c, y, \omega')} \leq -\frac{u_y(c, y, \omega)}{u_c(c, y, \omega)}.$$

This assumption is commonly referred to as the *Spence-Mirrlees single crossing property*.<sup>11</sup> It implies that utility-maximizing choices are such that higher types end up having higher incomes than lower types, and, in particular, that this ordering does not depend on the tax system. Thus, type  $\omega'$  chooses weakly higher earnings than type  $\omega < \omega'$  not only under an initial tax schedule  $T_0$  but also under any alternative tax schedule  $T_1$ .

The set  $\Omega$  is taken to be a compact subset of the non-negative real numbers,  $\Omega = [\underline{\omega}, \bar{\omega}] \subset \mathbb{R}_+$ . The cross-section distribution of types in the population is represented by a cumulative distribution function  $F$  with density  $f$ .

We allow for income effects. Specifically, we assume that leisure is a non-inferior good. If individuals experience an increase in an exogenous source of income  $e$ , they do not become more eager to work. More formally, we assume that for any pair  $(c, y)$ , any  $\omega$ , and any  $e' > e$ ,

$$-\frac{u_y(c + e, y, \omega)}{u_c(c + e, y, \omega)} \leq -\frac{u_y(c + e', y, \omega)}{u_c(c + e', y, \omega)}. \quad (1)$$

The assumptions introduced so far are preserved by monotone transformations of the individuals' utility functions. Our analysis of politically feasible reforms does not require anything else, i.e. it is based on an ordinal interpretation of the utility function  $u$ . When performing welfare comparisons, we invoke the additional assumption that an individual's marginal utility of consumption  $u_c(c, y, \omega)$  is both non-increasing in  $c$  and non-increasing in  $\omega$ , i.e.  $u_{cc}(c, y, \omega) \leq 0$  and  $u_{c\omega}(c, y, \omega) \leq 0$ .<sup>12</sup> These assumptions will enable us to show that the marginal consumption utility of low income types is not less than the marginal consumption utility of high income types.

The *Spence-Mirrlees single crossing property* is an ordinal property, i.e. a property that is preserved by monotone transformations of the utility function  $u$ . Higher types have flatter indifference curves. The index  $\omega$  represents this order. Any monotone transformation of  $\omega$  represents this order as well. Thus, as long as we only invoke the ordinal properties of preferences, many representations of an individuals' type are possible. This observation will give us a degree of freedom when we bring our analysis of politically feasible reforms to the data.

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<sup>11</sup>The existing literature frequently invokes a utility function  $U : \mathbb{R}_+^2 \rightarrow \mathbb{R}$  so that  $u(c, y, \omega) = U(c, \frac{y}{\omega})$  and interprets  $\omega$  as an hourly wage and  $l = \frac{y}{\omega}$  as the time that an individual needs to generate a pre-tax-income of  $y$ , see e.g. Mirrlees (1971) or Diamond (1998). Our analysis is consistent with this specification but does not require it.

<sup>12</sup>These assumptions hold for any utility function that is additively separable between utility from consumption on the one hand and costs of effort on the other. With a non-separable utility function of the form  $u(c, y, \omega) = U(c, \frac{y}{\omega})$ ,  $u_{c\omega}(c, y, \omega) \leq 0$  holds provided that  $U_{cl}(c, \frac{y}{\omega}) \geq 0$  so that working harder makes one more eager to consume. Seade (1982) refers to  $U_{cl}(c, \frac{y}{\omega}) \geq 0$  as non-Edgeworth complementarity of leisure and consumption.

## 4.2 Tax reforms

Individuals are confronted with a predetermined income tax schedule  $T_0$  that assigns a (possibly negative) tax payment  $T_0(y)$  to every level of pre-tax income  $y \in \mathbb{R}_+$ . Under the initial tax system individuals with no income receive a transfer equal to  $c_0 \geq 0$ . We assume that  $T_0$  is everywhere differentiable so that marginal tax rates are well-defined for all levels of income. We also assume that  $y - T_0(y)$  is a non-decreasing function of  $y$  and that  $T_0(0) = 0$ .

A reform induces a new tax schedule  $T_1$  that is derived from  $T_0$  so that, for any level of pre-tax income  $y$ ,  $T_1(y) = T_0(y) + \tau h(y)$ , where  $\tau$  is a scalar and  $h$  is a function. We represent a reform by the pair  $(\tau, h)$  where  $\tau$  measures the size the reform. A small reform, for instance, has  $\tau$  close to zero. Without loss of generality, we focus on reforms such that  $y - T_1(y)$  is non-decreasing. The reform induces a change in tax revenue denoted by  $\Delta^R(\tau, h)$ . For now we assume that this additional tax revenue is used to increase the basic consumption level  $c_0$ . Alternatives are considered in Section 7.

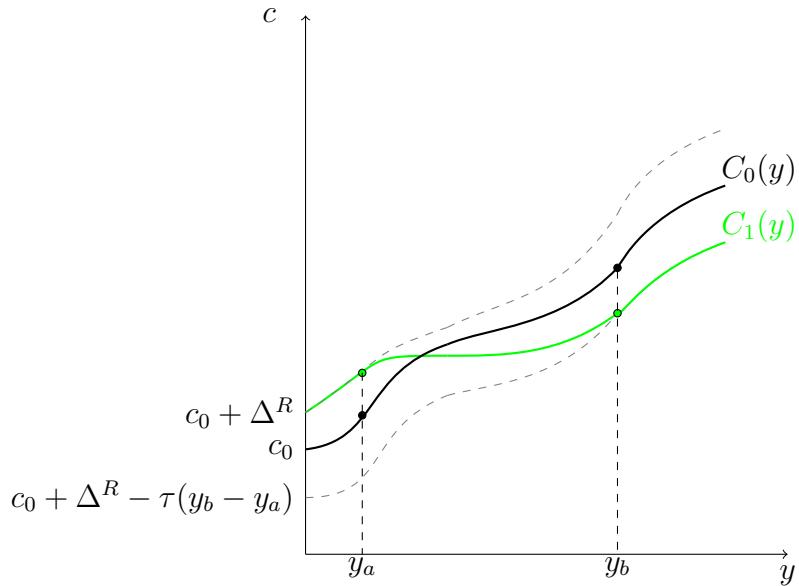


Figure 6: A simple reform

**Simple reforms.** Some of our results follow from looking at a special class of reforms. For this class, there exists a first threshold level of income  $y_a$ , so that the new and the old tax schedule coincide for all income levels below the threshold,  $T_0(y) = T_1(y)$  for all  $y \leq y_a$ . There exists a second threshold  $y_b > y_a$  so that, for all incomes between  $y_a$  and  $y_b$ , marginal tax rates are increased (or decreased) by  $\tau$ ,  $T'_0(y) + \tau = T'_1(y)$  for all  $y \in (y_a, y_b)$ . For all incomes above  $y_b$ , marginal tax rates coincide, so that  $T'_0(y) = T'_1(y)$

for all  $y \geq y_b$ . Hence, the function  $h$  is such that

$$h(y) = \begin{cases} 0, & \text{if } y \leq y_a, \\ y - y_a, & \text{if } y_a < y < y_b, \\ y_b - y_a, & \text{if } y \geq y_b. \end{cases}$$

For reforms of this type we will write  $(\tau, y_a, y_b)$  rather than  $(\tau, h)$ . Figure 6 shows how a reform in the  $(\tau, y_a, y_b)$ -class that generates positive tax revenue,  $\Delta^R > 0$ , affects the combinations of consumption  $c$  and earnings  $y$  that are available to individuals. Specifically, the figure shows the curves

$$C_0(y) = c_0 + y - T_0(y), \quad \text{and} \quad C_1(y) = c_0 + \Delta^R + y - T_0(y) - \tau h(y).$$

For incomes below  $y_a$  and above  $y_b$  the curves have the same slopes. The basic transfer increases by  $\Delta^R$  so that more consumption is available at income levels smaller than  $y_a$ . Less consumption is available at income levels larger than  $y_b$ . In Figure 6 we assume that, at these income levels, the loss from the additional tax payment  $\tau(y_a - y_b)$  exceeds the gain from the increase of the basic transfer.<sup>13</sup> Between  $y_a$  and  $y_b$  the increased marginal tax rate implies that the consumption schedule becomes flatter.

Simple reforms play a prominent role in the literature, see e.g. Saez (2001). By contrast, analyses that use functional derivatives to analyze tax perturbations rest on the assumption that both pre- and post-reform earnings are characterized by first order conditions, see Golosov et al. (2014). Simple reforms cannot be approached directly with this approach because they induce a discontinuity in marginal tax rates. Therefore, we present a detailed analysis of the behavioral responses to simple reforms in part A of the Appendix. The formal analysis that follows applies both to reforms in the  $(\tau, y_a, y_b)$ -class and to reforms where pre- and post-reform earnings follow from first order conditions.

**Notation.** To describe the implications of reforms for measures of revenue, welfare and political support it proves useful to introduce the following optimization problem: choose  $y$  so as to maximize

$$u(c_0 + e + y - T_0(y) - \tau h(y), y, \omega), \quad (2)$$

where  $e$  is a source of income that is exogenous from the individual's perspective. We assume that this optimization problem has, for each type  $\omega$ , a unique solution that we denote by  $y^*(e, \tau, \omega)$ .<sup>14</sup> The corresponding indirect utility level is denoted by  $V(e, \tau, \omega)$ . Armed with this notation we can express the reform-induced change in tax revenue as

$$\Delta^R(\tau, h) := \int_{\underline{\omega}}^{\bar{\omega}} \{T_1(y^*(\Delta^R(\tau, h), \tau, \omega)) - T_0(y^*(0, 0, \omega))\} f(\omega) d\omega.$$

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<sup>13</sup>Otherwise the reform would be Pareto-improving, leading to additional consumption at all levels of income.

<sup>14</sup>For ease of exposition, we ignore the non-negativity constraint on  $y$  in the body of the text and relegate this extension to part C in the Appendix. There, we clarify how the analysis has to be modified if there is a set of unemployed individuals whose labor market participation might be affected by a reform.

The reform-induced change in indirect utility for a type  $\omega$  individual is given by

$$\Delta^V(\omega | \tau, h) := V(\Delta^R(\tau, h), \tau, \omega) - V(0, 0, \omega).$$

**Pareto-improving reforms.** A reform  $(\tau, h)$  is said to be Pareto-improving if, for all  $\omega \in \Omega$ ,  $\Delta^V(\omega | \tau, h) \geq 0$ , and if this inequality is strict for some  $\omega \in \Omega$ .

**Welfare-improving reforms.** We consider a class of social welfare functions with weights that are a non-increasing function  $g : \Omega \rightarrow \mathbb{R}_+$ . The welfare change that is induced by a reform is given by

$$\Delta^W(\tau, h) := \int_{\underline{\omega}}^{\bar{\omega}} g(\omega) \Delta^V(\omega | \tau, h) f(\omega) d\omega.$$

A reform  $(\tau, h)$  is said to be welfare-improving if  $\Delta^W(\tau, h) > 0$ .

**Political support for reforms.** Political support for the reform is measured by the mass of individuals who are made better if the initial tax schedule  $T_0$  is replaced by  $T_1$ ,

$$S(\tau, h) := \int_{\underline{\omega}}^{\bar{\omega}} \mathbf{1}\{\Delta^V(\omega | \tau, h) > 0\} f(\omega) d\omega,$$

where  $\mathbf{1}\{\cdot\}$  is the indicator function. A reform  $(\tau, h)$  is supported by a majority of the population if  $S(\tau, h) \geq \frac{1}{2}$ . We call such reforms politically feasible.

### 4.3 Types and earnings

We repeatedly invoke the function  $\tilde{y}^0 : \Omega \rightarrow \mathbb{R}_+$  where  $\tilde{y}^0(\omega) = y^*(0, 0, \omega)$  gives earnings as a function of type in the status quo. We denote the inverse of this function by  $\tilde{\omega}^0$  so that  $\tilde{\omega}^0(y)$  is the type who earns an income of  $y$  in the status quo. By the Spence-Mirrlees single crossing property the function  $\tilde{y}^0$  is weakly increasing. The existence of its inverse  $\tilde{\omega}^0$  requires in addition that, under the status quo schedule  $T_0$ , there is no bunching so that different types choose different levels of earnings.

**Assumption 1** *The function  $\tilde{y}^0$  is strictly increasing and continuous.*

It is not difficult to relax this assumption. However, taking account of bunching in the status quo requires additional steps in the formal analysis that we relegate to part C of the Appendix. This extension is relevant because empirically observed tax schedules frequently have kinks and hence give rise to bunching, see e.g. Saez (2010) and Kleven (2016). That said, we focus on a status quo without bunching in the body of the text for expositional clarity.

In much of the literature following Mirrlees (1971) types are identified with hourly wages. Our framework is consistent with this approach, but is also compatible with others. In particular, if  $\tilde{y}^0$  is a strictly increasing function then we can also identify

an individual's type with the individual's income in the status quo. This is particularly useful for empirical applications. Information on the status quo distribution of incomes is often more easily available than information on alternative measures of productive ability.

## 5 Median voter theorems for monotonic reforms

The focus on monotonic reforms enables a characterization of reforms that are politically feasible. As we show in this section, checking whether or not a reform is supported by a majority of individuals is, with some qualifications, the same as checking whether or not the taxpayer with median income is a beneficiary of the reform. We begin with an analysis of small reforms and turn to large reforms subsequently.

### 5.1 Small reforms

We say that an individual of type  $\omega$  benefits from a marginal increase of  $\tau$  if, at  $\tau = 0$ ,

$$\Delta_\tau^V(\omega | \tau, h) := \frac{d}{d\tau} V(\Delta^R(\tau, h), \tau, \omega) > 0 .$$

If this derivative is negative, we say that the individual benefits from a decrease of  $\tau$ .

**Theorem 1** *Let  $h$  be a monotonic function. The following statements are equivalent:*

1. *The median voter benefits from a small reform.*
2. *There is a majority of voters who benefit from a small reform.*

To obtain an intuitive understanding of Theorem 1, consider a policy-space in which individuals trade-off increased transfers and increased taxes. The following Lemma provides a characterization of preferences over such reforms.

**Lemma 1** *Consider a  $\tau$ - $\Delta^R$  diagram and let  $s^0(\omega)$  be the slope of a type  $\omega$  individual's indifference curve through point  $(\tau, \Delta^R) = (0, 0)$ . For any  $\omega$ ,  $s^0(\omega) = h(\tilde{y}^0(\omega))$ .*

Consider, for the purpose of illustration, a reform in the  $(\tau, y_a, y_b)$ -class. Individuals who choose earnings below  $y_a$  are not affected by the increase of tax rates. As a consequence,  $s^0(\omega) = h(\tilde{y}^0(\omega)) = 0$  which means that they are indifferent between a tax increase  $\tau > 0$  and increased transfers  $\Delta^R \geq 0$  only if  $\Delta^R = 0$ . As soon as  $\tau > 0$  and  $\Delta^R > 0$ , they are no longer indifferent, but benefit from the reform. Individuals with higher levels of income are affected by the increase of the marginal tax rate, and would be made worse off by any reform with  $\tau > 0$  and  $\Delta^R = 0$ . Keeping them indifferent requires  $\Delta^R > 0$  as reflected by the observation that  $s^0(\omega) = h(\tilde{y}^0(\omega)) > 0$ . Moreover, if  $h$  is a non-decreasing function of  $y$ , the higher an individual's income the larger is the increase in  $\Delta^R$  that is needed in order to compensate the individual for an increase of marginal tax rates. Noting that  $\tilde{y}^0(\omega)$  is a non-decreasing function of  $\omega$  by the Spence-Mirrlees single crossing property, we obtain the following Corollary to Lemma 1.

**Corollary 1** Suppose that  $h$  is a non-decreasing function of  $y$ . Then  $\omega' > \omega$ , implies

$$\tilde{y}^0(\omega) \leq \tilde{y}^0(\omega') \quad \text{and} \quad s^0(\omega) \leq s^0(\omega') .$$

Corollary 1 establishes a single-crossing property for indifference curves in a  $\tau\Delta^R$ -space, see Figure 7 (right panel). The indifference curve of a richer individual is steeper than the indifference curve of a poorer individual. Thus, if  $h$  is a non-decreasing function of  $y$ , it is more difficult to convince richer individuals that a reform that involves higher taxes and higher transfers is worthwhile. This is the driving force behind Theorem 1: if the median voter likes such a reform, then anybody who earns less will also like it so that the supporters of the reform constitute a majority. If the median voter prefers the status quo over the reform, then anybody who earns more also prefers the status quo. Then, the opponents of the reform constitute a majority.

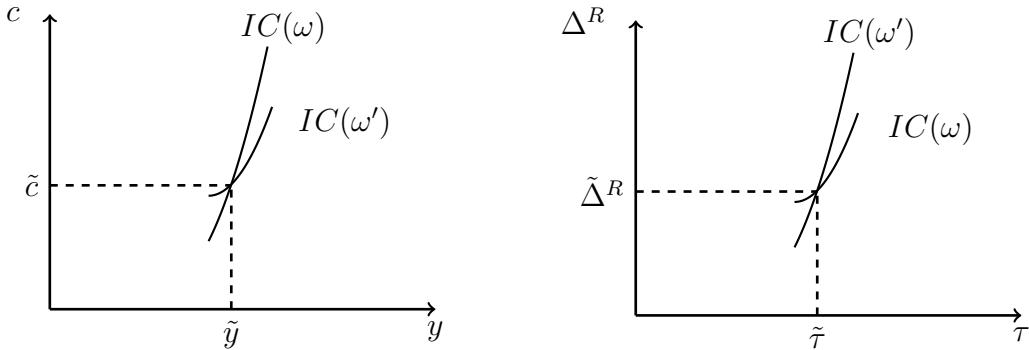


Figure 7: Single-crossing properties

Figure 7 shows indifference curves of two types  $\omega'$  and  $\omega$  with  $\omega' > \omega$ . The figure on the left illustrates the Spence-Mirrlees single-crossing property: in a  $y$ - $c$  space lower types have steeper indifference curves as they are less willing to increase their earnings in exchange for a given increase of their consumption level. The figure on the right shows indifference curves in a  $\tau\Delta^R$  space: here, lower types have flatter indifference curves indicating that they are more willing to accept an increase of marginal taxes in exchange for increased transfers.

The following Corollary uses these insights to provide a characterization of Pareto-improving reforms and of reforms that are more controversial as they come with winners and losers.

**Corollary 2** Let  $h$  be a non-decreasing function.

- (i) A small reform  $(\tau, h)$  with  $\tau > 0$  is Pareto-improving if and only if the richest individual is not made worse off, i.e. if and only if  $\Delta_\tau^V(\bar{\omega} | 0, h) \geq 0$ .
- (ii) A small reform  $(\tau, h)$  with  $\tau < 0$  is Pareto-improving if and only if the poorest individual is not made worse off, i.e. if and only if  $\Delta_\tau^V(\underline{\omega} | 0, h) \leq 0$ .

- (iii) A small reform  $(\tau, h)$  with  $\tau > 0$  benefits voters in the bottom  $x$  per cent and harms voters in the top  $1 - x$  per cent if and only if  $\Delta_\tau^V(\omega^x | 0, h) = 0$ , where  $\omega^x$  satisfies  $F(\omega^x) = x$ .

According to part (i) of Corollary 2, tax increases are Pareto-improving if and only if the individuals with top incomes benefit. According to part (ii), tax cuts are Pareto-improving if and only if they are in the interest of those with minimal income. Part (iii) characterizes a reform that involves tax increases and which splits the population into beneficiaries and opponents. If the individuals who just make it, say, to the top 10 per cent are indifferent then all individuals who belong to the bottom ninety percent are winners and individuals in the top 10 per cent are losers.

**Non-monotonic reforms.** As shown in the previous section, not all conceivable reforms are such that  $h$  is monotone for all levels of income. For such reforms we cannot prove an equivalence of support by the median voter and support by a majority of individuals. The following Proposition states a weaker result: it gives conditions under which support of the median voter is a sufficient condition for political feasibility.

**Proposition 1** Let  $\tilde{y}^{0M}$  be median income in the status quo.

1. Let  $h$  be non-decreasing for  $y \geq \tilde{y}^{0M}$ . If the median voter benefits from a small reform with  $\tau < 0$ , then it is politically feasible.
2. Let  $h$  be non-decreasing for  $y \leq \tilde{y}^{0M}$ . If the poorest voter benefits from a small reform with  $\tau < 0$ , then it is politically feasible.

The first part of Proposition 1 covers reforms that are monotonic and involve tax cuts that are more sizable for richer individuals, as under the Reagan and Bush tax cuts, see Figure 3. A way of making sure that such a reform is appealing to a majority of voters is to have the median voter among the beneficiaries. If, from the median voter's perspective, the reduced tax burden outweighs the loss of tax revenue, then everybody with above median income benefits from the reform.

The second part applies the same logic to tax cuts for low incomes. If the poorest individuals benefit from a tax cut and  $h$  is non-decreasing for below median incomes, then individuals with incomes closer to the median benefit even more. Individuals with below median incomes then constitute a majority in favor of the reform. This case applies, in particular, to reforms so that  $T_1 - T_0$  is negative and decreasing for incomes below a threshold  $\hat{y}$ , as for the reforms by Clinton and Obama, see Figure 5. In this case, political feasibility is ensured by putting the threshold (weakly) above the median, so that everybody with below median income is a beneficiary of the reform.

Heathcote et al. (2017) focus on income tax systems in a class that has a constant rate of progressivity. A tax system in this class takes the form  $T(y) = y - \lambda y^{1-\rho}$  where the parameter  $\rho$  is the measure of progressivity and the parameter  $\lambda$  affects the level of

taxation. An increase of the rate of progressivity can be viewed as a reform  $(\tau, h)$  so that  $h$  is increasing for incomes above a threshold  $\hat{y}$  and decreasing for incomes below  $\hat{y}$ .<sup>15</sup> As a consequence, such a reform is monotonic either below or above the median.

The following Proposition that we state without proof demonstrates that the same logic applies to reforms that involve higher taxes and higher transfers - rather than lower taxes and lower transfers.

**Proposition 2** *Let  $\tilde{y}^{0M}$  be median income in the status quo.*

1. *Let  $h$  be non-decreasing for  $y \leq \tilde{y}^{0M}$ . If the median voter benefits from a small reform with  $\tau > 0$ , then it is politically feasible.*
2. *Let  $h$  be non-decreasing for  $y \geq \tilde{y}^{0M}$ . If the richest voter benefits from a small reform with  $\tau > 0$ , then it is politically feasible.*

## 5.2 Large reforms

We can evaluate the gains or losses from large reforms simply by integrating over the gains and losses from small reforms since

$$\Delta^V(\omega | \tau, h) = \int_0^\tau \Delta_\tau^V(\omega | s, h) ds . \quad (3)$$

The following Lemma provides a characterization of the function  $\Delta_\tau^V$ . The Lemma does not require that a small reform is a departure from the status quo schedule with  $\tau = 0$ . It allows for the possibility that  $\tau$  has already been raised from 0 to some value  $\tau' > 0$  and considers the implications of a further increase of  $\tau$ .

**Lemma 2** *For all  $\omega$ ,*

$$\Delta_\tau^V(\omega | \tau', h) = \tilde{u}_c^1(\omega) (\Delta_\tau^R(\tau', h) - h(\tilde{y}^1(\omega))) , \quad (4)$$

where  $\tilde{u}_c^1(\omega) := u_c(c_0 + \Delta^R(\tau', h) + \tilde{y}^1(\omega) - T_1(\tilde{y}^1(\omega)), \tilde{y}^1(\omega), \omega)$  is a shorthand for the marginal utility of consumption that a type  $\omega$  individual realizes after the reform and  $\tilde{y}^1(\omega) := y^*(\Delta^R(\tau', h), \tau', \omega)$  is the corresponding earnings level.

If  $h$  is a monotonic function, then a reform's impact on available consumption, as measured by  $\Delta_\tau^R(\tau', h) - h(\tilde{y}^1(\omega))$ , is a monotonic functions of  $\omega$ . These observations enable us to provide an extension of Theorem 1 to large reforms: if every marginal increase of  $\tau$  yields a gain  $\Delta_\tau^R(\tau', h) - h(\tilde{y}^1(\omega))$  that is larger for less productive types, then a discrete change of  $\tau$  also yields a gain that is larger for less productive types. As a consequence, if the median voter benefits if  $\tau$  is raised from zero to some level  $\tau' > 0$ , then anyone

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<sup>15</sup>Formally, let the status quo be a tax system  $T_0$  with  $T_0(y) = y - \lambda_0 y^{1-\rho_0}$ . Consider the move to a new tax system  $T_1$  with  $T_1(y) = y - \lambda_1 y^{1-\rho_1}$  and  $\rho_1 > \rho_0$ . Then  $\tau h(y) = T_1(y) - T_0(y) = \lambda_0 y^{1-\rho_0} - \lambda_1 y^{1-\rho_1}$ . This expression is strictly increasing in  $y$  for  $y > \hat{y} := \left( \frac{\lambda_1(1-\rho_1)}{\lambda_0(1-\tau_0)} \right)^{\frac{1}{\rho_1-\rho_0}}$  and strictly decreasing for  $y < \hat{y}$ .

with below-median income will also benefit. If the median voter does not benefit, then anyone with above-median income will also oppose the reform. Consequently, a reform is politically feasible if and only if it is in the median voter's interest.

**Proposition 3** *Let  $h$  be a monotonic function.*

1. *Consider a reform  $(\tau, h)$  so that for all  $\tau' \in (0, \tau)$ ,  $\Delta_\tau^V(\omega^M | \tau', h) > 0$ , then this reform is politically feasible.*
2. *Consider a reform  $(\tau, h)$  so that for all  $\tau' \in (0, \tau)$ ,  $\Delta_\tau^V(\omega^M | \tau', h) < 0$ , then this reform is politically infeasible.*

Propositions 1 and 2 also extend to large reforms with similar qualifications.

### 5.3 Welfare implications of reforms

If we use the utility function  $u$  for an interpersonal comparison of utilities – as is standard in the literature on optimal taxation in the tradition of Mirrlees (1971) – we can, for instance, compare the utility gains that “the poor” realize if the tax system is reformed to those that are realized by “middle-class” or “rich” voters.

**Lemma 3** *For any pair  $(\omega, \omega')$  with  $\omega < \omega'$ ,  $\tilde{u}_c^1(\omega) \geq \tilde{u}_c^1(\omega')$ .*

According to Lemma 3, individuals with low types – who have less income than high types because of the Spence-Mirrlees single crossing property – are also more deserving in the sense that they benefit more from additional consumption. Utilitarian welfare would therefore increase if “the rich” consumed less and “the poor” consumed more. Together with Lemma 2, Lemma 3 also enables an interpersonal comparison of gains and losses from large tax reforms provided that  $h$  is non-decreasing: consider a type  $\omega'$  and a reform  $(\tau, h)$  so that for all  $\tau' \in (0, \tau)$ ,  $\Delta_\tau^V(\omega' | \tau', h) > 0$ . The utility gain of a type  $\omega'$  individual is given by

$$\begin{aligned}\Delta^V(\omega' | \tau, h) &= \int_0^\tau \Delta_\tau^V(\omega' | s, h) ds \\ &= \int_0^\tau \tilde{u}_c^1(\omega') (\Delta_\tau^R(s, h) - h(\tilde{y}^1(\omega'))) ds.\end{aligned}$$

Now consider an individual with type  $\omega < \omega'$ . Then  $\tilde{u}_c^1(\omega) \geq \tilde{u}_c^1(\omega')$  and  $\tilde{y}^1(\omega) \leq \tilde{y}^1(\omega')$  imply

$$\int_0^\tau \tilde{u}_c^1(\omega) (\Delta_\tau^R(s, h) - h(\tilde{y}^1(\omega))) ds \geq \int_0^\tau \tilde{u}_c^1(\omega') (\Delta_\tau^R(s, h) - h(\tilde{y}^1(\omega'))) ds$$

and hence  $\Delta^V(\omega | \tau, h) \geq \Delta^V(\omega' | \tau, h)$ , i.e. low types realize larger utility gains than high types. The following Proposition summarizes this discussion.

**Proposition 4** *Consider a reform  $(\tau, h)$  so that for all  $\tau' \in (0, \tau)$ ,  $\Delta_\tau^V(\omega' | \tau', h) > 0$ . Then  $\omega \leq \omega'$  implies  $\Delta^V(\omega | \tau, h) \geq \Delta^V(\omega' | \tau, h)$ .*

According to Proposition 4, the welfare gains from a reform are a monotonic: if middle income types benefit from a reform that involves higher taxes, then low income types realize even larger benefits.

## 6 Detecting politically feasible reforms

By our median voter theorems, in order to understand whether there are politically feasible reforms of a tax system we need to understand whether or not it can be reformed in a way that makes the voter with median income better off. But how do we tell whether or not a given tax system admits reforms that are in the median voter's interest? In this section, we first provide a characterization of such reforms (Theorem 2), and then develop a sufficient statistics approach that makes it possible to identify them empirically. Our analysis enable us to identify tax schedules that can be reformed in such a way that the requirements of political feasibility and welfare improvements are both met. Put differently, it makes it possible to identify tax schedules that are inefficient in the sense that the scope for politically feasible welfare improvements has not been exhausted. Throughout, we focus on simple reforms and provide sufficient conditions under which such reforms are politically feasible and/ or welfare improving.

### 6.1 Pareto-efficient tax systems and politically feasible reforms

Theorem 2 below provides a characterization of simple reforms that are politically feasible. Before we state the theorem, we introduce some terminology. A tax schedule  $T_0$  is Pareto-efficient if there is no Pareto-improving reform. If it is Pareto-efficient, then for all  $y_a$  and  $y_b$ ,

$$y_b - y_a \geq \Delta_\tau^R(0, y_a, y_b) \geq 0 ,$$

where  $\Delta_\tau^R(0, y_a, y_b)$  is the marginal change in tax revenue that results as we slightly rise  $\tau$  above 0, while keeping  $y_a$  and  $y_b$  fix. If we had instead  $\Delta_\tau^R(0, y_a, y_b) < 0$ , then a small reform  $(\tau, y_a, y_b)$  with  $\tau < 0$  would be Pareto-improving: all individuals would benefit from increased transfers and individuals with an income above  $y_a$  would, in addition, benefit from a tax cut. With  $y_b - y_a < \Delta_\tau^R(0, y_a, y_b)$ , a small reform  $(\tau, y_a, y_b)$  with  $\tau > 0$  would be Pareto-improving: all individuals would benefit from increased transfers. Individuals with an income above  $y_a$  would not benefit as much because of increased marginal tax rates. They would still be net beneficiaries because the increase of the tax burden was dominated by the increase of transfers. Under a Pareto-efficient tax system there is no scope for such reforms. We say that  $T_0$  is an *interior Pareto-optimum* if, for all  $y_a$  and  $y_b$ ,

$$y_b - y_a > \Delta_\tau^R(0, y_a, y_b) > 0 .$$

**Theorem 2** *Suppose that  $T_0$  is an interior Pareto-optimum.*

- (i) *For  $y_0 < \tilde{y}^{0M}$ , there is a simple reform  $(\tau, y_a, y_b)$  with  $y_a < y_0 < y_b$  and  $\tau < 0$  that is politically feasible.*

- (ii) For  $y_0 > \tilde{y}^{0M}$ , there is a simple reform  $(\tau, y_a, y_b)$  with  $y_a < y_0 < y_b$  and  $\tau > 0$  that is politically feasible.

According to Theorem 2, if the status quo is an interior Pareto-optimum, reforms that involve a shift towards lower marginal tax rates for below median incomes and reforms that involve a shift towards higher marginal tax rates for above median incomes are politically feasible. With an interior Pareto-optimum, a lowering of marginal taxes for incomes between  $y_a$  and  $y_b$  comes with a loss of tax revenue. For individuals with incomes above  $y_b$  the reduction of their tax burden outweighs the loss of transfer income so that they benefit from such a reform. If  $y_b$  is smaller than the median income, this applies to all individuals with an income (weakly) above the median. Hence, the reform is politically feasible. By the same logic, an increase of marginal taxes for incomes between  $y_a$  and  $y_b$  generates additional tax revenue. If  $y_a$  is chosen so that  $y_a \geq \tilde{y}^{0M}$ , only individuals with above median income have to pay higher taxes with the consequence that all individuals with below median income, and hence a majority, benefit from the reform.

Theorem 2 allows us to identify politically feasible reforms. To see whether a given status quo in tax policy admits politically feasible reforms, we simply need to check whether the status quo is an interior Pareto-optimum. This in turn requires a characterization of Pareto bounds for tax rates. In the following, we will provide such a characterization. It takes the form of sufficient statistics formulas for the Pareto bounds that are associated with a given status quo in tax policy.

## 6.2 An upper bound for marginal tax rates

If a tax system has rates that are inefficiently high, a Pareto-improving tax cut is possible; i.e., there is a simple reform  $(\tau, y_a, y_b)$  with  $\tau < 0$  so that

$$\Delta_\tau^R(\tau, y_a, y_b) \leq 0 . \quad (5)$$

Proposition 5 below provides sufficient conditions for the existence of a reform that satisfies (5). More specifically, it states a separate condition for every level of income. If, at income level  $y' = \tilde{y}^0(\omega')$ , the marginal tax rate  $T'_0(y')$  exceeds an upper bound  $\mathcal{D}^{up}(y')$ , formally defined below, then there exists a Pareto-improving tax cut for incomes close to  $y'$ . The function  $\mathcal{D}^{up}$  is therefore the upper Pareto bound for marginal tax rates.

The upper bound is shaped by the taxpayers' behavioral responses as captured by the partial derivatives of the function  $y^*$  and by the distributions of types and earnings. We denote by  $y_e^*(0, 0, \omega)$  the behavioral response of a type  $\omega$ -individual to increased transfers. This expression equals 0 if there are no income effects. The behavioral response to increased marginal tax rates is denoted by  $y_\tau^*(0, 0, \omega)$ . It captures the substitution effect associated with a change of marginal tax rates. For types close to  $\omega'$ , this expression is negative, as we also show formally in the proof of Proposition 5: earnings go up in response to a tax cut. Finally,  $y_\omega^*(0, 0, \omega)$  is the marginal change in income associated

with a higher type. By the Spence-Mirrlees single crossing property and our assumption that there is no bunching, this expression is strictly positive.

Income effects interact with marginal tax rates. They enter the upper Pareto bound  $\mathcal{D}^{up}$  via the expression

$$\tilde{I}_0(\omega') := E [T'_0(\tilde{y}^0(\omega')) y_e^*(0, 0, \omega') \mid \omega \geq \omega'] . \quad (6)$$

The sign of this expression depends on the sign of marginal tax rates in the status quo. If the status quo is a first best schedule with marginal tax rates of zero everywhere, then  $\tilde{I}_0(\omega') = 0$ . If, by contrast, marginal tax rates are positive for types above  $\omega'$ , then  $\tilde{I}_0(\omega_0) < 0$ . The term  $-\tilde{I}_0(\omega_0) > 0$  then captures that individuals with an income above  $y_b$  generate higher earnings after a  $(\tau, y_a, y_b)$ -reform that involves a tax increase,  $\tau > 0$ .

Finally, the Pareto bound depends on the inverse hazard rate of the type distribution at  $\omega'$ ,  $\frac{1-F(\omega')}{f(\omega')}$ . The inverse hazard rate gives the ratio of individuals with types above  $\omega'$ ,  $1 - F(\omega')$ , whose tax payments go down in response to a cut of marginal tax rates, to those whose earnings expand because of the substitution effect, as measured by the density at  $\omega'$ ,  $f(\omega')$ . The smaller this ratio, the tighter the Pareto bound.

**Proposition 5** *Let*

$$\mathcal{D}^{up}(y') := -\frac{1-F(\tilde{\omega}^0(y'))}{f(\tilde{\omega}^0(y'))} \left(1 - \tilde{I}_0(\tilde{\omega}^0(y'))\right) \frac{y_\omega^*(0, 0, \tilde{\omega}^0(y'))}{y_\tau^*(0, 0, \tilde{\omega}^0(y'))} .$$

*Suppose that there is an income level  $y'$  so that  $T'_0(y') > \mathcal{D}^{up}(y')$ . Then there exists a revenue-increasing reform  $(\tau, y_a, y_b)$  with  $\tau < 0$ , and  $y_a < y' < y_b$ .*

If there are no income effects, the upper bound  $\mathcal{D}^{up}$  coincides with the revenue-maximizing or Rawlsian income tax schedule for all levels of income where the latter does not give rise to bunching. With income effects this is not generally the case. The reason is that  $\mathcal{D}^{up}$  depends on the status quo schedule  $T_0$  via  $\tilde{I}_0$ . We discuss below how one can use  $\mathcal{D}^{up}$  to detect inefficiently high tax rates in empirical work. We first sketch its derivation.

**Sketch of Proof.** The change in tax revenue  $\Delta^R(\tau, y_a, y_b)$  satisfies the fixed point equation

$$\Delta^R(\tau, y_a, y_b) = \int_{\underline{\omega}}^{\bar{\omega}} \{T_1(y^*(\Delta^R(\tau, y_a, y_b), \tau, \omega)) - T_0(y^*(0, 0, \omega))\} f(\omega) d\omega .$$

Starting from this equation, we use the implicit function theorem and our analysis of behavioral responses to simple reforms in Appendix A to derive an expression for  $\Delta_\tau^R(\tau, y_a, y_b)$ .<sup>16</sup> We then evaluate this expression at the tax policy that prevails in the status quo, i.e. for  $\tau = 0$ , and obtain

$$\Delta_\tau^R(0, y_a, y_b) = \frac{1}{1 - I_0} \mathcal{R}(y_a, y_b),$$

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<sup>16</sup>We are effectively computing a Gâteaux derivative of tax revenue in direction  $h$ .

where  $I_0 := \tilde{I}_0(\underline{\omega})$  is our measure of income effects applied to the population at large. The multiplier  $\frac{1}{1-I_0}$  captures the behavioral responses due to the income effects that come from changing the intercept of the consumption schedule. Further,

$$\begin{aligned}\mathcal{R}(y_a, y_b) &= \int_{\tilde{\omega}^0(y_a)}^{\tilde{\omega}^0(y_b)} \{T'_0(y^*(0, 0, \omega)) y_\tau^*(0, 0, \omega) + y^*(0, 0, \omega) - y_a\} f(\omega) d\omega \\ &\quad + (y_b - y_a) \left\{ 1 - F(\tilde{\omega}^0(y_b)) - \int_{\tilde{\omega}^0(y_b)}^{\tilde{\omega}} T'_0(y^*(0, 0, \omega)) y_e^*(0, 0, \omega) f(\omega) d\omega \right\}.\end{aligned}$$

The first term on the right hand side gives the change in tax revenue that comes from individuals with incomes between  $y_a$  and  $y_b$ , driven by the behavioral response to the change in marginal tax rates and the mechanical effect according to which these individuals pay more taxes on incomes exceeding  $y_a$ . The second term gives the change in tax revenue that comes from individuals with incomes exceeding  $y_b$ , again consisting of a mechanical effect and the income effects that are associated with the increase of the tax burden by  $y_b - y_a$ .

Note that  $\Delta_\tau^R(0, y_a, y_a) = \frac{1}{1-I_0} \mathcal{R}(y_a, y_a) = 0$ . A small increase of marginal tax rates does not generate additional revenue if applied only to a null set of agents. However, if the cross derivative  $\Delta_{\tau y_b}^R(0, y_a, y_a) = \frac{1}{1-I_0} \mathcal{R}_{y_b}(y_a, y_a)$  is negative, then  $\Delta_\tau^R(0, y_a, y_b)$  turns negative, if starting from  $y_a = y_b$ , we marginally increase  $y_b$ . Straightforward computations yield:

$$\begin{aligned}\mathcal{R}_{y_b}(y_a, y_a) &= \left( \frac{d\tilde{\omega}^0(y_b)}{dy_b} \right)_{|y_b=y_a} T'_0(y^*(0, 0, \tilde{\omega}^0(y_a))) y_\tau^*(0, 0, \tilde{\omega}^0(y_a)) f(\tilde{\omega}^0(y_a)) \\ &\quad + (1 - \tilde{I}_0(\tilde{\omega}^0(y_a))) (1 - F(\tilde{\omega}^0(y_a))).\end{aligned}$$

Hence, if this expression is negative we can increase tax revenue by decreasing marginal tax rates in a neighborhood of  $y_a$ . Using

$$y_\omega^*(0, 0, \tilde{\omega}^0(y_a))^{-1} = \left( \frac{d\tilde{\omega}^0(y_b)}{dy_b} \right)_{|y_b=y_a},$$

the statement  $\mathcal{R}_{y_b}(y_a, y_a) < 0$  is easily seen to be equivalent to  $T'_0(y_a) > \mathcal{D}^{up}(y_a)$ , as claimed in the Proposition 5 for  $y_a = y'$ .

**From theory to data.** As we will now demonstrate, the characterization of the upper Pareto bound in Proposition 5 can be used to detect whether a given tax system has tax rates that are inefficiently high. To this end, we will first recast the upper Pareto bound in terms of sufficient statistics that can easily be related to data. We will then provide concrete empirical illustrations.

Sufficient statistics approaches characterize the effects of tax policy by means of elasticities that describe behavioral responses.<sup>17</sup> This route is also available here. To see this, define the elasticity of type  $\omega$ 's earnings with respect to the net-of-tax rate  $1 - T'(\cdot)$  and with respect to the skill index  $\omega$ , respectively, as

$$\tilde{\varepsilon}^0(\omega') \equiv \frac{1 - T'_0(\tilde{y}^0(\omega'))}{\tilde{y}^0(\omega')} y_\tau^*(0, 0, \omega'), \quad \text{and} \quad \tilde{\alpha}^0(\omega') \equiv \frac{\omega'}{\tilde{y}^0(\omega')} y_\omega^*(0, 0, \omega'). \quad (7)$$

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<sup>17</sup>See, e.g., Saez (2001), Chetty (2009), or Kleven (2018).

We can also write

$$\tilde{I}_0(\omega') = E \left[ T'_0(\tilde{\eta}^0(\omega)) \frac{\tilde{y}^0(\omega)}{c_0} \tilde{\eta}^0(\omega) \mid \omega \geq \omega' \right], \quad (8)$$

where  $\tilde{\eta}^0(\omega)$  is the elasticity of type  $\omega$ 's earnings with respect to the intercept of the consumption schedule.

**Corollary 3** *Let*

$$\tilde{D}^{up}(y') := -\frac{1 - F(\tilde{\omega}^0(y'))}{f(\tilde{\omega}^0(y')) \tilde{\omega}^0(y')} \left(1 - \tilde{I}_0(\tilde{\omega}^0(y'))\right) \frac{\tilde{\alpha}^0(\tilde{\omega}^0(y'))}{\tilde{\varepsilon}^0(\tilde{\omega}^0(y'))}.$$

Suppose there is an income level  $y'$  so that  $\frac{T'_0(y')}{1-T'_0(y')} > \tilde{D}^{up}(y')$ . Then there exists a tax-revenue-increasing reform  $(\tau, y_a, y_b)$  with  $\tau < 0$ , and  $y_a < y' < y_b$ .

The following remark that we state without proof clarifies the implications of frequently invoked functional form assumptions for the sufficient statistics formula in Corollary 3.

**Remark 1** *If the utility function  $u$  takes the special form  $u(c, y, \omega) = U(c, \frac{y}{\omega})$ , then  $\frac{y}{\omega}$  can be interpreted as labor supply in hours and the ratio  $\frac{\tilde{\alpha}^0(\omega)}{\tilde{\varepsilon}^0(\omega)}$  can be written as  $\frac{\tilde{\alpha}^0(\omega)}{\tilde{\varepsilon}^0(\omega)} = -\left(1 + \frac{1}{\epsilon^0(\omega)}\right)$ , where  $\epsilon^0(\omega)$  is the elasticity of hours worked with respect to the net wage rate, for an individual with wage rate  $\omega$ . If preferences are such that  $u(c, y, \omega) = c - \left(\frac{y}{\omega}\right)^{1+\frac{1}{\epsilon}}$  for a fixed parameter  $\epsilon$ , then, for all  $\omega$ ,  $\frac{\tilde{\alpha}^0(\omega)}{\tilde{\varepsilon}^0(\omega)} = -\left(1 + \frac{1}{\epsilon}\right)$ .*

The sufficient statistics formula in Corollary 3 is based on a generic notion of an individual's type. For an empirical application, one needs to specify what is meant by a type and a cross-section distribution of types in terms of data. One possible approach is to identify an individual's type with the individual's income in the status quo. On the assumption that the status quo does not give rise to bunching, income in the status quo is a monotonic transform and therefore also an admissible representation of an individual's type. We will employ this approach in all empirical applications that we provide.<sup>18</sup> For simplicity, we also frequently invoke the assumption that preferences take the quasi-linear form  $u(c, y, \omega) = c - \left(\frac{y}{\omega}\right)^{1+\frac{1}{\epsilon}}$  introduced in Remark 1. The test whether marginal taxes for incomes close to  $y'$  are inefficiently high then simply requires to check whether or not the ratio  $\frac{T'_0(y')}{1-T'_0(y')}$  exceeds

$$\tilde{D}^{up}(y') = \frac{1 - F_Y(y')}{f_Y(y')} \frac{1}{\epsilon},$$

where  $F_Y$  is the *cdf* and  $f_Y$  the density associated with the status quo distribution of incomes. The parameter  $\epsilon$  then also admits an interpretation as the elasticity of taxable income (ETI) with respect to the net-of-tax rate. There is a rich literature on

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<sup>18</sup>An extended analysis that allows for bunching can be found in Appendix C.

the estimation of this elasticity.<sup>19</sup> Obviously, the estimate affects the tightness of the Pareto-bound. The smaller the elasticity, the more permissive is the Pareto bound and the more difficult it is to detect tax rates that are inefficiently high. For our empirical illustrations we are not taking a stance on what the correct estimate is. Instead, we draw the Pareto bounds for hypothetical ETI estimates with the property that the status quo tax schedule comes close to the bound. As a consequence, ETI estimates that exceed such a cutoff imply a violation of Pareto efficiency, whereas lower estimates imply that marginal tax rates in the status quo are not inefficiently high.

For illustration, Figure 8 plots the sufficient statistic  $\tilde{\mathcal{D}}^{up}$  and the values of  $\frac{T'_0}{1-T'_0}$  that are implied by the US tax systems in the years 2012 and 2013.<sup>20</sup> The most significant change of the tax code under the US tax reforms in 2012 and 2013 is an increase of the top tax rate, relevant for incomes above \$400000, from 35% to 39.6%.<sup>21</sup> The figure on the left assume an ETI of 1.2, the figures on the right an ETI of 1.4. Thus, the cutoff is around 1.4. For higher estimates, the 2012 US tax system admits Pareto-improving reforms, for lower estimates it is an interior Pareto-optimum.

Similar graphs for the reforms that took place in 2012 and 2013 in France and the United Kingdom can be found in Appendix D.<sup>22</sup> The reform in the United Kingdom involves a reduction of the top tax rate from 50 to 45 percent. The ETI cutoff is much lower than the one previously found for the United States. For instance, with an ETI of 0.4 the reform is diagnosed as Pareto-improving, bringing excessive tax rates on the rich back to the range of interior Pareto optima. For an ETI of 0.6 the reform is neither Pareto-improving, nor politically feasible. Lower taxes on the rich are then not in the interest of a majority of taxpayers, but only in the interest of the minority of taxpayers who pay the top rate. Like the United States, France also increased the taxes on high incomes in 2012 and 2013. For France, the critical value of the ETI is around 0.8.

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<sup>19</sup>See e.g. Saez, Slemrod and Giertz (2012) for a survey or Blomquist and Newey (2017) for a recent paper on the methodology of ETI estimation. Further recent references on ETI estimates include Saez (2017) and Mertens and Olea (2018). Cabannes, Houtré and Landais (2014) present estimates based on French data. Adam, Browne, Phillips and Roantree (2017) use data from the United Kingdom.

<sup>20</sup>Data on the distribution of taxable income is taken from the World Wealth and Income Data base. The database can be accessed on [wid.world](http://wid.world).

<sup>21</sup>See Saez (2017) for a detailed description of all changes in the tax code.

<sup>22</sup>Data on the distributions of taxable income for France is again taken from the World Wealth and Income Data base. For the United Kingdom we use data provided by Her Majesty's Revenue & Customs (HMRC).

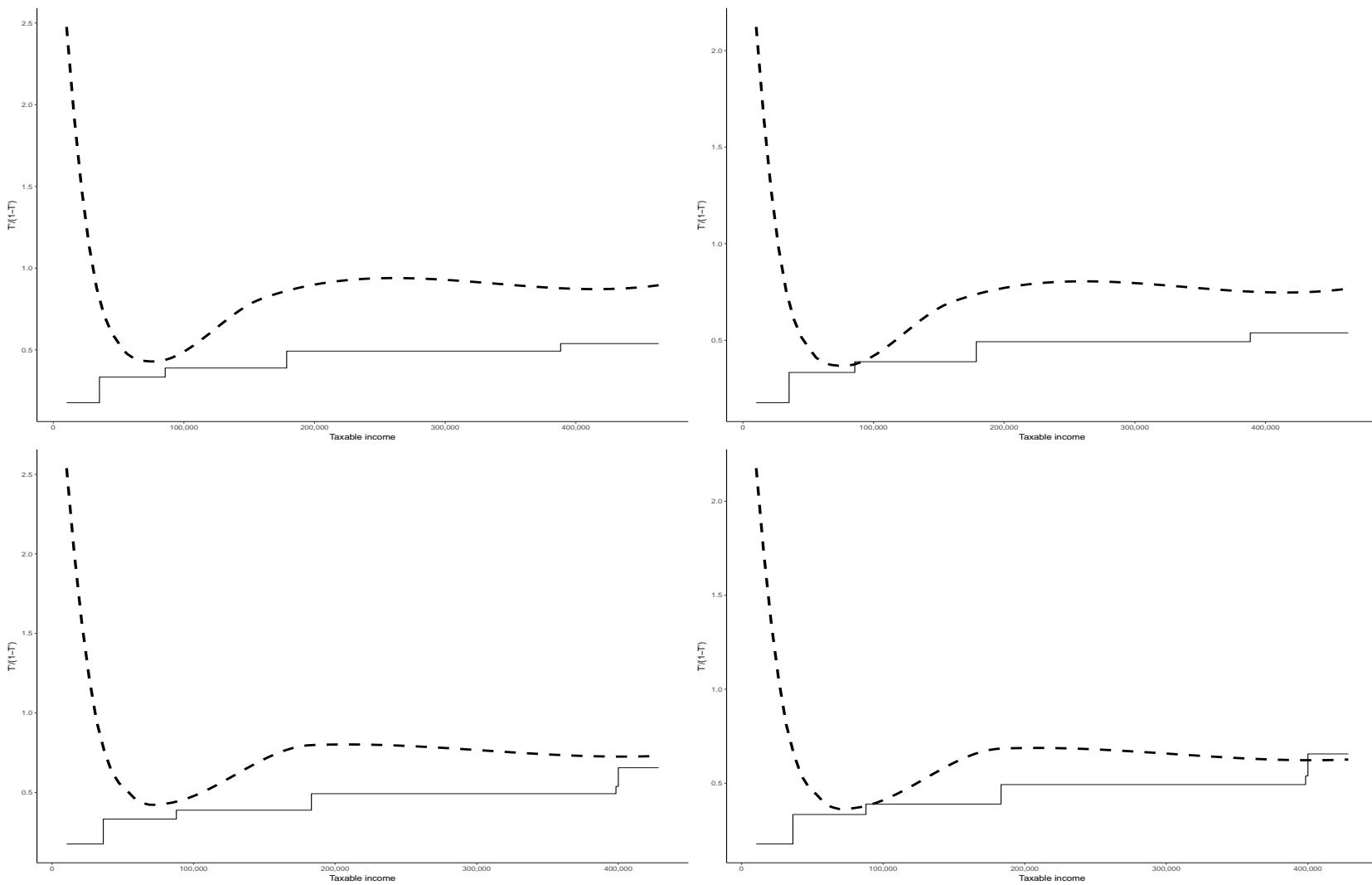


Figure 8: Upper Pareto bounds for the US income tax in 2012 and 2013

Figure 8 relates the upper Pareto bounds  $\tilde{D}^{up}$  (dashed line) to the US income tax system in 2012 (upper half) and 2013 (lower half). The figures on the left are drawn for an ETI of 1.2, the figures on the right for an ETI of 1.4.

### 6.3 A lower bound for marginal tax rates

The following Proposition derives conditions under which reforms  $(\tau, y_a, y_b)$  with  $\tau > 0$  are Pareto-improving. The function  $\mathcal{D}^{low}$ , defined below, is a lower bound for marginal tax rates. If marginal tax rates are below, then an increase is Pareto-improving. It is the counterpart to the upper bound  $\mathcal{D}^{up}$  in Proposition 5.

**Proposition 6** *Let*

$$\mathcal{D}^{low}(y) := \frac{1}{f(\tilde{\omega}^0(y))} \left\{ \left(1 - \tilde{I}_0(\tilde{\omega}^0(y))\right) F(\tilde{\omega}^0(y)) + \left(\tilde{I}_0(\tilde{\omega}^0(y)) - I_0\right) \right\} \frac{y_\omega^*(0, 0, \tilde{\omega}^0(y))}{y_\tau^*(0, 0, \tilde{\omega}^0(y))}.$$

*Suppose that there is an income level  $y'$  such that  $T'_0(y') < \mathcal{D}^{low}(y')$ . Then there exists a Pareto-improving reform  $(\tau, y_a, y_b)$  with  $\tau > 0$ , and  $y_a < y_0 < y_b$ .*

With  $I_0 > 0$ ,  $\mathcal{D}^{low}(y)$  is negative for low incomes. Thus,  $\mathcal{D}^{low}$  can be interpreted as a Pareto-bound on earnings subsidies. If those subsidies imply marginal tax rates lower than those stipulated by  $\mathcal{D}^{low}$ , then a reduction of these subsidies is Pareto-improving.

The following Corollary contains sufficient statistics for Pareto-improving tax increases. It is the counterpart to Corollary 3.

**Corollary 4** *Let*

$$\tilde{\mathcal{D}}^{low}(y) := \frac{1}{f(\tilde{\omega}^0(y)) \tilde{\omega}^0(y)} \left\{ \left(1 - \tilde{I}_0(\tilde{\omega}^0(y))\right) F(\tilde{\omega}^0(y)) + \left(\tilde{I}_0(\tilde{\omega}^0(y)) - I_0\right) \right\} \frac{\tilde{\alpha}^0(\tilde{\omega}^0(y))}{\tilde{\varepsilon}^0(\tilde{\omega}^0(y))}.$$

*Suppose that there is an income level  $y'$  such that  $\frac{T'_0(y')}{1-T'_0(y')} < \tilde{\mathcal{D}}^{low}(y')$ . Then there exists a Pareto-improving reform  $(\tau, y_a, y_b)$  with  $\tau > 0$ , and  $y_a < y' < y_b$ .*

Again, these sufficient statistics lend themselves to an empirical test of whether a given tax system has marginal tax rates that are inefficiently low. For preferences that take the quasi-linear form  $u(c, y, \omega) = c - (\frac{y}{\omega})^{1+\frac{1}{\epsilon}}$ , the lower Pareto bound is given by

$$\tilde{\mathcal{D}}^{low}(y') = -\frac{F_Y(y')}{f_Y(y')} \frac{1}{\epsilon},$$

where  $F_Y$  is the *cdf* and  $f_Y$  the density associated with the status quo distribution of incomes, and  $\epsilon$  is the ETI with respect to the net-of-tax rate.

**An empirical illustration.** Figure 9 relates the sufficient statistic  $\tilde{\mathcal{D}}^{low}$  to the US income tax in the years 2012 and 2013. The figure assumes an ETI value of 1.2.<sup>23</sup> As the figures illustrate,  $\tilde{\mathcal{D}}^{low}$  is decreasing for incomes exceeding a threshold. Marginal tax rates are non-decreasing for incomes exceeding another threshold. Hence, a violation of Pareto-efficiency can be detected for low incomes, if at all. The figure therefore focusses on low and middle incomes.

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<sup>23</sup>Similar Figures for France and the United Kingdom can be found in part D of the Appendix.

The left column of Figure 9 constructs the tax schedule based on the information contained in the OECD database. The OECD, however, does not include information on earnings subsidies. We emphasized before that the lower Pareto bound can be used to evaluate the efficiency of earnings subsidies – such as the Earned Income Tax Credit (EITC) in the United States – that seek to encourage labor force participation.<sup>24</sup> The right column therefore uses the more detailed information provided by the NBER TAXSIM database and shows the schedule that applies to a single without dependents who is eligible for the EITC program. Figure 9 shows marginal tax rates that are not inefficiently low. While the bounds are drawn for an ETI of 1.2, any plausible assumption about this elasticity would support this conclusion.<sup>25</sup>

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<sup>24</sup>The Working Tax Credit in the UK, or the Prime d’activité in France are similar programs.

<sup>25</sup>An ETI as high as 25 would be needed for a violation of Pareto-efficiency.

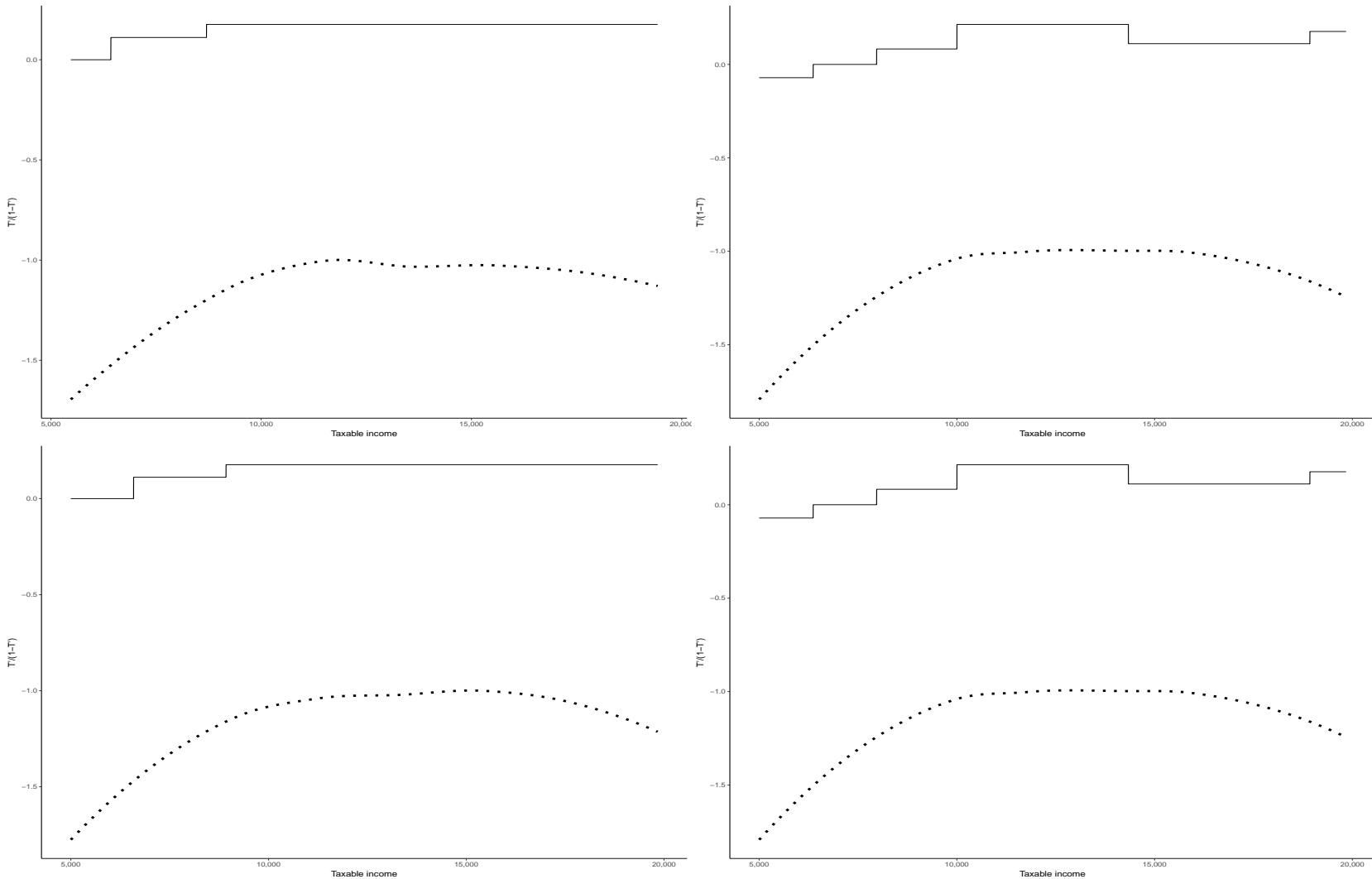


Figure 9: Lower Pareto bounds for the US income tax in 2012 and 2013

Figure 9 relates the lower Pareto bounds  $\tilde{D}^{low}$  (dotted line) to the US income tax system in 2012 (first row) and 2013 (second row) for an ETI of 1.2. The figures on the left are drawn for the statutory schedule taken from the OECD database and the figures on the right represent the full schedule with earning subsidies (EITC) for singles without dependents taken from the NBER TAXSIM database.

## 6.4 Politically feasible reforms

Propositions 5 and 6 characterize Pareto bounds for marginal tax rates. By Theorem 2, for a status quo such that marginal tax rates are between those bounds, tax increases for above median incomes and tax cuts for below median incomes are politically feasible. The following Proposition summarizes these findings.

### Proposition 7

1. Let  $y' < \tilde{y}^{0M}$  and  $T'_0(y') > \mathcal{D}^{low}(y')$ . Then there is a politically feasible reform  $(\tau, y_a, y_b)$  with  $\tau < 0$ , and  $y_a < y' < y_b$ .
2. Let  $y' > \tilde{y}^{0M}$  and  $T'_0(y') < \mathcal{D}^{up}(y')$ . Then there is a politically feasible reform  $(\tau, y_a, y_b)$  with  $\tau > 0$ , and  $y_a < y' < y_b$ .

According to Proposition 7 political economy forces push for low marginal tax rates for below median incomes and for high marginal tax rates for above median incomes. Any attempt to move from the low marginal tax rates to the high marginal tax rates in a continuous fashion will imply a strong increase of marginal rates for incomes in a neighborhood of the median. This increase is the price to be paid for having marginal tax rates close to  $\mathcal{D}^{low}$  for incomes below the median and for having marginal tax rates close to  $\mathcal{D}^{up}$  for incomes above the median. This suggests an explanation for the observation that some real-world tax schedules are very steep for incomes close to the median.<sup>26</sup> Figure 10 presents the tax schedules that are relevant for singles without dependents for the United States and France in 2012, respectively, taking account of the earnings subsidies that apply for this group. There is a region where the earnings subsidies are phased out with the implication of a very pronounced increase of marginal tax rates concentrated in a narrow range of incomes.<sup>27</sup> Moreover, the steep region is, by and large, in the vicinity of median income. In 2012 the median income for the equal-split distribution is around 18000 euros in France and \$24200 in the US (*wid.world* database).<sup>28</sup>

## 6.5 Politically feasible welfare-improvements

We now clarify the relation between politically feasible reforms and welfare-improving reforms. We are particularly interested in identifying reforms that are both politically

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<sup>26</sup>Germans refer to this as the “Mittelstandsbauch” (*middle class belly*) in the income tax schedule, for complementary evidence from the Netherlands see Jacobs et al. (2017).

<sup>27</sup>The earnings subsidies for other groups, such as e.g. single mothers with children, are larger than those for singles without dependents with the implication that the phase-out region is even steeper for these groups.

<sup>28</sup>These observations are not sensitive to the notion of median income that is employed. For the US, median individual income is \$21,157 and median household income \$50,348 in 2012 (March CPS database). The numbers are 21679 euros and 16716 euros for France for fiscal households (*foyer fiscaux*) and single without dependents, respectively (CASD ERFS database).

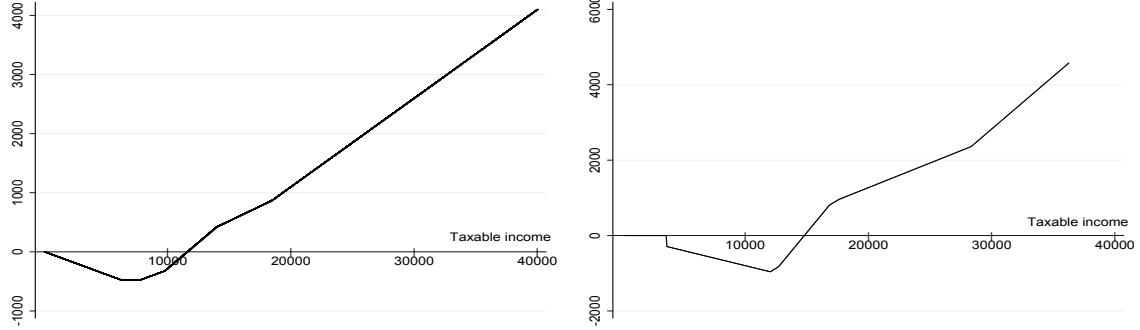


Figure 10: Income tax schedules for singles without dependants from micro-simulation models for the US (left figure) and France (right figure) in 2012

Figure 10 is based on the information provided by NBER TAXSIM simulator (accessible on <http://users.nber.org/taxsim/taxsim9/>) and the TAXIPP simulator (not accessible online, description available on <https://www.ipp.eu/en/tools/taxipp-micro-simulation/project/>).

feasible and welfare-improving.<sup>29</sup>

**Welfare improvements.** We begin with a Proposition that clarifies the conditions under which, for a given specification of welfare weights  $g$ , a small tax reform yields an increase in welfare. The following notation enables us to state the Proposition in a concise way. We define

$$\gamma_0 := \int_{\underline{\omega}}^{\bar{\omega}} g(\omega) \tilde{u}_c^0(\omega) f(\omega) d\omega ,$$

where  $\tilde{u}_c^0(\omega)$  is a shorthand for the marginal utility of consumption that a type  $\omega$  individual realizes under the status quo, and

$$\Gamma_0(\omega') := \int_{\omega'}^{\bar{\omega}} g(\omega) \tilde{u}_c^0(\omega) \frac{f(\omega)}{1 - F(\omega')} d\omega .$$

Thus,  $\gamma_0$  can be viewed as an average welfare weight in the status quo. It is obtained by multiplying each type's exogenous weight  $g(\omega)$  with the marginal utility of consumption in the status quo  $\tilde{u}_c^0(\omega)$ , and then computing a population average. By contrast,  $\Gamma_0(\omega')$  gives the average welfare weight of individuals with types above  $\omega'$ . Note that  $\gamma_0 = \Gamma_0(\underline{\omega})$  and that  $\Gamma_0$  is a non-increasing function.

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<sup>29</sup>Our analysis of welfare-improving reforms can also be related to a literature that seeks to identify society's social welfare function empirically – see, for instance, Christiansen and Jansen (1978), Blundell, Brewer, Haan and Shephard (2009), Bourguignon and Spadaro (2012), Bargain, Dolls, Neumann, Peichl and Siegloch (2011), Hendren (2014), Zoutman, Jacobs and Jongen (2014), Lockwood and Weinzierl (2016), or Bastani and Lundberg (2017). Through the lens of our model, this literature can alternatively be interpreted as identifying the set of social welfare functions for which a given reform would be welfare-improving.

**Proposition 8** *Let*

$$\mathcal{D}_g^W(y) := -\frac{1-F(\tilde{\omega}^0(y))}{f(\tilde{\omega}^0(y))} \Phi_0(\tilde{\omega}^0(y)) \frac{y_\omega^*(0,0,\tilde{\omega}^0(y))}{y_\tau^*(0,0,\tilde{\omega}^0(y))},$$

where  $\Phi_0(\omega) := 1 - \tilde{I}_0(\omega) - (1 - I_0) \frac{\Gamma_0(\omega)}{\gamma_0}$ .

1. Suppose there is an income level  $y'$  so that  $T'_0(y') < \mathcal{D}_g^W(y')$ . Then there exists a welfare-increasing reform  $(\tau, y_a, y_b)$  with  $\tau > 0$ , and  $y_a < y' < y_b$ .
2. Suppose there is an income level  $y'$  so that  $T'_0(y') > \mathcal{D}_g^W(y')$ . Then there exists a welfare-increasing reform  $(\tau, y_a, y_b)$  with  $\tau < 0$ , and  $y_a < y' < y_b$ .

The corresponding sufficient statistic  $\tilde{\mathcal{D}}_g^W$  admits an easy interpretation if there are no income effects so that the utility function is quasi-linear in private goods consumption and if the costs of productive effort are iso-elastic. In this case, we have  $I_0 = 0$ ,  $\tilde{I}_0(\omega) = 0$ , and  $\tilde{u}_c^0(\omega) = 1$  for all  $\omega$ . This implies, in particular, that  $\gamma_0 = 1$ ,  $\Gamma_0(\omega) = G(\omega) := \int_{\omega}^{\bar{\omega}} g(s) \frac{f(s)}{1-F(s)} ds$ , and  $\Phi_0(\omega) = 1 - G(\omega)$ , for all  $\omega$ . Consequently,

$$\tilde{\mathcal{D}}_g^W(y) = \frac{1-F(\tilde{\omega}^0(y))}{f(\tilde{\omega}^0(y)) \tilde{\omega}^0(y)} \left(1 - G(\tilde{\omega}^0(y))\right) \left(1 + \frac{1}{\epsilon}\right),$$

where the right-hand side of this equation is the *ABC*-formula due to Diamond (1998).

**Politically feasible welfare improvements.** Propositions 5 - 8 provide us with a characterization of the conditions under which a status quo tax policy admits reforms that are politically feasible *or* welfare-improving. Looking at the intersection of these conditions and using the fact that, for all  $y$ ,  $\mathcal{D}^{low}(y) \leq \mathcal{D}_g^W(y) \leq \mathcal{D}^{up}(y)$ , yields sufficient conditions for the existence of reforms that are politically feasible *and* welfare improving. The following Proposition states these conditions. A formal proof is omitted.

**Proposition 9**

1. Let  $y' < \tilde{y}^{0M}$  and  $T'_0(y') > \mathcal{D}_g^W(y')$ . Then there is a politically feasible and welfare-improving reform  $(\tau, y_a, y_b)$  with  $\tau < 0$ , and  $y_a < y' < y_b$ .
2. Let  $y' > \tilde{y}^{0M}$  and  $T'_0(y') < \mathcal{D}_g^W(y')$ . Then there is a politically feasible reform  $(\tau, y_a, y_b)$  with  $\tau > 0$ , and  $y_a < y' < y_b$ .

For below median incomes, lowering tax rates is politically feasible by Proposition 7. It is also welfare-improving, if, in the status quo, tax rates exceed  $\mathcal{D}_g^W$ . For above median incomes, raising taxes is politically feasible. This is welfare-improving if tax rates initially fall short of  $\mathcal{D}_g^W$ .

Proposition 9 states sufficient conditions for the existence of welfare-improving and politically feasible reforms. This raises the question of necessary conditions. Proposition 9 has been derived from focussing on “small” reforms, i.e., on small increases of marginal

tax rates applied to a small range of incomes. The arguments in the proofs of Propositions 5 - 8 imply that these conditions are also necessary in the following sense: if either

$$y' < \tilde{y}^{0M} \quad \text{and} \quad T'_0(y') \leq \mathcal{D}_g^W(y') ,$$

or

$$y' > \tilde{y}^{0M} \quad \text{and} \quad T'_0(y') \geq \mathcal{D}_g^W(y') ,$$

then there is no “small” reform for incomes close to  $y'$  that is both welfare-improving and politically feasible.<sup>30</sup>

The analysis suggests that existing tax schedules might be viewed as resulting from a compromise between concerns for welfare-maximization on the one hand, and concerns for political support on the other. If the maximization of political support was the only force in the determination of tax policy, we would expect to see tax rates close to the revenue-maximizing rate  $\mathcal{D}^{up}$  for incomes above the median and negative rates close to  $\mathcal{D}^{low}$  for incomes below the median. Concerns for welfare dampen these effects. A welfare-maximizing approach will generally yield higher marginal tax rates for incomes below the median and lower marginal tax rates for incomes above the median.

Our analysis also raises a question. Diamond (1998) and Saez (2001) have argued that, for plausible specifications of welfare weights, existing tax schedules have marginal tax rates for high incomes that are too low. Our analysis suggests that an increase of these tax rates is not only welfare-improving but also politically feasible. Why don’t we see more reforms that involve higher tax rates for the rich? Proposition 1 provides a possible answer to this question: reforms that involve tax cuts that are larger for richer taxpayers may as well prove to be politically feasible. For such reforms political feasibility requires that the median voter is included in the set of those who benefit from the tax cuts.

	$y < \tilde{y}^{0M}$			$y > \tilde{y}^{0M}$		
	Pareto	Political	Welfare	Pareto	Political	Welfare
$T'_0(y) > \mathcal{D}^{up}(y)$	↓	↓	↓	↓	↓	↓
$\mathcal{D}^{up}(y) > T'_0(y) > \mathcal{D}_g^W(y)$	-	↓	↓	-	↑	↓
$\mathcal{D}_g^W(y) > T'_0(y) > \mathcal{D}^{low}(y)$	-	↓	↑	-	↑	↑
$T'_0(y) < \mathcal{D}^{low}(y)$	↑	↑	↑	↑	↑	↑

Table 2: Summary of Propositions 5 - 9.

An Arrow pointing upwards (resp. downwards) indicates that raising (resp. lowering) marginal tax rates for incomes in a neighborhood of  $y$  is Pareto-improving, politically feasible, or welfare-improving. The symbol “-” indicates that changes of marginal tax rates are neither Pareto-improving nor Pareto-damaging.

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<sup>30</sup>We omit a more formal version of this statement that would require  $\varepsilon$ - $\delta$ -arguments.

Table 2 provides a summary of Propositions 5 - 9: according to the first line, if a tax system is such that the marginal tax rate at income  $y$  exceeds the upper Pareto bound, then lowering marginal tax rates for incomes in a neighborhood of  $y$  is Pareto-improving, welfare-improving, and politically feasible. Analogously, according to the last line, if tax rates are inefficiently low in the status quo, then increased rates are Pareto-improving, welfare-improving and politically feasible. The second and third line consider tax reforms that are not Pareto-improving. For below median incomes, only tax cuts are politically feasible. If marginal tax rates are too high according to a given welfare function, then there is scope for a politically feasible welfare-improvement. Otherwise, there is a conflict between what is politically feasible and what is desirable from a welfare perspective. For above median incomes, only higher tax rates are politically feasible. Thus, there is scope for a politically feasible welfare improvement if and only if moving towards higher rates is also welfare-improving.

## 7 Extensions

In this section we show that the median voter theorem for small monotonic reforms (Theorem 1) applies to models with more than one source of heterogeneity among individuals. Again, we show that a small tax reform is preferred by a majority of taxpayers if and only if it is preferred by the taxpayer with median income. Throughout, we stick to the assumption that individuals differ in their productive abilities  $\omega$ . We introduce a second consumption good and the possibility of heterogeneity in preferences over consumption goods in Section 7.1. We use this framework to discuss whether the introduction of distortionary taxes on savings is politically feasible. In Section 7.2 we consider fixed costs of labor market participation as an additional source of heterogeneity.<sup>31</sup> In Section 7.3 we assume that individuals differ in their valuation of increased public spending.<sup>32</sup> Finally, in Section 7.4, individuals differ by how much of their income is due to luck as in Alesina and Angeletos (2005).

### 7.1 Political support for taxes on savings

We now suppose that there are two consumption goods. We refer to them as food and savings, respectively. An individual's budget constraint now reads as

$$c_f + c_s + T_{0s}(c_s) + \tau_s h_s(c_s) \leq c_0 + y - T_0(y) - \tau h(y) . \quad (9)$$

The variables on the right-hand side of the budget constraint have been defined before. On the left-hand side,  $c_f$  denotes food consumption and  $c_s$  savings. In the status quo savings are taxed according to a possibly non-linear savings-tax function  $T_{0s}$ . A reform

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<sup>31</sup>See Saez (2002), Choné and Laroque (2011), and Jacquet, Lehmann and Van der Linden (2013).

<sup>32</sup>See Broadway and Keen (1993), Hellwig (2004), Bierbrauer (2014), or Weinzierl (2018).

replaces both the status quo income tax schedule  $T_0$  by  $T_1 = T_0 + \tau h$  and the status quo savings tax schedule  $T_{0s}$  by  $T_{1s} = T_{0s} + \tau_s h_s$ . We maintain the assumption that the functions  $h$  and  $h_s$  are non-decreasing and focus on revenue neutral reforms so that either  $\tau > 0$  and  $\tau_s < 0$  or  $\tau < 0$  and  $\tau_s > 0$ .

Preferences of individuals are given by a utility function  $u(v(c_f, c_s, \beta), y, \omega)$ , where  $v$  is a subutility function that assigns consumption utility to any consumption bundle  $(c_f, c_s)$ . The marginal rate of substitution between food and savings depends on a parameter  $\beta$ . We do not assume a priori that  $\beta$  is the same for all individuals. Under this assumption, however, the utility function  $u$  has the properties under which an efficient tax system does not involve distortionary commodity taxes, see Atkinson-Stiglitz (1976), or Laroque (2005) for a more elementary proof. Distortionary taxes on savings are then undesirable from a welfare-perspective.

Individuals choose  $c_f$ ,  $c_s$  and  $y$  to maximize utility subject to the budget constraint above. We denote the utility maximizing choices by  $c_f^*(\tau_s, \tau, \beta, \omega)$ ,  $c_s^*(\tau_s, \tau, \beta, \omega)$  and the corresponding level of indirect utility by  $V(\tau_s, \tau, \beta, \omega)$ . The slope of an indifference curve in a  $\tau$ - $\tau_s$  diagram determines the individuals' willingness to accept higher savings taxes in return for lower taxes on current earnings. The following Lemma provides a characterization of this marginal rate of substitution in a neighborhood of the status quo. Let

$$s(\tau, \tau_s, \beta, \omega) = -\frac{V_\tau(\tau_s, \tau, \beta, \omega)}{V_{\tau_s}(\tau_s, \tau, \beta, \omega)}$$

be the slope of an individual's indifference curve in a  $\tau$ - $\tau_s$  diagram. The slope in the status quo is denoted by  $s^0(\omega, \beta)$ . We denote the individual's food consumption, savings and earnings in the status quo by  $\tilde{c}_f^0(\omega, \beta)$ ,  $\tilde{c}_s^0(\omega, \beta)$  and  $\tilde{y}^0(\omega, \beta)$ , respectively.

**Lemma 4** *In the status quo the slope of a type  $(\omega, \beta)$ -individual's indifference curve in a  $\tau$ - $\tau_s$  diagram is given by*

$$s^0(\omega, \beta) = -\frac{h(\tilde{y}^0(\omega, \beta))}{h_s(\tilde{c}_s^0(\omega, \beta))}.$$

The proof of Lemma 4 can be found in the Appendix. The Lemma provides a generalization of Roy's identity that is useful for an analysis of non-linear tax systems. As is well known, with linear tax systems, the marginal effect of, say, an increased savings tax on indirect utility is equal to  $-\lambda^* c_s^*(\cdot)$ , where  $\lambda^*$  is the multiplier on the individual's budget constraint, also referred to as the marginal utility of income. Analogously, the increase of a linear income tax affects indirect utility via  $-\lambda^* y^*(\cdot)$  so that the slope of an indifference curve in a  $\tau_s$ - $\tau$ -diagram would be equal to the earnings-savings-ratio  $-\frac{y^*(\cdot)}{c_s^*(\cdot)}$ . Allowing for non-linear tax systems and non-linear perturbations implies that the simple earnings-savings-ratio is replaced by  $-\frac{h(y^*(\cdot))}{h_s(c_s^*(\cdot))}$ .

Consider a reform that involves an increase in the savings tax rate  $d\tau_s > 0$  and a reduction of taxes on income  $d\tau < 0$ . We say that a type  $(\omega, \beta)$ -individual strictly prefers a small reform with increased savings taxes over the status quo if

$$V_{\tau_s}(0, 0, \beta, \omega) d\tau_s + V_\tau(0, 0, \beta, \omega) d\tau > 0 ,$$

or, equivalently, if

$$\frac{d\tau_s}{d\tau} > s^0(\omega, \beta) = -\frac{h(\tilde{y}^0(\omega, \beta))}{h_s(\tilde{c}_s^0(\omega, \beta))} . \quad (10)$$

Since  $h_s$  is an increasing function, this condition is, ceteris paribus, easier to satisfy if the individual has little savings in the status quo.<sup>33</sup>

Different types will typically differ in their generalized earnings-savings-ratio  $s^0(\omega, \beta)$  and we can order types according to this one-dimensional index. Let  $(\omega, \beta)^{0M}$  be the type with the median value of  $s^0(\omega, \beta)$ . The following proposition extends Theorem 1. It asserts that a small reform is politically feasible if and only if it is supported by the median type  $(\omega, \beta)^{0M}$ .

**Proposition 10** *For a given status quo tax policy and a given pair of non-decreasing functions  $h$  and  $h_s$ , the following statements are equivalent:*

1. *Type  $(\omega, \beta)^{0M}$  prefers a small reform with increased savings taxes over the status quo.*
2. *There is a majority of individuals who prefer a small reform with increased savings taxes over the status quo.*

As Theorem 1, Proposition 10 exploits the observation that individuals can be ordered according to a one-dimensional statistic that pins down whether or not they benefit from a tax reform. This makes it possible to prove a median-voter theorem for reforms that remain in a neighborhood of the status quo. There is also an important difference to Theorem 1. With only one-dimensional heterogeneity, there is a monotonic relation between types and earnings so that the identity of the type with median income does not

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<sup>33</sup>The ratio  $\frac{d\tau_s}{d\tau}$  on the left-hand side of inequality (10) is determined as follow: Let  $\Delta^{R_s}(\tau_s, \tau)$  be the change of revenue from savings taxes and  $\Delta^R(\tau_s, \tau)$  the change of revenue from income taxation due to the reform. Revenue-neutrality requires that

$$\Delta_{\tau_s}^{R_s}(\tau_s, \tau) d\tau_s + \Delta_\tau^{R_s}(\tau_s, \tau) d\tau + \Delta_{\tau_s}^R(\tau_s, \tau) d\tau_s + \Delta_\tau^R(\tau_s, \tau) d\tau = 0 ,$$

or, equivalently, that

$$\frac{d\tau_s}{d\tau} = -\frac{\Delta_\tau^R(\tau_s, \tau) + \Delta_\tau^{R_s}(\tau_s, \tau)}{\Delta_{\tau_s}^{R_s}(\tau_s, \tau) + \Delta_{\tau_s}^R(\tau_s, \tau)} ,$$

which has to be evaluated for  $(\tau_s, \tau) = (0, 0)$ . We assume that this expression is well-defined and takes a finite negative value.

depend on the status quo. Whatever the tax system, the person with the median income is the person with the median type  $\omega^M$ . Here, by contrast, we allow for heterogeneity both in productive abilities and in preferences over consumption goods. The type with the median value of the generalized earnings-savings-ratio  $s^0(\omega, \beta)$  will then typically depend on the status quo tax system. This does not pose a problem if we focus on small reforms. In this case, preferences over reforms follow from the generalized earnings-savings-ratios in the status quo, and a small reform is preferred by a majority of individuals if and only if it is preferred by the individual with the median ratio.

## 7.2 Fixed costs of labor market participation

With fixed costs of labor market participation individuals derive utility  $u(c - \theta \mathbf{1}_{y>0}, y, \omega)$  from a  $(c, y)$ -pair. Fixed costs  $\theta$  absorb some of the individual's after-tax income if the individual becomes active on the labor market, e.g. because of additional child care expenses. As before, there is an initial status quo tax schedule under which earnings are transformed into after-tax income according to the schedule  $C_0$  with  $C_0(y) = c_0 + y - T_0(y)$ . After a reform, the schedule is

$$C_1(y) = c_0 + \Delta^R + y - T_0(y) - \tau h(y),$$

where  $h$  is a non-decreasing function of  $y$ . We denote by  $y^*(\Delta^R, \tau, \omega, \theta)$  the solution to

$$\max_y u(C_1(y) - \theta \mathbf{1}_{y>0}, y, \omega),$$

and the corresponding level of indirect utility by  $V(\Delta^R, \tau, \omega, \theta)$ . We proceed analogously for other variables: what has been a function of  $\omega$  in previous sections is now a function of  $\omega$  and  $\theta$ .

For a given function  $h$ , the marginal gain that is realized by an individual with type  $(\omega, \theta)$  if the tax rate  $\tau$  is increased, is given by the following analogue to equation (4),

$$\Delta_\tau^V(\omega, \theta | \tau, h) = \tilde{u}_c^1(\omega, \theta) (\Delta_\tau^R(\tau, h) - h(\tilde{y}^1(\omega, \theta))), \quad (11)$$

where  $\tilde{u}_c^1(\omega, \theta)$  is the marginal utility of consumption realized by a type  $(\omega, \theta)$ -individual after the reform, and  $\tilde{y}^1(\omega, \theta)$  are the individual's post-reform earnings. At  $\tau = 0$ , we can also write

$$\Delta_\tau^V(\omega, \theta | 0, h) = \tilde{u}_c^0(\omega, \theta) (\Delta_\tau^R(0, h) - h(\tilde{y}^0(\omega, \theta))), \quad (12)$$

where  $\tilde{u}_c^0(\omega, \theta)$  and  $\tilde{y}^0(\omega, \theta)$  are, respectively, marginal utility of consumption and earnings in the status quo.

For a given status quo tax policy and a given function  $h$  we say that type  $(\omega, \theta)$  strictly prefers a small tax reform over the status quo if  $\Delta_\tau^V(\omega, \theta | 0, h) > 0$ . The status quo median voter strictly prefers a small reform if  $\Delta_\tau^V((\omega, \theta)^{0M} | 0, h) > 0$ , where  $\tilde{y}^{0M}$  is the median of the distribution of earnings in the status quo and  $(\omega, \theta)^{0M}$  is the corresponding type; i.e.  $\tilde{y}^0((\omega, \theta)^{0M}) = \tilde{y}^{0M}$ .

**Proposition 11** *For a given status quo tax policy and a monotonic function  $h$ , the following statements are equivalent:*

1. *Type  $(\omega, \theta)^{0M}$  prefers a small reform over the status quo.*
2. *There is a majority of individuals who prefer a small reform over the status quo.*

Proposition 11 exploits that the slope of a type  $(\omega, \theta)$  individual's indifference curve through a point  $(\tau, \Delta^R)$ ,

$$s(\tau, \Delta^R, \omega, \theta) = h(y^*(\Delta^R, \tau, \omega, \theta)) .$$

is a function of the individual's income. As in the basic Mirrleesian setup, the interpretation is that individuals with a higher income are more difficult to convince that a reform that involves tax increases ( $\tau > 0$ ) is worthwhile. A difference to the Mirrleesian setup is, however, that there is no monotonic relation between types and earnings. In the presence of income effects, and for a given level of  $\omega$ ,  $y^*$  will increase in  $\theta$  as long as  $\theta$  is below a threshold  $\hat{\theta}(\omega)$  and be equal to 0 for  $\theta$  above the threshold. Moreover, the threshold is affected by tax policy. This implies that there is no longer a fixed type whose income is equal to the median income whatever the tax schedule. As in Proposition 10, this does not pose a problem if we focus on small reforms, i.e. on small deviations from  $(\tau, \Delta^R) = (0, 0)$ . In this case, preferences over reforms follow from the relation between types and earnings in the status quo, and a small reform is preferred by a majority of individuals if and only if it is preferred by the individual with the median level of income in the status quo.

### 7.3 Public-goods preferences

Suppose that the change in revenue  $\Delta^R$  is used to increase or decrease spending on publicly provided goods. The post-reform consumption schedule is then given by

$$C_1(y) = c_0 + y - T_0(y) - \tau h(y) ,$$

We assume that individuals differ with respect to their public-goods preferences. Now the parameter  $\theta$  is a measure of an individual's willingness to give up private goods consumption in exchange for more public goods. More specifically, we assume that individual utility is

$$u(\theta(R^0 + \Delta^R) + C_1(y), y, \omega) ,$$

where  $R^0$  is spending on publicly provided goods in the status quo. Again, we denote by  $y^*(\Delta^R, \tau, \omega, \theta)$  the solution to

$$\max_y u(\theta(R^0 + \Delta^R) + C_1(y), y, \omega)$$

and indirect utility by  $V(\Delta^R, \tau, \omega, \theta)$ . By the envelope theorem, the slope of a type  $(\omega, \theta)$  individual's indifference curve through point  $(\tau, \Delta^R)$  is now given by

$$s(\tau, \Delta^R, \omega, \theta) = \frac{h(y^*(\Delta^R, \tau, \omega, \theta))}{\theta}.$$

This marginal rate of substitution gives the increase in public-goods provision that an individual requires as a compensation for an increase of marginal tax rates. *Ceteris paribus*, individuals with a lower income and individuals with a higher public-goods preference require less of a compensation, i.e. they have a higher willingness to pay higher taxes for increased public-goods provision. If we focus on small reforms we observe, again, that if a type  $(\omega, \theta)$ -individual benefits from a small tax-increase, then the same is true for any type  $(\omega', \theta')$  with

$$\frac{h(\tilde{y}^0(\omega, \theta))}{\theta} \geq \frac{h(\tilde{y}^0(\omega', \theta'))}{\theta'}.$$

By the arguments in the proof of Proposition 11, a small reform with  $\tau > 0$  is preferred by a majority of individuals if and only if

$$\left( \frac{h(\tilde{y}^0(\omega, \theta))}{\theta} \right)^{0M} < \Delta_\tau^R(0, h),$$

where  $\left( \frac{h(\tilde{y}^0(\omega, \theta))}{\theta} \right)^{0M}$  is the median willingness to pay higher taxes for increased public spending in the status quo.

## 7.4 Fairness and politically feasible reforms

The validity of our approach does not depend on the assumption that voting behavior is driven by narrow self-interest. To illustrate this insight, we analyze politically feasible reforms in the context of a model in which social preferences determine political support for redistributive taxation. Specifically, we adopt the framework of Alesina and Angeletos (2005). Alesina and Angeletos assume that individual incomes can be due to luck or effort and that preferences over tax policies include a motive to tax income that is due to luck more heavily than income that is due to effort. Alesina and Angeletos focus, however, on linear tax systems.

There are two periods. When young individuals choose a level of human capital  $k$ . When old individuals choose productive effort or labor supply  $l$ . Pre-tax income is determined by

$$y = \pi(l, k) + \eta,$$

where  $\pi$  is a production function that is increasing in both arguments and  $\eta$  is a random source of income, also referred to as luck. An individual's life-time utility is written as  $u(c, l, k, \omega)$ . Utility is increasing in the first argument. It is decreasing in the second and third argument to capture the effort costs of labor supply and human capital investments,

respectively. Effort costs are decreasing in  $\omega$ . More formally, lower types have steeper indifference curves both in a  $(c, l)$ -space and in a  $(c, k)$ -space. We consider reforms that lead to a consumption schedule

$$C_1(y) = c_0 + \Delta^R + y - T_0(y) - \tau h(y) .$$

We assume that individuals first observe how lucky they are and then choose how hard they work, i.e. given a realization of  $\eta$  and given the predetermined level of  $k$ , individuals choose  $l$  so as to maximize

$$u(C_1(\pi(l, k) + \eta), l, k, \omega) .$$

We denote the solution to this problem by  $l^*(\Delta^R, \tau, \omega, \eta, k)$ . Indirect utility is denoted by  $V(\Delta^R, \tau, \omega, \eta, k)$ . As of  $t = 1$ , there is multi-dimensional heterogeneity among individuals: they differ in their type  $\omega$ , in their realization of luck  $\eta$  and possibly also in their human capital  $k$ .

In Alesina and Angeletos (2005) preferences over reforms have a selfish and fairness component. The indirect utility function  $V$  shapes the individuals' selfish preferences over reforms. The analysis of these selfish preferences can proceed along similar lines as the extension that considered fixed costs of labor market participation. Selfish preferences over small reforms follow from the relation between types and earnings in the status quo, and a small reform makes a majority better off if and only if it is beneficial for the individual with the median level of income in the status quo. More formally, let  $\tilde{y}^0(\omega, \eta, k) := y^*(0, 0, \omega, \eta, k)$  be a shorthand for the earnings of a type  $(\omega, \eta, k)$ -individual in the status quo and recall that the sign of

$$s(0, 0, \omega, \eta, k) = h(\tilde{y}^0(\omega, \eta, k))$$

determines whether an individual benefits from a small tax reform. Specifically, suppose that  $h$  is a non-decreasing function and denote by  $y^{0M}$  the median level of income in the status quo and by  $(\omega, \eta, k)^{0M}$  the corresponding type. A majority of individuals is – according to their selfish preferences – made better off if and only if the median voter benefits from the reform,

$$s^0((\omega, \eta, k)^{0M}) = h(\tilde{y}^{0M}) < \Delta_\tau^R(0, h) .$$

In their formalization of social preferences, Alesina and Angeletos (2005) view  $\pi(l, k)$  as a reference income. It is the part of income that is due to effort as opposed to luck. A tax reform affects the share of  $y = \pi(l, k) + \eta$  that individuals can keep for themselves. After the reform, the difference between disposable income and the reference income is given by<sup>34</sup>

$$C_1(y) - \pi(l, k) = \eta - T_0(\pi(l, k) + \eta) - \tau h(\pi(l, k) + \eta) .$$

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<sup>34</sup>The analysis in Alesina and Angeletos (2005) looks at a special case of this. They focus on a status quo equal to the laissez-faire schedule so that  $T_0(y) = 0$ , for all  $y$ , and a reform that introduces a linear tax schedule, i.e.  $h(y) = y$ , for all  $y$ . Under these assumptions, we have  $\eta - T_0(\pi(l, k) + \eta) - \tau h(\pi(l, k) + \eta) = (1 - \tau)\eta + \tau\pi(l, k)$ .

A social preferences for fair taxes is then equated with a desire to minimize the variance of  $\eta - T_0(\pi(l, k) + \eta) - \tau h(\pi(l, k) + \eta)$  taking into account that  $k$  and  $l$  are endogenous variables.<sup>35</sup> Denote this variance henceforth by  $\Sigma(\Delta^R, \tau)$ . Any one individual is assumed to evaluate a tax reform according to

$$V(\Delta^R, \tau, \omega, \eta, k) - \rho \Sigma(\Delta^R, \tau),$$

where  $\rho$  is the weight on fairness considerations which is assumed to be the same for all individuals. Therefore, heterogeneity in preferences over reforms is entirely due to heterogeneity in selfish preferences. Consequently, the finding that a small reform is preferred by a majority of taxpayers if and only if it is preferred by the voter with median income in the status quo is not affected by the inclusion of a demand for fair taxes.

## 8 Concluding remarks

This paper develops a framework for a political economy analysis of tax reforms. With non-linear tax systems, the policy-space is multi-dimensional with the implication that political economy forces are difficult to characterize. One of our main results is that this difficulty can be overcome by looking at tax reforms such that the change in tax payments is a monotonic function of income. We show that, with such a policy space, a reform is preferred by a majority of taxpayers if and only if it is preferred by the taxpayer with median income. This implies that tax decreases for incomes below the median and tax increases for incomes above the median are politically feasible.

Our analysis identifies, for each level of income, Pareto-bounds for marginal tax rates. It also identifies reforms that would be preferred by a majority of voters. Moreover, we derive sufficient statistics that make it possible to bring this framework to the data. This enables us to diagnose what types of reforms are possible under a given status quo tax policy. Future research might use this framework to complement existing studies on the history of income taxation. For instance, Scheve and Stasavage (2016) study whether tax systems have become more progressive in response to increases in inequality or in response to extensions of the franchise. Their analysis compares tax policies that have been adopted at different points in time, or by countries with different institutions. It does not include an analysis of the reforms that appear to have been politically feasible or welfare-improving in a given year for a given country and a given status quo tax schedule. The framework that is developed in this paper lends itself to such an analysis.

We develop the main argument in the context of a classical Mirrleesian model of income taxation. We also show that our analysis of small monotonic reforms extends to

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<sup>35</sup>Human capital investment is a function of effort costs  $\omega$  and the expectations  $(\Delta^{Re}, \tau^e)$  of the young on the tax reforms that will be adopted when they are old.

models with multi-dimensional heterogeneity of individuals, such as models with variable and fixed costs of labor market participation, models that include heterogeneity in preferences over public goods, or models that include an investment in human capital.

The focus on monotonic reforms has enabled us to prove a median voter theorem for non-linear tax systems. Moreover, the use of the perturbation method made it possible to identify directions for politically feasible reforms by means of sufficient statistics. The logic of this argument is not tied to non-linear tax systems and we conjecture that a similar analysis is possible for other multi-dimensional policy domains. What is needed, however, is that a focus on monotonic reforms is justified. For taxation, monotonicity holds provided that tax cuts or increases are monotonic functions of income – a property that is satisfied by most reforms in actual tax policy.

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# Appendix

## A Behavioral responses to simple reforms

A simple reform  $(\tau, y_a, y_b)$ -class involves jumps of marginal tax rates at  $y_a$  and  $y_b$ . Lemmas A.1-A.3 below clarify the behavioral responses to these discontinuities. Figure 11 provides an illustration.

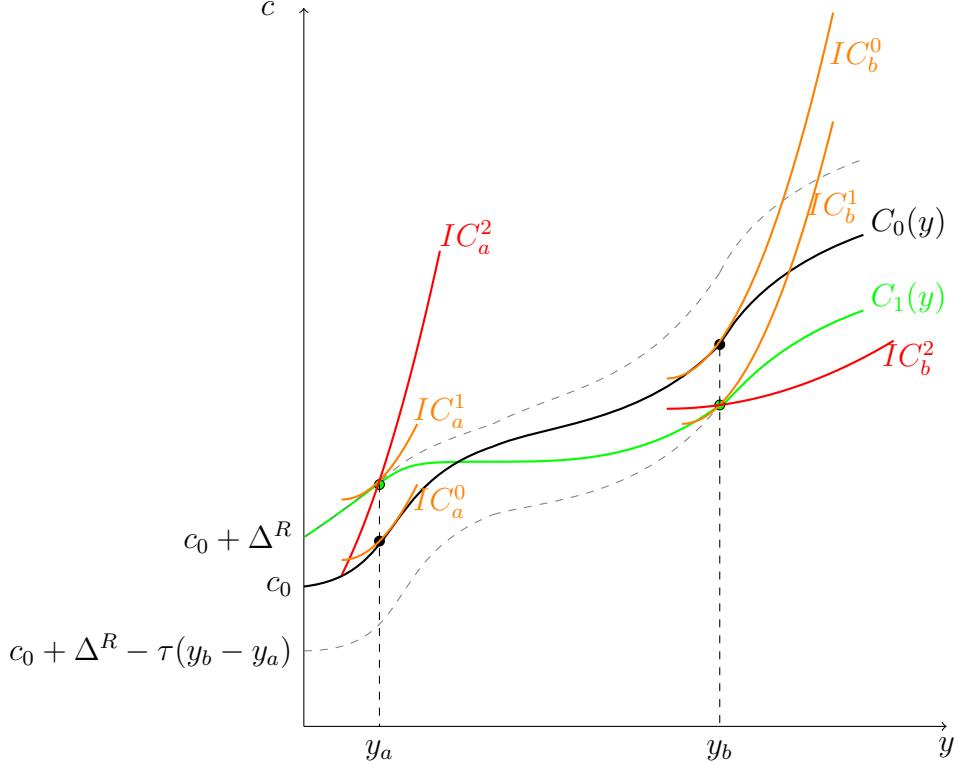


Figure 11:

The figure depicts the following curves:

- (a)  $C_0(y) = c_0 + y - T_0(y)$ ,  $C_1(y) = c_0 + \Delta^R + y - T_0(y) - \tau h(y)$ ,
- (b)  $IC_b^0$  indifference curve of type  $\omega_b(0)$  through  $y_b$  under  $C_0(y)$ ,  $IC_b^1$  curve through  $y_b$  under  $C_1(y)$  when there are no income effects,  $IC_b^2$  curve through  $y_b$  under  $C_1(y)$  with income effects;
- (c)  $IC_a^0$  indifference curve of type  $\omega_a(0)$  through  $y_a$  under  $C_0(y)$ ,  $IC_a^1$  curve through  $y_a$  under  $C_1(y)$  when there are no income effects,  $IC_a^2$  curve through  $y_a$  under  $C_1(y)$  if there are income effects.

The Spence-Mirrlees single crossing property implies that, under any tax schedule, more productive individuals choose a higher level of pre-tax income. Thus,  $\omega' > \omega$  implies that

$$y^*(\Delta^R(\tau', y_a, y_b), \tau', \omega') \geq y^*(\Delta^R(\tau', y_a, y_b), \tau', \omega) ,$$

for  $\tau' \in \{0, \tau\}$ . We can therefore define threshold types  $\omega_a(\tau')$  and  $\omega_b(\tau')$  so that

$$\omega < \omega_a(\tau') \quad \text{implies} \quad y^*(\Delta^R(\tau', y_a, y_b), \tau', \omega) \leq y_a ,$$

$$\omega \in (\omega_a(\tau'), \omega_b(\tau')) \quad \text{implies} \quad y^*(\Delta^R(\tau', y_a, y_b), \tau', \omega) \in (y_a, y_b) ,$$

and

$$\omega > \omega_b(\tau') \text{ implies } y^*(\Delta^R(\tau', y_a, y_b), \tau', \omega) \geq y_b.$$

The following Lemma asserts that, for a reform that involves an increase of the marginal tax rate,  $\tau > 0$ , type  $\omega_a(0)$  who chooses an income of  $y_a$  before the reform does not choose a level of income above  $y_a$  after the reform. Analogously, if marginal taxes go down, type  $\omega_b(0)$  does not choose an income above  $y_b$  after the reform. For a reform with  $\tau > 0$ , the logic is as follows: After the reform, because of the transfer  $\Delta^R$ , a type  $\omega_a(0)$ -individual is, at income level  $y_a$ , less eager to work more. Working more also is less attractive after the reform because of the increased marginal tax rates for incomes above  $y_a$ . Thus, both the income and the substitution effect associated with the reform make it less attractive for a type  $\omega_a(0)$ -individual to increase her income above  $y_a$ . The individual will therefore either stay at  $y_a$  or decrease her income after the reform. Only higher types will end up with an income of  $y_a$  after the reform, which implies  $\omega_a(0) \leq \omega_a(\tau)$ .

### **Lemma A.1**

1. Consider a reform so that  $\tau > 0$  and  $\Delta^R(\tau, y_a, y_b) > 0$ . Then  $\omega_a(\tau) \geq \omega_a(0)$ .
2. Consider a reform so that  $\tau < 0$  and  $\Delta^R(\tau, y_a, y_b) < 0$ . Then  $\omega_b(\tau) \geq \omega_b(0)$ .

**Proof** We only prove the first statement. The proof of the second statement follows from an analogous argument. Under the initial tax schedule  $T_0$  an individual with type  $\omega_a(0)$  prefers  $y_a$  over all income levels  $y \geq y_a$ . We argue that the same is true under the new tax schedule  $T_1$ . This proves that  $\omega_a(\tau) \geq \omega_a(0)$ . Consider a  $y$ - $c$ -diagram and the indifference curve of a type  $\omega_a(0)$ -type through the point  $(y_a, c_0 + y_a - T_0(y_a))$ . By definition of type  $\omega_a(0)$ , all points  $(y, c_0 + y - T_0(y))$  with  $y > y_a$  lie below this indifference curve under the initial tax schedule. Under the new schedule  $T_1$ , this individual receives a lump-sum transfer  $\Delta^R > 0$ . Hence, the indifference curve through  $(y_a, c_0 + \Delta^R + y_a - T_0(y_a))$  is at least as steep as the indifference curve through  $(y_a, c_0 + y_a - T_0(y_a))$ . Thus, the individual prefers  $(y_a, c_0 + \Delta^R + y_a - T_0(y_a))$  over all points  $(y, c_0 + \Delta^R + y - T_0(y))$  with  $y > y_a$ . Since for all  $y > y_a$ ,  $T_1(y) > T_0(y)$ , the individual also prefers  $(y_a, c_0 + \Delta^R + y_a - T_0(y_a))$  over all points  $(y, c_0 + \Delta^R + y - T_1(y))$  with  $y > y_a$ .  $\square$

According to the next lemma a reform induces bunching of individuals who face an upward jump of marginal tax rates after the reform. Specifically, a reform with  $\tau > 0$  will induce bunching at  $y_a$  because marginal tax rates jump upwards at income level  $y_a$ . A reform with  $\tau < 0$  will induce bunching at  $y_b$  because marginal tax rates jump upwards at income level  $y_b$ .

### **Lemma A.2**

1. Consider a reform so that  $\tau > 0$  and  $\Delta^R(\tau, y_a, y_b) > 0$ . Then there is a set of types  $[\omega_a(\tau), \bar{\omega}_a(\tau)]$  who bunch at  $y_a$  after the reform.
2. Consider a reform so that  $\tau < 0$  and  $\Delta^R(\tau, y_a, y_b) < 0$ . Then there is a set of types  $[\omega_b(\tau), \bar{\omega}_b(\tau)]$  who bunch at  $y_b$  after the reform.

**Proof** Consider a reform that involves an increase of marginal tax rates  $\tau > 0$ . If before the reform, all types were choosing an income level  $y^*(0, 0, \omega)$  that satisfies the first order condition

$$u_c(\cdot)(1 - T'_0(\cdot)) + u_y(\cdot) = 0 ,$$

then, after the reform there will be a set of types  $(\omega_a(\tau), \bar{\omega}(\tau)]$  who will now bunch at  $y_a$ . These individuals chose an income level above  $y_a$  before the reform. After the reform they will find that

$$u_c(\cdot)(1 - T'_0(\cdot) - \tau) + u_y(\cdot) < 0 ,$$

for all  $y \in (y_a, y^*(0, 0, \omega)]$  and therefore prefer  $y_a$  over any income in this range. At the same time, they will find that

$$u_c(\cdot)(1 - T'_0(\cdot)) + u_y(\cdot) > 0 ,$$

so that there is also no incentive to choose an income level lower than  $y_a$ . Hence, the types who bunch are those for which at  $y_a$ ,

$$1 - T'_0(y_a) > -\frac{u_y(\cdot)}{u_c(\cdot)} > 1 - T'_0(y_a) - \tau .$$

An analogous argument implies that for a reform that involves a decrease in marginal tax rates,  $\tau < 0$ , there will be a set of types  $[\omega_b(\tau), \bar{\omega}_b(\tau)]$  who bunch at  $y_b$  after the reform.  $\square$

Individuals who bunch at  $y_a$  after a reform with  $\tau > 0$  neither have an incentive to increase their earnings above  $y_a$  since

$$u_c(\cdot)(1 - T'_1(y_a)) + u_y(\cdot) = u_c(\cdot)(1 - T'_0(y_a) - \tau) + u_y(\cdot) \leq 0$$

nor an incentive to lower their earnings since

$$u_c(\cdot)(1 - T'_0(y_a)) + u_y(\cdot) \geq 0 .$$

By contrast, there are no individuals who bunch at  $y_b$  after a reform that involves increased marginal tax rates. Individuals who do not have an incentive to increase their income at  $y_b$  since

$$u_c(\cdot)(1 - T'_1(y_b)) + u_y(\cdot) = u_c(\cdot)(1 - T'_0(y_b)) + u_y(\cdot) \leq 0$$

definitely have an incentive to lower their income as

$$u_c(\cdot)(1 - T'_0(y_b) - \tau) + u_y(\cdot) < 0 .$$

An analogous argument implies that no one will bunch at  $y_a$  after a reform that involves a decrease of marginal tax rates.

The next Lemma establishes that, in the absence of income effects, for a reform with  $\tau > 0$ , individuals who choose an income above  $y_b$  after the reform also chose an income above  $y_b$  before the reform. We will subsequently discuss why these statements need no longer be true if there are income effects.

**Lemma A.3** Suppose that there are no income effects, i.e. for all  $(c, y, \omega)$  and any pair  $(e, e')$  with  $e' > e$ ,

$$-\frac{u_y(c+e, y, \omega)}{u_c(c+e, y, \omega)} = -\frac{u_y(c+e', y, \omega)}{u_c(c+e', y, \omega)}.$$

1. Consider a reform so that  $\tau > 0$  and  $\Delta^R(\tau, y_a, y_b) > 0$ . Then  $\omega_b(\tau) \geq \omega_b(0)$ .
2. Consider a reform so that  $\tau < 0$  and  $\Delta^R(\tau, y_a, y_b) < 0$ . Then  $\omega_a(\tau) \geq \omega_a(0)$ .

**Proof** We only prove the first statement. The proof of the second statement follows from an analogous argument. Under the initial tax schedule  $T_0$  an individual with type  $\omega_b(0)$  prefers  $y_b$  over all income levels  $y \geq y_b$ . We argue that the same is true under the new tax schedule  $T_1$ . This proves that  $\omega_b(\tau) \geq \omega_b(0)$ . Consider a  $y$ - $c$ -diagram and the indifference curve of a type  $\omega_b(0)$ -type through the point  $(y_b, c_0 + y_b - T_0(y_b))$ . By definition of type  $\omega_b(0)$ , all points  $(y, c_0 + y - T_0(y))$  with  $y > y_b$  lie below this indifference curve under the initial tax schedule. Under the new schedule  $T_1$ , this individual receives a lump-sum transfer  $\Delta^R - \tau(y_b - y_a) < 0$ . Without income effects, the indifference curve through  $(y_b, c_0 + \Delta^R - \tau(y_b - y_a) + y_b - T_0(y_b))$  has the same slope as the indifference curve through  $(y_b, c_0 + y_b - T_0(y_b))$ . Thus, the individual prefers  $(y_b, c_0 + \Delta^R - \tau(y_b - y_a) + y_b - T_0(y_b))$  over all points  $(y, c_0 + \Delta^R - \tau(y_b - y_a) + y - T_0(y))$  with  $y > y_b$ , or equivalently over all points  $(y, c_0 + \Delta^R + y - T_1(y))$  with  $y > y_b$ .  $\square$

If there are no income effects, then, after a reform with  $\tau > 0$ , an individual with type  $\omega_b(0)$  prefers an income level of  $y_b$  over any income above  $y_b$  before and after the reform since (i) the indifference curve through  $(c_0 + y_b - T_0(y_b), y_b)$  has the same slope as the indifference curve through  $(c_0 + \Delta^R + y_b - T_1(y_b), y_b)$  and (ii) for  $y > y_b$ ,  $T'_0(y) = T'_1(y)$  so that the incentives to increase income above  $y_b$  are unaffected by the reform. The individual has, however, an incentive to lower  $y$  since, because of the increased marginal tax rate, working less has become cheaper; i.e. it is no longer associated with as big a reduction of consumption. Thus,  $\omega_b(0) \leq \omega_b(\tau)$  if there are no income effects. Figure 11 provides an illustration. With income effects there is also an opposing force since the individual also has to pay additional taxes  $\tau(y_b - y_a) - \Delta^R$  which tends to flatten the indifference curve through  $y_b$ . Thus, there may both be income levels below  $y_b$  and income levels above  $y_b$  that the individual prefers over  $y_b$ . If the indifference curve flattens a lot, the individual will end up choosing  $y > y_b$  after the reform which implies that  $\omega_b(\tau) < \omega_b(0)$ .

A reform  $(\tau, y_a, y_b)$  with  $\tau > 0$  may yield to a discontinuity in the earnings function  $y^*(\Delta^R, \tau, \cdot)$  at  $\omega_b(\tau) = \sup\{\omega \mid y^*(\Delta^R, \tau, \omega) \leq y_b\}$ , see Figure 12 below for an illustration. The reform leads to a kink in the  $C_1$ -schedule at  $y_b$ . Depending on the shape of indifference curves, this may imply that

$$\lim_{\omega \uparrow \omega_b(\tau)} y^*(\Delta^R(\tau, y_a, y_b), \tau, \omega) < \lim_{\omega \downarrow \omega_b(\tau)} y^*(\Delta^R(\tau, y_a, y_b), \tau, \omega).$$

Analogously, a reform  $(\tau, y_a, y_b)$  with  $\tau < 0$  may come with a discontinuity in the earnings function  $y^*(\Delta^R, \tau, \cdot)$  at  $\omega_a(\tau)$ .

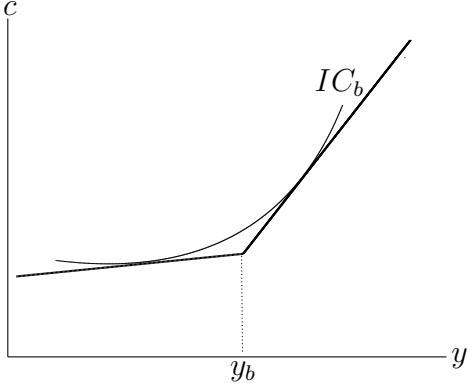


Figure 12: The possibility of a hole at  $y_b$

The figure illustrates a reform  $(\tau, y_a, y_b)$  with  $\tau > 0$  and a situation in which type  $\omega_b(\tau)$  is indifferent between an income level below  $y_b$  and an income level above  $y_b$ . Consequently, individuals with a type below  $\omega_b(\tau)$  choose an income strictly smaller than  $y_b$  and individuals with a type above choose an income strictly larger than  $y_b$ . The earnings function  $y^*(\Delta^R, \tau, \cdot)$  therefore exhibits a discontinuity at  $\omega_b(\tau)$  and the income distribution therefore has zero mass in a neighborhood of  $y_b$ .

## B Proofs

### Proof of Theorem 1

*Step 1.* We show that

$$\Delta_\tau^V(\omega | 0, h) = \tilde{u}_c^0(\omega) (\Delta_\tau^R(0, h) - h(\tilde{y}^0(\omega))) ,$$

for all  $\omega \in \Omega$ , where  $\tilde{u}_c^0(\omega)$  is a shorthand for the marginal utility of consumption that a type  $\omega$ -individual realizes in the status quo.

For individuals whose behavior is characterized by a first-order condition, this follows from a straightforward application of the envelope theorem. If we consider reforms in the  $(\tau, y_a, y_b)$ -class we have to take account of the possibility of bunching. Consider the case with  $\tau > 0$ . The case  $\tau < 0$  is analogous. With  $\tau > 0$ , there will be individuals who bunch at  $y_a$ . For these individuals, marginal changes of  $\Delta^R$  and  $\tau$  do not trigger an adjustment of the chosen level of earnings. Hence,

$$\begin{aligned} \Delta_\tau^V(\omega | \tau, h) &= \frac{d}{d\tau} u(c_0 + \Delta^R(\tau, h) + y_a - T_0(y_a) - \tau h(y_a), y_a, \omega) \\ &= u_c(c_0 + \Delta^R(\tau, h) + y_a - T_0(y_a) - \tau h(y_a), y_a, \omega) (\Delta_\tau^R(\tau, h) - h(y_a)) \end{aligned}$$

By assumption, the status quo tax schedule does not induce bunching. Thus, at  $\tau = 0$ , this expression applies only to type  $\omega_a$  with  $\tilde{y}^0(\omega_a) = y_a$ . This proves that

$$\Delta_\tau^V(\omega | 0, h) = \tilde{u}_c^0(\omega) (\Delta_\tau^R(0, h) - h(\tilde{y}^0(\omega))) ,$$

holds for all  $\omega$ .

*Step 2.* Suppose that  $h$  is a non-decreasing function. An analogous argument applies if  $h$  is non-increasing. We show that  $\Delta_\tau^V(\omega^M | 0, h) > 0$  implies  $\Delta_\tau^V(\omega | 0, h) > 0$  for a majority of individuals. By Step 1,  $\Delta_\tau^V(\omega^M | 0, h) > 0$  holds if and only if  $\Delta_\tau^R(0, h) - h(\tilde{y}^0(\omega^M)) > 0$ . As  $h$

and  $\tilde{y}$  are non-decreasing functions, this implies  $\Delta_\tau^R(0, h) - h(\tilde{y}^0(\omega)) > 0$ , for all  $\omega \leq \omega^M$ , and hence  $\Delta_\tau^V(\omega | 0, h) > 0$  for all  $\omega \leq \omega^M$ .

*Step 3.* Suppose that  $h$  is a non-decreasing function. An analogous argument applies if  $h$  is non-increasing. We show that  $\Delta_\tau^V(\omega^M | 0, h) \leq 0$  implies  $\Delta_\tau^V(\omega | 0, h) \leq 0$  for a majority of individuals. By Step 1,  $\Delta_\tau^V(\omega^M | 0, h) \leq 0$  holds if and only if  $\Delta_\tau^R(0, h) - h(\tilde{y}^0(\omega^M)) \leq 0$ . As  $h$  and  $\tilde{y}$  are non-decreasing functions, this implies  $\Delta_\tau^R(0, h) - h(\tilde{y}^0(\omega)) \leq 0$ , for all  $\omega \geq \omega^M$ , and hence  $\Delta_\tau^V(\omega | 0, h) \leq 0$  for all  $\omega \geq \omega^M$ .

## Proof of Lemma 1

For individuals whose behavior is characterized by a first-order condition, the utility realized under a reform that involves a change in the lump-sum-transfer by  $\Delta^R$  and a change of marginal taxes by  $\tau$  is given by  $V(\Delta^R, \tau, \omega)$ . The marginal rate of substitution between  $\tau$  and  $\Delta^R$  is therefore given by

$$\left( \frac{d\Delta^R}{d\tau} \right)_{|dV=0} = -\frac{V_\tau(\Delta^R, \tau, \omega)}{V_e(\Delta^R, \tau, \omega)}.$$

By the envelope theorem,

$$V_\tau(\Delta^R, \tau, \omega) = -u_c(\cdot) h(y^*(\Delta^R, \tau, \omega)) \quad \text{and} \quad V_e(\Delta^R, \tau, \omega) = u_c(\cdot).$$

Thus,

$$\left( \frac{d\Delta^R}{d\tau} \right)_{|dV=0} = h(y^*(\Delta^R, \tau, \omega)).$$

If we consider reforms in the  $(\tau, y_a, y_b)$ -class we have to take account of the possibility of bunching. We only consider the case with  $\tau > 0$ . The case  $\tau < 0$  is analogous. With  $\tau > 0$ , there will be individuals who bunch at  $y_a$ . For these individuals, marginal changes of  $\Delta^R$  and  $\tau$  do not trigger an adjustment of the chosen level of earnings. Hence, a marginal change of  $\Delta^R$  yields a change in utility equal to  $u_c(\cdot)$ . The change in utility due to a change in the marginal tax rate is given by  $-u_c(\cdot) h(y_a) = 0$ . Again, the marginal rate of substitution equals  $h(y_a) = 0$ .

## Proof of Proposition 1

From Step 1 in the proof of Theorem 1 we know that

$$\Delta_\tau^V(\omega | 0, h) = \tilde{u}_c^0(\omega) (\Delta_\tau^R(0, h) - h(\tilde{y}^0(\omega))) .$$

To proof the first statement in the Proposition, let

$$\Delta_\tau^V(\omega^M | 0, h) = \tilde{u}_c^0(\omega^M) (\Delta_\tau^R(0, h) - h(\tilde{y}^0(\omega^M))) < 0 ,$$

so that the median voter benefits from a small tax cut, i.e. a small reduction of  $\tau$ .

With  $h$  non-decreasing for  $y \geq \tilde{y}^{0M}$ , this implies that

$$\Delta_\tau^V(\omega | 0, h) = \tilde{u}_c^0(\omega) (\Delta_\tau^R(0, h) - h(\tilde{y}^0(\omega))) < 0 ,$$

for all  $\omega \geq \omega^M$ . Hence a majority of the population benefits from the tax cut.

The second statement in the Proposition follows from the same argument: If the poorest individual benefits from a tax cut and individuals with incomes closer to the median also benefit as  $h$  is non-decreasing for below median incomes, then there is majority support for the reform.

## Proof of Lemma 2

We seek to show that, for all  $\omega$ ,

$$\Delta_\tau^V(\omega | \tau, h) = \tilde{u}_c^1(\omega) (\Delta_\tau^R(\tau, h) - h(\tilde{y}^1(\omega))) .$$

Suppose that  $\tau > 0$ . The case  $\tau < 0$  is analogous. We first consider reforms  $(\tau, h)$  under which individual earnings are characterized by a first order condition. In this case, the envelope theorem implies

$$\Delta_\tau^V(\omega | \tau, h) = u_c(\cdot) (\Delta_\tau^R(\tau, h) - h(\tilde{y}^1(\omega))) ,$$

where  $u_c(\cdot)$  is marginal utility evaluated at  $c = c_0 + \Delta^R(\tau, h) + \tilde{y}^1(\omega) - T_1(\tilde{y}^1(\omega))$  and  $\tilde{y}^1(\omega) = y^*(\Delta^R(\tau, h), \tau, \omega)$ .

We now consider a reform that belongs to the  $(\tau, y_a, y_b)$ -class. The above reasoning extends to types with  $\omega \leq \omega_a(\tau)$ ,  $\omega \in [\bar{\omega}_a(\tau), \omega_b(\tau)]$  and  $\omega \geq \omega_b(\tau)$  whose behavior is characterized by a first order condition. Individuals with types in  $[\omega_a(\tau), \bar{\omega}_a(\tau)]$  bunch at income level  $y_a$  after the reform so that

$$\Delta_\tau^V(\omega | \tau, y_a, y_b) = u(c_0 + \Delta^R(\tau, y_a, y_b) + y_a - T_0(y_a), y_a, \omega) - V(0, 0, \omega) .$$

Hence,

$$\Delta_\tau^V(\omega | \tau, y_a, y_b) = u_c(\cdot) \Delta_\tau^R(\tau, y_a, y_b) ,$$

where  $u_c(\cdot)$  is marginal utility evaluated at  $c = c_0 + \Delta^R(\tau, y_a, y_b) + y - T_0(y)$  and  $y = y_a$ . Since  $h(y_a) = 0$ , this implies, in particular,

$$\Delta_\tau^V(\omega | \tau, y_a, y_b) = u_c(\cdot) (\Delta_\tau^R(\tau, y_a, y_b) - h(y_a)) .$$

## Proof of Proposition 3

Suppose that  $h$  is a non-decreasing function. The proof for  $h$  non-increasing is analogous. By (4),

$$\Delta_\tau^V(\omega^M | \tau', h) > 0$$

for all  $\tau' \in (0, \tau)$ , implies

$$\Delta_\tau^R(\tau', h) - h(\tilde{y}^1(\omega^M)) > 0 ,$$

for all  $\tau' \in (0, \tau)$ . By the Spence-Mirrlees single crossing property  $\tilde{y}^1$  is a non-decreasing function. Hence,  $\omega \leq \omega^M$  implies

$$\Delta_\tau^R(\tau', h) - h(\tilde{y}^1(\omega)) > 0 ,$$

for all  $\tau' \in (0, \tau)$  and, therefore, by (3) and (4)

$$\Delta_\tau^V(\omega | \tau', h) > 0 ,$$

again, for all  $\tau' \in (0, \tau)$ . Thus, the reform is supported by a majority of the population. This proves the first statement in Proposition 3. The second statement follows from an analogous argument.

## Proof of Lemma 3

We show that, for all  $\omega$ ,

$$\tilde{u}_c^{1'}(\omega) := \frac{\partial}{\partial \omega} \tilde{u}_c^1(\omega) \leq 0 .$$

Define

$$\tilde{y}^{1'}(\omega) := \frac{\partial}{\partial \omega} \tilde{y}^1(\omega) .$$

By the Spence-Mirrlees single crossing property,  $\tilde{y}^1$  is a non-decreasing function:  $\tilde{y}^{1'}(\omega) \geq 0$  for all  $\omega$ . Recall that, by definition,

$$\tilde{u}_c^1(\omega) = u_c(c_0 + \Delta^R + \tilde{y}^1(\omega) - T_1(\tilde{y}^1(\omega)), \tilde{y}^1(\omega), \omega) .$$

Hence,

$$\tilde{u}_c^{1'}(\omega) = (u_{cc}(\cdot)(1 - T'_1(\cdot)) + u_{cy}(\cdot)) \tilde{y}^{1'}(\omega) + u_{c\omega}(\cdot) .$$

If the individual is bunching, then  $\tilde{y}^{1'}(\omega) = 0$ , which implies

$$\tilde{u}_c^{1'}(\omega) = u_{c\omega}(\cdot) \leq 0 .$$

It remains to be shown that  $\tilde{u}_c^{1'}(\omega) \leq 0$  also holds if the individual is not bunching. If the individual is not bunching,  $\tilde{y}^1(\omega)$  satisfies the first order condition

$$1 - T'_1(\cdot) = -\frac{u_y(\cdot)}{u_c(\cdot)} .$$

Hence,

$$\tilde{u}_c^{1'}(\omega) = \left( -u_{cc}(\cdot) \frac{u_y(\cdot)}{u_c(\cdot)} + u_{cy}(\cdot) \right) \tilde{y}^{1'}(\omega) + u_{c\omega}(\cdot) ,$$

where, because of (1),

$$-u_{cc}(\cdot) \frac{u_y(\cdot)}{u_c(\cdot)} + u_{cy}(\cdot) \leq 0 .$$

To see this, note that we can also express condition (1) as the requirement that, for any combination of  $c, y, e$  and  $\omega$ , the derivative of  $-\frac{u_y(c+e,y,\omega)}{u_c(c+e,y,\omega)}$  with respect to  $e$  is non-negative. This yields the inequality above.

## Proof of Theorem 2

For a small reform in the  $(\tau, y_a, y_b)$ -class, evaluated at the status quo, equation (4) implies that

$$\Delta_\tau^V(\omega | 0, y_a, y_b) = \tilde{u}_c^0(\omega) (\Delta_\tau^R(0, y_a, y_b) - h(\tilde{y}^0(\omega))) , \quad (13)$$

To prove the first statement in Theorem 2, suppose that  $y_0 < \tilde{y}^0(w^M)$ . Choose  $y_a$  and  $y_b$  so that  $y_a < y_0 < y_b < \tilde{y}^0(w^M)$ . For a reform in the  $(\tau, y_a, y_b)$ -class,  $y_b < \tilde{y}^0(w^M)$  implies  $h(\tilde{y}^0(\omega)) = y_b - y_a$ , for all  $\omega \geq \omega^M$ . Since  $\Delta_\tau^R(0, y_a, y_b) < y_b - y_a$ , it follows that  $\Delta_\tau^V(\omega | 0, y_a, y_b) < 0$ , for all  $\omega \geq \omega^M$ , which implies that a small tax cut makes a majority of individuals better off.

To prove the second statement, suppose that  $y_0 > \tilde{y}^0(w^M)$  and choose  $y_a$  and  $y_b$  so that  $\tilde{y}^0(w^M) < y_a < y_0 < y_b$ . For a reform in the  $(\tau, y_a, y_b)$ -class,  $\tilde{y}^0(w^M) < y_a$  implies  $h(\tilde{y}^0(\omega)) = 0$ , for all  $\omega \leq \omega^M$ . Hence, if  $\Delta_\tau^R(0, y_a, y_b) > 0$ , then  $\Delta_\tau^V(\omega | 0, y_a, y_b) > 0$ , for all  $\omega \leq \omega^M$ , which implies that a small tax raise makes a majority of individuals better off.

## Proof of Proposition 5

We clarify the conditions under which a small reform in the  $(\tau, y_a, y_b)$ -class yields an increase of tax revenue. Before we turn to small reforms, we look at reforms that are large in that both  $|\tau|$  and  $y_b - y_a$  are bounded away from zero. We begin by looking at the implications of such a large tax reform for tax revenue. Throughout we look at reforms with  $\tau > 0$  that induce bunching at  $y_a$  and, possibly, a hole at  $y_b$ . The proof clarifies the conditions under which such a reform does not generate additional tax revenue, with the implication that tax rates in the status quo are inefficiently high.

We begin with a characterization of  $\Delta_\tau^R(\tau, y_a, y_b)$ , i.e. of the change in tax revenue that is implied by a marginal change of the tax rate  $\tau$ . We will be particularly interested in evaluating this expression at  $\tau = 0$  since  $\Delta_0^R(0, y_a, y_b)$  gives the increase in tax revenue from a small increase of marginal tax rates for incomes in the interval  $[y_a, y_b]$ . For now, we maintain the assumption that  $y_b - y_a$  is bounded away from zero.

### Lemma B.1

$$\Delta_\tau^R(0, y_a, y_b) = \frac{1}{1 - I_0} \mathcal{R}(y_a, y_b),$$

where  $I_0$  and  $\mathcal{R}(y_a, y_b)$  are defined in the body of the text.

**Proof of Lemma B.1.** It proves useful to look at different subsets of the population separately. The change in tax revenue that is due to individuals with types  $\omega \leq \omega_a(\tau)$  who chose an income level smaller or equal to  $y_a$  after the reform is given by

$$\begin{aligned} \Delta^{R1}(\tau, y_a, y_b) &:= \int_{\underline{\omega}}^{\omega_a(\tau)} \{T_1(y^*(\Delta^R(\cdot), \tau, \omega)) - T_0(y^*(0, 0, \omega))\} f(\omega) d\omega \\ &= \int_{\underline{\omega}}^{\omega_a(\tau)} \{T_0(y^*(\Delta^R(\cdot), \tau, \omega)) - T_0(y^*(0, 0, \omega))\} f(\omega) d\omega \\ &= \int_{\underline{\omega}}^{\omega_a(\tau)} \{T_0(y^*(\Delta^R(\cdot), 0, \omega)) - T_0(y^*(0, 0, \omega))\} f(\omega) d\omega, \end{aligned}$$

where the last equality uses the fact that the behavior of individuals with types below  $\omega_a(\tau)$  is affected only by  $\Delta^R(\cdot)$  but not by the increased marginal tax rates for individuals with incomes in  $[y_a, y_b]$ . The change in tax revenue that comes from individuals with types in  $[\omega_a(\tau), \bar{\omega}_a(\tau)]$  who bunch at an income level of  $y_a$  after the reform equals

$$\begin{aligned} \Delta^{R2}(\tau, y_a, y_b) &:= \int_{\omega_a(\tau)}^{\bar{\omega}_a(\tau)} \{T_1(y_a) - T_0(y^*(0, 0, \omega))\} f(\omega) d\omega \\ &= \int_{\omega_a(\tau)}^{\bar{\omega}_a(\tau)} \{T_0(y_a) - T_0(y^*(0, 0, \omega))\} f(\omega) d\omega. \end{aligned}$$

Individuals with types in  $(\bar{\omega}_a(\tau), \omega_b(\tau))$  choose an income level between  $y_a$  and  $y_b$  after the reform. The change in tax revenue that can be attributed to them equals

$$\begin{aligned} \Delta^{R3}(\tau, y_a, y_b) &:= \int_{\bar{\omega}_a(\tau)}^{\omega_b(\tau)} \{T_1(y^*(\Delta^R(\cdot), \tau, \omega)) - T_0(y^*(0, 0, \omega))\} f(\omega) d\omega \\ &= \int_{\bar{\omega}_a(\tau)}^{\omega_b(\tau)} \{T_0(y^*(\Delta^R(\cdot), \tau, \omega)) + \tau(y^*(\Delta^R(\cdot), \tau, \omega) - y_a) - T_0(y^*(0, 0, \omega))\} f(\omega) d\omega. \end{aligned}$$

Finally, the change in tax revenue that comes from individuals with types above  $\omega_b(\tau)$  who choose an income larger than  $y_b$  after the reform is given by

$$\begin{aligned} \Delta^{R4}(\tau, y_a, y_b) &:= \int_{\omega_b(\tau)}^{\bar{\omega}} \{T_1(y^*(\Delta^R(\cdot), \tau, \omega)) - T_0(y^*(0, 0, \omega))\} f(\omega) d\omega \\ &= \int_{\omega_b(\tau)}^{\bar{\omega}} \{T_0(y^*(\Delta^R(\cdot), \tau, \omega)) + \tau(y_b - y_a) - T_0(y^*(0, 0, \omega))\} f(\omega) d\omega \\ &= \int_{\omega_b(\tau)}^{\bar{\omega}} \{T_0(y^*(\Delta^R(\cdot) - \tau(y_b - y_a), 0, \omega)) + \tau(y_b - y_a) - T_0(y^*(0, 0, \omega))\} f(\omega) d\omega, \end{aligned}$$

where the last equality uses the fact that the behavior of individuals with types above  $\omega_b(\tau)$  is affected only by the transfer  $\Delta^R(\cdot) - \tau(y_b - y_a)$  but not by the increased marginal tax rates for individuals with incomes in  $[y_a, y_b]$ , see Figure 1. To sum up,

$$\Delta^R(\tau, y_a, y_b) = \Delta^{R1}(\tau, y_a, y_b) + \Delta^{R2}(\tau, y_a, y_b) + \Delta^{R3}(\tau, y_a, y_b) + \Delta^{R4}(\tau, y_a, y_b) .$$

We now provide a characterization of  $\Delta_\tau^R(\tau, y_a, y_b)$ , i.e. of the change in tax revenue that is implied by a marginal change of the tax rate  $\tau$ . We will be particularly interested in evaluating this expression at  $\tau = 0$  since  $\Delta_\tau^R(0, y_a, y_b)$  gives the increase in tax revenue from a small increase of marginal tax rates for incomes in the interval  $[y_a, y_b]$ . For now, we maintain the assumption that  $y_b - y_a$  is bounded away from zero. Since

$$\Delta_\tau^R(\tau, y_a, y_b) = \Delta_\tau^{R1}(\tau, y_a, y_b) + \Delta_\tau^{R2}(\tau, y_a, y_b) + \Delta_\tau^{R3}(\tau, y_a, y_b) + \Delta_\tau^{R4}(\tau, y_a, y_b) ,$$

we can characterize  $\Delta_\tau^R(\tau, y_a, y_b)$  by looking at each subset of types separately. For instance, it is straightforward to verify that

$$\begin{aligned} \Delta_\tau^{R1}(\tau, y_a, y_b) &= \Delta_\tau^R(\tau, y_a, y_b) \int_{\underline{\omega}}^{\omega_a(\tau)} T'_0(y^*(\Delta^R(\cdot), 0, \omega)) y_e^*(\Delta^R(\cdot), 0, \omega) f(\omega) d\omega \\ &\quad + \{T_0(y_a) - T_0(y^*(0, 0, \omega_a(\tau)))\} f(\omega_a(\tau)) \omega'_a(\tau) \end{aligned}$$

where  $\omega'_a(\tau)$  is the derivative of  $\omega_a(\tau)$ . Analogously we derive expressions for  $\Delta_\tau^{R2}(\tau, y_a, y_b)$ ,  $\Delta_\tau^{R3}(\tau, y_a, y_b)$ ,  $\Delta_\tau^{R4}(\tau, y_a, y_b)$ . This is tedious, but straightforward. It yields an expression for  $\Delta_\tau^R(\tau, y_a, y_b)$ . If we evaluate this expression at  $\tau = 0$  and use Assumption 1 – so that there is neither bunching at  $y_a$  nor a hole at  $y_b$  under the initial tax schedule  $T_0$  – we obtain

$$\begin{aligned} \Delta_\tau^R(0, y_a, y_b) &= \Delta_\tau^R(0, y_a, y_b) I_0 \\ &\quad \int_{\tilde{\omega}^0(y_a)}^{\tilde{\omega}^0(y_b)} \{T'_0(y^*(0, 0, \omega)) y_\tau^*(0, 0, \omega) + y^*(0, 0, \omega) - y_a\} f(\omega) d\omega \\ &\quad + (y_b - y_a) \left\{ 1 - F(\tilde{\omega}^0(y_b)) - \int_{\tilde{\omega}^0(y_b)}^{\bar{\omega}} T'_0(y^*(0, 0, \omega)) y_e^*(0, 0, \omega) f(\omega) d\omega \right\} , \end{aligned}$$

where  $I_0 := \tilde{I}(\underline{\omega})$  is the income effect measure  $\tilde{I}$  defined in the body of the text, see equation (6), applied to the population at large. Equivalently,

$$\Delta_\tau^R(0, y_a, y_b) = \frac{1}{1 - I_0} \mathcal{R}(y_a, y_b),$$

where  $\mathcal{R}(y_a, y_b)$  has also been defined in the body of the text.  $\square$

**Small reforms.** A small reform is defined by two properties: It involves a small change of marginal tax rates and this change applies to a small range of incomes. We now investigate the conditions under which such a reform increases tax revenue. Obviously

$$\Delta_\tau^R(0, y_a, y_a) = \frac{1}{1 - I_0} \mathcal{R}(y_a, y_a) = 0 .$$

Hence, if

$$\Delta_{\tau y_b}^R(0, y_a, y_a) = \frac{1}{1 - I_0} \mathcal{R}_{y_b}(y_a, y_a) > 0 ,$$

then  $\Delta_\tau^R(0, y_a, y_b)$  turns positive, if starting from  $y_a = y_b$ , we marginally increase  $y_b$ . Straightforward computations yield:

$$\begin{aligned} \mathcal{R}_{y_b}(y_a, y_a) &= \frac{d\tilde{\omega}^0(y_a)}{dy} T'_0(y^*(0, 0, \tilde{\omega}^0(y_a))) y_\tau^*(0, 0, \tilde{\omega}^0(y_a)) f(\tilde{\omega}^0(y_a)) \\ &\quad + (1 - \tilde{I}_0(\tilde{\omega}^0(y_a))) (1 - F(\tilde{\omega}^0(y_a))) , \end{aligned} \tag{14}$$

where  $y_\omega^*(0, 0, \tilde{\omega}^0(y_a)) = \left(\frac{d\tilde{\omega}^0(y_a)}{dy}\right)^{-1} > 0$ . We summarize these observations in the following Lemma:

**Lemma B.2** Suppose that, under tax schedule  $T_0$ , there is an income level  $y'$  and a type  $\omega'$  with  $y^*(0, 0, \omega') = y'$  so that

$$T'_0(y^*(0, 0, \omega')) y_\tau^*(0, 0, \omega') f(\omega') + (1 - \tilde{I}_0(\omega')) y_\omega^*(0, 0, \omega') (1 - F(\omega')) > 0.$$

Then there exists a revenue-increasing tax reform  $(\tau, y_a, y_b)$  with  $\tau > 0$ ,  $y_a < y' < y_b$ .

**On the sign of  $y_\tau^*(0, 0, \omega')$ .** We demonstrate that in Lemma B.2 above, we have  $y_\tau^*(0, 0, \omega') < 0$ . To see this, consider the optimization problem

$$\max_y u(c_0 + y - T_0(y) - \tau(y - y_a), y, \omega)$$

for type  $\omega'$ . By assumption  $y^*(0, 0, \omega') \in (y_a, y_b)$ . We argue that for any  $\tau$  so that  $y^*(0, \tau, \omega') \in (y_a, y_b)$  and  $y_b - y_a$  sufficiently small, we have  $y_\tau^*(0, 0, \omega') < 0$ . The first order condition of the optimization problem is

$$u_c(\cdot)(1 - T'_0(\cdot) - \tau) + u_y(\cdot) = 0.$$

The second order condition is, assuming a unique optimum,

$$B := u_{cc}(\cdot)(1 - T'_0(\cdot) - \tau)^2 + 2u_{cy}(\cdot)(1 - T'_0(\cdot) - \tau) + u_{yy}(\cdot) - u_c(\cdot)T''_0(\cdot) < 0.$$

From totally differentiating the first order condition with respect to  $c_0$ , we obtain

$$y_e^*(0, \tau, \omega_0) = -\frac{u_{cc}(\cdot)(1 - T'_0(\cdot) - \tau) + u_{cy}(\cdot)}{B} \leq 0.$$

This expression is non-positive by our assumptions on the utility function that ensure that leisure is a non-inferior good. From totally differentiating the first order condition with respect to  $\tau$ , and upon collecting terms, we obtain

$$y_\tau^*(0, \tau, \omega_0) = \frac{u_c(\cdot)}{B} - (y^*(0, \tau, \omega_0) - y_a)y_e^*(0, \tau, \omega_0),$$

and hence

$$y_\tau^*(0, 0, \omega_0) = \frac{u_c(\cdot)}{B} - (y^*(0, 0, \omega_0) - y_a)y_e^*(0, 0, \omega_0),$$

which is the familiar decomposition of a behavioral response into a substitution and an income effect. As  $y_b$  approaches  $y_a$ ,  $y^*(0, 0, \omega_0)$  also approaches  $y_a$  so that the income effect vanishes. Again, this is a familiar result: For small price changes, observed behavioral responses are well approximated by compensated or Hicksian behavioral responses.

The observation that for  $y_a$  close to  $y_b$ , we may, without loss of generality, assume that  $y_\tau^*(0, 0, \omega') < 0$ , enables us to rewrite Lemma B.2.

**Lemma B.3** Suppose that, under tax schedule  $T_0$ , there is an income level  $y'$  and a type  $\omega'$  with  $y^*(0, 0, \omega') = y'$  so that

$$T'_0(y') < -\frac{1 - F(\omega')}{f(\omega')} \left(1 - \tilde{I}_0(\omega')\right) \frac{y_\omega^*(0, 0, \omega')}{y_\tau^*(0, 0, \omega')}.$$

Then there exists a tax-revenue-increasing reform  $(\tau, y_a, y_b)$  with  $\tau > 0$ , and  $y_a < y' < y_b$ .

The right-hand-side of this inequality equals  $\mathcal{D}^{up}(y')$  for  $y'$  such that  $\tilde{\omega}^0(y') = \omega'$ . By the same logic, if  $T'(0)(y') > \mathcal{D}^{up}(y')$ , there exists a tax-revenue-increasing reform  $(\tau, y_a, y_b)$  with  $\tau < 0$ , and  $y_a < y' < y_b$ . This proves Proposition 5.

## Proof of Proposition 6

Starting from  $\tau = 0$  a tax increase is Pareto-improving if  $\Delta_\tau^R(0, y_a, y_b) - (y_b - y_a) \geq 0$ , where we recall that

$$\Delta_\tau^R(0, y_a, y_b) = \frac{1}{1 - I_0} \mathcal{R}(y_a, y_b).$$

Applying the logic in the proof of Proposition 5 one more time, a small reform is Pareto-improving provided that

$$\frac{1}{1 - I_0} \mathcal{R}_{y_b}(y_a, y_a) \geq 1.$$

Using equation (14) and  $y_\omega^*(0, 0, \tilde{\omega}^0(y_a)) = \left(\frac{d\tilde{\omega}^0(y_a)}{dy}\right)^{-1} > 0$ , this condition can be equivalently written as

$$T'_0(y^*(0, 0, \omega_a)) y_\tau^*(0, 0, \tilde{\omega}^0(y_a)) f(\omega_a) + (1 - \tilde{I}_0(\tilde{\omega}^0(y_a))) y_\omega^*(0, 0, \tilde{\omega}^0(y_a)) (1 - F(\tilde{\omega}^0(y_a))) > y_\omega^*(0, 0, \tilde{\omega}^0(y_a))(1 - I_0).$$

If this inequality holds we can Pareto-improve by increasing marginal tax rates in a neighborhood of  $y_a$ . Upon noting that  $y_\tau^*(0, 0, \tilde{\omega}^0(y_a)) < 0$ , this is easily seen to be equivalent to the claim  $T'_0(y_a) < \mathcal{D}^{low}(y_a)$  in Proposition 6.

## Proof of Proposition 8

We only prove the first statement in Proposition 8, the second follows from an analogous argument. We first look at the welfare implications of a small change of the marginal tax rates applied to an income bracket with positive length. Subsequently, we let the length of the interval vanish. Recall that  $\Delta_\tau^V(\omega | 0, y_a, y_b) = \tilde{u}_c^0(\omega) (\Delta_\tau^R(0, y_a, y_b) - h(\tilde{y}^1(\omega)))$  and  $\Delta_\tau^R(0, y_a, y_b) = \frac{1}{1 - I_0} \mathcal{R}(y_a, y_b)$ . Hence,

$$\begin{aligned} \Delta_\tau^W(0, y_a, y_b) &= \gamma_0 \Delta_\tau^R(0, y_a, y_b) - \int_{\tilde{\omega}^0(y_a)}^{\tilde{\omega}^0(y_b)} g(\omega) \tilde{u}_c^0(\cdot) \{y^*(0, 0, \omega) - y_a\} f(\omega) d\omega \\ &\quad - (1 - F(\tilde{\omega}^0(y_b))) (y_b - y_a) \Gamma_0(\tilde{\omega}^0(y_b)), \\ &= \frac{\gamma_0}{1 - I_0} \mathcal{R}(y_a, y_b) - \int_{\tilde{\omega}^0(y_a)}^{\tilde{\omega}^0(y_b)} g(\omega) \tilde{u}_c^0(\cdot) \{y^*(0, 0, \omega) - y_a\} f(\omega) d\omega \\ &\quad - (1 - F(\tilde{\omega}^0(y_b))) (y_b - y_a) \Gamma_0(\tilde{\omega}^0(y_b)). \end{aligned}$$

We now investigate under which conditions a marginal increase of  $\tau$  over a small interval of types increases welfare. As  $\Delta_\tau^W(0, y_a, y_a) = 0$ , if  $\Delta_{\tau y_b}^W(0, y_a, y_a) > 0$ ,  $\Delta_\tau^W(0, y_a, y_b)$  turns positive, if starting from  $y_a = y_b$ , we marginally increase  $y_b$ . Straightforward computations yield

$$\Delta_{\tau y_b}^W(0, y_a, y_a) = \frac{\gamma_0}{1 - I_0} \mathcal{R}_{y_b}(y_a, y_a) - (1 - F(\tilde{\omega}^0(y_b))) \Gamma_0(\tilde{\omega}^0(y_b)).$$

By equation (14) and  $\frac{d\tilde{\omega}^0(y_a)}{dy} = y_\omega^*(0, 0, \tilde{\omega}^0(y_a))^{-1}$ ,

$$\begin{aligned} \mathcal{R}_{y_b}(y_a, y_a) &= T'_0(y^*(0, 0, \tilde{\omega}^0(y_a))) \frac{y_\tau^*(0, 0, \tilde{\omega}^0(y_a))}{y_\omega^*(0, 0, \tilde{\omega}^0(y_a))} f(\tilde{\omega}^0(y_a)) \\ &\quad + (1 - \tilde{I}_0(\tilde{\omega}^0(y_a))) (1 - F(\tilde{\omega}^0(y_a))), \end{aligned}$$

so that we can write

$$\Delta_{\tau y_b}^W(0, y_a, y_a) = \frac{\gamma_0}{1-I_0} \left\{ T'_0(y^*(0, 0, \tilde{\omega}^0(y_a))) \frac{y_\tau^*(0, 0, \tilde{\omega}^0(y_a))}{y_\omega^*(0, 0, \tilde{\omega}^0(y_a))} f(\omega_a) + (1 - F(\tilde{\omega}^0(y_a))) \Phi_0(\tilde{\omega}^0(y_a)) \right\},$$

where

$$\Phi_0(\tilde{\omega}^0(y_a)) = 1 - \tilde{I}_0(\tilde{\omega}^0(y_a)) - (1 - I_0) \frac{\Gamma_0(\tilde{\omega}^0(y_a))}{\gamma_0}.$$

Using this expression and the fact that  $y_\tau^*(0, 0, \tilde{\omega}^0(y_a)) < 0$ , we obtain the characterization of welfare-increasing reforms in the first statement of Proposition 8.

## Proof of Proposition 10

We first show that a small reform is strictly supported by a majority of the population if it is strictly preferred by the median voter. Suppose that  $\frac{d\tau_s}{d\tau} > s^0((\omega, \beta)^{0M})$ . This also implies  $\frac{d\tau_s}{d\tau} > s^0(\omega, \beta)$ , for all individuals with  $s^0((\omega, \beta)^{0M}) \geq s^0(\omega, \beta)$ . By the definition of the status quo median voter  $(\omega, \beta)^{0M}$ , the mass of taxpayers with this property is equal to  $\frac{1}{2}$ . Hence, the reform is supported by a majority of the population. Second, we show that the status quo is weakly preferred by a majority of individuals if it is weakly preferred by the status quo median voter. Suppose that the status quo is weakly preferred by the median voter so that  $\frac{d\tau_s}{d\tau} \leq s^0((\omega, \beta)^{0M})$ . This also implies  $\frac{d\tau_s}{d\tau} \leq s^0(\omega, \beta)$ , for all types  $(\omega, \beta)$  so that  $s^0((\omega, \beta)^{0M}) \leq s^0(\omega, \beta)$ . By the definition of  $(\omega, \beta)^{0M}$  the mass of taxpayers with this property is equal to  $\frac{1}{2}$ . Hence, the status quo is weakly preferred by a majority of individuals.

## Proof of Proposition 11

We focus without loss of generality on tax increases, i.e.  $\tau > 0$  and on a non-decreasing function  $h$ . We first show that a small reform is strictly supported by a majority of the population if it is strictly preferred by the median voter. Suppose that  $\Delta_\tau^V((\omega, \theta)^{0M} | 0, h) > 0$ . Since  $\tilde{u}_c^0(\cdot) > 0$ , this implies

$$\Delta_\tau^R(0, h) - h(y^{0M}) > 0.$$

Since  $h$  is a non-decreasing function, this also implies

$$\Delta_\tau^R(0, h) - h(\tilde{y}^0(\omega, \theta)) > 0,$$

for all  $(\omega, \theta)$  so that  $\tilde{y}^0(\omega, \theta) \leq y^{0M}$ . By definition of the status quo median voter, the mass of taxpayers with  $\tilde{y}^0(\omega, \theta) \leq y^{0M}$  is equal to  $\frac{1}{2}$ . Hence, the reform is supported by a majority of the population. Second, we show that the status quo is weakly preferred by a majority of individuals if it is weakly preferred by the status quo median voter. Suppose that the status quo is weakly preferred by the median voter so that

$$\Delta_\tau^R(0, h) - h(y^{0M}) \leq 0.$$

Since  $h$  is a non-decreasing function, this also implies

$$\Delta_\tau^R(0, h) - h(\tilde{y}^0(\omega, \theta)) \leq 0,$$

for all  $(\omega, \theta)$  so that  $\tilde{y}^0(\omega, \theta) \leq y^{0M}$ . By definition of the status quo median voter, the mass of taxpayers with  $\tilde{y}^0(\omega, \theta) \leq y^{0M}$  is equal to  $\frac{1}{2}$ . Hence, the status quo is weakly preferred by a majority of individuals.

## C Bunching and non-negativity constraints

### C.1 Bunching

Proposition C.1 below extends Proposition 5 in the body of the text so as to allow for bunching in the characterization of revenue-increasing reforms. We leave the extensions of Propositions 6, 7 and 8 to the reader. These extensions simply require to replace the function  $\tilde{\omega}^0$  by its analog  $\tilde{\omega}^{0b} : y \mapsto \max\{\omega \mid y^*(0, 0, \omega) = y\}$  that takes account of the possibility of bunching.

**Proposition C.1** Define  $\mathcal{D}^{up,b}(y) := -\frac{1-F(\tilde{\omega}^{0b}(y))}{f(\tilde{\omega}^{0b}(y))} \left(1 - \tilde{I}_0(\tilde{\omega}^{0b}(y))\right) \frac{y_\omega^*(0, 0, \tilde{\omega}^{0b}(y))}{y_\tau^*(0, 0, \tilde{\omega}^{0b}(y))}$ .

1. Suppose that there is an income level  $y_0$  so that  $T'_0(y_0) < \mathcal{D}^{up,b}(y_0)$ . Then there exists a revenue-increasing reform with  $\tau > 0$ , and  $y_a < y_0 < y_b$ .
2. Suppose that there is an income level  $y_0$  so that  $T'_0(y_0) > \mathcal{D}^{up,b}(y_0)$ . Then there exists a revenue-increasing reform with  $\tau < 0$ , and  $y_a < y_0 < y_b$ .

**Proof.** We prove only the first statement in the Proposition. The proof of the second statement is analogous.

We consider a reform in the  $(\tau, y_a, y_b)$ -class with  $\tau > 0$  and assume that, prior to the reform, there is a set of types  $[\underline{\omega}_a, \bar{\omega}_a]$  who bunch at  $y_a$ . More formally, for all  $\omega \in [\underline{\omega}_a, \bar{\omega}_a]$ ,  $y^*(0, 0, \omega) = \tilde{y}^0(\omega) = y_a$ . We also assume, without loss of generality, that there is no further bunching between  $y_a$  and  $y_b$  so that the function  $\tilde{y}^0$  is strictly increasing for  $\omega > \bar{\omega}_a(0)$ . The reform will affect the set of types who bunch at  $y_a$  and we denote by  $\underline{\omega}_a(\tau)$  and  $\bar{\omega}_a(\tau)$  the minimal and the maximal type, respectively, who chose an earnings level of  $y_a$  after the reform. The type who chooses  $y_b$  after the reform is denoted by  $\omega_b(\tau)$ .

We first look at the implication of such a reform for tax revenue. Again, we decompose the change in tax revenue.

$$\Delta^R(\tau, y_a, y_b) = \Delta^{R1}(\tau, y_a, y_b) + \Delta^{R2}(\tau, y_a, y_b) + \Delta^{R3}(\tau, y_a, y_b) + \Delta^{R4}(\tau, y_a, y_b),$$

where

$$\Delta^{R1}(\tau, y_a, y_b) = \int_{\underline{\omega}}^{\underline{\omega}_a(\tau)} \{T_1(y^*(\Delta^R(\cdot), \tau, \omega)) - T_0(\tilde{y}^0(\omega))\} f(\omega) d\omega,$$

$$\Delta^{R2}(\tau, y_a, y_b) = \int_{\underline{\omega}_a(\tau)}^{\bar{\omega}_a(\tau)} \{T_1(y_a) - T_0(\tilde{y}^0(\omega))\} f(\omega) d\omega,$$

$$\Delta^{R3}(\tau, y_a, y_b) = \int_{\bar{\omega}_a(\tau)}^{\omega_b(\tau)} \{T_1(y^*(\Delta^R(\cdot), \tau, \omega)) - T_0(\tilde{y}^0(\omega))\} f(\omega) d\omega,$$

and

$$\Delta^{R4}(\tau, y_a, y_b) = \int_{\omega_b(\tau)}^{\bar{\omega}} \{T_1(y^*(\Delta^R(\cdot), \tau, \omega)) - T_0(\tilde{y}^0(\omega))\} f(\omega) d\omega .$$

Following the same steps as in the proof of Proposition 5, we obtain a characterization of  $\Delta_\tau^R(0, y_a, y_b)$ ,

$$\Delta_\tau^R(0, y_a, y_b) = \frac{1}{1 - I_0^b} \mathcal{R}^b(y_a, y_b) ,$$

where

$$I_0^b = \int_{\underline{\omega}}^{\omega_a(0)} T'_0(\cdot) y_e^*(0, 0, \omega) f(\omega) d\omega + \int_{\bar{\omega}_a(0)}^{\bar{\omega}} T'_0(\cdot) y_e^*(0, 0, \omega) f(\omega) d\omega ,$$

and

$$\begin{aligned} \mathcal{R}^b(y_a, y_b) &= \int_{\tilde{\omega}^{0b}(y_a)}^{\tilde{\omega}^{0b}(y_b)} \{T'_0(\cdot) y_\tau^*(0, 0, \omega) + y^*(0, 0, \omega) - y_a\} f(\omega) d\omega \\ &\quad + (y_b - y_a)(1 - F(\tilde{\omega}^{0b}(y_b))) - (y_b - y_a) \int_{\tilde{\omega}^{0b}(y_b)}^{\bar{\omega}} T'_0(\cdot) y_e^*(0, 0, \omega) f(\omega) d\omega . \end{aligned}$$

The superscripts  $b$  indicate that these expressions are the analogs to  $I_0$  and  $\mathcal{R}$  in the proof of Proposition 5 that take account of bunching. Note, however that  $\mathcal{R}^b(y_a, y_b) = \mathcal{R}(y_a, y_b)$  as there is no further bunching above  $y_a$ .

By the same arguments as in the proof of Proposition 5, if  $\mathcal{R}_{y_b}(y_a, y_a) > 0$ , a small increase of marginal taxes close to  $y_a$  leads to an increase of tax revenue. The condition under which this inequality holds have been characterized in the proof of Proposition 5. It holds if if

$$T'_0(y_a) < -\frac{1 - F(\tilde{\omega}^{0b}(y_a))}{f(\tilde{\omega}^{0b}(y_a))} \left(1 - \tilde{I}_0(\tilde{\omega}^{0b}(y_a))\right) \frac{y_\omega^*(0, 0, \tilde{\omega}^{0b}(y_a))}{y_\tau^*(0, 0, \tilde{\omega}^{0b}(y_a))} ,$$

or, equivalently,

$$T'_0(y_a) < \mathcal{D}^{up,b}(\tilde{\omega}^{0b}(y_a)) ,$$

which proves statement 1. in Proposition C.1.

## C.2 Non-negativity constraints

Binding non-negativity constraints on earnings are a particular type of bunching. The behavioral responses to a reform in the  $(\tau, y_a, y_b)$ -class with  $y_a > 0$  and  $\tau > 0$  then look as follows: There is a participation cutoff  $\hat{\omega}^P(\tau)$  so that individuals with  $\omega \leq \hat{\omega}^P(\tau)$ , choose earnings of zero after the reform, individuals with  $\omega \in (\hat{\omega}^P(\tau), \omega_a(\tau))$  choose  $y \in (0, y_a)$  after the reform, individuals with  $\omega \in [\omega_a(\tau), \bar{\omega}_a(\tau)]$  choose  $y = y_a$  after the reform, individuals with  $\omega \in (\bar{\omega}_a(\tau), \omega_b(\tau))$  choose  $y \in (y_a, y_b)$ , and individuals with  $\omega \geq \omega_b(\tau)$  choose  $y \geq y_b$ .

We leave it to the reader to verify that a small reform in the  $(\tau, y_a, y_b)$ -class raises revenue if and only if the conditions in Proposition 5 are fulfilled. While the accounting of behavioral responses has to include individuals with no income, the analysis in the end boils down to an analysis of the conditions under which  $\mathcal{R}_{y_b}(y_a, y_a) > 0$  holds, just as in the proof of Proposition 5. If we consider instead a reform in the  $(\tau, y_a, y_b)$ -class with  $y_a = 0$ , we modify marginal tax rates at a point of bunching. In this case the conditions in Proposition C.1 evaluated for  $y_a = 0$  clarify whether such a reform raises tax revenue. Once the revenue implications are clear, extensions of Propositions 6, 7 and 8 that allow for binding non-negativity constraints can be obtained along the same lines as in the body of the text.

## D Monotonic reforms and Pareto bounds

This Appendix contains additional information on our empirical analysis. Specifically, Section D.1 provides more details on the descriptive statistics in the main text that document the frequency of monotonic reforms in actual tax policy. These descriptive statistics are based on an analysis of changes of the statutory income tax system. For the example of the United States, Section D.2 discusses the prevalence of monotonic reforms using an alternative and more detailed data source, the NBER TAXSIM micro-simulation model. Finally, in Section D.3, we provide additional examples that illustrate how our analysis can be used to construct Pareto bounds for marginal tax rates.

### D.1 Descriptive statistics on tax reforms based on OECD data

The OECD provides annual data on key parameters of the statutory personal income tax systems of its member countries (central governments).<sup>36</sup> In particular, it documents personal income tax rates for wage income and the taxable income thresholds at which these statutory rates apply. The information is applicable for a single person without dependents. We use this information to construct the corresponding tax function. A reform takes place if this tax function changes from one year to the next. The OECD also reports personal allowances and tax credits, and we include these parameters in our tax functions. In many countries these allowances are equivalent to having a first bracket with a marginal tax rate of zero, see, for instance, Belgium, Estonia, Japan, Spain, the United Kingdom, or the United States. In other countries tax credits are equivalent to a first bracket with a marginal tax rate of zero, see, for instance, the Czech Republic, Italy, or the Netherlands.<sup>37</sup> In the supplementary material for this paper we present separate statistics for different OECD countries. More specifically, the following countries are covered: Australia, Austria, Belgium, Canada, Chile, Czech Republic, Denmark, Estonia, Finland, France, Greece, Hungary, Iceland, Ireland, Israel, Italy, Japan, Korea, Latvia, Luxembourg, Mexico, Netherlands, New Zealand, Norway, Poland, Portugal, Slovak Republic, Spain, Sweden, Switzerland, Turkey, United Kingdom, United States. We excluded Slovenia because of an inconsistency in the OECD database for this country and Germany because of an incorrect representation of the German tax system in the OECD database.<sup>38</sup>

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<sup>36</sup>The database provided by the OECD is Table I.1. Central government personal income tax rates and thresholds accessible on <http://stats.oecd.org/Index.aspx?DataSetCode=TABLEI1>.

<sup>37</sup>Additional details on the methodology applied by the OECD is accessible on <http://www.oecd.org/ctp/tax-policy/personal-income-tax-rates-explanatory-annex.pdf>.

<sup>38</sup>By and large, this does not affect the overall frequency of monotonic reforms. If we include Germany and base the analysis on data from the German Federal Ministry of Finance, accessible on <https://www.bmf-steuerrechner.de/index.xhtml;jsessionid=46D8EC6083BF2573A42C23A2B03B49DF>, then 80% of the reforms in OECD countries are found to be monotonic. When Germany is excluded the number is 78%.

## D.2 OECD data and the NBER TAXSIM database

The NBER TAXSIM database provides more detailed information on the US federal income tax than the OECD.<sup>39</sup> Examples of features that are disregarded by the OECD but included in the NBER TAXSIM database are personal exemptions and the earned income tax credit.

For the period 1981 – 2016, we compare the results based on the tax functions defined with OECD database to those that are obtained by focussing on the “Federal income tax liability including capital gains rates, surtaxes, alternative minimum taxes (AMT) and refundable and non-refundable credits” (*fittax* variable) for “single or head of household (unmarried)” childless taxpayer with only wage and salary income (including self-employment - *pwages* variable).<sup>40</sup> We left the age of the individual undetermined but we assumed that the taxpayer is eligible for the EITC and the full AMT exclusion but not for any age exemption or supplemental standard deduction.

The number of reforms that are monotonic over the whole range of incomes becomes small when we include all parameters of the tax system that are included in the NBER TAXSIM database: it drops to 6% – instead of the 80% based on OECD database. There are now 33 non-monotonic reforms, 10 of those involve only small non-monotonicities (1982, 1985, 1986, 1987, 1988, 2001, 2002, 2003, 2010, 2012) and 18 are monotonic for above or below median incomes (1989, 1990, 1991, 1992, 1993, 1998, 1999, 2000, 2004, 2005, 2006, 2007, 2008, 2011, 2013, 2014, 2015, 2016), leaving only five cases that do not seem to be covered by the median voter results for monotonic tax reforms that we present in the main text.

The reforms that were monotonic under the OECD definition of the tax function and that are no longer monotonic under the TAXSIM definition are the ones of the years: 1982, 1985, 1986, 1989, 1990, 1992, 1994, 1996, 1997, 1998, 1999, 2001, 2002, 2003, 2004, 2005, 2006, 2007, 2008, 2009, 2010, 2011, 2012, 2014, 2015, 2016. Figures 13, 14, and 15 document some of these examples. Figures 13 and 14 show examples of tax reforms for which the OECD and the NBER TAXSIM databases display the same monotonicity properties. In these figures, the left column displays the reform-induced change in tax payments over the whole range of incomes and the right column zooms in on low incomes. Figure 15 shows examples of reforms which are monotonic over the whole range of incomes according to the OECD but not according to the NBER TAXSIM database.

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<sup>39</sup>The NBER TAXSIM (Version 9) database is accessible on <http://users.nber.org/taxsim/taxsim9/>.

<sup>40</sup>All other sources of income or possible deductions are set to 0.

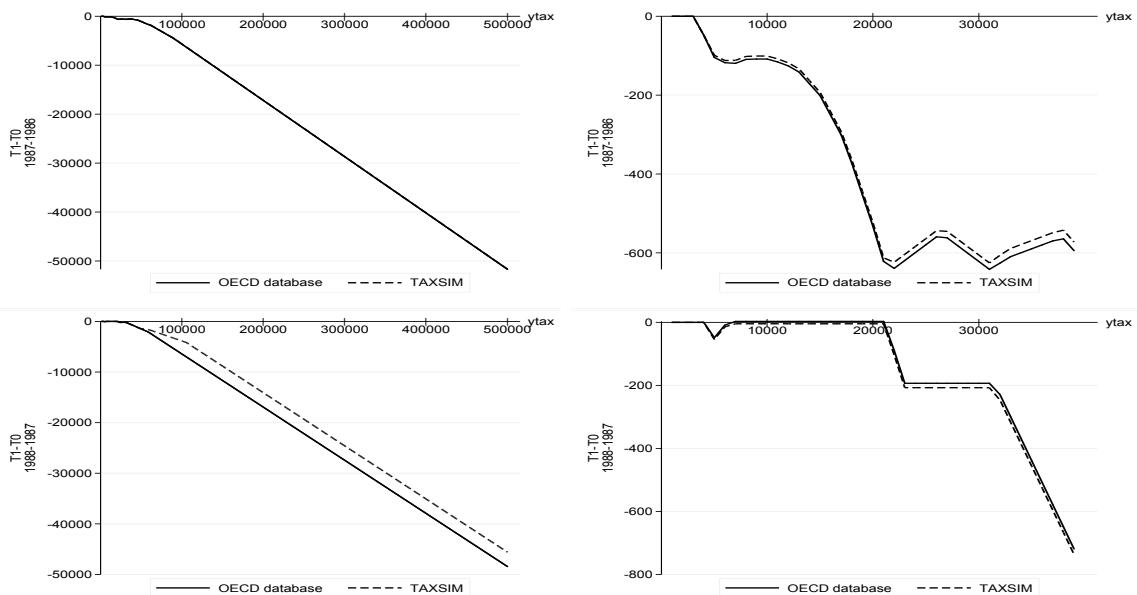


Figure 13: Reagan tax cuts: 1987 and 1988

Figure 13 shows the TRA'86 reforms that were implemented in the years 1987 and 1988.

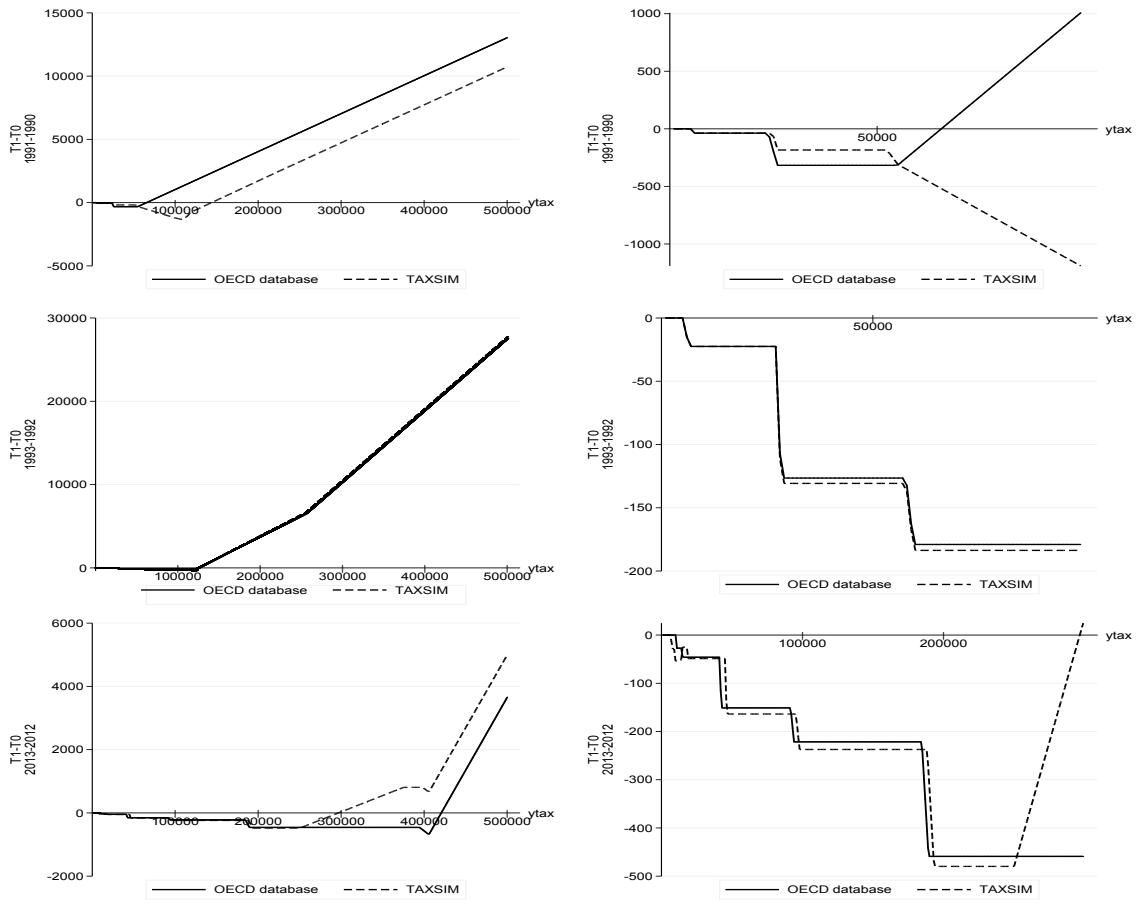


Figure 14: Reforms of the US income tax in 1991, 1993, and 2013

Figure 14 shows the reforms were implemented in years 1991 (first row), 1993 (OBRA'93, second row) and 2013 (third row).

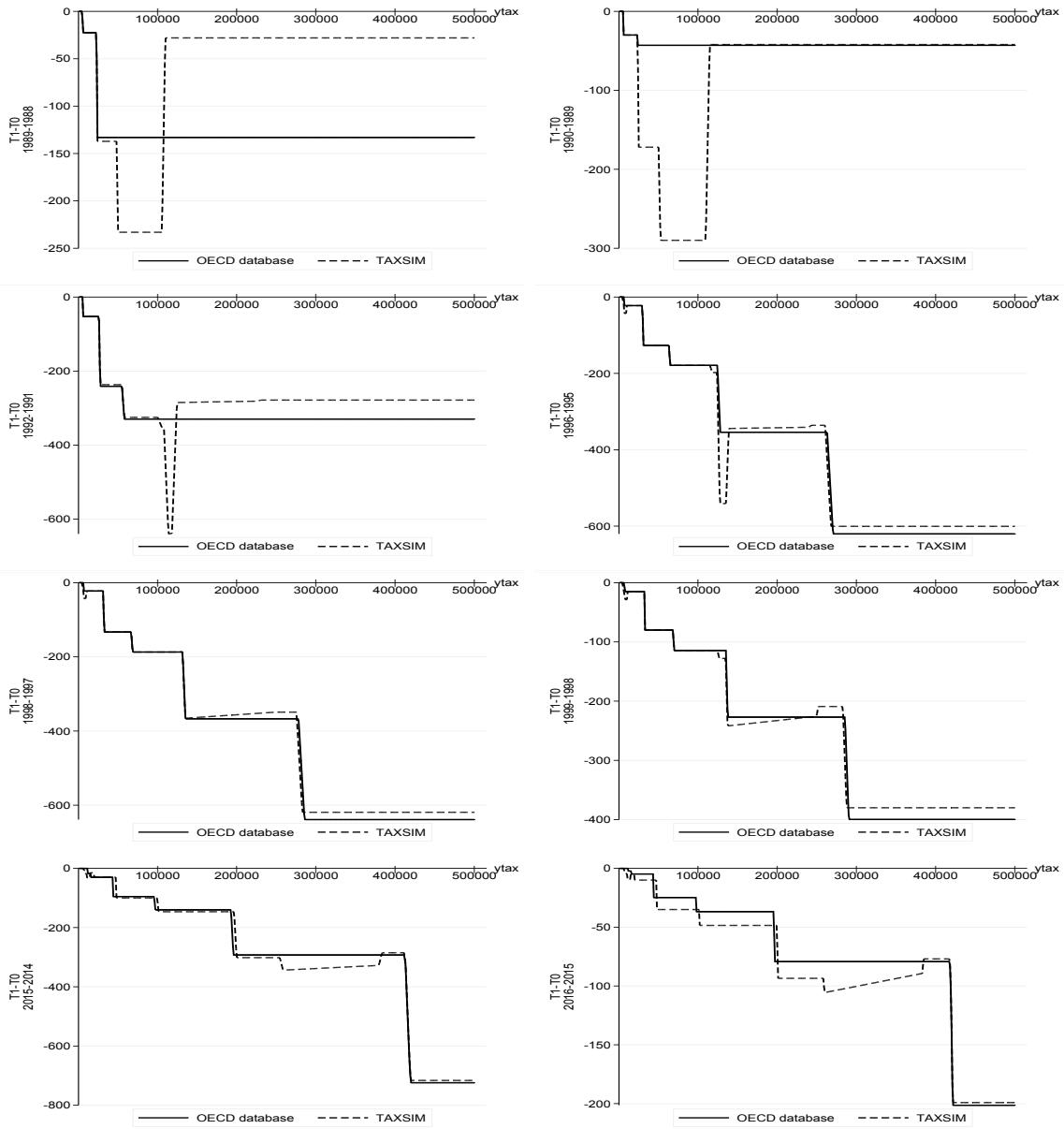


Figure 15: Monotonic reforms under OECD definition / non-monotonic under TAXSIM definition - US income tax

Figure 15 shows the reforms that were implemented in years 1989 and 1990 (first row), 1992 and 1996 (second row), 1996 and 1998 (third row), and 2015 and 2016 (fourth row).

### D.3 Constructing Pareto bounds: Additional Examples

We construct Pareto bounds for the French and the UK income tax system for the years 2012 and 2013. This complements the analysis for the US that is presented in the main text. The exercise is meant to be illustrative in the sense of showing how our theoretical approach can be brought to the data. It is not meant as a substantive analysis of the efficiency of the income tax systems in France, the UK or the US. The latter would warrant additional investments in data quality. Here, this would lead us astray.

An essential input for this construction is data on the distribution of incomes. We use data from the World Wealth and Income Database for France and the US,<sup>41</sup> and from HM Revenue & Customs (HMRC) for the UK.<sup>42</sup> More precisely, the US and France are joint-taxation countries and we use the data of the income distribution for equal-split adults.<sup>43</sup> For the UK we use the tax unit (as defined by the tax law) as the observation unit since taxation is individual-based. We apply the methodology developed by Blanchet, Fournier and Piketty (2017) to obtain the density and the cumulative distribution function characterizing the income distribution.<sup>44</sup>

Another essential input is data on statutory tax rates. Here, we use again the OECD tax database for the US, data from the Institut des Politiques Publiques available at [http : //www.ipp.eu/](http://www.ipp.eu/) for France, and data provided by the United Kingdom HM Revenue & Customs available at [https : //www.gov.uk/government/collections/tax-structure-and-parameters-statistics](https://www.gov.uk/government/collections/tax-structure-and-parameters-statistics). Figures 16 and 17 show the relationship between the current schedule and the upper Pareto bounds in the UK and France, respectively, in 2012 and 2013.

Figures 18 and 19 show the relationship between the current schedule and the lower Pareto bounds in the UK and France, respectively, in 2012. Due to problems of data availability, the UK schedule does not incorporate earnings subsidies.

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<sup>41</sup>The database can be accessed on *wid.world*.

<sup>42</sup>The database comes from Table 2.5 Income tax liabilities by income range accessible on [https : //www.gov.uk/government/statistics/income-tax-liabilities-by-income-range](https://www.gov.uk/government/statistics/income-tax-liabilities-by-income-range).

<sup>43</sup>For the methodology, see Alvaredo, Atkinson, Chancel, Piketty, Saez and Zucman (2016). For robustness, we compared the results with the Pareto bounds for tax units instead of equal-split adults and find that the cutoff values for the ETI are in the same range.

<sup>44</sup>This methodology implies that the distribution for the UK has a Pareto-tail. Our exercise is meant to illustrate the construction of Pareto bounds. For this purpose, we do not take a stance on which method for constructing the income distribution is most appropriate.

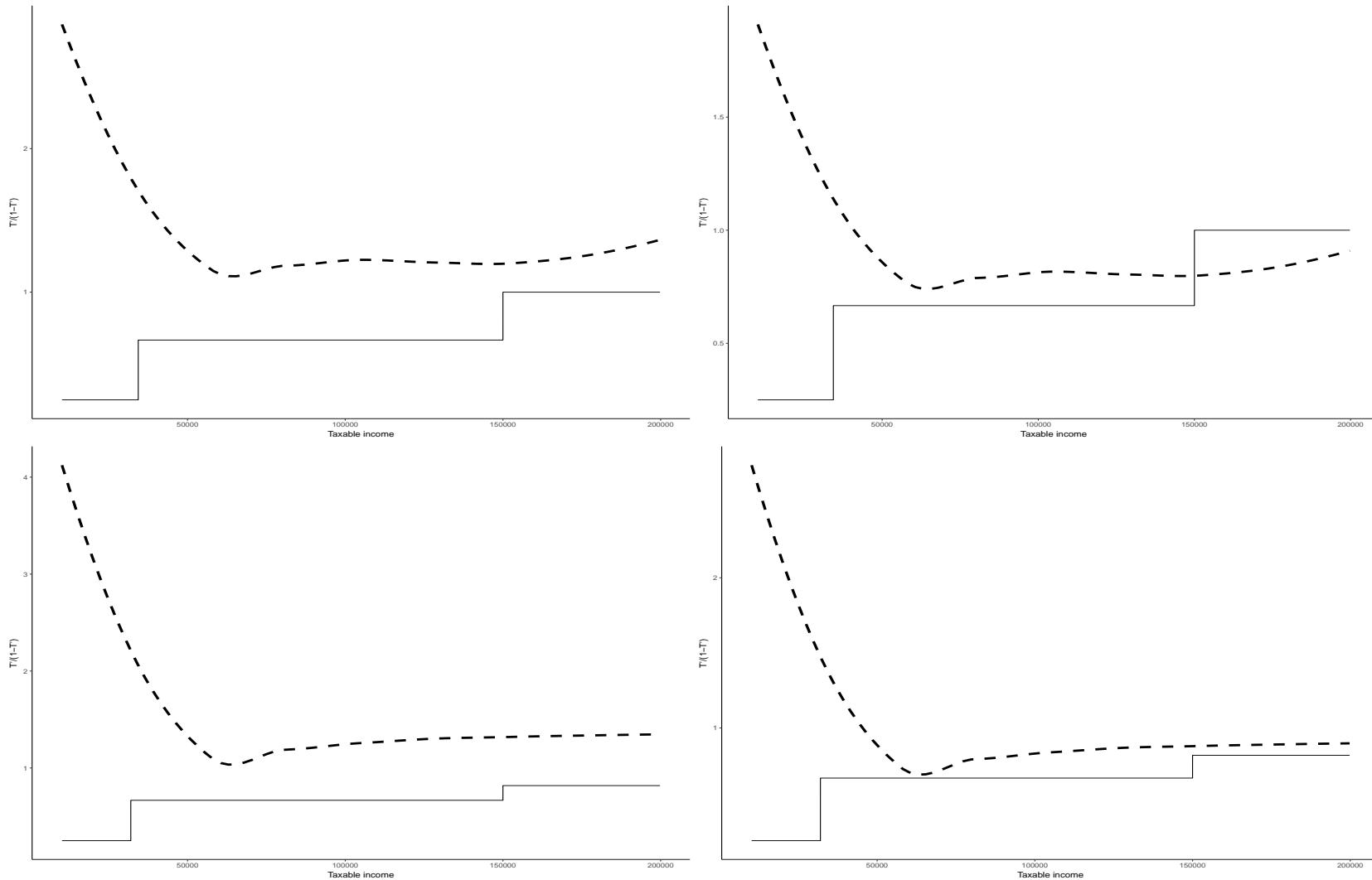


Figure 16: Upper Pareto bounds for the UK income tax in 2012 and 2013

Figure 16 relates the upper Pareto bounds  $\tilde{\mathcal{D}}^{up}$  (dashed line) to the UK income tax system in 2012 and 2013. The figures on the left are drawn for an ETI of 0.4, the figures on the right for an ETI of 0.6. Figure 16 is based on the information provided by the United Kingdom HM Revenue & Customs (available at <https://www.gov.uk/government/collections/tax-structure-and-parameters-statistics> and <https://www.gov.uk/government/statistics/income-tax-liabilities-by-income-range--2>).

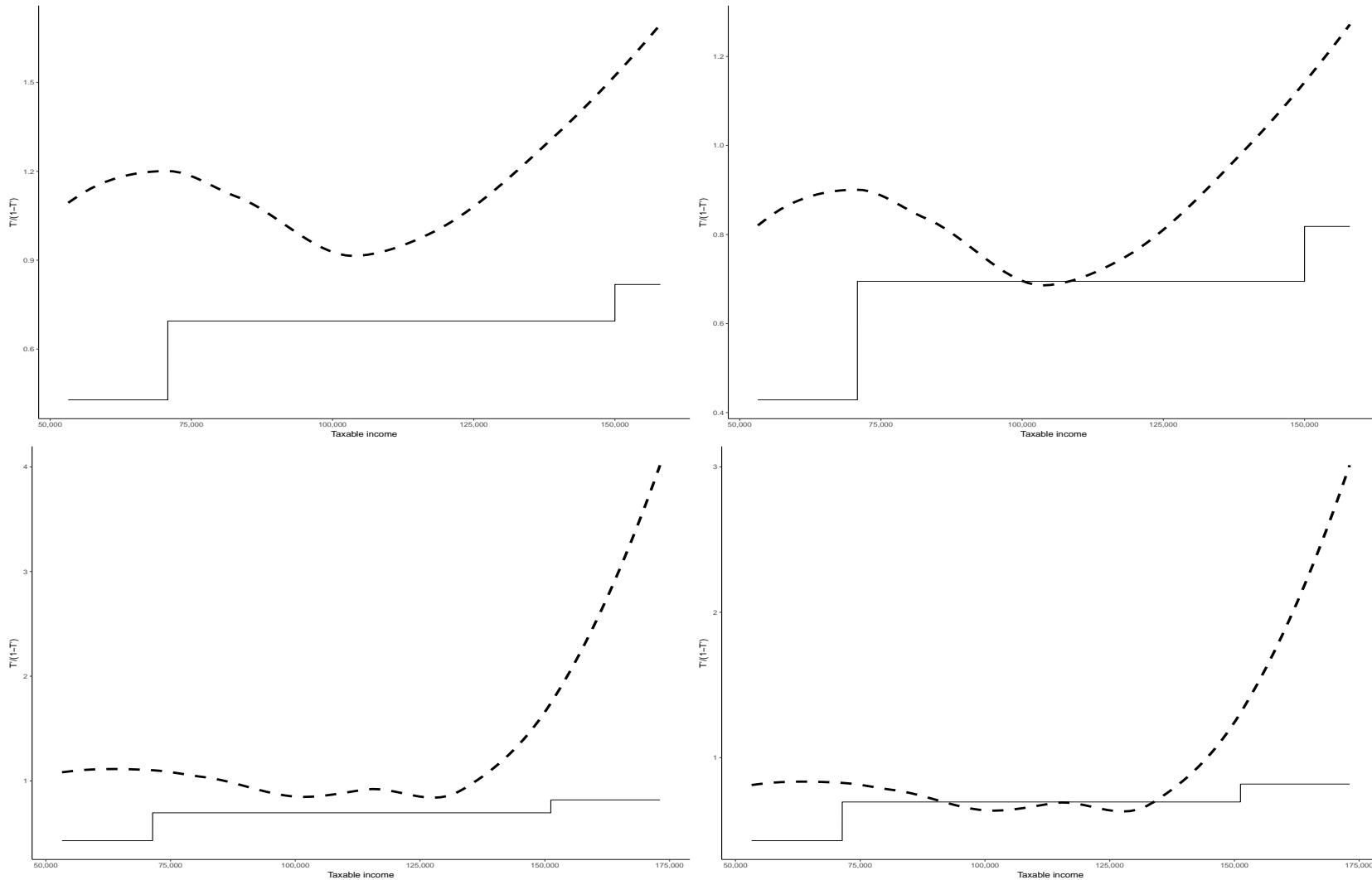
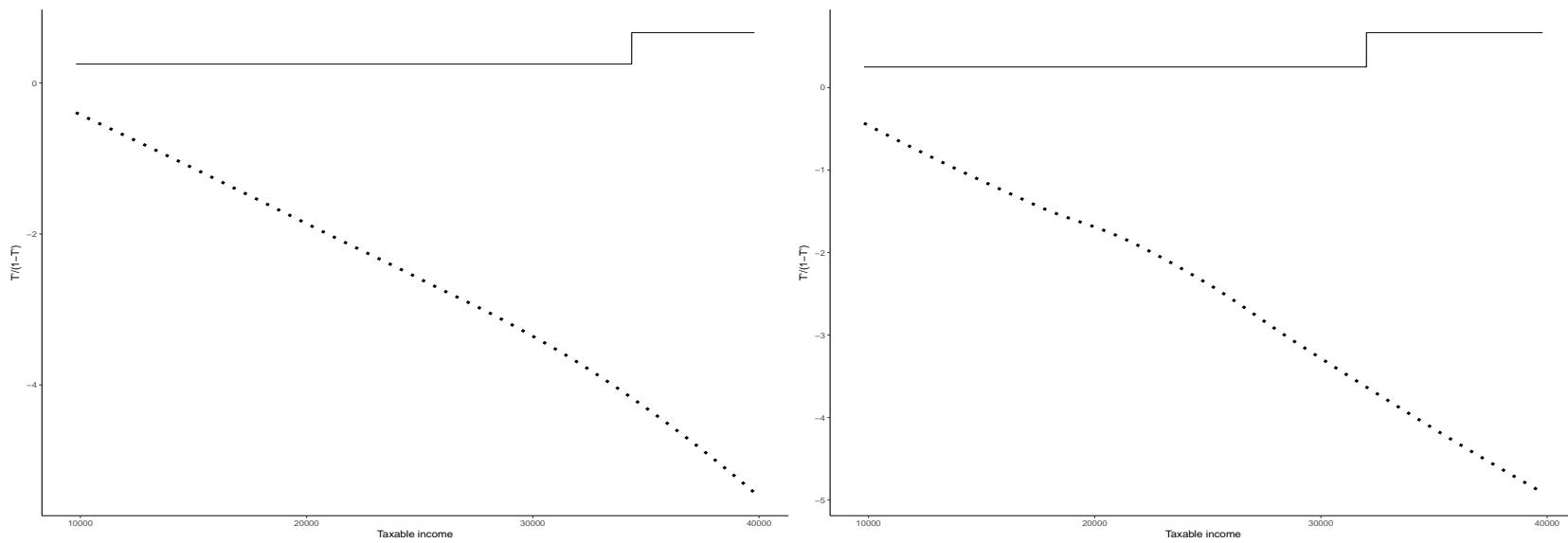


Figure 17: Upper Pareto bounds for the French income tax in 2012 and 2013

Figure 17 relates the upper Pareto bounds  $\tilde{D}^{up}$  (dashed line) to the French income tax system in 2012 and 2013. The figures on the left are drawn for an ETI of 0.6, the figures on the right for an ETI of 0.8. Figure 17 is based on the information provided by Institut des Politiques Publiques (available at <http://www.ipp.eu/>) and the World Wealth and Income database (available at <http://wid.world/data/>).



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Figure 18: Lower Pareto bounds for the UK income tax in 2012 and 2013

Figure 18 relates the lower Pareto bounds  $\tilde{\mathcal{D}}^{low}$  (dotted line) to the UK income tax system in 2012 (first row) and 2013 (second row) for an ETI of 0.4.

Figure 18 is based on the information provided by the United Kingdom HM Revenue & Customs (available at <https://www.gov.uk/government/collections/tax-structure-and-parameters-statistics> and <https://www.gov.uk/government/statistics/income-tax-liabilities-by-income-range--2>).

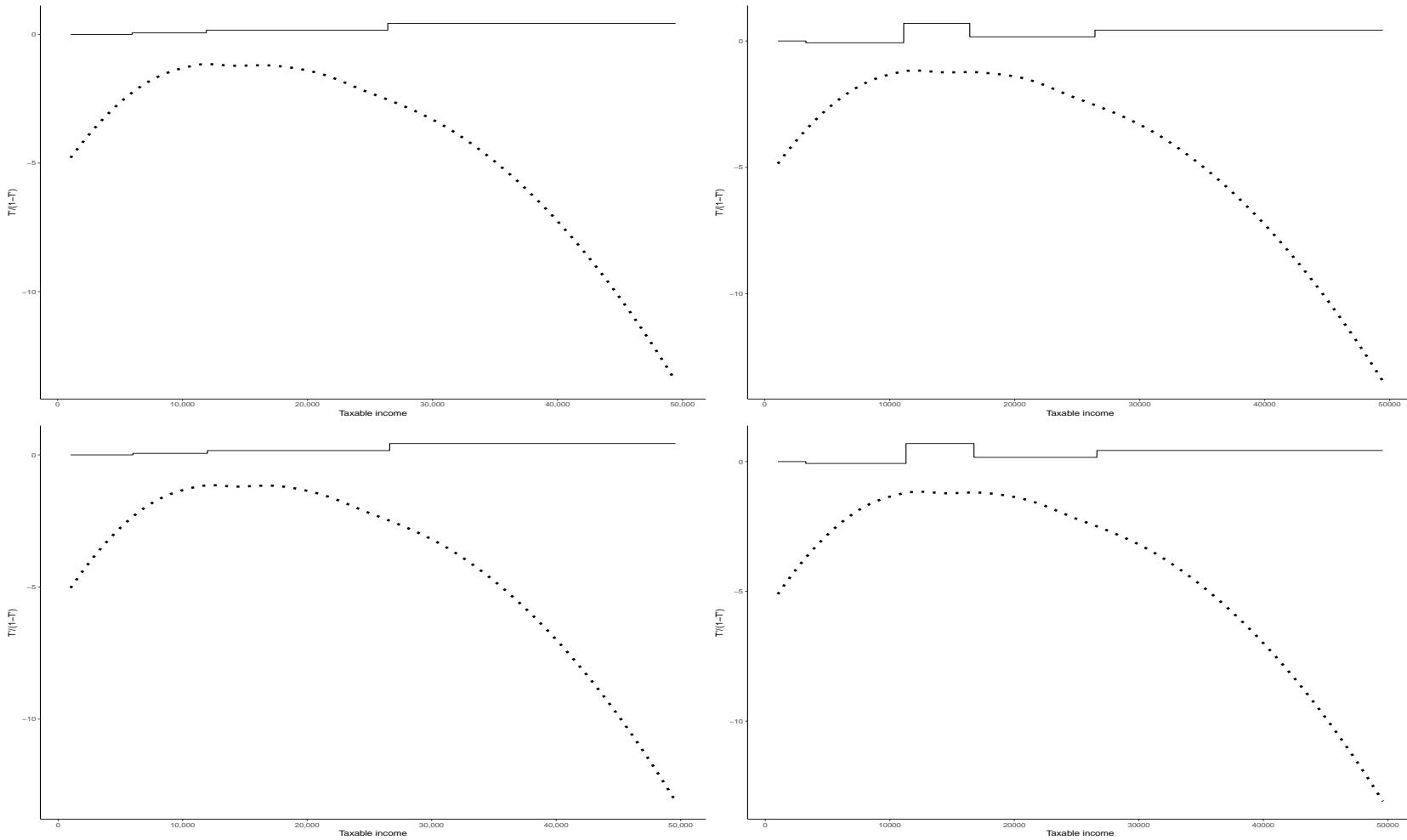


Figure 19: Lower Pareto bounds for the French income tax in 2012 and 2013

Figure 19 relates the lower Pareto bounds  $\tilde{D}^{low}$  (dotted line) to the French income tax system in 2012 (first row) and 2013 (second row) for an ETI of 0.6. The figures on the left are drawn for the statutory schedule taken from the OECD database and the one on the right represents the full schedule with earning subsidies for singles without dependents taken from the TAXIPP database. Figure 19 is based on the information provided by OECD database (accessible on <http://www.oecd.org/tax/tax-policy/tax-database.htm>), the Institut des Politiques Publiques (available at <http://www.ipp.eu/>), and the World Wealth and Income database (accessible on <http://wid.world/data/>).