Pareto-improving tax reforms and the Earned Income Tax Credit[*]

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August 24, 2021

Abstract

This paper provides necessary and sufficient conditions for the existence of Pareto-improving tax reforms. The conditions can be expressed as sufficient statistics and have a wide range of potential applications in public finance. We discuss one such application in detail: the introduction of the Earned Income Tax Credit (EITC) in the US. We find that the EITC can be viewed as a response to an inefficiency in the tax and transfer system prevailing at the time. This adds a new perspective to the literature on why the EITC is a good idea, emphasizing Pareto improvements rather than equity-efficiency trade-offs.

Keywords: Tax reforms; Non-linear income taxation; Optimal taxation; Earned Income Tax Credits; Pareto Efficiency.

JEL classification: C72; D72; D82; H21.

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1 Introduction

This paper presents new results on Pareto-efficient income taxation. Specifically, it provides necessary and sufficient conditions for the existence of a Pareto-improving reform direction. The analysis is based on a general framework that nests prominent models of taxation as special cases. We foresee a range of potential applications in public finance that combine our characterization with sufficient statistics for the revenue implications of tax reforms.

We discuss one such application in detail, the introduction of the Earned Income Tax Credit (EITC) in the US in the mid-1970s. For this application, we derive sufficient statistics from a model with fixed costs of labor market participation and variable costs of productive effort.[1] We find that the introduction of the EITC can be viewed as a response to an inefficiency in the tax and transfer system prevailing at the time. The judgment that the introduction of an EITC was a "good idea" is remarkably robust: it holds for any Paretian welfare function and for all empirically plausible values of labor supply elasticities at the intensive and the extensive margin.

A theory of Pareto-improving tax reforms. Our theoretical analysis is motivated by two observations: first, past reforms of the EITC typically involved *two brackets*, a phase-in range with lower marginal tax rates and a phase-out range with higher tax rates. Second, an observation on the typical thought experiment in the literature that uses perturbation methods for a characterization of optimal tax systems: it analyzes the welfare implications of lowering or raising the marginal tax rates in *one bracket*.[2]

These observations raise the question whether reforms with two brackets can do "more" than reforms involving a single bracket. Suppose that a given tax system

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[1] This framework is prominent in the literature that studies the EITC from an optimal tax perspective – see Saez (2002), Choné and Laroque (2011), Jacquet, Lehmann and Van der Linden (2013), or Hansen (2021).

[2] See Piketty (1997), Saez (2001), Golosov, Tsyvinski and Werquin (2014), or Jacquet and Lehmann (2021).

is "one-bracket-efficient" in the sense that there does not exist a Pareto-improving one-bracket reform. Can there be reforms with two brackets that make everyone better off? We show that the answer is "yes", i.e., reforms involving two brackets can achieve more than reforms with one bracket.

This leads to the next question. Suppose that the scope for Pareto-improving two-bracket reforms has been exhausted. Can there be reforms with three or even more brackets that make everyone better off? We show that the answer is "no", i.e., if there is no Pareto-improving reform involving one or two brackets, then there is no Pareto-improving element in the set of continuous reform directions.

These findings are derived from a generic static model of taxation: Individuals derive utility from consumption and the generation of income requires costly effort. They face a budget constraint that is shaped by a predetermined non-linear tax-transfer system. We consider reforms of this tax system so that marginal tax rates are changed simultaneously in an arbitrary number of income brackets. Also, there is full flexibility in terms of locating those brackets in the range of possible incomes. We then focus on the limit case of small reforms, involving marginal changes of tax rates over finitely many brackets of infinitesimal length. The interpretation is that we consider directions for reform in a neighborhood of a given status quo.

Making use of the theory. Our results provide guidance for the design of tax systems. There are two broad insights: *"Two is more than one!"*, one should not miss the additional opportunities that come with two-bracket reforms. *"Two is enough!"*, one does not miss reform opportunities by focusing on reforms with one or two brackets.

How can one use these insights? More specifically, how can one figure out whether a given tax and transfer system admits a Pareto-improving reform? Our analysis yields a test function that gives, for each income level y, the revenue implication $\mathcal{R}(y)$ of a small one-bracket reform in a neighborhood of y. The test for Pareto efficiency then makes use of the following insights:

1. There is no Pareto-improving one-bracket reform if and only if the function $y \mapsto \mathcal{R}(y)$ is bounded from below by 0 and from above by 1. These bounds

admit an interpretation as Laffer conditions which, respectively, indicate whether marginal tax rates are inefficiently high or inefficiently low.

2. There is no Pareto-improving two-bracket reform if and only if the function $y \mapsto \mathcal{R}(y)$ is non-increasing. A violation of this monotonicity condition implies that the tax system can be Pareto improved by an EITC-like reform – i.e. a two bracket reform with a phase-in and a phase-out range.

Thus, all that is needed to test for Pareto efficiency are sufficient-statistics formulas for the revenue implications of small tax reforms, each involving a modification of the marginal tax rate in a single bracket. The literature using perturbation methods in optimal taxation provides many examples of such sufficient-statistics formulas. Upon squaring our results with the formulas from that literature, one obtains a simple and complete test for Pareto efficiency. For concreteness, we present such sufficient-statistics formulas for a Mirrleesian model of income taxation with behavioral responses only at the intensive margin, and an extended model that also involves fixed costs of labor market participation.

When a tax system fails the Pareto test, this raises further questions. Is it possible to identify Pareto-improving direction for reform? Put differently, is it possible to identify "progress in tax policy"? Moreover, is it possible to measure how inefficient a given tax system is? Such a measure makes it possible to relate an inefficiency prevailing in one part of the tax system to another one prevailing in some other part of the tax system. For instance, a question that arises in the context of our application is whether the inefficiency in the tax system faced by childless singles was smaller than the inefficiency in the tax system for single parents.

We provide answers to these questions along the following lines: As we show, diagnosing whether an arbitrary tax perturbation is Pareto-improving is straightforward once an estimate of the reform's revenue implications is available. We also show how such an estimate can be obtained when the revenue implications of simple one-bracket reforms are known, or, equivalently, when the function $y \mapsto \mathcal{R}(y)$ is known. Thus, the information that is needed to test for Pareto efficiency can

also be used to identify Pareto-improving directions for reform.

We also propose a money-metric measure of how inefficient a given tax system is. Specifically, we characterize the Pareto-improving direction for reform that yields the largest revenue gain above what is needed to ensure that no agent is made worse off. When this maximal revenue gain is small, the tax system is close to the Pareto frontier, otherwise it is not. Again, this measure can be computed when the function $y \mapsto \mathcal{R}(y)$ is known.

The introduction of the EITC. We look at the introduction of the EITC in the US through the lens of this framework. The introduction of the EITC in 1975 was a substantial policy change for many low-income households, see, e.g., Bastian (2020). It was meant as a response to excessively high marginal tax rates for families that depended on welfare. We use this setting as a testbed for our approach. Specifically, we derive the requisite sufficient statistics from a model with behavioral responses at the intensive and the extensive margin. Thereby, we obtain a test function $y \mapsto \mathcal{R}(y)$, which we then use to investigate whether or not the system prevailing at the time was Pareto-efficient, and whether reforms of the EITC in subsequent years went into a Pareto-improving direction.

We find that, prior to the introduction of the EITC, the function $y \mapsto \mathcal{R}(y)$ was increasing over certain income ranges, indicating the existence of a Pareto-improving two-bracket reform. The introduction of the EITC did not fully remove these inefficiencies, and left room for further Pareto improvements by means of two-bracket reforms. The first EITC expansion in 1979 then went into a Pareto-improving direction. These findings are shown to be robust with respect to alternative assumptions about the behavioral responses to taxation, in particular the extensive margin and intensive margin elasticities of labor supply. Thus, both the introduction and the subsequent expansion of the EITC can be rationalized through the lens of our framework.

The EITC and the theory of optimal taxation. Previous literature on the desirability of the EITC has used an optimal tax approach, thereby providing an

answer to the following question: Are negative marginal taxes, or, equivalently, earnings subsidies for the "working poor" part of a tax policy that maximizes a utilitarian social welfare function? Providing a positive answer is not straightforward. The workhorse of the optimal tax literature, the Mirrlees (1971) model, stipulates non-negative marginal tax rates for all levels of income.[3] Thus, the EITC is a challenge for the theory of optimal taxation. In response to that challenge, Saez (2002) suggested the use of an extended version of the Mirrlees model that includes fixed costs of labor market participation and gives rise to behavioral responses both at the intensive and the extensive margin. With such a framework, there are conditions under which the EITC can be justified as being part of a welfare-maximizing policy, see Saez (2002) and Hansen (2021).[4]

Our analysis complements these findings by focusing on the tax and transfer system that prevailed when the EITC was introduced, and by taking a tax reform perspective. This relates our approach to a literature in public finance that emphasizes the analysis of reforms, i.e., of incremental changes of a given system, as opposed to an analysis of optimal tax systems.[5] With a tax reform perspective, we find that the EITC can be rationalized under weaker conditions than with an optimal tax perspective. First, we find that the introduction of the EITC was Pareto-improving, and not just utilitarian-welfare-improving. Second, when exploring alternative assumptions about intensive and extensive margin elasticities, we find that the EITC was Pareto-improving even without behavioral responses at the extensive margin. Thus, the introduction of the EITC was a good idea – even under the behavioral assumptions of the basic Mirrlees model.[6]

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[3]Negative marginal tax rates can be rationalized in the basic version of the Mirrlees model only with a welfare function that has non-monotonic welfare weights, e.g., one that assigns higher weights to people with intermediate incomes than to people with low incomes; see Stiglitz (1982), Choné and Laroque (2010), or Brett and Weymark (2017).

[4]For a non-utilitarian assessment based on generalized weights, see Saez and Stantcheva (2016).

[5]See Piketty (1997), Saez (2001), Golosov et al. (2014), or Jacquet and Lehmann (2021). For earlier contributions to the analysis of tax reforms, see Feldstein (1976), Weymark (1981) and the review in Guesnerie (1995).

[6]Kleven (2019) suggests that previous estimates of extensive margin elasticities were too high.

Outline. The remainder is organized as follows. The next section discusses the related literature. Section 3 contains our theory of Pareto-improving tax reforms which yields a characterization of necessary and sufficient conditions for the Pareto efficiency of a tax system. Section 4 shows how can identify Pareto-improving reform directions. It also introduces a quantitative measure that makes it possible to compare different tax systems according to how inefficient they are. Section 5 contains the application of these conditions to the introduction of the EITC in the US. Formal proofs are relegated to the Appendix.

2   Related literature

We build on and extend the existing literature on Pareto-efficient non-linear taxation. Previous literature has generalized the notion of a Laffer bound to non-linear tax schedules, see Stiglitz (1982), Brito, Hamilton, Slutsky and Stiglitz (1990) and, more recently, Badel and Huggett (2017).[7] Bierbrauer, Boyer and Peichl (2021) show that there is not only an upper Pareto bound, but also a lower Pareto bound for marginal tax rates. This lower bound is relevant for an assessment of earnings subsidies: if the bound is violated, then a reduction of these subsidies is Pareto-improving. Werning (2007) and Lorenz and Sachs (2016) develop a test for the Pareto efficiency of a given status quo tax schedule that involves a differential equation that describes how marginal tax rates change along the income distribution.[8] Failures of Pareto efficiency are also identified by the literature on the

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While this debate has a bearing on the desirability of the EITC from an optimal tax perspective, it is of no consequence for our conclusion that the introduction of the EITC was a reform that went into a Pareto-improving direction.

[7]There is a literature deriving the second-best Pareto frontier for a two-type Mirrlees model with contributions by Stantcheva (2014), Bierbrauer and Boyer (2014), and Bastani, Blomquist and Micheletto (2020). See, for reviews, Stiglitz (1987) and Boadway and Keen (2000).

[8]Werning (2007) considers a Mirrleesian framework where behavioral responses to taxation arise only at the intensive margin. Lorenz and Sachs (2016) consider, in addition, extensive margin responses. For related work, see also Blundell and Shephard (2012), Scheuer (2014), Koehne and Sachs (2019), or Hendren (2020).

inverse tax problem.[9] This literature identifies the welfare function that is maximized by an observed tax schedule. If this approach yields a welfare function with negative weights, this indicates that the tax policy under study is incompatible with the maximization of a Paretian social welfare function.[10]

All these contributions have in common that they focus on necessary conditions for Pareto efficiency. They do not provide sufficient conditions, i.e., there is no way of checking whether a given tax schedule satisfies *all* the conditions that are needed for Pareto efficiency. Instead, any one of these papers looks at a particular subset of these conditions. If the condition under consideration is violated, one can conclude that the given tax system can be reformed in a Pareto-improving way. If instead the condition is satisfied, one cannot conclude that the tax system is Pareto-efficient. The possibility of some other Pareto-improving reform remains.

In our approach, we consider an arbitrary number of brackets that can be distributed in an arbitrary way over the range of possible incomes. Allowing for a larger class of reforms than the previous literature enables us to show that, taken together, the conditions in Bierbrauer et al. (2021), on the one hand, and the conditions by Werning (2007) and Lorenz and Sachs (2016), on the other, imply Pareto efficiency.[11]

We moreover present sufficient-statistics formulas that can be used to check whether the necessary and sufficient conditions for the existence of Pareto-improving reform directions are satisfied. This relates our analysis to a broad literature employing sufficient statistics for policy evaluation, see Chetty (2009) and Kleven (2021) for reviews of this approach.

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[9]See, e.g., Christiansen and Jansen (1978), Blundell, Brewer, Haan and Shephard (2009), Bourguignon and Spadaro (2012), Bargain, Dolls, Neumann, Peichl and Siegloch (2011), Jacobs, Jongen and Zoutman (2017), Lockwood and Weinzierl (2016), or Hendren (2020).

[10]In part E of the Supplementary Material, we explain in detail what the conditions for Pareto efficiency in Theorems 1 and 2 imply for the inverse tax problem.

[11]A qualification needs to be added: Werning (2007) and Lorenz and Sachs (2016) impose assumptions of differentiability that are not needed in our approach. We get back to the differential equations in Werning (2007) and Lorenz and Sachs (2016) after presenting our main results in Section 3.

Our analysis of the introduction of the EITC in the US draws on the literature that provides estimates of the behavioral responses involved.[12] There is a range of estimates and some of the conventional wisdom in the literature has recently been challenged.[13] Specifically, Bastian (2020) estimates the behavioral responses to the 1975 EITC introduction, the reform that we focus on as well. Bastian and Jones (2021) provide an econometric analysis of the extent to which EITC expansions since the 1990s were self-financing. Their analysis takes account of a wide range of potential fiscal externalities associated with an expansion of the EITC.

3   Pareto-improving tax reforms

In this section, we present results on Pareto-efficient income taxation and Pareto-improving tax reforms. These results are general in the sense that they are not tied to a specific setup, such as a Mirrleesian model or a model with fixed costs of labor market participation.

3.1   The model

We consider an economy with a continuum of individuals. Individuals value consumption c and generate earnings y. The generation of earnings comes with effort costs that depend on a vector of individual characteristics $\theta \in \Theta \subset \mathbb{R}^n$. The cross-section distribution of θ is assumed to be atomless and represented by a cumulative distribution function F. Preferences are represented by the utility function $u : \mathbb{R}_+^2 \times \Theta \to \mathbb{R}$. Thus, $u(c, y, \theta)$ is the utility that a type θ individual derives from a bundle (c, y). The function u is continuously differentiable

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[12]Prominent references are Eissa and Liebman (1996), Meyer and Rosenbaum (2001), Moffitt (2003), Eissa and Hoynes (2004), and Blundell (2006). For surveys, see Hotz and Scholz (2003), Nichols and Rothstein (2015), and Hoynes (2019).

[13]For single mothers, early papers such as Meyer and Rosenbaum (2001) found large participation elasticities (sometimes above 1) along with only small responses at the intensive margin, while more recent papers such as Bastian (2020) and Kleven (2019) find smaller participation elasticities.

and increasing in the first argument, with partial derivative denoted by u_c. It is decreasing in the second argument y.

We assume that a single-crossing condition holds in one dimension of the type space, Θ_j: If type (θ_j, θ_{-j}) weakly prefers a bundle (c, y) to another bundle $(c', y') < (c, y)$, then type (θ'_j, θ_{-j}) with $\theta'_j > \theta_j$ strictly prefers (c, y) to (c', y'). This assumption implies that the individuals' earnings are increasing in θ_j.

Two special cases of this setup are of particular interest. First, a utility function that is quasilinear in consumption and has iso-elastic effort costs, in combination with a one-dimensional type space, i.e.,

$$ u(c, y, \theta) = c - \frac{1}{1 + \frac{1}{\varepsilon}} \left(\frac{y}{\theta} \right)^{1 + \frac{1}{\varepsilon}} , $$

where $\theta \in \Theta \subset \mathbb{R}_+$. The analysis of Diamond (1998) is based on this framework. In this specification, the type θ is a measure of productive ability, often identified with an hourly wage. This case is of pedagogical interest. It is the simplest framework that we can use for purposes of illustration. Second, a model with multidimensional heterogeneity due to fixed and variable effort costs of productive effort, and behavioral responses both at the intensive and the extensive margin. The analysis in Section 4 is based on such a framework.

There is a status quo tax policy. It is represented by a parameter c_0 and a tax function T_0, which jointly define the budget set $C_0(y) = c_0 + y - T_0(y)$ that individuals face. The parameter c_0 is the intercept of this consumption schedule. It is the transfer to individuals with no earnings. Without loss of generality, we let $T_0(0) = 0$.[14] We assume that T_0 is continuous. Otherwise, it can be an arbitrary non-linear tax function, possibly with kinks. Before the reform, individuals solve

$$ \max_{y \in \mathcal{Y}} \quad u(C_0(y), y, \theta) , $$

where $\mathcal{Y} = [0, \bar{y}]$ is a set of feasible earnings level.

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[14] In the literature, $T_0(y)$ is often referred to as the *participation tax*; see, e.g., Kleven (2014). This reflects that $T_0(y)$ is the additional tax payment of a person with earnings of y, relative to a person with no earnings. Alternatively, we could represent the status quo by a tax function $\tilde{T}_0$ so that $\tilde{T}_0(y) := -c_0 + T_0(y)$ with the implication that $\tilde{T}_0(0) = -c_0$. We find it more convenient to separate the transfer c_0 from the tax function.

A tax reform replaces T_0 by a new tax function T_1 so that $T_1 = T_0 + \tau\, h$. The scalar τ is a measure of the size of the tax reform and the function h gives the direction of the tax reform. Again, h is assumed to be a continuous function with $h(0) = 0$. For a given income y, the change in the tax burden due to the reform is therefore given by $T_1(y) - T_0(y) = \tau\, h(y)$. After the reform, individuals solve

$$\max_{y \in \mathcal{Y}} \quad u(C_1(y), y, \theta) \,, \tag{1}$$

where $C_1(y) = c_1 + y - T_0(y) - \tau\, h(y)$, and c_1 is the intercept after the reform. We denote the reform-induced changes in tax revenue by $R(\tau, h)$ and assume that it is absorbed by the intercept so that

$$c_1 = c_0 + R(\tau, h) \,.$$

Thus, any change in tax revenue is redistributed in a lump-sum fashion.

The change in tax revenue $R(\tau, h)$ is an endogenous object that depends on the behavioral responses to taxation. To see how it is determined, let $y^*(e, \tau, h, \theta)$ be the solution to (1), where

$$C_1(y) = c_0 + e + y - T_0(y) - \tau\, h(y) \,,$$

and e is a source of income that is exogenous from the individual's perspective. Also, let $y_0(\theta) := y^*(0, 0, h, \theta)$ be a shorthand for income in the status quo.[15] Then, $R(\tau, h)$ solves

$$R(\tau, h) = \mathbf{E}\left[T_1(y^*(R(\tau, h), \tau, h, \theta)) - T_0(y_0(\theta))\right] \,, \tag{2}$$

where the operator $\mathbf{E}$ indicates that we compute a population average using the distribution F.[16]

We denote by $v(\tau, h, \theta)$ the indirect utility that a type θ individual realizes after a tax reform (τ, h). We can use the analysis of *"Envelope theorems for arbitrary*

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[15] There may be types for whom the utility-maximization problem in (1) has multiple solutions. The function y^* is then taken to select one of them. How this selection is done is inconsequential for the analysis that follows.

[16] Brouwer's fixed point theorem can be used to establish the existence of a solution to this fixed point equation, in combination with conditions that ensure that $\mathbf{E}\left[T_1(y^*(e, \tau, h, \theta))\right]$ is continuous in e. This continuity is not immediate when the function y^* may exhibit jumps due

choice sets" in Milgrom and Segal (2002) to describe how individuals are affected by marginal changes of the reform intensity τ. Specifically, fix some type θ. Then, by Corollary 4 in Milgrom and Segal (2002),

$$\frac{d}{d\tau} \, v(\tau, h, \theta) = u_c(\cdot, \theta) \, [R_\tau(\tau, h) - h\,(y^*(\cdot))] \,, \tag{3}$$

where the marginal consumption utility of type θ, $u_c(\cdot, \theta)$, is evaluated at point $(C_1(y^*(\cdot)), y^*(\cdot))$, and $R_\tau(\tau, h)$ is the marginal effect of an increase in the reform intensity τ on tax revenue.[17] More formally, it is the Gateaux differential of tax revenue in direction h.[18] The envelope theorem covers cases in which the marginal tax rates (either in the status quo or after the reform) exhibit discontinuous jumps. It also applies when there are fixed costs of labor market participation, so that the utility function is, at $y = 0$, not continuous in y.

Equation (3) makes it possible to decompose the set of taxpayers into winners and losers of the tax reform. For concreteness, fix a reform direction h and suppose that, starting from the status quo policy, a small reform step has a positive impact on tax revenue, $R_\tau(0, h) > 0$. A taxpayer with type θ benefits from the reform if and only if this revenue gain outweighs the additional tax payment $h(y_0(\theta))$.

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to extensive margin responses or discontinuities in marginal tax rates. With a single crossing condition on preferences and an atomless type distribution, however, such jumps can be shown to wash out in the aggregate and therefore do not upset the continuity of $\mathbf{E}\,[T_1(y^*(e, \tau, h, \theta))]$ in e.

[17]For a type θ such that the utility-maximization problem in (1) has multiple solutions, the right-hand derivative of v is relevant for increases of τ and the left-hand derivative is relevant for decreases of τ.

[18]Our notation for Gateaux differentials is inspired by the one for partial derivatives. Conventions in mathematics are different. To make this explicit, let tax revenue $\mathcal{R}$ be a real-valued functional of the tax function T. Then, the Gateaux differential of tax revenue in direction h is formally defined as

$$\partial \mathcal{R}(T, h) := \lim_{\tau \to 0} \frac{\mathcal{R}(T + \tau\, h) - \mathcal{R}(T)}{\tau} \,,$$

where the left-hand side is the "typical" notation in the literature. Our notation can now be more formally introduced as $R_\tau(0, h) := \frac{d}{d\tau}\mathcal{R}(T_0 + \tau h)|_{\tau=0} = \partial\mathcal{R}(T_0, h)$. In Online Appendix A, we lay down further assumptions which guarantee the linearity of the Gateaux differential in the direction h, a property that is used in the proofs of Theorems 1 and 2.

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Hence, given reform direction h, a small increase of τ is Pareto-improving if

$$R_\tau(0,h) - \max_{y \in y_0(\Theta)} h(y) > 0 , \tag{4}$$

where $y_0(\Theta)$ is the image of function y_0.[19]

We say that there is no Pareto-improving direction in a class of reforms H if, for all functions $h \in H$,

$$R_\tau(\tau,h) - \max_{y \in y^*(\Theta)} h(y) < 0 . \tag{5}$$

The set H will be expanded as we go along. We first analyze classes of reforms with one or two income brackets in which marginal tax rates are changed. We then extend the results to tax reforms with finitely many brackets and, finally, cover the entire set of continuous reform directions h.

Single-bracket reforms.   A single-bracket reform is a pair (τ, h^s), where the function h^s is such that

$$h^s(y) = \begin{cases} 0, & \text{if } y \leq \hat{y} , \\ y - \hat{y}, & \text{if } y \in (\hat{y}, \hat{y} + \ell) , \\ \ell, & \text{if } y \geq \hat{y} + \ell . \end{cases}$$

for some threshold value of income $\hat{y}$, see Figure 1. Thus, a single-bracket reform is characterized by a triplet $(\tau, \ell, \hat{y})$, where $\hat{y}$ is the income level at which the bracket starts, ℓ is the length of the bracket and τ is the amount by which the marginal tax rate changes for incomes in the bracket.

After a one-bracket reform, the new tax schedule is given by

$$T_1(y) = T_0(y) + \tau h^s(y) = \begin{cases} T_0(y), & \text{if } y \leq \hat{y} , \\ T_0(y) + \tau(y - \hat{y}), & \text{if } y \in (\hat{y}, \hat{y} + \ell) , \\ T_0(y) + \tau\ell, & \text{if } y \geq \hat{y} + \ell . \end{cases}$$

Hence, the reform increases tax liabilities for all earnings above $\hat{y}$, with a maximum increase of $\tau\ell$. The marginal tax rate changes by τ for earnings in the bracket

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[19] A small reduction of τ for direction h is Pareto-improving if $R_\tau(0,h) - \min_{y \in y_0(\Theta)} h(y) < 0$, or, equivalently, if a small increase of τ for direction $-h$ is Pareto-improving.

$(\hat{y}, \hat{y} + \ell)$. It does not change for incomes above or below this bracket. Formally, the new schedule of marginal tax rates equals

$$T_1'(y) = T_0'(y) + \tau h^{s'}(y) = \begin{cases} T_0'(y), & \text{if } y \leq \hat{y} \,, \\ T_0'(y) + \tau, & \text{if } y \in (\hat{y}, \hat{y} + \ell) \,, \\ T_0'(y), & \text{if } y \geq \hat{y} + \ell \,. \end{cases}$$

(a) Reform direction h^s.   (b) Tax functions T_0 and T_1.
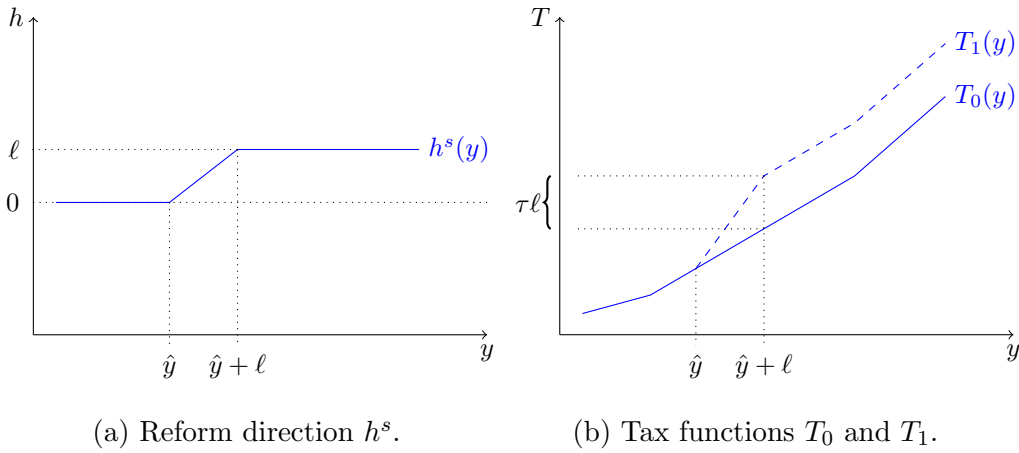
Figure 1: Income tax reforms with one bracket.

We will trace the welfare implications of multi-bracket reforms back to the properties of single-bracket reforms. It will prove convenient to have separate notation for the revenue implications of single-bracket reforms. For such reforms, we write $R^s(\tau, \ell, \hat{y})$ rather than $R(\tau, h^s)$. We write R^s_τ for the derivative of this function with respect to the first argument, and $R^s_{\tau\ell}$ for the cross-derivative with respect to the first and the second argument. It follows from first-order Taylor approximations that, for τ and ℓ close to zero,

$$\tau \, \ell \, R^s_{\tau\ell}(0, 0, y)$$

is a good approximation of $R^s(\tau, \ell, y)$, i.e., of the reform's revenue implications. Thus, the cross-derivative $R^s_{\tau\ell}$ can be interpreted as a measure of how much revenue can be raised by a small single-bracket reform. The function $y \mapsto R^s_{\tau\ell}(0, 0, y)$ is a recurrent theme in what follows. For a more concise notation, we will henceforth write $\mathcal{R}(y)$ rather than $R^s_{\tau\ell}(0, 0, y)$ and frequently refer to the function $y \mapsto \mathcal{R}(y)$.

Two-bracket reforms. A two-bracket reform combines two single-bracket reforms. Formally, it is a pair (τ, h_2), where the subscript of h_2 signifies a reform involving two brackets. The function h_2 is defined by

$$h_2(y) := \tau_1 \, h_1^s(y) + \tau_2 \, h_2^s(y) \, , \tag{6}$$

for

$$h_1^s(y) = \begin{cases} 0, & \text{if } y \le y_1 \, , \\ y - y_1, & \text{if } y \in (y_1, y_1 + \ell \, \ell_1) \, , \\ \ell \, \ell_1, & \text{if } y \ge y_1 + \ell \, \ell_1 \, . \end{cases}$$

and

$$h_2^s(y) = \begin{cases} 0, & \text{if } y \le y_2 \, , \\ y - y_2, & \text{if } y \in (y_2, y_2 + \ell \, \ell_2) \, , \\ \ell \, \ell_2, & \text{if } y \ge y_2 + \ell \, \ell_2 \, . \end{cases}$$

Thus, a two-bracket reform links two single-bracket reforms in a particular way: marginal tax rates change by $\tau \, \tau_1$ for incomes in the first bracket and by $\tau \, \tau_2$ for incomes in the second bracket. The first bracket has a length of $\ell \, \ell_1$, and the second bracket has a length of $\ell \, \ell_2$.
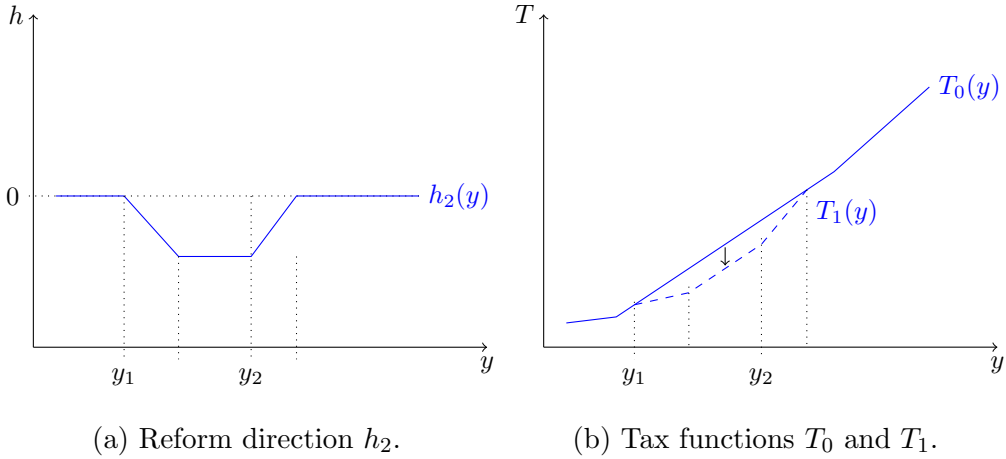

(a) Reform direction h_2.   (b) Tax functions T_0 and T_1.

Figure 2: A two-bracket tax cut.

The new tax schedule satisfies

$$T_1(y) = T_0(y) + \tau \, h_2(y) \, ,$$

and the new schedule of marginal tax rates equals

$$T_1'(y) = T_0'(y) + \tau\, h_2'(y)\ ,$$

where

$$h_2'(y) = \begin{cases} \tau_1, & \text{for} \quad y \in (y_1, y_1 + \ell\, \ell_1)\ , \\ \tau_2, & \text{for} \quad y \in (y_2, y_2 + \ell\, \ell_2)\ , \\ 0, & \text{for} \quad y \le y_1\ , y \in [y_1 + \ell\, \ell_1, y_2]\ , y \ge y_2 + \ell\, \ell_2\ . \end{cases}$$

In what follows, two-bracket reforms with $\tau_1 < 0$, $\tau_2 > 0$ and $\tau_1 \ell_1 + \tau_2 \ell_2 = 0$ are of particular interest. We refer to these reforms as *two-bracket tax cuts*. This choice of terminology reflects that these reforms do not increase anyone's tax burden and that all people with an income between the endpoints of the two brackets get a tax cut. Moreover, they involve a phase-in range where marginal taxes are reduced, and a subsequent phase-out range where marginal taxes are increased, see Figure 2.

Our construction of two-bracket reforms facilitates an analysis of the limit case $\tau \to 0$ and $\ell \to 0$, see Figure 3 for the case of a small two-bracket tax cut. As τ goes to zero, the ratio of the marginal tax rate changes is kept constant at $\frac{\tau_1}{\tau_2}$. Analogously, both brackets shrink when ℓ is send to zero, while the ratio of their lengths is kept constant at $\frac{\ell_1}{\ell_2}$.
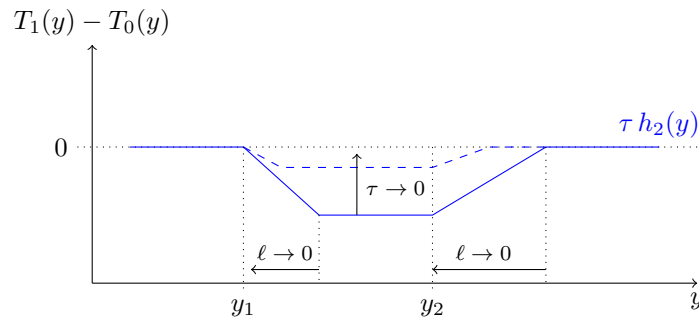

Figure 3: A small two-bracket tax cut.

Reforms with finitely many brackets. We extend the construction of two-bracket reforms to reforms with a finite number of brackets in the natural way: A

reform (τ, h_m) with m brackets is given by a collection of m single-bracket reforms such that $h_m(y) := \sum_{k=1}^{m} \tau_k\, h_k^s(y)$, where

$$
h_k^s(y) = \begin{cases} 0, & \text{if} \quad y \leq y_k\,, \\ y - y_k, & \text{if} \quad y \in (y_k, y_k + \ell\, \ell_k)\,, \\ \ell\, \ell_k, & \text{if} \quad y \geq y_k + \ell\, \ell_k\,. \end{cases}
$$

The reform induced tax schedule is then given by $T_1 = T_0 + \tau\, h_m$.

3.2 The main results

Under what conditions is it possible to make everyone better off by increasing or lowering the marginal tax rates in a finite number of income brackets? Theorems 1 and 2 provide answers to this question.

Theorem 1 *If T_0 is a Pareto-efficient tax system, then the function $y \mapsto \mathcal{R}(y)$ is bounded from below by 0, bounded from above by 1, and non-increasing.*

Theorem 1 states necessary conditions for the Pareto efficiency of a tax system. The first condition is that $\mathcal{R}(y) \geq 0$ for all y. Hence, a one-bracket reform involving an increase of marginal tax rates must not lead to a loss of tax revenue. If the condition was violated, it would be possible to raise revenue by means of a tax cut, and such a reform would be Pareto-improving. The logic is familiar from analyses of the Laffer curve. The second condition is that $\mathcal{R}(y) \leq 1$ for all y. It is a mirror image of the first condition. If it was violated, it would be possible to raise so much revenue by increasing marginal tax rates that even those who suffer most from the tax increase would be compensated. If T_0 is a Pareto-efficient tax system, there must be no scope for such a Pareto improvement.

The following Proposition clarifies what reform options exist when the function $y \mapsto \mathcal{R}(y)$ is increasing over some range.

Proposition 1 *If there are two income levels y_1 and $y_2 > y_1$ such that $\mathcal{R}(y_2) > \mathcal{R}(y_1)$, there exists a Pareto-improving two-bracket tax cut with $\tau > 0$ and $\ell > 0$.*

In light of Proposition 1, Theorem 1 provides a characterization of two necessary conditions for Pareto efficiency: first, there must not be a Pareto improvement in the class of one-bracket reforms. Second, there must not be a Pareto improvement in the class of two-bracket tax cuts. As we show formally in the proof of Proposition 1, if $y \mapsto \mathcal{R}(y)$ is increasing, a two-bracket tax cut between incomes y_1 and y_2 is self-financing: the revenue loss due to a reduction of marginal tax rates in the first bracket is more than offset by the revenue gain from the increase of marginal tax rates in the second bracket. Thus, the condition that $y \mapsto \mathcal{R}(y)$ must be non-increasing is an analogue to the condition $\mathcal{R}(y) \geq 0$ for all y. The latter rules out self-financing tax cuts for one-bracket reforms. The former does so for two-bracket reforms.

Theorem 1 and Proposition 1 show that there may exist Pareto-improving two-bracket reforms, even when no Pareto-improving one-bracket reform can be found. Given this finding, one might conjecture that there is no hope to obtain a concise characterization of Pareto-efficient tax systems: even if one found a condition ruling out Pareto-improving two-bracket reforms, there would still be the possibility of a Pareto-improving three-bracket reform. If one had eliminated those, one would still have to deal with four-bracket reforms, and so on. Theorem 2 shows that this is not the case: ruling out Pareto-improving one- and two-bracket reforms is sufficient for Pareto efficiency.

Theorem 2 *If the function $y \mapsto \mathcal{R}(y)$ is bounded from below by 0, bounded from above by 1, and non-increasing then there is no Pareto-improving direction in the class of reforms with finitely many brackets.*

According to Theorem 2, ruling out Pareto-improving one- and two-bracket reforms guarantees Pareto efficiency. The Theorem implies, in particular, that any tax system that can be Pareto-improved by a tax reform that affects three or more brackets, can also be Pareto-improved by a tax reform that affects at most two brackets.

Why is Theorem 2 true? The class of m-bracket reforms gives rise to many possible combinations of rate increases and rate cuts. There could be increases in all odd brackets and decreases in all even brackets; there could be increases for high incomes and decreases for low incomes, etc. The difficulty in the proof is to show that none of these combinations can be Pareto-improving under the conditions of Theorem 2. Here we give an intuition that covers some of these cases.

Assume first that marginal tax rates go up in the first bracket. With $\mathcal{R}$ throughout below 1, this yields a revenue gain that is, however, not large enough to compensate people with incomes above the first bracket who now face an increased tax burden. Thus, additional revenue is needed and this requires to increase marginal taxes in some other bracket k higher up in the income distribution. With $\mathcal{R}(y) < 1$, if we raise enough revenue in bracket k so as to compensate the people below, this will only add to making people with incomes above bracket k worse off, and so on if we add further brackets in which taxes go up. Moreover, there is no way to overcome this by mixing in brackets with tax cuts. When $\mathcal{R}(y) > 0$, for all y, this only aggravates the difficulty of drumming up enough revenue for the compensation of those who face higher taxes. Thus, there is no Pareto improvement with a rate increase in bracket 1.

How about having instead a first bracket with a tax cut? People with incomes above this bracket are then made better off; but, with $\mathcal{R}$ positive throughout, this creates a revenue loss that is harmful for anyone else. Adding further brackets where tax rates are lowered leads to an even more substantial revenue loss. We can now try to offset this effect with brackets in which tax rates go up. Assume that we increase marginal taxes in the second bracket just enough to make sure that agents with incomes above this bracket face neither a tax cut not a tax increase, $\tau_2 \ell_2 = -\tau_1 \ell_1$ (hence, we employ a two-bracket tax cut). With $y \mapsto \mathcal{R}(y)$ decreasing, the revenue gain due the second bracket is not large enough to make up for the revenue loss from the first bracket. Hence, we have to raise marginal taxes in the second bracket – or in further brackets – even more. But then, the reform

raises the overall tax burden for people further up in the income distribution. This neither is Pareto-improving. Hence, irrespectively of whether we have a tax increase or a tax cut in the first bracket, those who are hit hardest by an increase of their tax liability cannot be made better off.

Continuous reform directions.   So far, our results were restricted to the class of tax reforms with finitely many brackets, thereby excluding, e.g., continuously differentiable reform directions. The following corollary extends Theorem 2 to cover the entire class of continuous reform directions. It exploits that any continuous function $h : \mathcal{Y} \to \mathbb{R}$ can be approximated arbitrarily well by an m-bracket reform with m sufficiently large.

Corollary 1 *If $y \mapsto \mathcal{R}(y)$ is bounded from below by 0, bounded from above by 1, and non-increasing, there is no Pareto-improving direction in the class of continuous functions $h : \mathcal{Y} \to \mathbb{R}$.*

According to Corollary 1, any tax system that can be Pareto-improved by any continuous reform, can also be Pareto-improved by a tax reform that affects at most two brackets.

An alternative characterization.   As we discuss in part D of the Online Appendix, when earnings are bounded away from zero and bounded from above, it is possible to obtain a more parsimonious characterization of sufficient conditions for Pareto efficiency: The monotonicity condition on $y \mapsto \mathcal{R}(y)$ is then sufficient for Pareto efficiency.[20] This said, for our application of interest, the introduction of the EITC, incentives for labor market participation play a key role. In this context, an assumption that everybody has strictly positive earnings would not be appropriate.

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[20]We are grateful to an anonymous referee for pointing to this possibility.

3.3 Discussion

Sufficient statistics for $\mathcal{R}$. All previous results are expressed using the function $y \mapsto \mathcal{R}(y)$. Thus, given an estimate of this function, our approach allows to check whether a tax system of interest is Pareto-efficient. Different models of taxation give rise to different versions of the function $y \mapsto \mathcal{R}(y)$. The concrete specification will depend on the application of interest and on a choice of what model to use for this application. We illustrate this with two examples. The formulas that follow are derived in part F of the Supplementary Material.

First, consider the model of Diamond (1998) with $u(c, y, \theta) = c - \frac{1}{1 + \frac{1}{\epsilon}} \left(\frac{y}{\theta} \right)^{1 + \frac{1}{\epsilon}}$, where $\theta \in \Theta \subset \mathbb{R}_+$ is a measure of productivity and the parameter ϵ pins down the labor supply elasticity at the intensive margin. For this model, the revenue implications of a small one-bracket reform at income y are given by

$$\mathcal{R}(y) = 1 - F_y(y) - \varepsilon_0(y) \, y \, f_y(y) \, \frac{T_0'(y)}{1 - T_0'(y)} \quad , \tag{7}$$

where F_y ist the *cdf* of the earnings distribution, and $\varepsilon_0 : y \mapsto \varepsilon_0(y)$ is a function which gives, for each level of y, the intensive-margin elasticity of earnings with respect to the retention rate $1 - T_0'(y)$.

Second, the literature on the desirability of earnings subsidies for the "working poor" suggests the use of a framework with taxpayers who differ both in the variable costs of productive effort and in the fixed costs of labor market participation.[21] We present a general framework and derive a sufficient-statistics formula characterizing $y \mapsto \mathcal{R}(y)$ in the Supplementary Material (see Proposition 4 in part F).[22] For ease of exposition, we focus here on the case of quasi-linear preferences and iso-elastic effort costs. This is also the specification that we will use in our

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[21] A similar framework is also used in the literature on optimal pension and retirement policies, see, e.g., Golosov, Shourideh, Troshkin and Tsyvinski (2013), Michau (2014), and Shourideh and Troshkin (2017).

[22] This derivation is of stand-alone-interest in that it is based on a general specification of preferences, allowing for income effects, monetary or psychic fixed costs of labor market participation and complementarities between consumption and leisure.

benchmark analysis of the EITC in the subsequent section. Hence, suppose that

$$u(c, y, \omega, \gamma) = c - \frac{1}{1 + \frac{1}{\epsilon}} \left(\frac{y}{\omega} \right)^{1 + \frac{1}{\epsilon}} - \gamma \, \mathbb{1}_{y>0} \ ,$$

where ω and γ are, respectively, interpreted as a taxpayer's variable and fixed cost type. Thus, an individual's type θ is now taken to be a pair $\theta = (\omega, \gamma)$ and $\Theta = \Omega \times \Gamma$. Then,

$$\mathcal{R}(y) = 1 - F_y(y) - \varepsilon_0(y) \, y \, f_y(y) \, \frac{T_0'(y)}{1 - T_0'(y)} - \int_y^\infty f_y(y') \, \pi_0(y') \, \frac{T_0(y')}{y' - T_0(y')} \, dy' \ , \quad (8)$$

where $\pi_0(y)$ is an extensive-margin (participation) elasticity. It measures the percentage of individuals with an income of y who leave the labor market when their after-tax income $y - T_0(y)$ is decreased by one percent. This formula has first been derived by Jacquet et al. (2013) and also appears in Lorenz and Sachs (2016).

A further comment on related literature. Both Werning (2007) and Lorenz and Sachs (2016) present tests for Pareto efficiency that involve differential inequalities. Their approach can be illustrated with the setup of Diamond (1998). Recall that, in this case, $\mathcal{R}(y)$ is given by equation (7). If the expressions on the right hand side of (7) are taken to be differentiable in y, then the condition that $y \mapsto \mathcal{R}(y)$ must be non-increasing can be formulated as a differential equation that involves the derivative of the ratio $\frac{T_0'(y)}{1 - T_0'(y)}$ and the derivative of the inverse hazard rate $\frac{f_y(y)}{1 - F_y(y)}$. Both Werning (2007) and Lorenz and Sachs (2016) present such equations. Their findings are implied by our result in Theorem 1 that the monotonicity of the function $y \mapsto \mathcal{R}(y)$ is necessary for Pareto efficiency.

Tagging. Our analysis can be extended to allow for tagging.[23] Suppose that the population can be divided into separate groups and that it is publicly observable to which group a person belongs. The tax and transfer system may then treat individuals who belong to different groups differently. For instance, transfers and earnings subsidies for lone mothers may be larger than those for childless individuals. The above analysis of Pareto-efficient taxation can then be applied separately

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[23]The seminal reference is Akerlof (1978). For a review, see Piketty and Saez (2013).

for each group. This implies, in particular, that revenue changes due to a tax reform that affects one group are rebated lump sum in this group.[24]

4 Evaluating tax reforms

Below, we apply the necessary and sufficient conditions for the Pareto efficiency of a tax system to the introduction of the EITC in the 1970s. We will find that there were indeed inefficiencies in the tax-and-transfer system prevailing at the time. Such a finding raises further questions: Did the 1975 EITC reform have a Pareto-improving direction? Relatedly, did the reform make the pre-existing inefficiency smaller? These questions require tools for an evaluation of tax reforms. We introduce them in this section. Proposition 2 below clarifies how one can check whether a tax reform has a Pareto-improving direction. We, moreover, introduce a money-metric measure of how inefficient a tax system is. This measure can be used to substantiate a statement such as "the inefficiency in the tax-and-transfer system for childless singles is small in comparison to the inefficiency prevailing in the one for single parents."

Pareto-improving directions. The following Proposition clarifies how one can check whether a specific tax reform has a Pareto-improving direction. It can also be used to evaluate whether reforms observed in the past had a Pareto-improving direction.

Proposition 2 *A reform in direction h is Pareto-improving if*

$$R_\tau(0, h) = \int_{\mathcal{Y}} h'(y) \, \mathcal{R}(y) \, dy \quad > \quad \max_{y \in y_0(\Theta)} h(y) \,. \tag{9}$$

Once a reform direction h is specified, inequality (9) can be used to determine whether it is Pareto-improving. The equation on the left-hand side shows that,

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[24]This is without loss of generality. Redistributing the revenue gains from a self-financing tax reform among various groups can only make it more difficult to realize a Pareto improvement. There are then less resources that can be used to compensate those adversely affected by the tax reform.

with the function $y \mapsto \mathcal{R}(y)$ at hand, one obtains an estimate of the reform's impact on tax revenue in a straightforward way: it is a weighted average of the revenue implications of one-bracket reforms where the weights are given by the reform-induced changes of marginal tax rates, $y \mapsto h'(y)$. A reform with direction h and step size τ changes the marginal tax at income y by $\tau h'(y)$; hence, $h'(y)$ is the change in marginal tax rates per unit change of τ. According to (9), if the overall revenue gain is large enough to compensate even those agents who face the largest tax increase, then direction h is Pareto-improving.

Measuring the size of inefficiencies. When a tax system (T_0, c_0) is not Pareto-efficient, there is a set of Pareto-improving reform directions. Is there a way to judge whether one these reform directions is *better* than another? One possibility is to order reform directions according to $R_\tau(0, h) - \max_{y \in y(\Theta)} h(y)$, the tax revenue gain in excess of what is needed to compensate the agents facing the largest tax increase.[25] With this measure, the *best* reform direction h maximizes

$$\int_0^{\bar{y}} h'(y)\mathcal{R}(y)dy - \max_{y \in y_0(\Theta)} \int_0^y h'(z)dz$$

over the set of continuous functions $h : \mathcal{Y} \to \mathbb{R}$. This problem is linear in the function h' which gives the reform-induced change in marginal tax rates. So, to ensure the existence of a solution we also impose the constraint that the changes in marginal tax rates must be bounded, $|h'(y)| \leq a$ for each $y \in \mathcal{Y}$. Henceforth, h_R denotes the solution to this problem and we refer to it as the *revenue-maximizing tax cut.*

Proposition 3 in Online Appendix B characterizes the revenue-maximizing tax cut for a tax system that is inefficient because the revenue function $y \mapsto \mathcal{R}(y)$ is increasing over some income range.[26] As we show, the best reform direction h_R is then a two-bracket tax cut that affects marginal tax rates over an income range $[y_s, y_t]$. Specifically, it reduces marginal taxes in a phase-in range, going from y_s

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[25]Tax revenue is a metric that is used in consumer choice theory when evaluating policy or price changes using compensating or equivalent variations. It is also used in Werning (2007)'s approach to Pareto-efficient taxation.

[26]This is the relevant scenario in our application to the 1974 US tax-and-transfer system below.

to $\frac{1}{2}(y_s + y_t)$, and increases marginal taxes in a phase-out range that begins at $\frac{1}{2}(y_s + y_t)$ and ends at y_t.[27] The impact on tax revenue can be written as

$$a\, I(T_0, c_0) = a\, \left[\int_{\frac{1}{2}(y_s+y_t)}^{y_t} \mathcal{R}(y)dy - \int_{y_s}^{\frac{1}{2}(y_s+y_t)} \mathcal{R}(y)dy \right] . \tag{10}$$

It equals the difference between the revenue gain in the phase-out range and the revenue loss in the phase-in range. Note that the term denoted by $I(T_0, c_0)$ does not depend on the parameter a that bounds the change in marginal tax rates. Hence, $I(T_0, c_0)$ admits an interpretation as a scale-invariant measure of how inefficient the tax system is. We can therefore use it to rank different tax systems according to how inefficient they are: Tax system (T_A, c_A) is more inefficient than tax system (T_B, c_B) if $I(T_A, c_A) > I(T_B, c_B)$. Thus, the more inefficient a tax system, the larger are the potential revenue gains from reforming it.[28]

Formally, $I(T_0, c_0)$ gives the slope of net revenue in direction h_R. For later reference, we note that, as an implication, a Taylor approximation can be used to obtain an estimate of the revenue implications of a reform (τ, h_R) with τ strictly positive, but close to zero:

$$R(\tau, h_R) - \max_{y \in y_0(\Theta)} \tau\, h_R(y) \simeq \tau\, a\, I(T_0, c_0) . \tag{11}$$

We apply this formula in the subsequent section where we discuss how close the actual reforms of the EITC came to the benchmark h_R. We then look at reforms that change marginal taxes rates by at most one percentage point. This requires to choose the parameters τ and a such that $\tau\, a = 0.01$.

5 Application: The introduction of the EITC

We now relate our insights on Pareto-improving tax reforms to the 1975 introduction of the EITC and its subsequent expansion. After describing the EITC

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[27]The income levels y_s and y_t are optimally chosen. As we show in Online Appendix B, optimality requires that $\mathcal{R}(y_s) = \mathcal{R}(\frac{y_s+y_t}{2}) = \mathcal{R}(y_t)$.

[28]The revenue gain $I(T_0, c_0)$ can also be interpreted as a measure of distance from the Pareto frontier: it is zero for any Pareto-efficient tax system, and positive for any inefficient tax system.

1975 reform, we first use Theorem 1 in combination with the sufficient-statistics formula in (8) to show that the US tax-transfer system was not Pareto-efficient prior to the introduction of the EITC in 1974. We then apply Proposition 2 to check whether the direction of the 1975 EITC reform was Pareto-improving, and compare it to the revenue-maximizing tax cut for single parents at the time. We also show that, based on our inefficiency measure, the 1974 tax system for single parents was more inefficient than the one for childless singles.

5.1 Background on the EITC

The introduction of the EITC in 1975 was a response to a "poverty trap". In the 1960s, new welfare programs had been introduced as part of President Johnson's "war on poverty". On the one hand, the new programs provided more generous benefits to families with low incomes. On the other hand, these benefits were phased out in a way that implied high effective marginal tax rates for many low-income families, exceeding 70% in many cases (see Figure 4 below). In the following decade, the share of welfare recipients increased substantially. By the early 1970s, finding ways out of the "poverty trap" by an increase of work incentives was considered a pressing concern.[29]

The US Congress enacted the EITC as a temporary policy for the year 1975.[30] As described in Bastian (2020), this was a substantial policy change that affected a large share of the population.[31] It was set up as a refundable tax credit that was phased in at a marginal rate of 10% for taxpayers with less than 4,000 USD annual income, giving a maximum credit of 400 USD. The credit was then phased out at a marginal rate of 10% for incomes between 4,000 and 8,000 USD. Taxpayers with incomes above 8,000 USD were not eligible. The program was initially restricted to

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[29]Detailed reviews of the debates at the time can be found in Ventry (2000), Moffitt (2003), or Nichols and Rothstein (2015).

[30]While the program was initially introduced under the name *Earned Income Credit*, it was soon relabeled to its current name *Earned Income Tax Credit*.

[31]According to CPS data, about 50% of single parents and 43% of childless singles had earned incomes in the EITC range (i.e., strictly positive and below 8,000 USD).

working taxpayers with dependent children. Later, the EITC became a permanent policy. Over the following decades, there were several expansions.[32]

5.2 Calibration

We focus on two subgroups of the population, single parents and childless singles. In 1975, the EITC was introduced for the former, but not for the latter. Our analysis below will rationalize this policy choice: we will show that there was clearly scope for a Pareto-improving reform of the tax and transfer system for single parents, whereas no equally strong case can be made for childless singles. Our benchmark analysis is based on the formula for $y \mapsto \mathcal{R}(y)$ in equation (8).[33]

We use data on the most important elements of the US tax-and-transfer system for the tax year 1974 and later (see Table 1 in part C of the Supplementary Material for details). Specifically, we take account of the federal income tax and the two largest welfare programs, Aid for Families with Dependent Children (AFDC) and Supplementary Nutrition Assistance Programs (SNAP, also known as Food Stamps). The details of AFDC varied across states, so that a unified treatment for the US at large is not possible. We therefore focus on California, the state with the largest population both in the 1970s and today. Moreover, taxes and welfare transfers differed with respect to the number of children. In the following, we focus on the subgroup of single parents with two children.[34]

Figure 4 shows effective marginal tax rates, $y \mapsto T_0'(y)$, and participation tax rates, $y \mapsto \frac{T_0(y)}{y}$, for single parents (left panel) and for childless singles (right

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[32]For example, more generous credits for parents with two or more children in 1991, and for parents with three or more children in 2009. In 1994, US authorities also introduced a more modest EITC for childless workers. In 2002, the eligibility thresholds were expanded for married taxpayers. See Hoynes (2019) for a review.

[33]This formula is based on a specification of preferences so that there are no income effects. We allow for income effects when we explore the robustness of our findings.

[34]In our data, the median number of children in single-parent households was two, while the arithmetic mean was 2.2. As we show in part C of the Supplementary Material, an analysis for the subgroups of parents with less or more children yields similar conclusions.

panel) before the reform in 1974.[35] At low incomes, both marginal tax rates and participation tax rates were much higher for single parents than for childless singles. The reason is that, for single parents, the phasing-out of AFDC and SNAP transfers implied an income range with exceptionally high marginal tax rates well above 70% and participation tax rates above 60%. This was not the case for childless singles. The dotted vertical lines in both panels of Figure 4 indicate the income range that was affected by the introduction of the EITC in 1975: It reduced marginal taxes in the phase-in range between 0 and 4,000 USD (first dotted line) and raised marginal taxes in the phase-out range between 4,000 and 8,000 USD (second dotted line).
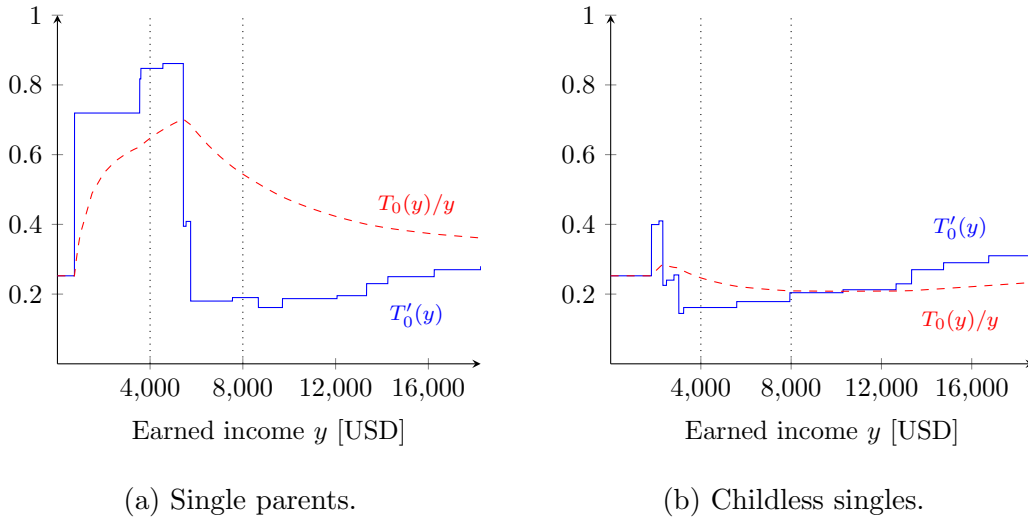

(a) Single parents.

(b) Childless singles.

Figure 4: US tax-transfer schedules 1974, single parents and childless singles.
Notes: Figure 4 shows the 1974 effective marginal tax $T_0'(y)$ (blue lines) and participation tax rate $T_0(y)/y$ (red lines) for single parents (left panel) and for childless singles (right panel) as functions of earned income in 1974 USD. The introduction of the EITC in 1975 decreased marginal taxes between 0 and 4,000 USD (first dotted vertical line) and increased them between 4,000 and 8,000 USD (second dotted vertical line).
Source: Authors' calculations (see part C of the Supplementary Material for details).

We estimate the 1974 income distributions in both subgroups based on data

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[35]Recall that we define $T_0(y)$ to capture the participation tax at income y, i.e., the tax payment at income y relative to the tax payment at zero income. In the literature, the ratio $T_0(y)/y$ is commonly referred to as the participation tax rate, see, e.g., Kleven (2014).

from the March 1975 Current Population Survey (CPS). For this purpose, we consider the sample of non-married individuals aged 25 to 60 who do neither co-habit with an unmarried spouse nor with another adult family member. We partition this sample into childless singles and single parents.[36] In line with the EITC rules, we consider as earned income the sum of wage income and self-employment income. Single parents with strictly positive earned incomes below $8,000$ USD were eligible for the EITC.[37] For our benchmark analysis, we estimate the income distributions for both groups using a non-parametric kernel density estimation.[38]

We draw on a rich literature providing estimates of labor supply elasticities. Our benchmark analysis for childless singles is based on the elasticities suggested by Chetty, Guren, Manoli and Weber (2013): an intensive-margin elasticity of 0.33, and a participation elasticity that equals 0.25 on average. For single parents, we also use an intensive-margin elasticity of 0.33, and an average participation elasticity of 0.58, as estimated by Bastian (2020) based on the 1975 EITC reform.[39] For both groups, we assume, in line with the empirical evidence, that participation elasticities decline with income (see part C of the Supplementary Material for details).

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[36] For the benchmark analysis reported below, we estimated the income distribution based on the set of single parents with any number of children ($N = 1,494$). As a robustness check, we estimate this distribution for the smaller sample of single parents with exactly two children ($N = 453$). In part C of the Supplementary Material, we show that this is inconsequential for the results of our analysis.

[37] Eligibility for EITC, AFDC and SNAP also involved assets and capital income tests. According to CPS data, more than 90% of single parents satisfied these tests. In our main analysis, we therefore ignore them (i.e., we assume that all single parents satisfy the tests). Part C of the Supplementary Material provides further details and shows that our qualitative results remain unchanged if we explicitly account for the assets and capital income tests.

[38] Our results are not affected if we instead use parametric estimates of the income distribution, see part C of the Supplementary Material for further details.

[39] Kleven (2019) finds an extensive margin elasticity close to zero for single parents based on later EITC reforms. We consider this case in our sensitivity analysis below.

5.3 Empirical results

In the following, we present our benchmark calibrations of the revenue functions $\mathcal{R}_{sp}(y)$ and $\mathcal{R}_{cs}(y)$ for *s*ingle *p*arents and *c*hildless *s*ingles, respectively. We then use these functions to investigate, first, whether the US tax-and-transfer system was Pareto-efficient prior to the EITC introduction and, second, whether the 1974 EITC reform for single parents was a reform in a Pareto-improving direction. Third, we characterize the best available reform direction at the time, i.e., the revenue-maximizing Pareto improvement.

Was the 1974 US tax-and-transfer system Pareto-efficient? Figure 5 plots the revenue functions $y \mapsto \mathcal{R}_{sp}(y)$ and $y \mapsto \mathcal{R}_{cs}(y)$ for our benchmark calibration of the 1974 US tax system. Specifically, the solid blue line depicts the revenue function $\mathcal{R}_{sp}(y)$ for single parents, while the teal line depicts the revenue function $\mathcal{R}_{cs}(y)$ for childless singles.

For single parents, function $\mathcal{R}_{sp}(y)$ does not satisfy the necessary conditions for Pareto efficiency in Theorem 1. First, it attains negative values for incomes between approximately $1,500$ and $5,400$ USD. This implies that one-bracket tax cuts in this income range would have been self-financing and Pareto-improving. Second, $y \mapsto \mathcal{R}_{sp}(y)$ is increasing in the income range between $5,000$ and $6,000$ USD, thereby violating the monotonicity condition. Hence, there was room for Pareto-improving two-bracket tax cuts, resembling the EITC.

For childless singles, the revenue response function $y \mapsto \mathcal{R}_{cs}(y)$ is throughout between 0 and 1, so that there was no scope for a Pareto-improving reform involving only a single bracket. By contrast, the monotonicity condition on $y \mapsto \mathcal{R}_{cs}(y)$ is violated as the teal line is slightly increasing in the range between $2,000$ and $3,000$ USD. Again, this indicates the possibility of Pareto-improving two-bracket reforms. That said, the visual impression is that the scope for such a Pareto improvement was more limited for childless singles than for single parents. Below, we confirm this conjecture using the inefficiency measure introduced in Section 4 above.
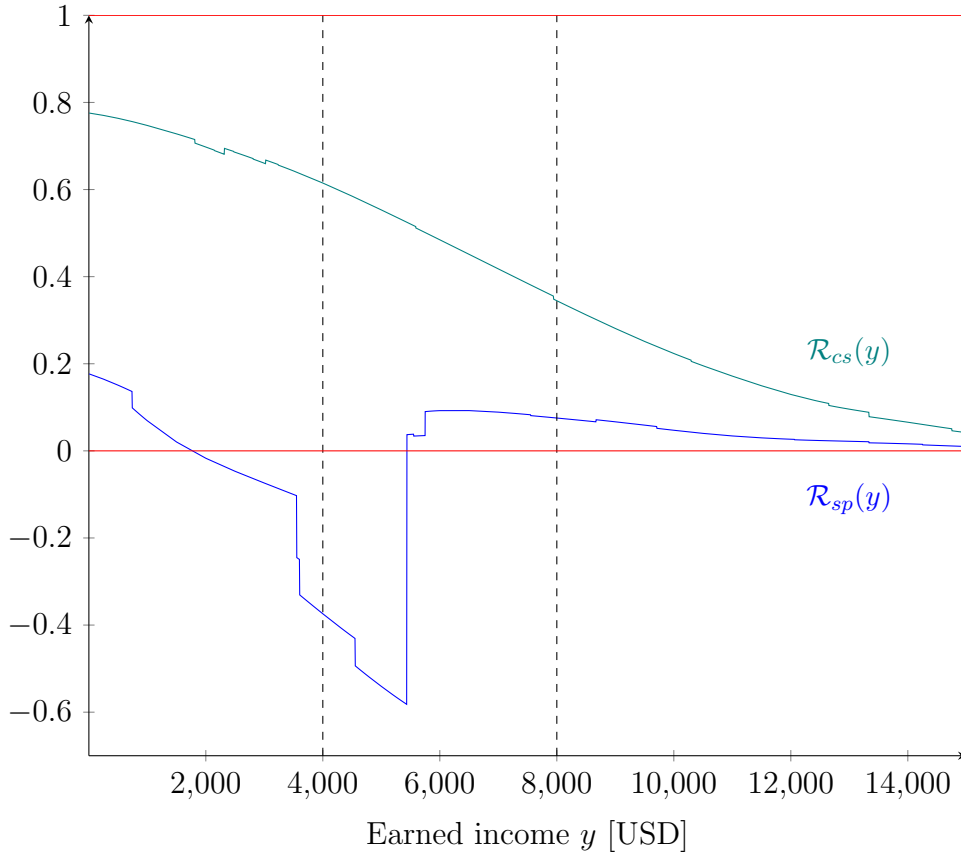
Figure 5: Pareto test of the 1974 US tax-transfer system.

Notes: Figure 5 shows the revenue functions for single parents $y \mapsto \mathcal{R}_{sp}(y)$ (blue) and childless singles $y \mapsto \mathcal{R}_{cs}(y)$ (teal) in 1974 as functions of earned income for our benchmark calibration: intensive-margin elasticities of 0.33 for both groups, average participation elasticities of 0.58 for single parents and 0.25 for childless singles. The vertical dashed lines mark the endpoints of the phase-in range at 4,000 USD and the phase-out range at 8,000 USD of the 1974 EITC.

Source: Authors' calculations (see part C of the Supplementary Material for details).
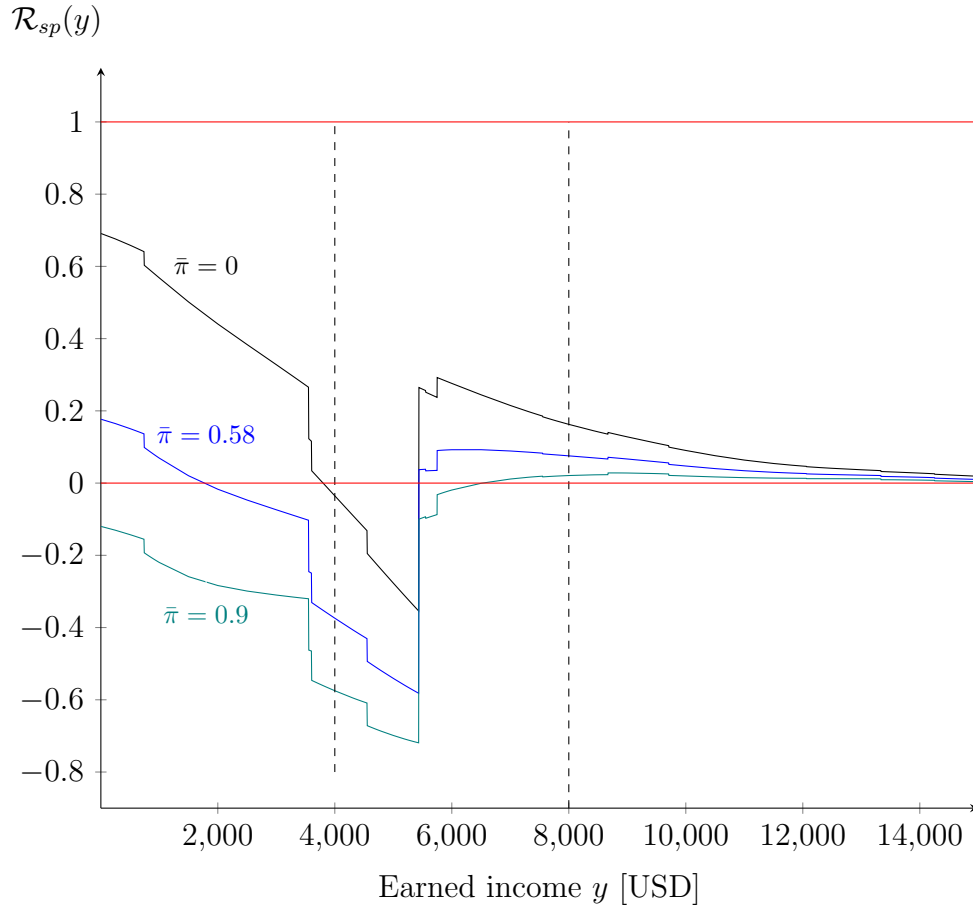
Figure 6: Pareto test of 1974 US tax system, different participation elasticities.

Notes: Figure 6 shows the revenue function $\mathcal{R}_{sp}(y)$ for single parents in 1974, assuming an average participation elasticity of 0.58 (blue line, benchmark), a higher participation elasticity of 0.9 (teal line) and a case without extensive-margin responses (black line). The intensive-margin elasticity is held at the benchmark level of 0.33. The dashed vertical lines mark the endpoints of the phase-in range at $4,000$ USD and the phase-out range at $8,000$ USD of the 1975 EITC.

Source: Authors' calculations (see part C of the Supplementary Material for details).

Sensitivity analysis. The finding that it was possible to realize a Pareto improvement by means of a two-bracket tax cut for single parents is robust in various dimensions. For instance, Figure 6 explores alternative assumptions about behavioral responses at the extensive margin: It shows the revenue function $y \mapsto \mathcal{R}_{sp}(y)$ for our benchmark calibration with an average participation elasticity of 0.58 (blue line), for a case with a larger participation elasticity of 0.9 (teal line), and for the limit case with a participation elasticity of zero (black line). The figure suggests that the scope for Pareto-improving reforms is larger, the more strongly labor supply responds at the extensive margin. But this scope does not vanish if the participation elasticity is zero. This observation is interesting in the light of the discussion about the EITC from an optimal tax perspective, where positive extensive-margin elasticities are frequently considered necessary for the desirability of an EITC, see, e.g., Saez (2002) or Hansen (2021). As we show here, with a tax reform perspective applied to the tax and transfer system as of 1974, the introduction of the EITC can be rationalized even when there are no behavioral responses at the extensive margin.[40]

Was the 1975 EITC reform Pareto-improving? Figure 5 shows that it was possible to Pareto-improve the US tax-transfer system by a two-bracket tax cut, i.e., by the introduction of *some EITC*. A separate question is whether the EITC reform that *actually took place* went into a Pareto-improving direction. To answer this question, we make use of the conditions in Proposition 2 which combine an estimate of the revenue effects with information on the reform direction, referred to as $\tilde{h}_{75}$ below. Specifically, the 1975 EITC reform reduced marginal taxes at all incomes below $4,000$ USD by 10 percentage points, and increased marginal taxes at all incomes between $4,000$ and $8,000$ USD by the same amount. It did not increase tax liabilities at any income level. This reform had a Pareto-improving

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[40]Part C.2 of the Supplementary Material contains further robustness checks that consider, e.g., other assumptions about intensive-margin elasticities, income effects, single parents with one or three children, alternative estimates of the income distribution, or other representations of the tax-transfer system.

direction if the condition

$$R_\tau(0, \tilde{h}_{75}) = -\int_0^{4,000} \mathcal{R}_{sp}(y)dy + \int_{4,000}^{8,000} \mathcal{R}_{sp}(y)dy \quad \geq \quad 0 \qquad (12)$$

is satisfied. Whether this inequality holds depends on the details of the calibration. For the benchmark case with an intensive-margin elasticity of 0.33 and an average participation elasticity of 0.58, the reform was not Pareto-improving. For a participation elasticity above 0.84, by contrast, it was Pareto-improving.[41]

The revenue-maximizing tax cut. What would have been the revenue-maximizing tax cut in 1975 and how does it relate to the reform that actually was taken? Under our benchmark calibration, the former is a two-bracket tax cut that reduces marginal tax rates between $1,248$ and $5,748$ USD, and increases marginal taxes between $5,748$ and $10,248$ USD. Thus, the optimal reform would have been a version of the EITC that involved a wider range of incomes, and also higher incomes than the actual 1975 EITC. With the revenue gain from this reform, it would have been possible to pay an additional lump-sum transfer of 12.6 USD per percentage-point change in marginal taxes to each single parent (corresponding to 71 USD in 2021). Figure 7 illustrates this reform and its revenue implications graphically. For childless singles, the corresponding number is much smaller, namely 1 cent per percentage point change in marginal taxes. This confirms the conjecture above that the inefficiency in the tax and transfer system for childless singles was orders of magnitude smaller than the one for single parents.

Did subsequent reforms improve the EITC? Since its introduction, the EITC was repeatedly reformed and expanded in two major ways. First, the range of eligible incomes was enlarged in several steps, starting with the 1979 reform. Second, benefits were made dependent on family size, with larger benefits for larger families; e.g., in 1991 and 2009. Did these reforms Pareto-improve the EITC or, put differently, was there progress in US tax policy for people with low

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[41]Some studies find participation elasticities in this range, see, e.g., Meyer and Rosenbaum (2001).
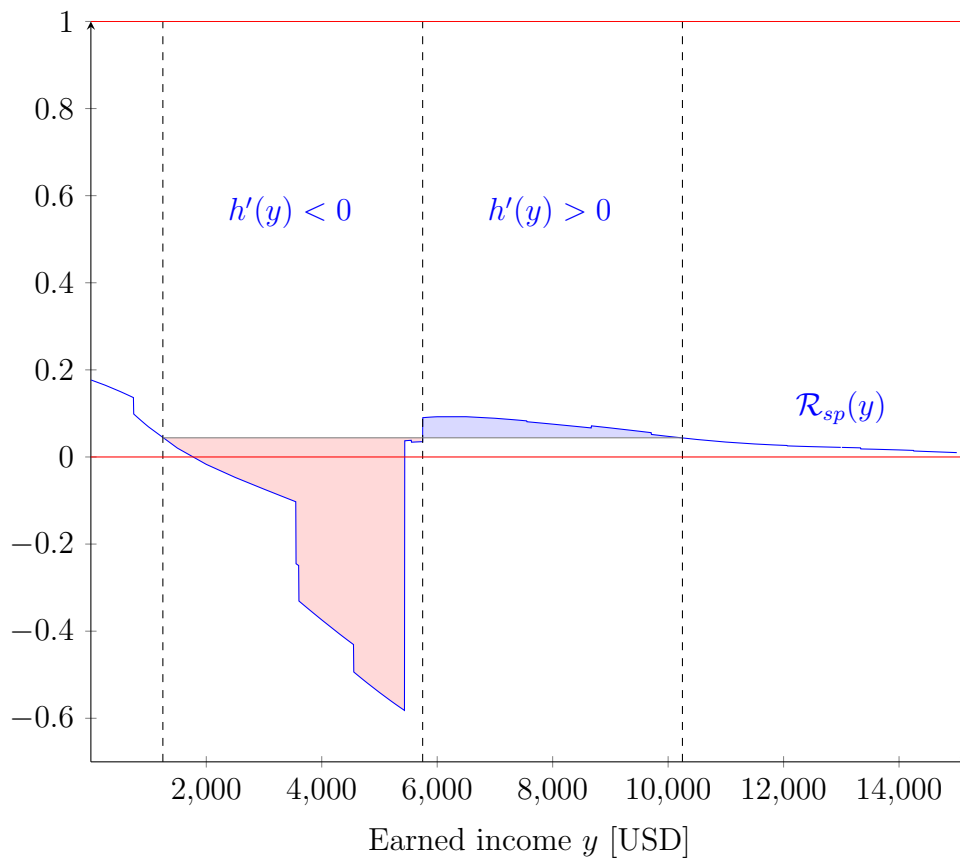
Figure 7: The revenue-maximizing tax cut for single parents.

Notes: Figure 7 shows the revenue function $y \mapsto \mathcal{R}_{sp}(y)$ for single parents given our benchmark calibration. The revenue-maximizing tax cut would have reduced marginal taxes between $1,248$ (first dashed line) and $5,748$ USD (second dashed line) and increased them between $5,748$ and $10,248$ USD (third dashed line). The sum of both shaded areas represents the available revenue gain per single-parent family.

Source: Authors' calculations (see part C of the Supplementary Material for details).

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incomes? Our approach to study Pareto-improving tax reforms can also be used to answer these questions. The following paragraphs summarize our findings; a more detailed analysis can be found in part C of the Supplementary Material.

First, we study the efficiency of the US tax-transfer system between the EITC introduction in 1975 and the first EITC expansion in 1979. As of 1974, the revenue-maximizing tax cut would have reduced marginal taxes between $1,248$ and $5,748$ USD. The 1975 EITC only reduced marginal taxes below $4,000$ USD, however, and increased them between $4,000$ and $8,000$ USD. Thus, the actual reform aggravated inefficiencies at the bottom of the phase-out range. Correspondingly, the 1975 US tax-transfer system remained Pareto-inefficient, and there was further scope for Pareto-improving reforms at higher incomes. In 1979, the US government indeed extended the EITC to higher income levels – by means of a two-bracket tax cut affecting incomes between $4,000$ and $10,000$ USD. We demonstrate that, based on our benchmark calibration, the 1979 EITC reform actually had a Pareto-improving direction. Thus, while the initial version of the EITC was suboptimal, its design was improved subsequently.

Second, we look into the desirability of making the EITC provisions dependent on the number of children. We find that, for each group, the introduction of an EITC schedule in 1974 was Pareto-improving. The scope for Pareto improvements was even larger with differentiated schedules that were more generous for larger families. The 1974 EITC did not condition on the number of children. More than a decade later, the US government improved the design of the EITC in this dimension, introducing more generous tax credits for families with two and more children in 1991, and for families with three and more children in 2009.

6   Concluding Remarks

A key lesson from this paper is that tax reforms with two brackets – one in which tax rates are lowered, and one in which tax rates are increased – deserve particular attention. Our theoretical results show that such reforms can make every one better off, even if no simple one-bracket tax reform can. Moreover, a

tax system is Pareto-efficient if there is no Pareto improvement in the class of tax reforms that affect at most two brackets. Our study of the EITC shows that such reforms have also been successfully used in practice.[42] Finally, we show that sufficient-statistics formulas can be used to identify Pareto-improving two-bracket reforms in practice.

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[42]Interestingly, the US tax and transfer system of 2018 has a pattern of inefficiencies that is qualitatively similar to those in the mid 70s, but quantitatively less significant, see part G of the Supplementary Material.

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Online Appendix (not for publication)

A Proofs

A.1 Proof of Theorem 1 and Proposition 1.

To prove Theorem 1, we proceed in two steps. We first show that the non-existence of a Pareto-improving one-bracket reform implies that $\mathcal{R}(y) \in [0,1]$ for all y. We then show that the non-existence of a Pareto-improving two-bracket reform implies that $y \mapsto \mathcal{R}(y)$ is non-increasing.

Reforms with one bracket. Adapting inequality (5) to the case of a one-bracket reform we find that a small reform with $\tau > 0$ that increases marginal tax rates from the status quo is Pareto-improving if, for some $\ell > 0$,

$$R_\tau^s(0, \ell, \hat{y}) - \ell > 0 \,, \tag{13}$$

i.e., if marginal revenue gains are so large that even those agents are made better off whose tax bill increases by the maximal amount of $\max_y \ h^s(y) = \ell$. For a one-bracket reform with $\tau < 0$, we have to compare the derivative $R_\tau^s(0, \ell, \hat{y})$ with $\min_y \ h^s(y) = 0$, see Footnote 19. Consequently, a small one-bracket reform that reduces marginal tax rates is Pareto-improving if

$$R_\tau^s(0, \ell, \hat{y}) < 0 \,, \tag{14}$$

so that a tax cut leads to larger tax revenues, a logic familiar from analyses of the Laffer curve.

Below we exploit the fact that a one-bracket reform on a bracket of length zero does not affect tax revenue, i.e., $R_\tau^s(0, 0, \hat{y}) = 0$ for any $\hat{y}$. To see this, recall that the new tax schedule satisfies $T_1(y) = T_0(y)$ for any $y \leq \hat{y}$, and $T_1(y) = T_0(y) + \tau \ell$ for any $y \geq \hat{y} + \ell$. For a one-bracket reform on a bracket of length $\ell = 0$, the new tax schedule is thus identical to the status quo tax schedule, $T_1(y) = T_0(y)$ for all y, independent of the step size τ. Hence, a variation in τ neither affects the budget set C_1 nor individual behavior y^*, nor tax revenue $R^s(\tau, 0, \hat{y})$.

Lemma 1

(i) *If $\mathcal{R}(\hat{y}) - 1 > 0$ for some $\hat{y} \in \mathcal{Y}$, there exists a Pareto-improving one-bracket reform with $\tau > 0$ and $\ell > 0$.*

(ii) *If $\mathcal{R}(\hat{y}) < 0$ for some $\hat{y} \in \mathcal{Y}$, there exists a Pareto-improving one-bracket reform with $\tau < 0$ and $\ell > 0$.*

(iii) *If there is no Pareto-improving reform, then $\mathcal{R}(\hat{y}) \in [0, 1]$ for all $\hat{y}$.*

Proof of Lemma 1. As explained above, we have $R_\tau^s(0, 0, \hat{y}) = 0$. If $\mathcal{R}(\hat{y}) = R_{\tau\ell}^s(0, 0, \hat{y}) > 1$, this implies that $R_\tau^s(0, \ell, \hat{y}) - \ell$ turns positive if, starting from $\ell = 0$, the length of the interval is slightly increased. This proves (i). Analogously, if $\mathcal{R}(\hat{y}) = R_{\tau\ell}^s(0, 0, \hat{y}) < 0$, this implies that $R_\tau^s(0, \ell, \hat{y})$ turns negative if, starting from $\ell = 0$, the length of the interval is slightly increased. This proves (ii). Thus, necessary conditions for the non-existence of a Pareto-improving one-bracket reform are $\mathcal{R}(\hat{y}) \leq 1$ and $\mathcal{R}(\hat{y}) \geq 0$. This proves (iii).

Reforms with two brackets. Lemma 1 above gives necessary conditions for the existence of Pareto-improving reforms with a single bracket. The following Lemma gives the analogue for the case of two-bracket reforms. In particular, we show that, if $y \mapsto \mathcal{R}(y)$ is increasing, the combination of two reforms – each of which would not be Pareto-improving on a stand alone basis – yields a Pareto improvement. For this purpose, we denote by $R(\tau, h_2)$ the change in tax revenue due to a joint reform with two brackets, where $h_2 = h_1^s + h_2^s$ is composed of two single bracket reforms.

Proof of Proposition 1. Fix two income levels y_1 and y_2 such that $y_2 > y_1$ and $\mathcal{R}(y_2) > \mathcal{R}(y_1)$. We now construct a Pareto-improving two-bracket reform with the parameters $\{(y_1, \tau_1, \ell_1, y_2, \tau_2, \ell_2, \tau, \ell)\}$. In particular, let $\tau_1 < 0$, $\tau_2 > 0$, and $\tau_1 \ell_1 + \tau_2 \ell_2 = 0 > \tau_1 \ell_1$. This implies that $\max_y h(y) = 0$. By the linearity of the

Gateaux differential,[43] we moreover find that

$$
\begin{aligned}
R_\tau(0, h_2) &= \tau_1 R_\tau^s(0, \ell\,\ell_1, y_1) + \tau_2 R_\tau^s(0, \ell\,\ell_2, y_2) \text{ , and} \\
R_{\tau\ell}(0, h_2) &= \tau_1 \ell_1 R_{\tau\ell}^s(0, 0, y_1) + \tau_2 \ell_2 R_{\tau\ell}^s(0, 0, y_2) \\
&= \tau_2 \ell_2 \left[\mathcal{R}(y_2) - \mathcal{R}(y_1) \right] > 0 \ .
\end{aligned}
$$

Hence, there exists $\hat{\ell} > 0$ such that $R_\tau(0, h_2) - \max_y h(y) > 0$ for all $\ell \in (0, \hat{\ell})$. Finally, by equation (4), this implies that for a reform as constructed above with $\ell \in (0, \hat{\ell})$, a small increase in τ is Pareto-improving.

Suppose that $\mathcal{R}(y_1)$ and $\mathcal{R}(y_2)$ are between 0 and 1. Then, there is no Pareto-improving one-bracket reform for incomes close to y_1 or close to y_2. If $\mathcal{R}(y_1) < \mathcal{R}(y_2)$, however, there is still scope for a Pareto improvement that involves two brackets.

A.2   Proof of Theorem 2

A reform with an arbitrary number m of brackets can be characterized as a collection

$$
\{(y_k, \tau\,\tau_k, \ell\,\ell_k)\}_{k=1}^m
$$

of one-bracket reforms, where the marginal tax in the kth bracket is changed by $\tau\,\tau_k$ and length of the kth bracket is given by $\ell\,\ell_k$. As before, the parameters (τ, ℓ) determine the size of the reform and the overall revenue is denoted by $R(\tau, h_m)$. The following lemma states sufficient conditions for the existence of a Pareto-improving reform with m brackets.

---

[43] Gateaux differentials are not linear in general. To clarify the conditions under which they are, for $0 < \bar{y}, a < \infty$, let $\tau \in [-a, a]$ and $h \in \mathcal{H} := (\mathcal{C}[0, \bar{y}], \|\cdot\|_\infty)$, where $\|\cdot\|_\infty$ denotes the sup norm. We define the operator

$$
\mathcal{R}: \quad \mathcal{H} \to \mathcal{K}: \quad h \mapsto \mathcal{R}(\tau, h),
$$

where $\mathcal{K} := (\mathcal{C}_b([-a, a]), \|\cdot\|_\infty)$ and $\mathcal{C}_b([-a, a])$ denotes the set of bounded continuous real functions defined on $[-a, a]$. Note that $\mathcal{H}$ and $\mathcal{K}$ are Banach spaces. In this setting, the Gateaux differential of $R_\tau(\tau, \cdot)$ is linear (Zorn (1945); Theorem 2.3).

Lemma 2 *Consider a collection $\{(y_k, \tau \ \tau_k, \ell \ \ell_k)\}_{k=1}^{m}$ of simple reforms. Let $\tau_0 \ l_0 = 0$. There is a reform (τ, h_m) with $\tau > 0$ and $\ell > 0$ that is Pareto-improving if*

$$\sum_{k=1}^{m} \tau_k \ \ell_k \ \mathcal{R}(y_k) - \max_{j \in \{0,1,...,m\}} \ \sum_{k=0}^{j} \tau_k \ \ell_k \quad > \quad 0 \ .$$

Proof The linearity of the Gateaux differential implies that

$$R_\tau(0, h_m) \quad = \quad \sum_{k=1}^{m} \tau_k R_\tau^s(0, \ell \ \ell_k, y_k) \ , \ \text{and}$$

$$R_{\tau\ell}(0, h_m) \quad = \quad \sum_{k=1}^{m} \tau_k \ \ell_k \ \mathcal{R}(y_k) \ .$$

Moreover,

$$\max_y \ h_m(y) \quad = \quad \ell \max_{j \in \{0,1,...,m\}} \ \sum_{k=0}^{j} \tau_k \ \ell_k \ .$$

As shown above, $R_\tau(0, h_m)$ equals zero for a reform with $\ell = 0$ such that all brackets have length zero. Hence, if the condition in the lemma is satisfied, there exists a $\hat{\ell} > 0$ such that $R_\tau(0, h_m) - \max_y \ h_m(y) > 0$ for all $\ell \in (0, \hat{\ell})$. By equation (4), this implies that for such an m-bracket reform (τ, h_m) with $\ell \in (0, \hat{\ell})$, a small increase in the step size τ is Pareto-improving.

Lemma 2 states sufficient conditions for the existence of Pareto-improving reforms. If we limit attention to small reforms these conditions are also necessary, i.e., if they do not hold there is no small reform that is Pareto-improving. The following lemma shows that, if the conditions in Theorem 2 hold, the condition in Lemma 2 is violated for any collection of m single bracket reforms. Consequently, there is no small Pareto-improving m-bracket reform.

Lemma 3 *Suppose that the function $y \mapsto \mathcal{R}(y)$ is bounded from below by 0, bounded from above by 1 and non-increasing. Let $\tau_0 \ l_0 = 0$. Then,*

$$\sum_{k=1}^{m} \tau_k \ \ell_k \ \mathcal{R}(y_k) - \max_{j \in \{0,1,...,m\}} \ \sum_{k=0}^{j} \tau_k \ \ell_k \quad \leq \quad 0 \tag{15}$$

for any collection $\{(y_k, \tau_j \ \tau_k, \ell_j \ \ell_k)\}_{k=1}^{m}$, and for any $m \geq 1$.

Proof of Lemma 3. Let j^* be a bracket in which the function h_m achieves a maximum, $j^* := \operatorname{argmax}_j \quad \sum_{k=0}^{j} \tau_k \, \ell_k$. Note that this implies that $\sum_{k=z}^{j^*} \tau_k \, \ell_k \geq 0$ for any $z \in \{0, \ldots, j^*\}$ and $\sum_{k=j^*+1}^{z} \tau_k \, \ell_k \leq 0$ for any $z \in \{j^* + 1, \ldots, m\}$; otherwise j^* would not be a maximizer.

Step 1. We verify the following claim: Suppose that $j^* > 0$ and that

$$\sum_{k=z}^{j^*} \tau_k \, \ell_k \, \mathcal{R}(y_k) \quad \leq \quad \mathcal{R}(y_z) \sum_{k=z}^{j^*} \tau_k \, \ell_k \tag{16}$$

holds for some $z \in \{1, \ldots, j^*\}$. Then, if $z > 1$, we also have

$$\sum_{k=z-1}^{j^*} \tau_k \, \ell_k \, \mathcal{R}(y_k) \quad = \quad \tau_{z-1} \, l_{z-1} \, \mathcal{R}(y_{z-1}) + \sum_{k=z}^{j^*} \tau_k \, \ell_k \, \mathcal{R}(y_k)$$

$$\leq \quad \tau_{z-1} \, l_{z-1} \, \mathcal{R}(y_{z-1}) + \underbrace{\mathcal{R}(y_z)}_{\leq \mathcal{R}(y_{z-1})} \underbrace{\sum_{k=z}^{j^*} \tau_k \, \ell_k}_{\geq 0}$$

$$\leq \quad \mathcal{R}(y_{z-1}) \sum_{k=z-1}^{j^*} \tau_k \, \ell_k \, .$$

Condition (16) is obviously satisfied for $z = j^*$. Hence, a repeated application of the preceding argument yields

$$\sum_{k=1}^{j^*} \tau_k \, \ell_k \, \mathcal{R}(y_k) \quad \leq \quad \underbrace{\mathcal{R}(y_1)}_{\in [0,1]} \underbrace{\sum_{k=1}^{j^*} \tau_k \, \ell_k}_{\geq 0} \quad \leq \quad \sum_{k=1}^{j^*} \tau_k \, \ell_k \quad = \quad \sum_{k=0}^{j^*} \tau_k \, \ell_k \, . \tag{17}$$

Step 2. An analogous argument implies that

$$\sum_{k=j^*+1}^{m} \tau_k \, \ell_k \, \mathcal{R}(y_k) \quad \leq \quad \mathcal{R}(y_m) \underbrace{\sum_{k=j^*+1}^{m} \tau_k \, \ell_k}_{\leq 0} \quad \leq \quad 0 \, . \tag{18}$$

Step 3. Together (17) and (18) imply that, if $j^* \in \{1, \ldots, m-1\}$,

$$R_{\tau\ell}(0, h_m) \quad = \quad \sum_{k=1}^{j^*} \tau_k \, \ell_k \, \mathcal{R}(y_k) + \sum_{k=j^*+1}^{m} \tau_k \, \ell_k \, \mathcal{R}(y_k) \quad \leq \quad \sum_{k=0}^{j^*} \tau_k \, \ell_k \, , \tag{19}$$

which proves (15). Note that the cases $j^* = 0$ and $j^* = m$ are also covered. With $j^* = 0$, $\sum_{k=1}^{j^*} \tau_k \, \ell_k \, \mathcal{R}(y_k)$ does not enter the chain of inequalities and (18) directly implies (15). With $j^* = m$, $\sum_{k=j^*+1}^{m} \tau_k \, \ell_k \, \mathcal{R}(y_k)$ does not enter and (17) directly implies (15).

A.3 Proof of Corollary 1 and Proposition 2

Take any continuous reform direction h on $[0, \bar{y}]$. We approximate h with a piece-wise linear reform direction h_m that involves m one-bracket reforms $(\tau_k, h_k^s)_{k=1}^m$, so that $h_m(y) = \sum_{k=1}^m \tau_k \, h_k^s(y)$. Throughout, we let $\ell = 1$ and divide the domain $[0, \bar{y}]$ into m brackets of equal length $\ell_k = \frac{1}{m}\bar{y}$, starting at incomes $y_1 = 0$, $y_2 = \frac{1}{m}\bar{y}$, ..., and $y_m = \frac{m-1}{m}\bar{y}$. Thus, we have m adjacent brackets – a special case of our general formalism, which also allows for gaps between the brackets where marginal tax rates change. For any k, we then let

$$\tau_k = \frac{h(y_{k+1}) - h(y_k)}{\ell_k} \;, \quad \text{where we set} \quad y_{m+1} = \bar{y}.$$

This yields an approximation of h by a piecewise linear function. The construction is illustrated in Figure 8. By choosing m sufficiently large, the piecewise linear function h_m approximates h in the sense that, for any $\varepsilon > 0$, there exists $\hat{m}(\varepsilon)$ so that for any $m > \hat{m}(\varepsilon)$,

$$\sup_{y \in \mathcal{Y}} \; | \, h(y) - h_m(y) \, | \quad < \quad \varepsilon \;.$$

For later reference, we note that this implies in particular that, for any y^* that maximizes $h(y)$ over $\mathcal{Y}$, we have

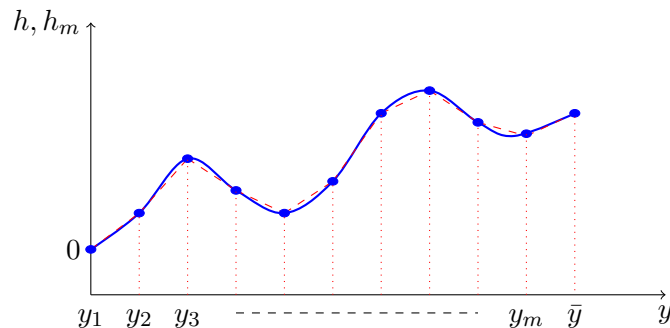$$h(y^*) - h_m(y^*) < \varepsilon \;. \tag{20}$$

Figure 8: Approximation of function h (solid, blue) by piecewise linear function h_m (dashed, red).

The Gateaux differential is linear in the reform direction h and hence continuous. We therefore have

$$\lim_{m \to \infty} R_\tau(0, h_m) = R_\tau(0, h) , \tag{21}$$

i.e., the Gateaux differential for direction h_m converges to the Gateaux differential for direction h. We now provide a characterization of $\lim_{m \to \infty} R_\tau(0, h_m)$. By the linearity of the Gateaux differential, we have

$$R_\tau(0, h_m) = \sum_{k=1}^{m} \tau_k R_\tau(0, h_k^s) = \sum_{k=1}^{m} \tau_k R_\tau^s(0, \ell_k, y_k) .$$

For m large and $\ell_k = \frac{\bar{y}}{m}$ close to zero, a first-order Taylor approximation moreover gives

$$R_\tau^s(0, \ell_k, y_k) \approx \ell_k \, R_{\tau\ell}^s(0, 0, y_k) = \ell_k \, \mathcal{R}(y_k) .$$

The approximation is perfect in the limit case $m \to \infty$ or, equivalently, $\ell_k = \frac{\bar{y}}{m} \to 0$. Therefore,

$$\begin{aligned}
\lim_{m \to \infty} R_\tau(0, h_m) &= \lim_{m \to \infty} \sum_{k=1}^{m} \tau_k \ell_k \mathcal{R}(y_k) \\
&= \lim_{m \to \infty} \sum_{k=1}^{m} [y_{k+1} - y_k] \, \tau_k \, \mathcal{R}(y_k) \\
&= \int_{y \in \mathcal{Y}} h'(y) \, \mathcal{R}(y) \, dy ,
\end{aligned}$$

where the last term is the Riemann integral that gives the marginal revenue effect of a reform in direction h, and $h' : y \mapsto h'(y)$ is the change of the marginal tax rate at income y due to a unit increase in τ. To see this, note first that the term in the second line is the limit of a Riemann sum, the latter involving the step function $y \mapsto \tau_k \, \mathcal{R}(y_k)$ for $y \in [y_k, y_k + \ell_k]$. Second note that both h and h_m are continuous functions on a compact interval. Hence, they have a bounded variation and are therefore differentiable almost everywhere. Moreover, we have $\lim_{m \to \infty} \ell_k = 0$ and therefore, for any y in the interior of bracket k,

$$\tau_k = h_m'(y) \longrightarrow_{m \to \infty} h'(y) .$$

This completes the derivation of Equation (9) which appears in Proposition 2.

We now show that the conclusion of Theorem 2 extends to all continuous reform directions. We first note that (21) implies that for any $\varepsilon > 0$, there is an $\tilde{m}(\varepsilon) \in \mathbb{R}$ such that, for $m > \tilde{m}(\varepsilon)$

$$R_\tau(0, h) - R_\tau(0, h_m) \quad < \quad \varepsilon . \tag{22}$$

To complete the proof of Corollary 1, we proceed by contradiction. Suppose that the conditions of Theorem 2 apply and that there is a continuous reform direction h that is Pareto-improving, i.e., that satisfies

$$R_\tau(0, h) - \max_{y \in \mathcal{Y}} h(y) \quad = \quad 2\varepsilon ,$$

for some $\varepsilon > 0$. Then, by (20) and (22), for $m > \max\{\hat{m}(\varepsilon), \tilde{m}(\varepsilon)\}$, there is also an m-bracket reform such that $R_\tau(0, h_m) - \max_{y \in \mathcal{Y}} h_m(y) > 0$, i.e., that is Pareto-improving. But this is impossible by Theorem 2. The assumption that there is a Pareto-improving direction h in the class of continuous functions on $[0, \bar{y}]$ has therefore led to a contradiction and must be false.

B   The revenue-maximizing tax cut

Fix a status quo tax-transfer system (T_0, c_0) and a corresponding revenue function $\mathcal{R}(y)$. By Proposition 2, a reform direction h is Pareto-improving if

$$\Pi(h) \quad := \quad \int_0^{\bar{y}} h'(y)\mathcal{R}(y)dy - \max_{y \in y_0(\Theta)} \int_0^y h'(z)dz , \tag{23}$$

is strictly positive. Then, a small reform in direction h provides a net revenue gain in excess to what is needed to compensate the agents facing the largest tax increase. In the following, we solve for the reform direction h_R that maximizes Π over the set of functions such that $h' : [0, \bar{y}] \to [-a, a]$ for some fixed $a > 0$. To simplify the exposition, we impose further assumptions that are satisfied in the context of our application in Section 5 (see Figure 7).

Assumption 1 *Let $y_0(\Theta) = [0, \bar{y}]$. There is a unique pair of income levels (y_s, y_t) with $0 < y_s < y_t < \bar{y}$ such that $\mathcal{R}(y)$ is strictly decreasing on $(0, y_s)$ and on $(y_t, \bar{y})$. Let $y_m = \frac{y_s + y_t}{2}$. There is a number $r \in (0, 1)$ so that:*

(i) $\mathcal{R}(y_s) = \mathcal{R}(y_t) = r$,

(ii) $\mathcal{R}(y) \in (r, 1)$ *for each* $y \in (0, y_s)$ *and for each* $y \in (y_m, y_t)$,

(iii) $\mathcal{R}(y) < r$ *for each* $y \in (y_s, y_m)$,

(iv) $\mathcal{R}(y) \in [0, r)$ *for each* $y \in (y_t, \bar{y})$,

In words: The monotonicity condition on $y \mapsto \mathcal{R}(y)$, see Theorems 1 and 2, is satisfied for incomes lower than y_s and for incomes higher than y_t, but violated for incomes between y_s and y_t. By (i), at y_s and y_t, marginal tax rates also satisfy the boundedness conditions in those Theorems. By (ii), for incomes lower than y_s and for incomes in the designated phase-out range (y_m, y_t), marginal tax rates are not inefficiently low. By (iii), for incomes in the designated phase-in range, the revenue potential is bounded by the one at y_s: y_s is a maximum of $y \mapsto \mathcal{R}(y)$ over this range of incomes. Finally, by (iv), for incomes larger than y_t, marginal tax rates are not inefficiently high and, over this income range, $y \mapsto \mathcal{R}(y)$ obtains a maximum at y_t.

Proposition 3 *Fix $a > 0$. Under Assumption 1, the revenue-maximizing tax cut h_R is, for any $a > 0$, given by a two-bracket tax cut with*

$$
h_R'(y) = \begin{cases} 0 & \text{for} \quad y \in [0, y_s) \ , \\ -a & \text{for} \quad y \in (y_s, y_m) \ , \\ a & \text{for} \quad y \in (y_m, y_t) \ , \\ 0 & \text{for} \quad y \in (y_t, \bar{y}] \ . \end{cases}
\tag{24}
$$

Moreover, $\Pi(h_R) = aI(T_0, c_0)$ with

$$
I(T_0, c_0) = \int_{y_m}^{y_t} \mathcal{R}(y) dy - \int_{y_s}^{y_m} \mathcal{R}(y) dy \ .
\tag{25}
$$

Proof To solve for the revenue-maximizing tax cut, we proceed in three steps. First, we show that $\max_{y \in y_0(\Theta)} h_R(y)$ equals zero. Second, we solve for the reform that maximizes $\Pi(h)$ subject to (i) $\max_{y \in y_0(\Theta)} h(y) = 0$, (ii) $h'(y) \in [-a, a]$ for all $y \in [y_s, y_t)$, and (iii) the additional restriction that $h'(y) = 0$ for all incomes below y_s and above y_t. Third, we show that the solution to this more restricted problem also solves the original maximization problem.

Step 1. The normalization that system $T(0) = 0$ for any tax system that we consider, also implies $h(0) = 0$ for any reform direction that we consider. Therefore $\max_{y \in y_0(\Theta)} h_R(y) \geq 0$. To show that $\max_{y \in y_0(\Theta)} h_R(y) = 0$, we provide a proof by contradiction. For this purpose, assume that there is some $\varphi > 0$ such that $\max_{y \in y_0(\Theta)} h_R(y) = \varphi$ and denote by y^* the lowest income level such that $h_R(y) = \varphi$. Then, $y^* > 0$ and there must be an income $y' \in (0, y^*)$ such that $h_R(y) > 0$ for all $y \in (y', y^*)$. Consider a perturbed reform h_ε such that $h'_\varepsilon(y) = h'_R(y) - \varepsilon$ for all incomes (y', y^*) and $h'_\varepsilon(y) = h'_R(y)$ for all other incomes. The net revenue gain from this perturbed reform is

$$\Pi(h_\varepsilon) = \int_0^{\bar{y}} h'_\varepsilon(y)\mathcal{R}(y)dy - \varphi + \varepsilon(y^* - y') \ .$$

The derivative of $\Pi(h_\varepsilon)$ with respect to ε is

$$\frac{d\Pi(h_\varepsilon)}{d\varepsilon}\Big|_{\varepsilon=0} = -\int_{y'}^{y^*} \mathcal{R}(y)dy + (y^* - y') > 0 \ ,$$

where the positive sign follows because $\mathcal{R}(y) < 1$ for any $y \in (0, \bar{y})$ by Assumption 1. This contradicts that assumption that Π obtains a maximum at h_R.

Step 2. Consider the problem to maximize $\Pi(h)$ over the set of functions h such that (i) $h'(y) \in [-a, a]$ for all $y \in (y_s, y_t)$, (ii) $h'(y) = 0$ for all $y \leq y_s$ and all $y \geq y_t$ and (iii) $h(y) = \int_0^y h'(z)dz = \int_{y_s}^y h'(z)dz \leq 0$ for all $y \in (y_s, y_t)$. Note that the function given in (24) satisfies these constraints.

We now consider a Lagrangian for a more relaxed problem that takes only the constraint $h(y_t) \leq 0$ into account. We argue below that a solution to this relaxed problem satisfies (i)-(iii).

$$\mathcal{L}(t) = \int_{y_s}^{y_t} h'(y)\mathcal{R}(y)dy - \mu \int_{y_s}^{y_t} h'(y)dy \ ,$$

where μ is a Lagrange multiplier. The solution to this restricted problem is given by a function $\tilde{h} : (y_s, y_t) \to [-a, a]$ and a value $\tilde{\mu}$ of the multiplier. For any $y \in (y_s, y_t)$, the derivative of $\mathcal{L}$ with respect to $h'(y)$ is given by

$$\frac{\partial\mathcal{L}}{\partial h'(y)} = \mathcal{R}(y) - \tilde{\mu} \ .$$

As the Lagrangian in linear in each $h'(y)$, the solution involves $\tilde{h}'(y)$ equal to the lower bound $-a$ for all y such that $\mathcal{R}(y) < \tilde{\mu}$, and $\tilde{h}'(y)$ equal to the upper bound a for all y such that $\mathcal{R}(y) > \tilde{\mu}$. Under Assumption 1, this is only consistent with $\tilde{h}(y_t) = \int_{y_s}^{y_t} \tilde{h}'(y)dy = 0$ if $\tilde{\mu} = r = \mathcal{R}(y_s)$. Then, $\tilde{h}'(y) = -a$ for all $y \in (y_s, y_m)$ and $\tilde{h}'(y) = a$ for all $y \in (y_m, y_t)$. Hence, $\tilde{h}$ equals the function given in (24), so it satisfies (i)-(iii). Consequently,

$$\Pi(\tilde{h}) = \int_{y_s}^{y_m} -a\mathcal{R}(y)dy + \int_{y_m}^{y_t} a\mathcal{R}(y)dy = aI(T_0, c_0) \,,$$

with $I(T_0, c_0)$ given in (25). We also note that, as $\tilde{\mu}$ is strictly positive, the constraint $\tilde{h}(y_t) \leq 0$ is binding.

Step 3. It remains to show that we cannot increase Π further by allowing $h'(y) \in \{-a, a\}$ for incomes below y_s and above y_t, while respecting the constraint $\int_0^y h(z)dz \leq 0$ for all $y \in [0, \bar{y}]$. A repeated application of the arguments in Step 2, once for incomes below y_s, and once for incomes above y_t, exploiting the monotonicity of $y \mapsto \mathcal{R}(y)$ over these income ranges, shows that any candidate solution to this problem will take the form

$$h'_R(y) = \begin{cases} 0 & \text{for} & y < y_\alpha \,, \\ -a & \text{for} & y \in (y_\alpha, y_m) \,, \\ a & \text{for} & y \in (y_m, y_\beta) \,, \\ 0 & \text{for} & y > y_\beta \,. \end{cases}$$

where $y_\alpha \leq y_s$ and $y_\beta \geq y_t$. The constraint $h(\bar{y}) = \int_0^{\bar{y}} h_R(y)dy \leq 0$ is only satisfied if $y_\beta - y_m \leq y_m - y_\alpha$. Finally, choosing y_α and y_β to maximize $\Pi(h_R)$ subject to $y_\alpha \leq y_s$ and $y_\beta \geq y_t$ and $y_\beta - y_m \leq y_m - y_\alpha$, shows that $y_\alpha = y_s$ and $y_\beta = y_t$ is an optimal choice.

Supplementary Material (not for publication)

C Empirical analysis

In this section, we first provide additional background information on the 1974 US tax-and-transfer system. Second, we conduct various robustness checks on the result that there was scope for a Pareto improvement by means of a two bracket tax cut. Third, we refine our analysis by looking separately at single parents who differ in the number of children and discuss the desirability of tagging in this dimension. Finally, we analyze the US tax-and-transfer system for single parents in the post-reform years 1975 to 1978 and analyze the scope for further improvements of the EITC.

C.1 Data description and benchmark calibration

This section provides a description of our data and explains the choices for our benchmark calibration. We start with a description of the 1974 US tax-and-transfer system. Subsequently, we describe how we obtained estimates of the relevant income distributions and our benchmark assumptions on behavioral responses to tax reforms.

Status quo tax function: the US tax and transfer system in 1974. We take account of the federal income tax and the two largest welfare programs, Aid for Families with Dependent Children (AFDC) and Supplementary Nutrition Assistance Programs (SNAP, also called Food Stamps). ADFC was available only for single parents and varied to some extent across US states. We focus on the AFDC rules in California. SNAP was a federal program that was available both for single parents and childless singles, but more generous for single parents. Programs for single parents also depended on the number of children. In our benchmark analysis, we focus on the tax-transfer schedule for single parents with two children. In our data, the median number of children in single-parent households was two, and the arithmetic mean was about 2.2. Figure 4 in the main text depicts the

Table 1: Sources for US tax-transfer system, 1974-1978.

Information	Years	Sources
Income tax	1974-1975	Internal Revenue Service, "Instructions for Form 1040", years 1974, 1975. URLs: `https://www.irs.gov/pub/irs_prior/i1040--1974.pdf`; `www.irs.gov/pub/irs_prior/i1040--1975.pdf`.
	1974-1978	Internal Revenue Service, "Statistics of Income, Individual income tax returns", years 1974, 1975, 1978. URLs: `https://www.irs.gov/pub/irs-soi/74inar.pdf`; `https://www.irs.gov/pub/irs-soi/75inar.pdf`; `www.irs.gov/pub/irs-soi/78inar.pdf`.
EITC	1975, 1978	Tax Policy Center, "Earned Income Tax Credit Parameters, 1975-2021". URL: `https://www.taxpolicycenter.org/statistics/eitc-parameters`.
AFDC	1974-1978	Office of the Assistant Secretary for Planning and Evaluation, "Aid to Families with Dependent Children. The Baseline", 1998. URL: `https://aspe.hhs.gov/basic-report/aid-families-dependent-children-baseline`
	1974	US Department of Health, Education and Welfare, "Aid to Families with Dependent Children: Standards for Basic Needs, July 1974", 1974. URL: `https://hdl.handle.net/2027/mdp.39015088906634`.
	1975, 1978	TRIM3 project, "TRIM3 AFCD Rules". Accessible at `trim3.urban.org`.
SNAP	1974-1975	US Bureau of the Census, "Characteristics of Households Purchasing Food Stamps. Current Population Reports", Series 9-23, No. 61, 1976. URL: `https://www2.census.gov/library/publications/1976/demographics/p23-061.pdf`.
	1978	Federal Register Vol. 43, No. 95, May 16, 1978. Accessible at `https://www.govinfo.gov/app/collection/fr`.
SNAP, Payroll tax	1974-1978	Social Security Administration, "Annual Statistical Supplement. Section 2: Program Provisions and SSA Administrative Data", 2010. URL: `https://www.ssa.gov/policy/docs/statcomps/supplement/2010/2a1-2a7.html`.

effective tax rates for this subgroup of single parents and for childless singles. For the years 1975 and later, we also account for the Earned Income Tax Credit. Table 1 below depicts the sources we use for computing the US tax and transfer systems for single parents and childless singles in the years 1974 to 1978.

Income distribution. We estimate the 1974 income distributions based on Current Population Survey (CPS) data, using the Annual Social and Economics Supplement from the March 1975 wave. We proceed in the same way for the years 1975 and 1978. Specifically, we consider for each year the sample of non-married individuals aged 25 to 60 who do neither co-habit with an unmarried spouse nor with another adult family member. We partition this sample into childless singles and single parents. For the benchmark analysis reported in the main text, we estimate the earned income distribution based on the set of all single parents (i.e., with any number of children). In the subsequent analysis, we also look into the income distributions in the subsets of single parents with one, two, and three children.

In line with the EITC rules, we consider as earned income the sum of (self-reported) wage income and self-employment income. In this sample, 30.9% of single parents and 14.8% of childless singles have zero or negative incomes, while 49.9% of single parents and 43% of childless singles had strictly positive incomes below 8,000 USD, the eligibility threshold of the EITC. For our benchmark analysis, we estimate the distributions of earned income for both groups using a non-parametric kernel density estimation with a Gaussian kernel. In the benchmark, we use bandwidths of 997 USD for single parents and 1,200 USD for childless singles, following Silverman's rule. Figure 9 shows the estimated pdf and the cdf of both income distributions.

Behavioral responses to taxation. We draw on a rich literature providing estimates of labor supply responses at the intensive and the extensive margin, see the discussions in Saez, Slemrod and Giertz (2012) or Chetty et al. (2013). Based on a meta-study and focusing on population-wide averages, Chetty et al. (2013)
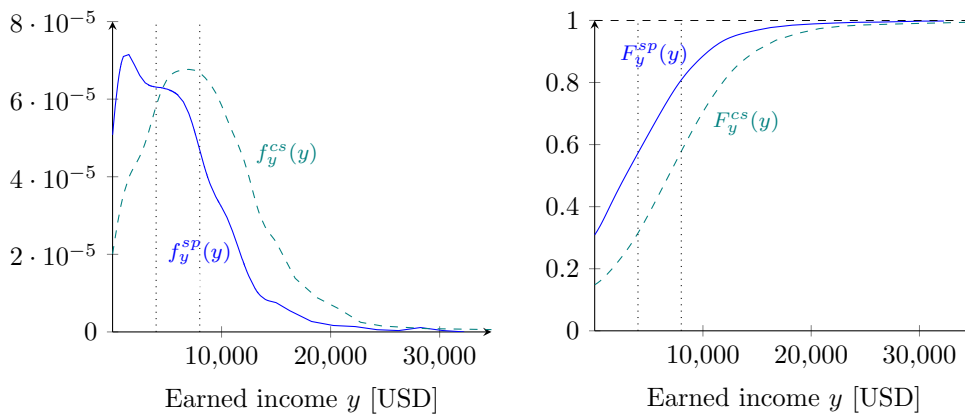
(a) Probability density functions.　　(b) Cumulative distribution functions.

Figure 9: Income distributions of single parents and childless singles, US 1974.

Notes: Figure 9 shows the kernel estimates of the US income distributions among single parents (solid blue lines) and childless singles (dashed teal lines) in 1974. Panel (a) depicts the probability density functions; panel (b) depicts the cumulative distribution functions of both income distributions. The dotted vertical lines mark the endpoints of the phase-in range at $4,000$ USD and the phase-out range at $8,000$ USD of the 1975 EITC.

Source: Authors' calculations (see Table 1 for details).

suggest an intensive-margin elasticity of labor supply with respect to the net-of-tax rate of 0.33, and an extensive-margin elasticity with respect to net labor income of 0.25. Bargain, Dolls, Neumann, Peichl and Siegloch (2014) provide similar estimates for a sample of childless singles. For single parents, various studies find larger responses at the extensive margin. Specifically, Bastian (2020) estimates labor supply responses of single mothers to the 1975 EITC introduction, and finds an average participation elasticity of 0.58. Most earlier studies find similar or even larger participation responses by single mothers, e.g., Meyer and Rosenbaum (2001). In contrast, Kleven (2019) recently estimated a participation elasticity close to zero based on EITC reforms in the 1990s. Besides, several studies find that persons with little formal education and low incomes respond more strongly at the extensive margin than persons with higher education and higher incomes – see, e.g., Juhn, Murphy and Topel (1991), Juhn, Murphy and Topel (2002), Meghir and Phillips (2010). There is only limited empirical evidence on the relevance of income effects for labor supply. Recent evidence by Cesarini, Lindqvist, Notowidigdo and Östling (2017) based on Swedish lottery winners suggests (i) a marginal propensity to earn out of unearned income (MPE) of -0.08, (ii) with about two thirds of income effects arising at the intensive margin, one third at the extensive margin, and (iii) with little heterogeneity in income effects along the income distribution. Imbens, Rubin and Sacerdote (2001) report similar estimates with an MPE of -0.11 for the US, see also Holtz-Eakin, Joulfaian and Rosen (1993).

In our benchmark calibration, we assume an average participation elasticity of 0.58 for single parents and 0.25 for childless singles, and an intensive-margin elasticity of 0.33 for both subgroups, following Bastian (2020) and Chetty et al. (2013). Moreover, we assume that participation elasticities are decreasing with income in both groups, according to the function $\pi_0(y) = \pi_a - \pi_b \ (y/\tilde{y})^{1/2}$, where $\tilde{y}$ equals 50,000 USD. Similar assumptions are employed by Jacquet et al. (2013) and Hansen (2021). For single parents, we assume that the participation elasticity falls from 0.67 at very low incomes to 0.4 at incomes above 50,000 USD (i.e., $\pi_a = 0.67$, $\pi_b = 0.27$), giving rise to an average value of 0.58. For childless singles,

we assume π to fall from 0.4 to 0.1 (i.e., $\pi_a = 0.4$, $\pi_b = 0.3$), giving rise to an average value around 0.25. In the benchmark calibration, we leave out income effects. In the sensitivity analysis below, we consider a large range of alternative assumptions on labor supply elasticities.

C.2 Sensitivity analysis

In the following, we provide an extensive sensitivity analysis. We start by considering alternative assumptions on the behavioral responses to tax reforms. Then, we repeat our analysis using alternative estimates of the income distributions among single parents. Finally, we consider alternative representations of the US tax-transfer system. We find that our main results are robust to variations in all these dimensions: The 1974 US tax-transfer system for single parents was Pareto-inefficient, and there existed Pareto-improving tax reforms with two brackets similar to the 1975 EITC introduction.

Labor supply elasticities. Figure 10 plots the revenue function $y \mapsto \mathcal{R}_{sp}(y)$ under different assumptions on the extensive-margin elasticity π_0. This aspect deserves particular attention because the optimal tax literature has established the result that an EITC with negative marginal taxes can only be optimal with labor supply responses at the extensive margin, see Saez (2002) and Hansen (2021). Moreover, Kleven (2019) recently challenged the conventional view that the participation responses of single parents are particularly large. In Figure 10, we therefore show the revenue function $y \mapsto \mathcal{R}_{sp}(y)$ for a range of different assumptions on participation elasticities. The blue line is our benchmark case in which the participation elasticity is falling from 0.67 at very low incomes to 0.4 at high incomes, giving an average elasticity of $\bar{\pi} = 0.58$. Additionally, the black line is a case where the elasticity is zero at all income levels; the brown line depicts a case where the elasticity is falling from 0.3 to 0.1 ($\bar{\pi} = 0.23$); the teal line shows a case where it is falling from 1 to 0.75 ($\bar{\pi} = 0.92$). Finally, the red line depicts a case with an elasticity of 0.4 at all income levels. In all these cases, the intensive-margin
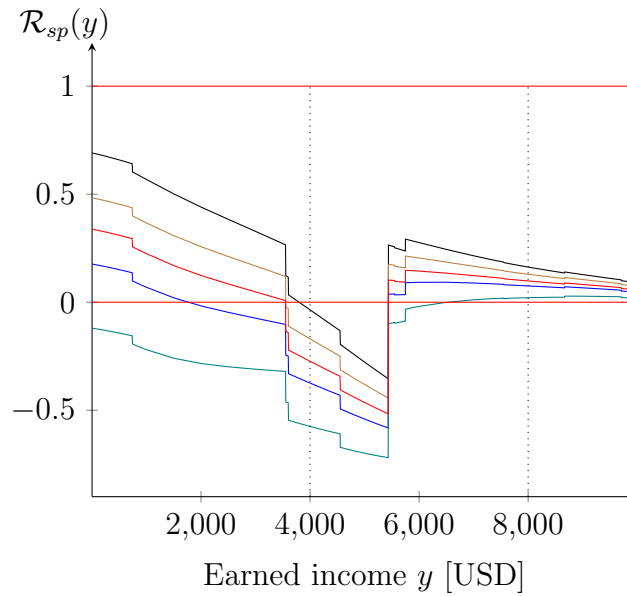
Figure 10: Pareto test of the 1974 US tax-transfer system, different participation elasticities.

Notes: Figure 10 shows the function $y \mapsto \mathcal{R}_{sp}(y)$ for single parents in 1974, assuming different participation elasticities: constant at 0 (black line), falling with income from 0.3 to 0.1 (brown line), falling from 0.67 to 0.4 (blue line), falling from 1 to 0.75 (teal line). The dotted vertical lines mark the endpoints of the phase-in range at $4,000$ USD and the phase-out range at $8,000$ USD of the 1975 EITC.

Source: Authors' calculations (see Table 1 for details).

elasticity is held constant at $\varepsilon = 0.33$, as in our benchmark calibration.

The qualitative result is the same in all cases: for single parents, the 1974 US tax system was not Pareto-efficient. There existed Pareto-improving tax cuts with one and two brackets. This is even true in the limit case of vanishing labor supply responses at the extensive margin. Hence, the finding that an EITC-like two-bracket tax cut for low-income earners was Pareto-improving holds irrespective of what we assume about the strength of behavioral responses to taxation at the extensive margin. Quantitatively, they play a role, though. Higher participation elasticities imply larger revenue gains from a two-bracket tax cut; i.e., higher participation elasticities make an EITC-like reform more attractive (see Figure 10).
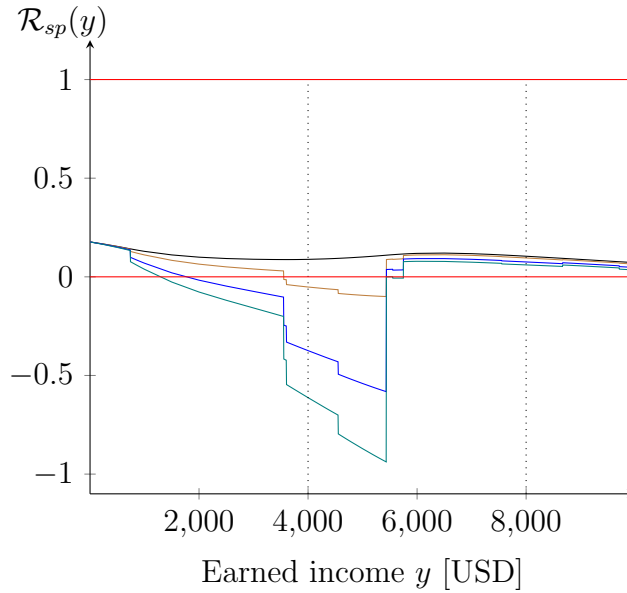

Figure 11: Pareto test of the 1974 US tax-transfer system, different intensive-margin elasticities.

Notes: Figure 11 shows the marginal revenue function $\mathcal{R}_{sp}(y)$ for single parents in 1974, assuming an intensive-margin elasticity of 0 (brown line), 0.1 (black line), 0.33 (blue line, benchmark) and 0.5 (teal line). The dotted vertical lines mark the endpoints of the phase-in range at $4,000$ USD and the phase-out range at $8,000$ USD of the 1975 EITC.

Source: Authors' calculations (see Table 1 for details).

Figure 11 plots the revenue function $y \mapsto \mathcal{R}_{sp}(y)$ for different levels of the intensive-margin elasticity ε between 0 and 0.5, while holding the average participation elasticity at the benchmark level of 0.58. We find that the 1974 US tax system for single parents violated the monotonicity condition for Pareto efficiency for any level of the intensive-margin elasticity in this range. Again, this even remains true in the limit case where labor supply does not respond at the intensive margin, i.e., for $\varepsilon = 0$. More specifically, we find the inefficiencies in the 1974 tax-transfer system to be quantitatively more pronounced with higher levels of the intensive-margin elasticity.
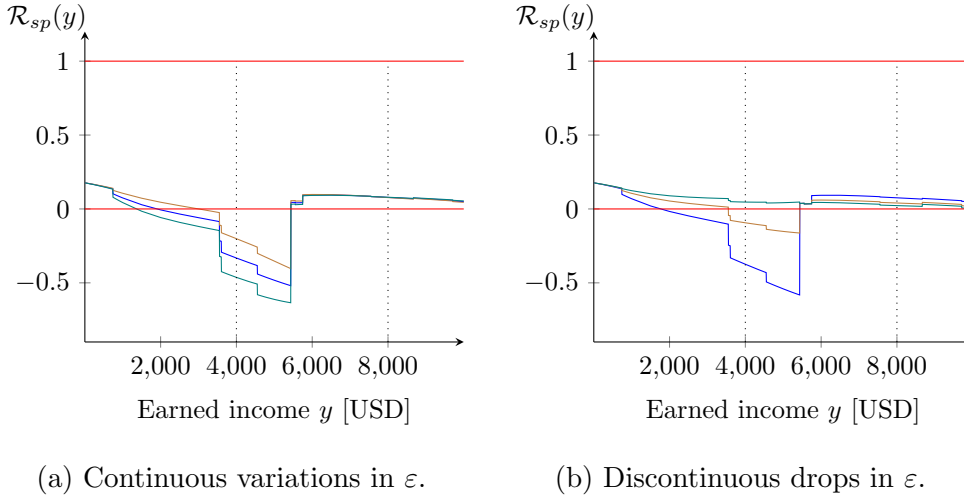

(a) Continuous variations in ε.   (b) Discontinuous drops in ε.

Figure 12: Pareto test of the 1974 US tax-transfer system, heterogeneous intensive-margin elasticities.

Notes: Figure 12a shows the marginal revenue function $\mathcal{R}_{sp}(y)$ for single parents in 1974, assuming that the intensive-margin elasticity ε varies with income: falling from 0.5 to 0.1 (brown line), or increasing from 0.1 to 0.5 (teal line), or equal to 0.3 at all income levels. Figure 12b shows function $\mathcal{R}_{sp}(y)$ under the assumption that the intensive-margin elasticity ε jumps in two steps from 0.13 over 0.33 to 0.73 (brown line), or from 0.03 over 0.33 to 0.93 (teal line), or equals 0.33 at all income levels (blue line, benchmark). In all cases, the average participation elasticity is held at the benchmark level of 0.58. The dotted vertical lines mark the endpoints of the phase-in range at $4,000$ USD and the phase-out range at $8,000$ USD of the 1975 EITC.

Source: Authors' calculations (see Table 1 for details).

In Figure 12, we plot the revenue function $y \mapsto \mathcal{R}_{sp}(y)$ under the assumption that the intensive-margin elasticity ε varies with income. The left panel compares a case where ε is equal to 0.3 at all income levels (blue line) with cases where ε is linearly increasing from 0.1 at the bottom to 0.5 at incomes above $15,000$ USD (teal line), or decreasing from 0.5 at the bottom to 0.1 at incomes above $15,000$ USD (brown line). As can easily be seen, in all cases, the revenue function violates the monotonicity condition from Theorem 1 so that there is room for a Pareto improvement by means of a symmetric two-bracket tax cut. In an attempt to identify the limits of this robustness analysis, we also explore some extreme cases in which the elasticity ε varies with income in a discontinuous way. Specifically, the right panel compares the benchmark case where ε is equal to 0.33 at all income levels (blue line) with two extreme cases. The brown line shows a case with an elasticity of 0.13 at the bottom, an upward jump to 0.33 at income level $5,433$ USD, and a further jump to 0.73 at $5,478$ USD. Our qualitative results remain unchanged in this case. The teal line shows a case where both upward jumps are even more pronounced, from 0.03 at the bottom over 0.33 to 0.93 at the top. In this second case, the revenue function $y \mapsto \mathcal{R}_{sp}(y)$ gets close to satisfying the monotonicity condition for a Pareto-efficient tax schedule (see Figure 12b).

Income effects. Finally, we consider a model in which income effects in labor supply arise due to concave consumption utility

$$u(c, y, \omega, \gamma) = \psi(c) - k(y, \gamma) - \gamma \, \mathbb{1}_{y>0} \, ,$$

with $\psi'(c) > 0$, $\psi''(c) < 0$ for all $c > 0$. With income effects, our sufficient-statistics formula for the revenue function $y \mapsto \mathcal{R}(y)$ becomes

$$
\begin{aligned}
\mathcal{R}(y) \;=\; (1 - \mathcal{M}_0)^{-1} &\left\{ 1 - F_y(y) - f_y(y)\, y\, \varepsilon(y)\, \frac{T_0'(y)}{1 - T_0'(y)} \right. \\
&\left. - \int_y^{\bar{y}} f_y(y')\pi(y') \frac{T_0(y')}{y' - T_0(y')} dy' - \int_y^{\bar{y}} f_y(y')\eta(y')T_0'(y')dy' \right\} \, , \quad (26)
\end{aligned}
$$

where the multiplier term $\mathcal{M}_0$ is given by

$$\mathcal{M}_0 = \int_0^{\bar{y}} f_y(y)\eta(y))T_0'(y)dy + \int_0^{\bar{y}} f_y(y)\nu(y)T_0(y)dy \, . \quad (27)$$

In this formula, $\eta(y)$ captures income effects at the intensive margin (i.e., the effect of an increase of the base transfer c_0 on the earnings of workers with pre-reform income y); $\nu(y)$ captures income effects at the extensive margin (i.e., the effect of an increase of the base transfer c_0 on the employment rate of agents who choose to earn y when becoming active on the labor market). With concave consumption utility and additively separable preferences, $\eta(y)$ and $\nu(y)$ are both negative. Our benchmark case with quasi-linear utility is nested with $\eta(y) = \nu(y) = 0$. We formally derive a version of formula (26) in part F of the Supplementary Material below. Specifically, Proposition 4 gives the revenue function $y \mapsto \mathcal{R}(y)$ in terms of the model's primitives based on a general framework that allows for complementaries between consumption and leisure.

By (26), income effects matter for the revenue effect of a one-bracket tax increase at income y in two ways. First, they increase labor supply at the intensive margin for all agents with income above y, for whom the reform has reduced net income directly. This effect is captured by the last term in (26). With positive marginal taxes, this implies an upward shift of the revenue function. Second, when the revenue gain is rebated lump sum, this spurs further income effects at both margins all across the income distribution. This is captured by the term $\mathcal{M}_0$ in (27). With positive marginal taxes and positive participation tax rates, income effects make $\mathcal{M}_0$ negative and decrease the multiplier $(1 - \mathcal{M}_0)^{-1}$. Consequently, relative to the benchmark without income effects, the revenue function $y \mapsto \mathcal{R}(y)$ shifts towards the horizontal axis, i.e., its absolute value is decreased.

Figure 13 compares the revenue function $\mathcal{R}_{sp}(y)$ for our benchmark calibration to two scenarios with income effects. The teal line shows the revenue function $y \mapsto \mathcal{R}_{sp}(y)$ for a case with $\eta(y) = -0.08$ and $\nu(y) = -8 \times 10^{-6}$, corresponding to an MPE of -0.1 close to the estimates by Cesarini et al. (2017) and Imbens et al. (2001). The brown line shows $y \mapsto \mathcal{R}_{sp}(y)$ for a larger MPE of -0.4, corresponding to $\eta(y) = -0.24$ and $\nu(y) = -2.4 \times 10^{-5}$. The blue line is the benchmark calibration without income effects. In all cases, we maintain our benchmark assumptions about substitution effects at both margins, i.e., $\varepsilon = 0.33$ and $\bar{\pi} = 0.58$. Figure 13
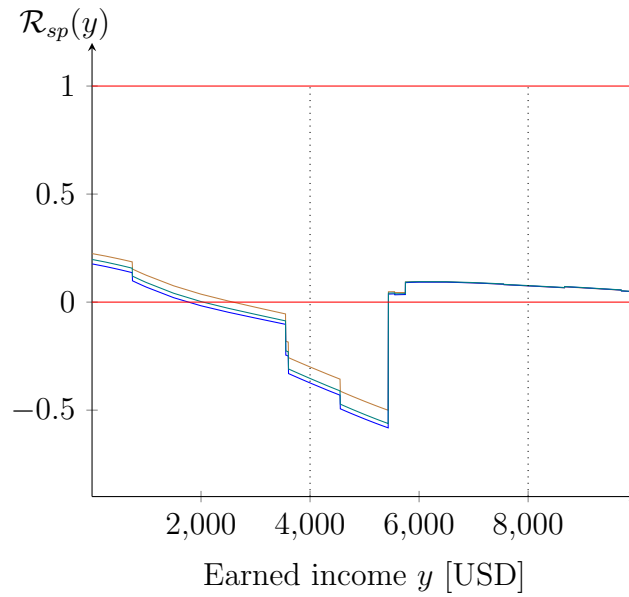
Figure 13: Pareto test of the 1974 US tax-transfer system, accounting for income effects.

Notes: Figure 10 shows the revenue function for single parents $y \mapsto \mathcal{R}_{sp}(y)$ in 1974 as a function of earned income, assuming income effects at both margins with an MPE of -0.08 (teal line) or -0.24 (brown line). For comparison, the blue line shows the benchmark case without income effects. The dotted vertical lines mark the endpoints of the phase-in range at $4,000$ USD and the phase-out range at $8,000$ USD of the 1975 EITC.

Source: Authors' calculations (see Table 1 for details).

demonstrates that our conclusion on the desirability of an EITC is robust to the consideration for income effects.

Summing up, our sensitivity analysis demonstrates that our main result does not hinge on the specifics of our benchmark calibration. The 1974 US tax and transfer system for single parents was not Pareto-efficient and there existed Pareto-improving reforms akin to the introduction of the EITC. This conclusion is obtained for a wide range of empirically plausible assumptions about labor supply responses at the intensive and extensive margin.
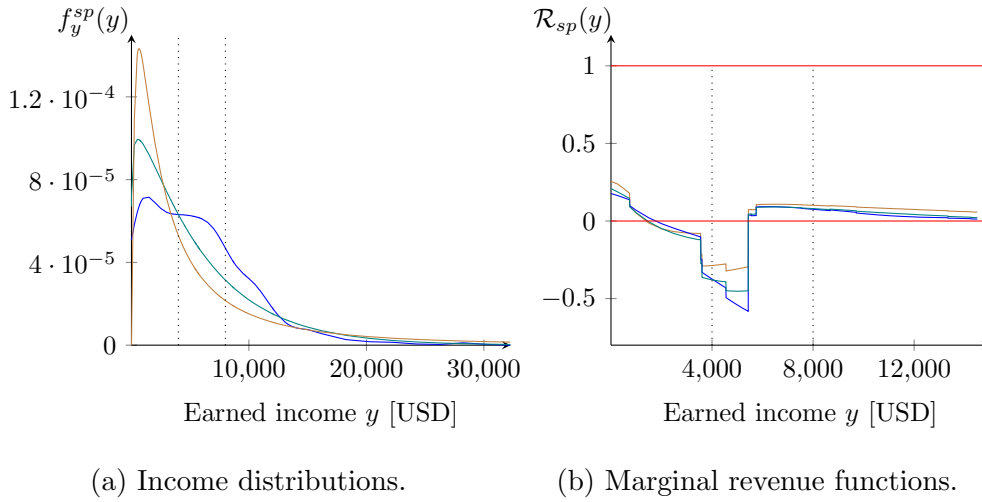

(a) Income distributions.   (b) Marginal revenue functions.

Figure 14: Pareto test of 1974 US tax system for single parents, parametric income distributions.

Notes: Figure 14a shows alternative estimates of the income distribution for single parents in 1974: kernel estimation (blue, benchmark), gamma distribution (teal), lognormal distribution (brown). Figure 14b shows the revenue function $y \mapsto \mathcal{R}_{sp}(y)$ for single parents in 1974, based on the kernel estimation (blue), gamma distribution (teal), and lognormal distribution (brown), using benchmark values for the elasticities at both margins. The dotted vertical lines mark the endpoints of the phase-in range at $4,000$ USD and the phase-out range at $8,000$ USD of the 1975 EITC.

Source: Authors' calculations (see Table 1 for details).

Alternative estimates of the income distribution. As alternatives to our benchmark calibration, we first verified that variations in the bandwidth choice

for our kernel estimates do not substantially affect our results. We also fitted gamma and lognormal distributions to the earned income data in the March 1975 CPS. Figure 14 shows versions of the revenue function that result with these parametric distributions. The differences to the benchmark case are small.
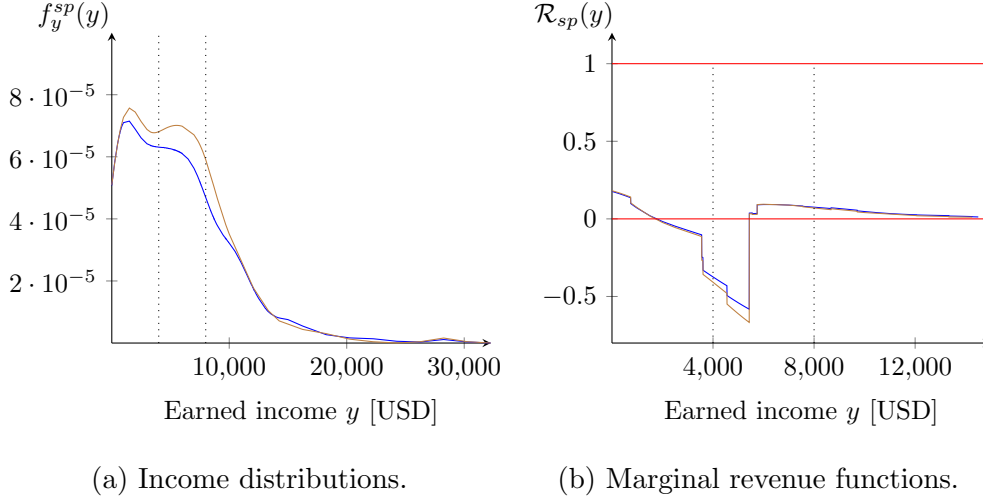

(a) Income distributions.

(b) Marginal revenue functions.

Figure 15: Pareto test of 1974 US tax system for single parents, different samples. **Notes:** Figure 15a shows the pdfs of the distributions of earned income based on the samples of all single parents (blue, benchmark) versus single parents with exactly two children (brown line) on the basis of the March 1975 CPS. Figure 15b gives the revenue function $y \mapsto \mathcal{R}_{sp}(y)$ for single parents in 1974, based on the estimated income distributions for all single parents (blue) versus single parents with exactly two children (brown), using benchmark values for the elasticities at both margins. The dotted vertical lines mark the endpoints of the phase-in range at $4,000$ USD and the phase-out range at $8,000$ USD of the 1975 EITC.
Source: Authors' calculations (see Table 1 for details).

For the benchmark analysis reported in the main text, we estimate the earned income distribution based on the set of all single parents ($N = 1,494$), with no conditioning on the number of children. As an alternative, we also considered non-parametric kernel estimates of the income distribution for the smaller sample of single parents with exactly two children ($N = 453$). Figure 15 shows that this does not make much of a difference.

Alternative representations of the tax and transfer system. As final robustness checks, we consider three alternative representations of the 1974 US tax and transfer system for single parents. In all three cases, our results are virtually unchanged. First, we take into account the wealth and assets tests for welfare recipients. In 1974, only families with liquid assets below $1,500$ USD were eligible for welfare transfers (AFDC, SNAP). Using the approximation of liquid assets suggested by Giannarelli (1992), we find that about 8% of single parents with earned incomes below the relevant thresholds failed the programs' assets test. Moreover, single parents were only eligible for the EITC if the sum of their earned income and capital income was below $8,000$ USD. In the March 1975 CPS data, only 0.9% of the single parents with incomes in the EITC range lost eligibility due to high capital incomes. Hence, we find that about 7% of single parents were eligible for the EITC, but not for welfare programs.

Our benchmark analysis in the main text ignores the assets test. Thereby, we provide an answer to the question whether, in 1974, introducing an EITC with the same assets tests as AFDC and SNAP would have been Pareto-improving. Alternatively, one can ask whether the tax-transfer system would have been Pareto-improved by a tax reform without an assets test, conditional on the assets test for welfare recipients being in place. To answer this question, the brown line in Figure 16a shows a version of the revenue function $y \mapsto \mathcal{R}_{sp}(y)$ under the assumption that 7% of single parents were not eligible for AFDC and SNAP, while all other single parents were eligible. For comparison, the blue line shows $y \mapsto \mathcal{R}_{sp}(y)$ for the benchmark case where all single parents are eligible for welfare, and the dashed teal line shows $y \mapsto \mathcal{R}_{sp}(y)$ for the counterfactual case where no single parent was eligible for welfare. As Figure 16a shows, taking into account the wealth test leaves our results qualitatively unchanged: There was scope for Pareto improvements by means of both one-bracket and two-bracket reforms.

Second, we take into account social security contributions (payroll taxes) to compute the effective tax rates for single parents. By contrast, our benchmark analysis takes into account the statutory income tax and welfare transfers, but
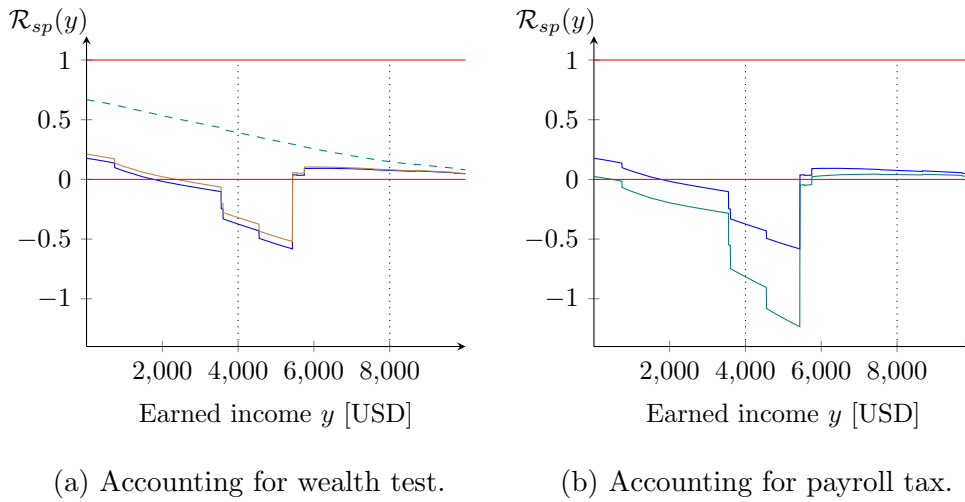
(a) Accounting for wealth test.   (b) Accounting for payroll tax.

Figure 16: Pareto test of 1974 US tax system, accounting for wealth test or payroll taxes.

Notes: Figure 16a shows the revenue function for single parents $y \mapsto \mathcal{R}_{sp}(y)$ taking into account the wealth test for AFDC and SNAP eligibility (brown line) versus ignoring it as in the benchmark (blue line). The dashed teal line shows $y \mapsto \mathcal{R}_{sp}(y)$ ignoring AFDC and SNAP, i.e., based on the statutory income tax schedule alone. Figure 16b plots $y \mapsto \mathcal{R}_{sp}(y)$ taking into account the employee share of social security contributions (teal line) versus ignoring them as in the benchmark (blue line). The dotted vertical lines mark the endpoints of the phase-in range at $4,000$ USD and the phase-out range at $8,000$ USD of the 1975 EITC.

Source: Authors' calculations (see Table 1 for details).

ignores payroll taxes. Specifically, Figure 16b illustrates the effect of taking into account the employee share of payroll taxes, which was 5.85% at the time. The teal line gives the revenue function $y \mapsto \mathcal{R}_{sp}(y)$ for this case, while the blue line shows the benchmark case without payroll taxes. With payroll taxes, we find even larger inefficiencies in the 1974 US tax-transfer system for single parents, and two-bracket tax cuts for low-income earners become even more attractive. Taking into account both the employee share and the employer share of payroll taxes would reinforce this effect.
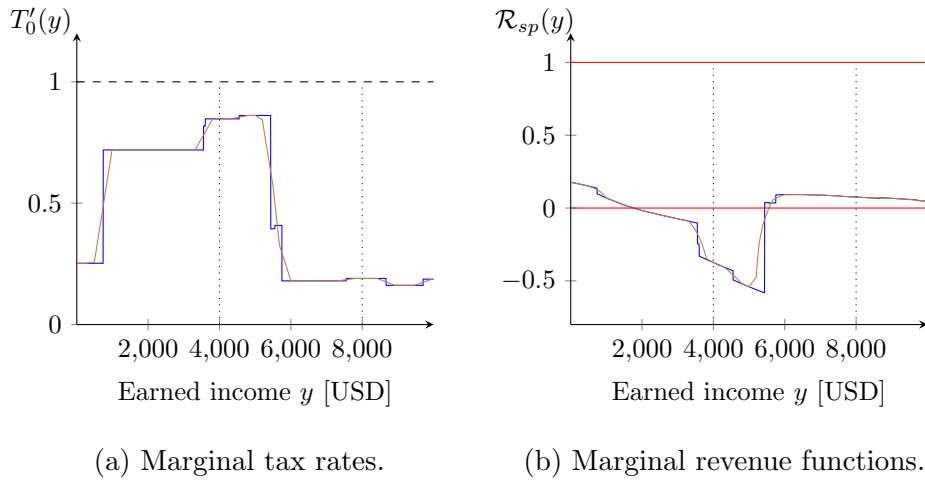

(a) Marginal tax rates.   (b) Marginal revenue functions.

Figure 17: Pareto test of 1974 US tax system, using smoothed tax schedule.

Notes: Figure 17a shows a smoothed version of the marginal tax rates for single parents (teal line) versus the statutory marginal tax rates (blue line). Figure 17b shows the revenue function $y \mapsto \mathcal{R}_{sp}(y)$ based on the smoothed tax schedule (teal line) versus the statutory tax schedule as in the benchmark (blue line). The dotted vertical lines mark the endpoints of the phase-in range at $4,000$ USD and the phase-out range at $8,000$ USD of the 1975 EITC.

Source: Authors' calculations (see Table 1 for details).

Third, we consider a smoothed version of the 1974 tax-transfer schedule for single parents that eliminates kinks. Such smoothed tax schedules are commonly used in the literature on implicit marginal welfare weights. Specifically, Figure 17a compares the statutory marginal tax $T_0'(y)$ (blue line) with a smooth alternative that is given by the average marginal tax in intervals of length 500 USD,

$[T_0(y+250) - T_0(y-250)]/500$ (brown line). In Figure 17b, the brown line shows the revenue function $y \mapsto \mathcal{R}_{sp}(y)$ based on the smoothed schedule and the corresponding participation taxes, while the blue line shows $y \mapsto \mathcal{R}_{sp}(y)$ based on the statutory (benchmark) tax schedule. Again, the differences between these curves are small.

C.3 Heterogeneity with respect to household size

In this section, we take a closer look at single parents who differ in the number of children. In 1975, a single EITC schedule was introduced for all single parents, irrespective of the household size. Decades later, the US authorities introduced additional, more generous EITC schedules for single parents with two or more children (in 1991), and for single parents with three or more children (in 2009). Our analysis provides two insights. First, the 1974 US tax-transfer system was not Pareto-efficient, irrespective of what subgroup of single parents we consider. Second, the size of the inefficiencies differed across these subgroups: Using the inefficiency measure introduced in Section 4, the 1974 tax-transfer system was more inefficient for single parents with three children than for those with two children, and the least inefficient for single parents with one child. Correspondingly, the best available tax reforms differed across subgroups, and the introduction of differentiated EITC schedules would have made sense already in the mid-1970s.

Figure 18 demonstrates pronounced heterogeneity among single parents in two dimensions. First, Figure 18a shows that the 1974 US tax-transfer schedules differed substantially depending on the number of children: For single parents with more children, effective marginal taxes were much higher and these high marginal tax rates were concentrated on higher incomes. Specifically, marginal taxes above 70% applied for single parents with one child up to annual incomes around 4, 500 USD; for those with two children up to annual incomes around 5, 400 USD; for those with three children up to annual incomes around 6, 600 USD. The main reason for these differences is that the maximum amount of welfare transfers (AFDC, SNAP) was increasing in family size. As the transfers were
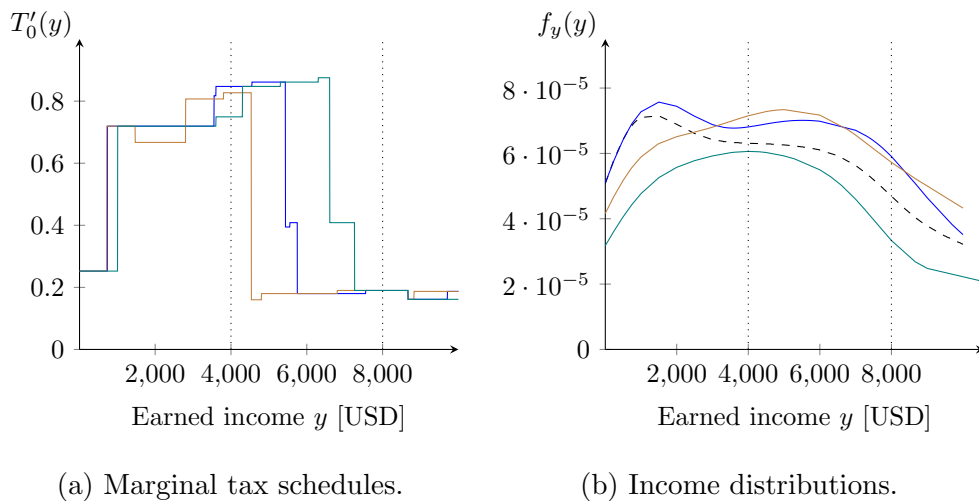
(a) Marginal tax schedules.   (b) Income distributions.

Figure 18: Tax rates and income distributions for single parents with 1, 2, and 3 children.

Notes: Figure 18 shows the tax and transfer schedules and the income distributions among single parents with one child (brown lines), two children (blue lines), and three children (teal lines). Specifically, Figure 18a shows the marginal taxes $y \mapsto T_0'(y)$ in 1974. Figure 18b shows the kernel estimates of the 1974 pdf $y \mapsto f_y(y)$ of earned incomes; the dashed black line plots the estimated pdf among all single parents. In both panels, the dotted vertical lines mark the endpoints of the phase-in range at $4,000$ USD and the phase-out range at $8,000$ USD of the 1975 EITC.

**Source:** Authors' calculations (see Table 1 for details).

phased out at the same rates, the phasing-out of welfare transfers occurred at higher incomes for families with more children. Second, Figure 18b shows that there were large differences in the distributions of earned income across single parents: Parents with more children had on average lower incomes. Specifically, it shows the kernel estimates of the pdfs of earned income based on CPS March 1975 data, using a Gaussian kernel with a bandwidth of 1200 for single parents with one and two children and a bandwidth of 1400 for single parents with three children. For comparison, the dashed black line shows the estimated pdf of the income distribution among all single parents. Among single parents with positive incomes, the average incomes were $5,037$ USD for those with one child; $4,165$ USD for those with two children; and $3,550$ USD for those with three children. The shares of single parents without any earned income were 20.9% (one child); 24.9% (two children); and 43.5% (three children).

To perform separate tests for Pareto efficiency of the 1974 US tax system in these subgroups, Figure 19 shows the revenue function $y \mapsto \mathcal{R}_{sp}(y)$ for single parents with one child in the upper left panel, for those with two children in the upper right panel, and for those with three children in the lower panel. For all cases, we assume an intensive-margin elasticity of 0.33 and participation elasticities that decrease with income from 0.67 to 0.4 as in the benchmark calibration. First, Figure 19 clarifies that the 1974 US tax-transfer schedules were not Pareto-efficient for any subgroup of single parents. All panels show that there were Pareto-improving tax reforms with both one and two brackets. Second, the relevant income ranges differed: With more children, the inefficiencies arise at higher income levels and cover a wider range of incomes.

The inefficiency measure developed in Section 4 confirms the visual impression from Figure 19. For single parents with one child, the revenue-maximizing tax cut was a two-bracket tax cut that reduced marginal taxes in the income range $2,003$ to $4,533$ USD and increased them in the income range $4,533$ to $7,063$ USD, accompanied by two further two-bracket tax cuts in the income ranges $1,365$ to $1,575$ USD and $8,559$ to $8,775$ USD. This reform allowed to raise the base
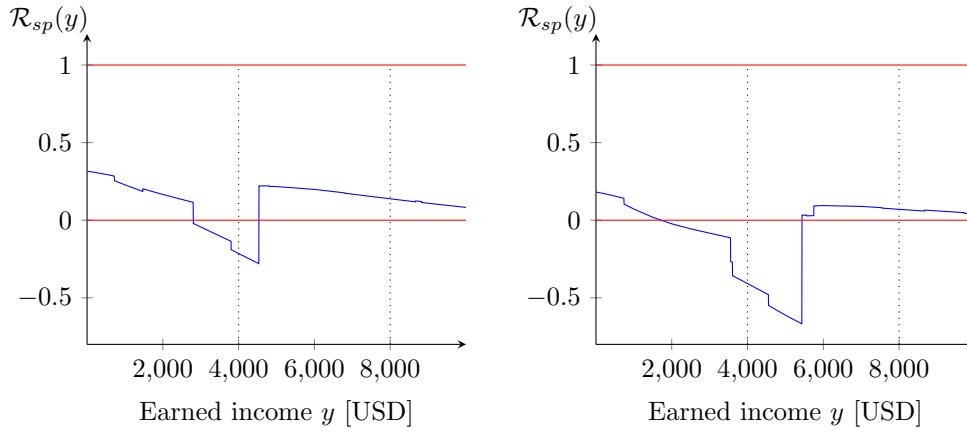
(a) Single parents with one child.

(b) Single parents with two children.
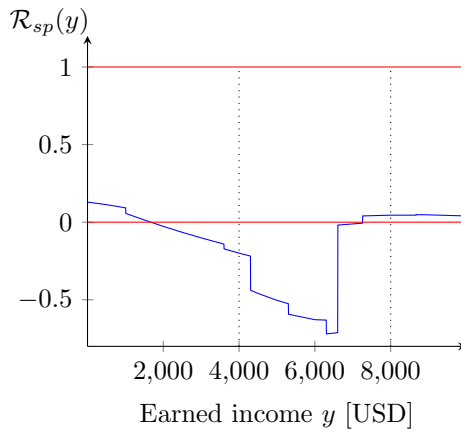
(c) Single parents with three children.

Figure 19: Pareto test of 1974 US tax system, single parents with 1, 2, and 3 children.

Notes: Figure 19 shows the revenue functions $y \mapsto \mathcal{R}_{sp}(y)$ in 1974 for single parents with one child (upper left panel), single parents with two children (upper right panel), and single parents with three children (lower panel). In all panels, the dotted vertical lines mark the endpoints of the phase-in range at $4,000$ USD and the phase-out range at $8,000$ USD of the 1975 EITC.

Source: Authors' calculations (see Table 1 for details).

transfer to each agent in this group by 6.45 USD per percentage point change in the marginal tax rates, with more than 99% of that amount coming from the income range $2,003$ to $7,063$ USD.

For single parents with two children, the revenue-maximizing Pareto-improvement was a two-bracket tax cut that reduced marginal taxes in the income range $1,305$ to $5,748$ USD and increased marginal taxes in the income range $5,748$ to $10,191$ USD. This reform allowed to raise the base transfer to each household in this group by 13.64 USD per percentage point change in marginal taxes.

For single parents with three children, finally, the revenue-maximizing tax cut was a two-bracket tax cut that reduced marginal taxes in the income range $1,339$ to $7,257$ USD and increased them in the income range $7,257$ to $13,175$ USD. It allowed to raise the base transfer to single parents with three children by 18.22 USD per percentage point change in marginal taxes. For completeness, we note that this heterogeneity is mainly driven by the differences in the tax-transfer schedules shown in Figure 18a. If we compute the revenue functions $y \mapsto \mathcal{R}_{sp}(y)$ using the same income distribution for all subgroups of single parents, the inefficiencies differ even more in size than if we use subgroup-specific income distributions as in Figure 19.

Summing up, we find that the inefficiencies in the 1974 US tax-transfer system were substantially larger for single parents with more children. Moreover, the best available reforms differed across the subgroups: For lone parents with more children, EITC-like reforms on much wider and higher income ranges were Pareto-improving than for those with less children. We conclude that, already in 1974, the introduction of differentiated EITC schedules – as implemented decades later – would have been desirable.

C.4   Results for further post-reform years

We now extend our analysis to the US tax-transfer systems after the 1975 EITC introduction. We start by explaining how the effective marginal tax-transfer schedules for single parents with two children changed from 1974 to 1975. We also

provide a graphical decomposition of the overall change due to adjustments of the statutory income tax, changes of welfare transfers, and changes due to the introduction of the EITC. Second, we demonstrate that the 1975 tax-transfer schedule was not Pareto-efficient. We then show that these inefficiencies remained and even increased until 1978, partly due to adjustments of the tax schedule and the welfare transfers that were independent of the EITC. Third, we take a closer look at the tax-transfer system for single parents in 1978, when the first expansion of the EITC as of 1979 was decided. We find that the 1979 EITC reform went into a Pareto-improving direction.

Figure 20 illustrates the 1975 changes in the effective tax-transfer schedule for single parents. The upper left panel compares the marginal taxes for low-income earners in 1974 (blue line) and 1975 (orange line). The other three panels associate these changes to the elements of the tax-transfer system: the statutory income tax, the EITC, and the welfare transfers AFDC and SNAP. In the statutory income tax, there were minor adjustments in the standard deduction and a disregard for personal exemptions. This implied a reduction of marginal taxes in the lowest tax bracket (see Figure 20b in the upper right panel). The introduction of the EITC reduced the marginal tax by 10 percentage points between 0 and 4,000 USD, and increased the marginal tax by 10 percentage points between 4,000 and 8,000 USD (see Figure 20c in the lower left panel). Finally, adjustments in the AFDC and SNAP parameters implied increases in the maximum transfer amounts and in the eligibility thresholds. As a result, both transfers were phased out over wider income ranges, and marginal tax rates increased substantially between 5,400 and 6,100 USD (see Figure 20d in the lower right panel.) We note that the EITC introduction implied the most substantial tax changes for single parents, but the AFDC and SNAP adjustments also played a role.

In Section 5 in the main text, we study whether the 1975 EITC introduction went into a Pareto-improving direction, holding all other elements of the tax-transfer system fixed. This is the reform we seek to evaluate, and therefore it would not be appropriate to evaluate several changes in the tax and transfer system
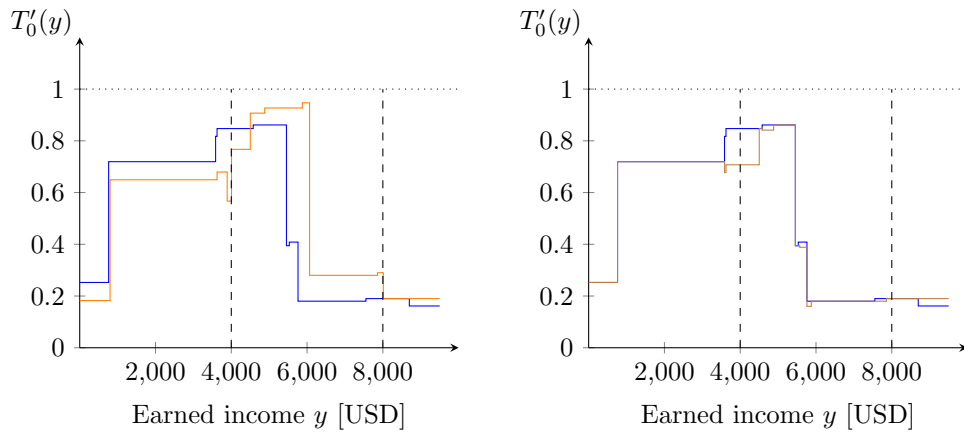
(a) Marginal taxes 1974 versus 1975.

(b) Changes in statutory income tax.
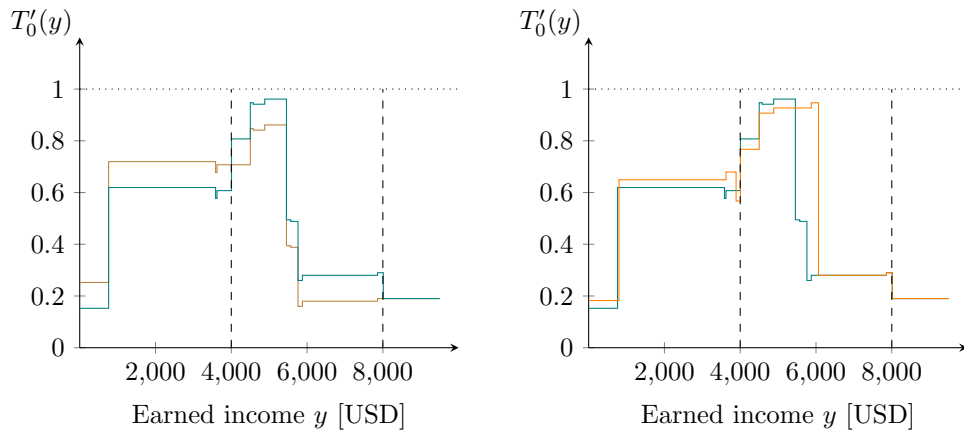
(c) Changes due to EITC.

(d) Changes in AFDC and SNAP.

Figure 20: Changes in effective marginal taxes for single parents, 1974 to 1975.

Notes: Figure 20a compares the effective marginal taxes for single parents with two children in 1974 (blue line) and 1975 (orange line). Figures 20b to 20d decompose these differences into three elements of the tax-transfer systems. Figure 20b shows how the marginal tax in 1974 was changed by adjustments of the statutory income tax (brown line vs. blue line); Figure 20c shows the effect of the EITC introduction (teal line vs. brown line); Figure 20d accounts for changes in AFDC and SNAP parameters (orange line vs. teal line). The dotted vertical lines mark the endpoints of the phase-in range at $4,000$ USD and the phase-out range at $8,000$ USD of the 1975 EITC.

Source: Authors' calculations (see Table 1 for details).

simultaneously. We find that this reform was not Pareto-improving according to our benchmark calibration, while a similar two-bracket tax cut in an alternative income range would have been. In particular, the 1975 EITC further increased the excessively high marginal taxes between $4,000$ and $5,750$ USD, while a reduction would have been Pareto-improving. As can be seen from Figure 20a, the same pattern emerges if we take account of all changes in the tax-transfer schedule between 1974 and 1975. This suggests that, due to the suboptimal design of the EITC, the 1975 US tax schedule neither was Pareto-efficient.

We confirm this hypothesis using the conditions for Pareto efficiency in Theorems 1 and 2 applied to the tax and transfer system for single parents prevailing after the introduction of the EITC. Figure 21 provides a graphical illustration of this test for 1975, right after the EITC introduction, and for 1978, before the first EITC expansion. It shows that the US tax-transfer system remained inefficient throughout this time. More specifically, the upper right panel shows the revenue function $y \mapsto \mathcal{R}_{sp}(y)$ for the 1975 tax system. On the one hand, it shows that the conditions for Pareto efficiency were now violated on a smaller income range than in 1974. In 1975, the revenue-maximizing tax cut was a two-bracket reform that reduced marginal taxes between $2,525$ and $6,045$ USD, and increased them between $6,045$ and $9,565$ USD (i.e., on a smaller interval than in 1974). On the other hand, function $y \mapsto \mathcal{R}_{sp}(y)$ attains even more negative values for some incomes. As a result, we find that the post-EITC tax system was even more inefficient than the pre-EITC tax system: In 1975, a two-bracket tax cut that changed marginal taxes by one percentage point allowed to increase the base transfer to single parents 21.9 USD. In 1974, only an increase in the base transfer by 12.6 USD was feasible.

From 1975 to 1978, the income range with inefficiencies widened further, eventually covering the majority of the phase-out range of the EITC between $4,000$ and $8,000$ USD (see Figure 21). This increase was driven partly by secular changes such as wage growth, and partly by a series of minor adjustments in deductions, income disregards and AFDC/SNAP parameters. In 1978, finally, the tax-transfer

(a) Marginal tax 1975.

(b) Revenue function 1975.

(c) Marginal tax 1978.

(d) Revenue function 1978.
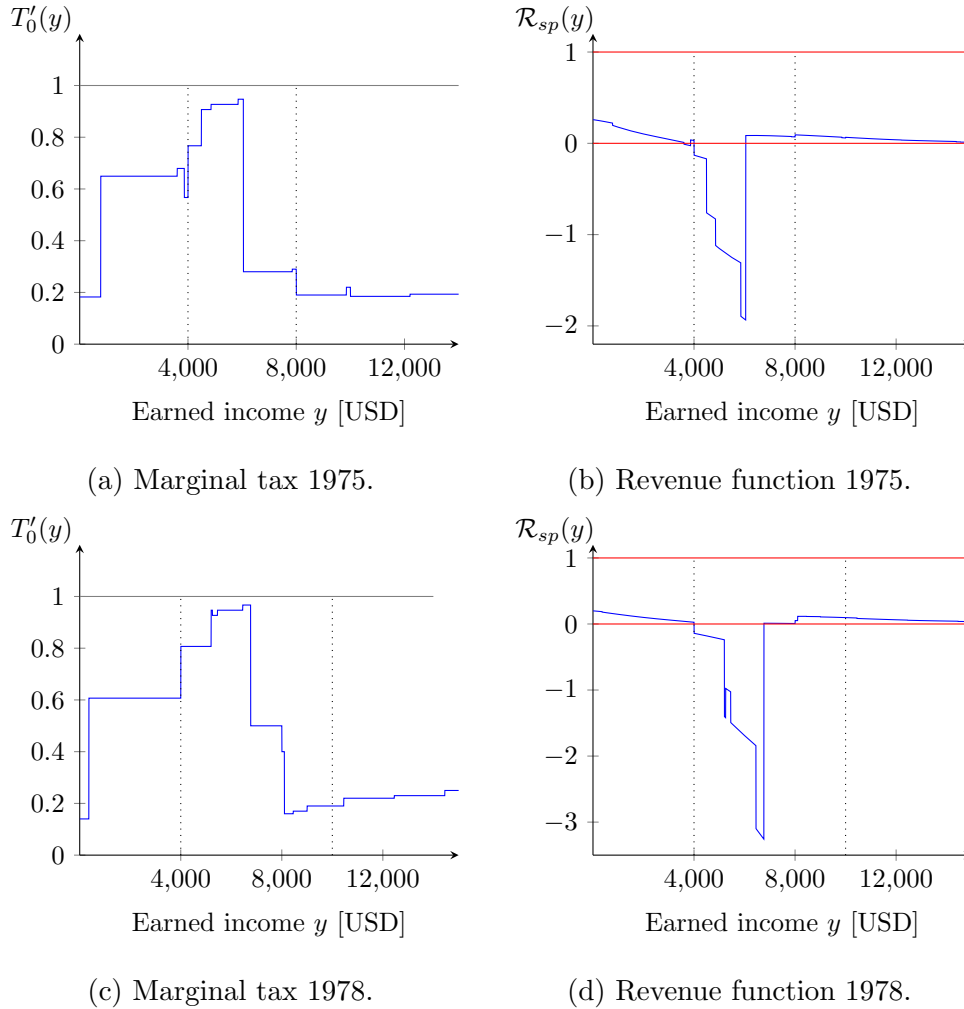
Figure 21: Pareto test of US tax-transfer system, 1975 and 1978.

Notes: Figure 21 shows the marginal tax rates (left panels) and the revenue function $y \mapsto \mathcal{R}_{sp}(y)$ (right panels) for single parents in the years 1975 and 1978. In the upper panels, the vertical dotted lines mark the endpoints of the phase-in range at $4,000$ USD and the phase-out range at $8,000$ USD of the 1975 EITC. In the lower panels, the dashed vertical lines at $4,000$ and $10,000$ USD mark the income range that was affected by the 1979 EITC expansion.

Source: Authors' calculations (see Table 1 for details).

system involved larger inefficiencies than both in 1974 and 1975. It was possible to increase the base transfer to each single parent by up to 35.4 USD by a two-bracket tax cut that changed marginal taxes by only one percentage point. Specifically, this revenue-maximizing tax cut required to reduce marginal taxes between $3,256$ and $8,100$ USD and to increase them between $8,100$ and $12,944$ USD.

Interestingly, the 1979 EITC reform was fairly similar to this revenue-maximizing tax cut. The US government strongly expanded the EITC in the tax year 1979, using a two-bracket tax cut with the following properties: The phase-in range was expanded from $4,000$ to $5,000$ USD; a plateau range between $5,000$ and $6,000$ USD was introduced and the phase-out range was adjusted and now went from $6,000$ to $10,000$ USD. While the phase-in rate was kept at 10%, the phase-out rate was raised from 10% to 12.5%. Altogether, the 1979 EITC reform was a tax cut in direction

$$
\tilde{h}_{79}(y) = \begin{cases}
-0.2 & \text{for} \quad y \in [4.000; 5.000) \,, \\
-0.1 & \text{for} \quad y \in [5.000; 6.000) \,, \\
0.025 & \text{for} \quad y \in [6.000; 8.000) \,, \\
0.125 & \text{for} \quad y \in [8.000; 10.000) \,, \\
0 & \text{for} \quad y \notin [4.000, 10.000) \,.
\end{cases}
$$

To check whether this reform went into a Pareto-improving direction, we evaluate condition (9) using the reform direction $y \mapsto \tilde{h}_{79}(y)$ and the 1978 revenue function $y \mapsto \mathcal{R}_{sp}(y)$. Based on our benchmark calibration, we find that the 1979 EITC reform direction was indeed Pareto-improving: It raised tax revenue and allowed to increase the base transfer to all single parents, while directly reducing the tax liabilities for all taxpayers with earned incomes between $4,000$ and $10,000$ USD.

To sum up, we have demonstrated that the 1974 EITC reform did neither go in a Pareto-improving direction nor did it restore a Pareto-efficient tax system; mainly because the 1974 EITC was too limited in the income range that it covered. By contrast, the 1979 reform expanded the EITC to incomes up to $10,000$ USD, thereby correcting the initial weaknesses and achieving a Pareto improvement.

D Pareto efficiency when earnings are bounded away from zero and bounded from above

Suppose that $y \mapsto \mathcal{R}(y)$ is non-increasing over $\mathcal{Y}$, so that there is no scope for a Pareto improvement by a two-bracket reform. Then, by Theorem 2 and Corollary 1, the status quo tax system is Pareto-efficient if $\mathcal{R}$ is bounded from above by 1 for the lowest incomes, and bounded from below by 0 for the highest incomes. Hence, our Pareto test only requires to verify one-bracket-efficiency at the very top and the very bottom. Under more restrictive assumptions, even the conditions for one-bracket-efficiency at the extremes become dispensable as we show in the following corollary. For this purpose, we denote by $y_{min} := \inf y_0(\Theta)$ the infimum of the income levels, and by $y_{max} := \sup y_0(\Theta)$ the supremum of the income levels chosen under the status quo tax policy.

Corollary 2 *If $y_{min} > 0$, $y_{max} < \bar{y}$, and $y \mapsto \mathcal{R}(y)$ is non-increasing, then there is no Pareto-improving direction in the class of continuous functions.*

Under the conditions of Corollary 2, the monotonicity of $y \mapsto \mathcal{R}(y)$ is sufficient for Pareto efficiency, i.e., there is no need to invoke the requirements that this function must be bounded from below by 0 and from above by 1. The conditions are that all types choose their incomes in some interior subset of $\mathcal{Y}$. In this case, as we show formally below, we have $\mathcal{R}(y) = 1$ for all $y \in [0, y_{min})$, and $\mathcal{R}(y) = 0$ for all $y \in (y_{max}, \bar{y}]$. Consequently, all three sufficient conditions in Theorem 2 are satisfied if $y \mapsto \mathcal{R}(y)$ is non-increasing.[44]

The conditions in Corollary 2 hold, for instance, in a Mirrleesian model of income taxation with only intensive-margin responses when Inada conditions ensure positive and bounded incomes for everybody.[45] By contrast, there is typically a

---

[44]Note that Corollary 2 provides a more compact expression of our sufficient conditions, but it should not be interpreted as showing that two-bracket reforms can achieve strictly more than one-bracket reforms. In particular, a two-bracket reform with one bracket below y_{min} or above y_{max} is economically equivalent to a one-bracket reform.

[45]Technically, this also requires that the parameter $\bar{y}$ in $\mathcal{Y} = [0, \bar{y}]$ is chosen so large that this upper bound does not interfere with individual choices.

positive mass of taxpayers with zero income in models with an intensive and an extensive margin. In this case, the requirement of boundedness does not follow from the requirement of monotonicity. Put differently, with a mass of non-working people, there can exist Pareto-improving reforms with one bracket even if there is no Pareto-improving reform with two brackets.

D.1 Proof of Corollary 2

Recall that $y_0(\Theta)$ is the image function of y_0, i.e., the set of income levels that are individually optimal for some type in Θ given the status quo tax system. The infimum of this set is denoted by y_{min} and the supremum by y_{max}.

Lemma 4 *If $y_{min} > 0$, then $\mathcal{R}(\hat{y}) = 1$ for any $\hat{y} \in [0, y_{min})$.*

Proof Fix a one-bracket reform $(\tau, \ell, \hat{y})$ such that $\hat{y} \geq 0$, $\hat{y} + \ell < y_{min}$ and assume that $\tau \ell$ is close to zero. This implies that, at any income level $y \geq y_{min}$, the tax burden increases by $\tau \ell$. We now argue that there are no behavioral responses to such a reform. More specifically, we show that "no behavioral responses" is consistent with both utility-maximizing behavior and the government budget constraint. When there are no behavioral responses and $y_0(\theta) > \hat{y} + \ell$ for all θ, this implies that the change of aggregate tax revenue equals $R^s(\tau, \ell, \hat{y}) = \tau \ell$. Since this additional tax revenue is rebated lump-sum, this also implies that all taxpayers receive additional transfers of $\tau \ell$. Hence, for any income in $y^*(\Theta)$, the additional tax payment and the additional transfer cancel each other out, implying that taxpayers face the same budget set before and after this reform, $C_1(y) = C_0(y)$. Moreover, for $\tau \ell$ sufficiently small, incomes smaller than y_{min} remain dominated by $y_0(\theta) \geq y_{min}$ for each θ. From $R^s(\tau, \ell, \hat{y}) = \tau \ell$, we obtain $R^s_{\tau \ell}(0, 0, \hat{y}) = 1$, which completes the proof. $\square$

Figure 22a illustrates these arguments: The solid blue line depicts the status quo budget set $C_0(y)$, the dashed blue line shows the upward shift by $\tau \ell$ in the post-reform budget set $C_1(y)$ for incomes below y_k. The red line, finally, shows

an indifference curve of the lowest-earning type θ such that $y_0(\theta) = y_{min}$ before the reform. As apparent from Figure 22a, the type continues to prefer y_{min} to any lower income and does not change her behavior as long as $\tau\,\ell$ is small enough.
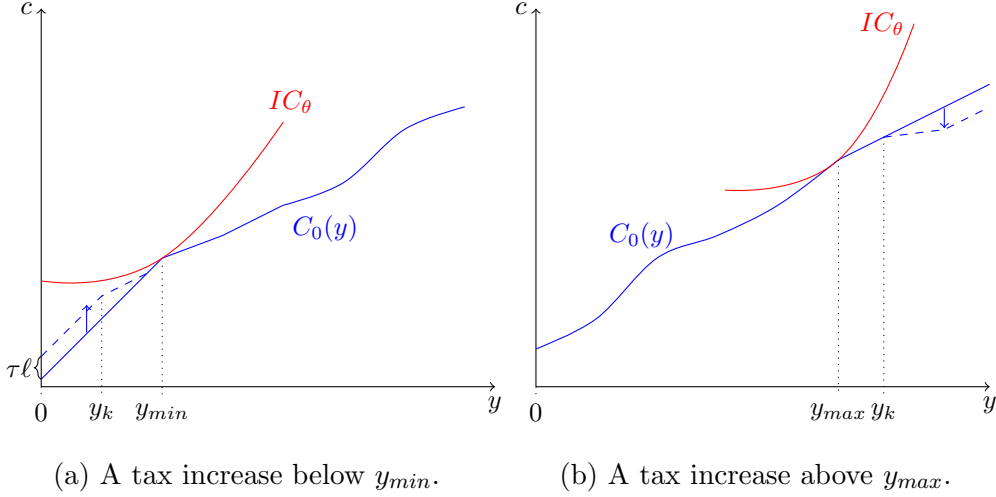

(a) A tax increase below y_{min}.   (b) A tax increase above y_{max}.

Figure 22: Illustration of small one-bracket reforms below y_{min} and above y_{max}.

Lemma 5 *If $y_{max} < \bar{y}$, then $\mathcal{R}(y) = 0$ for any $y \in (y_{max}, \bar{y}]$.*

Proof Fix a one-bracket reform $(\tau, \ell, \hat{y})$ such that $\hat{y} \in (y_{max}, \bar{y})$ and $\tau\,\ell > 0$. This reform does not change the tax burden at any income $y \leq y_{max}$. It increases the tax burden for incomes above $\hat{y}$ that were already dominated by some income below y_{max} for each type θ prior to the reform. We now argue that there are no behavioral responses to such a reform. More specifically, we show that "no behavioral responses" is consistent with both utility-maximizing behavior and the government budget constraint. When there are no behavioral responses and $y_0(\theta) < \hat{y}$ for each θ, this implies that the aggregate tax revenue does not change, $R^s(\tau_k, \ell_k, y_k) = 0$. This also implies that the base transfer is not changed, $c_1 = c_0$. Hence, for any income $y \leq \hat{y}$, taxpayers face the same budget set before and after this reform, $C_1(y) = C_0(y)$. Moreover, because of the tax increase for incomes larger than $\hat{y}$, these incomes remain dominated by some income below y_{max} for each type θ. From $R^s(\tau, \ell, \hat{y}) = 0$, we obtain $\mathcal{R}(\hat{y}) = R^s_{\tau\ell}(0, 0, \hat{y}) = 0$, which completes the proof. $\qquad\square$

Figure 22b illustrates these arguments. Again, the solid blue line depicts the status quo budget set $C_0(y)$, and the dashed blue line depicts the downward shift in the budget set due to the reform. The red depicts an indifference curve of a type θ who chooses the highest income y_{max} before the reform. As apparent from Figure 22b, the type continues to prefer y_{max} to any higher income and does not change her behavior after the reform.

Corollary 2 follows from combining Theorem 2 with Lemmas 4 and 5. Thus, if $y_{min} > 0$ and $y_{max} < \bar{y}$, monotonicity of $\mathcal{R}$ is a sufficient condition for the non-existence of Pareto-improving tax reforms. The lower and upper bounds on $\mathcal{R}$ become dispensable. Put differently, whenever there is a Pareto-improving reform with one bracket, there is also a Pareto-improving two-bracket tax cut.

Now suppose that the conditions in Corollary 2 are violated, so that either $y_{min} = 0$ or $y_{max} = \bar{y}$. In particular, the case with a mass of people with zero incomes is both empirically relevant and a typical outcome in the model with labor supply responses at the intensive and the extensive margin that we focus on in Section 5. In this case, it is possible that $\mathcal{R}(y)$ is monotonically decreasing over $(0, \bar{y})$ even though $\mathcal{R}(y) > 1$ for positive incomes close to 0. Then, there exists a Pareto-improving one-bracket reform, but no such reforms with two brackets. Hence, one-bracket efficiency at the bottom remains a substantive constraint.

In contrast, in the case $y_{max} = \bar{y}$, we can simply raise the upper threshold $\bar{y}$ of the set of feasible incomes to make sure that all people choose incomes below $\bar{y}$. After this adjustment, $\mathcal{R}(y) = 0$ for all incomes between y_{max} and the adjusted level of $\bar{y}$ by Lemma 5. Then, $\mathcal{R}(y) < 0$ for some income level close to $\bar{y}$ implies a violation of the condition that $y \mapsto \mathcal{R}(y)$ must be non-increasing over $[0, \bar{y}]$. Hence, whenever there is a Pareto-improving reform that reduces marginal taxes in one bracket of incomes below y_{max}, there is also a Pareto-improving two-bracket reform. In particular, any reform that combines the previously mentioned one-bracket tax cut with a tax increase in some bracket of incomes above y_{max} will do the job. It should be noted, however, that such a two-bracket reform is economically equivalent to a one-bracket reform by the arguments given in the proof of

Lemma 5: The second bracket has no effect on behavior as no type will choose an income in or above the second bracket.

E Pareto-improving tax reforms imply negative weights in the "inverse tax problem"

The literature on the "inverse tax problem" looks at observed tax policies to infer a government's redistributive concerns, using a revealed-preferences logic.[46] Specifically, it inverts the necessary (first-order) conditions for a welfare maximum to identify the government's "implicit marginal welfare weights" for people with different income levels, i.e., the marginal welfare weights that make the observed tax system optimal conditional on a given set of labor supply elasticities and an income distribution. How is this exercise related to Pareto-efficient taxation? It is a necessary condition for the Pareto efficiency of a tax system that it maximizes a Paretian social welfare function, i.e., an objective that is increasing in the individual utility of each type. Thus, if there is an observed tax system for which the "implicit welfare weights" at some incomes are negative, then this tax system cannot be Pareto-efficient. This suggests that necessary conditions for the non-negativity of implicit welfare weights are at the same time necessary conditions for Pareto efficiency. In the following, confirm this conjecture: we show that, whenever our approach detects a Pareto inefficiency, the implicit weights for people in certain income ranges are indeed negative.

Welfare implications. Let there be a social welfare function,

$$\mathcal{W}(\tau, h) := \mathbf{E}\left[g(\theta)\, v(\tau, h, \theta)\right]\ ,$$

where $g : \theta \mapsto g(\theta)$ specifies weights for different types of individuals. Upon using (3), the welfare implications of a refomr in direction h away from a given the status

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[46]See, e.g., Christiansen and Jansen (1978), Blundell et al. (2009), Bourguignon and Spadaro (2012), Bargain et al. (2011), Jacobs et al. (2017), Lockwood and Weinzierl (2016), or Hendren (2020).

quo can be written as

$$\mathcal{W}_\tau(0, h) = \lambda_0 \left\{ R_\tau(0, h) - \frac{1}{\lambda_0} \mathbf{E}_y \left[\tilde{g}(y) \, h(y) \right] \right\} , \tag{28}$$

where $\lambda_0 = \mathbf{E}\left[g(\theta) \, u_c(\cdot) \right]$ is a welfare-weighted population average of the marginal utility of consumption, evaluated at the status quo policy with $\tau = 0$. The expression $\mathbf{E}_y \left[\tilde{g}(y) \, h(y) \right]$, by contrast, looks only at people whose tax burden is affected by the reform, i.e., people with $h[y_0(\theta)] \neq 0$. Specifically, $\tilde{g}(y) = \mathbf{E}\left[g(\theta) \, u_c(\cdot) \mid y_0(\theta) = y \right]$ is commonly referred to as the *marginal social welfare weight* of people with income y. Under appropriate regularity assumptions, the status quo tax system is welfare-maximizing if and only if $\mathcal{W}_\tau(0, h) = 0$ for any admissible direction h.

Pareto-improving reforms and the inverse tax problem. The canonical optimal tax problem is to find the tax policy that maximizes a given social welfare function. The literature on the *inverse tax problem*, by contrast, takes the tax policy (T_0, c_0) as given and tries to find the function $\tilde{g} : y \mapsto \tilde{g}(y)$ such that the first-order conditions are satisfied, i.e., $\mathcal{W}_\tau(0, h) = 0$ for any direction h. For example, Jacobs et al. (2017) show how, upon smoothing the tax schedule and income distribution, $\tilde{g}(y)$ can be recovered pointwise for each income level y. The solution $y \mapsto \tilde{g}(y)$ to this inverted problem can be interpreted as giving *implicit welfare weights* if the first-order conditions characterize a global welfare maximum, see Bourguignon and Spadaro (2012) for a detailed analysis.

As we show now, the existence of Pareto-improving reforms according to Theorem 1 implies that the implicit welfare weights are negative at certain income levels, and hence contradicts the maximization of a Paretian social welfare function.

Suppose that $\mathcal{R}(y_1) < 0$ so that a small one-bracket tax cut at income level y_1 is Pareto-improving. The marginal welfare implications of a single-bracket reform h^s at income y_1 are given by

$$\mathcal{W}_{\tau\ell}(0, h^s) = \lambda_0 \left\{ \mathcal{R}(y_1) - \frac{1}{\lambda_0} \left[1 - F_y(y_1) \right] \mathbf{E}_y \left[\tilde{g}(y) \mid y > y_1 \right] \right\} .$$

If the first-order condition for welfare-maximization is satisfied, $\mathcal{W}_{\tau\ell}(0, h^s) = 0$, then we have

$$\mathbf{E}_y\left[\tilde{g}(y) \mid y > y_1\right] < 0 ,$$

i.e., on average, the implicit welfare weights of people with incomes above y_1 are negative.

Analogously, suppose that $\mathcal{R}(y_1) > 1$ so that a small one-bracket tax increase at income level y_1 is Pareto-improving. Then, the first-order condition requires that

$$\mathbf{E}_y\left[\tilde{g}(y) \mid y < y_1\right] < 0 ,$$

i.e., on average, the implicit welfare weights of people with incomes below y_1 are negative.

Finally, assume that a small two-bracket tax cut between incomes y_1 and $y_2 > y_1$ is Pareto-improving, $\mathcal{R}(y_2) > \mathcal{R}(y_1)$. The marginal welfare implications of such a two-bracket reform h_2 with $\tau_1 < 0$, $\tau_2 > 0$ and $\tau_1\,\ell_1 + \tau_2\,\ell_2 = 0$ are

$$
\begin{aligned}
\mathcal{W}_{\tau\ell}(0, h_2) &= \lambda_0 \Big\{ \tau_1\,\ell_1\,\mathcal{R}(y_1) + \tau_2\,\ell_2\,\mathcal{R}(y_2) \\
&\quad - \frac{1}{\lambda_0}\tau_1\,\ell_1\left[1 - F_y(y_1)\right]\ \mathbf{E}_y\left[\tilde{g}(y) \mid y > y_1\right] \\
&\quad - \frac{1}{\lambda_0}\tau_2\,\ell_2\left[1 - F_y(y_2)\right]\ \mathbf{E}_y\left[\tilde{g}(y) \mid y < y_2\right] \Big\} \\
&= \lambda_0\,\tau_2\,\ell_2 \Big\{ \mathcal{R}(y_2) - \mathcal{R}(y_1) \\
&\quad + \frac{1}{\lambda_0}\left[F_y(y_2) - F_y(y_1)\right]\ \mathbf{E}_y\left[\tilde{g}(y) \mid y \in (y_1, y_2)\right] \Big\} .
\end{aligned}
$$

If such a reform satisfies the first-order condition, $\mathcal{W}_{\tau\ell}(0, h_2) = 0$, then

$$\mathbf{E}_y\left[\tilde{g}(y) \mid y \in (y_1, y_2)\right] < 0 ,$$

i.e., the implicit welfare weights of people with incomes between y_1 and y_2 are negative.

To sum up, whenever one of the necessary Pareto conditions from Theorem 1 is violated, the implicit welfare weights of people in some part of the income distribution are negative. This implies a contradiction to the maximization of a Paretian social welfare function.

F   Sufficient statistics with behavioral responses at the intensive and the extensive margin

We consider a setting with two-dimensional heterogeneity. Individuals differ both in fixed and variable costs associated with the generation of income. Such a framework has been suggested by the literature that analyzes earnings subsidies from an optimal-tax perspective (see, e.g., Saez (2002), Jacquet et al. (2013), or Hansen (2021)). The analysis in this part of the Supplementary Material yields a characterization of $y \mapsto R_{\tau\ell}^s(0,0,y)$ that depends on labor supply elasticities at the intensive and the extensive margin.

Henceforth, fixed costs are captured by a parameter γ, variable costs by a parameter ω. Thus, we write $\theta = (\omega, \gamma)$ for an individual's type. Variable cost types and fixed cost types belong, respectively, to subsets of the positive reals that we denote by $\Omega = [\underline{\omega}, \overline{\omega}]$ and $\Gamma = [\underline{\gamma}, \overline{\gamma}]$. The joint distribution is denoted by F. The utility that an individual with type (ω, γ) derives from a (c,y)-pair that involves positive earnings is denoted by $u(c, y, \omega, \gamma)$. We denote by $u^{no}(c_0)$ the utility of individuals with no earnings. The function u^{no} is assumed to be increasing and weakly concave.

Variable costs.   To capture variable costs, we assume that preferences satisfy the Spence-Mirrlees single-crossing property: Consider two individuals with the same fixed cost type γ, and an arbitrary point in the (c,y)-space with $y > 0$. At any such point, an individual with a higher ω-type has a flatter indifference curve. The interpretation is that, due to her lower variable costs, she needs less compensation for a marginal increase of her earnings. Formally, for any given γ, $\omega' > \omega$ implies that

$$ -\frac{u_y(c, y, \omega', \gamma)}{u_c(c, y, \omega', \gamma)} < -\frac{u_y(c, y, \omega, \gamma)}{u_c(c, y, \omega, \gamma)} \ . $$

for any pair (c,y) with $y > 0$.

Let $C : y \mapsto C(y)$ be a non-decreasing function, interpreted as the boundary of a budget set that individuals face. An implication of the Spence-Mirrlees single

crossing property is as follows: Consider two individuals who differ only in the variable cost type. If type ω weakly prefers an earnings level y' over an earnings level $y < y'$, then any type $\omega' > \omega$ strictly prefers y' over y. More formally, for any γ, any pair ω', ω with $\omega' > \omega$, and any pair y', y with $y' > y$,

$$u(C(y'), y', \omega, \gamma) \geq u(C(y), y, \omega, \gamma) \quad \text{implies} \quad u(C(y'), y', \omega', \gamma) > u(C(y), y, \omega', \gamma) \,.$$

Fixed costs. Fixed costs affect the compensation that individuals demand for positive earnings. Let $\pi(c, y, \omega, \gamma)$ be such that

$$u(c + \pi(c, y, \omega, \gamma), y, \omega, \gamma) = u^{no}(c_0) \,.$$

We assume that π is an increasing function of γ.[47]

An implication is as follows: Consider two individuals who differ only in their fixed-cost type γ. Given a non-decreasing consumption schedule $C : y \mapsto C(y)$, if type γ prefers an earnings level of 0 over an earnings level of $y > 0$, then any individual with a fixed-cost type $\gamma' > \gamma$, will also prefer 0 over y; for any ω, any pair γ', γ with $\gamma' > \gamma$, and any $y > 0$,

$$u^{no}(c_0) \geq u(C(y), y, \omega, \gamma) \quad \text{implies} \quad u^{no}(c_0) > u(C(y), y, \omega, \gamma') \,.$$

The Spence-Mirrlees single-crossing property preserves monotonicity of choices in variable costs. For a given continuous consumption schedule $C_0 : y \mapsto C_0(y)$, $y_0(\omega, \gamma)$ is the utility-maximizing choice of type (ω, γ). By the Spence-Mirrlees single-crossing property, $\omega' > \omega$ implies $y_0(\omega', \gamma) \geq y_0(\omega, \gamma)$. In particular, $y_0(\omega', \gamma) = 0$ implies $y_0(\omega, \gamma) = 0$. Thus, for any given γ, there is a cutoff type $\hat{\omega}_0(\gamma)$ so that $\omega < \hat{\omega}_0(\gamma)$ implies $y_0(\omega, \gamma) = 0$, whereas $\omega \geq \hat{\omega}_0(\gamma)$ implies $y_0(\omega, \gamma) > 0$.

The earnings function will generally exhibit an upward jump at $\hat{\omega}(\gamma)$. With C continuous, raising y slightly above 0 comes only with a small gain in consumption

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[47]This property holds for various preference specifications that have been explored in the literature. In particular, it holds for separable utility functions of the form $u(c, y, \omega, \gamma) = \tilde{u}(c, y, \omega) - \gamma \, \mathbf{1}_{y>0}$, where $\mathbf{1}$ is the indicator function. It also holds for specifications with monetary fixed costs $u(c, y, \omega, \gamma) = \tilde{u}(c - \gamma \, \mathbf{1}_{y>0}, y, \omega)$. The two classes coincide if the function $\tilde{u}$ is quasi-linear in c.

utility, but an upward jump of effort costs. Thus, a significant increase of earnings is needed to have a gain in consumption utility that offsets these effort costs. Moreover, by our assumption on fixed costs, $\gamma' > \gamma$ implies that $\hat{\omega}(\gamma') \geq \hat{\omega}(\gamma)$.

The earnings function y_0 is bounded away from zero for all (ω, γ) with $\omega > \hat{\omega}(\gamma)$. Over this domain, we take y_0 to be a non-decreasing function of γ. Thus, $y_0(\omega, \gamma) > 0$, $y_0(\omega, \gamma') > 0$ and $\gamma' > \gamma$ imply that $y_0(\omega, \gamma') \geq y_0(\omega, \gamma)$.[48]

For a given reform direction h, we denote by $y^*(e, \tau, h, \omega, \gamma)$ the solution to the problem

$$\max_{y \geq 0} \ u(c_0 + e + y - T_0(y) - \tau \ h(y), y, \omega, \gamma)$$

and the corresponding indirect utility by $v(e, \tau, h, \omega, \gamma)$. The parameter e stands for a source of income that is exogenous from an individuals' perspective. In the subsequent analysis, e will be equal to the change in tax revenues, $e = R(\tau, h)$. If $y^*(e, \tau, h, \omega, \gamma) = 0$, then $v(e, \tau, h, \omega, \gamma) = u^{no}(c_0 + e)$.

The earnings function y^* exhibits a discontinuity at $\hat{\omega}(\cdot)$. Earnings are zero for types below $\hat{\omega}(\cdot)$ and bounded away from zero for types above. Individuals with type $\hat{\omega}(\cdot)$ are indifferent between earnings of zero and a strictly positive earnings level. It is convenient to assume that these individuals have positive earnings. Thus, we assume that

$$y^*(R(\cdot), \tau, \hat{\omega}(\cdot), \gamma) > 0 \ .$$

Intensive-margin responses. For one-bracket reforms, the derivative of the function y^*_τ with respect to τ gives how earnings respond to small changes in marginal tax rates for incomes that lie in that bracket. These are the behavioral responses at the intensive margin.

Extensive-margin responses. For a given reform direction h, we view the cutoff type $\hat{\omega}$ not only as a function of γ, but also as a function of the size of the

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[48]For separable utility functions of the form $u(c, y, \omega, \gamma) = \tilde{u}(c, y, \omega) - \gamma \ \mathbf{1}_{y>0}$, $y_0(\omega, \gamma) > 0$, $y_0(\omega, \gamma') > 0$ and $\gamma' > \gamma$ imply that $y_0(\omega, \gamma') = y_0(\omega, \gamma)$. For specifications $u(c, y, \omega, \gamma) = \tilde{u}(c - \gamma \ \mathbf{1}_{y>0}, y, \omega)$ with concave consumption utility, $y_0(\omega, \gamma) > 0$, $y_0(\omega, \gamma') > 0$ and $\gamma' > \gamma$ imply that $y_0(\omega, \gamma') > y_0(\omega, \gamma)$.

reform as measured by τ. Formally, for given γ, the cutoff type $\hat{\omega}(\tau, \gamma)$ is defined as the value of ω that solves

$$u^{no}(c_0 + R(\tau, h)) = v(\tau, \omega, \gamma) .$$

The effect of a small change of the reform intensity τ on the cutoff type $\hat{\omega}$ is obtained by computing a total differential of this equation. This yields, invoking again the envelope theorem,

$$u_c^{no}(\cdot)\, R_\tau(\tau, h) = u_c(\cdot)\, (R_\tau(\tau, h) - h(\cdot)) + u_\omega(\cdot)\, \hat{\omega}_\tau(\tau, \gamma) ,$$

where the functions u_c, u_ω and h are evaluated at $y = y^*(R(\cdot), \tau, \hat{\omega}(\cdot), \gamma)$. Equivalently,

$$\hat{\omega}_\tau(\tau, \gamma) = \frac{u_c^{no}(\cdot)}{u_\omega(\cdot)} R_\tau(\tau, h) - \frac{u_c(\cdot)}{u_\omega(\cdot)} (R_\tau(\tau, h) - h(\cdot)) . \tag{29}$$

To interpret these expressions, consider the following thought experiment: a fraction $F_\Omega(\hat{\omega}(\tau, \gamma) \mid \gamma)$ of individuals with fixed cost type γ has zero earnings, where $F_\Omega(\cdot \mid \gamma)$ is the distribution of variable cost types ω conditional on the fixed cost type being γ. Consider a small increase of transfers only for the unemployed, the marginal effect on $F_\Omega(\cdot \mid \gamma)$ is given by

$$P^{no}(\tau, \gamma) := f_\Omega(\hat{\omega}(\tau, \gamma) \mid \gamma)\, \frac{u_c^{no}(\cdot)}{u_\omega(\cdot)} , \tag{30}$$

where the letter P is chosen to indicate a marginal effect on Participation. Alternatively, the effect of a transfer only to those with positive earnings is given by

$$P(\tau, \gamma) = f_\Omega(\hat{\omega}(\tau, \gamma) \mid \gamma)\, \frac{u_c(\cdot)}{u_\omega(\cdot)} . \tag{31}$$

Thus, assuming that the distribution F and these marginal effects are known is equivalent to assuming that the extensive-margin elasticities and semi-elasticities that are ubiquitous in the related literature are known.

Notation. It is convenient to use a shorthand for endogenous variables at the status quo. For instance, we will occasionally write $\hat{\omega}_0(\gamma) := \hat{\omega}(0, \gamma)$ for the

type at the participation margin, among those with fixed cost type γ. We write $y_0(\omega, \gamma) := y^*(0, 0, h, \omega, \gamma)$ for income in the status quo, and similarly for other variables. Given a fixed-cost type γ, we denote by $\omega_0(y, \gamma)$ the variable cost type who chooses earnings of y in the status quo. If we evaluate partial derivatives at the status quo, we occasionally write $\hat{\omega}_{0\tau}(\gamma)$ or $y_{0e}(\omega, \gamma)$ and so on. We, moreover, write $\hat{y}_0(\gamma) := y_0(\hat{\omega}_0(\gamma), \gamma)$ for the status quo income of the cutoff type among those with fixed costs of γ. Finally,

$$\mathcal{K}(\omega_0(y, \gamma) \mid \gamma) = E_\Omega \left[T_0'(y_0(s, \gamma)) \, y_{0e}(s, \gamma) \mid s \geq \omega_0(y, \gamma), \gamma \right] \, ,$$

is a measure of the size of income effects among those individuals with fixed cost type γ who have earnings exceeding y.

Revenue implications of reforms with one bracket. By our analysis in the previous section, to understand whether a given tax system can be reformed in a Pareto-improving way, we need to check whether the function $y \mapsto R_{\tau\ell}^s(0, 0, y)$ is bounded from below by 0, bounded from above by 1, and non-increasing. The following proposition provides a characterization of this function for the given setup with variable and fixed costs of productive effort. A version of this function in terms of empirically estimable objects can be found in (26) in part C of the Supplementary Material.

Proposition 4 *Suppose that, for any given γ, $\omega_0(y, \gamma)$ is strictly increasing in y, whenever $y > 0$. Also suppose that, for all (ω, γ), income in the status quo satisfies the first-order conditions of utility-maximization whenever $y_0(\omega, \gamma) > 0$. Then,*

$$R_{\tau\ell}^s(0, 0, y) = (1 - \mathcal{M}_0)^{-1} \left(\mathcal{I}(y) - \mathcal{X}(y) \right) ,$$

where

$$\mathcal{M}_0 = E_\Gamma \left[(1 - F(\hat{\omega}_0(\gamma) \mid \gamma)) \, \mathcal{K}(\hat{\omega}_0(\gamma) \mid \gamma) - T_0(\hat{y}_0^*(\gamma)) \, (P_0^{no}(\gamma) - P_0(\gamma)) \right] ,$$

$$\begin{aligned} \mathcal{I}(y) \;=\; & T_0'(y) \, E_\Gamma \left[f(\omega_0(y, \gamma) \mid \gamma) \tfrac{y_{0\tau}(\omega_0(y,\gamma),\gamma)}{y_{0\omega}(\omega_0(y,\gamma),\gamma)} \right] \\ & + E_\Gamma \left[(1 - \mathcal{K}(\omega_0(y, \gamma) \mid \gamma))(1 - F(\omega_0(y, \gamma) \mid \gamma)) \right] , \end{aligned}$$

and

$$\mathcal{X}(y) = \int_{\underline{\gamma}}^{\overline{\gamma}} T_0(\hat{y}_0(\gamma)) \, P_0(\gamma) \, \mathbf{1}(\hat{y}_0(\gamma) \geq y) \, f_{\Gamma}(\gamma) d\gamma \; .$$

The proof of Proposition 4 can be found below. Proposition 4 shows that the revenue effect of a small single-bracket reform at income level y, $R_{\tau\ell}^s(0,0,y)$, can be decomposed into a term $\mathcal{I}(y)$ that includes a mechanical effect and an intensive-margin effect, an extensive-margin effect $\mathcal{X}(y)$, and a multiplier $\mathcal{M}_0$. The assumptions that $\omega_0(y,\gamma)$ is strictly increasing in y and that individual behavior can be described by first-order conditions are made for ease of exposition. They avoid complications due to bunching.

The term $\mathcal{I}(y)$ consists of a mechanical effect, a behavioral response from an income effect and a behavioral response from a substitution effect. The mechanical effect is that individuals with an income larger than y now pay additional taxes. This yields a revenue gain that is proportional to the mass of these people, $E_{\Gamma}\left[1 - F(\omega_0(y,\gamma) \mid \gamma)\right]$. With income effects, these people also seek to make up for the fact that the tax reform makes them poorer and they respond with an increase of their earnings. Together the mechanical and the income effect amount to $E_{\Gamma}\left[(1 - \mathcal{K}(\omega_0(y,\gamma) \mid \gamma))(1 - F(\omega_0(y,\gamma) \mid \gamma))\right]$. The substitution effect is relevant for people with an income of y. They have less of an incentive to exert productive effort when the marginal tax rate for incomes close to y is increased. This is captured by the expression

$$T_0'(y) \, E_{\Gamma}\left[f(\omega_0(y,\gamma) \mid \gamma) \frac{y_{0\tau}(\omega_0(y,\gamma),\gamma)}{y_{0\omega}(\omega_0(y,\gamma),\gamma)}\right] \; .$$

Alternatively, this term can be written as

$$\frac{T_0'(y)}{1 - T_0'(y)} \left(1 + \frac{1}{\varepsilon_0(y)}\right) E_{\Gamma}\left[f(\omega_0(y,\gamma) \mid \gamma)\right] \; ,$$

where $\varepsilon_0(y)$ is the elasticity of earnings with respect to the net-of-tax rate among people with pre-reform earnings of y.

The extensive-margin effect is shaped by the employment response of those individuals who are close to indifferent between staying out of the labor market and entering. More specifically, $\mathcal{X}(y)$ gives an average for all types who choose

earnings of at least y when entering the labor force. A change of the marginal rates in a bracket that begins at y has no effect on individuals who only consider incomes lower than y. For those who consider an income of y or above, there is a negative effect on participation and this tends to lower the revenue that is raised by such a reform. Naturally, the effect of this employment response on tax revenue depends on the tax payment that these individuals pay when entering the labor force and which is lost when they stay out.

The multiplier $\mathcal{M}_0$ mitigates all previously discussed effects. It reflects income effects at both margins, which appear if utility is non-linear in consumption. In this case, a tax reform that yields a revenue gain and hence increased lump-sum transfers has a negative effect on the earnings of market participants at the intensive margin. This effect is captured by the first component of $\mathcal{M}_0$,

$$E_\Gamma \left[(1 - F(\hat{\omega}_0(\gamma) \mid \gamma)) \, \mathcal{K}(\hat{\omega}_0(\gamma) \mid \gamma) \right] .$$

In addition, there are also income effects at the extensive margin. Specifically, the increased lump-sum transfers makes non-participation more attractive with concave consumption utility such that $(P_0^{no}(\gamma) - P_0(\gamma)) > 0$. This effect is captured by the second component in $\mathcal{M}_0$. It implies a loss of tax revenue if $T_0(y) > 0$.

F.1 Proof of Proposition 4

The change in tax revenue associated with a one-bracket reform (τ, ℓ, y_a) can be written as

$$R^s(\tau, \ell, y_a) = \mathbb{E}_\Gamma \left[\mathcal{R}^s(\tau, \ell, y_a \mid \gamma) \right] ,$$

where $\mathcal{R}^s(\tau, \ell, y_a \mid \gamma)$ is the change in tax revenue due to individuals with a given fixed cost type γ, and $\mathbb{E}_\Gamma$ is an expectations operator that indicates the computation of a population average using the marginal distribution F_Γ of fixed costs. Also, note that

$$
\begin{aligned}
\mathcal{R}^s(\tau, \ell, y_a \mid \gamma) &= \mathbb{E}_\Omega \left[T_1(y^*(R^s(\cdot), \tau, h, \omega, \gamma) - T_0(y_0(\omega, \gamma)) \mid \gamma \right] \\
&= \int_{\underline{\omega}}^{\overline{\omega}} \{ T_1(y^*(R^s(\cdot), \tau, h, \omega, \gamma) - T_0(y_0(\omega, \gamma)) \} f_\Omega(\omega \mid \gamma) \, d\omega ,
\end{aligned}
$$

where $f_\Omega(\cdot \mid \gamma)$ is the density of the conditional distribution of ω for given γ. The change in revenue associated with a marginal change of τ can be written as

$$R_\tau^s(\tau, \ell, y_a) = \mathbb{E}_\Gamma \left[\mathcal{R}_\tau^s(\tau, \ell, y_a \mid \gamma) \right] , \tag{32}$$

where

$$
\begin{aligned}
\mathcal{R}_\tau^s(\tau, \ell, y_a \mid \gamma) &= \frac{d}{d\tau} \int_{\underline{\omega}}^{\overline{\omega}} T_1(y^*(R^s(\cdot), \tau, h, \omega, \gamma) f_\Omega(\omega \mid \gamma) \, d\omega \\
&= \frac{d}{d\tau} \int_{\hat{\omega}(\tau,\gamma)}^{\overline{\omega}} T_1(y^*(R^s(\cdot), \tau, h, \omega, \gamma) f_\Omega(\omega \mid \gamma) \, d\omega .
\end{aligned}
$$

By standard arguments,

$$
\begin{aligned}
\mathcal{R}_\tau^s(\tau, \ell, y_a \mid \gamma) &= \frac{d}{d\tau} \int_{\hat{\omega}(\tau,\gamma)}^{\overline{\omega}} T_1(y^*(R^s(\cdot), \tau, h, \omega, \delta) f_\Omega(\omega \mid \gamma) \, d\omega \\
&= -T_1(y^*(R^s(\cdot), \tau, h, \hat{\omega}(\cdot), \gamma)) f(\hat{\omega}(\cdot) \mid \gamma) \, \hat{\omega}_\tau(\cdot) \\
&\quad + \int_{\hat{\omega}(\tau,\gamma)}^{\overline{\omega}} \frac{d}{d\tau} T_1(y^*(R^s(\cdot), \tau, h, \omega, \gamma) f_\Omega(\omega \mid \gamma) \, d\omega , \quad (33)
\end{aligned}
$$

i.e., the change of tax revenues can be decomposed into a change that comes from extensive-margin responses and a change that comes from intensive-margin responses to the tax reform.

A small change of the marginal tax rate. Equations (32), (33) and (29) imply that

$$R_\tau^s(0, \ell, y_a) = \mathbb{E}_\Gamma \left[\mathcal{R}_\tau^s(0, \ell, y_a \mid \gamma) \right] , \tag{34}$$

where

$$
\begin{aligned}
\mathcal{R}_\tau^s(0, \ell, y_a \mid \gamma) &= -T_0(y_0(\hat{\omega}_0(\gamma), \gamma)) f(\hat{\omega}_0(\gamma) \mid \gamma) \, \hat{\omega}_{0\tau}(\cdot) \\
&\quad + \int_{\hat{\omega}_0(\gamma)}^{\overline{\omega}} \left(\frac{d}{d\tau} T_1(y^*(R^s(\cdot), \tau, h, \omega, \gamma) \right)_{|\tau=0} f_\Omega(\omega \mid \gamma) \, d\omega ,
\end{aligned}
$$
$$\tag{35}$$

and

$$\hat{\omega}_{0\tau} = \frac{u_{0c}^{no}(\cdot)}{u_{0\omega}(\cdot)} R_\tau^s(0, \ell, y_a) - \frac{u_{0c}(\cdot)}{u_{0\omega}(\cdot)} \left(R_\tau^s(0, \ell, y_a) - h(y_0(\hat{\omega}_0(\gamma), \gamma)) \right) . \tag{36}$$

By the arguments in Bierbrauer and Boyer (2018),

$$\int_{\hat{\omega}_0(\gamma)}^{\overline{\omega}} \left(\frac{d}{d\tau} T_1(y^*(R^s(\cdot), \tau, \omega, \delta)) \right)_{|\tau=0} f_\Omega(\omega \mid \gamma) \, d\omega$$
$$= R^s_\tau(0, \ell, y_a) \, (1 - F_\Omega(\hat{\omega}_0(\gamma) \mid \gamma)) \, \mathcal{K}(\hat{\omega}_0(\gamma) \mid \gamma) + \mathcal{J}(\ell, y_a \mid \gamma) \tag{37}$$

where

$$\mathcal{K}(\hat{\omega}_0(\gamma) \mid \gamma) = E_\Omega \left[T_0'(y_0(s, \gamma)) \, y_{0e}(s, \gamma) \mid s \geq \hat{\omega}_0(\gamma), \gamma \right] \, ,$$

as defined in the body of the text, and

$$\mathcal{J}(\ell, y_a \mid \gamma) = \int_{\omega_0(y_a \mid \gamma)}^{\omega_0(y_a + \ell \mid \gamma)} T_0'(y_0(\omega, \gamma)) \, y_{0\tau}(\omega, \gamma) \, f(\omega \mid \gamma) d\omega$$
$$+ \int_{\omega_0(y_a \mid \gamma)}^{\omega_0(y_a + \ell \mid \gamma)} (y_0(\omega, \gamma) - y_a) \, [1 - T_0'(y_0(\omega, \gamma)) \, y_{0e}(\omega, \gamma)] \, f(\omega \mid \gamma) d\omega$$
$$+ \ell \left(1 - F(\omega_0(y_a + \ell \mid \gamma)) \right.$$
$$\left. - \int_{\omega_0(y_a + \ell \mid \gamma)}^{\overline{\omega}} T_0'(y_0(\omega, \gamma)) \, y_{0e}(\omega, \gamma) f(\omega \mid \gamma) d\omega \right)$$

and $\omega_0(y_a \mid \gamma)$ and $\omega_0(y_a + \ell \mid \gamma)$ are, respectively, the ω-types who choose income levels of y_a and $y_a + \ell$ in the status quo. Equations (30), (31), (35), (36) and (37) imply that

$$R^s_\tau(0, \ell, y_a \mid \gamma) = R^s_\tau(0, \ell, y_a) \, M_0(\gamma)$$
$$- T_0(y_0(\hat{\omega}_0(\gamma), \gamma)) \, P_0(\gamma) \, h(y_0(\hat{\omega}_0(\gamma), \gamma))$$
$$+ \mathcal{J}(\ell, y_a \mid \gamma) \tag{38}$$

where the multiplier $M_0(\gamma)$ is given by

$$M_0(\gamma) := (1 - F(\hat{\omega}_0(\gamma)) \mid \gamma) \, \mathcal{K}(\hat{\omega}_0(\gamma) \mid \gamma) - T_0(y_0(\hat{\omega}^0(\gamma), \gamma)) \, (P_0^{no}(\gamma) - P_0(\gamma)) \, .$$

We let $\mathcal{M}_0 = E_\Gamma[M_0(\gamma)]$. Then, equations (34) and (38) imply that

$$R^s_\tau(0, \ell, y_a) = (1 - \mathcal{M}_0)^{-1} \left(E_\Gamma[\mathcal{J}(\ell, y_a \mid \gamma)] - E_\Gamma[\mathcal{Z}(\ell, y_a \mid \gamma)] \right), \tag{39}$$

where

$$\mathcal{Z}(\ell, y_a \mid \gamma) = T_0(y_0(\hat{\omega}_0(\gamma), \gamma)) \, P_0(\gamma) \, h(y_0(\hat{\omega}_0(\gamma), \gamma)) \, .$$

If y_a is a very high, the tax reform affects only high incomes. Plausibly, y_a is then also above the income level that individuals at the extensive margin would consider, i.e. $y_a > \max_\gamma y_0(\hat{\omega}_0(\gamma), \gamma)$. In this case, $h(y_0(\hat{\omega}_0(\gamma), \gamma)) = 0$ and hence $\mathcal{Z}(\ell, y_a \mid \gamma) = 0$.

A small change of the marginal tax rate for a narrow bracket. We are interested in determining how much additional tax revenue a reform generates that involves a small change of marginal tax rates for incomes in a narrow bracket. To this end, we provide a characterization of $R^s_{\tau\ell}(0,0,y_a)$, i.e., of the cross-derivative of tax revenue with respect to the change of the marginal tax rate τ and the length of the bracket ℓ at the status quo. To understand the logic of this exercise, note that our previous derivations imply that $R^s_\tau(0,0,y_a) = 0$, i.e., if the length of the bracket is zero, the marginal tax rate remains unchanged at each income level. Consequently, there is no effect on tax revenue. If, however, $R^s_{\tau\ell}(0,0,y_a) > 0$, then moving from $\ell = 0$ to some $\ell = \varepsilon$ for $\varepsilon > 0$ but small, implies that $R^s_\tau(0,\varepsilon,y_a) > 0$, so that a small change of the marginal tax rate then has a positive effect on revenue. More formally, if $R^s_{\tau\ell}(0,0,y_a) > 0$ there exist $\delta > 0$ and $\varepsilon > 0$ so that $R^s(\delta,\varepsilon,y_a) > 0$.

It follows from equation (39) that

$$R^s_{\tau\ell}(0,0,y_a) = (1 - \mathcal{M}_0)^{-1}\Big(E_\Gamma[\mathcal{J}_\ell(0,y_a \mid \gamma)] - E_\Gamma[\mathcal{Z}_\ell(0,y_a \mid \gamma)] \Big), \qquad (40)$$

where, by the arguments in Bierbrauer and Boyer (2018),

$$
\begin{aligned}
E_\Gamma[\mathcal{J}_\ell(0,y_a \mid \gamma)] &= T'_0(y_a) E_\Gamma\left[f(\omega_0(y_a,\gamma) \mid \gamma)\frac{y_{0\tau}(\omega_0(y_a,\gamma),\gamma)}{y_{0\omega}(\omega_0(y_a,\gamma),\gamma)} \right] \\
&\quad + E_\Gamma\left[(1 - \mathcal{K}(\omega_0(y_a,\gamma) \mid \gamma))(1 - F(\omega_0(y_a,\gamma) \mid \gamma)) \right]. (41)
\end{aligned}
$$

We now work towards a characterization of $\mathcal{Z}_\ell(0,y_a) := E_\Gamma[\mathcal{Z}_\ell(0,y_a \mid \gamma)]$. Suppose that $y_a \le \max_\gamma y_0(\hat\omega_0(\gamma),\gamma)$ and denote by $\gamma_0(y_a)$ and $\gamma_0(y_a + \ell)$, respectively, the types at the extensive margin who earn, respectively, incomes of y_a and $y_b = y_a + \ell$, i.e.

$$y_0(\hat\omega_0(\gamma_0(y_a)),\gamma_0(y_a)) = y_a, \quad \text{and} \quad y_0(\hat\omega_0(\gamma_0(y_a + \ell)),\gamma_0(y_a + \ell)) = y_a + \ell.$$

Armed with this notation, we can write

$$
\begin{aligned}
E_\Gamma[\mathcal{Z}_\ell(\ell,y_a \mid \gamma)] &= \int_{\gamma_0(y_a)}^{\gamma_0(y_a+\ell)} T_0(\hat y_0(\gamma)) P_0(\gamma) (\hat y_0(\gamma) - y_a) f_\Gamma(\gamma) d\gamma \\
&\quad + \ell \int_{\gamma_0(y_a+\ell)}^{\overline\gamma} T_0(\hat y_0(\gamma)) P_0(\gamma) f_\Gamma(\gamma) d\gamma,
\end{aligned}
$$

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where $\hat{y}_0(\gamma)$ is a shorthand for $y_0(\hat{\omega}_0(\gamma), \gamma)$. Straightforward computations yield

$$\mathcal{Z}_\ell(0, y_a) = \int_{\gamma_0(y_a)}^{\overline{\gamma}} T_0(\hat{y}_0(\gamma)) \; P_0(\gamma) f_\Gamma(\gamma) d\gamma \; .$$

Now, to accommodate both the case $y_a \leq \max_\gamma \; \hat{y}_0(\gamma)$ and the case $y_a > \max_\gamma \; \hat{y}_0(\gamma)$, we will write henceforth

$$\mathcal{Z}_\ell(0, y_a) = \int_{\underline{\gamma}}^{\overline{\gamma}} T_0(\hat{y}_0(\gamma)) \; P_0(\gamma) \; \mathbf{1}(\hat{y}_0(\gamma) \geq y_a) \; f_\Gamma(\gamma) d\gamma \; .$$

Proposition 4 as stated in the text now follows upon adopting the more concise notation

$$\mathcal{I}(y) := \mathcal{J}_\ell(0, y) \quad \text{and} \quad \mathcal{X}(y) := \mathcal{Z}_\ell(0, y) \; ,$$

for any earnings level y.

F.2 Implications of Proposition 4

F.2.1 Diamond's model

With quasi-linear in consumption preferences and without fixed costs of labor market participation, we have

$$\mathcal{M}_0 = 0 \; , \quad \text{and} \quad \mathcal{X}(y) = 0 \; ,$$

and

$$\mathcal{I}(y) = T_0'(y) \; f(\omega_0(y)) \; \frac{y_{0\tau}(\omega_0(y))}{y_{0\omega}(\omega_0(y))} + 1 - F(\omega_0(y)) \; . \tag{42}$$

This can be rewritten as

$$\mathcal{I}(y) = -\frac{T_0'(y)}{1 - T_0'(y)} \; f(\omega_0(y)) \; \left(1 + \frac{1}{\varepsilon_0(y)}\right)^{-1} \omega_0(y) + 1 - F(\omega_0(y)) \; . \tag{43}$$

Getting from equation (42) to equation (43) requires to invoke the first-order condition characterizing $y_0(\omega)$ for the purpose of deriving comparative statics results that yield a characterization of $y_{0\tau}(\omega_0(y))$ and $y_{0\omega}(\omega_0(y))$. Thus,

$$R_{\tau\ell}^s(0, 0, y) = -\frac{T_0'(y)}{1 - T_0'(y)} \; f(\omega_0(y)) \; \left(1 + \frac{1}{\varepsilon_0(y)}\right)^{-1} \omega_0(y) + 1 - F(\omega_0(y)) \; ,$$

which is equation (7) in the main text using that

$$F_y(y) = F(\omega_0(y)) \quad \text{and} \quad f_y(y) = f(\omega_0(y)) \; \omega_0'(y) = f(\omega_0(y)) \; \frac{1}{y_{0\omega}(\omega_0(y))} \; .$$

and that, for any y in $y_0(\Omega)$, $\varepsilon_0(y) = -\frac{y}{1-T_0'(y)} \; y_{0\tau}(\cdot)$.

F.2.2 Fixed costs as an extension of Diamond's model

We now consider an extension of Diamond (1998) that includes fixed costs of labor market participation. Preferences are now given by

$$u(c, y, \omega, \gamma) = c - \frac{1}{1 + \frac{1}{\epsilon}} \left(\frac{y}{\omega}\right)^{1 + \frac{1}{\epsilon}} - \gamma \, \mathbb{1}_{y > 0} \,,$$

where ϵ is a parameter. Again, the absence of income effect implies that $\mathcal{M}_0 = 0$. We can, once more, rewrite $\mathcal{I}(y)$ using the distribution of incomes F_y so that

$$F_y(y) = E_\Gamma \left[F_\Omega(\omega_0(y, \gamma) \mid \gamma) \right] \,.$$

This yields

$$\mathcal{I}(y) = -\frac{T_0'(y)}{1 - T_0'(y)} \, \varepsilon_0(y) \, y f_y(y) \, + 1 - F_y(y) \,.$$

While this expression looks exactly as in Diamond's model, the distribution of incomes F_y is now shaped by the joint distribution of fixed and variable costs. We also rewrite

$$\mathcal{X}(y) = \int_{\underline{\gamma}}^{\overline{\gamma}} T_0(\hat{y}_0(\gamma)) \, P_0(\gamma) \, \mathbf{1}(\hat{y}_0(\gamma) \geq y) \, f_\Gamma(\gamma) \, d\gamma \,.$$

in a way that is more handy in the context of our application: We first note that, with quasi-linear in consumption preferences, $P_0(\gamma) = f_\Omega(\hat{\omega}_0(\gamma) \mid \gamma)$, and therefore

$$\mathcal{X}(y) = \int_{\underline{\gamma}}^{\overline{\gamma}} T_0(\hat{y}_0(\gamma)) \, \mathbf{1}(\hat{y}_0(\gamma) \geq y) \, f_\Omega(\hat{\omega}_0(\gamma) \mid \gamma) \, f_\Gamma(\gamma) \, d\gamma \,.$$

We then note that $\hat{y}_0$ is an increasing function.[49] Thus, if we denote by $\gamma_0(y)$ be value of γ for which $\hat{y}_0(\gamma) = y$, then we can write $\mathcal{X}(y)$ as

$$\mathcal{X}(y) = \int_{\gamma_0(y)}^{\overline{\gamma}} T_0(\hat{y}_0(\gamma)) \, f_\Omega(\hat{\omega}_0(\gamma) \mid \gamma) \, f_\Gamma(\gamma) d\gamma \,.$$

---

[49]This follows from the following observations. First, recall that, by definition, $\hat{y}_0(\gamma) = y_0(\hat{\omega}_0(\gamma), \gamma)$. Second, with quasi-linear in consumption preferences, the function $y_0 : (\omega, \gamma) \mapsto y_0(\omega, \gamma)$ is increasing in the first argument and constant in the second argument. Third, $\hat{\omega}_0 : \gamma \mapsto \hat{\omega}_0(\gamma)$ is increasing, as higher rents from labor market participation are needed to offset larger fixed costs.

We seek an interpretation of $\mathcal{X}(y)$ in terms of extensive-margin elasticities. Therefore, consider the following thought experiment: For pre-tax incomes in an interval $[y_0, y]$, after-tax incomes are marginally decreased. Overall employment is then reduced by

$$L_E(y_0, y) = \int_{\gamma_0(y_0)}^{\gamma_0(y)} \frac{f_\Omega(\hat{\omega}_0(\gamma) \mid \gamma)}{u_\omega(\ )} f_\Gamma(\gamma) \, d\gamma .$$

Denote the derivative of $L_E(y_0, y)$ with respect to y by $l_E(y)$ and apply Leibnitz rule to derive

$$l_E(y) = f_\Omega(\hat{\omega}_0(\gamma_0(y)) \mid \gamma_0(y)) \, u_\omega(\ )^{-1} \, f_\Gamma(\gamma_0(y)) \, \gamma_0'(y) .$$

Using the fundamental theorem of calculus, we can now also write

$$L_E(y_0, y) = \int_{y_0}^{y} l_E(y') \, dy' .$$

Thus, $l_E(y)$ is a marginal effect. One that measures the mass of people flowing out of the pool of people who earn y and into the pool of people who earn zero, when consumption for people earning y is slightly reduced. We will now rewrite $\mathcal{X}(y)$ in a way that highlights the significance of this employment response. We perform a substitution using

$$y' = \hat{y}_0(\gamma) \quad \text{and} \quad dy' = \frac{\partial \, \hat{y}_0^*(\gamma)}{\partial \, \gamma} \, d\gamma .$$

Also, note that $\gamma_0 : y' \mapsto \gamma_0(y')$ is the inverse of the function $\hat{y}_0$, so that

$$\gamma_0'(y') \, dy' = d\gamma$$

Finally, note that $\hat{y}_0(\gamma_0(y')) = y'$. Thus,

$$
\begin{aligned}
\mathcal{X}(y) &= \int_{y}^{\bar{y}_0} T_0(y') \frac{f_\Omega(\hat{\omega}_0(\gamma_0(y')) \mid \gamma_0(y'))}{u_\omega(\ )} f_\Gamma(\gamma_0(y')) \, \gamma_0'(y') \, dy' \\
&= \int_{y}^{\bar{y}_0} T_0(y') \, l_E(y') \, dy' \\
&= \int_{y}^{\bar{y}_0} \frac{T_0(y')}{y' - T_0(y')} \frac{l_E(y')}{f_y(y')} \, (y' - T_0(y')) \, f_y(y') \, dy' \\
&= \int_{y}^{\bar{y}_0} \frac{T_0(y')}{y' - T_0(y')} \, \pi_0(y') \, f_y(y') \, dy' ,
\end{aligned}
$$

where $\overline{y}_0$ is the highest level of y in the support of F_y. We interpret

$$\pi_0(y') := \frac{l_E(y')}{f_y(y')} \ (y' - T_0(y'))$$

as an extensive-margin elasticity, it relates a percentage change in the mass of people at pre-tax income y due to extensive-margin responses to a percentage change in their after-tax labor income $y - T_0(y)$. The literature typically refers to $\mathbb{E}_y\left[\pi_0(y')\right]$ as the participation elasticity.

Upon collecting terms, and upon assuming an unbounded distribution of income in the status quo, we obtain

$$
\begin{aligned}
R_{\tau\ell}^s(0,0,y) &= 1 - F_y(y) - \varepsilon_0(y) \, y \, f_y(y) \, \frac{T_0'(y)}{1 - T_0'(y)} \\
&\quad - \int_y^\infty f_y(y') \, \pi_0(y') \, \frac{T_0(y')}{y' - T_0(y')} \, dy' \, .
\end{aligned}
\tag{44}
$$

Note that (44) coincides with equation (8) in the body of the text.

G   Analysis of the 2018 US tax-transfer system

There were many reforms of the US tax-and-transfer system since the mid-1970s. To get a sense of whether there was progress in the US tax policy over time, this section studies the 2018 tax-and-transfer system for single parents. We find that it was still not Pareto-efficient. But, based on the inefficiency measure introduced in Section 4, the inefficiencies in the current tax system are quantitatively an order of magnitude smaller than those in the mid-1970s.

Calibration.   Again, our analysis focuses on the group of single parents with two children in California. We parametrize the revenue function given in (8) for the 2018 tax system that applied to this group.

We take account of the federal income tax and various transfer and welfare programs: the federal Earned Income Tax Credit, the Child Tax Credit (CTC, a partly refundable tax credit introduced in 1998), SNAP and Temporary Assistance for Needy Families (TANF, the successor of AFDC). Table 2 below presents a summary of the sources that we use for this purpose. As of 2018, the maximum

Table 2: Sources for US tax-transfer system, 2018.

Information	Sources
Income tax	Internal Revenue Service, "1040 Instructions, Tax Year 2018", 2019. Accessible at `https://www.irs.gov/pub/irs-dft/i1040gi--dft.pdf`.
EITC	Tax Policy Center, "Earned Income Tax Credit Parameters, 1975-2018", 2018. Accessible at `https://www.taxpolicycenter.org/file/178859`.
AFDC	CCWRO, "CalWORKs (also known as AFDC/TANF)", 2018. Accessible at `https://www.ccwro.org/advocateresources/public-assistance-table`.
	City and County of San Francisco Human Services Agency, "CalWORKS Eligibility Handbook", 2018. URL: `https://www.sfhsa.org/file/6406`.
SNAP	New Mexico Department of Human Services, "Income Eligibility Guidelines for SNAP and Financial Assistance", 2018. URL: `https://www.hsd.state.nm.us/uploads/FileLinks/26463f122f47474487faee4922e09ce8/ISD_017_Income_Eligibility_Guidelines_for_SNAP_and_Financial_Assistance__FFY18_1.pdf`.

EITC amounts to $5,716$ USD for single taxpayers with two children. It is phased in at a rate of 0.4 for annual incomes below $14,290$ USD and phased out at a rate of 0.2106 for incomes between $18,660$ and $45,802$ USD.

Figure 23 shows the effective marginal tax, $T_0'(y)$, and the participation tax rate, $T_0(y)/y$, for single parents that result from the interplay of these programs. In 2018, effective marginal tax rates for low incomes are much lower than in the 1970s. Most notably, they are negative due to the EITC phase-in for incomes below $2,700$ USD (only), and close to 60% in an income range between $14,300$

and $18,500$ USD. In contrast to the 1974 tax system, the end of the EITC phase-in range in now closely aligned to the full phasing out of transfers (TANF, SNAP) around $18,000$ USD.
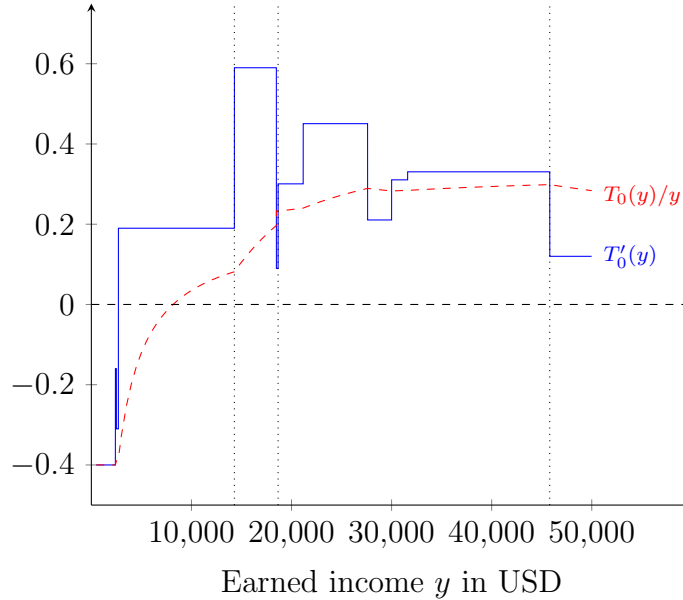

Figure 23: US income tax and transfer schedule in 2018 for single parents.

Notes: Figure 23 shows the 2018 effective marginal tax $T_0'(y)$ (solid blue line) and the participation tax rate $T_0(y)/y$ (dashed red line) for single parents with two children as functions of earned income in 2018 USD. The dotted vertical lines mark the endpoint of the phase-in range at $14,290$ USD, the starting point of the phase-out range at $18,660$ USD, and the endpoint of the phase-out range at $45,802$ USD.

Source: Authors' calculations (see Table 2 for details).

We estimate the income distribution among single parents based on data from the March 2019 wave of the CPS. As before, we use a non-parametric kernel density estimation. The share of single parents with no income has gone down considerably since the mid-1970s. In 2018, it amounted to 15.2%, about half of the share in the 1970s. Around 54.1% of single parents had strictly positive incomes below $45,802$ USD and were, therefore, eligible for the EITC. In our data, 81.4% of single parents were female.

For the behavioral responses to taxation, we stick to the benchmark case with an intensive-margin elasticity of 0.33 and participation elasticities that are de-

creasing from 0.67 for very low income levels to 0.4 for incomes above 50.000 USD.

Empirical results. Figure 24 plots the revenue function $y \mapsto \mathcal{R}_{sp}(y)$ for the US tax-and-transfer system for single parents in 2018. Remember that, for the 1974 US tax system, this revenue function violated two necessary conditions for Pareto efficiency: It was negative at incomes between $2,000$ and $5,500$ USD, and increasing in the range between $5,000$ and $6,000$ USD. Hence, there existed both Pareto-improving one-bracket reforms and Pareto-improving two-bracket reforms. By contrast, the 2018 version of $y \mapsto \mathcal{R}_{sp}(y)$ is bounded from below by 0 and from above by 1. Hence, we no longer find Pareto improvements by means of one-bracket reforms. But there are still several income ranges where the function is increasing. This indicates that, even today, there exist Pareto-improving tax reforms with two brackets.

The visual impression from Figure 24 is that, despite these non-monotonicities, the inefficiencies in the 2018 US tax system are smaller in magnitude than those in the 1974 tax system. To verify this conjecture, we apply the method introduced in Section 4 to find the revenue-maximizing tax cut – the *best* Pareto-improving reform at the time – and use the net revenue gain from this reform as a measure of the current tax system's inefficiency. We find that, in 2018, the revenue-maximizing tax cut was given by a combination of the following two-bracket tax cuts: (i) one between $15,990$ and $20,980$ USD that includes the starting point of the EITC phase-in range, (ii) one between $25,200$ and $30,000$ USD, and (iii) one between $42,359$ and $49,245$ USD that includes the EITC eligibility threshold. Based on our calibration, this reform would have allowed to raise the base transfer to each single-parent household by about three USD (in 2018 units) for each percentage-point change in marginal tax rates. By contrast, the revenue-maximizing tax cut in 1974 would have allowed to raise the base transfer to single parents by about 12.7 USD in 1974 units, or about 64 USD in 2018 units, per percentage-point change in marginal taxes. We conclude that the set of US tax reforms since the 1970s did not fully eliminate the inefficiencies in the tax-transfer system, but reduced them
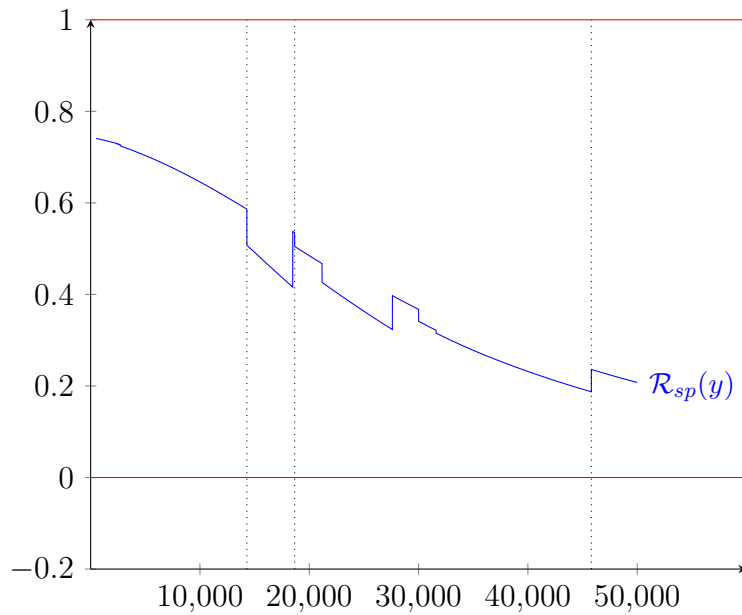
Figure 24: Pareto test of the 2018 tax system for single parents.

Notes: Figure 24 shows the revenue function $y \mapsto \mathcal{R}_{sp}(y)$ for single parents in 2018, assuming an intensive-margin elasticity of 0.33 and an participation elasticity that is decreasing from 0.67 at very low incomes to 0.4 at high incomes. The dotted vertical lines mark the endpoint of the phase-in range at $14,290$ USD, the starting point of the phase-out range at $18,660$ USD, and the endpoint of the phase-out range at $45,802$ USD.

Source: Authors' calculations (see Table 2 for details).

substantially.

Finally, note that these results for the 2018 US tax system are robust in several dimensions. Most importantly, the recent empirical evidence suggests that labor supply elasticities have decreased in magnitude over time, especially those related to the extensive margin. We therefore also considered alternative calibrations with smaller labor supply elasticities at both margins. Qualitatively, our results remain unchanged: the marginal revenue function satisfies the lower and upper boundary conditions, but violates the monotonicity condition. Hence, we conclude that the 2018 US tax-and-transfer system for single parents continues to be Pareto-inefficient, but these inefficiencies are an order of magnitude smaller than those in the mid-1970s.