Is a market-based approach to climate policy desirable?*

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Abstract

A uniform price for carbon is at the center of market-based approaches to climate policy. Actual climate policy, by contrast, has many sector-specific rules. This paper studies the desirability of the market-based approach using tools from the theory of optimal taxation. It is found that (i) a market-based approach is efficient in that it allows to reach emission targets at minimal costs, and (ii) and that departures from it can be justified by equity concerns. In the light of this equity-efficiency tradeoff, a justification of the market-based approach can be given, but it involves indifference with respect to the distributive consequences of climate policy.

Keywords: Climate policy, equity-efficiency trade-off, optimal taxation.

JEL codes: D63, H21, H22, Q58

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1 Introduction

Proponents of a market-based approach to climate policy demand a uniform price on CO2 emissions. With a "correct" price, i.e. one that reflects the social cost of carbon, sector-specific rules are superfluous. They are even harmful because the same emission reduction could be achieved at a lower cost to society. Actual climate policy, by contrast, is a mix of sector specific green taxes, sector-specific cap-and-trade-systems and sector specific regulation.[1] For the proponents of the market-based approach, this plethora of sector-specific rules appears as a political failure, an inability of the political process to reach climate policy targets in an efficient way. Against this background, this paper studies the desirability of the market-based approach to climate policy using tools from the theory of optimal taxation. We analyze a model in which individuals differ in their incomes and in their preferences for green versus brown consumption goods. The technologies of firms are endogenous. Their incentives to reduce the emission intensity of their production depend on the CO2 prices they are facing. This framework nests as special cases the Mirrleesian model of optimal income taxation, Ramsey's model of optimal sector-specific taxation and the partial equilibrium model due to Weitzman that is typically used to justify the market-based approach. We clarify that there are somewhat restrictive conditions under which the market-based approach is justified in the sense that any departure from it implies a violation of Pareto-efficiency. More generally, however, the market-based approach cannot be justified without an explicit value judgment. In particular, with "laissez-fairish" welfare weights the market-based approach seems attractive. Concerns for the distributive consequences of climate policy cannot be captured by such weights, however. With such concerns, sector specific rules and hence a departure from the market-based approach can be justified. Thus, as a main result, this paper shows that a sector-specific approach to climate policy has a foundation which is as good or as bad as the one of the market-based approach: Both are justifiable and in both cases the justification requires an explicit value judgment.

Sketch of the formal analysis. We consider an economy that has three sectors, one produces an unspecific consumption good, one produces a green good and one produces a brown good. The brown and the green good are imperfect substitutes. Firms have decreasing returns to scale and realize inframarginal profits. Their production comes with emissions and they can exert $R\&D$ effort to reduce the emission intensity of their production. Individuals receive labor income and differ in productive abilities, as in

[1] To give examples, the European Union emission trading system (EU ETS) covers electricity and heat generation, aluminium, cement, and steel works, amongst other sectors. The EU is planning to introduce a separate emission trading system (EU ETS II) covering buildings and road transport. The EU moreover has CO2 emission performance standards for cars and vans. In addition there are measures at the national level. Germany, for instance, has green taxes covering fossil fuels and electricity. One could go on.

Mirrleesian models of income taxation. In addition, they differ in their willingness to pay for green consumption goods. Thus, there is a joint distribution of incomes and preferences for green consumption goods. Individuals also own the firms in the economy and differ in their share in the economy's total profits. Some individuals mostly have labour income, others mostly have "capital income". Possibly, individuals also differ in their portfolios, e.g. with some individuals having larger stakes in the brown sector, and others having larger stakes in the green sector. The government has access to a non-linear income tax. It's policy choices are constraint by the need to reach an economy-wide emission target. To reach the target, firms in the unspecific sector, the green sector and the brown sector need to reduce emissions. The analysis focusses on the question whether it is optimal to reach the emission target with sector-specific rules such as sector-specific emission targets or sector specific taxes. Alternatively, there is a price on emissions that is uniform across sectors and, if anything, a uniform consumption tax. If that's the case, we say that climate policy takes a market-based approach.

There are two special cases of this environment in which the market-based approach is clearly desirable, in the sense that any departure from it implies a violation of Pareto-efficiency. In the first special case, all individual characteristics – i.e. preferences for brown versus green consumption goods, productive abilities and "capital incomes" – are observable. In the second special case, productive abilities are the only source of heterogeneity amongst individuals and, moreover, assumed to be private information. Hence, in the second case, all individuals are assumed to have the same consumption preferences and to receive the same "capital incomes." These benchmarks are derived from a primal approach: Allocations are chosen so as to minimize overall emissions subject to the requirements (i) of physical feasibility, (ii) to reach a predetermined profile of utilities and (iii) of incentive compatibility when productive abilities are taken to be private information.

The main part of the analysis takes a dual approach, however. At the heart of this analysis is how market outcomes change when climate policy deviates from the market-based approach. Sector-specific taxes drive a wedge between consumer and producer prices and there are numerous general equilibrium effects. For instance, labor incomes depend not just on the properties of the income tax, but also on consumer prices which change in response to changes of emission or commodity taxes. Also, a tax that increases consumer prices in one sector shifts the excess demand curves in other sectors with repercussions for the whole vector of equilibrium prices. Thereby it also affects the firms' profits and tax revenue. A key step in the analysis is to show that competitive equilibria exist and are unique and lend themselves to a comparative statics analysis. This provides the basis for studying the welfare implications of departures from the market-based approach to climate policy.

The test for the desirability of the market based approach then proceeds as follows: We consider a competitive equilibrium that results under uniform commodity taxation

and with a price for emissions that is uniform across sectors. We then employ a generic social welfare function to evaluate deviations from this policy. Admissible deviations are those that respect the government's emission target. The welfare implications of an admissible deviation are captured by a sufficient statistics formula that highlights efficiency losses from sector-specific taxes, but also distributive effects across individuals with different consumption preferences or different sources and levels of "capital income." The extent to which these distributive effects are desirable then depends inter alia on the specification of welfare weights. When there are no welfare gains from a deviation, we say that the market based approach is desirable. Otherwise, it is not.

The main result. We show that the market-based approach is desirable when two conditions are fulfilled. First, a condition of *distributive indifference*. It holds when the welfare weights are the same across all individuals, irrespectively of whether their "capital" or labor incomes are high or low, and irrespective of whether or not they spend much of their disposable income on green or brown goods. Second, the elasticities of the individual's labour incomes with respect to tax induced changes of consumer prices need to be negligible. Note that these conditions are different in nature. The first condition involves an explicit value judgment. The second condition is a hypothesis on how the economy works. If both conditions are fulfilled, the market based approach is justified.

Together the two conditions are sufficient for a justification of the market-based approach. This leads to the question whether they are necessary. The answer is "no". First, as explained above, when individuals differ only in productive abilities, the market-based approach is desirable even when the welfare weights of "the poor" are higher than the welfare weights of "the rich." Second, even if the two conditions are not satisfied, it is conceivable that an empirical application of the sufficient statistics test reveals negligible gains from a reform towards a more sector-specific climate policy. Whether or not this is the case turns the question of this paper into a pragmatic one: are magnitudes such that a deviation from the market-based approach is really paying off? This is then no longer a question on the principles of climate policy.

Related literature. This paper combines ideas from different strands of the literatures.

The literature on the regulation of externalities often times uses a partial equilibrium model. The seminal paper by Weitzman (1974) is a prominent example. Firms differ in their marginal costs of abatement. There is a social benefit of abatement that is independent of which firm is incurring the costs of abatement. With a tax on emissions or prices for tradable emission permits, firms will expand abatement up to the point where the marginal costs of abatement are equal to the tax or the price of an emission permit. If the tax or the price is equal to the marginal social benefit of abatement, a first-best outcome results. In particular, marginal costs of abatement are equalized across firms, with the implication that the economy-wide costs of abatement are minimized. In

this framework, the case for a market-based approach to climate policy is compelling. Treating firms in different sectors differently can only make things worse.

In the partial equilibrium model, avoiding emissions is the firms' only activity. This paper enriches this framework. Firms are producing consumption goods and emissions are a by-product. The firms' incentives to avoid emissions therefore depend not only on how high green taxes or prices for emission permits are. They also depend on the demand for their final product. In the model introduced below, an increase in demand goes together with an increased effort to avoid emissions. Therefore also the commodity tax system – which affects the incentives of consumers to buy one good or another – matters for climate policy. The comparative statics properties of this framework are broadly consistent with the patterns documented in Känzig (2023). At the firm level, employment, output and emissions are all decreasing when carbon prices go up.

General equilibrium effects are a topic of its own in the analysis of tax incidence. A seminal paper that focussed on corporate taxation is Harberger (1962). More recently, Sachs et al. (2020) focussed on general equilibrium effects in the labour market. Bovenberg and Goulder (1996) present a second best analysis of optimal environmental taxes. Their setting also gives rise to general equilibrium effects. An important difference to the analysis in this paper is that Bovenberg and Goulder analyze a model with a representative household. Thus, their framework is not suited to studying the distributive implications of climate policy. Distributive issues are important in the analysis of optimal environmental taxes by Cremer et al. (1998) and, more recently, Pai and Strack (2022). Their settings, however, do not give rise to general equilibrium effects.

The alternative to a market-based approach to climate policy is one that is sector-specific. Optimal sector specific taxes are characterized in Ramsey models of taxation, albeit with the objective to generate a predetermined level of tax revenue. Here, by contrast, the focus is on reaching a target level of emissions. Diamond (1975) has used the Ramsey approach to characterize a welfare-maximizing commodity tax system, taking into account that some goods are consumed in larger proportion by those with a low marginal utility of income ("the rich") and others by those with a high marginal utility of income ("the poor"). Even though this paper is not using a Ramsey approach, but allows for non-linear income taxation, some of the welfare implications of differential commodity taxation resemble those identified by Diamond.

As is well known, the Ramsey approach has been criticized by Atkinson and Stiglitz (1976). When consumption preferences are such that all individuals would spend a given amount of disposable income in the same way, then any tax system that involves differential commodity taxes is dominated by one that relies only on income taxation, see Laroque (2005) for a simple proof. A related result is proven in this paper. The market-based approach to climate policy dominates any sector-specific approach under the following assumption: Individuals differ only in their productive abilities as in Mirrleesian models of income taxation. As an implication, there is then no heterogeneity in preferences for

green versus brown consumption goods and, moreover, all individuals receive the same "capital income", or, equivalently, all profits are taxed at 100 percent. This result is interesting as a benchmark, but the underlying assumptions are not empirically plausible. Hellwig and Werquin (2023) trace out some of the empirical implications that are implied by the assumptions of Atkinson-Stiglitz and show that they are inconsistent with actual consumption data. Ferey et al. (2022) present optimal tax formulas for non-linear income and consumption taxes that apply when the Atkinson-Stiglitz assumptions are not satisfied. This paper follows Saez (2002) in the modelling of consumption spending: Utility is additively separable between consumption utility on the one hand and effort costs on the other. Moreover, the consumption utility part may be different for different individuals. While this nests the Atkinson-Stiglitz specification as a special case, it allows for heterogeneity in consumption preferences; for instance, it allows for the possibility that incomes and preferences for green rather than brown consumption goods are positively correlated.

While this paper uses arguments and techniques from the literature on optimal taxation, there are still notable differences to the workhorse models in this literature. Here, the government is constraint by the need to reach an emission target. Also, firms make profits and make an effort to reduce emissions and these choices depend on the properties of the tax system. Firm profits are a source of income for some, but not for all households. Thus, there is inequality in incomes beyond the inequality in labour incomes. A difference between this paper and much of the related literature in public finance is, moreover, that there is no attempt to characterize an optimal tax system. Instead, the question is whether a particular benchmark, the market-based approach to climate policy, is desirable. Thus, the focus is on the welfare-implications of deviating from uniform taxes or uniform prices for emission permits. There is no derivation of optimal tax formulas.

Finally, much the literature on optimal commodity taxes uses a mechanism design approach. This makes it possible to characterize optimal tax systems without ad hoc assumptions on the functional form of an optimal tax schedule. It detaches the normative theory, however, from its positive counterpart. There is then no explicit characterization of how market prices and quantities respond to changes of the tax system. This paper uses the dual approach and therefore contains such a characterization.

Outline. The remainder of the paper is organized as follows. The next section introduces the model. Section 3 presents benchmark results for specific versions of the general setup. In those cases, a market-based approach to climate policy is clearly desirable. The main result of the paper can be found in Section 4. The last section contains concluding remarks. Formal proofs are relegated to the Appendix.

2 The model

2.1 Households

Preferences. There is a unit mass of individuals or households. Individuals have preferences that are represented by a utility function

$$u(x_c, \chi(\beta x_g, x_b)) - k(y_l, \omega) \ .$$

The consumption utility u depends on two arguments, the quantity consumed of the unspecific consumption good x_c, and a subutility χ which results from the combination of green and brown consumption goods, x_g and x_b. The brown and the green good are assumed to be imperfect substitutes. As an example, think of x_g as "kilometers travelled by train", of x_b as "kilometers travelled by plane" and of χ as the subutility from travelling. The function χ depends on a parameter β so that the willingness to pay for the green good is increasing in β. The functions u and χ are both assumed to be homothetic. In parts of the analysis – in particular, for the proof of equilibrium existence and uniqueness – we invoke the functional form assumptions

$$u(x_c, \chi(\beta x_g, x_b)) = x_c^{1-\nu} \chi(\beta x_g, x_b)^{\nu} \ ,$$

and

$$\chi(\beta x_g, x_b) = \left(\beta x_g^{1-\varepsilon_\chi} + x_b^{1-\varepsilon_\chi} \right)^{\frac{1}{1-\varepsilon_\chi}} \ .$$

We denote by y_l an individual's labour supply, k is an effort cost function, and ω is a measure of productive abilities that affects the marginal effort costs. We assume that the cross-derivative k_{12} is negative so that higher ω-types have lower marginal effort costs. Otherwise, k is assumed to satisfy the usual Inada conditions.

Budget constraint. We denote the vector of consumer prices by $q = (q_c, q_g, q_b)$. Producer prices are given by $p = (p_c, p_g, p_b)$. Commodity taxes drive a wedge between consumer and producer prices so that

$$q_c = (1 + t_c)\, p_c \quad q_g = (1 + t_g)\, p_g \quad \text{and} \quad q_b = (1 + t_b)\, p_b \ .$$

We are, inter alia, interested in the desirability of differential commodity taxation. Thus, we consider the possibility to tax green and brown consumption at rates that are different from t_c. When individuals supply y_l units of labor, they realize a gross labor income of $p_w\, y_l$, where p_w is the wage rate. Labor income is taxed according to a non-linear income tax schedule: $T_l : p_w\, y_l \mapsto T_l(p_w\, y_l)$. The tax schedule T_l is assumed to be twice continuously differentiable. Possibly, individuals also realize "capital income" from the shares they hold in the economy's firms. We write $s = (s_c, s_g, s_b)$ for a generic portfolio and $\Pi = (\Pi_c, \Pi_g, \Pi_b)'$ for the column vector that list the profits realized in the different

sectors in the economy. A generic "capital income" can then be written as the scalar product $s\,\Pi$. The government redistributes net tax revenues $\mathcal{R}$ in a lump-sum fashion. Possibly, this revenue is generated by taxes on CO2 emissions, discussed in more detail below. Taking all this into account an individual's budget constraint reads as

$$q_c\, x_c + q_g\, x_g + q_b\, x_b \quad \leq \quad p_w\, y_l - T_l(p_w\, y_l) + s\; \Pi^E + \mathcal{R}^E \;. \tag{1}$$

When individuals choose labour supply and consumption demand they hold expectations about the profits and the tax revenue that will contribute to their disposable income, as indicated by the superscript E. When we formally state the definition of an equilibrium below, we will add the requirement that these expectations are correct.

Utility maximization. Individuals choose $x = (x_c, x_g, x_b)$ and y to maximize utility subject to the budget constraint in (1). It will prove useful to decompose this problem into an inner and an outer problem. The inner problem is to maximize $u(x_c, \chi(\beta x_g, x_b))$ for a given level of disposable income c. Hence, the budget constraint for the inner problem is

$$q_c\, x_c + q_g\, x_g + q_b\, x_b \quad \leq \quad c \;.$$

For given tax policy, the solution $x^* = (x_c^*, x_g^*, x_b^*)$ to this problem depends on the prices of consumption goods $q = (q_c, q_g, q_b)$, the preference parameter β, the disposable income c and the commodity tax system $t_x = (t_c, t_g, t_b)$. The indirect utility function v is defined by

$$v(c, \beta, q) = u(x_c^*(c, \beta, q), \chi(\beta x_g^*(c, \beta, q), x_b^*(c, \beta, q))) \;.$$

The outer problem is to choose c and y to maximize

$$v(c, \beta, q) - k(y, \omega)$$

subject to

$$c = p_w\, y_l - T_l(p_w\, y_l) + s\; \Pi^E + \mathcal{R}^E$$

Let $\theta = (\beta, \omega, s)$ be a shorthand for an individual's type. The solution to the outer problem can be written as $c^*(\theta, \Pi^E, \mathcal{R}^E, q_x, p_w, T_l)$ and $y_l^*(\theta, \Pi^E, \mathcal{R}^E, q_x, p_w, T_l)$. Individual demand for the various consumption goods is obtained by inserting $c^*(\theta, \Pi^E, \mathcal{R}^E, q_x, p_w, T_l)$ for c in $x_c^*(\beta, c, q)$, $x_g^*(\beta, c, q)$ and $x_b^*(\beta, c, q)$.

Comparative statics of individual choices. With u homothetic the inner problem can be written as: Choose $z_c = \frac{x_c}{c}$, $z_g = \frac{x_g}{c}$ and $z_b = \frac{x_b}{c}$ to maximize $c\; u(z_c, \chi(z_g, z_b))$ subject to

$$q_c\, z_n + q_g\, z_g + q_b\, z_b \quad \leq \quad 1 \;. \tag{2}$$

The optimal choices of $z = (z_n, z_g, z_b)$ then depend only on the consumer prices q and the preference parameter β. Indirect utility is therefore given by

$$v(c, \beta, q) = c\, u(z_c^*(\beta, q), \chi(z_g^*(\beta, q), z_b^*(\beta, q))) =: c\, \tilde{v}(\beta, q)\ . \tag{3}$$

We henceforth refer to $\tilde{v}(\beta, q_x)$ as the marginal utility of disposable income. The following Lemma (that we state without proof) gives implications of equation (3).

Lemma 1 *Suppose that $p_w\, y_l - T_l(p_w\, y_l)$ is a non-decreasing function of y_l.*

i) The marginal utility of disposable income is constant:

$$v_c(c, \beta, q) = \tilde{v}(\beta, q) \quad \text{and} \quad v_{cc}(c, \beta, q) = 0\,,$$

where v_c and v_{cc} denote, respectively, the first and the second derivative of the indirect utility function with respect to the level of disposable income.

ii) The utility-maximizing labour supply y_l^* does not depend on s_c, s_g, s_b, Π^E and $\mathcal{R}^E$.

iii) The marginal utility of disposable income is increasing in β and decreasing in the consumer prices q_c, q_g, and q_b.

iv) The utility-maximizing level of disposable income c^* and y_l^* are non-decreasing functions of β and non-increasing functions of q_c, q_g, and q_b.

v) c^* and y_l^* are non-decreasing functions of ω.

Thus, the "capital income" that individuals realize and the tax revenues that the government might redistribute are without consequence for individual labour supply. Still there are income effects. If an individual's disposable income goes up, the consumption of all goods scales up.[2] Put differently, the composition of the consumption basket remains the same as individuals get poorer or richer. Only the size of the basket changes. Heterogeneity in the composition of the basket is then only due to heterogeneity in the preference for green consumption goods, parameterized by β. Higher values of β imply a higher marginal utility of disposable income. This increases earnings incentives. Thus, ceteris paribus, individuals with a higher taste for green goods do not earn less than people with a higher taste for brown goods. By the same logic, higher consumer prices lower the marginal utility of disposable income and thus reduce earnings incentives. Finally, as in Mirrleesian models of income taxation, earnings incentives increase in productive abilities, so that both disposable income and labor earnings are non-decreasing functions of ω.

[2]As is well known, with homothetic preferences, Engel curves – describing how the demand for various consumption goods varies with disposable income– are linear.

Aggregate labor supply, Aggregate consumption demand. Let Φ_θ be the *cdf* that describes the joint distribution of productive abilities ω, preferences for green consumption β and "capital incomes" s. Aggregate labor supply can then be written as

$$Y_l(\Pi^E, \mathcal{R}^E, q, p_w, T_l) = \mathbf{E}_\theta \left[y_l^*(\theta, \Pi^E, \mathcal{R}^E, q, p_w, T_l) \right] \ ,$$

where the operator $\mathbf{E}_\theta$ indicates the computation of an expectation using the distribution Φ_θ. We define the aggregate demand for the different consumption goods analogously. Henceforth $X_c(\Pi^E, \mathcal{R}^E, q, p_w, T_l)$, $X_g(\Pi^E, \mathcal{R}^E, q, p_w, T_l)$ and $X_b(\Pi^E, \mathcal{R}^E, q, p_w, T_l)$ denote, respectively, aggregate demand for the unspecific consumption good, the green and the brown good.

2.2 Firms

There are three sectors in the economy, indexed by $j \in \{c, g, b\}$, where c stands for the sector producing the unspecific consumption good, g stands for the green sector and b for the brown sector. The firms in any one sector j produce the sector's final output good, using labor l as the only input. In addition, they can invest resources r to reduce the emission intensity of their production. Firms differ in the cost of the investment that is needed to reduce the emission intensity of their production. This is meant to capture that it is easy for some firms to reduce the emissions intensity of their production, for others it is difficult. The heterogeneity of firms in terms of their abatement costs is important for the justification of the market-based approach. The presumption is that the market-based approach is efficient in that it gives firms with low abatement incentives to cut emissions, whereas firms with high abatement costs pay the price of carbon.

Profit-maximization. The profit-maximization problem of a generic firm in sector j is to choose l and r to maximize

$$p_j \ \alpha \ f_j(l) - p_w \ l - t_{je} \Big(e_{j0} - a_j(r) \Big) \ \alpha f_j(l) - p_c \ \gamma \ r \ , \quad \text{with} \quad a_j(0) = 0.$$

We now explain the various terms that enter this expression. A firm in sector j, sells goods at a price p_j to the market. The production function f_j is assumed to satisfy the usual Inada conditions. We will sometimes refer to is an iso-elastic production function

$$f_j(l) = \frac{1}{1 - \frac{1}{\sigma_j}} l^{1 - \frac{1}{\sigma_j}}$$

where σ_j is the elasticity of substitution for firms in sector j. Firms in a given sector j are assumed to differ in their factor productivity α and we denote by Φ_α^j the *cdf* that describes the cross-section distribution of α in sector j. The wage bill of a firm that hires l units of labor is $p_w l$.

Emissions of CO2 are a byproduct of production. The parameter e_{j0} gives the emission intensity of a firm in sector j if it does nothing – as captured by the subscript 0 – to

avoid emissions. The possibility of emission avoidance is captured by the function a_j. It is non-negative, increasing in r and concave. It is bounded from above by e_{j0} and satisfies the Inada conditions. The more resources r are devoted to emission avoidance, the lower are the emissions $e_{j0} - a_j(r)$ per unit of output. Emissions in sector j are taxed at rate t_{je}. From the firm's perspective, the tax rate t_{je} can equivalently be interpreted as a price for an emission permit. In this interpretation, a firm combines two factors of production, labor and emission permits, in the production of its final output good. Thus, in the given setting, the classical question on the desirability of *production efficiency*[3] can be posed as the question whether an optimal policy should distort the relative prices of labor inputs and emission permits away from some first-best benchmark. Below, we will get to the question what an appropriate first best benchmark is in this case.

The parameter γ is a measure of how many resources a firm needs to invest to achieve a given level of emissions reduction. Firms with high γ have a high cost of emission avoidance. We denote by Φ_γ^j the *cdf* that describes the cross-section distribution of γ in sector j. The *cdf* Φ_j describes the joint distribution of α and γ in sector j. We treat the unspecific consumption good as a multi-purpose good that can be used both for consumption and for investments into emission avoidance. Thus, a firm that wants to reduce its emission intensity by $a_j(r)$ needs to spend $\gamma\, r$.

A firm's decision how much labor to hire and hence how much to produce and its decision how much to invest into emission avoidance are interdependent. Consider the first order conditions that characterize the profit maximizing choices l^* and r^*. The first order condition for the choice of labor inputs is

$$\Big(p_j \;- t_{je}(e_{j0} - a_j(r^*))\Big)\; \alpha\; f_j'(\,l^*) \quad = \quad p_w\ , \tag{4}$$

and the first order condition for emission avoidance is

$$\frac{p_c\,\gamma}{a_j'(r^*)\;\alpha\;f_j(l^*)} \quad = \quad t_{je}\ . \tag{5}$$

These are two equations in two unknowns. In the absence of emissions taxes, the first order condition in (4) is the familiar condition that the value of the marginal product of labor is equal to the wage rate. With an emissions tax, the value p_j is reduced by the emission costs that come with an expansion of employment and production. These costs are lower the more the firm invests into emission avoidance. Thus, a higher level of r^* goes together with a higher level of l^*. The first order condition in (5) also gives rise to a complementarity between output and employment on the one hand and emission avoidance on the other. It has, on the left hand side, the marginal cost of avoiding one unit of emissions and, on the right hand side, the price of an emission permit, or equivalently, the taxes that can be saved when one unit of emissions is avoided. An inspection of (5) shows that a higher level of l^* reduces the marginal cost of avoidance.

[3]See Diamond and Mirrlees (1971).

Hence, firms who opt for a larger scale of production also devote more resources to the avoidance of emissions.

Comparative statics of firm behavior: Output, Employment, Investment and Emissions. To understand how form behavior changes when taxes and prices change it is useful to decompose the firm's profit-maximization problem into an inner and an outer problem. For the inner problem, the employment level l is taken as given and the firm chooses r to maximize

$$-t_{je}(e_{j0} - a_j(r))\, \alpha f_j(l) - p_c\, \gamma\, r$$

The solution to this problem is denoted by $r^*(l, p_c, t_{je}, \gamma)$. It is straightforward to verify that r^* is increasing in l and t_{je} and decreasing in p_c and γ. The outer problem then is to choose l to maximize

$$p_j\, \alpha\, f_j(l) - \ p_w l - t_{je}\, (e_{j0} - a_j(r^*(l, \cdot)))\, \alpha f_j(l)) - p_c\, \gamma\, r^*(l, \cdot)\ .$$

We denote the solution to this problem by $l^*(p_j, p_w, p_c, t_{je}, \gamma, \alpha)$. It is straightforward to verify that l^* is increasing in p_j and α and decreasing in p_w. The complementarity of the investment and the labour choice implies, moreover, that l^* is decreasing in p_c and γ, as r^* is decreasing in these variables.

Lemma 2 *Let $f_j(l) = \frac{1}{1-\frac{1}{\sigma_j}} l^{1-\frac{1}{\sigma_j}}$. Denote profit-maximizing emissions by*

$$\mathbf{e}_j^*(p_j, p_w, p_c, t_{je}, \gamma, \alpha) := \Big(e_{j0} - a(r^*(l^*(\cdot), \cdot))\Big)\ \alpha\ f_j(l^*(\cdot))\ .$$

The function $\mathbf{e}_j^*$ has the same comparative statics properties as the function l^*: It is increasing in p_j and α and decreasing in p_w and t_{je}. It is decreasing in p_c and γ.

Thus, essentially, at the level of an individual firm, the comparative statics of emissions, output, employment and investment all have the same sign. Anything that makes the firm expand output, employment and investment also implies more emissions. Anything that makes the firm reduce emissions goes together with a down-scaling of all its economic activities.

These comparative statics results can be related to discussions on whether the "green transformation" of the economy – the change of technologies so that production processes become cleaner – can be a source of economic growth. Through the lens of the model, the answer is "no" if the comparison is to a benchmark economy that has emissions which are too high. A change of policy that brings down emissions will then also bring down output, employment and investments into greener technologies. The answer is "yes" if the comparison is to a benchmark economy that has to cut emissions while operating with fixed technologies. The possibility to invest then implies that output and employment are higher than they would otherwise be.

What distinguishes the green sector from the brown sector? So far the sector names "unspecific", "green" and "brown" have been labels with no meaning. There are different conceivable ways to distinguish sectors according to how dirty they are. One conceivable order is according to how dirty they are. For instance, if for any given r,

$$e_{b0} - a_b(r) \quad > \quad e_{c0} - a_c(r) \quad > \quad e_{g0} - a_g(r) \ ,$$

then, emissions per unit of output are largest in the brown sector and smallest in the green sector. An alternative is to order them according to their marginal cost of avoidance. If for any given α, γ, l and r,

$$\frac{\gamma}{a_b'(r)\ \alpha\ f_b(l)} \quad > \quad \frac{\gamma}{a_c'(r)\ \alpha\ f_c(l)} \quad > \quad \frac{\gamma}{a_g'(r)\ \alpha\ f_g(l)} \ ,$$

then the marginal avoidance costs are lowest in the green sector and highest in the brown sector. The analysis in this paper does not presume that it is possible to order sectors in this way, but it is consistent with such a possibility.

Aggregation. For later use in the analysis of competitive equilibria, we define labor demand, goods supply and the demand for emission permits both at the sector and the aggregate level. We start from with individual firm behavior. The choices of a firm in sector j depend on its characteristics $\theta_j = (\alpha, \gamma)$, the prices (p_j, p_w) it is facing and the sector specific tax t_{je}. We denote, respectively, by $l^*(\theta_j, p_j, p_w, t_{je})$ and $r^*(\theta_j, p_j, p_w, t_{je})$ the firm's labour demand and the resources that it invests to avoid emissions. The firm's supply of good j is then given by

$$y_j^*(\theta_j, p_j, p_w, t_{je}) = \alpha f_j(l^*(\theta_j, p_j, p_w, t_{je}))$$

and its emissions are equal to

$$\mathbf{e}_j^*(\theta_j, p_j, p_w, t_{je}) = \Big(e_{j0} - a_j(r^*(\theta_j, p_j, p_w, t_{je}))\Big)\ y_j^*(\theta_j, p_j, p_w, t_{je}) \ ,$$

where

$$\gamma\, r^*(\theta_j, p_j, p_w, t_{je})$$

is the firm's investment into a greener technology, measured in expenditures for the unspecific consumption good. Profits are then given by

$$\begin{aligned} \pi_j(\theta_j, p_j, p_w, t_{je}) \quad &= \quad p_j\ y_j^*(\theta_j, p_j, p_w, t_{je}) - p_w\ l^*(\theta_j, p_j, p_w, t_{je}) \\ &\qquad - p_c\ \gamma\ r^*(\theta_j, p_j, p_w, t_{je}) - t_{je}\ \mathbf{e}^*(\theta_j, p_j, p_w, t_{je}) \ . \end{aligned}$$

Labor demand, goods supply, emissions and profits at the sector level. Let Φ_j be the *cdf* that represents the distribution of firm characteristics in sector j. Total labor demand by firms in sector j is denoted by

$$L_j(p_j, p_w, t_{je}) = \mathbf{E}_j\left[l^*(\theta_j, p_j, p_w, t_{je})\right] \ .$$

Analogously we define by $Y_j(p_j, p_w, t_{je})$ the sector's goods supply, by $\mathcal{E}_j(p_j, p_w, t_{je})$ the sector's demand for emission permits and by $R_j(p_j, p_w, t_{je})$ the sector's demand for the unspecific consumption good. Profits in sector j are denoted by $\Pi_j(p_j, p_w, t_{je})$.

Aggregate labor demand and the aggregate demand for emission permits. Aggregate labor demand is given by

$$L(p, p_w, t_e) = L_c(p_c, p_w, t_{ce}) + L_g(p_c, p_g, p_w, t_{ge}) + L_b(p_c, p_b, p_w, t_{be})$$

where $t_e = (t_{ce}, t_{be}, t_{ge})$ is the collection of sector-specific emissions taxes. Analogously, we denote the overall demand for emission permits by $\mathcal{E}(p, t_e)$, the resources devoted to the greening of technologies by $R(p, t_e)$ and by

$$\Pi(p, p_w, t_e) = \Big(\Pi_c(p_c, p_w, t_{ce}), \Pi_g(p_c, p_g, p_w, t_{ge}), \Pi_b(p_c, p_b, p_w, t_{jb})\Big)$$

the vector of sectoral profits.

2.3 Government

Overall tax policy consists of collection of taxes that appear in the individuals' budget constraints $(t_c, t_b.t_g, T_l)$ and the emission taxes $t_e = (t_{ce}, t_{be}, t_{ge})$ that affect the choices of firms. Differential commodity taxation is reflected in the possibility to tax green and brown consumption goods at rates that differ from t_c. Sector-specific taxation is captured by the possibility to tax CO2 emissions at sector-specific rates. We use $\mathcal{T} = (t_c, t_b.t_g, T_l, t_e)$ as a shorthand for overall tax policy. We assume that there is a national emission target $\bar{\mathcal{E}}$

$$\mathcal{E}(p, p_w, t_e) \leq \bar{\mathcal{E}} \ . \tag{6}$$

and that the government considers only policies which reach this target.[4] Tax revenue, of any, is rebated lump sum. Given a tax policy, a price system, and expectations about

[4]In political practice, it is disputable whether the national emission targets associated with the Paris climate conference are really binding.

tax revenues and profits Π^E and $\mathcal{R}^E$, aggregate tax revenue $\mathcal{R}(p, p_w, \mathcal{T})$ is given by

$$\begin{aligned} \mathcal{R}(p, p_w, \mathcal{T}) \quad = \quad & t_c \, p_c \, X_c(\Pi^E, \mathcal{R}^E, q, p_w, T_l) \\ & + \quad t_g \, p_g \, X_g(\Pi^E, \mathcal{R}^E, q, p_w, T_l) \\ & + \quad t_b \, p_b \, X_b(\Pi^E, \mathcal{R}^E, q, p_w, T_l) \\ & + \quad \mathbf{E}_\theta \Big[T_l(p_w \; y_l^*(\theta, \Pi^E, \mathcal{R}^E, q, p_w, T_l)) \Big] \\ & + \quad \textstyle\sum_{j \in \{c,b,g\}} \; t_{je} \; \mathcal{E}_j(p_j, p_w, t_{je}) \; . \end{aligned} \tag{7}$$

By the following Lemma, there is and only one level of expected tax revenue $\mathcal{R}^E$ that is "correct", i.e. consistent with the way in which actual tax revenue depends on expected tax revenue. The proof follows from an application of Brouwer's fixed point theorem.

Lemma 3 *Suppose that there is an upper bound $\bar{\mathcal{R}}$ on the tax revenue that can be collected. Suppose that all consumption goods are normal goods. Then, for given prices p, p_w, tax policy $\mathcal{T}$ and expected profits Π^E, there is one and only level of tax revenue so that $\mathcal{R}(p, p_w, \mathcal{T}) = \mathcal{R}^E$.*

2.4 Equilibrium

Given a tax policy $\mathcal{T}$, a price system (p_w, p_c, p_g, p_b) is an equilibrium price system if the following conditions are met: The labor market clears,

$$L(p, p_w t_e) = Y_l(\Pi^E, \mathcal{R}^E, q, p_w, T_l) \; ,$$

the goods markets clear

$$X_c(\Pi^E, \mathcal{R}^E, q, p_w, T_l) + R(p, p_w, t_e) = Y_c(p_c, p_w, t_{ce}) \; ,$$

$$X_g(\Pi^E, \mathcal{R}^E, q, p_w, T_l) = Y_g(p_g, p_c, p_w, t_{ge}) \; ,$$

and

$$X_b(\Pi^E, \mathcal{R}^E, q, p_w, T_l) = Y_b(p_b, p_c, p_w, t_{be}) \; ,$$

and expectations are correct

$$\mathcal{R}^E = \mathcal{R}(p, p_w, \mathcal{T}) \quad \text{and} \quad \Pi^E = \Pi(p, p_w, t_e) \, ,$$

where $R(p, p_w T)$ is defined in equation (7) and $\Pi(p, p_w, t_e)$ is the vector that lists aggregate profits in the different sectors of the economy.

Lemma 4 *If the goods markets clear and expectations are correct, then the labor market clears.*

The lemma, which is simply a version of Walras' law for the given economy, simplifies the equilibrium characterization. If all goods markets clear and expectations are correct, the labor market clears too, so that the equilibrium characterization can focus on the goods market clearing conditions.

In the following we provide a proof of existence and uniqueness under the assumptions that the consumption utility function u is Cobb-Douglas, i.e.

$$u(x_c, \chi(\beta x_g, x_b)) = x_c^{1-\nu} \chi(\beta x_g, x_b)^{\nu} \ .$$

and that the subutility χ gives rise to a constant elasticity of substitution,

$$\chi(\beta, x_g, x_b) = \left(\beta x_g^{1-\varepsilon_\chi} + x_b^{1-\varepsilon_\chi}\right)^{\frac{1}{1-\varepsilon_\chi}} \ .$$

Proposition 1 *Under these assumptions, the following is true: There exists $\bar{\nu}$ so that for $\nu < \bar{\nu}$, for any vector of tax rates, there is a unique equilibrium price vector.*

The assumption that ν is small implies that labor market outcomes, disposable incomes and earnings do not depend much on the prices of the green and the brown good. Under these assumptions, the excess demand functions constructed in the proof of the Proposition have the property that they are monotonically decreasing in the "own" price, while still depending, in a parametric way, on the prices of the other goods. The properties of demand implied by the assumption of a constant elasticity of substitution then imply the existence of a unique equilibrium price vector. More specifically, the formal argument in the proof proceeds as follows: We take the wage rate as the numeraire, so that only the prices p_c, p_g and p_b need to be determined in equilibrium. We then fix p_b and p_c at arbitrary levels and show that there is unique price p_g that clears the market for the green good. As we vary p_b, this partial equilibrium value of p_g adjusts. So, there is a possibility to vary both p_b and p_g while keeping the market for the green good in partial equilibrium. We further observe that any variation that involves a higher/ lower level of p_b lowers/ increases excess demand in the market for the brown good. Thus, we can bring the market for the brown good into partial equilibrium, while maintaining the partial equilibrium in the market for the green good. All this holds for arbitrary values of p_c. As a final step we bring p_c to the level that clears the market for the unspecific consumption good while adjusting p_g and p_b so that both the market for the green good and the market for the brown good both remain in partial equilibrium. Ultimately we have found a general equilibrium price vector in this way. The monotonicity of excess demand functions in their "own" price implies, moreover, that this general equilibrium price vector is unique.

Proposition 1 serves a modest purpose. It assures us that there is a way to specify the primitives of the model so that equilibria exist and are unique. A more general

proof of existence and uniqueness does not seem to be available. By the Sonnenschein-Mantel-Debreu theorem, excess demand functions have little structure in general. For an exchange economy with two goods and CES consumer preferences a proof of existence and uniqueness can be found in Mas-Colell et al. (1995). Proposition 1 extends this result in various ways: There is production, and there are profits in equilibrium, there are three rather than two consumption goods, there are linear taxes on both inputs (emission permits) and outputs and there is a non-linear income tax.

3 Benchmarks

3.1 First-best

Let there be a given utility profile $U_0 : \theta \mapsto U_0(\theta)$. We say that an allocation is first best if it is physically feasible and reaches this utility profile with minimal emissions. Thus, a first-best allocation solves the following problem: What has to be chosen are labor supply $y_l : \theta \mapsto y_l(\theta)$ and consumption levels $x_c : \theta \mapsto x_c(\theta)$, $x_b : \theta \mapsto x_b(\theta)$ and $x_g : \theta \mapsto x_g(\theta)$ for the different types of households. In addition, for every sector $j \in \{c, b, g\}$ and every type of firm $\theta_j = (\alpha_j, \gamma_j)$ in that sector, labor inputs and resources devoted to the abatement of emissions need to be chosen. This is captured by the functions $l_j : \theta_j \mapsto l_j(\theta_j)$ and $r_j : \theta_j \mapsto r_j(\theta_j)$. The objective is to minimize

$$\sum_{j\in\{c,b,g\}} \mathcal{E}_j \quad = \quad \sum_{j\in\{c,b,g\}} \mathbf{E}_j \left[\Big(e_{0j} - a_j(r_j(\theta_j)\Big) \alpha_j f_j(l_j(\theta_j)) \right]$$

subject to the following constraints: First, the chosen allocation needs to reach utility profile U_0. Formally, for all θ,

$$u(x_c(\theta), \chi(\beta x_g(\theta), x_b(\theta))) - k(y(\theta), \omega) = U_0(\theta) \ . \tag{8}$$

Second, the labour used up in the production process is bounded from above by the amount that households make available,

$$\sum_{j\in\{c,b,g\}} \mathbf{E}_j \left[l_j(\theta_j) \right] \quad \leq \quad \mathbf{E}_\theta \left[y(\theta) \right] \ . \tag{9}$$

Third, aggregate consumption is bounded by the production sector's (net) output of the various goods. For the unspecific consumption good this requires that

$$\mathbf{E}_\omega[x_c(\omega)] \quad \leq \quad \mathbf{E}_c[\alpha_c f_c(l_c(\theta_c))] - \sum_{j\in\{c,b,g\}} \mathbf{E}_j[\gamma_j r_j(\theta_j)] \ . \tag{10}$$

For the green and the brown good the constraints are, respectively,

$$\mathbf{E}_\omega[x_g(\omega)] \quad \leq \quad \mathbf{E}_g[\alpha_g f_g(l_g(\theta_g))] \quad \text{and} \quad \mathbf{E}_\omega[x_b(\omega)] \quad \leq \quad \mathbf{E}_b[\alpha_b f_b(l_b(\theta_b))] \ . \tag{11}$$

Proposition 2 *At a solution to a first-best problem:*

i) The marginal costs of emission avoidance are equalized: For any $j, k \in \{c, g, b\}$, and any pair $\theta_j = (\alpha_j, \gamma_j)$ and $\theta_k = (\alpha_k, \gamma_k)$,

$$\frac{\gamma_j}{a_j'(r_j(\theta_j))\alpha_j f_j(l_j(\theta_j))} = \frac{\gamma_k}{a_k'(r_k(\theta_k))\alpha_k f_k(l_k(\theta_k))} \ . \tag{12}$$

ii) The marginal rates of substitution between any pair of consumption goods are equalized across households.

iii) The marginal rates of substitution between consumption goods and effort costs are equalized across households.

We omit a formal proof, which can be obtained with standard arguments from a Lagrangean approach. Upon relating the conditions that characterize a first-best allocation to those that characterize a competitive equilibrium allocation we obtain the following Corollary.

Corollary 1 *With sector specific CO2 prices or differential commodity taxation or non-linear income taxation, a competitive equilibrium allocation is not a first best-allocation.*

As is well known, with private information on preferences or abilities, first best allocations that involve redistribution in favor of "the poor" are typically not incentive-compatible. First-best allocations that are incentive-compatible have distributive implications which may be deemed problematic. This is the root of the equity-efficiency trade-off in the Mirrleesian theory of optimal taxation. It is concerned with a second-best problem, welfare-maximization over the set of incentive-compatible allocation. As we will now see, when incentive compatibility constraints need to address only private information in productive abilities, then every-second best allocation is compatible with the market-based approach.

3.2 Second-best with heterogeneity only in productive abilities

Assumption 1 *Suppose that all individuals have the same preferences over consumption goods, i.e. β is the same for all. Also suppose that all individuals have identical claims on the profits generated in the economy, i.e. s is the same for all. Thus, individuals differ only in their productive abilities ω.*

Under Assumption 1, there is heterogeneity only in productive abilities. Labor earnings and household consumption can therefore more simply be described as functions of ω, rather than as functions of the triple $\theta = (\theta, s, \omega)$. When productive abilities are, moreover, private information incentive compatibility constraints need to be respected. Let $U_0 : \omega \mapsto U_0(\omega)$ be a given utility profile. Incentive compatibility requires that for every pair ω and ω' in the set of ability types Ω, we have

$$U(\omega) \quad \geq \quad u_0(\omega') - k(y_l(\omega'), \omega) \ , \tag{13}$$

where $u_0 = \omega \mapsto u_0(\omega)$ is the profile of consumption utilities. The second-best problem is otherwise as the first-best problem stated above; i.e. an allocation is chosen to minimize total emissions subject to the constraints of physical feasibility and the requirement to reach a given profile of utilities.

Proposition 3 *Under Assumption 1, at a solution to a second-best problem:*

i) The marginal costs of emission avoidance are equalized: For any $j, k \in \{c, g, b\}$, and any pair $\theta_j = (\alpha_j, \gamma_j)$ and $\theta_k = (\alpha_k, \gamma_k)$,

$$\frac{\gamma_j}{a_j'(r_j(\theta_j))\alpha_j f_j(l_j(\theta_j))} = \frac{\gamma_k}{a_k'(r_k(\theta_k))\alpha_k f_k(l_k(\theta_k))} \, . \tag{14}$$

ii) The marginal rates of substitution between any pair of consumption goods are equalized across households.

Corollary 2 *Under Assumption 1, a competitive equilibrium allocation with sector-specific emission taxes or differential commodity taxation is not a second best-allocation.*

Non-linear income taxation is no impediment for reaching a second-best outcome. While a second-best outcome requires that marginal rates of substitution between consumption goods are not distorted away from the marginal rates of transformation prevailing in the production sector, the marginal rate of substitution between consumption utility and effort costs can be distorted by a non-linear income tax. Second-best allocations under Assumption 1 may therefore involve substantial redistribution.

Assumption 1 is interesting as a benchmark, but it is not empirically plausible. Empirically, income and wealth are correlated and green tastes seem to be more prevalent among "the rich." Therefore, in the next section, we revisit the question on the desirability of the market-based approach without imposing Assumption 1.

4 A Test for the desirability of the market-based approach to climate policy

The benchmark results of the previous section where obtained through a primal approach. That is, allocations where chosen subject to feasibility and, possibly, incentive constraints. With the primal approach, market prices are not explicitly modelled as functions of tax policy. We now return to the competitive equilibria that were defined in Section 2 and take a dual approach; i.e. we will study what a deviation from the market-based approach implies for market prices and equilibrium quantities, including emissions.

Admissible deviations from the market-based approach. The test for the desirability of the market-based approach then proceeds as follows: We consider a competitive

equilibrium with (i) uniform emissions taxes, (ii) uniform commodity taxes, (iii) an arbitrary income tax schedule which are specified such that the emission target is reached. We then consider deviations from uniform emission taxes and/ or uniform commodity taxes and evaluate them with alternative social welfare functions. The evaluation focusses on deviations that respect the emission target; i.e. we will consider only deviations to policies $\mathcal{T}$ with the property that

$$\mathcal{E}(p^*(\mathcal{T}), t_e) = \bar{\mathcal{E}} \; , \tag{15}$$

where, here and henceforth, we suppress the dependence of endogenous variables on the wage rate p_w. By Walras's law (recall Lemma 4), we can set $p_w = 1$ without loss of generality.

Measures of social welfare. An additive utilitarian social welfare measure is given by

$$\mathcal{W} = \mathbf{E}_\theta[g(\theta)\; U(\theta)] \; ,$$

where

$$U(\theta) = u(x_c(\theta), \chi(\beta, x_g(\theta), x_b(\theta))) - k(y_l(\theta), \omega) \; .$$

is the utility realized by a household of type θ, and $g : \theta \mapsto g(\theta)$ specifies welfare weights as a function of the individuals' types.

As will become clear, in the given setting, generalized welfare weights, see Saez and Stantcheva (2016), have an intuitive appeal. Assume that the welfare weights carry an additional argument, the marginal utility of disposable income. Specifically, let

$$g(\tilde{v}(\beta, q_x), \theta) = \frac{1}{\tilde{v}(\beta, q_x)} \tilde{g}(\theta) \; .$$

When used for policy evaluation, such weights imply that individuals with different preferences for green vs brown consumption goods but an equal disposable income have the same weight.

For later use, we introduce the following shorthands: For the social marginal utility of disposable income of type θ we write

$$\mathbf{g}(\tilde{v}(\beta, q_x), \theta) := g(\tilde{v}(\beta, q_x), \theta)\; \tilde{v}(\beta, q_x) \; .$$

The population average of this quantity, sometimes also referred to as the marginal value of public funds, is denoted by $\bar{\mathbf{g}} := \mathbf{E}_\theta[\mathbf{g}(\tilde{v}(\cdot), \theta)]$. Finally, the social marginal utility of income for the recipients of "capital income" from sector j is $\bar{\mathbf{g}}_{\Pi^j} := \mathbf{E}_\theta[\mathbf{g}(\tilde{v}(\cdot), \theta)\; s_j]$.

The test. The test now proceeds as follows: let $\tau_1 \in \{t_c, t_g, t_b, t_{ce}, t_{ge}, t_{be}\}$ and $\tau_2 \in \{t_c, t_g, t_b, t_{ce}, t_{ge}, t_{be}\}$ be two different tax rates. A policy change that respects the emission target needs to satisfy

$$\mathcal{E}_{\tau_1} d\tau_1 \ + \ \mathcal{E}_{\tau_2} d\tau_2 = 0$$

or

$$\frac{d\tau_2}{d\tau_1} \quad = \quad -\frac{\mathcal{E}_{\tau_1}}{\mathcal{E}_{\tau_2}} \ ,$$

where $\mathcal{E}_{\tau_1}$ and $\mathcal{E}_{\tau_2}$ are, respectively, total differentials. They give the marginal impact on total emissions when the levels of the tax instruments τ_1 and τ_2 slightly increases.[5] For the sake of the argument, let $d\tau_1 > 0$ and $d\tau_2 < 0$. The welfare-implications of such a policy change are positive if

$$\mathcal{W}_{\tau_1} d\tau_1 \ + \ \mathcal{W}_{\tau_2} d\tau_2 > 0$$

or, equivalently, if

$$\mathcal{W}_{\tau_1} \ - \ \mathcal{W}_{\tau_2} \left(\frac{\mathcal{E}_{\tau_1}}{\mathcal{E}_{\tau_2}} \right) > 0 \ ,$$

where $\mathcal{W}_{\tau_1}$ and $\mathcal{W}_{\tau_2}$ are, respectively, total differentials of a given welfare measure.

Note that this test does not involve the solution of an optimal tax problem. There still is a similarity. At a solution to an optimal tax problem with the requirement to reach a given emission target, the ratio $\frac{\mathcal{E}_{\tau_1}}{\mathcal{E}_{\tau_2}}$ has to be proportional to the ratio $\frac{\mathcal{W}_{\tau_1}}{\mathcal{W}_{\tau_2}}$.[6] The test exploits the observation that a lack of proportionality gives us the possibility to reach a higher level of welfare while respecting the emission target. The test can be performed, however, without having to take on board regularity conditions which ensure that a solution to an optimal tax problem is well defined. We only need to evaluate deviations from a given competitive equilibrium allocation. In particular, we can proceed with no need to discuss the thorny and somewhat esoteric issue what welfare weights would look like at an optimal welfare-maximizing allocation.

Welfare implications of policy changes. The following Lemma is the key ingredient for our ability to actually perform the test. It gives the welfare implications of policy changes starting from an arbitrary competitive equilibrium allocation. This characterization of welfare implications takes the form of a sufficient statistics formula; that is, welfare implications can be computed when market outcomes (prices and quantities) are known, when estimates of the elasticities are available that capture how market outcomes change when taxes change, and when welfare weights have been specified.

[5]The total differential is the sum of a direct effect that a tax increase may have on emissions and of an effect that comes from changes of equilibrium prices in response to the change of the tax rate.

[6]This can be shown using a Lagrangean approach.

Lemma 5 *For* $\tau \in \{t_c, t_b, t_g, t_{ce}, t_{be}, t_{ge}\}$,

$$\begin{aligned}
\mathcal{W}_\tau &= -\sum_j \frac{dq_j^*(\mathcal{T})}{d\tau}\; Cov(\mathbf{g}(\tilde{v}(\cdot),\theta), x_j^*(\cdot)) \\
&+ \bar{\mathbf{g}}\; \sum_j (q_j^*(\mathcal{T}) - p_j^*(\mathcal{T}))X_{j\tau}^*(\cdot) \\
&+ \frac{dp_c^*(\mathcal{T})}{d\tau_j}\Big((\bar{\mathbf{g}}_{\Pi_c} - \bar{\mathbf{g}})Y_c^*(\cdot) - \sum_j (\bar{\mathbf{g}}_{\Pi_j} - \bar{\mathbf{g}})\mathbf{E}_j[\gamma_j\; r_j^*(\cdot)]\Big) \\
&+ \frac{dp_g^*(\mathcal{T})}{d\tau}\; (\bar{\mathbf{g}}_{\Pi_g} - \bar{\mathbf{g}})X_g^*(\cdot) \quad + \frac{dp_b^*(\mathcal{T})}{d\tau}\; (\bar{\mathbf{g}}_{\Pi_b} - \bar{\mathbf{g}})X_b^*(\cdot) \\
&+ \bar{\mathbf{g}}\; \mathbf{E}_\theta[T_l'(y_l^*(\cdot,\theta))y_{l\tau}^*(\cdot,\theta)] \\
&+ \bar{\mathbf{g}}\; \sum_j t_{je}\; \mathcal{E}_{j\tau}^*(\cdot) \\
&+ \sum_j \mathbf{I}(\tau = \tau_{je})(\bar{\mathbf{g}} - \bar{\mathbf{g}}_{\Pi_j})\mathcal{E}_j^*(\cdot)
\end{aligned}$$

where $X_{c\tau}^*$, $X_{g\tau}^*$ *and* $X_{b\tau}^*$ *are total differentials of equilibrium quantities,* $y_{l\tau}^*(\cdot,\theta)$ *is the total differential of equilibrium labor supply for an individual of type* θ *and* $\mathcal{E}_{j\tau}^*$ *is the total differential of equilibrium emissions in sector* j.

The Lemma highlights that a change of taxes has distributive effects and it involves efficiency losses. The distributive effects involve term that relates the social marginal utility of income of a subset of individuals to the population average. Efficiency losses, by contrast, are due to the changes of equilibrium quantities.

Distributive effects. More specifically, suppose that the consumer price of good j goes up when some tax rate changes. When good j is mainly consumed by households with a low social marginal utility of income, so that $\text{Cov}(\mathbf{g}(\tilde{v}(\cdot),\theta), x_j^*(\cdot)) < 0$, then the fact that these households have to pay more for their consumption tends to raise the welfare measure. For instance, when there is a positive correlation of the taste for green goods, as measured by β, and labour income, as measured by ω, and if welfare weights simply depend on disposable income, then this covariance is positive for the brown consumption good and negative for the green consumption good. The signs would be reversed with welfare weights that are "green" in the sense that people with a higher share of the green good in their consumption basket receive more weight than people with a larger share of the brown good.

Alternatively, suppose that the producer price of good j goes down. This tends to increase welfare if those who receive "capital income" from sector j have a welfare weight that is lower than the one of the average person. This is the case when there is a positive correlation between "capital income", as measured by s_j, and labour income and when welfare weights are monotonic in disposable income. Alternatively, with welfare weights that are pro-business in the sense that people with "capital incomes" receive higher

weights than the average person, a reduction of producer prices and hence profit margins tends to lower welfare.

The term

$$\sum_j \mathbf{I}(\tau = \tau_{je})(\bar{\mathbf{g}} - \bar{\mathbf{g}}_{\Pi_j})\mathcal{E}_j^*(\cdot)$$

shows that the same logic applies to the evaluation of higher taxes on emissions. If the business owners who have to pay these taxes receive below average weights, this is a plus for welfare, otherwise it is a minus.

Efficiency losses. The term

$$\bar{\mathbf{g}} \sum_j (q_j^*(\mathcal{T}) - p_j^*(\mathcal{T}))X_{j\tau}^*(\cdot)$$

is the general equilibrium analogue to Harberger's famous triangle. Commodity taxes drive a wedge between consumer and producer prices. Therefore they crowd out economic transactions that would be mutually beneficial with lower taxes: $X_{j\tau}^*(\cdot)$ is a measure of the volume of transactions that are lost in the market for good j when taxes change and $q_j^*(\mathcal{T}) - p_j^*(\mathcal{T})$ is a per unit measure of the gains from trade that are lost as a consequence.

The term

$$\bar{\mathbf{g}} \sum_j t_{je}\ \mathcal{E}_{j\tau}^*(\cdot)$$

captures that the volume of emissions changes when taxes change. While a reduction of emissions helps to reach the emissions target, it also implies a loss of tax revenue and hence of welfare.

Finally, changes of the tax system affect consumer prices and thereby the marginal utility of income. This affects earnings incentives on the labor market and hence the tax revenue that comes through the taxation of labour incomes. This is what is picked up by

$$\bar{\mathbf{g}}\ \mathbf{E}_\theta[T_l'(y_l^*(\cdot,\theta))y_{l\tau}^*(\cdot,\theta)]\ .$$

Sufficient conditions for the desirability of the market-based approach. Lemma 5 is based on an arbitrary competitive equilibrium. We now specialize this further and consider the competitive equilibrium that results when climate policy takes a market-based approach. We assume that carbon taxes and commodity taxes are uniform. Thus, there is a number $\bar{t}_e$, so that $t_{je} = \bar{t}_e$, for all j. We also let $t_c = t_g = t_b = 0$.[7] Consequently, efficiency losses from commodity taxation vanish

$$\bar{\mathbf{g}} \sum_j (q_j^*(T) - p_j^*(T))X_{j\tau}^*(\cdot) \quad = \quad 0\ .$$

[7]Consider the budget constraint of the inner problem: $q_c x_c + q_g x_g + q_b x_b \leq c$. With uniform commodity taxation – i.e. $t_c = t_g = t_b = t$ – this can be written as $p_c x_c + p_g x_g + p_b x_b \leq \frac{c}{1+t}$ where $c = p_w\, y_l - T_l(p_w\, y_l) + s\ \Pi^E + \mathcal{R}^E$. Thus, upon adjusting T_l, s and $\bar{g}$ we can reinterpret the status quo as one that has no commodity taxation at all.

We now add a condition of *distributive indifference:* Welfare weights are the same for all individuals; i.e. for all θ,

$$\mathbf{g}(\tilde{v}(\beta, q_x), \theta) = \bar{\mathbf{g}} \; .$$

If this condition holds, only aggregate disposable income matters for welfare. An additional euro of disposable income in the hands of "the rich" is then as valuable as an additional euro in the hands of "the poor". Consequently, all distributive terms disappear from $\mathcal{W}_\tau$ and we are left with

$$\mathcal{W}_\tau \;=\; \bar{\mathbf{g}} \left(\mathbf{E}_\theta[T_l'(y_l^*(\cdot,\theta)) y_{l\tau}^*(\cdot,\theta)] \quad + \quad t_e \, \mathcal{E}_\tau^*(\cdot) \right) ,$$

where $\mathcal{E}_\tau^*(\cdot) = \sum_j \; \mathcal{E}_{j\tau}^*(\cdot)$.

Finally, we add the assumption consumption or emissions taxes do not distort labour supply, so that, for all θ, $y_{l\tau}^*(\cdot,\theta) = 0$, and hence

$$\mathbf{E}_\theta[T_l'(y_l^*(\cdot,\theta)) y_{l\tau}^*(\cdot,\theta)] = 0 \; .$$

It is an empirical question whether or not this assumption is a good approximation for how individuals behave. If the taxes primarily affect goods that have a small budget share, then it is conceivable that labour supply does not respond much.[8] Under this assumption, we have that

$$\mathcal{W}_\tau \;=\; \bar{\mathbf{g}} \, t_e \, \mathcal{E}_\tau^*(\cdot) \; ,$$

and the test for the desirability of the market-based approach yields

$$\mathcal{W}_{\tau_1} \;-\; \mathcal{W}_{\tau_2} \left(\frac{\mathcal{E}_{\tau_1}}{\mathcal{E}_{\tau_2}} \right) = 0 \; .$$

Hence, there is no reason to deviate from the market-based approach. The following Proposition summarizes this discussion.

Proposition 4 *Consider a competitive equilibrium induced by a market-based approach to climate policy. Under the assumptions of*

i) distributive indifference and

ii) no distortions of labour supply

there are no welfare gains from departing from the market-based approach.

[8] As argued in the proof of Proposition 1, with

$$u(x_c, \chi(\beta x_g, x_b)) = x_c^{1-\nu} \chi(\beta x_g, x_b)^\nu \quad \text{and} \quad \chi(\beta, x_g, x_b) = \left(\beta x_g^{1-\varepsilon_\chi} + x_b^{1-\varepsilon_\chi} \right)^{\frac{1}{1-\varepsilon_\chi}} \; ,$$

in the limit case $\nu \to 0$, the marginal utility of income depends only on q_c. Thus, for ν very small, changes of q_c are akin to changes of marginal tax rates, and therefore one would expect a behavioral response, whereas changes of q_g and q_b are almost without consequence for the marginal utility of income and hence shouldn't have an impact on labour supply.

Can the market-based approach be desirable under alternative assumptions? Proposition 4 gives sufficient conditions for the desirability of the market-based approach. This raises the question whether these conditions are also necessary. The answer is "no." By Proposition 3, with heterogeneity only in productive abilities, the market based approach is desirable even when welfare weights for "the poor" are larger than the welfare weights for the "rich."

Moreover, even when there is a non-degenerate joint distribution of preferences for green consumption, "capital incomes" and labor incomes, an empirical application of the sufficient statistics approach may bring the result that the welfare gains of departing from the market-based approach are small. To illustrate this possibility, note that we can write the welfare implication of a tax change as

$$\mathcal{W}_\tau \quad = \quad \mathcal{W}_\tau^{net} \; + \; \bar{\mathbf{g}} \sum_j t_{je} \frac{d\mathcal{E}_j^*(\cdot)}{d\tau} \; ,$$

where $\mathcal{W}_\tau^{net}$ is the welfare impact net of its impact on the level of emissions. Using this notation, the condition for a desirability of moving away from the market-based approach can also be written as

$$\mathcal{W}_{\tau_1}^{net} - \mathcal{W}_{\tau_2}^{net} \left(\frac{\mathcal{E}_{\tau_1}}{\mathcal{E}_{\tau_2}} \right) \quad > \quad 0 \; .$$

The assumptions of i) distributive indifference and ii) no distortions of labour supply imply that $\mathcal{W}_\tau^{net} = 0$, so that no such gains exist. It is logically possible that

$$\mathcal{W}_{\tau_1}^{net} - \mathcal{W}_{\tau_2}^{net} \left(\frac{\mathcal{E}_{\tau_1}}{\mathcal{E}_{\tau_2}} \right)$$

is (close to) zero even if $\mathcal{W}_{\tau_1}^{net} \neq 0$ and $\mathcal{W}_{\tau_2}^{net} \neq 0$. Whether or not this is the case can only be answered by bringing the theory to data.

5 Concluding remarks

This paper has shown that climate policy is confronted with an equity-efficiency trade-off. A uniform price on carbon is efficient in the sense that it allows to reach national emission targets at minimal costs. At the same time, deviations to a sector-specific climate policy can be justified by distributive concerns. While such a deviation has an efficiency cost it may improve social welfare.

A market-based approach to climate policy has advantages of simplicity and accountability. Those are not captured by the welfare analysis that is presented in this paper. With a uniform price for carbon there is a clear mapping between one policy instrument and one policy goal. It is then very clear what need to be done when emission targets are missed. The price for carbon needs to go up.

Still, as we have argued in this paper, the distributive implications of such an approach may be perceived as unfair. Possibly this is an explanation for the lack of political support and the protests that are spurred by plans for more ambitious climate policies. Reaching emission targets in a politically feasible way may therefore require a sector-specific approach. A more systematic analysis of the conditions under which climate policy can attract majority support is a topic fur future research.

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A Proofs

Proof of Lemma 2

The first order condition of the outer problem can be rewritten as

$$(e_0 - a(r))\alpha f_j'(l^*) \quad = \quad \frac{p_j\, \alpha f'(l^*) - p_w}{t_{je}} \,,$$

or, using that $f_j(l) = \frac{1}{1-\frac{1}{\sigma_j}} l^{1-\frac{1}{\sigma_j}}$,

$$(e_0 - a(r))\alpha f_j(l^*)\frac{1-\frac{1}{\sigma_j}}{l^*} \quad = \quad \frac{p_j\, \alpha f_j(l^*)\frac{1-\frac{1}{\sigma_j}}{l^*} - p_w}{t_{je}} \,.$$

Hence

$$\mathbf{e}^* \quad = \quad \frac{p_j\, \alpha f(l^*) - p_w l^* \left(1-\frac{1}{\sigma_j}\right)^{-1}}{t_{je}}$$

Note that $\mathbf{e}^*$ is increasing in l^*: The derivative of the right hand side with respect to l^* equals

$$\frac{1}{t_{je}}\left(p_j\alpha f_j'(l^*) - p_w\left(1-\frac{1}{\sigma_j}\right)^{-1}\right) > \frac{1}{t_{je}}\left(p_j\alpha f_j'(l^*) - p_w\right) > 0$$

Thus, emissions go up if (i) p_j increases, (ii) p_w decreases, or (iii) t_{je} decreases. The direct effect and the effect via l^* have the same sign. All other parameters affect emissions only via l^*. In any case, the effect on l^* has the same sign as the effect on $\mathbf{e}^*$. □

Proof of Lemma 3

Inserting $\mathcal{R}(p, p_w, \mathcal{T})$ for $\mathcal{R}^E$ in the right hand side of equation (7) turns this equation into a fixed point equation that can also be written as

$$G(\rho) := \rho \; ,$$

where

$$\begin{aligned} G(\rho) \;\; &= \;\; t_c \, p_c \, X_c(\Pi^E, \rho, q, p_w, T_l) \\ &+ t_g \, p_g \, X_g(\Pi^E, \rho, q, p_w, T_l) \\ &+ t_b \, p_b \, X_b(\Pi^E, \rho, q, p_w, T_l) \\ &+ \mathbf{E}_\theta \Big[T_l(p_w \; y_l^*(\theta, \Pi^E, \rho, q, p_w, T_I)) \Big] \\ &+ \textstyle\sum_{j \in \{c,b,g\}} \; t_{je} \; \mathcal{E}_j(p_j, p_w, t_{je}) \; . \end{aligned}$$

<u>*Step 1.*</u> We first show that, for all ρ, $G'(\rho) < 1$. To see this, note that $G(\rho)$ can also be written as a sum of the tax revenue due to individuals and the sum of tax revenue due to firms.

$$G(\rho) = G_I(\rho) + G_F \; ,$$

where

$$G_F \;\; = \;\; \textstyle\sum_{j \in \{c,b,g\}} \; t_{je} \; \mathcal{E}_j(p_j, p_w, t_{je}) \; .$$

does not depend on ρ and

$$\begin{aligned} G_I(\rho) \;\; &:= \;\; t_c \, p_c \, X_c(\Pi^E, \rho, q, p_w, T_l) \\ &+ t_g \, p_g \, X_g(\Pi^E, \rho, q, p_w, T_l) \\ &+ t_b \, p_b \, X_b(\Pi^E, \rho, q, p_w, T_l) \\ &+ \mathbf{E}_\theta \Big[T_l(p_w \; y_l^*(\theta, \Pi^E, \rho, q, p_w, T_I)) \Big] \; . \end{aligned}$$

can also be written as

$$G_I(\rho) \;\; := \;\; \mathbf{E}_\theta \Big[T(\theta, \rho) \Big] \; ,$$

where

$$T(\theta, \rho) \;\; = \;\; T_l(p_w \; y_l^*(\theta, \Pi^E, \rho, q, p_w, T_I)) + \sum_{j=c,g,b} t_j \; p_j \; x_j^*(\beta, c^*(\theta, \Pi^E, \rho, \cdot), q)$$

are the tax payments of a type θ-individual, interpreted as a function of exogenous tax revenue ρ, holding fixed the economy's price and tax system. From the individual's budget constraint we have that

$$C(\rho, \theta) + T(\theta, \rho) = \mathcal{I}(\theta, \rho) + \rho \ , \tag{16}$$

where

$$\mathcal{I}(\theta, \rho) = p_w \ y_l^*(\theta, \Pi^E, \rho, p_w, q, T_l) - T_l(y_l^*(\theta, \Pi^E, \rho, p_w, q, T_l)) + s \ \Pi(p, p_w, t_e)$$

is the sum of the individual's net labor and "capital income" and

$$\begin{aligned} C(\rho, \theta) \quad &= \quad p_c \ x_c^*(\beta, c^*(\theta, \Pi^E, \rho, \cdot), q_x) \\ &\quad + p_g \ x_g^*(\beta, c^*(\theta, \Pi^E, \rho, \cdot), q_x) \\ &\quad + p_b \ x_b^*(\beta, c^*(\theta, \Pi^E, \rho, \cdot), q_x) \end{aligned}$$

are the individual's net expenditures on consumption goods. Equation (16) implies that

$$T_\rho(\theta, \rho) = 1 - C_\rho(\theta, \rho) + \mathcal{I}_\rho(\theta, \rho) \ .$$

Form the assumption that all goods are normal goods it follows that $C_\rho(\theta, \rho) > 0$. The fact that the marginal utility of disposable income is constant, see Lemma 1, can be shown to imply that y_l^* does not depend on ρ, with the consequence that $\mathcal{I}_\rho(\theta, \rho) = 0$. Thus, we have that

$$T_\rho(\theta, \rho) < 1 \ ,$$

and as a consequnce

$$G'(\rho) \quad = \quad \mathbf{E}_\theta \Big[T_\rho(\theta, \rho) \Big] \quad < \quad 1 \ .$$

Step 2. Under the given assumptions G is a continuous function on a bounded domain $\rho \in [0, \bar{R}]$. By Brouwer's fixed point theorem, there is a solution to the equation $G(\rho) = \rho$.

Step 3. It remains to be shown that this solution is unique. Step 1 implies that $G(\rho) - \rho$ is a decreasing function. Moreover $G(\rho) \geq 0$, with $G(0) = 0$ indicating that taxes are prohibitive so that no tax revenue is collected and $G(0) > 0$ indicating that there is positive tax revenue even if none of that revenue is rebated lump sum and individuals spend only their labor and "capital" incomes. If $G(0) = 0$ then $\rho = 0$ is the unique solution to the fixed point equation. If $G(0) > 0$ there is a unique $\rho > 0$ solving the fixed point equation $G(\rho) = \rho$. □

A.1 Proof of Lemma 4

The labor market clearing condition can be equivalently written as

$$p_w \left[Y_l(\Pi^E, \mathcal{R}^E, q, p_w, T_l) - L(p, p_w, t_e)\right] = 0 \ ,$$

or, as

$$\mathbf{E}_\theta \Big[p_w y_l^*(\cdot)\Big] - p_w \sum_j \mathbf{E}_j \Big[l^*(\cdot)\Big] = 0 \ .$$

Using the individuals' budget constraints and the definition of profits, this can be equivalently written as

$$\begin{aligned}
&\mathbf{E}_\theta \Big[p_c(1+t_c)x_c^*(\cdot) + p_g(1+t_g)x_g^*(\cdot) + p_b(1+t_b)x_b^*(\cdot) + T_l(p_w y_l^*(\cdot)) - s\Pi^E - \mathcal{R}^E\Big] \\
&- \textstyle\sum_j \mathbf{E}_j \Big[p_j \alpha f_j(l^*(\cdot)) - t_{je}\ \mathbf{e}_j^*(\cdot) - \gamma r^*(\cdot) - \pi^j(\cdot)\Big] \\
&= 0 \ .
\end{aligned}$$

This in turn is equivalent to

$$\begin{aligned}
&\mathcal{R} - \mathcal{R}^E \quad + \quad \Pi - \Pi^E \\
&+ p_c \Big[X_c(\Pi^E, \mathcal{R}^E, q, p_w, T_l) + R(p, T_f) - Y_c(p_c, p_w, t_{ce})\Big] \\
&+ p_g \Big[X_g(\Pi^E, \mathcal{R}^E, q, p_w, T_l) - Y_g(p_g, p_c, p_w, t_{ge})\Big] \\
&+ p_b \Big[X_b(\Pi^E, \mathcal{R}^E, q, p_w, T_l) - Y_b(p_b, p_c, p_w, t_{be})\Big] \\
&= 0 \ .
\end{aligned}$$

□

A.2 Proof of Proposition 1

Choice of the numeraire. By Lemma 4 labor market clearing is implied when the three goods markets clear. Thus, to prove existence and uniqueness we can focus without loss of generality on the goods markets. There are four prices that enter the three goods-market clearing conditions. Henceforth, and without loss of generality, we set the wage rate p_w equal to 1.

A.2.1 Firm Behavior and its implications for supply and demand

As explained above, the profit-maximization problem of a firm in sector j can be decomposed in an inner and an outer problem. The inner problem is to choose an investment into emission avoidance r for a given level of employment l and hence a given level of output. The outer problem then is to choose the level of labor demand. We recall this decomposition here as it is useful to determine how the firms' supply and its demand of the unspecific consumption good depend on the economy's price system.

The inner problem. Given l, choose r to maximize

$$-t_{je}\,(e_{j0} - a_j(r))\alpha f_j(l) - p_c\,\gamma\,r$$

The solution to this problem is denoted by $r^*(l, p_c, t_{je}, \gamma)$. It is straightforward to verify that r^* is increasing in l and t_{je} and decreasing in p_c and γ.

The outer problem. The outer problem is to choose l to maximize

$$p_j\;\alpha\;f_j(l) - t_{je}(e_{j0} - a_j(r^*(l, \cdot)))\alpha f_j(l) - p_c\;\gamma\;r^*(l, \cdot)\;.$$

We denote the solution to this problem by $l^*(p_j, p_c, t_{je}, \gamma, \alpha)$. It is straightforward to verify that l^* is increasing in p_j. The complementarity of the investment and the labour choice implies, moreover, that l^* is decreasing in p_c and γ, as r^* is decreasing in these variables.

Implications. (i) Holding fixed p_c, the supply in the green sector increases in p_g and the supply of the brown sector increases in p_b. (ii) Every sector's demand of the unspecific consumption good for investment purposes decreases in p_c. (iii) The net supply of the unspecific consumption good (= supply minus own demand for investment purposes) is increasing in p_c.

A.2.2 Consumption demand with Coub Douglas and CES preferences

The inner problem. With Cobb Douglas and CES preferences, the inner problem can be written as: Choose z_c, z_g and z_b to maximize

$$z_c^{1-\nu}\left(\beta z_g^{1-\varepsilon_\chi} + z_b^{1-\varepsilon_\chi}\right)^{\frac{\nu}{1-\varepsilon_\chi}}$$

subject to

$$q_c\;z_c + q_g\;z_g + q_b\;z_b = 1\;.$$

The solution is

$$z_c^*(q_c) = \frac{1-\nu}{q_c}\;,\quad z_g^*(q_g, q_b) = \frac{\nu}{q_g}\;\alpha_g(\beta, q_b, q_g)\quad\text{and}\quad z_b^*(q_g, q_b) = \frac{\nu}{q_b}\;\alpha_b(\beta, q_b, q_g)\;,$$

where

$$\alpha_g\left(\beta, \frac{q_b}{q_g}\right) := \frac{\beta^{\frac{1}{\varepsilon_\chi}}\left(\frac{q_b}{q_g}\right)^{\frac{1-\varepsilon_\chi}{\varepsilon_\chi}}}{1+\beta^{\frac{1}{\varepsilon_\chi}}\left(\frac{q_b}{q_g}\right)^{\frac{1-\varepsilon_\chi}{\varepsilon_\chi}}}\quad\text{and}\quad \alpha_b\left(\beta, \frac{q_b}{q_g}\right) := 1 - \alpha_g\left(\beta, \frac{q_b}{q_g}\right)\;.$$

Thus, a fraction $1-\nu$ of disposable income is spent on the unspecific consumption good, a fraction $\nu\;\alpha_g\left(\beta, \frac{q_b}{q_g}\right)$ is spent on the green consumption good and a fraction $\nu\;\alpha_g\left(\beta, \frac{q_b}{q_g}\right)$ is spent on the brown consumption good.

The indirect utility $\tilde{v}(\beta, q_x)$ associated with a solution to the inner problem is obtained by inserting z_c^*, z_g^* and z_b^* into the objective of the inner problem. It follows from Roy's identity that $\tilde{v}(\beta, q_x)$ is decreasing in q_c, q_g and q_b. Using L'Hospital's rule one can show, moreover, that

$$\lim_{\nu \to 0} \tilde{v}(\beta, q_x) = \frac{1}{q_c} \; .$$

Thus, when the budget shares of the green and the brown good become small, then indirect utility no longer depends on the prices of these goods.

The outer problem. The problem is to choose c and y to maximize

$$c \; \tilde{v}(\beta, q_x) - k(y_l, \omega) \quad \text{s.t.} \quad c = y_l - T_l(y_l) + s\Pi^E + \mathcal{R}^E \; .$$

Note that the utility maximizing earnings level does neither depend on Π^E nor on $\mathcal{R}^E$. Thus, we can write $y_l^*(\tilde{v}(\beta, q_x), \omega, T_l)$ and

$$c^*(\tilde{v}(\beta, q_x), \omega, T_l, s\Pi^E + \mathcal{R}^E) = n_l(y_l^*(\tilde{v}(\beta, q_x), \omega, T_l) + s\Pi^E + \mathcal{R}^E,$$

where we refer to $n_l(y_l) := y_l - T_l(y_l)$ as the net labor income function. Note that c^* is a decreasing function of $\tilde{v}$ and hence a decreasing function of q_c, q_g and q_b. Also note that c^* depends continuously on the marginal utility of consumption. Therefore

$$\lim_{\nu \to 0} c^*(\tilde{v}(\beta, q_x), \omega, T_l, s\Pi^E + \mathcal{R}^E) = c^* \left(\frac{1}{q_c}, \omega, T_l, s\Pi^E + \mathcal{R}^E \right) \; ,$$

so that labour supply and disposable income – i.e. the solutions to the outer problem – do not depend on the prices of the green and the brown consumption good.

Consumption demand. Individual demand for the unspecific consumption good is given by

$$x_c^*(\cdot) = \frac{1-\nu}{q_c} \; c^*(\cdot) \; .$$

Individual demand for the green good is given by

$$x_g^*(\cdot) = \frac{\nu}{q_g} \; \alpha_g(\beta, q_b, q_g) \; c^*(\cdot) \; ,$$

and individual demand for the brown good is given by

$$x_b^*(\cdot) = \frac{\nu}{q_b} \; \alpha_b(\beta, q_b, q_g) \; c^*(\cdot) \; .$$

Aggregate demand can therefore be written as

$$X_c = \frac{1-\nu}{q_c} \; \mathbf{E}_\theta \left[c^*(\cdot) \right] \; ,$$

$$X_g = \frac{\nu}{q_g} \; \mathbf{E}_\theta \left[\alpha_g(\beta, q_b, q_g) \; c^*(\cdot) \right]$$

and

$$X_b = \frac{\nu}{q_b} \; \mathbf{E}_\theta \left[\alpha_b(\beta, q_b, q_g) \; c^*(\cdot) \right]$$

A.2.3 Excess demand functions

Let $\tilde{v}(\beta, t, p)$ be a shorthand for $\tilde{v}(\beta, (1+t_c)p_c, (1+t_g)p_g, (1+t_b)p_b)$. We also introduce the shorthands

$$A_g(p_c, p_g, p_b) := \mathbf{E}\left[\alpha_g\left(\beta, \frac{(1+t_b)p_b}{(1+t_g)p_g}\right)\ c^*(\tilde{v}(\beta, t, p), \cdot)\right] ,$$

$$A_b(p_c, p_g, p_b) := \mathbf{E}\left[\alpha_b\left(\beta, \frac{(1+t_b)p_b}{(1+t_g)p_g}\right)\ c^*(\tilde{v}(\beta, t, p), \cdot)\right] ,$$

and

$$A_c(p_c, p_g, p_b) := \mathbf{E}\left[\ c^*(\tilde{v}(\beta, t, p), \cdot)\right] .$$

Armed with this notation, we define the excess demand functions

$$Z_g(p_c, p_g, p_b) := \frac{\nu}{p_g} - \frac{(1+t_g)Y_g(p_g, p_c, t_{ge})}{A_g(p_c, p_g, p_b)} ,$$

$$Z_g(p_c, p_g, p_b) := \frac{\nu}{p_b} - \frac{(1+t_b)Y_b(p_g, p_c, t_{be})}{A_b(p_c, p_g, p_b)} ,$$

and

$$Z_c(p_c, p_g, p_b) := \frac{1-\nu}{p_c} - \frac{(1+t_c)Y_c^{net}(p_c, p_g, p_b, t_e)}{A_c(p_c, p_g, p_b)} ,$$

where Y_c^{net} is the production sector's net supply of the unspecific consumption goods.

A.2.4 Existence and uniqueness

Existence and uniqueness follow from the observations below.

Observation 1. Given p_c and p_b, the excess demand function Z_g is decreasing in p_g and there is a unique value of p_g so that $Z_g(p_c, p_g, p_b) = 0$.

Observation 2. Given p_c and p_g, the excess demand function Z_b is decreasing in p_b and there is a unique value of p_b so that $Z_b(p_c, p_g, p_b) = 0$.

Observation 3. If ν is sufficiently small, then the sign of

$$\frac{d}{dp_j}\left[\alpha_b\left(\beta, \frac{(1+t_b)p_b}{(1+t_g)p_g}\right)\ c^*(\tilde{v}(\beta, (1+t_c)p_c, (1+t_g)p_g, (1+t_b)p_b), \cdot)\right]$$

is equal to the sign of

$$\frac{d}{dp_j}\alpha_b\left(\beta, \frac{(1+t_b)p_b}{(1+t_g)p_g}\right) ,$$

for $p_j \in \{p_g, p_b\}$.

Observation 4. If ν is sufficiently small, then the sign of

$$\frac{d}{dp_j}\left[\alpha_g(\beta,(1+t_b)p_b,(1+t_g)p_g)\; c^*(\tilde{v}(\beta,(1+t_c)p_c,(1+t_g)p_g,(1+t_b)p_b),\cdot)\right]$$

is equal to the sign of

$$\frac{d}{dp_j}\alpha_g(\beta,(1+t_b)p_b,(1+t_g)p_g)\ ,$$

for $p_j \in \{p_g, p_b\}$.

Observation 5. (Gross Substitutes.) If ν is sufficiently small, then Z_g is increasing in p_b and Z_b is increasing in p_g.

Observation 6. (Single crossing condition.) Fix p_c. Consider a p_b-p_g diagram. An iso-excess demand function for good $j = g, b$ has the slope

$$\left(\frac{dp_g}{dp_b}\right)_{dZ_j=0} = -\frac{Z_{j,p_b}}{Z_{j,p_g}}$$

If ν is sufficiently small, these iso-excess demand functions are upward sloping. Now suppose that any point in this diagram, Z^b is steeper than Z^g, which holds provided that

$$Z_{g,p_g} Z_{b,p_b} > Z_{g,p_b} Z_{b,p_g} \tag{17}$$

then a move toward higher prices for the brown good along the iso-excess demand curve for the green good, implies that the excess demand for the brown good goes down. Condition (17) can be shown to hold for ν small enough.

Observation 7. If condition (17) holds, then, for every p_c, there exist prices $p_g(p_c)$ and $p_b(p_c)$ so that

$$Z_g(p_c, p_g(p_c), p_b(p_c)) = 0 \quad \text{and} \quad Z_b(p_c, p_g(p_c), p_b(p_c)) = 0\ .$$

To see this: By observation 1, fix p_b at an arbitrary level and solve for the price p_g that clears the market for the green good. If at this pair of prices there is positive/ negative excess demand for the brown good, move along the iso-excess demand curve for the green good towards higher/ lower prices p_b. Eventually the excess demand for the brown good will fall/ rise to zero. This follows from the functional forms above.

Observation 8. For all p_g and p_b, the excess demand function is strictly decreasing in p_c and there is a price p_c so that $Z_c(p_c, p_g, p_b) = 0$. If the conditions detailed in Observation 7 are satisfied, one can vary p_c to clear the market for the unspecific consumption good, while keeping the markets for the green and the brown good in equilibrium. □

A.3 Proof of Proposition 3

Consider a given utility profile $U_0 : \omega \mapsto U_0(\omega)$, where

$$U_0(\omega) = u_0(\omega) - k(y_{l0}(\omega), \omega) \ , \tag{18}$$

and

$$u_0(\omega) := u(x_{0c}(\omega), \chi(\beta, x_{0g}(\omega), x_{0b}(\omega)))$$

is the consumption utility realized by a type ω-individual in the status quo. In the following we take the function U_0 as given and seek to find that allocation that reaches these utility levels with minimal emissions subject to the economy's resource constraint and the requirement of incentive compatibility.

Incentive compatibility. By standard arguments, incentive compatibility in the status quo holds if and only if

$$U_0'(\omega) = -k_2(y_{l0}(\omega), \omega)$$

and if the function y_{l0} is non-decreasing. Consequently, for $\underline{\omega} = \text{argmin}\Omega$, we have

$$U_0(\omega) = U_0(\underline{\omega}) - \int_{\underline{\omega}}^{\omega} k_2(y_{l0}(n), n) dn$$

and

$$u_0(\omega) = V_0(\omega) - k(y_{l0}(\omega), \omega) \ . \tag{19}$$

Thus, if we take the utility profile $U_0 : \omega \mapsto U_0(\omega)$ and hence also the derivative of this function $U_0' : \omega \mapsto U_0'(\omega)$ as given, then due to (18), we also take the profile $y_{l0} : \omega \mapsto y_{l0}(\omega)$ as given. With the functions U_0 and y_{l0} given, also the consumption utility profile $u_0 : \omega \mapsto u_0(\omega)$ is given.

Thus, for the problem to reach the status quo utilities with minimal emissions, we can as well assume that $y_{l0} : \omega \mapsto y_{l0}(\omega)$, and hence aggregate labor supply $Y_0 E_\omega[y_0(\omega)]$, as well as $u_0 : \omega \mapsto u_0(\omega)$ are predetermined. A solution to this problem that respects these constraints will be incentive compatible by construction.

The optimization problem. What has to be chosen are consumption levels $x_c : \omega \mapsto x_c(\omega)$, $x_b : \omega \mapsto x_b(\omega)$ and $x_g : \omega \mapsto x_g(\omega)$ for households that differ in productivity. In addition, for every sector $j \in \{c, b, g\}$ and every type of firm $\theta_j = (\alpha_j, \gamma_j)$ in that sector labor inputs and resources devoted to the abatement of emissions need to be chosen. Formally, the functions $l_j : \theta_j \mapsto l_j(\theta_j)$ and $r_j : \theta_j \mapsto r_j(\theta_j)$ need to be chosen.

The objective is to minimize

$$\sum_{j \in \{c,b,g\}} \mathcal{E}_j \quad = \quad \sum_{j \in \{c,b,g\}} \mathbf{E}_j \left[\Big(e_{0j} - a_j(r_j(\theta_j) \Big) \alpha_j f_j(l_j(\theta_j)) \right]$$

subject to the following constraints: First, the chosen allocation needs to generate the same consumption utility as the status quo allocation. Formally, for all ω,

$$u(x_c(\omega), \chi(\beta x_g(\omega), x_b(\omega))) = u_0(\omega) \ . \tag{20}$$

Second, the labour used up in the production process is bounded from above by Y_0, the amount that households make available in the status quo,

$$\sum_{j\in\{c,b,g\}} \mathbf{E}_j \left[l_j(\theta_j)\right] \quad \leq \quad Y_0 \ . \tag{21}$$

Third, aggregate consumption is bounded by the production sector's (net) output of the various goods. For the unspecific consumption good this requires that

$$\mathbf{E}_\omega[x_c(\omega)] \quad \leq \quad \mathbf{E}_c[\alpha_c f_c(l_c(\theta_c))] - \sum_{j\in\{c,b,g\}} \mathbf{E}_j[\gamma_j r_j(\theta_j)] \ . \tag{22}$$

For the green and the brown good the constraints are, respectively,

$$\mathbf{E}_\omega[x_g(\omega)] \quad \leq \quad \mathbf{E}_g[\alpha_g f_g(l_g(\theta_g))] \quad \text{and} \quad \mathbf{E}_\omega[x_b(\omega)] \quad \leq \quad \mathbf{E}_b[\alpha_b f_b(l_b(\theta_b))] \ . \tag{23}$$

Solving the problem. Consider the Lagrangean

$$\begin{aligned}
\mathcal{L} \ := \ & \textstyle\sum_{j\in\{c,b,g\}} \mathbf{E}_j \left[\Big(e_{0j} - a_j(r_j(\theta_j)\Big)\alpha_j f_j(l_j(\theta_j))\right] \\
& -\mathbf{E}_\omega \left[\mu(\omega)\Big(u_0(\omega) - u(x_c(\omega), \chi(\beta, x_g(\omega), x_b(\omega)))\Big)\right] \\
& -\lambda_l \Big(\textstyle\sum_{j\in\{c,b,g\}} \mathbf{E}_j \left[l_j(\theta_j)\right] \Big) \\
& -\lambda_c \Big(\mathbf{E}_\omega[x_c(\omega)] + \textstyle\sum_{j\in\{c,b,g\}} \mathbf{E}_j[\gamma_j r_j(\theta_j)] - \mathbf{E}_c[\alpha_c f_c(l_c(\theta_c))]\Big) \\
& -\lambda_g \Big(\mathbf{E}_\omega[x_g(\omega)] - \mathbf{E}_g[\alpha_g f_g(l_g(\theta_g))]\Big) \\
& -\lambda_b \Big(\mathbf{E}_\omega[x_b(\omega)] - \mathbf{E}_b[\alpha_b f_b(l_b(\theta_b))]\Big)
\end{aligned}$$

where $\mu(\omega) := \frac{\nu(\omega)}{\phi(\omega)}$ whith $\nu(\omega)$ the Lagrangean multiplier for the constraint in (20) and ϕ the density associated with the distribution of ω, and λ_l, λ_c, λ_g and λ_b are the multipliers associated with the resource constraints.

Proof of Lemma 5

Henceforth we denote the consumers' equilibrium prices by

$$q^*(\mathcal{T}) = (q_c^*(\mathcal{T}), q_g^*(\mathcal{T}), q_b^*(\mathcal{T}))$$

and the producers' equilibrium prices by

$$p^*(\mathcal{T}) = (p_c^*(\mathcal{T}), p_g^*(\mathcal{T}), p_b^*(\mathcal{T})) ,$$

where the tax system $\mathcal{T}$ consists of the non-linear income tax schedule T_l, the emission taxes $t_e = (t_{ce}, t_{ge}, t_{be})$ and the consumption taxes $t_x = (t_c, t_g, t_b)$.

Lemma 6 *Consider a generic tax rate $\tau \in \{t_c, t_g, t_b, t_{ce}, t_{ge}, t_{be}\}$. The marginal effect of a change of τ on type θ's indirect utility is given by*

$$V_\tau(\theta, \cdot) := \tilde{v}(\beta, q_x) \left(s\,\Pi_\tau + \mathcal{R}_\tau - \left(\frac{dq_c^*}{d\tau}\, x_c^*(\cdot) + \frac{dq_g^*}{d\tau}\, x_g^*(\cdot) + \frac{dq_b^*}{d\tau}\, x_b^*(\cdot) \right) \right)$$

where Π_τ and $\mathcal{R}_\tau$ are, respectively, total differentials of equilibrium profits and equilibrium tax revenue.

Proof. The utility realized by a generic type θ individual in a competitive equilibrium given tax policy $\mathcal{T}$ is given by

$$V(\theta, q^*(\mathcal{T}), T_l) := c^*(\cdot)\tilde{v}(\beta, q_x) - k(y_l^*(\cdot), \omega) \text{ where } c^* = y_l^* - T(y_l^*) + s\Pi + R ,$$

Note that y_l^* is a function of $\tilde{v}$ via the outer problem. The terms involving changes of labor earnings cancel, however, by the first order condition of the outer problem. By Roy's identity, the marginal effect of a change of τ on individual welfare equals

$$\tilde{v}(\beta, q_x) \left(s\,\Pi_\tau + \mathcal{R}_\tau - \left(\frac{dq_c^*}{d\tau}\, x_c^*(\cdot) + \frac{dq_g^*}{d\tau}\, x_g^*(\cdot) + \frac{dq_b^*}{d\tau}\, x_b^*(\cdot) \right) \right)$$

□

With a generic social welfare function, the corresponding change in social welfare is given by

$$W_\tau = \mathbf{E}_\theta \left[\mathbf{g}(\tilde{v}(\beta, q_x), \theta) \left(s\,\Pi_\tau + \mathcal{R}_\tau - \left(\frac{dq_c^*}{d\tau}\, x_c^*(\cdot) + \frac{dq_g^*}{d\tau}\, x_g^*(\cdot) + \frac{dq_b^*}{d\tau}\, x_b^*(\cdot) \right) \right) \right]$$

Note that the revenue effect $\mathcal{R}_\tau$ is weighted by $\bar{\mathbf{g}} := \mathbf{E}[\mathbf{g}(\tilde{v}(\cdot), \beta, \omega, s)]$ in the welfare function. Analogously, the effect on equilibrium profits in sector j is weighted by

$$\bar{\mathbf{g}}_{\Pi^j} := \mathbf{E}[\mathbf{g}(\tilde{v}(\cdot), \beta, \omega, s)\ \tilde{v}(\beta, q_x)\ s_j] .$$

Below we provide a more detailed characterization of these effects. First, we note, however, that

$$-\mathbf{E}_\theta \left[\mathbf{g}(\tilde{v}(\beta, q_x), \theta) \left(\frac{dq_c^*}{d\tau}\, x_c^*(\cdot) + \frac{dq_g^*}{d\tau}\, x_g^*(\cdot) + \frac{dq_b^*}{d\tau}\, x_b^*(\cdot) \right) \right]$$

can be rewritten as

$$-\mathbf{E}\Big[\mathbf{g}(\tilde{v}(\beta,q_x),\theta)\Big(\tfrac{dq_c^*}{d\tau}\ \ (x_c^*(\cdot)-X_c^*(\cdot))+\tfrac{dq_g^*}{d\tau}\ \ \big(x_g^*(\cdot)-X_g^*(\cdot)\big)+\tfrac{dq_b^*}{d\tau}\ \ (x_b^*(\cdot)-X_b^*(\cdot))\Big)\Big]$$

$$-\bar{\mathbf{g}}\ \tfrac{dq_c^*}{d\tau}\ X_c^*(\cdot)\ -\bar{\mathbf{g}}\ \tfrac{dq_g^*}{d\tau}\ X_g^*(\cdot)-\bar{\mathbf{g}}\ \tfrac{dq_b^*}{d\tau}\ X_b^*(\cdot)\ .$$

$$=-\tfrac{dq_c^*}{d\tau}\ \mathrm{Cov}(\mathbf{g}(\tilde{v}(\cdot),\theta),x_c^*(\cdot))-\bar{\mathbf{g}}\ \tfrac{dq_c^*}{d\tau}\ X_c^*(\cdot) \tag{24}$$

$$-\tfrac{dq_g^*}{d\tau}\ \mathrm{Cov}(\mathbf{g}(\tilde{v}(\cdot),\theta),x_g^*(\cdot))-\bar{\mathbf{g}}\ \tfrac{dq_g^*}{d\tau}\ X_g^*(\cdot)$$

$$-\tfrac{dq_b^*}{d\tau}\ \mathrm{Cov}(\mathbf{g}(\tilde{v}(\cdot),\theta),x_b^*(\cdot))-\bar{\mathbf{g}}\ \tfrac{dq_b^*}{d\tau}\ X_b^*(\cdot)\ .$$

Characterizing $\mathcal{R}_\tau$. Revenue can be written as

$$\begin{aligned}\mathcal{R}(\cdot) \;=\;& (q_c^*(\mathcal{T})-p_c^*(\mathcal{T}))X_c^*(\Pi(\cdot),\mathcal{R}(\cdot),q^*(\mathcal{T}))\\ &+(q_g^*(\mathcal{T})-p_g^*(\mathcal{T}))X_g^*(\Pi(\cdot),\mathcal{R}(\cdot),q^*(\mathcal{T}))\\ &+(q_b^*(\mathcal{T})-p_b^*(\mathcal{T}))X_b^*(\Pi(\cdot),\mathcal{R}(\cdot),q^*(\mathcal{T}))\\ &+\mathbf{E}[T_l(y_l^*(\cdot))]\\ &+\sum_{j=c,b,g} t_{je}\,\mathcal{E}_j^*(\cdot)\end{aligned}$$

For $\tau\in\{t_c,t_g,t_b\}$ we have

$$\begin{aligned}\mathcal{R}_\tau \;=\;& \left(\tfrac{dq_c^*(\mathcal{T})}{d\tau}-\tfrac{dp_c^*(\mathcal{T})}{d\tau}\right)X_c^*(\cdot)+(q_c^*(\mathcal{T})-p_c^*(\mathcal{T}))X_{c\tau}^*\\ &+\left(\tfrac{dq_g^*(\mathcal{T})}{d\tau}-\tfrac{dp_g^*(\mathcal{T})}{d\tau}\right)X_g^*(\cdot)+(q_g^*(\mathcal{T})-p_g^*(\mathcal{T}))X_{g\tau}^*\\ &\left(\tfrac{dq_b^*(\mathcal{T})}{d\tau}-\tfrac{dp_b^*(\mathcal{T})}{d\tau}\right)X_b^*(\cdot)+(q_b^*(\mathcal{T})-p_b^*(\mathcal{T}))X_{b\tau}^*\\ &+\mathbf{E}[T_l'(y_l^*(\cdot,\theta))y_{l\tau}^*(\cdot,\theta)]\\ &+\sum_{j=c,b,g} t_{je}\,\mathcal{E}_{j\tau}\end{aligned}$$

where $X_{c\tau}^*$, $X_{g\tau}^*$ and $X_{b\tau}^*$ are total differentials of equilibrium quantities, $y_{l\tau}^*(\cdot,\theta)$ is the total differential of equilibrium labor supply for an individual of type θ and $\mathcal{E}_{j\tau}$ is the total differential of equilibrium emissions in sector j. For $\tau\in\{t_{ce},t_{ge},t_{be}\}$, there is an additional term $\mathbf{I}(\tau=\tau_{je})\mathcal{E}_j^*(\cdot)$ in this expression. Hence, we can summarize: For $\tau\in\{t_c,t_g,t_b,t_{ce},t_{ge},t_{be}\}$, we have

$$
\begin{aligned}
\mathcal{R}_\tau \;=\; & \left(\tfrac{dq_c^*(\mathcal{T})}{d\tau} - \tfrac{dp_c^*(\mathcal{T})}{d\tau}\right) X_c^*(\cdot) + (q_c^*(\mathcal{T}) - p_c^*(\mathcal{T}))X_{c\tau}^* \\
& + \left(\tfrac{dq_g^*(\mathcal{T})}{d\tau} - \tfrac{dp_g^*(\mathcal{T})}{d\tau}\right) X_g^*(\cdot) + (q_g^*(\mathcal{T}) - p_g^*(\mathcal{T}))X_{g\tau}^* \\
& \left(\tfrac{dq_b^*(\mathcal{T})}{d\tau} - \tfrac{dp_b^*(\mathcal{T})}{d\tau}\right) X_b^*(\cdot) + (q_b^*(\mathcal{T}) - p_b^*(\mathcal{T}))X_{b\tau}^* \\
& + \mathbf{E}[T_l'(y_l^*(\cdot,\theta))y_{l\tau}^*(\cdot,\theta)] \\
& + \sum_{j=c,b,g} t_{je}\,\mathcal{E}_{j\tau} \\
& + \sum_{j=c,b,g} \mathbf{I}(\tau = \tau_{je})\mathcal{E}_j^*(\cdot)\;.
\end{aligned}
\tag{25}
$$

Characterizing $\Pi_{j\tau}$. For $\tau \in \{t_c, t_g, t_b\}$, by the envelope theorem, equilibrium profits are affected only via price changes. Thus, for $j \in \{c, g, b\}$, we have

$$
\Pi_{j,\tau} \;=\; \tfrac{dp_j^*(\mathcal{T})}{d\tau} Y_j^*(\cdot) - \tfrac{dp_c^*(\mathcal{T})}{d\tau}\mathbf{E}_j[\gamma\; r^*(\cdot)]\;. \tag{26}
$$

For $\tau \in \{t_{ce}, t_{ge}, t_{be}\}$, we have

$$
\Pi_{j,\tau} \;=\; \tfrac{dp_j^*(\mathcal{T})}{d\tau} Y_j^*(\cdot) - \tfrac{dp_c^*(\mathcal{T})}{d\tau}\mathbf{E}_j[\gamma\; r^*(\cdot)] - \mathbf{I}(\tau = \tau_{je})\mathcal{E}_j^*(\cdot)\;. \tag{27}
$$

Collecting terms. Upon collecting terms and upon making an obvious use of market-clearing conditions we find that

$$
\begin{aligned}
W_\tau \quad = \quad & - \tfrac{dq_c^*(\mathcal{T})}{d\tau} \operatorname{Cov}(\mathbf{g}(\tilde{v}(\cdot), \theta), x_c^*(\cdot)) \\
& + \tfrac{dp_c^*(\mathcal{T})}{d\tau_j} \Big((\bar{\mathbf{g}}_{\Pi_c} - \bar{\mathbf{g}})(Y_c^* - \mathbf{E}[\gamma \; r_c^*]) - \; (\bar{g}_{\Pi_g} - \bar{g})\mathbf{E}[\gamma \; r_g^*]) - (\bar{g}_{\Pi_b} - \bar{g})\mathbf{E}[\gamma \; r_b^*]) \Big) \\
& + \bar{\mathbf{g}} \; (q_c^*(\mathcal{T}) - p_c^*(\mathcal{T}))X_{c\tau}^*(\cdot) \\
& \\
& - \tfrac{dq_g^*(\mathcal{T})}{d\tau} \operatorname{Cov}(\mathbf{g}(\tilde{v}(\cdot), \theta), x_g^*(\cdot)) \\
& + \tfrac{dp_g^*(\mathcal{T})}{d\tau} \; (\bar{\mathbf{g}}_{\Pi_g} - \bar{\mathbf{g}})X_g^*(\cdot) \\
& + \bar{\mathbf{g}} \; (q_g^*(\mathcal{T}) - p_g^*(\mathcal{T}))X_{g\tau}^*(\cdot) \\
& \\
& - \tfrac{dq_b^*(\mathcal{T})}{d\tau} \operatorname{Cov}(\mathbf{g}(\tilde{v}(\cdot), \theta), x_b^*(\cdot)) \\
& + \tfrac{dp_b^*(\mathcal{T})}{d\tau} \; (\bar{\mathbf{g}}_{\Pi_b} - \bar{\mathbf{g}})X_b^*(\cdot) \\
& + \bar{\mathbf{g}} \; (q_b^*(T) - p_b^*(T))X_{b\tau}^*(\tau) \\
& \\
& + \bar{\mathbf{g}} \; \mathbf{E}_\theta[T_l'(y_l^*(\cdot, \theta))y_{l\tau}^*(\cdot, \theta)] \\
& + \bar{\mathbf{g}} \sum_{j=c,b,g} t_{je}\mathcal{E}_{j\tau}^*(\cdot) \\
& + \bar{\mathbf{g}} \sum_{j=c,b,g} \mathbf{I}(\tau = \tau_{je})\mathcal{E}_j^*(\cdot) \; ,
\end{aligned}
$$

which proves Lemma 5.