Appendix: A Parsimonious Model of Subjective Life Expectancy

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1 Decision-theoretic Foundations

1.1 The Environment

Formally, we consider a probability space $(\mu, \Omega, \mathcal{F})$ where μ denotes a subjective additive probability measure and the event space $\mathcal{F}$ is rich enough to express our assumptions that for any given number n of individuals (i) the agent knows for each individual whether it has survived until age m or not whereas (ii) the agent does not make any direct observations about the true parameter value of the individuals' survival probability. As state space we assume

$$\Omega = \times_{i=1}^{\infty} S_i \times \Pi,$$

with generic element $\omega = (s_1, s_2, ..., \pi_{j,m})$ such that for the sample-space $\times_{i=1}^{\infty} S_i$ holds $S_i = \{0, 1\}$ for $i = 1, 2, ...$ whereby the parameter-space $\Pi = [0, 1]$ collects all possible values of the individuals' "true" survival probability. The event space $\mathcal{F}$ is accordingly defined as follows. Endow each S_i with the discrete metric and denote by $\mathcal{S}_k$ the Borel σ-algebra in $\times_{i=1}^{k} S_i$ and by $\mathcal{S}_\infty$ the σ-algebra generated by $\mathcal{S}_1, \mathcal{S}_2, ...$. Similarly, endow Π with the Euclidean metric and denote by $\mathcal{B}$ the Borel σ-algebra in Π. Our event space $\mathcal{F}$ is then defined as the standard product algebra $\mathcal{S}_\infty \otimes \mathcal{B}$.

Define by $\tilde{\pi}_{j,m} : \Omega \to [0, 1]$ such that $\tilde{\pi}_{j,m}(s_1, s_2, ..., \pi_{j,m}) = \pi_{j,m}$ the $\mathcal{F}$-measurable coordinate random variable that assigns to every state of the world the true probability of surviving from age j to m. As in the main text we will now drop subscripts

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and reintroduce them below. For our purpose it is furthermore convenient to denote by $\boldsymbol{\pi}$ the event in $\mathcal{F}$ such that $\pi \in \Pi$ is the true probability, i.e.,

$$\boldsymbol{\pi} = \{\omega \in \Omega \mid \tilde{\pi}(\omega) = \pi\}.$$

As the agent's prior over $\tilde{\pi}$ is given as a Beta distribution with parameters $\alpha, \beta > 0$ we have

$$\mu(\boldsymbol{\pi}) = K_{\alpha,\beta}\pi^{\alpha-1}(1-\pi)^{\beta-1}$$

where $K_{\alpha,\beta} = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}$ is a normalizing constant whereby the gamma function is defined as $\Gamma(y) = \int\limits_0^\infty x^{y-1}e^{-x}dx$ for $y > 0$.

Let $X_i : \Omega \to \{0,1\}$, with $i = 1, 2, ...$, denote the $\mathcal{S}_i$-measurable coordinate random variable so that

$$X_i(s_1, ..., s_i, ..., \pi) = s_i$$

whereby we interpret $\{\omega \in \Omega \mid X_i(\omega) = 0\}$ as the event in $\mathcal{F}$ that individual i has not survived until age m whereas $\{\omega \in \Omega \mid X_i(\omega) = 1\}$ denotes the complement event that individual i has survived until age m. We assume that, conditional on the parameter-value π, each X_i is Bernoulli distributed with probabilities

$$\mu(\{\omega \in \Omega \mid X_i(\omega) = x\} \mid \boldsymbol{\pi}) = \pi^x(1-\pi)^{1-x} \text{ for } x \in \{0,1\}.$$

Furthermore, denote by $I_n : \Omega \to \{0, ..., n\}$ the $\mathcal{S}_n$-measurable random variable counting the number of surviving individuals in a sample of size n, i.e., $I_n = \sum_{i=1}^n X_i$. Hence, $\mathbf{I}_n^k$, i.e., the event in $\mathcal{F}$ so that k out of n individuals have survived until age m, is given by

$$\mathbf{I}_n^k = \{\omega \in \Omega \mid I_n(\omega) = k\}.$$

Since the X_i are i.i.d. Bernoulli distributed, each I_n is, conditional on the parameter-value π, binomially distributed with probabilities

$$\mu\left(\mathbf{I}_n^k \mid \boldsymbol{\pi}\right) = \binom{n}{k}\pi^k(1-\pi)^{n-k} \text{ for } k \in \{0, ..., n\}.$$

By Bayes' rule we then obtain the following posterior probability that π is the true value conditional on information $\mathbf{I}_n^k$

$$\begin{aligned}
\mu\left(\boldsymbol{\pi} \mid \mathbf{I}_n^k\right) &= \frac{\mu\left(\boldsymbol{\pi} \cap \mathbf{I}_n^k\right)}{\mu\left(\mathbf{I}_n^k\right)} \\
&= \frac{\mu\left(\mathbf{I}_n^k \mid \boldsymbol{\pi}\right)\mu(\boldsymbol{\pi})}{\int_{[0,1]}\mu\left(\mathbf{I}_n^k \mid \boldsymbol{\pi}\right)\mu(\boldsymbol{\pi})\,d\pi} \\
&= K_{\alpha+k,\beta+n-k}\pi^{\alpha+k-1}(1-\pi)^{\beta+n-k-1}.
\end{aligned}$$

Furthermore, observe that the unconditional probability of receiving information $\mathbf{I}_n^k$ is given by

$$\begin{aligned}
\mu\left(\mathbf{I}_n^k\right) &= \frac{\mu\left(\mathbf{I}_n^k \mid \boldsymbol{\pi}\right)\mu\left(\boldsymbol{\pi}\right)}{\mu\left(\boldsymbol{\pi} \mid \mathbf{I}_n^k\right)} \\
&= \frac{\binom{n}{k}\pi^k\left(1-\pi\right)^{n-k}\mu\left(\boldsymbol{\pi}\right)\cdot K_{\alpha,\beta}\pi^{\alpha-1}\left(1-\pi\right)^{\beta-1}}{K_{\alpha+k,\beta+n-k}\pi^{\alpha+k-1}\left(1-\pi\right)^{\beta+n-k-1}} \\
&= \binom{n}{k}\frac{K_{\alpha,\beta}}{K_{\alpha+k,\beta+n-k}} \\
&= \binom{n}{k}\frac{\Gamma\left(\alpha+\beta\right)}{\Gamma\left(\alpha\right)\Gamma\left(\beta\right)}\frac{\Gamma\left(\alpha+k\right)\Gamma\left(\beta+n-k\right)}{\Gamma\left(\alpha+\beta+n\right)} \\
&= \binom{n}{k}\frac{\Gamma\left(\alpha+\beta\right)}{\Gamma\left(\alpha+\beta+n\right)}\cdot\frac{\Gamma\left(\alpha+k\right)}{\Gamma\left(\alpha\right)}\cdot\frac{\Gamma\left(\beta+n-k\right)}{\Gamma\left(\beta\right)} \\
&= \binom{n}{k}\frac{\Gamma\left(\alpha+\beta\right)}{\left(\alpha+\beta+n-1\right)\cdot\ldots\left(\alpha+\beta\right)\cdot\Gamma\left(\alpha+\beta\right)} \\
&\quad\cdot\frac{\Gamma\left(\alpha+k-1\right)\cdot\ldots\cdot\alpha\cdot\Gamma\left(\alpha\right)}{\Gamma\left(\alpha\right)} \\
&\quad\cdot\frac{\Gamma\left(\beta+n-k-1\right)\cdot\ldots\cdot\beta\cdot\Gamma\left(\beta\right)}{\Gamma\left(\beta\right)} \\
&= \binom{n}{k}\frac{\left(\alpha+k-1\right)\cdot\ldots\cdot\alpha\cdot\left(\beta+n-k-1\right)\cdot\ldots\cdot\beta}{\left(\alpha+\beta+n-1\right)\cdot\ldots\cdot\left(\alpha+\beta\right)},
\end{aligned}$$

whereby the last equality readily follows from the fact that $\Gamma\left(x\right)=\left(x-1\right)\cdot\Gamma\left(x-1\right)$ for $x>1$ (Rudin 1976, Theorem 8.18).

1.2 Ambiguous Beliefs

Definition 1. *For a given measurable space $(\Omega,\mathcal{F})$ the neo-additive capacity, ν, is defined, for some $\delta,\lambda\in[0,1]$ by*

$$\nu\left(A\right)=\delta\cdot\left(\lambda\cdot\omega^o\left(A\right)+\left(1-\lambda\right)\cdot\omega^p\left(A\right)\right)+\left(1-\delta\right)\cdot\mu\left(A\right) \tag{1}$$

for all $A\in\mathcal{F}$ whereby μ is some additive probability measure and we have for the non-additive capacities ω^o

$$\begin{aligned}
\omega^o\left(A\right) &= 1 \text{ if } A\neq\emptyset \\
\omega^o\left(A\right) &= 0 \text{ if } A=\emptyset
\end{aligned}$$

and ω^p *respectively*

$$\omega^p(A) = 0 \text{ if } A \neq \Omega$$
$$\omega^p(A) = 1 \text{ if } A = \Omega.$$

For an $\mathcal{F}$-measurable bounded real function f, it can be shown that the Choquet expected value of f with respect to a neo-additive capacity ν is given as

$$E[f,\nu] = \delta \cdot (\lambda \cdot \sup f + (1-\lambda) \cdot \inf f) + (1-\delta) \cdot E[f,\mu],$$

cf. Schmeidler (1986).

In the context of survival expectations, we are interested in the agent's belief about event A according to which she is alive at some target age m. Under the assumption that there is always the possibility to reach age m, the event A cannot be the null event, implying $\omega^o(A) = 1$. On the other hand, we also stipulate that there is always the possibility to die before reaching age m so that A cannot be the universal event either, implying $\omega^p(A) = 0$. As a consequence, the agent's belief to survive until age m in (1) simplifies to

$$\nu(A) = \delta \cdot \lambda + (1-\delta) \cdot \mu(A).$$

According to our interpretation, the additive probability $\mu(A)$ in (1.2) stands in for the agent's "rational" part of her survival beliefs. Under the rational expectations paradigm the subjective additive probability measure μ must, first, coincide with the "true" probability distribution and, second, the agent must not be ambiguous about her subjective belief, i.e., $\delta = 0$. However, we do not only assume that the representative agent is ambiguous about her subjective belief, $\delta \neq 0$, but also that the subjective probability μ may deviate from its objective counterpart.

1.3 Updating of Ambiguous Beliefs

Definition 2. *The generalized Bayesian update rule for determining the conditional capacity* $\nu(A \mid B)$, $B \in \mathcal{F}$, *for a given prior capacity* ν *is given as follows: for all* $A \in \mathcal{F}$,

$$\nu(A \mid B) = \frac{\nu(A \cap B)}{\nu(A \cap B) + 1 - \nu(A \cup \neg B)}.$$

Observation 1. *Let* $\mu(B) > 0$. *An application of the generalized Bayesian update rule to a neo-additive prior results in the posterior belief*

$$\nu(A \mid B) = \delta_B \cdot \lambda + (1-\delta_B) \cdot \mu(A \mid B)$$

whereby

$$\delta_B = \frac{\delta}{\delta + (1-\delta) \cdot \mu(B)}.$$

Proof. Let $A, B \notin \{\emptyset, \Omega\}$ and $A \cap B \neq \varnothing$. Then

$$\begin{aligned}
\nu(A \mid B) &= \frac{\delta \cdot \lambda + (1-\delta) \cdot \mu(A \cap B)}{\delta \cdot \lambda + (1-\delta) \cdot \mu(A \cap B) + 1 - (\delta \cdot \lambda + (1-\delta) \cdot \mu(A \cup \neg B))} \\
&= \frac{\delta \cdot \lambda + (1-\delta) \cdot \mu(A \cap B)}{1 + (1-\delta) \cdot (\mu(A \cap B) - \mu(A \cup \neg B))} \\
&= \frac{\delta \cdot \lambda + (1-\delta) \cdot \mu(A \cap B)}{1 + (1-\delta) \cdot (\mu(A \cap B) - \mu(A) - \mu(\neg B) + \mu(A \cap \neg B))} \\
&= \frac{\delta \cdot \lambda + (1-\delta) \cdot \mu(A \cap B)}{1 + (1-\delta) \cdot (-\mu(\neg B))} \\
&= \frac{\delta \cdot \lambda + (1-\delta) \cdot \mu(A \cap B)}{\delta + (1-\delta) \cdot \mu(B)} \\
&= \delta_B \cdot \lambda + (1-\delta_B) \cdot \mu(A \mid B).
\end{aligned}$$

□

2 Data

According to our model two different types of data are required for the empirical analysis: (i) subjective conditional beliefs to live until target age and (ii) predicted objective conditional probabilities to live from age r to age $r+1$ for all $r = j, \ldots, m-1$. We here describe our data sources and the methodologies we apply to construct these data.

2.1 HRS Data

The HRS is a national representative panel survey of individuals aged 50 and older and their spouses. In addition to respondents from eligible birth years, the survey interviewed the spouses or partners of the respondents, regardless of age. Thus, some (mostly female) individuals are younger than 50 and few, younger than 40. In our application we focus on the target group of the HRS and therefore only look at individuals of age 50 and older. Some respondents of the above question were 90 years old at the time of interview. We do not include these observations in our analysis.

Younger HRS interviewees were also asked about their probabilities to live until age 75. Some of these respondents have given inconsistent answers at certain points of time as their self-reported probabilities to live until 75 are lower than the self-reported probabilities to live until 80 or 85. We excluded these cases of evidently inconsistent answering patterns. Furthermore, in some cases, individuals reported the same probability to live until age 75 as to live until age 80 or 85. As this answering pattern may be due to pure rounding and is not strictly inconsistent with our theoretical model, we keep these observations in the sample.

The presence of focal point answers in our data is discussed in Subsection 4.3 of our paper.

2.2 Cohort Life Tables

We adopt the Lee-Carter procedure (Lee and Carter 1992) to estimate trends in mortality and to project survival rates into the future. The procedure allows us to describe and to project the development of age-specific mortality rates over time within a parsimonious framework. Basically, the model splits mortality rates into age-specific components that are constant over time and a time varying survival index capturing the development of mortality. Then, one can extrapolate the time series of the mortality index by means of a suitable time series model. Future age-specific mortality rates can be recovered by linking the projected mortality index to the age-specific components.

To describe the methodology, we now introduce a time index t. Following Lee and Carter (1992) we decompose the average objective age-specific survival probability in period t as

$$\log(\pi^{\star}_{t,r,r+1}) = a_r + b_r d_t$$

where a_r and b_r are the age-specific constants and where d_t is the time specific factor. We opt for a parsimonious representation of the time series process of d_t and assume that d_t follows a unit root process with drift

$$d_t = \theta + d_{t-1} + \epsilon_t.$$

where $\epsilon \sim \mathcal{N}(0, \sigma^2_{\epsilon})$.

We assign objective survival probabilities to each respondent in our HRS panel in each wave $\tau \in \{2000, 2002, 2004\}$ as follows. We estimate for each wave τ, sex specific values of $\hat{a}_r$, $\hat{b}_r$, $\hat{\theta}$, $\hat{\sigma}_{\epsilon}$ and calculate predicted values of $\hat{\pi}^{\star}_{t,r,r+1}$ using data only until period τ. We then proceed to the next wave and update the objective information also using the data for the two years in between periods τ and $\tau+2$. Our predictions of future objective survival probabilities, $\hat{\pi}^{\star}_{t,r,r+1}$, are calculated by iterating forward on

$$\hat{d}_t = \hat{\theta} + \hat{d}_{t-1}$$

and

$$\hat{\pi}^{\star}_{t,r,r+1} = \exp\left(\hat{a}_r + \hat{b}_r \hat{k}_t\right).$$

While we ignore uncertainty of our estimates of the age-vectors a_r and b_r, we account for uncertainty of the objective data by calculating standard deviations and confidence intervals of $\hat{\theta}$ by bootstrapping. This uncertainty is also reflected in our estimates. Table 1 reports the sex and wave specific point estimates $\hat{\theta}$ and the respective standard deviations. Estimated parameter values for waves 1, 2 and 3 are based on population data from HMD and SSA for $1900 - 2000$, $1900 - 2002$ and $1900 - 2004$, respectively.

Table 1: Parameter estimates of $\hat{\theta}$

	Men		Women	
	$\hat{\theta}$	$\hat{\sigma}(\theta)$	$\hat{\theta}$	$\hat{\sigma}(\theta)$
wave 1	-1.4186	0.5336	-1.8586	0.5339
wave 2	-1.4123	0.5426	-1.8287	0.5336
wave 3	-1.4518	0.4927	-1.8462	0.5103

Notes: Standard errors of $\hat{\theta}$ are calculated from 500 bootstrap iterations.
Source: Own calculations based on SSA and HMD.

Figure 1 shows data on, and predicted values for, the remaining life expectancy at age 65 for wave 2002. The dashed lines are the bootstrapped 95% confidence intervals. The new information on objective survival probabilities between waves only leads to small changes in these predictions. Results for other years are therefore not shown. Furthermore, life expectancy at birth and the remaining life-expectancies at other ages display similar trends whereby the trend is increasing with age.

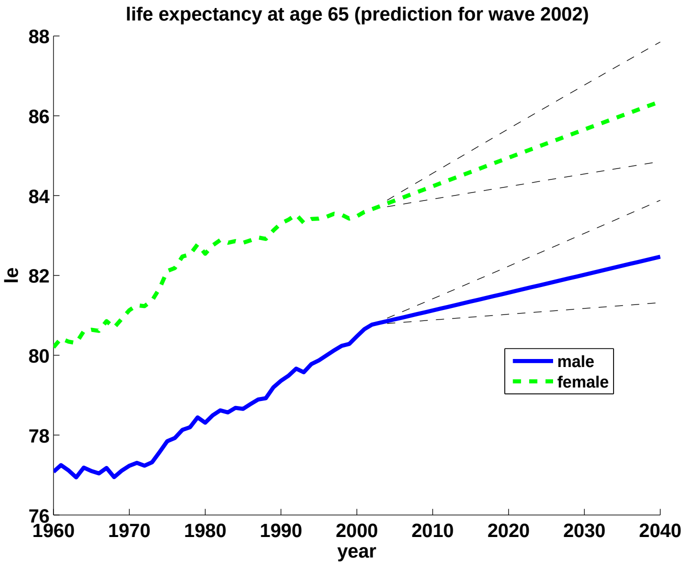
2.3 Focal Point Answers

2.4 Hazard rates

We show hazard rates in Figure 3, for men in panel (a) and for women in panel (b) between waves 2002 and 2004 for our full sample. The wiggles in the HRS data (dashed lines) are a consequence of relatively small sample size. Evidently, the HRS hazard rates correspond with the mortality rates in the population. The pattern is similar for the hazard rates between waves 2000 and 2002 (and also for our sample corrected by focal point answers) and therefore not shown.[1]

[1] If anything, we find for ages above 75 slightly higher mortality rates in the HRS between waves 2000 and 2002 than in the population which gives even more support to our interpretation of the data as "optimism" at higher ages.

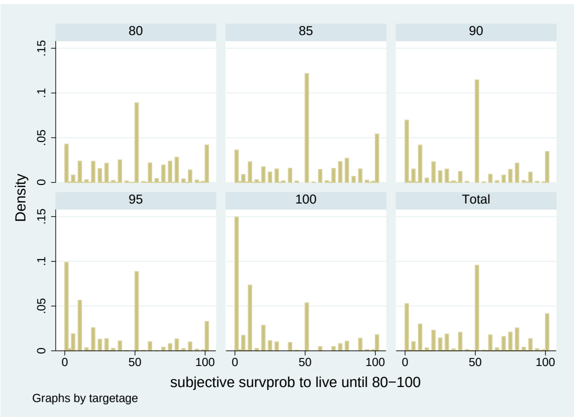
Figure 1: Predicted life expectancy at age 65 in year 2002



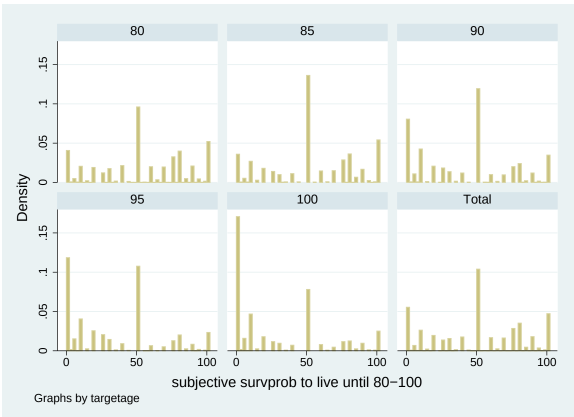
Notes: Thin dashed lines are 95% confidence intervals obtained from 500 bootstrap iterations.
Source: Own calculations based on HMD and SSA data.

Figure 2: Answer pattern

(a) Men



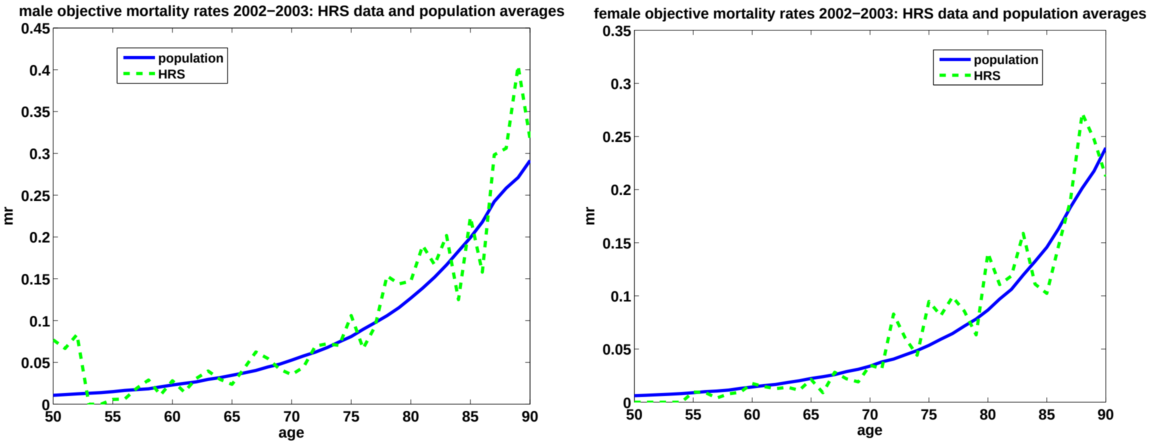
(b) Women



Source: Own calculations based on HRS data.

Figure 3: Objective survival rates in 2002-2003: HRS data versus population averages

(a) Men (b) Women



Notes: Solid line: population wide hazard rates (mortality rates) for 2002-2003. Dashed line: HRS hazard rates (mortality rates) between waves 2002 and 2004.
Source: Own calculations based on HRS, SSA and HMD data.

2.5 Cohort Effects

To accommodate the criticism that cohort effects may drive the patterns in the data, Figure 4 presents the subjective beliefs for various cohorts. As there are no clear-cut gaps between the respective line segments that represent birth cohorts, this stylized evidence can not be regarded as an indication for relevant cohort effects.

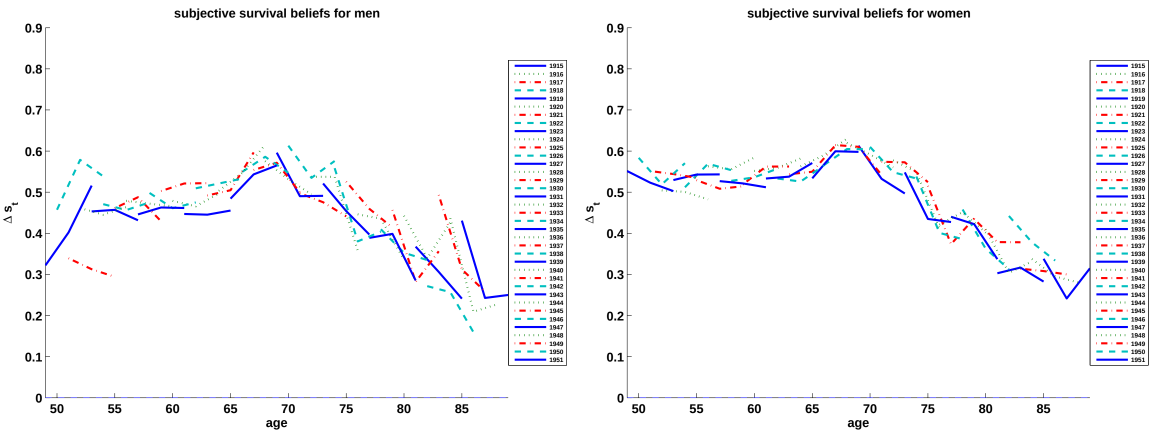
3 Additional Results

3.1 Speed of the Learning Process

We investigate sensitivity of our results with respect to the speed of the Bayesian learning process. That is, we consider a specification in which the initial age is 20 as in our baseline results but the speed of the learning process is now ten times faster in that we assume $n(h) = 10 \cdot h$. Results for this specification, reported in Table 2, indicate that the speed of the learning process interacts with our estimate of the degree of likelihood insensitivity, δ, whereas the other parameters are roughly unaffected. More precisely, we find that, when speed of learning is ten times faster than in our baseline specification, the estimate of parameter δ is about 10 times lower. Again, this mechanically follows from the specification of the learning model. Interpretation of the point estimate of ϕ is affected. $\phi = 0.89$ for men ($\phi = 0.9$

Figure 4: Subjective survival expectations by cohorts

(a) Men (b) Women



Notes: These graphs display subjective beliefs for various birth cohorts.

Source: Own calculations based on HRS.

for women) means that a person with one year of experience at age 20 estimates the additive probability to survive from age 50 to age 80 (for which $m - j = 30$) to be $\frac{2\phi^{m-j}+10\cdot h}{2+10\cdot h} \cdot 100\% = \frac{2\phi^{30}+10}{12} \cdot 100\% = 92.4\%$ (92.2%) of the objective data. Resulting degrees of likelihood insensitivity and the implied curvature parameter of the probability weighting function are shown in Figure 5. These results do not differ much from the baseline specification.

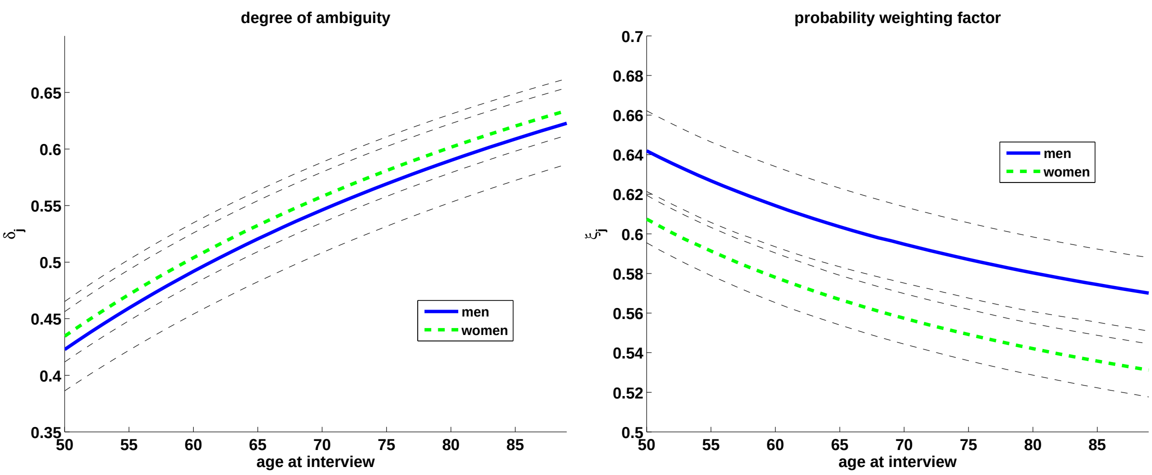
Table 2: Parameter estimates: Higher learning speed

	Men			Women		
	$\hat{\Psi}$	$\hat{\sigma}(\Psi)$	$\widehat{CI}(\Psi)$	$\hat{\Psi}$	$\hat{\sigma}(\Psi)$	$\widehat{CI}(\Psi)$
Initial bias: ϕ	0.893	0.002	[0.890 0.897]	0.901	0.002	[0.899 0.906]
Likelihood insens.: δ	0.002	0.000	[0.002 0.003]	0.002	0.000	[0.002 0.003]
Degree of optimism: λ	0.438	0.011	[0.417 0.459]	0.386	0.011	[0.364 0.409]
R^2	0.039	0.003	[0.032 0.046]	0.062	0.003	[0.056 0.067]
$\bar{R}^2$	0.776	0.037	[0.662 0.812]	0.929	0.011	[0.890 0.931]

Notes: These results are based on a specification of our model with $n(h) = 10h$ rather than $n(h) = h$. $\hat{\Psi}$ are point estimates of model parameters, $\hat{\sigma}(\Psi)$ is the respective standard deviation and $\widehat{CI}(\Psi)$ is the respective 95% confidence interval. Standard errors are calculated by bootstrapping the subjective and objective survival probabilities by drawing with replacement in 500 bootstrap iterations.
Source: Own calculations based on HRS, SSA and HMD data.

Figure 5: Likelihood insensitivity and implied probability weighting factor: Higher learning speed

(a) Likelihood insensitivity (δ_h) (b) Implied probability weighting factor (ξ_h)



Notes: Thin dashed lines are 95% confidence intervals obtained from 500 bootstrap iterations.
Source: Own calculations based on HRS, HMD and SSA data.

3.2 Decreasing Marginal Experience

We build on the earlier specification of the experience function and now combine it with decreasing marginal experience by specifying $n(h) = 10 \cdot \sqrt{h}$. Results for this specification are summarized in Table 3 and Figure 6. Implied estimates of initial ambiguity are $\delta = 0.016$ for both sexes and are thereby closer to our benchmark results. This is so because the overall level of ambiguity for biological ages $[50, \ldots, 90]$ is similar to the other specifications but increases less, see Figure 6. Otherwise, results are not affected much.

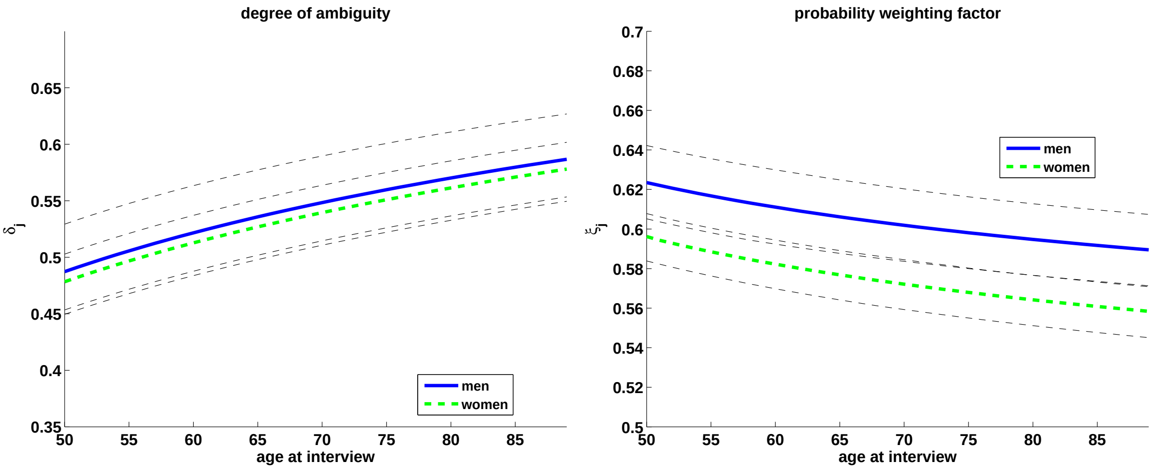
Table 3: Parameter estimates: Decreasing marginal experience

	Men			Women		
	$\hat{\Psi}$	$\hat{\sigma}(\Psi)$	$\widehat{CI}(\Psi)$	$\hat{\Psi}$	$\hat{\sigma}(\Psi)$	$\widehat{CI}(\Psi)$
Initial bias: ϕ	0.918	0.002	[0.915 0.921]	0.922	0.002	[0.919 0.926]
Likelihood insens.: δ	0.016	0.001	[0.014 0.019]	0.016	0.001	[0.014 0.018]
Degree of optimism: λ	0.451	0.011	[0.430 0.470]	0.399	0.012	[0.374 0.423]
R^2	0.041	0.003	[0.035 0.048]	0.064	0.003	[0.058 0.070]
$\bar{R}^2$	0.805	0.035	[0.695 0.835]	0.950	0.010	[0.913 0.950]

Notes: These results are based on a specification of our model with $n(h) = 10\sqrt{h}$ rather than $n(h) = h$. $\hat{\Psi}$ are point estimates of model parameters, $\hat{\sigma}(\Psi)$ is the respective standard deviation and $\widehat{CI}(\Psi)$ is the respective 95% confidence interval. Standard errors are calculated by bootstrapping the subjective and objective survival probabilities by drawing with replacement in 500 bootstrap iterations.
Source: Own calculations based on HRS, SSA and HMD data.

Figure 6: Likelihood insensitivity and implied probability weighting factor: Decreasing marginal experience

(a) Likelihood insensitivity (δ_h) (b) Implied probability weighting factor (ξ_h)



Notes: Thin dashed lines are 95% confidence intervals obtained from 500 bootstrap iterations.
Source: Own calculations based on HRS, HMD and SSA data.

3.3 Hump-Shaped Experience Experience

Finally, we specify a hump-shaped experience function of the form $n(h) = \beta_0 + \beta_1 \cdot h + \beta_2 \cdot h^2$. This specification is a simple reduced form representation of a learning model in which depreciation of memory dominates at some point. We determine parameters $\{\beta_i\}_{i=1}^3$ such that the level of the experience function peaks at age 65 ($h = 46$). We further require the same level as for the square root specification at ages 50 and 65 ($h = 31$ and $h = 46$). This gives the conditions $n(31) = \beta_0 + \beta_1 \cdot 31 + \beta_2 \cdot 31^2 = 10$, $n(46) = \beta_0 + \beta_1 \cdot 46 + \beta_2 \cdot 46^2 = 10 \cdot \sqrt{46}$ and $n'(46) = \beta_1 + \beta_2 \cdot 2 \cdot 46 = 0$. Results are summarized in Table 4 and Figure 7. The hump-shaped experience function translates into a hump-shaped likelihood insensitivity. This also implies a u-shaped curvature parameter of the associated probability weighting function.

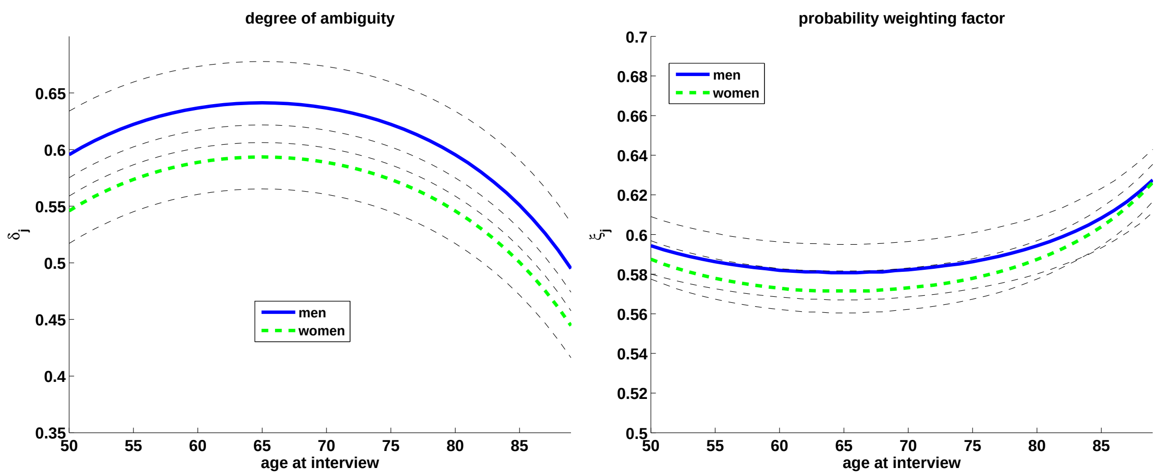
Table 4: Parameter estimates: Hump-shaped experience

	Men			Women		
	$\hat{\Psi}$	$\hat{\sigma}(\Psi)$	$\widehat{CI}(\Psi)$	$\hat{\Psi}$	$\hat{\sigma}(\Psi)$	$\widehat{CI}(\Psi)$
Initial bias: ϕ	0.939	0.002	[0.934 0.943]	0.935	0.006	[0.918 0.941]
Likelihood insens.: δ	0.025	0.002	[0.022 0.030]	0.021	0.001	[0.019 0.023]
Degree of optimism: λ	0.462	0.008	[0.446 0.477]	0.426	0.012	[0.401 0.450]
R^2	0.043	0.003	[0.036 0.049]	0.064	0.003	[0.058 0.070]
$\bar{R}^2$	0.817	0.036	[0.702 0.845]	0.946	0.012	[0.900 0.949]

Notes: These results are based on a specification of our model with $n(h) = \beta_0 + \beta_1 \cdot h + \beta_2 \cdot h^2$ rather than $n(h) = h$. $\hat{\Psi}$ are point estimates of model parameters, $\hat{\sigma}(\Psi)$ is the respective standard deviation and $\widehat{CI}(\Psi)$ is the respective 95% confidence interval. Standard errors are calculated by bootstrapping the subjective and objective survival probabilities by drawing with replacement in 500 bootstrap iterations.
Source: Own calculations based on HRS, SSA and HMD data.

Figure 7: Likelihood insensitivity and implied probability weighting factor: Hump-shaped experience

(a) Likelihood insensitivity (δ_h) (b) Implied probability weighting factor (ξ_h)



Notes: Thin dashed lines are 95% confidence intervals obtained from 500 bootstrap iterations.
Source: Own calculations based on HRS, HMD and SSA data.

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