Welfare-Enhancing Distributional Effects of Central Bank Asset Purchases[1]

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Abstract

This paper examines distributional effects of central bank interventions in secondary markets. We develop a model with limited contract enforcement and idiosyncratic risk, where unusually large disruptions in financial markets are absent and conventional monetary policies are neutral. In contrast, purchases of debt securities at above-market prices affect the equilibrium allocation by shifting the relevant real interest rates for lenders and borrowers in different directions. We show that central bank asset purchases can enhance social welfare via changes in market prices that stimulate borrowing and address pecuniary externalities induced by a collateral constraint. Adding aggregate risk, we further find that asset purchases should be conducted in a countercyclical way to stimulate borrowing particularly in times when asset prices fall.

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1 Introduction

Should central banks intervene in secondary markets for private debt securities? The US Federal Reserve (Fed) and the European Central Bank (ECB), which have traditionally traded money in exchange for (short-term) treasuries, have recently included large-scale purchases of private debt securities in secondary markets into their set of policy instruments.[3] While previous studies have focussed on beneficial effects of unconventional monetary policies in times of stressed financial markets,[4] this paper examines if central banks should intervene in secondary markets for private debt securities even in tranquil times. As a novel contribution, we show that asset purchases can affect market prices in a way that stimulates borrowing and addresses pecuniary externalities, which is not possible under conventional monetary policies (like changes in the short-term interest rate or in the inflation target). The analysis thus shows that asset purchases can in general serve as a useful complement to conventional monetary policy measures.

The impact of asset purchases on relative prices has distributional consequences, i.e. different effects on holders and issuers of debt, which are in principle ambiguous. If, for example, a central bank offers a favorable price for specific assets in secondary markets, one might suppose that primarily agents who hold and sell these assets (i.e. savers or lenders) gain from this intervention. This argument, however, neglects that these agents, who receive liquid funds (central bank money) in exchange for less liquid assets, might further use/invest the proceeds, such that market prices and other participants in financial markets are also affected. Thus, in tranquil times, when neither asset liquidation nor liquidity hoarding is urgent, the price effects from central bank asset purchases are central and their distributional consequences are non-trivial. Concretely, borrowers might gain when the pass though of favorable prices of central bank asset purchases reduce the costs of borrowing, which has, for example, also been observed as a consequence of recent US Federal Reserve asset purchases programmes.[5]

This paper shows that central bank interventions in secondary markets can exert welfare enhancing effects on prices and the real allocation, even when financial markets are not stressed in an extraordinary way. Our analysis is nevertheless based on the view that financial markets are characterized by imperfections, such that policy interventions that alter prices and quantities in financial markets can potentially affect the allocation in a beneficial way. The main novel contribution is that price effects of secondary market interventions, which cannot be induced by

[3] For example, the Fed purchased large positions of MBS in 2009 and 2012, while the ECB is currently still purchasing private debt securities. For example, purchases of asset-backed securities by the ECB were introduced in 2014 and expected to "facilitate credit conditions" (see ECB press release of 2nd Oct, 2014).

[4] Several studies have that unconventional monetary policy measures like direct central bank lending and purchases of equity or treasuries, which differ interventions in secondary market for private debt securities, can be beneficial when agents face extraordinary high costs of financial intermediation or of asset liquidation (see, e.g. Curdia and Woodford, 2011, Gertler and Karadi, 2011, Del Negro et al., 2016, or Woodford, 2016).

[5] See for example Hancock and Passmore (2011) who report that Fed's MBS purchases in 2008 do not only affect MBS yields, but also reduced mortgage rates.

conventional monetary policies, can address pecuniary externalities and can particularly benefit constrained borrowers. Specifically, we show that lenders (i.e. the holders of eligible assets) can be incentivized to increase their supply of funds by the central bank offering an above-market price for debt securities. This leads to a wedge between the effective real interest rate for borrowers and lenders, which policy makers can use to internalize pecuniary externalities induced by financial constraints (as discussed in Bianchi, 2011, or Davila and Korinek, 2017). In fact, the real interest rate for borrowers falls, in accordance with empirical evidence on recent asset purchases programmes,[6] whereas the lenders' effective real return on debt securities rises with central bank interventions. Thus, asset purchases can induce borrowers' consumption to increase relatively to lenders' consumption, which tends to enhance social welfare when borrowers are constrained.[7]

We apply a simple incomplete market model where private agents face idiosyncratic preference shocks and borrow/lend among each other in terms of secured debt. To isolate the effects of financial market interventions and to present the main novel results in a transparent way, we abstract from financial intermediation, endogenous production, and aggregate risk (for the baseline scenario), implying that conventional monetary policy actions do not affect the equilibrium allocation. Agents differ with regard to their valuation of non-durable consumption goods, for which money serves as a mean of payment (as in Lucas and Stokey, 1987), giving rise to borrowing/lending in terms of money. As the main friction, we consider that contract enforcement is limited, such that lending relies on the borrower's ability to pledge collateral (as in Kiyotaki and Moore, 1997). Likewise, the central bank supplies money only against eligible assets, which would solely consist of treasury securities under a conventional monetary policy regime. Here, we further account for the possibility of central bank purchases of collateralized loans in secondary markets.[8] When the monetary policy rate, i.e. price of money in terms of eligible assets, is set below the marginal rate of intertemporal substitution, eligible assets are scarce and money supply is effectively rationed, such that Wallace's (1981) irrelevance result of open market operations does not apply. Individually rational lenders then participate in asset purchases programs if the central bank offers an above-market price, while they are willing to supply the proceeds at a lower loan rate to borrowers.

To facilitate aggregation and to enable the derivation of analytical results, we apply linear-quadratic preferences and define a competitive equilibrium with a representative lender and a representative borrower. In contrast to conventional monetary policy measures (e.g. changes in

[6] Specifically, the loan rate falls by a reduction in the liquidity premium, which accords to empirical evidence on price effects of US Federal Reserve asset purchases (see Gagnon et al., 2011). The behavior of the liquidity premium is further consistent with Krishnamurthy and Vissing-Jorgensen's (2012) findings.

[7] Studies on distributional effects of monetary policy have until now focussed on conventional policies (see, e.g., Berentsen et. al, 2005, Algan and Ragot, 2010, Lippi et al., 2015, Auclert, 2016, or Garriga et al., 2016), which are, by construction, neutral in our model.

[8] In contrast to related studies on unconventional policies (see, for example, Curdia and Woodford, 2011, or Gertler and Karadi, 2011), the central bank does – as in reality – not directly trade with ultimate borrowers.

the policy rate or in the inflation target), asset purchases can affect the allocation in a welfare-enhancing way by changing the real interest rates of borrowers and lenders in different ways. To isolate these effects, we first examine a non-monetary intervention that asymmetrically affects the relevant real interest rates of borrowers and lenders, and can be examined in a more straightforward way than an asset purchase regime (where the central bank has more instruments at its disposal). Specifically, we show that a borrowing subsidy, which is financed by a lump-sum tax on borrowers and, therefore, only affects marginal costs of borrowing, can enhance social welfare by addressing pecuniary externalities induced by the collateral constraint. Precisely, agents do not account for the price of collateral to increase with borrowers' demand, which relates to the pecuniary externalities examined in Bianchi (2011) or Davila and Korinek (2017) and can be internalized by a Pigouvian tax/subsidy.[9] We then establish that a central bank can exactly replicate the Pigouvian subsidy by an asset purchase policy where secured loans are purchased at an above-market price. Further, we show that – compared to the constrained efficient allocation under Pigouvian subsidy – central bank asset purchases can implement welfare-dominating allocations. The reason is that the central bank does not just directly alter real interest rates (like the Pigouvian subsidy), but further changes the total supply of money that is available for lending. While an asset purchase policy endows the policy maker with more instruments, first best cannot be implemented,[10] since borrowing can in general not be stimulated without distorting relative prices.

To provide numerical examples for welfare-enhancing policies, we apply a CRRA utility function. While the latter facilitates the calibration of the model, we rely on pooled end-of-period funds within households (as in Lucas and Stokey, 1987, or Shi, 1997) when defining a competitive equilibrium with representative borrowers and lenders. For this equilibrium, which differs from the previous one solely by agents utility, we confirm the main results derived before. We further introduce aggregate risk in form of a stochastic aggregate endowment and examine welfare effects of state-contingent purchases of secured loans. We find that asset purchases should be conducted in a countercyclical way, such that borrowing is stimulated in adverse states. The reason is that under adverse shocks borrowers suffer not only from a reduction in endowment, but also from a decline in the price of collateral (i.e. the price of housing). Countercyclical asset purchases then stimulate (dampen) borrowing and thus borrowers' consumption in situations where the borrowing capacity is reduced (enhanced). Thus, the central bank can in this way support prudential policies that aim at reducing the economy's vulnerability in crisis times by reducing debt ex-ante (see, for example, Stein, 2012); an analysis of this interaction being an issue for future research.

[9] As a crucial difference to our analysis, Bianchi (2011) and Davila and Korinek (2017) focus on ex-ante policies, whereas our analysis examines policies that can take the state of the economy in every period into account. While we show that borrowing subsidies can enhance welfare in this case, the pecuniary externality in our model would also justify to disincentivize borrowing if we restrict our attention to ex-ante interventions (see Section 4 for a discussion).

[10] To ensure a non-trivial policy analysis, we assume that fiscal policy supplies debt securities in an exogenous (suboptimal) way, which does not support the implementation of the first best equilibrium.

Our analysis of central bank purchases of private debt securities in secondary markets relates to studies on other types of unconventional monetary policies by Curdia and Woodford (2011) and Gertler and Karadi (2011), who show that direct central bank lending to ultimate borrowers can be effective if financial market frictions are sufficiently severe. Using an estimated preferred habitat model, Chen et al. (2012) find that changing the composition of treasury debt as under US Federal Reserve large scale asset purchase programs during the financial crisis had moderate GDP growth and inflation effects. Del Negro et al. (2016) examine government purchases of equity in response to an adverse shock to resaleability and show that the introduction of this type of policy after 2008 have prevented the US economy from a repeat of the Great Depression. Woodford (2016) applies a model with fire sale externalities and a positive probability of crisis states to assess the impact of central bank purchases of long-term treasuries on financial market stability. In contrast to our paper, these studies do not examine private debt purchases in secondary markets, focus on the case of stressed financial markets (like in the recent financial crisis), and they do not derive distributional consequences of central bank interventions. Our paper further relates to Araújo et al. (2013), who show that asset purchases can exert ambiguous welfare effects under endogenous collateral constraints. In contrast to our paper, where asset purchases affect prices via a liquidity premium stemming from the role of money as a means of payment, there is no special role of currency in their model. The liquidity premium effects, by which central bank debt purchases alters asset prices in our model, are similar to the effects of central bank treasury purchases on the term premium in Williamson (2016). The specification of central bank operations in our paper closely relates to Schabert (2015), who examines welfare gains from money rationing in a New Keynesian model without idiosyncratic shocks and with frictionless financial markets. Our analysis of borrowing subsidies relates to Correia et al. (2015), who apply a representative agent model with frictional intermediation due to costly enforcement and show that credit subsidies are desirable and they are – in contrast to our analysis – superior to monetary policy measures. Finally, our analysis of state contingent asset purchases relates to the analysis of ex-post (monetary or fiscal) policies that Jeanne and Korinek (2017) examine under pecuniary externalites induced by borrowing constraints.

In Section 2 we present the model. In Section 3, we provide analytical results on welfare-enhancing financial market interventions. In Section 4, we present some numerical examples and analyze state-contingent asset purchases under aggregate risk. Section 5 concludes.

2 The model

In this Section, we develop an incomplete markets model with idiosyncratic preference shocks and limited contract enforcement. Large parts of the model are specified in a deliberately simple way, while it exhibits features that we view as necessary to suitably account for the way central banks have implemented asset purchase programmes. Specifically, money is assumed to be special, as it serves as the exclusive means of payment for non-durable goods and debt contracts are only available in nominal terms. To focus on the effects of asset purchases, we disregard endogenous production, price rigidities, and aggregate risk (in this Section), such that conventional monetary policy actions are neutral. We further abstract from financial intermediation, for convenience, while we model the supply of central bank money in a detailed way (as in Schabert, 2015). Concretely, money is supplied by the central bank only in exchange for eligible assets, which solely consist of short-run treasury securities under a conventional monetary policy regime. Our particular focus is on the market for loans, where agents can – due to limited enforcement of debt contracts – only borrow against collateral and where the central bank can influence prices by purchasing secured loans from lenders (in a secondary market), which is not equivalent to direct central bank lending to ultimate borrowers in our model.[11]

2.1 Overview

The economy consists of households, a central bank, and a government. Households enter a period with money holdings and government bonds, and dispose of an exogenously given endowment of a non-durable good. They can further hold a durable good, which is supplied at a fixed amount. At the beginning of each period, open market operations are conducted, where the central bank sells or purchases assets outright or supplies money under repurchase agreements (repos) against treasury securities at the policy rate. Then, idiosyncratic preference shocks are realized and, subsequently, housing is traded. Households with a high realization of the preference shock tend to consume more than households with a low realization of the preference shock. Given that money serves as a means of payment for cash goods (non-durable goods), the former tend to borrow money from the latter. We assume that loan contracts cannot be enforced, such that only collateralized loans are feasible. As the primary object of our analysis, we consider that these collateralized loans might be purchased by the central bank from lenders, such that the proceeds are available to extend loan supply. After cash goods are traded, repos are settled and subsequently the asset market opens. In the asset market, borrowing agents repay secured loans, the government issues new bonds, and the central bank reinvests earnings from maturing bonds.

The central bank sets the price of money (i.e. the policy rate), decides on the amount of money that is supplied against treasuries in open market operations and via purchases of loans, and it transfers interest earnings to the government. The government issues one period bonds in an ad-hoc way and has access to lump-sum transfers.[12] The effects of asset purchases will rely on rationed money supply, i.e. on money being supplied by the central bank only against eligible

[11] Analyses of direct central bank lending can be found in Curdia and Woodford (2011) and Gertler and Karadi (2011).

[12] The government might further introduce a Pigouvian tax/subsidy as a means of financial market intervention, which will be examined in Section 3.2

assets that are not unboundedly available.[13] By setting the price of money below agents' marginal valuation of money, the central bank can induce a scarcity of money as well as of eligible assets (i.e. treasuries and collateralized loans), and can influence asset prices.

2.2 Private sector

There are infinitely many and infinitely lived households i of measure one, which are characterized by identical initial stocks of wealth. Their utility increases with consumption $c_{i,t}$ of a non-durable good and holdings of a durable good, i.e. housing $h_{i,t}$; the supply of the latter being normalized to one. Each household is endowed with y_i, where $y_{i,t} = y_t$ and y_t denotes aggregate endowment that is exogenously determined with mean one. Households can differ with regard to their marginal valuation of consumption of the non-durable good due to preference shocks $\epsilon_i > 0$, which are i.i.d. across households and time. The instantaneous utility function $u_{i,t}$ of a household i is given by

$$u_{i,t} = u(\epsilon_i, c_{i,t}, h_{i,t}), \tag{1}$$

where $h_{i,t}$ denotes the end-of-period stock of housing. We assume that $u_{i,t}$ is strictly increasing, concave, and separable in consumption and housing. The idiosyncratic shock ϵ_i exhibits two possible realizations, $\epsilon_i \in \{\epsilon_l, \epsilon_b\}$, with mean one, equal probabilities $\pi_\epsilon = 0.5$, and $\epsilon_l < \epsilon_b$. Households rely on money for purchases of non-durable goods, whereas we treat housing as a "credit good" (see Lucas and Stokey, 1987). They hold money $M^H_{i,t-1}$ at the beginning of each period and they can acquire additional money $I_{i,t}$ from the central bank, for which they hold eligible assets. Specifically, households can get money $I_{i,t}$ from the central bank in open market operations, where money is supplied against treasury securities $B_{i,t-1}$ discounted with the policy rate R^m_t:

$$0 \leq I_{i,t} \leq \kappa^B_t B_{i,t-1}/R^m_t. \tag{2}$$

The central bank supplies money against a fraction $\kappa^B_t \geq 0$ of randomly selected bonds under repurchase agreements as well as outright operations (see Section 2.3), implying that the non-negativity in (2) does not rule out deflationary paths. In contrast to purchases of private debt, purchases of public debt can affect the allocation only via an increase in the supply of money, while associated effects on the interest rate on treasuries will be irrelevant for the equilibrium allocation. When household i draws the realization ϵ_b (ϵ_l), which materializes after treasury open market transactions are conducted,[14] it is willing to consume more (less) than households who draw ϵ_l (ϵ_b). Hence, ϵ_b-type households tend to borrow an additional amount of money from ϵ_l-type households. We assume that borrowing and lending among households only takes place in

[13] Under money rationing, the central bank can simultaneously control the price and the amount of money, and can thereby implement welfare dominating allocations compared to policy regimes that satiate money demand (see Schabert, 2015).

[14] The assumption that preference shocks are realized after money is supplied in open market operations against treasuries is only relevant for the case where the money supply constraint (2) is not binding.

form of short-term nominal debt at the price $1/R_t^L$. Following Jermann and Quadrini (2012), we assume – for convenience – that loan contracts are signed at the beginning of the period and repaid at the end of each period. We account for the fact that debt repayment cannot be guaranteed and that enforcement of debt contracts is limited. Thus, agents have to pledge collateral to be able to borrow, i.e. loans are collateralized by the stock of borrowers' housing. Specifically, households can borrow the amount $-L_{i,t} > 0$ up to the liquidation value of their housing stock at the end of the period $h_{i,t}$ (when loans mature),

$$-L_{i,t} \leq zP_t q_t h_{i,t}, \tag{3}$$

where P_t denotes the aggregate price level, q_t the real housing price, and $z \in (0,1)$ the liquidation share of collateral. As the main object of our analysis, we consider that the possibility that the central bank purchases secured loans in addition to treasuries. In particular, after the preference shocks are realized and loan contracts are signed, the central bank offers money in exchange for a randomly selected fraction $\kappa_t \in [0,1]$ of secured loans at the price $1/R_t^m$:

$$0 \leq I_{i,t}^L \leq \kappa_t L_{i,t}/R_t^m. \tag{4}$$

By purchasing loans, the central bank can thus influence lenders' valuation of secured loans and can induce an increase in the amount of money that is available for loan supply. For this, the price $1/R_t^m$ that the central bank pays and its relation to the market price $1/R_t^L$ are obviously decisive. We assume that loan purchases are conducted in form of repos, where loans are repurchased by lenders before they mature (such that lenders earn the interest on loans). After loans are issued and asset purchases are conducted, the market for non-durables opens. Money is assumed to serve as the means of payment for non-durable goods, for which household i can use money holdings $M_{i,t-1}^H$ as well as new injections $I_{i,t}$ and $I_{i,t}^L$ plus/minus loans, such that the cash-in-advance constraint for household i is

$$P_t c_{i,t} \leq I_{i,t} + I_{i,t}^L + M_{i,t-1}^H - L_{i,t}/R_t^L. \tag{5}$$

It should be noted that the previous constraints (2)-(5) are affected by various prices, which are taken as given by private agents. In particular, they do not take into account that their behavior might affect the prices of housing q_t (see 3) and of loans $1/R_t^L$ (see 5). These pecuniary externalities, which will be relevant for the allocation of resources in equilibrium, can be addressed by policy interventions in a welfare-enhancing way (see Section 3.2).

Before the asset market opens, repurchase agreements are settled, i.e. agents buy back loans and treasuries under repos from the central bank, and transfers are paid. In the asset market, households repay intraperiod loans, invest in treasuries, and might trade assets among each other.

Thus, the budget constraint of household i is

$$\begin{aligned} & M_{i,t-1}^{H}+B_{i,t-1}+L_{i,t}\left(1-1/R_{t}^{L}\right)+P_{t}y_{i,t}+P_{t}\tau_{i,t} \\ & \geq M_{i,t}^{H}+\left(B_{i,t}/R_{t}\right)+\left(I_{i,t}+I_{i,t}^{L}\right)\left(R_{t}^{m}-1\right)+P_{t}c_{i,t}+P_{t}q_{t}\left(h_{i,t}-h_{i,t-1}\right), \end{aligned} \tag{6}$$

where $1/R_t$ denotes the price of treasuries and $\tau_{i,t}$ the lump-sum government transfer. Maximizing $E\sum_{t=0}^{\infty}\beta^t u_{i,t}$, where the discount factor satisfies $\beta\in(0,1)$, subject to (1)-(6) taking prices as given, leads to the following first order conditions for consumption, holdings of treasuries and money, and additional money from treasury open market operations $\forall i\in\{b,l\}$:

$$u'(\epsilon_i,c_{i,t})=\lambda_{i,t}+\psi_{i,t}, \tag{7}$$

$$\lambda_{i,t}=\beta R_t E_t\left[\left(\lambda_{i,t+1}+\kappa_{t+1}^{B}\eta_{i,t+1}\right)/\pi_{t+1}\right], \tag{8}$$

$$\lambda_{i,t}=\beta E_t\left[\left(\lambda_{i,t+1}+\psi_{i,t+1}\right)/\pi_{t+1}\right], \tag{9}$$

$$\overline{E}_t\psi_{i,t}=\left(R_t^m-1\right)\overline{E}_t\lambda_{i,t}+\overline{E}_t R_t^m\eta_{i,t}, \tag{10}$$

where π_t denotes the inflation rate and $\overline{E}_t$ the expectations at the beginning of period t before individual shocks are drawn. Further, $\lambda_{i,t}\geq 0$ is the multiplier on the asset market constraint (6), $\eta_{i,t}\geq 0$ the multiplier on the money supply constraint (2), and $\psi_{i,t}\geq 0$ the multiplier on the cash-in-advance constraint (5), where all constraints are expressed in real terms. Condition (8) indicates that the interest rate on government bonds is affected by a liquidity premium, stemming from the possibility to exchange a fraction κ_t^B of bonds in open market operations (see 2). Condition (10) for money supplied against treasuries reflects that idiosyncratic shocks are not revealed before treasury open market operations are initiated. Further, the following type-specific first order conditions for loans and housing have to be satisfied, for borrowers

$$\lambda_{i,t}\left(1-1/R_t^L\right)-\left(\psi_{i,t}/R_t^L\right)+\zeta_{i,t}=0, \tag{11}$$

$$u'(h_{i,t})+\zeta_{i,t}zq_t+\beta E_t q_{t+1}\lambda_{i,t+1}-q_t\lambda_{i,t}=0, \tag{12}$$

and for lenders, where we additionally consider the first order condition for money acquired from loan purchases $I_{l,t}^L$,

$$\lambda_{i,t}\left(1-1/R_t^L\right)-\left(\psi_{i,t}/R_t^L\right)+\mu_{i,t}\kappa_t=0, \tag{13}$$

$$u'(h_{i,t})+\beta E_t q_{t+1}\lambda_{i,t+1}-q_t\lambda_{i,t}=0, \tag{14}$$

$$-\lambda_{i,t}\left(1-1/R_t^m\right)+\left(\psi_{i,t}/R_t^m\right)-\mu_{i,t}=0, \tag{15}$$

Note that differences between the first order conditions for borrowers and lenders are due to the multiplier $\zeta_{i,t}\geq 0$ on the collateral constraint (3), which is only relevant for borrowers, and the multiplier $\mu_{i,t}\geq 0$ on the money supply constraint (4), which restricts loan purchases and is therefore only relevant for lenders. Condition (15), which describes the agents' willingness to sell loans to the central bank, is evidently also exclusively relevant for lenders. The conditions (11) and (13) further show that the multiplier on the cash-in-advance constraint (5) is positive if the loan rate R_t^L exceeds one, as the latter measures the relative price of cash goods. Further, the associated complementary slackness conditions,[15] as well as (2)-(5), (6) as an equality, and the associated transversality conditions hold.

Notably, $\lambda_{i,t} \geq 0$, $\psi_{i,t} \geq 0$, (10), and (15) imply that the policy rate is bounded from below by $R_t^m \geq 1$, if the money supply constraints (2) and (4) are not binding, $\eta_{i,t} = \mu_{i,t} = 0$. However, if there are binding, $\mu_{i,t} > 0$ and $\eta_{i,t} > 0$, which will be the case under an effective asset purchase policy (see Section 2.4), a policy rate below one, $R_t^m < 1$, is also feasible. Moreover, a nominal loan rate R_t^L below one is also feasible, which requires a binding collateral constraint $\zeta_{i,t} > 0$ (see 11). Hence, a zero lower bound on nominal interest rates does not generally apply in this model.

Combining (7) and (9) to $\frac{\psi_{i,t}}{u'(\epsilon_i, c_{i,t})} = 1 - \beta \frac{E_t[u'(\epsilon_i, c_{i,t+1})/\pi_{t+1}]}{u'(\epsilon_i, c_{i,t})}$ shows that the liquidity constraint (5) is binding when the nominal marginal rate of intertemporal substitution $\frac{u'(\epsilon_i, c_{i,t})}{\beta E_t(u'(\epsilon_i, c_{i,t+1})/\pi_{t+1})}$ exceeds one. Further, the money supply constraint (4) is binding, $\mu_{i,t} > 0$, implying that lenders are willing to refinance loans at the central bank to the maximum amount, when this allows to extract rents. This is the case when the policy rate R_t^m is lower than the loan rate R_t^L, which can be seen from combining (15) with (7), (9), and (18) to

$$\frac{\mu_{i,t}}{u'(\epsilon_i, c_{i,t})} = \frac{1}{1-\kappa} \left(\frac{1}{R_t^m} - \frac{1}{R_t^L} \right). \tag{16}$$

If, however, the policy rate equals the loan rate, $R_t^m = R_t^L$, lenders have no incentive to refinance loans at the central bank and (4) becomes slack (see 16). Thus, only if the central bank offers a price for loans $1/R_t^m$ that exceeds the market price $1/R_t^L$, lenders are willing to sell secured loans until the money supply constraint (4) is binding ($\mu_{i,t} > 0$). Notably, lenders can thereby be incentivized to raise their supply of loans, while they do not take into account the impact of asset purchases on market prices.

The conditions for loan demand (11) and loan supply (15) reveal that the credit market allocation can be affected by the borrowing constraint (for $\zeta_{i,t} > 0$) as well as by central bank loan purchases (for $\mu_{i,t} > 0$). The borrowers' demand condition for loans (11) can – by using (7), (9), and (15) – be rewritten as

$$\frac{1}{R_t^L} = \beta \frac{E_t\left[u'(\epsilon_i, c_{i,t+1})/\pi_{t+1}\right]}{u'(\epsilon_i, c_{i,t})} + \frac{\zeta_{i,t}}{u'(\epsilon_i, c_{i,t})}. \tag{17}$$

Hence, a positive multiplier $\zeta_{i,t}$ tends to raise the RHS of (17), implying a relative increase in current marginal utility of consumption, which can be mitigated by a lower loan rate. Put differently,

[15] Specifically, complementary slackness conditions are given by $\eta_{i,t}[\kappa_t^B b_{i,t-1}(\pi_t R_t^m)^{-1} - i_{i,t}] = 0$, $\zeta_{i,t}\left[zq_t h_{i,t} + l_{i,t}\right] = 0$, $\mu_{i,t}\left[\kappa_t l_{i,t}/R_t^m - i_{i,t}^L\right] = 0$, and $\psi_{i,t}\left[i_{i,t} + i_{i,t}^L + m_{i,t-1}^H - (l_{i,t}/R_t^L) - c_{i,t}\right] = 0$, where the real variables are given by $b_{i,t} = B_{i,t}/P_t$, $l_{i,t} = L_{i,t}/P_t$, $m_{i,t}^H = M_{i,t}^H/P_t$, $i_{i,t} = I_{i,t}/P_t$, $i_{i,t}^L = I_{i,t}^L/P_t$.

under a binding borrowing constraint (3) the borrowers' nominal marginal rate of intertemporal substitution exceeds the loan rate. Further, the lenders' loan supply condition (13) can – by using (7) and (9) – be written as $\frac{1}{R_t^L}\cdot\frac{1-\kappa_t R_t^L/R_t^m}{1-\kappa_t}=\beta\frac{E_t[u'(\epsilon_i,c_{i,t+1})/\pi_{t+1}]}{u'(\epsilon_i,c_{i,t})}$ or

$$\frac{1}{R_t^L}=\kappa_t\cdot\frac{1}{R_t^m}+(1-\kappa_t)\cdot\beta\frac{E_t\left[u'(\epsilon_i,c_{i,t+1})/\pi_{t+1}\right]}{u'(\epsilon_i,c_{i,t})}. \tag{18}$$

Condition (18) implies that the loan rate depends on the lender's nominal marginal rate of intertemporal substitution as well as on the policy rate R_t^m, if the central bank purchases loans, $\kappa_t>0$. According to (18), a higher share of purchased loans κ_t for a given policy rate $R_t^m<R_t^L$, or a lower policy rate R_t^m for a given share of purchased loans, $\kappa_t>0$, tend to reduce the loan rate, while the loan rate approaches the policy rate, $R_t^L\to R_t^m$, for $\kappa_t\to 1$. Further note that (7), (9), and (10) imply $\frac{\overline{E}_t\eta_{i,t}}{\overline{E}_t u'(\epsilon_i,c_{i,t})}=\frac{1}{R_t^m}-\beta\frac{\overline{E}_t[u'(\epsilon_i,c_{i,t+1})/\pi_{t+1}]}{\overline{E}_t u'(\epsilon_i,c_{i,t})}$, where the term $\frac{\beta\overline{E}_t[u'(\epsilon_i,c_{i,t+1})/\pi_{t+1}]}{\overline{E}_t u'(\epsilon_i,c_{i,t})}$ cannot be larger than the inverse of the loan rate $1/R_t^L$ (see 17 and 18). Thus, a policy rate satisfying $1\le R_t^m<R_t^L$ ensures that money is scarce, such that the liquidity constraint (5) is binding, and that agents liquidate all available bonds, such that the money supply constraint (2) is binding as well as (4). Given that money supply is then effectively constrained by the available amount of eligible assets, i.e. bonds and secured loans, this type of monetary policy implies money rationing. The central bank can then control the price of money (by setting R_t^m) as well as the amount of money by setting κ_t and κ_t^B. If however the central bank supplies money in an unrestricted way at the policy rate R_t^m, the nominal marginal rate of intertemporal substitution will be equal to the latter and asset purchases are irrelevant.

2.3 Public sector

The government issues nominal bonds at the price $1/R_t$ and pays lump-sum transfers τ_t, while we abstract from government spending and issuance of long-term debt. In Section 3.2, we further introduce a borrowing tax/subsidy as a means of financial market intervention, which is not specified here, for convenience. As described above, short-term government bonds serve as eligible assets for central bank operations. Hence, sufficiently large holdings of treasuries can in principle support self-insurance against illiquidity risk (see also Woodford, 1990) and thereby the implementation of the first best allocation. To ensure a non-trivial policy analysis, we assume that the supply of short-term government bonds does not support the implementation of the first best allocation and is exogenous to the state of the economy. Specifically, the total amount of short-term government bonds B_t^T grows at a rate $\Gamma>0$,

$$B_t^T=\Gamma B_{t-1}^T, \tag{19}$$

given $B_{-1}^T>0$. The government further receives seigniorage revenues τ_t^m from the central bank, such that its budget constraint reads $(B_t^T/R_t)+P_t\tau_t^m=B_{t-1}^T+P_t\tau_t$. Due to the existence of lump-sum transfers/taxes, which balance the budget, fiscal policy will be irrelevant for the equilibrium allocation, except for the supply of treasuries (19).[16]

The central bank supplies money in open market operations either outright or temporarily via repos against treasuries, M_t^H and M_t^R. It can further increase the supply of money by purchasing secured loans from lenders, I_t^L, i.e. it conducts repos where secured loans serve as collateral. At the beginning of each period, its holdings of treasuries and the stock of outstanding money are given by B_{t-1}^c and M_{t-1}^H. It then receives treasuries and loans in exchange for money. Before the asset market opens, where the central bank rolls over maturing assets, repos in terms of treasuries and secured loans are settled. Hence, its budget constraint reads $(B_t^c/R_t) - B_{t-1}^c + P_t\tau_t^m = R_t^m\left(M_t^H - M_{t-1}^H\right) + (R_t^m - 1)\left(I_t^L + M_t^R\right)$, showing that the central bank earns interest from bonds purchased outright and by supplying money in open market operations. The central bank transfers its interest earnings from asset holdings and from open market operations to the government, $P_t\tau_t^m = (1 - 1/R_t)\,B_t^c + R_t^m\left(M_t^H - M_{t-1}^H\right) + (R_t^m - 1)\left(I_t^L + M_t^R\right)$. Thus, the budget constraint implies that central bank asset holdings evolve according to $B_t^c - B_{t-1}^c = M_t^H - M_{t-1}^H$. Further assuming that initial values for its assets and liabilities satisfy $B_{-1}^c = M_{-1}^H$, gives the central bank balance sheet

$$B_t^c = M_t^H. \tag{20}$$

The central bank has four instruments at its disposal. It sets the policy rate R_t^m and can decide how much money to supply against a randomly selected fraction of treasuries, for which it can adjust $\kappa_t^B \in (0, 1]$. The central bank can further decide whether it supplies money in exchange for treasuries either outright or temporarily via repos. Specifically, it controls the ratio of treasury repos to outright purchases $\Omega_t > 0 : M_t^R = \Omega_t M_t^H$, where a sufficiently large value for Ω_t ensures that injections are always positive, $I_{i,t} > 0$. Finally, the central bank can decide to purchase loans, i.e. to supply money temporarily against secured loans under repos. In each period, it therefor decides on a randomly selected share of secured loans $\kappa_t \in [0, 1]$ that it is willing to exchange for money under repos. To give a preview, only policy regimes with loan purchases, $\kappa_t > 0$, will be non-neutral with regard to the equilibrium allocation, whereas conventional monetary policies, $\kappa_t = 0$, will be neutral.

2.4 Equilibrium properties

In equilibrium, agents' optimal plans are satisfied and prices adjust such that all markets clear: $0 = \sum_i l_{i,t}$, $h = \sum_i h_{i,t}$, $y = \sum_i c_{i,t}$, $m_t^H = \sum_i m_{i,t}^H$, $m_t^R = \sum_i m_{i,t}^R$, $b_t = \sum_i b_{i,t}$, and $b_t^T = b_t^c + b_t$, where $l_{i,t} = L_{i,t}/P_t$, $m_{i,t}^H = M_{i,t}^H/P_t$, $m_t^R = M_t^R/P_t$, $b_{i,t} = B_{i,t}/P_t$, $b_t = B_t/P_t$, $b_t^c = B_t^c/P_t$, and $b_t^T = B_t^T/P_t$. A definition of a competitive equilibrium is given in Appendix A. Before we examine policy effects on the equilibrium allocation, we describe the first best allocation, which maximizes

[16] Note that the growth rate Γ might affect the long-run inflation rate if the money supply constraint (2) is binding. As shown in Appendix C, the central bank can nonetheless implement a desired inflation target by suited long-run adjustments of its money supply instruments.

ex-ante social welfare

$$E\sum_{t=0}^{\infty}\beta^{t}\sum_{i}u_{i,t}, \tag{21}$$

s.t. $h = \sum_i h_{i,t}$, and $y = \sum_i c_{i,t}$ and serves as a reference case for the subsequent analysis. Applying the law of large numbers and indexing all agents drawing ϵ_l (ϵ_b) in period t with l (b), we can summarize the first best allocation as a set of sequences $\{c^*_{b,t}, c^*_{l,t}, h^*_{b,t}, h^*_{l,t}\}_{t=0}^{\infty}$ satisfying $h^*_{b,t} + h^*_{l,t} = h$, $c^*_{l,t} + c^*_{b,t} = y$,

$$u_c(\epsilon_b, c^*_{b,t}) = u_c(\epsilon_l, c^*_{l,t}), \text{ and } h^*_{b,t} = h^*_{l,t}. \tag{22}$$

Under the first best allocation, the marginal utilities of consumption and of the end-of-period stock of housing are identical for borrowers and lenders (see 22). This will typically not be the case in a competitive equilibrium where the borrowing constraint (3) is binding. Only if the equilibrium lending rate R^L_t were equal to zero and the supply of eligible assets sufficiently large, such that money were abundantly available, agents would be able to self-ensure against liquidity risk and the first best equilibrium would be implementable (see also Woodford, 1990). To provide a non-trivial analysis of policy interventions, this outcome is ruled out by imposing an ad-hoc specification for the supply of short-term government bonds (see 19). As an alternative way to demonstrate the welfare enhancing role of asset purchases, one could instead introduce additional frictions that render the Friedman rule undesirable/impossible, which are neglected here for convenience. Given that the first best allocation is not implementable, the equilibrium allocation of consumption and housing will be distorted by the collateral constraint (3) and the liquidity constraint (5). In the subsequent sections, we will show how asset purchases can (favorably) influence the allocation by inducing a redistribution of resources between borrowers and lenders, whereas conventional monetary policy measures, i.e., mere changes in the policy rate or the inflation target, are neutral.

For asset purchases to be relevant, money has to be supplied at a favorable price (see 16), which implies that access to money is effectively rationed by the available amount of assets eligible for central bank operations. Specifically, the central bank has to set the policy rate below the lender's marginal rate of intertemporal substitution, implying $R^m_t < R^L_t$ (see 18), to ration money supply. As discussed above, a policy rate below one, $R^m_t < 1$, is then also feasible (see 15). Under a non-rationed money supply, which is equivalent to the case where the central bank supplies money in a lump-sum way (as typically assumed in the literature), the money supply constraints (2) and (4) are slack and the loan rate is identical to the policy rate $R^L_t = R^m_t$. In this case, asset purchases are irrelevant (see 16). For the subsequent analysis, we will therefore separately discuss the two cases where money supply is rationed and where money supply is not rationed. The latter is the case under a conventional monetary policy regime, which is in fact ineffective, because it affects prices for all agents in a symmetric way. In contrast, under money rationing and asset purchases, interest rates for borrowers and lenders can be altered in distinct ways.

3 Analytical results

In this Section, we examine different types of policy interventions in an analytical way. In the first part of this Section, we impose some further assumptions, which facilitate aggregation and the derivation of analytical results, and we define a competitive equilibrium in terms of a representative borrower and a representative lender, where conventional monetary policies are shown to be neutral. In the subsequent part of this Section, we show how asset purchases can enhance welfare by addressing pecuniary externalities and by easing borrowing conditions. We firstly analyze the constrained efficient allocation that a social planer can implement by a Pigouvian tax/subsidy, which can be examined in a more straightforward way than an asset purchase regime, where the central bank has more instruments at its disposal. Secondly, we show that asset purchases can not only replicate the constrained efficient allocation, but can further increase the set of feasible allocations, including welfare-dominating allocations, by relaxing borrowing conditions.

3.1 Conventional monetary policy

Here, we examine the benchmark case of a conventional monetary policy, where the central bank sets the policy rate equal to the loan rate, $R_t^m = R_t^L$, such that both money supply constraints (2) and (4) are not binding ($\eta_{i,t} = \mu_{i,t} = 0$), implying that asset purchases are irrelevant (see 16). Even when the central bank were willing to buy loans, lenders would then not gain from selling loans and prices would not be affected. Given that we aim at disclosing the distributional and welfare effects of financial market interventions, we apply three assumptions that allow to derive the main results in an analytical way. We assume that preferences are given by a linear-quadratic form, which enables aggregation over individual choices. Once the competitive equilibrium is defined in terms of aggregate variables, we analytically derive the main results on policy interventions, which will be confirmed for alternative preferences (see Section 4).[17]

Assumption 1 *Instantaneous utility of households satisfies*

$$u(\epsilon_i, c_{i,t}, h_{i,t}) = \epsilon_i(\delta c_{i,t} - (1/2)c_{i,t}^2) + (\gamma h_{i,t} - (1/2)h_{i,t}^2), \tag{23}$$

where $\partial u/\partial c_{i,t} = u'(\epsilon_i, c_{i,t}) > 0$ *and* $\partial u/\partial h_{i,t} = u'(h_{i,t}) > 0$.

According to Assumption 1, the marginal utilities of consumption and housing are linear, $u_c(\epsilon_i, c_{i,t}) = \epsilon_i(\delta - c_{i,t})$ and $u_h(\epsilon_i, c_{i,t}, h_{i,t}) = \gamma - h_{i,t}$, where the parameters $\delta > 0$ and $\gamma > 0$ guarantee that the marginal utilities of consumption and housing are strictly positive in equilibrium. Under Assumption 1, the set of conditions that describe the behavior of agents' who draw ϵ_l in period t – indexed with (l, i, t) – is given by (6) holding as an equality, (7), (9), (14), $\epsilon_l(\delta - c_{l,i,t})/R_t^L = \beta E_t[0.5(\epsilon_l(\delta - c_{l,i,t+1}) + \epsilon_b(\delta - c_{b,i,t+1}))]/\pi_{t+1}$, and $c_{l,i,t} \leq 0.5\,(i_{l,i,t} + i_{b,i,t}) + m_{l,i,t-1}^H \pi_t^{-1} - l_{l,i,t}/R_t^L$,

[17] For the case of CRRA preferences, which are introduced in Section 4, aggregation will be enabled by pooling funds within households at the end of each period.

where the last condition, i.e. the cash-in-advance constraint, accounts for treasury open market operations being conducted before idiosyncratic shocks are drawn. Due to the linear-quadratic utility function, all conditions are linear in the agents' choice variables. The cash-in-advance constraint might, however, not be binding, which would be the case when the nominal interest rate equals one, $R_t^m = R_t^L = 1$ (see 13 for $\kappa_t = 0$). Given our specification of fiscal policy, the latter policy is not sufficient to implement the first best allocation, which would require agents to accumulate bonds and money to a sufficiently large amount to ensure the borrowing constraint never to be binding (see Section 2.2). To avoid indeterminacies due to a slack cash-in-advance constraint, we assume that the latter is just binding even when the nominal interest rate equals one and the associated multiplier equals zero, $\psi_{l,i,t} = 0$. Alternatively, one can assume that the Friedman rule does not hold exactly, but only as a limit (analogously, for example, to Gu et al., 2016).

Assumption 2 *Agents will hold money equal to the amount of planned nominal consumption expenditures even when the multiplier on the cash-in-advance constraint equals zero.*

It should be noted that Assumption 2 is made for convenience only and does not affect the main conclusions: If cash-in-advance constraints were not binding, monetary policy would apparently be irrelevant. As will be shown below, a conventional monetary policy will in fact also be irrelevant if the cash-in-advance constraint is binding (see Corollary 2). Assumption 2 will therefore not be decisive for the assessment of monetary policy. Under both Assumptions 1 and 2, we can easily aggregate by summing over all agents who draw ϵ_l in period t. Using the law of large numbers, all agents face the same probability (0.5) of drawing ϵ_l in period t, such that average holdings of money, bonds, and housing of these agents at the beginning of each period are identical. The resulting set of conditions for the representative lender are given in Appendix A.

Given that we are interested in analyzing policy interventions we restrict our attention to cases where the equilibrium allocation is inefficient due to a relevant distortion, which is here given by the collateral requirement originating from limited contract enforcement. For the equilibrium allocation to be inefficient, the borrowing constraint (3) therefore has to be binding, which is apparently more likely for a larger difference in the agents' valuation of consumption and for a lower liquidation value of collateral. To further facilitate aggregation, we restrict our attention to the case where the associated multiplier is strictly positive for all agents drawing ϵ_b, $\zeta_{b,i,t} > 0$, which can be guaranteed by a sufficiently large difference in agents' valuation of consumption relative to the liquidation value of collateral, $(\epsilon_b - \epsilon_l)/z$. The resulting set of conditions for the representative borrower are given in Appendix A.

Assumption 3 *The ratio $(\epsilon_b - \epsilon_l)/z$ is sufficiently large such that the borrowing constraint (3) is binding for all agents drawing ϵ_b.*

Let $x_{l,t} = 2\sum_{l,i} x_{l,i,t}$ ($x_{l,t} = 2\sum_{l,i} x_{l,i,t}$) be the value of any generic variable x_t of a representative agent drawing ϵ_l (ϵ_b) in period t. Applying the Assumptions 1, 2, and 3, we can define a competitive equilibrium in terms of a representative borrower and a representative lender as follows.

Definition 1 *A competitive equilibrium of the economy with a representative borrower and a representative lender under a conventional monetary policy regime is a set of sequences* $\{c_{b,t},\ c_{l,t},\ h_{b,t},\ q_t,\ \pi_t\}_{t=0}^{\infty}$ *satisfying*

$$\epsilon_l(\delta - c_{l,t}) = \beta 0.5 E_t \left[(\epsilon_l(\delta - c_{l,t+1}) + \epsilon_b(\delta - c_{b,t+1})) \left\{ R_t^L/\pi_{t+1} \right\} \right], \tag{24}$$

$$\left\{ R_t^L/q_t \right\} (2h_{b,t} - h)/z = \epsilon_b(\delta - c_{b,t}) - \beta 0.5 E_t[(\epsilon_b(\delta - c_{b,t+1}) + \epsilon_l(\delta - c_{l,t+1})) \left\{ R_t^L/\pi_{t+1} \right\}], \tag{25}$$

$$\epsilon_l(\delta - c_{l,t}) \left\{ q_t/R_t^L \right\} = \gamma - (h - h_{b,t}) + \beta E_t[\epsilon_l(\delta - c_{l,t+1}) \left\{ q_{t+1}/R_{t+1}^L \right\}], \tag{26}$$

$$c_{b,t} - c_{l,t} = z h_{b,t} 2 \left\{ q_t/R_t^L \right\}, \tag{27}$$

$$y_t = c_{b,t} + c_{l,t}, \tag{28}$$

and $R_t^L = R_t^m$*, for* $\{y_t\}_{t=0}^{\infty}$ *and a sequence* $\{R_t^m \geq 1\}_{t=0}^{\infty}$ *set by the central bank.*

An agent who draws a preference shock ϵ_b in period t, borrows money from other agents to increase its consumption possibilities. Given that the loan has to be repaid at the end of the period, it has less funds available at the beginning of period $t+1$. While idiosyncratic histories of shock realizations matter for individual net wealth positions, they do not matter for the aggregate behavior of borrowers/lenders, given that all agents – regardless of their net wealth position – face the same probability of drawing ϵ_b (ϵ_l) and their behavioral relations are linear. Further note that the multiplier on the borrowing constraint satisfies

$$\zeta_{b,t} = [\epsilon_b(\delta - c_{b,t}) - \epsilon_l(\delta - c_{l,t})]/R_t^L = (2h_{b,t} - h)/(zq_t) \geq 0, \tag{29}$$

indicating that both, the housing and the consumption choice (that would ideally satisfy $h_b = h_l$ and $\epsilon_l(\delta - c_{b,t}) = \epsilon_l(\delta - c_{l,t})$, see 22), are distorted by a binding borrowing constraint ($\zeta_{b,t} > 0$). On the one hand, the marginal utility of consumption is then larger for borrowers than for lenders, $\epsilon_b(\delta - c_{b,t}) > \epsilon_l(\delta - c_{l,t})$. On the other hand, borrowers' housing exceeds lenders' housing, $h_{b,t} > h/2$, as the former is characterized by a relatively higher valuation of housing due to its ability to serve as collateral. Given that the supply of non-durables and durables is exogenous, such that $h = h_{b,t}^* + h_{l,t}^*$, and $y = c_{l,t} + c_{b,t}$, (29) implies that the equilibrium allocation equals the first best allocation (see 22) when the borrowing constraint gets irrelevant, $\zeta_{b,t} \to 0$. The following corollary 1 highlights that the distortion due to the liquidity constraint (5) alone does not lead to an allocative inefficiency.

Corollary 1 *For the limiting case where the multiplier on the borrowing constraint approaches zero, the equilibrium allocation is identical with the first best allocation.*

Definition 1 reveals that the nominal interest rate and thus the policy rate only matters jointly with either the housing price or the inflation rate. Precisely, the conditions (24)-(28) impose restrictions on the allocation, $c_{b,t}, c_{l,t}$, and $h_{b,t}$, the ratio R_t^L/q_t, and the real interest rate R_t^L/π_{t+1} (see curly brackets in 24-27), but not separately on q_t, π_t, and R_t^L. Thus, conventional monetary policy measures, i.e. changes in the policy rate $R_t^m = R_t^L$, leave the allocation unaffected, while they affect the inflation rate and the relative price of housing. The latter effect is due to the liquidity constraint and the well-known inflation tax on cash goods (here, non-durables), which implies that higher interest rates reduce the demand for consumption and raise the demand for housing.

Corollary 2 *Under a conventional monetary policy regime, changes in the monetary policy rate do not affect the equilibrium allocation, while the housing price and the inflation rate increase with the nominal interest rate.*

The reason for the neutrality summarized in Corollary 2 is that conventional monetary policies can only affect equilibrium prices that are equally relevant for both agents, while the aggregate endowment with durable and non-durable goods is exogenously determined. Notably, asset purchases will instead drive a wedge between prices that are either relevant for borrowers or for lenders. Given that changes in the monetary policy instrument R_t^m under a conventional monetary policy regime do not affect the equilibrium allocation, the latter is time-invariant if there is no aggregate risk. To facilitate comparisons between the different policy experiments, we restrict our attention to the case of time-invariant policies in the subsequent analysis. In Section 4.3, where we introduce aggregate risk, we extend the analysis by considering state-contingent and thus time-varying policies.

3.2 Constrained efficiency under a Pigouvian tax

For the remainder of this section, we abstract from aggregate risk $y_t = y$ and focus on time-invariant financial market interventions, such that neither the allocation nor prices are time-varying. Given that conventional monetary policy is neutral (see Corollary 2), real effects stemming from unconventional monetary policies cannot be replicated by mere changes in the policy rate or in the inflation rate. We will show that asset purchases can enhance social welfare due to its effects on the relevant real interest rates for borrowers and lenders and on the relative price of collateral. To examine the former effects, we first examine a (non-monetary) financial market intervention with one instrument, which can be analyzed in a more straightforward way than an asset purchase regime (where the central bank has several instruments at its disposal): We consider a policy intervention that alters the cost of borrowing, while abstracting from direct redistributive measures. Specifically, we suppose that a planer can influence private borrowing by a Pigouvian tax/subsidy on debt τ^L and transfers/collects the funds in cash to/from the taxed agents in a lump-sum way. Hence, the borrower's effective real interest rate is given by

$$r_{b,t}^{\tau} = \frac{R_t^L/\pi_{t+1}}{1-\tau_t}, \tag{30}$$

and the borrower's loan price net of taxes is $(1-\tau^L)/R_t^L$ is financed by a lump-sum transfer/tax equal to $\tau_t^R = \tau^L l_t/R_t^L$.[18] This intervention affects the marginal costs of borrowing and can thereby correct for inefficiencies induced by externalities associated with financial market frictions, leading to a constrained efficient allocation.

Consider the competitive equilibrium as given in Definition 1 under time-invariant endogenous variables and with the Pigouvian tax/subsidy. The borrowers' consumption Euler equation (25) then changes to $(1-\tau^L)\epsilon_b(\delta - c_b) = \beta 0.5[(\epsilon_b(\delta - c_b) + \epsilon_l(\delta - c_l)) \left\{R^L/\pi\right\} + \left\{R^L/q\right\}(2h_b - h)/z$, and condition (26) implies the price of housing relative to consumption q/R_L to be negatively related to lenders' housing $h - h_b$ and positively related to lenders' consumption c_l,

$$\frac{q}{R^L} = \frac{\gamma - (h - h_b)}{(1-\beta)\epsilon_l(\delta - c_l)}. \tag{31}$$

Notably, an increase in the relative price q/R^L tends to raise the difference between consumption of borrowers and lenders (see 27), as it relaxes the impact of the collateral constraint (3) on borrowers' consumption. Yet, the impact of the demand for housing and consumption on the relative price q/R^L is not internalized by individuals, giving rise to inefficiencies induced by pecuniary externalities. For example, borrowers do not internalize that an increase in their housing (thus a decrease in lenders' housing) tends to increase the relative price q/R^L. Using condition (31) to substitute out q/R^L in the borrowers' consumption Euler equation and in (27), and (24) to substitute out the real interest rate, yields

$$(1-\tau^L)\epsilon_b(\delta - c_b) - \epsilon_l(\delta - c_l) = (2h_b - h)\,\frac{(1-\beta)\epsilon_l(\delta - c_l)}{z(\gamma - h + h_b)}. \tag{32}$$

Given that there is no time variation, the problem of a social planer, who maximizes social welfare (21) by controlling the tax/subsidy rate τ^L and respecting the borrowing constraint, can then – by using the primal approach – be summarized as

$$\max_{c_l,c_b,h_b} \{u(\epsilon_b, c_b, h_b) + u(\epsilon_l, c_l, h - h_b)\}/(1-\beta), \tag{33}$$

$$\text{s.t.} \quad y = c_l + c_b,\ c_b - c_l \leq 2zh_b \cdot \left(q/R^L\right), \text{ and (31).}$$

In contrast to private agents, the social planer takes into account that changes in the allocation alter the relative price q/R^L (see 31) and internalizes pecuniary externalities using the Pigouvian tax/subsidy. Thereby, the solution to this problem, where the social planer decides on private borrowing subject to the competitive equilibrium conditions, leads to a constrained efficient allocation. Given this constrained efficient allocation, (32) determines the associated tax/subsidy rate τ^L. The following proposition summarizes the main results.

[18] Thus, the tax/subsidy and the lump-sum transfers/taxes enter the budget constraint (6) and the goods market constraint (5) of borrowers.

Proposition 2 *The implementation of a constrained efficient allocation of the representative agents economy without aggregate risk requires a subsidy on borrowing, $\tau^L < 0$, if but not only if $z/(1-\beta) \geq 1$. Compared to the laissez-faire case ($\tau^L = 0$), the Pigouvian subsidy raises borrowers' consumption and housing as well as the real interest rate R^L/π, which is associated with a decline in lenders' consumption and housing.*

Proof. See Appendix B. ■

Proposition 2 implies that a financial market intervention that stimulates borrowing can enhance social welfare if $z/(1-\beta) \geq 1$. An increase in borrowing, which is induced by a Pigouvian subsidy $\tau^L < 0$, tends to increase borrowers' consumption and has to be supported by a larger stock of housing held by borrowers. As implied by (31) and $c_l = y - c_b$, the price of housing in terms of consumption, q/R^L, increases with the latter, $\partial(q/R^L)/\partial h_b > 0$, and decreases with the former, $\partial(q/R^L)/\partial c_b < 0$. If the impact of housing on the relative price q/R^L dominates, which can be shown to be the case under the following condition (see proof of Proposition 2)

$$\frac{(1-\tau^L)u_{c_b} - u_{c_l}}{u_{c_b} - u_{c_l}} = \frac{1 + \left[\partial(q/R^L)/\partial h_b\right] \cdot h_b/(q/R^L)}{1 + zh_b \cdot \left[\partial(q/R^L)/\partial c_l\right]} > 1, \tag{34}$$

where $\partial(q/R^L)/\partial c_l = u_{h_l}(1-\beta)^{-1}(-u_{c_l c_l})/u_{cl}^2 > 0$ and $\partial(q/R^L)/\partial h_b = (1-\beta)^{-1}(-u_{h_l h_l})/u_{c_l} > 0$, a constrained efficient allocation is associated with a borrowing subsidy $\tau^L < 0$. Given that housing is a durable good, permanent adjustments in housing demand are associated with large price changes, i.e. amplified by the multiplier $1/(1-\beta)$, such that condition (34) is ensured by $z/(1-\beta) \geq 1$, where $z \in (0,1)$ accounts for the assumption that the liquidation value of housing is less than one. In this case, which is likely to be satisfied by reasonable values for the parameters β and z (see Section 4.1), borrowing is inefficiently low in a competitive equilibrium, given that the private agents do not internalize the favorable effects of increased housing demand on the relative price q/R^L. The social planer can then correct for this pecuniary externality by a borrowing subsidy $\tau^L < 0$ (financed by a lump-sum tax on borrowers, $\tau_t^R = -\tau^L l_t/R_t^L > 0$), which induces agents to internalize changes in the relative price q/R^L. As summarized in Proposition 2, the subsidy causes agents to borrow more, leading to an increase in borrowers' consumption and housing. Notably, the subsidy tends to reduce the costs of borrowing r_b^τ (see 30), while it simultaneously raises the real interest rate R^L/π. The latter induces lenders to increase their supply of funds, such that their consumption and housing decreases.

Given that $z \geq 1-\beta$ is just a sufficient condition, a violation of this condition does not necessarily imply the opposite result. If however (34) is violated, the positive impact of an increase in borrowers' consumption and housing on the terms of borrowing (q/R^L) is reversed, which would require rather a tax on debt than a subsidy. It should be noted that the pecuniary externality that arises in this model, is related to the type of pecuniary externalities induced by borrowing or fire sale constraints as discussed in Bianchi (2011) or Davila and Korinek (2017). In contrast to these analyses, which focus on the prudential role of financial regulation, we abstract here from aggregate risk and consider a scenario where the collateral constraint is always binding for borrowers. Thus, the policy intervention discussed in this section cannot directly be compared to the type of ex-ante interventions recommended in studies where financial constraints only bind in adverse states and agents do not take the possibility of collateral price effects into account.[19]

3.3 Welfare-enhancing asset purchases

We now turn to the effects of central bank purchases of secured loans, $\kappa_t > 0$ (see 4). Firstly, we will show that an asset purchase policy can be equivalent to a Pigouvian subsidy that supports the constrained efficient allocation, which is characterized in Proposition 2, by manipulating the relevant interest rates for borrowers and lenders in different ways. Thus, asset purchases can enhance social welfare by addressing pecuniary externalities, which is not possible under a conventional monetary policy regime (see Corollary 2). Secondly, we will show that compared to the case of a Pigouvian subsidy (as described in Section 3.2) the central bank can enlarge the set of feasible equilibria and can even implement allocations that welfare-dominate the constrained efficient allocation by further inducing favorable borrowing conditions.

Given that asset purchases are not in general effective, as discussed in Section 2.4, the central bank has to offer an above market price for loans to affect relevant prices and the equilibrium allocation. Thus, it has to set the policy rate below the market loan rate $R_t^m < R_t^L$, which implies that money supply is effectively rationed by holdings of eligible collateral, i.e. the money supply constraints in terms of treasuries and secured loans (2) and (4) are binding (see 16),

$$i_{i,t} = \kappa_t^B 0.5 b_{t-1}/(\pi_t R_t^m) \text{ for } i \in \{l, b\} \quad \text{and} \quad i_{l,t}^L = \kappa_t l_t / R_t^m. \tag{35}$$

If, $R_t^m < R_t^L$, purchases of loans $\kappa_t > 0$ drive a wedge between the borrowers' and the lenders' effective real interest (loan) rate, such that the pecuniary externalities discussed in Section 3.2 can be addressed like with the Pigouvian subsidy $\tau^L < 0$. Concretely, under an asset purchase regime, the effective real return for a lender $r_{l,t}^{ap}$ is distorted by the term $\frac{1-\kappa_t}{1-\kappa_t R_t^L/R_t^m}$ (see 18), whereas the borrowers' real interest rate $r_{b,t}^{ap}$ is not directly affected by the policy instruments (in contrast to the Pigouvian subsidy, see 30):

$$r_{b,t}^{ap} = \frac{R_t^L}{\pi_{t+1}}, \text{ and } r_{l,t}^{ap} = \frac{R_t^L}{\pi_{t+1}} \frac{1-\kappa_t}{1-\kappa_t R_t^L/R_t^m}. \tag{36}$$

Moreover, by additionally purchasing eligible assets the central bank increases the overall amount of funds available for lending compared to the case of a conventional monetary policy. Using (35)

[19] Remarkably, the impact of housing and consumption demand on relative prices, as considered above, also implies that agents tend to "overborrow" today when there is a positive probability that future borrowing conditions worsen due to some exogenous impulse (as for example in Bianchi, 2011).

to rewrite the binding liquidity constraints (5) and taking differences yields

$$c_{b,t} - c_{l,t} = zh_{b,t}\left[2 - \kappa_t R_t^L/R_t^m\right]\left\{q_t/R_t^L\right\}, \tag{37}$$

where we substituted out loans with the binding borrowing constraint (63). Comparing (27) with (37), suggests that asset purchases adversely affect the difference between borrowers' and lenders' consumption in the first instance, as loan purchases endow lenders rather than borrowers with additional money. Hoverer, this increased supply of money affects market prices and can be lend to borrowers, such that the supply of loans tends to increase. Thus, to identify the ultimate impact on the equilibrium allocation, changes in the relative price q_t/R_t^L, which tends to increase under asset purchases (see below), have also to be taken into account.

Applying the same aggregation procedure as in Section 3.1, using (19), (20), and $B_t^T = B_t^c + B_t$, and eliminating the multiplier $\lambda_{l,t}$ with $\lambda_{l,t} = \beta E_t[0.5(\epsilon_l(\delta - c_{l,t+1}) + \epsilon_b(\delta - c_{b,t+1}))/\pi_{t+1}]$ in (55), we can summarize a competitive equilibrium under Assumptions 1-3 and money rationing ($R_t^m < R_t^L$) as follows.

Definition 2 *A competitive equilibrium of the representative agents economy under money rationing is a set of sequences* $\{c_{b,t}, c_{l,t}, h_{b,t}, m_t^H, b_t, q_t, R_t^L, \pi_t\}_{t=0}^{\infty}$ *satisfying (25), (28), (37),*

$$\epsilon_l(\delta - c_{l,t}) = \beta 0.5 E_t[(\epsilon_b(\delta - c_{b,t+1}) + \epsilon_l(\delta - c_{l,t+1}))(R_t^L/\pi_{t+1})]\frac{1-\kappa}{1-\kappa R_t^L/R_t^m}, \tag{38}$$

$$\begin{aligned} & q_t \beta E_t\left[0.5(\epsilon_l(\delta - c_{l,t+1}) + \epsilon_b(\delta - c_{b,t+1}))/\pi_{t+1}\right] \\ &= \gamma - (h - h_{b,t}) + \beta^2 E_t q_{t+1}\left[0.5(\epsilon_l(\delta - c_{l,t+2}) + \epsilon_b(\delta - c_{b,t+2}))/\pi_{t+2}\right], \end{aligned} \tag{39}$$

$$c_{b,t} = 0.5(1+\Omega_t)m_t^H + zq_t h_{b,t}/R_t^L \tag{40}$$

$$(1+\Omega_t)m_t^H = \kappa_t^B b_{t-1}\pi_t^{-1}/R_t^m + m_{t-1}^H \pi_t^{-1}, \tag{41}$$

$$b_t + m_t^H = \Gamma\left(b_{t-1} + m_{t-1}^H\right)/\pi_t, \tag{42}$$

and the transversality conditions, for $\{y_t\}_{t=0}^{\infty}$ *and sequences* $\{0 \leq \kappa_t < R_t^m/R_t^L,\ \kappa_t^B > 0,\ \Omega_t > 0,\ R_t^m < R_t^L\}_{t=0}^{\infty}$ *set by the central bank, given* $m_{-1}^H > 0$, *and* $b_{-1} > 0$.

Evidently, there are more instruments available for the central bank when it supplies money in a rationed way compared to a conventional monetary policy regime or a Pigouvian tax/subsidy. In fact, the fraction of bonds eligible for open market operations κ_t^B and the repo share Ω_t can be adjusted by the central bank to support a particular equilibrium allocation and associated prices, such that (40) is not a binding constraint for the policy maker. It should further be noted that the long-run inflation rate π in principle depends on the growth rate of treasuries Γ (see 42). Yet, the central bank can implement a desired inflation rate by suited adjustments of its instruments κ_t^B and Ω_t, as shown in Appendix C. Thus, the inflation rate actually serves as a choice variable of the central bank, while (40)-(42) can be ensured to be satisfied by suited choices of κ_t^B and Ω_t. Under money rationing, the central bank therefore has three instruments at its disposal, namely, the inflation rate π_t, the policy rate R_t^m, and the share of purchased loans κ_t.

Hence, a competitive equilibrium under money rationing can be summarized by a set of sequences $\{c_{b,t},\, c_{l,t},\, h_{b,t},\, q_t,\, R_t^L\}_{t=0}^{\infty}$ satisfying (25), (28), (37), (38), and (39) for a monetary policy setting $\{\kappa_t,\, R_t^m,\, \pi_t\}_{t=0}^{\infty}$. Notably, the inflation rate only affects the conditions (25) and (38), where it appears jointly with the loan rate to measure the real interest rate R_t^L/π_{t+1}. Now, suppose again that there is no aggregate risk and that monetary policy is time-invariant, $\pi_t = \pi \geq 0$, $R_t^m = R^m \in [1, R^L)$ and $\kappa_t = \kappa \geq 0$. To further simplify the notation, we introduce the price discount s_t, i.e. the ratio between the loan rate and the policy rate

$$s_t = R_t^L/R_t^m > 1,$$

which serves as a policy instrument (instead of R^m). Using that all variables are then time-invariant and substituting out q and R^L/π with (39) and (38) in (25) and (37), a competitive equilibrium under money rationing can be reduced to a set $\{c_b, c_l, h_b\}$ satisfying $y = c_l + c_b$,

$$\left[\frac{1-\kappa}{1-\kappa s}\right] u_{c_b} = u_{c_l}\left(1 + \frac{1-\beta}{z}\frac{u_{h_l} - u_{h_b}}{u_{h_l}}\right), \tag{43}$$

$$c_b - c_l = h_b \frac{z}{1-\beta}\frac{u_{h_l}}{u_{c_l}} \cdot \left[\frac{(1-\kappa)(2-\kappa s)}{(1-\kappa s)}\right], \tag{44}$$

(where $u_{c_b} = \epsilon_b(\delta - c_b)$, $u_{c_l} = \epsilon_l(\delta - c_l)$, $u_{h_l} = \gamma - (h - h_b)$, and $u_{h_b} = \gamma - h_b$) given the policy instruments $\kappa \in (0, 1/s)$ and $s = R^L/R^m > 1$. For a given allocation $\{c_b, c_l, h_b\}$ and policy $\{\kappa, s\}$, the borrowers' real interest rate r_b^{ap} is determined by (24). Notably, the inflation rate can still be chosen by the central bank. In fact, by raising the inflation rate, the central bank can induce an increase in the nominal loan rate R^L.

We can now easily show that the constrained efficient allocation can be implemented by a suited asset purchase regime setting $\{\kappa, s\}$. For this, we compare the latter with the corresponding conditions under the Pigouvian subsidy $\widetilde{\tau}^L < 0$ (see Proposition 2) that implements the constrained efficient allocation $\{c_l, c_b, h_b\}$ satisfying $y = c_l + c_b$,

$$\left[1 - \widetilde{\tau}^L\right] u_{c_b} = u_{c_l}\left(1 + \frac{1-\beta}{z}\frac{u_{h_l} - u_{h_b}}{u_{h_l}}\right), \tag{45}$$

$$c_b - c_l = h_b \frac{z}{1-\beta}\frac{u_{h_l}}{u_{c_l}} \cdot [2], \tag{46}$$

where (45) stems from (32), and (46) from combining (27) and (31). The comparison of the terms in square brackets in (43)-(44) with the corresponding terms in (45)-(46) immediately reveals that the implementation of the constrained efficient allocation under the Pigouvian subsidy requires the monetary policy instruments, s and κ, to satisfy $\frac{1-\kappa}{1-\kappa s} = 1 - \widetilde{\tau}^L$ and $\frac{(1-\kappa)(2-\kappa s)}{(1-\kappa s)} = 2$, and therefore

$$\kappa = -\widetilde{\tau}^L > 0 \text{ and } s = 2/(1 - \widetilde{\tau}^L). \tag{47}$$

Thus, the central bank can implement the constrained efficient allocation by purchasing loans up to a fraction κ that equals the subsidy rate and by offering a price $1/R^m = (1/R^L) \cdot 2/(1+\kappa)$ (see 47), which is in principle feasible for $\kappa \in (0, 1/s)$ and $s > 1$. This equivalence result is summarized in the following proposition.

Proposition 3 *Suppose that money supply is rationed and $z \geq 1 - \beta$. Then, the constrained efficient allocation under the Pigouvian subsidy can be implemented by the central bank via asset purchases.*

As shown above, the central bank can implement the constrained efficient allocation under the Pigouvian subsidy by purchasing loans, such that its instruments κ and s satisfy (47). In fact, the Pigouvian subsidy directly alters the effective borrowers' real interest rate r_b^τ, whereas asset purchases directly change the effective lenders' real interest rate r_l^{ap}. As in the case of the Pigouvian subsidy $\widetilde{\tau}^L < 0$, asset purchases tend to reduce the equilibrium real loan rate R^L/π, which is the relevant rate for borrowers (see 36), compared to the case without loan purchases (see also Section 4.2). At the same time the lender's effective real rate r_l^{ap} increases, such that the representative borrower (lender) consumes more (less) than without asset purchases (see Proposition 2).

An asset purchase policy can, however, not only affect the real interest rates of borrowers and lenders, r_b^{ap} and r_l^{ap}, in different ways (like the Pigouvian subsidy), but can further alter borrowing conditions. Yet, first best cannot be implemented with an asset purchase policy. To see this, recall that under first best $u_{c_b} = u_{c_l}$ holds (see 22). Condition (43) would in this case imply $\frac{1-\kappa}{1-\kappa s} > 1$ to equal $1 + \frac{1-\beta}{z} \frac{u_{h_l} - u_{h_b}}{u_{h_l}}$ and thus $u_{h_l} > u_{h_b} \Rightarrow h_b > h_l$, which violates the second requirement for a first best allocation (see 22). Intuitively, because an increase in borrowing has to be associated by a higher value of collateral, i.e. housing, asset purchases cannot implement the first best equilibrium.

Nevertheless, an asset purchase policy can implement allocations that welfare-dominates allocations under a Pigouvian subsidy. Importantly, the instruments $\{\kappa, s\}$ do not affect the private sector behavior in identical ways (see 43 and 44), since a higher price discount s (either induced by a reduction in the policy rate R^m or an increase in the inflation rate π) increases the amount of money supplied per loan, whereas a higher κ increases the fraction of purchased loans. To enhance social welfare compared to the constrained efficient allocation, the central bank can, on the one hand, ease the constraint imposed on borrowers' consumption (see 44) compared to the case of the Pigouvian subsidy (46) by setting $\{\kappa, s\}$ to satisfy

$$\frac{(1-\kappa)(2-\kappa s)}{(1-\kappa s)} > 2 \tag{48}$$

$\Leftrightarrow s > 2/(1+\kappa)$. Given that the borrowing constraint is effectively relaxed under (48), the consumption differential $c_b - c_l$, which is under a binding borrowing constraint inefficiently small, can be increased compared to the constrained efficient allocation under a Pigouvian subsidy (as described in Proposition 2). On the other hand, any change in the instruments also affects the "monetary subsidy rate" $\frac{1-\kappa}{1-\kappa s}$, which has a distortionary effect on the allocation via its direct impact on the relative price of housing q/R^L:

$$\frac{q}{R^L} = \frac{u_{h_l}}{(1-\beta)u_{c_l}} \cdot \frac{1-\kappa}{1-\kappa s}, \tag{49}$$

Since the terms $\frac{(1-\kappa)(2-\kappa s)}{1-\kappa s}$ and $\frac{1-\kappa}{1-\kappa s}$ are not identical, the central bank can use two channels for its two instrument at disposal $\{\kappa, s\}$. Specifically, it can in principle relax borrowing conditions by ensuring (48) and simultaneously steer relative prices in an efficient way considering (49), which includes addressing the pecuniary externality associated with the borrowing constraint. Since $\frac{(1-\kappa)(2-\kappa s)}{1-\kappa s}$ is monotonically increasing in s but not in κ, an optimal choice of both instruments would lead to an infinitely large value for s (with monotonically decreasing welfare gains, see Section 4.2). For the subsequent analysis, we therefore examine the problem for a given value for the price discount s and assess the optimal fraction of purchased loans κ. By treating one instrument as given, the optimal policy problem is then analogous to the problem in (2). Concretely, to manipulate relative prices in an (constrained) efficient way, the choice for κ for a given s has to satisfy

$$\frac{\frac{1-\kappa}{1-\kappa s} \cdot u_{c_b} - u_{c_l}}{u_{c_b} - u_{c_l}} = \frac{1 + h_b\left(-u_{h_l h_l}\right)/u_{h_l}}{1 + \frac{1}{1-\beta} h_b u_{h_l}\left(-u_{c_l c_l}\right) u_{c_l}^{-2} \cdot \frac{z}{2}\frac{(1-\kappa)(2-\kappa s)}{1-\kappa s}}, \tag{50}$$

which corresponds to the condition for the optimal borrowing subsidy (34). Hence, an asset purchase policy $\kappa \in (0, 1/s)$ and $s > 1$ that satisfies (48) and (50) implements an allocation that welfare-dominates the constrained efficient allocation under a Pigouvian subsidy. This result is summarized in the following proposition.

Proposition 4 *Suppose that money supply is rationed and there is no aggregate risk. Then, the first best equilibrium cannot be implemented, while the central bank can implement allocations via asset purchases that welfare-dominate allocations that are implementable under a Pigouvian subsidy.*

Notably, the monetary subsidy rate $\frac{1-\kappa}{1-\kappa s}$ is increasing in the fraction of purchased loans κ and in the price discount s. The factor $\frac{(1-\kappa)(2-\kappa s)}{1-\kappa s}$ that alters the tightness of the borrowing constraint (44) is also increasing in s, while it is increasing (decreasing) in κ for $(1-\sqrt{s-1})/s < \kappa < 1/s$, and decreasing for $\kappa < (1-\sqrt{s-1})/s$. Hence, if for example the central bank increases the price discount s, it raises the term $\frac{(1-\kappa)(2-\kappa s)}{(1-\kappa s)}$ and thereby relaxes the borrowing constraint. The fraction κ then has to be chosen to control for the distortion of the relative price (see 49). Condition (50) indicates that larger values for $\frac{(1-\kappa)(2-\kappa s)}{(1-\kappa s)}$ and thus an increase in borrowing demands a lower value for the monetary subsidy rate $\frac{1-\kappa}{1-\kappa s}$. Hence, social welfare can be enhanced compared to the constrained efficient allocation under a Pigouvian subsidy, by increasing s and reducing κ according to (50) compared to the case where the Pigouvian subsidy is replicated (47). In the subsequent section, where provide numerical analyses, we confirm these results.

4 Numerical results

In this section, we provide numerical examples illustrating the analytical results derived in the previous section. To facilitate the parametrization of the model, we introduce a more standard (CRRA) utility function. Applying such a utility function, however, implies that we cannot easily aggregate over individual households as in Section 3. To focus on the effects of central bank asset purchases, we again simplify the analysis and abstract from implications of an endogenous distribution of agents' net wealth. For this, we assume that funds are pooled within households at the end of each period, such that household members are identical at the beginning of each period before they split up into borrowers and lenders. An equilibrium in terms of a representative borrower and a representative lender then only differs from the previous version by non-linear – instead of linear – marginal utilities. In the last part of this section, we further introduce aggregate risk via a random aggregate endowment and demonstrate that state contingent asset purchases should be conducted in a countercyclical way.

4.1 A version with CRRA preferences

We consider infinitely many households of measure one, which consist of infinitely many members i. In each period, ex-ante identical household members draw the idiosyncratic preference shock, which induces some members to borrow and others to lend. Like Lucas and Stokey (1987) or Shi (1997), we assume that at the end of each period (after loans are repaid) household members obtain equal shares of total household wealth, such that they are again equally endowed before new preference shocks are drawn in the next period. Thus, aggregation is facilitated by a redistribution of wealth within each household (rather than by linearity of agents' behavioral relations). We assume that period utility of household member i is given by a separable CRRA utility function

$$u^{CRRA}(\epsilon_i, c_{i,t}, h_{i,t}) = \epsilon_i \frac{c_{i,t}^{1-\sigma} - 1}{1-\sigma} + \gamma \frac{h_{i,t}^{1-\sigma} - 1}{1-\sigma}, \text{ where } \gamma, \sigma > 0, \tag{51}$$

and $i \in \{b, l\}$, such that Assumption 1 (and thus 23) does not apply. We further allow for aggregate risk in form of a random process for aggregate endowment y_t, which will be examined in Section 4.3. Specifically, we assume that the log of aggregate endowment follows an AR1 process

$$\log y_t = \rho \log y_{t-1} + \varepsilon_{y,t}. \tag{52}$$

Otherwise, the model presented in Section 2 is unchanged, such that the competitive equilibrium in terms of a representative borrower and a representative lender is identical to those given in Definitions 1 and 2, except for marginal utilities being non-linear in this version (see Definition 4 in Appendix D). As in the case of linear-quadratic preferences, it can be shown that a constrained efficient allocation without aggregate risk can be implemented by a Pigouvian subsidy, $\tau^L < 0$ (see Appendix E), while the constrained efficient allocation can again be implemented via asset purchases, given that Proposition 3 apparently holds for both types of preferences (see 43-46).

To solve the model numerically, we have to assign values for the elasticity of intertemporal substitution σ, the discount factor β, the utility weight for housing γ, the liquidation value of collateral z, the degree of household heterogeneity $\Delta\epsilon = \epsilon_b - \epsilon_l$, the autocorrelation coefficient ρ of the AR1 process, and the standard deviation of the innovations σ_ε.[20] We interpret a model period as one year and calibrate the model consistent with postwar US data. We estimate the process (52) using (linearly detrended) annual US data for real gdp per capita (for 1947-2008), leading to $\rho = 0.752$ and $\sigma_\varepsilon = 0.0216$. The value for the elasticity of intertemporal substitution σ is set equal to 2, which is a typical value applied in business cycle studies. The constant liquidation value of collateral z is set equal to 0.55, which is similar to values applied in related studies (see Iacoviello, 2005, or Garriga et al., 2015). For the remaining three parameters, β, γ, and $\Delta\epsilon$, we apply values that allow to match three targets for the reference case without financial market interventions.[21] The first target is the mean share of installment loans to income of 21% (for 1998-2004, see Survey of Consumer Finances), which correspond to the specification in our model, where loans are demanded for consumption and not for housing. The second target is the mean yield on MBS of 6.6% for pre-2009 US data, taken from Hancock and Passmore (2011), which corresponds to the rate on secured loans $R^L - 1$. The third target is the cross sectional standard deviation of real log consumption of 0.64 (see De Giorgi and Gambetti, 2012). While it is not possible to exactly match all three targets, our choice $\beta = 0.8$, $\gamma = 0.002$, and $\Delta\epsilon = 0.76$ yields to a reasonable match given by $R^L = 1.06$, $(l/R^L)/y = 0.2$, and a standard deviation of real log consumption of 0.6.

4.2 Welfare gains of asset purchases without aggregate risk

We first consider the case without aggregate risk ($\sigma_\varepsilon = 0$) and compute the equilibrium allocation and associated prices for different types of policy regimes. As a references case, we consider a laissez-faire regime, i.e. where monetary policy is conducted in a conventional way and does therefore not affect the equilibrium allocation (see Corollary 2). Figure 1 shows how asset purchases, which require monetary policy instruments to satisfy $\kappa \in (0, 1/s)$ and $s > 1$, affect market prices and the equilibrium allocation. The effects are computed for a range of values of $\kappa \in (0, 0.75)$ and $s \in (1, 1.2)$, while the inflation rate is fixed at a value ($\pi - 1 = 7\%$) that implies a positive equilibrium loan rate, $R^L > 1$. All variables are expressed in terms of percentage deviations from their corresponding laissez faire values. Higher values for the share of purchased loans κ as well as a larger price discount s increase the effects of the central bank intervention, while their impact on prices and quantities is not unambiguous. As revealed in the first row of Figure 1, higher values

[20] We further have to assign values to the growth rate of treasuries Γ and for the repo share Ω. Given that both are not relevant for the equilibrium allocation under the current set of central bank instruments, we apply the values $\Gamma = \pi$ and $\Omega = 1$, for convenience.

[21] Notably, the data samples are not alligned due to limited data availability.

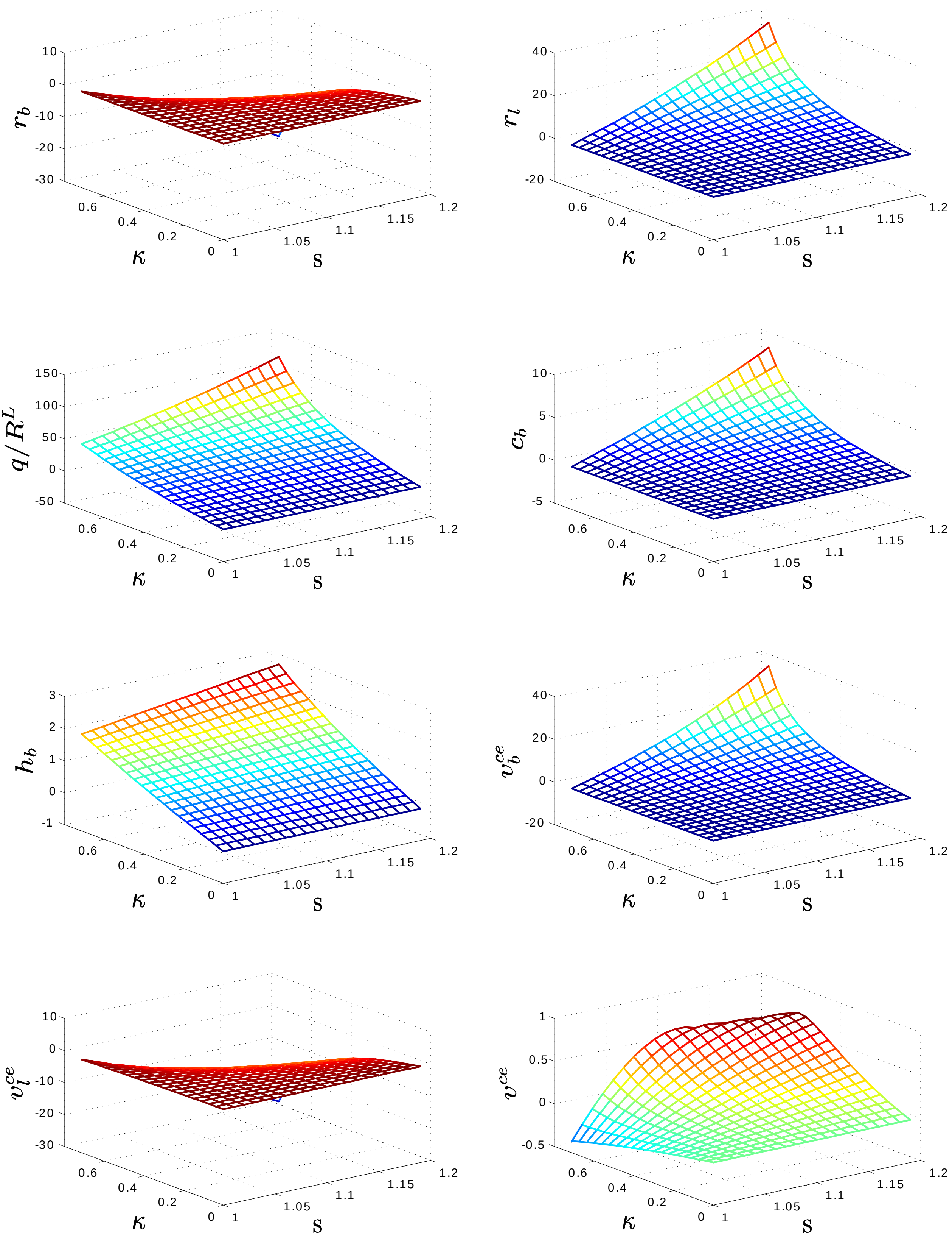

Figure 1: Effects of κ and s for $\pi = 1.07$ (in % deviations from laissez faire values)

for κ and s reduce the real rate for borrowers r_b, whereas they tend to raise the real rate for lender r_l. Notably, the latter is not the case for combinations of low price discounts s and high shares of purchased loans κ, where lenders receive a larger amount of cash at a small discount, such that they tend to consume more and their real rate r_l is below the laissez faire case. Overall, the price effects of the policy interventions can be relatively large, in particular for the relative housing price q/R^L, which can be more than twice as large as in the laissez faire case for a combination of high κ and high s. Consistently, borrowers consumption also tend to increase in these cases, except for combinations of a low s and a high κ. The effects on borrowers' housing reveal that a larger share of purchased loans κ tend to raise collateral demand, which is not generally the case for a larger discount s, where more money is supplied by the central bank per unit of loan.

In the last two columns of Figure 1 we present the welfare effects of asset purchases. For this, we compute the utility values for a representative borrower and a representative lender, $u^{CRRA}(\epsilon_b, c_b, h_b)$ and $u^{CRRA}(\epsilon_l, c_l, h_l)$, and present the ex-ante permanent consumption equivalents for their welfare $v_b^{ce} = \left[(1-\sigma)\, u^{CRRA}(\epsilon_b, c_b, h_b)\right]^{1/(1-\sigma)}$ and $v_l^{ce} = \left[(1-\sigma)\, u^{CRRA}(\epsilon_l, c_l, h_l)\right]^{1/(1-\sigma)}$, as well as for social welfare, $v^{ce} = [(1-\sigma)\, 0.5\{u^{CRRA}(\epsilon_l, c_l, h_l) + u^{CRRA}(\epsilon_b, c_b, h_b)\}]^{1/(1-\sigma)}$. Apparently, welfare of the representative lender falls with larger values for κ and s, whereas welfare of the representative borrower tends to increases. Notably, the welfare gain for borrowers increases by more than 30% compared to the laissez faire case. Yet, the impact on social welfare is much smaller as the two welfare components are affected in opposite ways. While social welfare tends to increase compared to the laissez faire case for larger values of κ and s, changes in social welfare are non-monotonic as the decline in lenders' welfare strongly increases for the largest combinations of κ and s. In total, the welfare gain does not exceed 1% of social welfare under laissez faire.

To see how the monetary policy instruments under asset purchases should be combined in the most efficient way, we refer to the analysis of Section 3.3. We again set the inflation rate, which shifts the associated nominal loan rate, equal to 1.07. We then vary the price discount $s > 1$ and compute a suited fraction of purchased loans κ according to the efficiency condition (50) as well as to (43), (44), and $y = c_l + c$, where we now apply the utility function (51). Figure 2 shows the macroeconomic effects of a change in s for two different values for the liquidation share of collateral, $z = 0.55$ (black solid line) and $z = 0.45$ (red dashed line), while we mark the associated values of the constrained efficient allocation under the Pigouvian subsidy with (blue) circles ($\kappa = -\widetilde{\tau}^L = 0.31$ for $z = 0.55$ and $\kappa = -\widetilde{\tau}^L = 0.38$ for $z = 0.45$). All values are now given in absolute terms, except for social welfare v^{ce}, which is again given in terms of percentage deviations from the corresponding laissez faire values.

As shown in the first line of Figure 2, a higher price discount s is accompanied by a higher value for the term $\frac{(1-\kappa)(2-\kappa s)}{1-\kappa s}$, which implies a relaxation of the borrowing constraint, and with a lower share of purchased loans κ, in accordance with the efficiency condition (50). The lower value of κ in fact reduces the monetary subsidy rate $\frac{1-\kappa}{1-\kappa s}$, to correct for distortionary effects on market prices

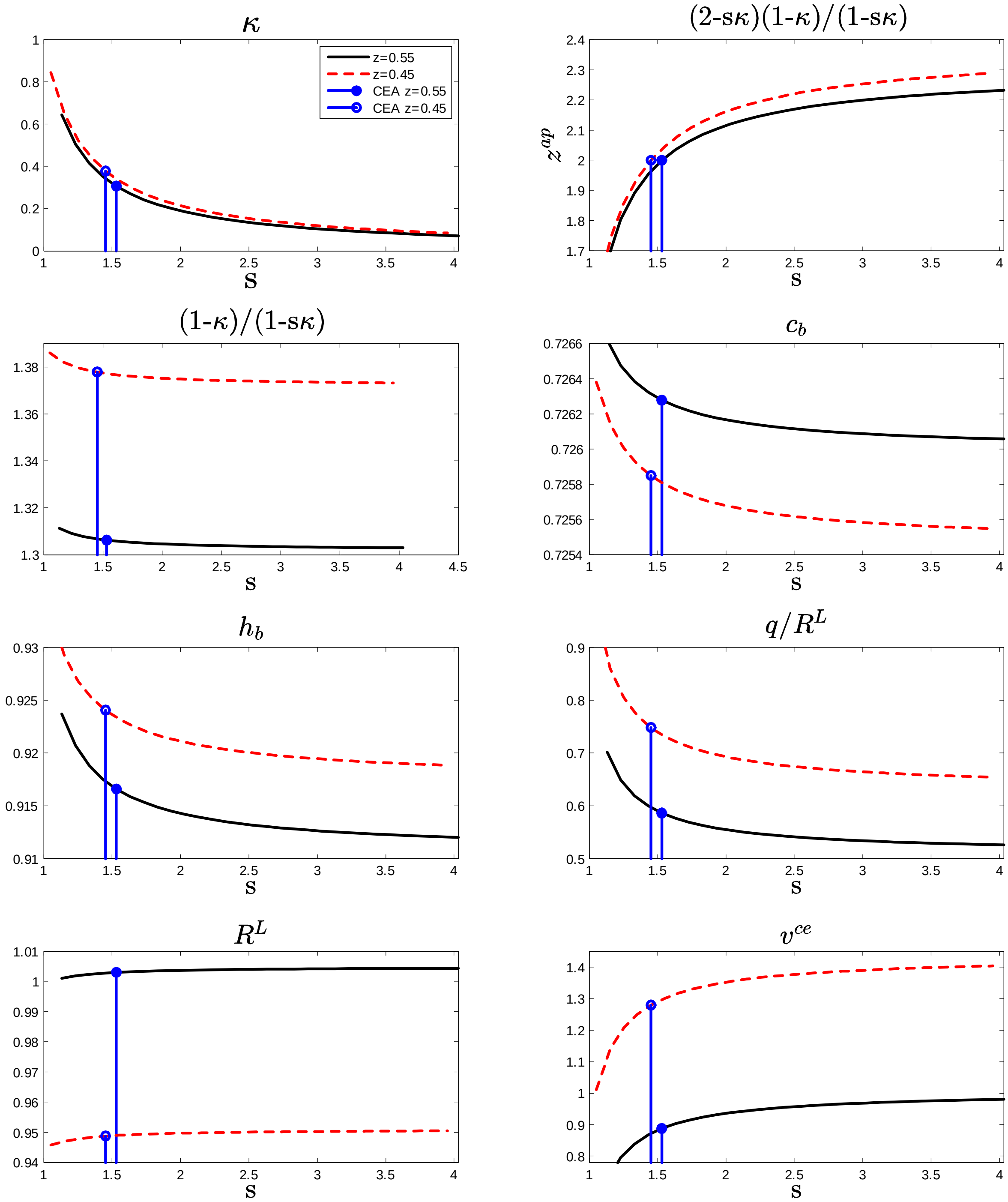

Figure 2: Equilibrium objects under asset purchases and the constrained efficient allocation for variations in s [Note: All values are given in absolute terms, except for social welfare, which is in % deviations from laissez faire values.]

and to address the pecuniary externality. As a consequence, the relative price of housing q/R^L in fact decreases with the price discount s (consistent with 49). Remarkably, borrowers' consumption and borrowers' housing also decrease with s, while the percentage changes in consumption are relatively small, i.e. more than 10 times smaller than for housing. Hence, the relaxation in the borrowing constraint induced by a higher price discount predominantly reduces the demand for collateral. Overall, an increase in the price discount s and the associated reduction in κ enhance social welfare compared the constrained efficient allocation under the Pigouvian subsidy by relaxing the collateral requirement. As shown in Proposition 4 and displayed in the last panel of Figure 2, monetary policy can thus raise the welfare gain compared to the constrained efficient allocation. Concretely, the welfare gain under the Pigouvian subsidy of about 0.9% (compared to laissez faire) can be increased up to 1%, with diminishing gains of larger values of the price discount s. For a more severe collateral constraint, (see red dashed line for $z = 0.45$), the effects of changes in the price discount are comparable, while, intuitively, the absolute welfare gain from policy interventions is larger.

4.3 Aggregate risk and state contingent asset purchases

In the final part of the analysis, we introduce aggregate risk, by considering a positive standard deviation σ_ε for the aggregate endowment process (52), and examine state contingent asset purchases. Due to aggregate endowment shocks, welfare losses stemming from credit market imperfections can be amplified in the short-run, i.e. when the economy deviates from a stationary equilibrium due to $\varepsilon_t \neq 0$. To understand the welfare enhancing role of state contingent interventions, consider first the laissez faire case. When the economy is hit by an adverse endowment shock, $\varepsilon_t < 0$, both types of agents (borrowers and lenders) have less non-durable goods available for consumption. As the lenders' demand for housing shifts downward (see 31), the housing price and thereby the value of collateral fall, which tends to tighten the borrowing capacity of agents. Thus, borrowers particularly suffer from the adverse shock, implying that their marginal utility of consumption increases relatively more than the lenders' marginal utility of consumption. In such a situation, a policy of stimulating borrowing can be welfare enhancing when it mitigates the decline in borrowers' consumption. This can in principle be achieved by an asset purchase regime that manipulates borrowing conditions in a favorable way.

To identify welfare enhancing state contingent asset purchases, we set-up the problem of a social planer under uncertainty (see Appendix F). As discussed in Section 4.2, an optimal choice for the fraction of purchased loans κ and the price discount s would lead to an infinitely large value for the latter, while the welfare gains of increasing s above the value that implements the constrained efficient allocation under the Pigouvian subsidy are limited (see Figure 2). Hence, for the subsequent analysis we focus on exactly the latter case and show how this asset purchase policy responds to aggregate shocks. Precisely, we set the means for the fraction of purchased

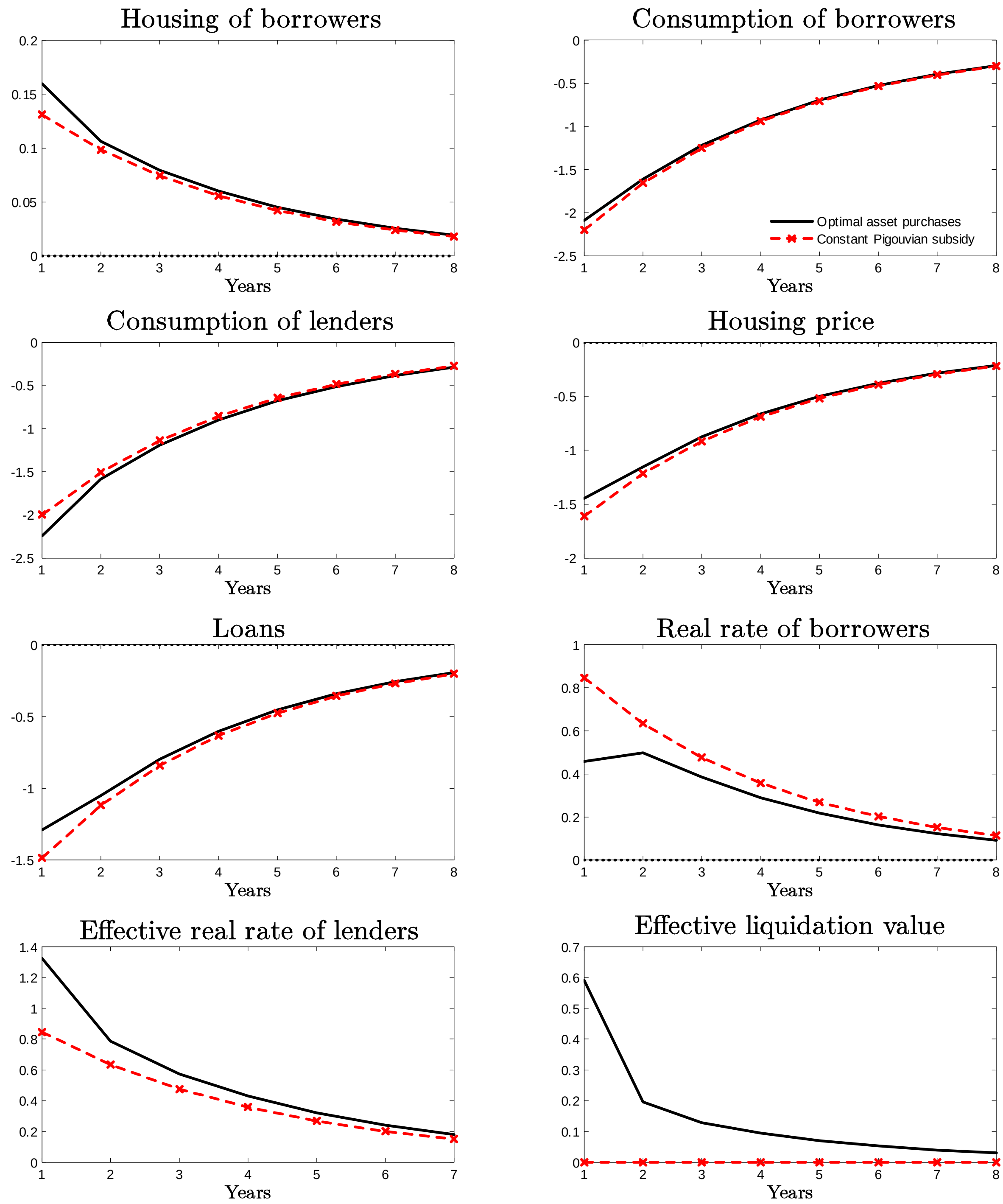

Figure 3: Responses to a minus one st.dev. aggregate endowment shock (in % deviations from a non-stochastic mean)

loans κ and of the price discount s to replicate the Pigouvian subsidy without aggregate risk (as in 47), while we set the price discount s_t in a state contingent way keeping κ_t constant. Following large parts of the literature on Ramsey policies, we disregard the issue of time-inconsistency and restrict our attention to time-invariant processes of the solution to the policy plan. To avoid the policy plan under commitment to exhibit a unit root, we introduce a fixed depreciation of housing to the fraction δ_h and an equally sized newly constructed supply of housing, to ensure a fixed supply h over time (see Appendix F).[22] Apparently, this assumption has neither an impact on the mechanism and on the main results. The model is then solved by applying a second order perturbation method (implemented in dynare).

Figure 3 presents impulse responses to a negative endowment shock by one standard deviation, which hits all agents equally. The black solid line shows the responses under the optimal asset purchase policy, while the red dashed line with crosses shows the responses for the case of a constant Pigouvian subsidy (with the identical long-run equilibrium allocation). Apparently, the adverse endowment shock reduces consumption of borrowers and lenders. While the differences in the responses of housing and consumption under both regimes are relatively small, differences in the interest rates are much more pronounced. The state contingent intervention via asset purchases apparently mitigates the adverse effects of the endowment shock on borrowers' consumption and housing by stimulating borrowing via a relaxation of the borrowing constraint, i.e. asset purchases raise the effective liquidation value of collateral $\widetilde{z}_t$, where

$$\widetilde{z}_t = \frac{z}{2}\frac{(1-\kappa)(2-\kappa s)}{(1-\kappa s)},$$

by varying the term $\frac{(1-\kappa)(2-\kappa s)}{1-\kappa s}$ (see last panel of Figure 3). Compared to the constant Pigouvian subsidy, the state contingent intervention reduces the increase in the real rate of borrowers $r^{ap}_{b,t}$ and amplifies the increase in the real rate of lenders $r^{ap}_{l,t}$. Thus, the state contingent asset purchase policy is countercyclical in the sense that it stimulates (dampens) borrowing in downturns (upturns).

To demonstrate how an asset purchase policy should respond to an exogenous worsening of financial conditions, we further examine an unexpected change in the liquidation value of collateral z_t.[23] A comparison of the responses under the state contingent asset purchase policy and under a constant Pigouvian subsidy (see Figure 4 in Appendix G), shows that an asset purchase policy can substantially mitigate the adverse effects of the liquidation value shock on loans and on borrowers' consumption. By raising the price discount s_t, which tends to lower (raise) the borrowers' (lenders') real interest rate, the central bank reduces the fall in loans, which is associated with an increase in borrowers' housing and thus in the price of housing. While borrowers' consumption decreases

[22] For the numerical analysis we set δ_h to 1%.

[23] Concretely, we assume that z_t is generated by $\log z_t = \rho \log z_{t-1} + (1-\rho)\log z_{t-1} + \varepsilon_{z,t}$, where the $\varepsilon's$ are i.i.d. with mean zero and standard deviations σ_ε (where the parameter values are adopted from Section 4.1, for convenience).

under a constant Pigouvian tax, the state contingent asset purchase policy is able to stabilize borrowers' consumption subsequent to the liquidation value shock. Overall, responses to both types of shocks imply that the ex-post asset purchase policy stimulates borrowing in adverse states and, symmetrically, mitigates the build-up of debt in favorable states of the economy. Hence, a state contingent asset purchase policy can be supportive for prudential financial regulations that aim at reducing the vulnerability in crisis times by reducing debt ex-ante.

5 Conclusion

This paper examines distributional effects of unconventional monetary policy that cannot be induced by conventional monetary policies. It is shown that an exchange of private debt securities against central bank money can enhance social welfare by stimulating the private debt market, which is particularly beneficial for borrowers facing relevant borrowing limits. We show that the central bank can incentivize (individually rational) lenders to enhance the supply of funds by purchasing debt securities at an above-market price. This causes the borrower's real interest rate to fall relative to the effective real interest of lenders, such that borrowers consume more and lenders less. Asset purchases can then enhance social by relaxing borrowing conditions and by addressing pecuniary externalities stemming from a borrowing constraint. These results are derived without referring to stressed financial markets or to a crisis scenario, implying that purchases of private debt securities can be a useful monetary policy instrument even in non-crisis times. We further show that under aggregate risk asset purchases should be conducted in a countercyclical way, which can be supportive for prudential (ex-ante) regulatory measures. The analysis of the interaction between ex-post interventions via asset purchases and prudential policies is beyond the scope of this paper and is left for future research.

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Appendix

A Competitive equilibrium

Definition 3 *A competitive equilibrium is a set of sequences* $\{c_{i,t},\ l_{i,t},\ i_{i,t},\ i^L_{i,t},\ \zeta_{i,t},\ \lambda_{i,t},\ h_{i,t},\ m^H_{i,t},\ b_{i,t},\ m^H_t,\ b_t,\ b^T_t,\ \pi_t,\ R^L_t,\ q_t\ \}_{t=0}^{\infty}$ *satisfying for all* $i \in [0,1]$

$$\begin{aligned}
\lambda_{i,t} &= \beta E_t[u'(\epsilon_i, c_{i,t+1})/\pi_{t+1}],\\
\frac{1}{R^L_t} &= \beta\frac{E_t\left[u'(\epsilon_i, c_{i,t+1})/\pi_{t+1}\right]}{u'(\epsilon_i, c_{i,t})} + \frac{\zeta_{i,t}}{u'(\epsilon_i, c_{i,t})} \ \ \text{or}\ \ \frac{1}{R^L_t} = \beta\frac{E_t\left[u'(\epsilon_i, c_{i,t+1})/\pi_{t+1}\right]}{u'(\epsilon_i, c_{i,t})}\cdot\frac{1-\kappa_t}{1-\kappa_t R^L_t/R^m_t},\\
c_{i,t} &= i_{i,t} + i^L_{i,t} + m^H_{i,t-1}\pi_t^{-1} - l_{i,t}/R^L_t \ \ \text{if}\ \psi_{i,t} > 0,\\
&\quad \text{or}\ c_{i,t} \leq i_{i,t} + i^L_{i,t} + m^H_{i,t-1}\pi_t^{-1} - l_{i,t}/R^L_t \ \ \text{if}\ \psi_{i,t} = 0,\\
R^m_t i_{i,t} &= \kappa^B_t b_{i,t-1}\pi_t^{-1} \quad \text{if}\ \eta_{i,t} > 0, \quad \text{or}\ R^m_t i_{i,t} < \kappa^B_t b_{i,t-1}\pi_t^{-1} \quad \text{if}\ \eta_{i,t} = 0,\\
R^m_t i^L_{i,t} &= \kappa_t l_{i,t} \quad \text{if}\ \mu_{i,t} > 0\ \text{or}\ R^m_t i^L_{i,t} \leq \kappa_t l_{i,t} \quad \text{if}\ \mu_{i,t} = 0,\\
-l_{i,t} &= z q_t h_{i,t} \quad \text{if}\ \zeta_{i,t} > 0,\ \text{or}\ -l_{i,t} \leq z_t q_t h_{i,t} \quad \text{if}\ \zeta_{i,t} = 0,\\
q_t\lambda_{i,t} &= u_{h,i,t} + \zeta_{i,t} z q_t + \beta E_t q_{t+1}\lambda_{i,t+1},\\
i_{i,t} &= (1+\Omega_t)m^H_{i,t} - m^H_{i,t-1}\pi_t^{-1},\\
b^T_t &= b_t + m^H_t,\\
b^T_t &= \Gamma b^T_{t-1}/\pi_t,
\end{aligned}$$

$0 = \sum_i l_{i,t},\ h = \sum_i h_{i,t},\ y = \sum_i c_{i,t},\ b_t = \sum_i b_{i,t},$ *and* $m^H_t = \sum_i m^H_{i,t}$, *where the multipliers* $\psi_{i,t}$, $\mu_{i,t}$, *and* $\eta_{i,t}$ *satisfy* $\psi_{i,t} = u'(\epsilon_i, c_{i,t}) - \beta E_t\left[u'(\epsilon_i, c_{i,t+1})/\pi_{t+1}\right] \geq 0$, $\mu_{i,t} = [(1/R^m_t) - (1/R^L_t)]u'(\epsilon_i, c_{i,t})/(1-\kappa) \geq 0$, *and* $\sum_i \eta_{i,t} = (\sum_i u'(\epsilon_i, c_{i,t})/R^m_t) - \beta E_t \sum_i\left[u'(\epsilon_i, c_{i,t+1})/\pi_{t+1}\right] \geq 0$, *the transversality conditions, a monetary policy setting* $\{R^m_t \geq 1,\ \kappa^B_t > 0,\ \kappa_t \in [0,1],\ \Omega_t > 0\}_{t=0}^{\infty}$, *given* $\Gamma > 0$, $\{y_t\}_{t=0}^{\infty}$, *and initial values* $m^H_{i,-1} = m^H_{-1} > 0$, $b_{i,-1} = b_{-1} > 0$, $h_{i,-1} = h_{-1} = 1$ *and* $b^T_{-1} > 0$.

Let $c_{l,t} = 2\sum_{l,i} c_{l,i,t}$, $h_{l,t} = 2\sum_{l,i} h_{l,i,t}$, $l_{l,t} = 2\sum_{l,i} l_{l,i,t}$, $\lambda_{l,t} = 2\sum_{l,i}\lambda_{l,i,t}$, $i_{l,t} = 2\sum_{l,i} i_{l,i,t}$, $c_{b,t} = 2\sum_{b,i} c_{b,i,t}$, $h_{b,t} = 2\sum_{b,i} h_{b,i,t}$, $l_{b,t} = 2\sum_{b,i} l_{b,i,t}$, $\lambda_{b,t} = 2\sum_{b,i}\lambda_{b,i,t}$, and $i_{b,t} = 2\sum_{b,i} i_{b,i,t}$. Based on the Assumptions 1 and 2 and the law of large numbers, the set of conditions that describe the behavior for a representative lender, i.e. a representative agent drawing ϵ_b in period t, is given by

$$\begin{aligned}
&0.5m^H_{t-1}\pi_t^{-1} + 0.5b_{t-1}\pi_t^{-1} + l_{l,t}\left(1 - 1/R^L_t\right) + 0.5y_t + 0.5\tau_t \qquad (53)\\
&= m^H_{l,t} + (b_{l,t}/R_t) + 0.5\left(i_{l,t} + i_{b,t}\right)\left(R^m_t - 1\right) + c_{l,t} + q_t\left(h_{l,t} - 0.5h\right)
\end{aligned}$$

$$\lambda_{l,t} = \epsilon_l(\delta - c_{l,t})/R^L_t, \qquad (54)$$

$$q_t\lambda_{l,t} = \gamma - h_{l,t} + \beta E_t q_{t+1}\lambda_{l,t+1}, \qquad (55)$$

$$\frac{\epsilon_l(\delta - c_{l,t})}{R^L_t} = \beta E_t\left[\frac{0.5(\epsilon_l(\delta - c_{l,t+1}) + \epsilon_b(\delta - c_{b,t+1}))}{\pi_{t+1}}\right], \qquad (56)$$

$$c_{l,t} = 0.5\left(i_{l,t} + i_{b,t}\right) + 0.5m^H_{t-1}\pi_t^{-1} - l_{l,t}/R^L_t, \qquad (57)$$

where $m^H_{l,t-1} = \Sigma_{l,i} m^H_{l,i,t-1} = 0.5m^H_{t-1}$, $b_{l,t-1} = \sum_{l,i} b_{l,i,t-1} = 0.5b_{t-1}$, and $h_{l,t-1} = \sum_{l,i} h_{l,i,t-1} =$

0.5h. Applying the Assumptions 1 and 3, the set of conditions describing the behavior of a representative borrower, i.e. a representative agent drawing ϵ_b in period t, is

$$\begin{aligned} & 0.5m_{t-1}^H\pi_t^{-1} + 0.5b_{t-1}\pi_t^{-1} + l_{b,t}\left(1 - 1/R_t^L\right) + 0.5y_t + 0.5\tau_t \\ &= m_{b,t}^H + (b_{b,t}/R_t) + 0.5\left(i_{l,t} + i_{b,t}\right)\left(R_t^m - 1\right) + c_{b,t} + q_t\left(h_{b,t} - 0.5h\right), \end{aligned} \tag{58}$$

$$\lambda_{b,t} = \beta E_t\left[0.5(\epsilon_l(\delta - c_{l,t+1}) + \epsilon_b(\delta - c_{b,t+1}))/\pi_{t+1}\right], \tag{59}$$

$$q_t\lambda_{b,t} = \gamma - h_{b,t} + \zeta_{b,t}zq_t + \beta E_t q_{t+1}\lambda_{b,t+1}, \tag{60}$$

$$\frac{\epsilon_b(\delta - c_{b,t})}{R_t^L} = \beta E_t\left[\frac{0.5(\epsilon_l(\delta - c_{l,t+1}) + \epsilon_b(\delta - c_{b,t+1}))}{\pi_{t+1}}\right] + \zeta_{b,t}, \tag{61}$$

$$c_{b,t} = 0.5\left(i_{l,t} + i_{b,t}\right) + 0.5m_{t-1}^H\pi_t^{-1} - l_{b,t}/R_t^L, \tag{62}$$

$$-l_{b,t} = zq_t h_{b,t}, \tag{63}$$

where $m_{b,t-1}^H = \Sigma_{b,i}m_{b,i,t-1}^H = 0.5m_{t-1}^H$, $b_{b,t-1} = \sum_{b,i} b_{b,i,t-1} = 0.5b_{t-1}$, and $h_{b,t-1} = \sum_{b,i} h_{b,i,t-1} = 0.5h$. Using that $h = h_{l,t} + h_{b,t}$, $l_t = l_{l,t} = -l_{b,t}$, and that (54), (56), and (59) imply $\lambda_t = \lambda_{b,t} = \lambda_{l,t}$, and substituting out $\zeta_{b,t}$, λ_t, and l_t, leads to the set of conditions (24)-(28).

B Proof of Proposition 2

The problem of the social planer problem (33), which aims at maximizing ex-ante social welfare, can be written as a static

$$\begin{aligned} \max_{c_b,c_l,h_b}\min_{\chi_1,\chi_2}(1-\beta)^{-1}\ &\{[u(\epsilon_b,c_b,h_b) + u(\epsilon_l,c_l,h-h_b)] \\ &+ \chi_1\left[y - c_b - c_l\right] + \chi_2\left[2zh_b\cdot(q/R^L) - c_b + c_l\right]\}, \end{aligned} \tag{64}$$

where $(q/R^L) = \frac{1}{1-\beta}u_{h_l}/u_{c_l}$. The first order conditions are $u_{c_b} = \chi_1 + \chi_2$,

$$\begin{aligned} u_{c_l} &= \chi_1 - \chi_2\left(1 + 2zh_b\cdot\left[\partial(q/R^L)/\partial c_l\right]\right), \\ u_{h_l} - u_{h_b} &= \chi_2\left(2z\cdot\left(q/R^L\right) + 2zh_b\cdot\left[\partial(q/R^L)/\partial h_b\right]\right), \end{aligned}$$

where $\partial(q/R^L)/\partial c_l = \frac{u_{h_l}}{1-\beta}(-u_{c_lc_l})/u_{cl}^2 > 0$ and $\partial(q/R^L)/\partial h_b = \frac{1}{1-\beta}(-u_{h_lh_l})/u_{c_l} > 0$. Substituting out the multipliers χ_1 and χ_2 as well as (q/R^L) with $(q/R^L) = \frac{1}{1-\beta}u_{h_l}/u_{c_l}$, we get the following condition for the constrained efficient allocation

$$\frac{u_{c_b} - u_{c_l}}{u_{h_l} - u_{h_b}}\frac{z}{1-\beta}\frac{u_{h_l}}{u_{c_l}} = \frac{1 + zh_b\cdot\left[\partial(q/R^L)/\partial c_l\right]}{1 + \left[\partial(q/R^L)/\partial h_b\right]\cdot h_b/(q/R^L)}. \tag{65}$$

To disclose the implications for the tax/subsidy rate, which is associated with this policy, we compare (65) with the competitive equilibrium condition (32), which can be rewritten as

$$\frac{(1-\tau^L)u_{c_b} - u_{c_l}}{u_{h_l} - u_{h_b}}\frac{z}{1-\beta}\frac{u_{h_l}}{u_{c_l}} = 1. \tag{66}$$

Apparently, the LHS of (66) differs from the LHS of (65) only by the tax rate τ^L, while the RHSs differ due to the derivatives of the relative price (q/R^L). For $z(q/R^L) \cdot \left[\partial(q/R^L)/\partial c_l\right] < \left[\partial(q/R^L)/\partial h_b\right] \Leftrightarrow$

$$z(q/R^L)\frac{-u_{c_lc_l}}{u_{c_l}} < \frac{-u_{h_lh_l}}{u_{h_l}}, \tag{67}$$

the RHS of (66) is smaller than one, implying a subsidy $\tau^L < 0$. Inserting the derivatives of the utility function (23), $u_{c_l} = \epsilon_l(\delta - c_l)$, $u_{c_lc_l} = -\epsilon_l$, $u_{h_l} = \gamma - (h - h_b)$, $u_{h_lh_l} = -1$, and using used the constraints $c_b - c_l = 2zh_b(q/R^L)$ and $y = c_l + c_b$, the inequality (67) can be rewritten as

$$\frac{1-\beta}{z}\epsilon_l\left(\frac{c_b - c_l}{2h_b}\right)^2 < 1.$$

Using that (29) implies $h_b \geq 0.5h$, $\epsilon_l < 1$, and that $y = h = 1$, we can conclude that $(1-\beta)/z \leq 1$ is a sufficient condition for (67) and thus for borrowing subsidy to be required for the implementation of the constrained efficient allocation $\tau^L < 0$.

We further seek to identify the impact of a subsidy on consumption and housing of the representative borrower. For this, we apply the competitive equilibrium conditions (28), (66), and $c_b - c_l = 2zh_b\frac{\gamma-(h-h_b)}{(1-\beta)\epsilon_l(\delta-c_l)}$, and substitute out c_l with $c_l = y - c_b$ to get $F(\boldsymbol{\tau}^L, h_b, c_b) = 0$ and $G(h_b, c_b) = 0$, where

$$\begin{aligned} F(\boldsymbol{\tau}^L, h_b, c_b) &= \frac{(1-\boldsymbol{\tau}^L)\cdot\epsilon_b(\delta - c_b) - \epsilon_l(\delta - y + c_b)}{(2h_b - h)(1-\beta)\epsilon_l(\delta - y + c_b)(1/z)} - \frac{1}{\gamma - h + h_b}, \\ G(h_b, c_b) &= 2zh_b\frac{\gamma - (h - h_b)}{(1-\beta)\epsilon_l(\delta - y + c_b)} - 2c_b + y. \end{aligned}$$

The partial derivatives of $G(h_b, c_b)$, where G_x abbreviates $\partial G/\partial x$, are given by

$$G_{h_b} = 2z\frac{2h_b - h + \gamma}{\epsilon_l(\delta - y + c_b)(1-\beta)} > 0, \quad G_{c_b} = -2\left(\frac{z}{\epsilon_l}\frac{h_b(\gamma - h + h_b)}{(1-\beta)(c_b - y + \delta)^2} + 1\right) < 0,$$

implying $\partial h_b/\partial c_b = -G_{c_b}/G_{h_b} > 0$. The partial derivatives of $F(\boldsymbol{\tau}^L, h_b, c_b)$ are given by

$$\begin{aligned} F_{\boldsymbol{\tau}^L} &= -\frac{\epsilon_b(\delta - c_b)}{\epsilon_l(1-\beta)(2h_b - h)(\delta - y + c_b)(1/z)} < 0, \; F_{h_b} = -\frac{2\gamma - h}{(2h_b - h)(\gamma - h + h_b)^2} < 0, \\ F_{c_b} &= -\frac{2\epsilon_b(\delta - y/2)\left(1 - \tau^L\right)}{\epsilon_l(\delta - y + c_b)^2(1-\beta)(2h_b - h)(1/z)} < 0. \end{aligned}$$

Thus, consumption of the representative borrower decreases with the tax rate, since

$$\partial c_b/\partial\tau^L = -(G_{h_b}F_{\tau^L})/(F_{c_b}G_{h_b} - F_{h_b}G_{c_b}) < 0.$$

Hence, introducing a subsidy $\tau^L < 0$ increases consumption and housing of the representative borrower (by $\partial h_b/\partial c_b > 0$). Given that consumption (housing) of lenders decreases for a given endowment (stock of housing), the lenders' consumption Euler equation (24), which can be written as $1 = \beta 0.5[1 + \epsilon_b(\delta - c_b)/(\epsilon_l(\delta - c_l))] \left(R^L/\pi\right)$, further implies that the real interest rate increases with the subsidy. ■

C Monetary policy and inflation

Suppose that government bonds are supplied at a rate that is not identical to the inflation target, $\Gamma \neq \pi^*$. Then, the total stock of bonds $b_t^T = b_t + m_t^H$ might grow or shrink in a long-run equilibrium at a constant rate Γ/π (see 42). The money demand condition (40) then requires for constant steady state values c_b, R^L, h_b, q, and z, that the term $\widetilde{m}_t = (1 + \Omega_t)m_t^H$ is also constant in the long-run. Combining (40), (41), and (42), leads to $\kappa_t^B b_t = R_t^m \pi_t[\widetilde{m}_t - \widetilde{m}_{t-1}(1+\Omega_{t-1})^{-1}\pi_t^{-1}]$ and $[b_t + \widetilde{m}_t/(1 + \Omega_t)] = \Gamma\left[b_{t-1} + \widetilde{m}_{t-1}/(1 + \Omega_{t-1})\right]/\pi_t$. Further, substituting out b_t, gives

$$\left[\frac{R_t^m \pi_t}{\kappa_t^B}\left(\widetilde{m}_t - \frac{\widetilde{m}_{t-1}\pi_t^{-1}}{1+\Omega_{t-1}}\right) + \frac{\widetilde{m}_t}{1+\Omega_t}\right] = \frac{\Gamma}{\pi_t}\left[\frac{R_{t-1}^m \pi_{t-1}}{\kappa_{t-1}^B}\left(\widetilde{m}_{t-1} - \frac{\widetilde{m}_{t-2}\pi_{t-1}^{-1}}{1+\Omega_{t-2}}\right) + \frac{\widetilde{m}_{t-1}}{1+\Omega_{t-1}}\right]. \quad (68)$$

Taking the limit $t \to \infty$ of both sides of (68), we can use that for a constant long-run inflation rate π and a constant policy rate R^m a steady state is characterized by a constant value for $\widetilde{m}_t$. The term in the square brackets in (68) grows/shrinks with the constant rate Γ/π. When the growth rate of bonds exceeds the inflation rate, $\Gamma > \pi$, this can be guaranteed by a permanently shrinking value for κ_t^B. Thus, the central bank can let κ_t^B grow at the rate π/Γ and can let the share of money supplied outright go to zero in the long-run, i.e. it can set κ_t^B and $1/\Omega_t$ according to $\lim_{t\to\infty} \kappa_t^B/\kappa_{t-1}^B = \pi/\Gamma < 1$ and $\lim_{t\to\infty} 1/\Omega_t = 0$ if $\Gamma > \pi$. For $\Gamma < \pi$, the term in the square bracket in (68) permanently shrinks, which can not be supported by a growing value κ_t^B without violating the restriction $\kappa_t^B \leq 1$. In this case, the central bank can let κ_t^B go to zero and can let the share $1/\Omega_t$ of money supplied outright grow in a long-run equilibrium. For $\pi = 1$ and $\Gamma < 1$, it can thus set κ_t^B and $1+1/\Omega_t$ in a steady state according to $\lim_{t\to\infty} (1 + 1/\Omega_t) / (1 + 1/\Omega_{t-1}) = 1/\Gamma > 1$ and $\lim_{t\to\infty} \kappa_t^B = 0$.

D A CRRA version with representative agents

Definition 4 *A competitive equilibrium of the economy with preferences satisfying (51) and wealth redistribution within households consists of a set of sequences* $\{c_{b,t},\ c_{l,t},\ \pi_t,\ R_t^L,\ h_{b,t},\ q_t,\ b_t,\ b_t^T,\ m_t^H\}_{t=0}^{\infty}$ *satisfying*

$$(1-\tau^L)\epsilon_b c_{b,t}^{-\sigma}/R_t^L = \beta E_t[0.5(\epsilon_b c_{b,t+1}^{-\sigma} + \epsilon_l c_{l,t+1}^{-\sigma})/\pi_{t+1}] + \gamma((h-h_{b,t})^{-\sigma} - h_{b,t}^{-\sigma})/[q_t z], \quad (69)$$

$$\epsilon_l c_{l,t}^{-\sigma}/R_t^L = \beta E_t[0.5(\epsilon_b c_{b,t+1}^{-\sigma} + \epsilon_l c_{l,t+1}^{-\sigma})/\pi_{t+1}]\frac{1-\kappa_t}{1-\kappa_t R_t^L/R_t^m}, \quad (70)$$

$$q_t \epsilon_l c_{l,t}^{-\sigma}\frac{1/R_t^L - \kappa_t/R_t^m}{1-\kappa_t} = \gamma\left(h-h_{b,t}\right)^{-\sigma} + \beta E_t\left[q_{t+1}\epsilon_l c_{l,t+1}^{-\sigma}\frac{1/R_{t+1}^L - \kappa_{t+1}/R_{t+1}^m}{1-\kappa_{t+1}}\right], \quad (71)$$

$$c_{b,t} - c_{l,t} \leq zq_t h_{b,t}\left[(2/R_t^L) - (\kappa_t/R_t^m)\right], \tag{72}$$

$$c_{l,t} + c_{b,t} = y_t, \tag{73}$$

$$0.5(1+\Omega_t)m_t^H \geq c_{b,t} + z_t q_t h_{b,t}/R_t^L, \tag{74}$$

$$\kappa_t^B b_{t-1}\pi_t^{-1}/R_t^m \geq (1+\Omega_t)m_t^H - m_{t-1}^H \pi_t^{-1}, \tag{75}$$

$$b_t^T = \Gamma b_{t-1}^T/\pi_t, \tag{76}$$

$$b_t^T = b_t + m_t^H, \tag{77}$$

the transversality conditions, a monetary policy setting $\{R_t^m \geq 1,\ \kappa_t \in [0,1],\ \kappa_t^B > 0,\ \Omega_t > 0\}_{t=0}^{\infty}$, *a tax/subsidy* τ^L, *given* $\{y_t\}_{t=0}^{\infty}$, $\Gamma > 0$, $b_{-1}^T > 0$, $b_{-1} > 0$, *and* $m_{-1}^H > 0$.

The first best allocation apparently satisfies $\epsilon_b c_{b,t}^{-\sigma} = \epsilon_l c_{l,t}^{-\sigma}$ and $h_{b,t} = h_{l,t} = 2h$. Under binding borrowing, liquidity, and money supply constraints, a competitive equilibrium without aggregate risk consists of a set $\{c_l,\ c_b,\ R^L,\ h_b,\ q\}$ satisfying

$$1/R^L = \beta\left(c_l^{\sigma}/\epsilon_l\right)0.5(\epsilon_b c_b^{-\sigma} + \epsilon_l c_l^{-\sigma})\pi^{-1}\frac{1-\kappa}{1-\kappa R^L/R^m}, \tag{78}$$

$$(1-\tau^L)\epsilon_b c_b^{-\sigma} = R^L\beta 0.5(\epsilon_b c_b^{-\sigma} + \epsilon_l c_l^{-\sigma})\pi^{-1} + R^L\left(\gamma/qz\right)\left((h-h_b)^{-\sigma} - h_b^{-\sigma}\right), \tag{79}$$

$$\gamma(h-h_b)^{-\sigma} = q\left(1-\beta\right)\epsilon_l c_l^{-\sigma}\frac{1/R^L - \kappa/R^m}{1-\kappa}, \tag{80}$$

$$c_b - c_l = zqh_b[(2/R^L) - (\kappa/R^m)], \tag{81}$$

$$y = c_l + c_b, \tag{82}$$

for a monetary policy setting $\{1 \leq R^m < R^L,\ \kappa \in [0,1),\ \pi > \beta\}$, and a tax/subsidy τ^L. Once the set $\{c_l,\ c_b,\ R^L,\ h_b,\ q\}$ is determined, the values m^H and b are given by $m^H = \left(c_b - zqh_b/R^L\right)\frac{1}{0.5(1+\Omega)}$ and $b = \frac{R^m\pi}{\kappa^B}\left(1+\Omega-\pi^{-1}\right)m^H$ given κ^B and Ω.

E Constrained efficiency under CRRA preferences

In this Appendix, we consider an economy under CRRA preferences and pooling of wealth within households as summarized in Definition 4. We will show that a constrained efficient allocation is again associated with a lump-sum financed borrowing subsidy, as already shown for the case of linear-quadratic preferences (see Proposition 2). Consider the economy as given in Definition 4 for $y_t = y$, $R^m = R^L$, and $\pi_t = \pi$. Given that conventional monetary policy measures do not affect the allocation and we restrict the tax/subsidy rate also to be constant, the equilibrium allocation and prices are time-invariant. Hence, the set $\{c_l,\ c_b,\ R^L,\ h_b,\ q\}$ has to satisfy (79), (82)

$$\epsilon_l c_l^{-\sigma}/R^L = \beta 0.5(\epsilon_b c_b^{-\sigma} + \epsilon_l c_l^{-\sigma})/\pi, \tag{83}$$

$$c_b - c_l \leq zqh_b 2/R^L, \tag{84}$$

$$\gamma(h-h_b)^{-\sigma} = q\beta(1-\beta)0.5(\epsilon_b c_b^{-\sigma} + \epsilon_l c_l^{-\sigma})/\pi, \tag{85}$$

given $\{\tau^L, \pi\}$. Substituting out the housing price q with (85) in (84), leads to

$$0 \leq zh_b 2 \frac{\gamma(h - h_{b,t})^{-\sigma_h}}{(1-\beta)\epsilon_l c_l^{-\sigma}} - c_b + c_l, \tag{86}$$

where we further used (83) to substitute out the real rate R^L/π.

Proposition 5 *Consider an economy without aggregate risk, with preferences satisfying (51), and wealth redistribution within households. The constrained efficient allocation can be implemented by a subsidy on borrowing, if but not only if $\epsilon_b/\epsilon_l \leq 3^\sigma$.*

Proof. The problem of a social planer, who aims at maximizing social welfare (21) by setting the tax/subsidy rate τ^L, can again be summarized as (64). Likewise, a constrained efficient allocation is associated with a borrowing subsidy if (67) is satisfied. Applying the partial derivatives of the CRRA utility function (51), $u_{c_l} = \epsilon_l c_l^{-\sigma}$, $u_{c_l c_l} = -\sigma u_{cl}/c_l$, $u_{h_l} = \gamma(h - h_b)^{-\sigma}$, $u_{h_l h_l} = -\sigma u_{hl}(h - h_b)^{-1}$, we can rewrite (67) as $((c_b/c_l) - 1)(h/h_b - 1) - 2 < 0$. Since the ratio c_b/c_l is smaller under a binding borrowing constraint than under first best, $(c_b^*/c_l^*) = (\epsilon_b/\epsilon_l)^{1/\sigma}$ and $h/2 \leq h_b \Leftrightarrow (h/h_b) - 1 \leq 1$ holds, we can conclude that $((c_b/c_l) - 1)(h/h_b - 1) - 2 < ((\epsilon_b/\epsilon_l)^{1/\sigma} - 1)(h/h_b - 1) - 2 \leq (\epsilon_b/\epsilon_l)^{1/\sigma} - 3$. Hence, the constrained efficient allocation requires a subsidy, $\tau^L < 0$, if but not only if the preference shock satisfies $\epsilon_b/\epsilon_l \leq 3^\sigma$. ■

F Asset purchases under aggregate risk

Suppose that housing depreciates every period at the rate δ_h, while new housing is constructed at the same rate, such that total supply again equals h. For an individual agent, the investment decision in housing is then described by

$$q_t \epsilon_l c_{l,t}^{-\sigma} \frac{1/R_t^L - \kappa_t/R_t^m}{1-\kappa_t} = \gamma (h - h_{b,t})^{-\sigma} + \beta E_t (1-\delta_h) q_{t+1} \epsilon_l c_{l,t+1}^{-\sigma} \frac{1/R_{t+1}^L - \kappa_{t+1}/R_{t+1}^m}{1-\kappa_{t+1}}, \tag{87}$$

instead of (71). For the analysis of an optimal asset purchase policy, we apply the conditions (69), (70), (72), (73), and (87) and define

$$\widetilde{z}_t = \frac{z}{2} \frac{(2 - \kappa_t s_t)(1-\kappa_t)}{1 - \kappa_t s_t}, \tag{88}$$

$$x_t = \frac{q_t}{R_t^L} \frac{1 - \kappa_t s_t}{1 - \kappa_t}, \tag{89}$$

where $s_t = R_t^L / R_t^m$. Further combining (69) and (70) to

$$\epsilon_b c_{b,t}^{-\sigma} \left(q_t/R_t^L\right) = \frac{1 - \kappa_t s_t}{1-\kappa_t} \epsilon_l c_{l,t}^{-\sigma} \left(q_t/R_t^L\right) + \gamma((h - h_{b,t})^{-\sigma} - h_{b,t}^{-\sigma})/z, \tag{90}$$

and recalling that the policy maker has two instruments at his disposal to adjust the two terms $\frac{(2-\kappa_t s_t)(1-\kappa_t)}{1-\kappa_t s_t}$ and $\frac{1-\kappa_t s_t}{1-\kappa_t}$, the constraints for an optimal choice of the set of sequences $\{c_{b,t}, c_{l,t}, h_{b,t} x_t\}_{t=0}^\infty$

are given by (73)

$$c_{b,t} - c_{l,t} \leq \widetilde{z} h_{b,t} 2 x_t, \tag{91}$$

$$\epsilon_l c_{l,t}^{-\sigma} x_t = \gamma (h - h_{b,t})^{-\sigma_h} + \beta E_t \epsilon_l c_{l,t+1}^{-\sigma} (1 - \delta_h) x_{t+1}, \tag{92}$$

while $\frac{1-\kappa_t s_t}{1-\kappa_t}$ has to be set according to (90). Hence, the planer problem under commitment can be summarized as $\max_{\{c_{b,t}, c_{l,t}, h_{b,t} x_t\}_{t=0}^{\infty}} E \sum_{t=0}^{\infty} \beta^t [u(\epsilon_b, c_{b,t}, h_{b,t}) + u(\epsilon_l, c_{l,t}, h - h_{b,t})]$ subject to (73), (91), and (92). Neglecting the conditions for period $t = 0$, the solution to the policy problem has to satisfy the following first order conditions

$$0 = \epsilon_b c_{b,t}^{-\sigma} - \mu_t - \lambda_t,$$
$$0 = \epsilon_l c_{l,t}^{-\sigma} + \mu_t - \lambda_t - \sigma \psi_t \epsilon_l c_{l,t}^{-\sigma-1} x_t + \sigma \psi_{t-1} (1 - \delta_h) \epsilon_l c_{l,t}^{-\sigma-1} x_t,$$
$$0 = \gamma \left(h_{b,t}^{-\sigma_h} - ((h - h_{b,t})^{-\sigma_h} \right) + \mu_t \widetilde{z} 2 x_t - \psi_t \sigma_h \gamma (h - h_{b,t})^{-\sigma_h - 1},$$
$$0 = \mu_t \widetilde{z} h_{b,t} 2 + \psi_t \epsilon_l c_{l,t}^{-\sigma} - \psi_{t-1} (1 - \delta_h) \epsilon_l c_{l,t}^{-\sigma},$$

where λ_t, μ_t, and ψ_t denote the multiplier on the constraints (73), (91), and (92), respectively. Notably, the last condition would imply a unit root in the multiplier ψ_t under a binding borrowing constraint, $\mu_t > 0$, if there were no depreciation of housing ($\delta_h = 0$). Eliminating λ_t and μ_t with the first two conditions, leads to

$$0 = \gamma \left(h_{b,t}^{-\sigma_h} - ((h - h_{b,t})^{-\sigma_h} \right) + \left(\epsilon_b c_{b,t}^{-\sigma} - \epsilon_l c_{l,t}^{-\sigma} \right) \widetilde{z} x_t \tag{93}$$
$$+ \left(\psi_t - \psi_{t-1} (1 - \delta_h) \right) \sigma \epsilon_l c_{l,t}^{-\sigma-1} x_t \widetilde{z} x_t - \psi_t \sigma_h \gamma (h - h_{b,t})^{-\sigma_h - 1} \ ,$$

$$0 = \left(\epsilon_b c_{b,t}^{-\sigma} - \epsilon_l c_{l,t}^{-\sigma} \right) \widetilde{z} h_{b,t} + \left(\psi_t - \psi_{t-1} (1 - \delta_h) \right) \left(\sigma \epsilon_l c_{l,t}^{-\sigma-1} x_t \widetilde{z} h_{b,t} + \epsilon_l c_{l,t}^{-\sigma} \right). \tag{94}$$

Hence, the allocation under the optimal plan is a set of sequences $\{h_{b,t}, c_{b,t}, c_{l,t}, x_t, s_t, \psi_t, \widetilde{z}_t\}_{t=0}^{\infty}$ satisfying (73), (88), (90), (91), (92), (93), and (94). Figure 3 then shows impulse responses for a fixed value of κ that implements the long-run constrained efficient allocation under the Pigouvian subsidy.

G Additional figures

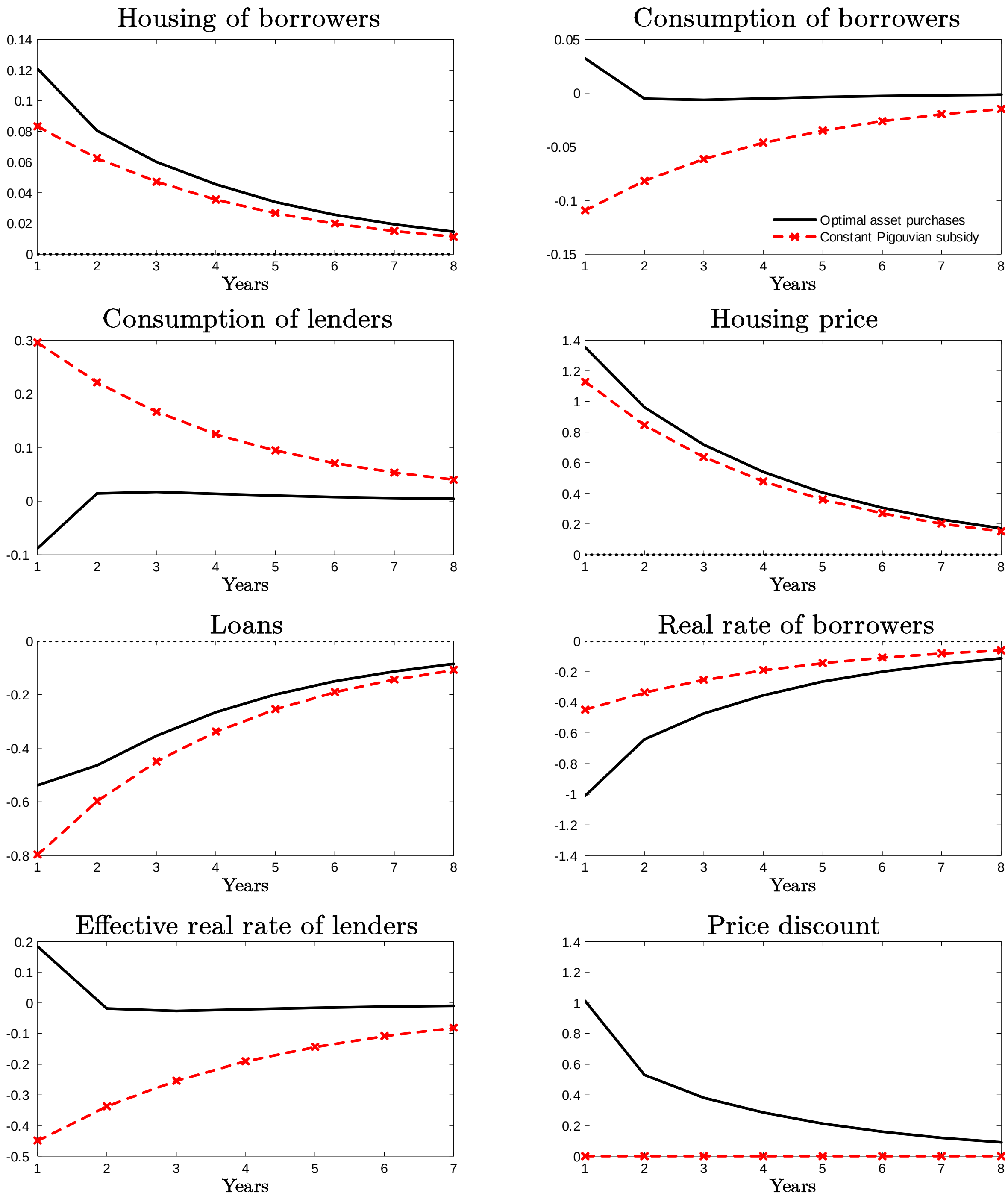

Figure 4: Responses to a minus one st.dev. liquidation value shock (in % deviations from a non-stochastic mean)