Mortgage Default during the U.S. Mortgage Crisis*

Thomas Schelkle†

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Abstract

Which theory can quantitatively explain the rise in mortgage default rates during the U.S. mortgage crisis? This paper finds that the double-trigger hypothesis attributing mortgage default to the joint occurrence of negative equity and a life event like unemployment is consistent with the evidence. In contrast a traditional frictionless default model strongly overpredicts the increase in default rates. The paper provides micro-foundations for double-trigger behavior in a model where unemployment may cause liquidity problems for the borrower. This framework implies that mortgage crises may be mitigated at a lower cost by bailing out borrowers instead of lenders.

JEL codes: E21, G21, D11

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†University of Cologne; Center for Macroeconomic Research; Albertus-Magnus-Platz; 50923 Köln; Germany; Email: schelkle@wiso.uni-koeln.de

1 Introduction

After the collapse of the house price boom in the United States residential mortgage delinquencies of both prime and subprime loans have increased substantially. The resulting losses of mortgage-backed-securities marked the start of the recent financial and economic crisis. These events highlight the importance of understanding the economic mechanisms triggering mortgage default and the rise in default rates. Insights into these issues may then inform political debates on how to prevent future mortgage crises or mitigate ones that have already started. This paper contributes to this research agenda by investigating what type of theoretical mortgage default model can quantitatively explain the observed rise in default rates in the Unites States between 2002 and 2010.

The paper considers the two main competing theories of mortgage default - the frictionless option-theoretic model and the double-trigger hypothesis. The traditional frictionless literature assumes that borrowers "ruthlessly" default on their mortgage to maximize their financial wealth, cf. for example Kau, Keenan, and Kim (1994), Kau, Keenan, Muller, and Epperson (1992, 1995) and the surveys of Quercia and Stegman (1992), Kau and Keenan (1995) and Vandell (1995). In this framework negative equity is a necessary, but not sufficient, condition for default. Instead there exists a threshold level of negative equity such that a rational wealth-maximizing agent will exercise the default option. This theory is frictionless in the sense of assuming a perfect credit market for unsecured credit such that default is unaffected by income fluctuations or liquidity problems.

The other main theory on mortgage default is the double-trigger hypothesis. This theory agrees that negative equity is a necessary condition for default. But it attributes default to the joint occurrence of negative equity and a life event like unemployment or divorce. The double-trigger hypothesis is well-known among mortgage researchers, cf. the discussions by Gerardi, Shapiro, and Willen (2007), Foote, Gerardi, and Willen (2008) and Foote, Gerardi, Goette, and Willen (2009). But in contrast to the well established and formalized frictionless default theory, it is usually only discussed verbally or with stylized models.

These two microeconomic theories are assessed quantitatively using data on prime fixed-rate mortgages with high initial loan to value ratios (above 95%).[1] The test exploits variation across cohorts of loans originated between 2002 and 2008. In the data cohorts of mortgage borrowers who experienced a more adverse path of average house prices, and therefore lower home equity, defaulted much more frequently. This variation in default rates across cohorts does not seem to be driven by other observable loan or borrower characteristics. *Qualitatively* both theories are consistent with this joint variation in home equity and default rates. However an estimation and simulation of reduced form models of the two theories reveals important *quantitative* differences between them.

Specifically, the estimation by a simulated method of moments procedure forces both models to match the default rates of the 2002 cohort, which they can both fit well. Given the estimated parameters representing the default threshold and frequency of life events, respectively, I then assess the ability of the models to predict the default rates of the 2003 to 2008 cohorts. In this out-of-sample exercise I find that the frictionless theory is excessively sensitive to changes in aggregate house prices and predicts a far too strong rise in default rates across cohorts. The leftward shift of the home equity distribution caused by the observed fall in aggregate house prices strongly increases the number of borrowers with extreme levels of negative equity. This moves too many borrowers over the estimated default threshold compared to the observed increase in default rates. In contrast, the double trigger hypothesis gives a surprisingly good fit to the observed rise in default rates. Its predictions are based on the borrowers who experience any level of negative equity, of which only a certain share defaults according to the estimated frequency of life events. The double-trigger theory then predicts an increase in default rates across cohorts proportional to the increase in the number of borrowers with any level of negative equity. Observed default rates exhibit such a pattern.

In the second part of the paper these insights guide the development of a parsimonious structural model of mortgage default. This dynamic stochastic partial equilibrium

[1] There are various data reasons that make a focus on highly leveraged borrowers advantageous, which are explained in detail in section 2. However I also document that the full sample of loans with all possible initial loan to value ratios exhibits very similar dynamics, though there are of course level differences. In an extension I also show explicitly that under plausible assumptions the results of the analysis generalize to loans with lower initial loan to value ratios.

model micro-founds the double-trigger hypothesis. Borrowers in the model face liquidity constraints and idiosyncratic unemployment shocks such that unemployed borrowers who have exhausted their buffer stock savings need to make painful cuts to consumption. This magnifies the cost of servicing the mortgage such that unemployment triggers default in a negative equity situation. The model also includes a direct utility flow from living in the bought house that prevents employed agents from defaulting after a strong fall of house prices. These features generate double-trigger behavior in the model. The calibrated model can quantitatively explain most of the observed rise in mortgage default as a consequence of falling aggregate house prices.

The structural model is then used for a first pass to formally analyze two possible mitigation policies in a mortgage crises that may help to stabilize the financial system. If the government desires to neutralize the losses of mortgage lenders from default, it could either bail out the lenders or mitigate the liquidity problems of homeowners who would otherwise default. Actual stabilization policy during the crisis arguably encompassed both types of measures. For instance the Troubled Asset Relief Program (TARP) represents an example of the former and the Home Affordable Modification Program (HAMP) of the latter type of policy. The analysis shows that a subsidy policy to homeowners is about $7-10$ times cheaper than a bailout of lenders in this model where default is partly driven by liquidity problems. Though these are only partial equilibrium results, they suggest a large potential to reduce the cost of mitigating a mortgage crisis for taxpayers.

The paper relates to different strands of the theoretical and empirical literature. The structural model of the paper builds on previous theoretical work by Campbell and Cocco (2003, 2015) and Corradin (2014) who also model liquidity constraints in a mortgage framework. Similar models are used to examine bail-out guarantees or the role of mortgage product innovation and falling house prices for the mortgage crisis by Chatterjee and Eyigungor (2015), Corbae and Quintin (2015), Jeske, Krueger, and Mitman (2013) and Garriga and Schlagenhauf (2009). Compared to this work I consider more parsimonious models, but the contribution of my paper is to compare theoretical default models in much more detail to empirical observations. For instance Campbell and Cocco (2015) only compare theoretical predictions on the difference between fixed- and variable rate mortgages to some broad patterns in the data. Chatterjee and Eyigungor (2015) and Corbae and Quintin (2015) conduct relatively stylized computational experiments with their models and investigate the fit only to data on aggregate default or foreclosure rates.

In contrast, I compare model predictions to detailed monthly data for several cohorts of loans. Since these cohorts experienced very different house price paths this provides much richer variation for an empirical assessment of theoretical default models. In these comparisons I simulate an empirically accurate home equity distribution within and between cohorts and apply the theoretical default mechanisms to them. I isolate the effect of falling house prices by keeping the type of loan contract in the model and the data constant. In contrast comparisons to aggregate data in prior work are prone to confound the effect of house prices with compositional effects with respect to loan cohorts or contract types. This more demanding comparison to the data reveals that including income shocks and liquidity constraints in a mortgage framework does not automatically lead to an empirically successful model. Instead only models where agents with substantial negative equity but no liquidity problems do not find it optimal to default will truly feature double-trigger behavior and an accurate sensitivity of default rates to changes in aggregate house prices. This is an important finding because capturing the economic default mechanism and an accurate house price sensitivity is of great relevance for the use of such models for policy or macroeconomic risk analysis. Finally I also apply this micro-founded and empirically tested double-trigger model to formally evaluate the cost differences between a bailout of lenders and subsidies to homeowners for mitigating a mortgage crisis.[2]

The paper is also related to a vast empirical literature that studies the determinants of mortgage default.[3] This literature provides a wealth of evidence that negative equity

[2]Hatchondo, Martinez, and Sánchez (2015) also use a structural default model to analyze policies preventing default. But the difference is that I first show that the structural model can capture the empirically observed rise in default rates during the crisis before conducting policy analysis. Furthermore, I analyze different policies. These authors investigate preventive policies such as stronger recourse or limits on the initial loan-to-value ratio. In contrast this paper studies mitigation policies which are only conducted once the crisis has already started in order to neutralize the losses of lenders.

[3]Studies within this extensive literature differ by research question, estimation method, data set and results. A detailed literature review that would do justice to these different contributions is unfortunately

or falling house prices are strong determinants of default. Some studies have also investigated the role of life events as triggers for default. Many studies found that state unemployment or divorce rates are correlated with default rates. Elul, Souleles, Chomsisengphet, Glennon, and Hunt (2010) provide evidence that variables measuring illiquidity and interactions between illiquidity and negative equity significantly affect default. Gerardi, Herkenhoff, Ohanian, and Willen (2013) show at the individual level that unemployment and income shocks increase the probability of default. My paper is motivated by these prior empirical results.[4] But it uses a very different methodology.

This prior literature sheds light on the theories by documenting merely statistical correlations between life events and defaults. Instead this paper includes the economic structure of the competing theories explicitly in the analysis.[5] I investigate whether the theories exhibit an empirically accurate sensitivity of default rates to changes in aggregate house prices. This reveals the excess sensitivity of a purely negative equity threshold based default theory and the accurate sensitivity of a double-trigger model. Thus the paper documents a novel set of facts on the relative merit of the two theories and their ability to explain the aggregate default dynamics during the mortgage crisis.

The paper is structured as follows. Section 2 describes the data and empirical facts on mortgages and house prices. Reduced-form models of the two theories are compared to the data in section 3. The structural model is developed in section 4 and parameterized in

beyond the scope of this paper. The pre-crisis literature is surveyed by Quercia and Stegman (1992) and Vandell (1995) and an example is the study by Deng, Quigley, and Van Order (2000). The U.S. mortgage crisis has then caused an enormous increase in empirical work on mortgage default. Examples of this empirical research include Amromin and Paulson (2009), Bajari, Chu, and Park (2010), Bhutta, Dokko, and Shan (2010), Demyanyk and Van Hemert (2011), Elul, Souleles, Chomsisengphet, Glennon, and Hunt (2010), Ferreira and Gyourko (2015), Foote, Gerardi, Goette, and Willen (2008), Foote, Gerardi, Goette, and Willen (2009), Foote, Gerardi, and Willen (2008), Fuster and Willen (2013), Gerardi, Herkenhoff, Ohanian, and Willen (2013), Gerardi, Lehnert, Sherlund, and Willen (2008), Gerardi, Shapiro, and Willen (2007), Ghent and Kudlyak (2011), Guiso, Sapienza, and Zingales (2013), Jagtiani and Lang (2011), Mayer, Pence, and Sherlund (2009), Mian and Sufi (2009), Palmer (2013) and Stanton and Wallace (2011), among others.

[4]Another interesting empirical fact is the great heterogeneity in default behavior for borrowers with the same level of negative equity (Quercia and Stegman 1992). A double-trigger model can rationalize this fact because life events, which are unobserved in all standard mortgage data sets, may account for the heterogeneity in default behavior of borrowers with the same level of negative equity.

[5]Bajari, Chu, and Park (2010) also estimate a model based on a simple default rule akin to my reduced form models in section 3. However they consider the different triggers separately, i.e. borrowers default either because of negative equity or a life event. Instead I investigate the theoretically more appealing double-trigger hypothesis which requires satisfaction of both conditions and also develop a structural model based on optimizing behavior.

section 5. The results of the structural model are presented in section 6. The structural model is applied for policy analysis in section 7 and section 8 concludes. An online appendix contains technical details and further results.

2 Data and Empirical Facts

This section presents the data on mortgages and house prices and the key facts of default rates and house price paths across cohorts that the paper attempts to explain. Furthermore it provides evidence on relatively stable loan and borrower characteristics across cohorts.

2.1 Data Sources and Main Facts

Information on mortgage contract characteristics and payment histories in the United States is based on the large loan-level data base of Lender Processing Services (LPS), also known as McDash data. I did not have access to the full loan-level data, but obtained information that was aggregated from the full data base. "Aggregate" here simply means that my data contain the average value of a certain variable for all loans in the data base that satisfy a set of conditions that I can specify. The data cover the time period from January 2002 until June 2010 at a monthly frequency and the analysis is focussed on loans originated between 2002 and 2008.

I restrict the sample to prime, first, fixed-rate, 30-years mortgages that have a standard amortization schedule (are not balloon mortgages). I focus on only one mortgage type because the structural model would have to be recomputed for each different mortgage contract. The selection is motivated by the fact these are the most common mortgage contracts. Furthermore mortgage distress was not primarily a subprime phenomenon, but equally concerned prime borrowers (Ferreira and Gyourko 2015). The data base contains around 23 million loans with these characteristics in 2010. I further focus the analysis on loans with an initial loan-to-value ratio (LTV) above 95%, which depending on the year represents about $20-30\%$ of all outstanding loans and around 20% of newly originated loans that satisfy the above restrictions.

Looking at loans within a narrow range of LTVs allows to generate a more accurate home equity distribution in simulations of the model. This is very important due to the highly non-linear relationship between default decisions and negative equity in the theoretical models, on which the conducted empirical test also relies heavily. Furthermore, the loans with a high LTV default most frequently, so it makes sense to focus an analysis of mortgage default on them. But the main reason for concentrating on this group is a data problem. In the LPS data only the LTV of the first mortgage is observed, but not the combined LTV of the first and a possible second mortgage.[6] Since the combined mortgage amount determines a borrower's home equity the fact that second mortgages are unobserved is a problem. In order to mitigate this data problem I thus focus on first mortgages with a very high LTV because these borrowers should be least likely to have a second mortgage on their home. Though these considerations make a focus on highly leveraged borrowers desirable, there is evidence that the general patterns and conclusions of the analysis apply more widely. Specifically, online appendix A documents that the full sample of all loans exhibits broadly the same dynamics across cohorts than this sample of highly leveraged borrowers, though there are of course various level differences. Furthermore, in appendix section B.4 I show that under plausible assumptions on second mortgages the main conclusions of the reduced-form exercise generalize to loans with an initial LTV of the first mortgage between 75% and 84%.

The data set contains for each loan cohort (defined by origination month) over time how many active loans are delinquent and the number of completed foreclosures. Following much of the empirical literature, I define a loan to be in default when it is 60 days or more past due, i.e. two payments have been missed. Cumulative default rates for a loan cohort are then constructed as the share of active loans that are 60 days or more delinquent times the share of initial loans that are still active plus the share of initial loans

[6]Elul, Souleles, Chomsisengphet, Glennon, and Hunt (2010) provide evidence that second mortgages are frequent and significantly affect the combined loan-to-value ratio. They report that on average 26% of all borrowers have a second mortgage and this adds on average 15% to the combined LTV. Unfortunately, they do not report a break-down of these statistics by the LTV of the first mortgage.

where foreclosure has already been completed.[7] However I also show in online appendices B.3 and C.5 that all my substantive results are robust to using an alternative default definition of 120 or more days past due, which represents even more serious delinquency.[8]

Information on house prices comes from the Federal Housing Finance Agency (FHFA). The monthly national and census division level repeat-purchase house price indices between 1991 and 2010 deflated by the Consumer Price Index (CPI) are used as measures of aggregate real house price movements. The simulations of this paper also contain realistic microeconomic house price distributions based on empirical estimates of their variance by the FHFA as discussed in section 3.4.

The key empirical facts on mortgage default rates and house prices across loan cohorts are presented in figure 1. Figure 1(a) shows the observed cumulative default rates for loan cohorts originated between 2002 and 2008 grouped by the year of origination.[9] Figure 1(b) presents the mean real house price paths for these cohorts of loans. These mean house price paths accurately account for the geographical composition across census divisions of the different loan cohorts. One observes that mortgage borrowers who experienced a more adverse path of average house price growth rates defaulted much more frequently. Explaining this variation quantitatively and using it to discriminate between the mentioned theories is the main aim of the paper.

A general problem of the empirical literature on mortgage default is the lack of loan-

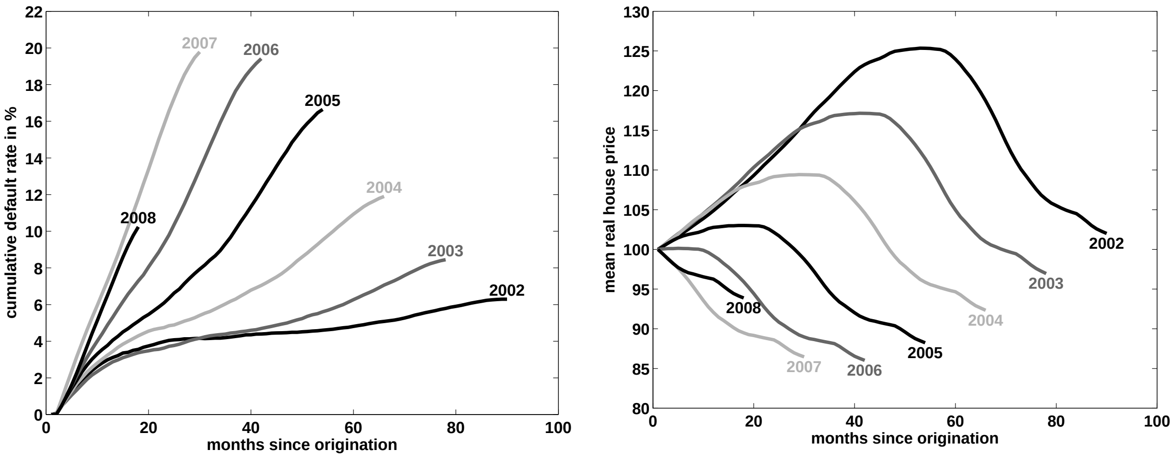
[7]The period of default is backdated by one month to capture the time when the first payment has been missed.

[8]The reason for also looking at a 120 days definition is that a considerable fraction of loans which are only 60 days past due will ultimately become current again. Evidence on cure rates is for example provided by Adelino, Gerardi, and Willen (2013) and on general transitions between different stages of delinquency by Herkenhoff and Ohanian (2013). However the theoretical literature on mortgage default, which I follow here, models default as a permanent mortgage termination. This means that there exists a certain tension between the theoretical and empirical literature with respect to the used concept of default. The rationale for also looking at a 120 days definition is then that more serious stages of delinquency are also much more permanent as documented empirically in Herkenhoff and Ohanian (2013). Thus this check addresses the potential concerns on the correspondence between theoretical and empirical concepts of default. In this check I find that my results are robust to using this alternative reasonable measure of default. Furthermore I have also investigated the effect of using a definition of default that requires a loan to be in foreclosure. This also generates similar results (which are available upon request) and does not resolve the empirical problems of a frictionless option model documented in section 3.

[9]In the data set and all the model simulations of the paper loan cohorts are defined by month of origination. However in all the graphs of the paper I group loan cohorts by year of origination and the shown curves for an origination year are averages of the underlying twelve cohorts defined by origination month.

Figure 1: Cumulative Default Rates and House Prices by Loan Cohort

(a) Cumulative Default Rates (b) Mean Real House Price Paths



level data that links borrowers' repayment behavior to their individual house prices and life events. The implications of unobserved individual unemployment spells for empirical investigations of mortgage default are also discussed by Gyourko and Tracy (2014). Though some studies cited in the introduction have made partial progress in this direction, such an ideal data set does still not exist. Given these general data problems this paper takes a different approach characterized by two main features. First it investigates at a more aggregate level of whole cohorts of loans how shifts in home equity distributions induced by changes to average house prices lead to changes in default rates. Second it exploits the economic structure of different theories and how their predictions on default specifically depend on home equity. This provides novel evidence on the house price sensitivity embedded in the competing theories and their ability to explain the aggregate dynamics of default during the mortgage crisis.

2.2 Loan Characteristics at Origination

Before proceeding to the analysis I briefly discuss one alternative explanation for the rise in default rates observed in figure 1(a). This explanation is that lending standards and loan quality deteriorated sharply before the mortgage crisis. Thus, I first present evidence that average loan quality is fairly stable across cohorts in my data set. One should also keep in mind that I only consider prime fixed-rate mortgages. Therefore any compositional shifts in the mortgage market towards subprime or variable rate mortgages do by construction not confound my analysis.

One concern is that the loan-to-value ratio (LTV) might have increased over time leaving a smaller buffer before borrowers experience negative equity. I only consider loans that have a LTV above 95% and thus limit this possibility to shifts within that class of loans. Within this class the average LTV is basically constant across cohorts and only fluctuates mildly around the average value of 98.2% as seen in the first row of table 1. In section 3 I even control for observed changes across cohorts in the within-cohort distribution of LTVs and find that these are irrelevant.

The second row of table 1 reports the average FICO credit score at origination of the different loan cohorts. These are very stable as well. To the extent that these credit scores are good measures of creditworthiness a significant deterioration in loan quality is not observable here.

Table 1 also contains information on the average mortgage rate that different cohorts face. A higher mortgage rate might make the loan as such less attractive to the borrower. There is some variation in this variable across cohorts. But the mortgage rate and default rates seem to be fairly uncorrelated across cohorts.

The average debt-to-income (DTI) ratio representing the share of the required mortgage payment in gross income is presented in the last row of table 1.[10] This has increased over time indicating that borrowers in later cohorts need to devote more of their gross income to service the mortgage. But the increase was quite modest.

These statistics show that there is no evidence in favor of a strong deterioration of lending standards over time in my data set of prime fixed-rate mortgages with a LTV above 95%. Furthermore in online appendix A I document that this stability of loan characteristics across cohorts applies more widely to the full loan sample of all possible initial LTVs, though there are of course level differences. Nevertheless these conclusions

[10]The data on the DTI is the only mortgage variable in the whole paper that is based on a somewhat different loan selection. The reason is that the DTI was not available in the tool that was used to aggregate and extract information from the LPS loan-level data set. Instead LPS provided me with a separate tabulation where it was not possible to use the same selection criteria. Specifically, the DTI information is for the same LTV class as the rest of the data, but it does not only cover prime, fixed-rate, 30-years mortgages. However the vast majority of loans in the LPS data are prime, fixed-rate mortgages and the modal maturity of these loans is 30 years, so this information should at least be a good approximation to the actual loan pool I consider.

Table 1: Average Loan Characteristics at Origination by Loan Cohort

Cohort	2002	2003	2004	2005	2006	2007	2008	All
LTV in %	98.2	98.3	98.2	98.3	98.4	98.1	97.8	98.2
FICO score	676	673	669	670	668	670	678	672
Mortg. rate in %	6.9	6.0	6.1	6.0	6.6	6.7	6.2	6.4
DTI in %	39	39	40	40	40	42	42	40

may be somewhat specific to the prime market.[11] However when thinking about the whole mortgage market one should keep in mind that the prime market is much larger than the subprime market, which even in its heyday constituted only about 20% of new loan originations (Joint Center for Housing Studies at Harvard University 2008, p. 4). Overall this evidence casts doubts on explanations of the mortgage crisis that rely solely on lax lending standards. Instead this paper shows that the fall in house prices can explain the rise in default rates within a suitable theoretical model.

3 Reduced Form Models

This section presents evidence on mortgage default from estimating and simulating two highly stylized models. These models represent the simplest possible reduced forms of a frictionless option-theoretic model (the "threshold" model) and the double-trigger hypothesis (the "shock" model). A key feature of the simulation is that it includes empirically accurate home equity distributions within and between cohorts emanating from variation in initial mortgage balances and house prices. The aim is to discriminate between pure forms of these theories in a relatively general way that is independent of the exact specification of the respective structural model. Several recent structural default models cited in the introduction are in a sense hybrids of these two theories. Thus one may also view this section as a general empirical inspection of the two key economic

[11] For example Demyanyk and Van Hemert (2011) present evidence that loan quality deteriorated in the subprime market. However Palmer (2013) finds that the fall in house prices is also the dominant cause of the rise in mortgage default in the subprime market. Similar to the statistics presented above Amromin and Paulson (2009) note that it is difficult to detect a deterioration in loan quality in the prime market. A particular advantage of my descriptive statistics is that they are based on all loans in the LPS data base satisfying my sample selection criteria. Other empirical studies using LPS data typically work with a 1% random sample such that their descriptive statistics are based on fewer observations.

mechanisms embedded in recent default models. The insights into the empirical performance of these default mechanisms then guide the development of a suitable structural economic model in the following section.

3.1 Model Setup

The paper considers individual borrowers who took out a fixed-rate 30-years mortgage. Each loan cohort defined by origination date consists of many borrowers who are indexed by $i = 1, \ldots, N$ and observed in periods $t = 1, \ldots, T$ after loan origination. Borrowers take a single decision each period and can either service the mortgage or default on the loan and "walk away" from the house. Denote the default decision of an individual borrower i in month t after origination by a set of dummy variables d_{it}. The variables d_{it} take the value 1 once the borrower has defaulted, and the value 0 in all periods prior to default. Thus it is sufficient to present default decision rules in period t for situations when the borrower has not defaulted yet.

For a fixed-rate mortgage the nominal mortgage balance M_{it} of borrower i evolves deterministically over time according to

$$M_{i,t+1} = (1 + r^m)M_{it} - m_i \tag{1}$$

where r^m is the monthly mortgage rate which is constant across individuals and m_i are fixed nominal monthly payments. As in the real world these fixed payments mostly cover mortgage interest early on in the contract and later a larger share goes towards the repayment of principal. The payments m_i are determined at the beginning of the contract and satisfy

$$m_i = \left[\sum_{t=1}^{T} \frac{1}{(1 + r^m)^t} \right]^{-1} M_{i0} \tag{2}$$

where M_{i0} is the initial loan amount and the loan has a maturity of $T = 360$ months. The initial loan amount is a function of the initial loan to value ratio LTV_i and initial house price P_{i0} and given by $M_{i0} = LTV_i \times P_{i0}$. Here borrowers are heterogenous with respect to the LTV. It is assumed that agents take decisions based on real variables. Thus it is useful to define the real mortgage balance as $M_{it}^{real} = \frac{M_{it}}{\Pi_t}$ where Π_t is the CPI and

$\Pi_0 = 1$. This assumption does not affect the results and the conclusions are identical when decisions are based on nominal variables.

The real house price P_{it} of a homeowner evolves stochastically over time as described in section 3.4 below. P_{i0} is normalized to 100.

The real home equity of a borrower is given by $P_{it} - M_{it}^{real}$, i.e. the difference between the value of the house and the outstanding mortgage amount. The two default theories differ in how their predictions depend on home equity.

3.2 The Threshold Model

The first model assumes that borrowers with negative equity default on their mortgage at the first time that the real value of equity falls below a certain threshold value. Therefore I call this the "threshold model". Here, I adopt the simplest possible specification with a threshold that is proportional to the initial house price and constant over time given by ϕP_{i0} where $\phi < 0$.[12] If in period t the borrower has not defaulted yet then the default decision in that period is described by

$$d_{it} = \begin{cases} 1, & \text{if } P_{it} - M_{it}^{real} < \phi P_{i0} \\ 0, & \text{otherwise} \end{cases} \tag{3}$$

This is a simple reduced-form of a frictionless option model. The corresponding structural model would derive the threshold parameter ϕ from optimizing behavior. For example the borrower might trade off the expected future capital gains on the house for the mortgage payments in excess of rents. Thus the parameter ϕ should depend crucially on expected future house prices. The fact that such expectations are hard to pin down is one important reason why comparing predictions of such a theory to empirical default behavior has been difficult. Here I remain agnostic about the exact trade-off and the value of ϕ and instead estimate it from the data.

[12] One could also argue for a specification with a threshold ϕP_{it} such that the equity threshold is proportional to the current house price. This is just a minor difference in functional form and all the main results below are basically identical when using such a specification instead. The only difference is that the estimated value of ϕ is then slightly larger in absolute magnitude.

3.3 The Shock Model

The second model assumes that borrowers with any level of negative equity only default on their mortgage when they also receive a default shock in that period. I call this the "shock model". Again I adopt the simplest possible specification. The probability to receive a default shock $\psi \in [0,1]$ is constant and default shocks are independently and identically distributed over time. If the borrower has not defaulted yet, the default decision in period t is determined by

$$d_{it} = \begin{cases} 1, & \text{if } P_{it} - M_{it}^{real} < 0 \text{ and the default shock occurs} \\ 0, & \text{otherwise} \end{cases} \tag{4}$$

This is a reduced-form of a double-trigger model. Here the default shock represents a life event that combined with negative equity triggers default. The parameter ψ represents the probability that a life event occurs and is estimated from the data. Possible examples for such a life event could be unemployment or divorce, but I again preserve generality here and remain agnostic about the exact nature of these events. Section 4 then provides a micro-founded double-trigger model where unemployment acts as the life event.

3.4 Simulation of House Prices

This section describes how house prices are modelled and simulated. This information applies also to the structural model in the following section. The general aim is to accurately capture realistic house price distributions within and between cohorts in the simulation based on the empirical procedures and estimates of the FHFA.

Throughout the paper the evolution of the real house price P_{it} of an individual house i in period t is modeled as

$$\ln(P_{it}) = \ln(P_{i,t-1}) + g_t^{agg} + g_{it}^{ind} \tag{5}$$

where the house price growth rate has two components, an aggregate component g_t^{agg} that is common to all houses and an individual component g_{it}^{ind} specific to the individual house. Including an individual component is important in order to accurately capture the house price variation between borrowers within a cohort. Without this feature theoretical models cannot explain any default during times of positive aggregate house price growth. The formulation is consistent with the approach used by the FHFA to estimate the house price index, cf. the description in Calhoun (1996).[13]

In equation (5) a census division index was suppressed for convenience. But the aggregate trend represented by g_t^{agg} and the moments of g_{it}^{ind} are in fact specific to the census division in which the house is located. Thus, this paper uses information on house prices at the census division level and the regional composition of loan cohorts to simulate house prices accurately. When drawing house prices the simulation draws are allocated across census divisions such that in each cohort the simulated sample has the same regional composition as in the mortgage data. The aggregate component g_t^{agg} represents the growth rate of the census division real house price index. In the simulation this component is taken directly from the FHFA data deflated by the CPI. The aggregate component generates the variation in mean house price paths across loan cohorts.

The individual component g_{it}^{ind} is unobserved. But the FHFA provides estimates of the variance and I use these to simulate a realistic microeconomic house price distribution. Specifically, it is assumed that the individual component g_{it}^{ind} is independent over time and individuals and normally distributed with mean zero and variance V_t. The variance of g_{it}^{ind} depends on the time since the house was bought. This is a realistic feature of the data and based on estimates of the FHFA. For simplicity the following exposition assumes that the house was bought in period 0 such that t is also the time since purchase. Using my own notation the FHFA specifies a quadratic formula in time for the variance of the total individual part of the house price change since purchase given by

$$\operatorname{Var}\left(\sum_{\tau=1}^{t} g_{i\tau}^{ind}\right) = \frac{\kappa}{3}t + \frac{\lambda}{9}t^2 \tag{6}$$

where an adjustment has been made for the fact that this paper operates at a monthly instead of a quarterly frequency. By the independence assumption the variance of g_{it}^{ind} is

[13] I use a slightly different notation relative to the FHFA because I want to use this equation in a dynamic optimization problem and simulations. In order to see how it is related, rewrite equation (5) as $\ln(P_{it}) = \ln(P_{i,0}) + \sum_{\tau=1}^{t} g_\tau^{agg} + \sum_{\tau=1}^{t} g_{i\tau}^{ind}$ where $\ln(P_{i,0}) + \sum_{\tau=1}^{t} g_\tau^{agg} = \beta_t + N_i$ and $\sum_{\tau=1}^{t} g_{i\tau}^{ind} = H_{it}$ give equation (1) in Calhoun (1996).

then given by

$$V_t = \text{Var}\left(g_{it}^{ind}\right) = \text{Var}\left(\sum_{\tau=1}^{t} g_{i\tau}^{ind}\right) - \text{Var}\left(\sum_{\tau=1}^{t-1} g_{i\tau}^{ind}\right) = \frac{\kappa}{3} + \frac{\lambda}{9}(2t-1). \tag{7}$$

The FHFA provides estimates of κ and λ at the census division level that I use to generate realistic distributions around the division level aggregate trends. The estimates of κ are positive and those of λ are negative and small in absolute magnitude. This implies that the variance of $\sum_{\tau=1}^{t} g_{i\tau}^{ind}$ increases less than linearly with time and the variance of a single g_{it}^{ind} is decreasing over time.[14]

3.5 Model Simulation, Estimation and Test

Conditional on the respective model parameters ϕ and ψ both models can be simulated for subsequent cohorts of loans originated each month between 2002 and 2008. For each cohort I draw $25,000$ individual histories of house prices and for the shock model also default shock histories. When computing the mortgage balance the mortgage rate is kept constant within a cohort and set equal to the respective cohort average. But borrowers within a cohort are heterogenous with respect to the initial LTV which varies in steps of one percentage point between 95% and 104% (the few loans with a higher LTV are subsumed in the 104% LTV bin). The frequency of these different loan-to-value ratios at origination is varied across cohorts as observed in the mortgage data. This means that possible changes to the average mortgage rate and the LTV distribution across cohorts are taken into account in the simulation. Data on the path of inflation rates from the CPI is used to compute the real mortgage balance. The decision rules are then applied to these shock histories and paths of the real mortgage balance. Therefore the simulation generates empirically accurate home equity distributions within and between cohorts by including this variation in initial mortgage balances, mortgage rates and house prices.

The idea of the estimation and test procedure is to estimate the unknown model parameters using only the default data of the cohort originated in 2002. The estimation employs a simulated method of moments procedure (McFadden (1989), Pakes and Pollard

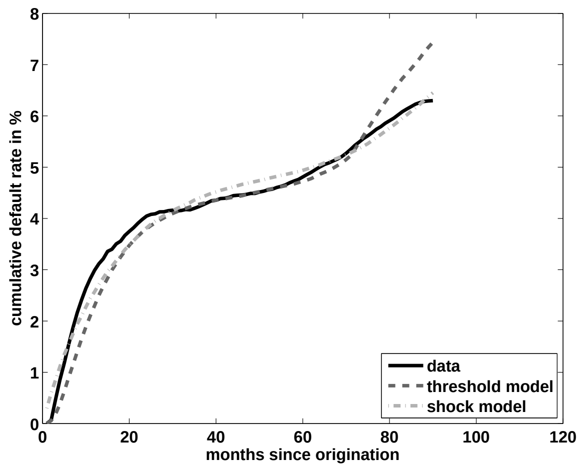
[14] On average across census divisions the estimates of κ and λ imply that the shock in the first month g_{i1}^{ind} has a standard deviation of about 2.49%, while after five years the standard deviation of g_{i60}^{ind} is around 2.37%. Hence the standard deviation of g_{it}^{ind} decreases relatively slowly over time.

(1989), Duffie and Singleton (1993)). The respective parameters ϕ and ψ are chosen such that the cumulative default rates for the 2002 cohort simulated from the model match as well as possible those observed in the data, cf. online appendix B.1 for details. Keeping the parameter values estimated for the 2002 cohort fixed, the test is based on out-of-sample predictions of the two models for the cohorts originated between 2003 and 2008. It consists of informally comparing simulated and empirically observed default rates for these remaining cohorts and checking which estimated model gives a better fit to the data.

3.6 Results

For the threshold model the negative equity default threshold ϕ is estimated as -11.1%. This means borrowers default as soon as they have a real value of negative equity of 11.1% of the initial house price. In contrast, for the shock model the default shock probability ψ is estimated to be 1.05% such that each period 1.05% of those borrowers with negative equity default on their loan. The fit of the two models to the cumulative default rate of the 2002 cohort is shown in figure 2. Both models fit this data very well.

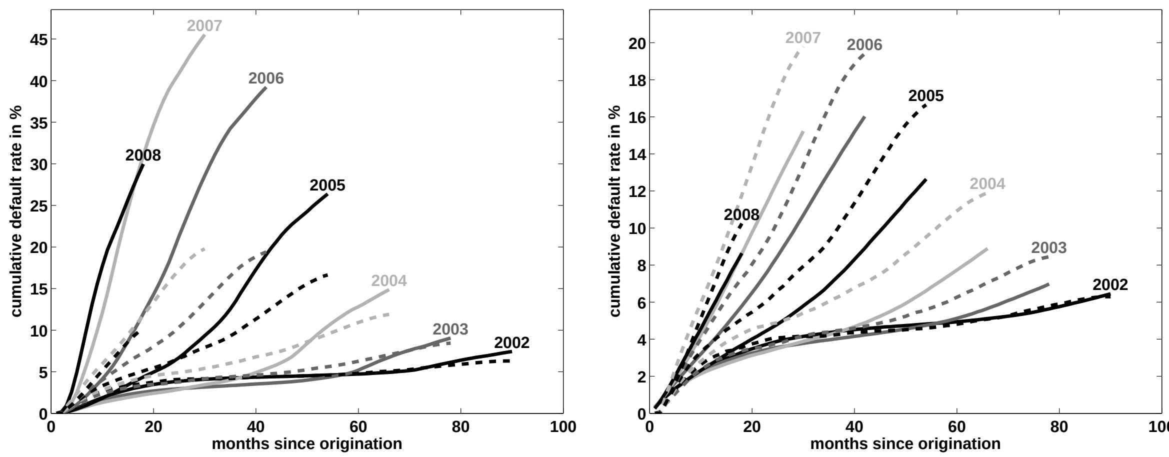
Figure 2: Cumulative Default Rate of 2002 Cohort: Models vs. Data



The next step is to test the two estimated models by checking how well they perform in predicting out-of-sample. Figure 3(a) shows the fit of the threshold model to the full sample of all cohorts between 2002 and 2008. The equivalent fit of the shock model is presented in figure 3(b). It turns out that the threshold model has severe empirical problems. When it is forced to match default rates of the 2002 cohort, it overpredicts default rates for the later cohorts in the simulation period by at least one order of magnitude. The threshold model is excessively sensitive to the shifts in the mean of the house price distribution observed in the data. In contrast, the shock model gives a good fit to the broad dynamics in the data. The impression from visually inspecting the graphs is also confirmed by simple measures of goodness of fit of the model predictions to the data. For instance the root mean squared error (mean absolute error) of the out-of-sample forecast is 4.1 (3.2) times higher for the threshold model than for the shock model.

Figure 3: Cumulative Default Rates of all Cohorts: Models (solid lines) vs. Data (dashed lines)

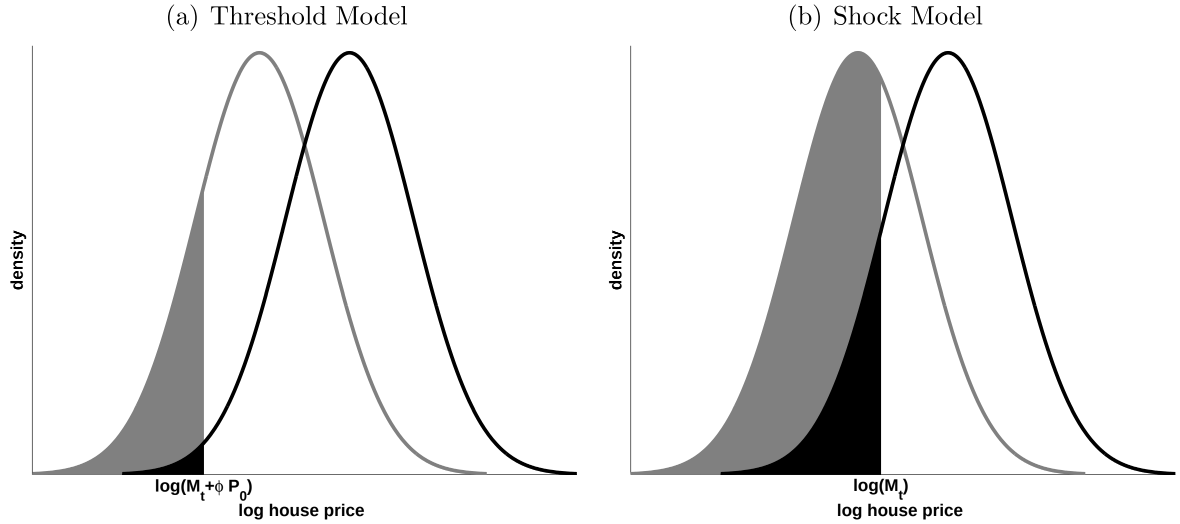
(a) Threshold Model (b) Shock Model



Admittedly the shock model generates a slightly too low increase in default rates. This could indicate that there is a small difference between the quality of borrowers of the 2002 cohort and the other cohorts though they appear to be similar based on observed characteristics. Such a conclusion is supported by the excellent fit to all but the 2002 cohort that one obtains when one estimates the model on all cohorts simultaneously, cf. online appendix B.2. Another potential explanation is that life events and in particular unemployment became more frequent during this time period.

The explanation for the difference between models is illustrated in figure 4. The figure shows the leftward shift of the (log) house price distribution from the black to the gray distribution due to a fall in aggregate house prices. The shock model predicts that out of all borrowers with negative equity, i.e. those with a (log) house price below the (log) mortgage balance, a fraction ψ defaults each period. In reaction to a leftward shift of the house price distribution the shock model predicts that the default rate should increase in proportion to the increase in the number of borrowers who experience any level of negative equity. In figure 4(b) this is represented by the ratio of the sum of the black and gray area to the original black area. It turns out that observed default rates approximately exhibit this pattern. But the threshold model is concerned with the far left tail of the house price and equity distribution. It predicts that all borrowers with a house price below the sum of the mortgage balance and ϕ times the initial house price default (remember that ϕ is negative). When the house price distribution shifts left the threshold model predicts that default rates should increase as much as the number of borrowers with extreme levels of negative equity represented by the shift from the black area to the sum of the black and gray area in figure 4(a). However the number of borrowers with such an extreme level of negative equity increases much faster than observed default rates. This difference between models drives the empirical results.

Figure 4: Illustration of Reaction of Models to Fall in Aggregate House Prices



I have conducted a large number of robustness checks to scrutinize these results, which are reported in detail in online appendices B.2, B.3 and B.4. These checks include estimating the models on cohorts other than the 2002 one, replacing the out-of-sample test with an in-sample test, abstracting from within cohort and cross cohort heterogeneity in initial LTVs and mortgage rates, assuming a different distribution of individual house price shocks, allowing threshold and shock parameters to vary over the course of the loan, using an alternative definition of default and extending the analysis to loans with a lower initial LTV. I find that the results are robust across all these specifications.[15]

Two conclusions can be drawn from the results of this section. First, an empirically successful structural model cannot rely on a single-trigger mechanism alone. Instead some feature other than house price shocks must play a role. This motivates the development of a structural double-trigger model in the following section.

A potential criticism of such a conclusion is that a change of the threshold parameter ϕ across cohorts (which I kept constant) may be able to resolve the documented problems of single-trigger models. Since the threshold model fails by overpredicting default rates of later cohorts this would require rational single-trigger borrowers to become more reluctant to default during the crisis. Accordingly, borrowers would need to have more optimistic expectations on future house prices during the housing bust than the housing boom. But such a change to expectations seems implausible given the economy entered a deep recession at that time. Furthermore it is also inconsistent with survey evidence on house price expectations (Case, Shiller, and Thompson 2012), which indicates that expectations became more pessimistic during this time period. Thus a threshold model with a time-varying expectations process would have even greater difficulties to explain observed default behavior during the crisis.

In a similar vein one may worry that ϕ could vary across cohorts due to changes in interest rates. Indeed the traditional option theoretic literature predicts that higher interest rates make the borrower more reluctant to default such that default only occurs at an even more extreme level of negative equity, cf. for example Kau, Keenan, Muller, and Epperson (1992). In order to shed light on this possibility I construct interest rate paths

[15] I have focussed on an out-of-sample exercise here because it documents the differing sensitivity of the theories to changes in aggregate house prices in a simple and intuitive way. However there are also reasons to consider the performance of the theories in an in-sample test which examines the fit of the two models when they are estimated on the data from all cohorts. Such an exercise is informative on the best possible fit to the data that both models can achieve. An in-sample investigation confirms or even strengthens the out-of-sample results, cf. figure B.1 in the appendix. The threshold model still has considerable problems to match the data even under these most favorable circumstances. In contrast, the shock model achieves an excellent fit to the data.

for different cohorts from data on annual nominal interest rates of ten year government bonds. This data shows that at a given time since origination the 2007 and 2008 cohorts faced lower nominal interest rates than the 2002 cohort. However according to the theory this interest rate variation should make the later cohorts even more willing to default such that their default threshold ϕ should be closer to zero compared to the one of the 2002 cohort. Since the benchmark threshold model already overpredicts default rates for the later cohorts this evidence suggests that changes to interest rates cannot resolve the empirical problems of frictionless option theory and instead would worsen them.

Another concern may be that the reduced form nature of the threshold model somehow biased the analysis against frictionless option theory even though the threshold model does preserve its key theoretical prediction. For this reason I have also investigated a structural single-trigger model that explicitly solves the stochastic optimization problem of a rational borrower in a frictionless world. This analysis shows that the conclusions across models when comparing a structural single-trigger model to the structural double-trigger model of the next section are the same as when comparing reduced form models, cf. online appendix C.6.

A second conclusion is that for a double-trigger model the increase in the fraction of borrowers with negative equity caused by the mean shift in house prices is sufficient to broadly explain the rise in default rates in this data set. Together with the evidence on the stability of loan characteristics in section 2 this supports the view that the fall in aggregate house prices is key for understanding the observed rise in default rates.

4 Structural Model

This section introduces a micro-founded partial equilibrium model of double-trigger behavior where unemployment acts as the life event that may trigger default together with negative equity. The aim of the structural model is to analyze whether and how unemployment may play this role, how well such a micro-founded model explains the rise in default rates during the crisis and to subsequently use the model for policy analysis in section 7. In the model a homeowner who bought a house with a fixed-rate mortgage each period chooses non-housing consumption and whether to stay in the house and service the mortgage or leave the house and terminate the mortgage. The mortgage can be terminated either by selling the house or defaulting on the loan by "walking away". The homeowner faces uncertainty on the future price of the house, unemployment shocks and a borrowing constraint for unsecured credit. It is also assumed that the homeowner derives a direct utility flow from the bought house. One period corresponds to one month. Throughout this section an individual index i is suppressed for convenience.

4.1 Mortgage Contract

The household took out a fixed rate mortgage with outstanding nominal balance M_0 and nominal mortgage rate r^m to finance the purchase of a house of price P_0 in period 0. Mortgage interest and principal have to be repaid over T periods in equal instalments of nominal value m that are fixed at the beginning of the contract and satisfy equation (2). Over time the outstanding nominal mortgage balance M_t evolves according to equation (1) as long as the household services the mortgage.

4.2 Preferences and Choices

Preferences are specified as in Campbell and Cocco (2003), but allow for a direct utility benefit of owning a house. Household decisions over the length of the mortgage contract are determined by maximizing expected utility given by

$$U = \mathrm{E}_0 \sum_{t=1}^{T} \beta^{t-1} \left(\frac{C_t^{1-\gamma}}{1-\gamma} + \theta \mathcal{I}(own_t) \right) + \beta^T \frac{W_{T+1}^{1-\gamma}}{1-\gamma} \tag{8}$$

which is derived from consumption C_t in periods 1 to T and remaining wealth W_{T+1} at the end of the contract.[16] The flow utility function from consumption is assumed to be of the CRRA form where γ denotes the parameter of relative risk aversion and the inverse of the intertemporal elasticity of substitution. β is the time discount factor. $\mathcal{I}(own_t)$ is

[16]Following Campbell and Cocco (2003), the specification in equation (8) implicitly assumes that the borrower maximizes utility only over the course of the mortgage contract because the continuation value is largely arbitrary. An obvious alternative is to extend the utility function to the remaining lifetime of the borrowers. One complication here is that I do not have any demographic information on the borrowers in my data set. However I have experimented with adding further time periods after the end of the mortgage contract and also including a retirement period. This had no significant effect on the results.

an indicator variable that is one if the agent owns a home in period t and zero otherwise. θ is a direct utility benefit from being a homeowner. This could reflect for example an emotional attachment to the specific house or the benefit that an owner cannot be asked to move out by a landlord as may happen to a renter. This direct utility flow is a very important feature to generate double-trigger behavior because it prevents employed agents from defaulting after a strong fall of house prices as explained in more detail in section 6.3 below.

In each period the homeowner has to decide how much to consume and on staying or leaving the house. If the agent wants to leave this can be done by either selling the house (and repaying the current mortgage balance) or defaulting on the loan by "walking away".[17]

4.3 Constraints

The dynamic budget constraint depends on the borrower's house tenure choice. For a homeowner who stays in the house it is given by

$$A_{t+1} = (1+r)\left(A_t + Y_t - \frac{m}{\Pi_t} + \tau r^m \frac{M_t}{\Pi_t} - C_t\right) \tag{9}$$

where A_t denotes real asset holdings and Y_t real net labor income in period t. The real interest rate on savings r is assumed to be constant over time. m is the nominal payment to service the mortgage. But the nominal mortgage interest $r^m M_t$ is tax deductable and τ is the tax rate. All nominal variables need to be deflated by the current price level for consumption goods Π_t to arrive at a budget constraint in terms of real variables. The presence of Π_t generates the "mortgage tilt effect". This means that due to inflation the real burden of the mortgage is highest at the beginning of the contract and then declines over time. It is assumed that the inflation rate π is constant over time and Π_t thus evolves according to $\Pi_{t+1} = (1+\pi)\Pi_t$ with $\Pi_0 = 1$.

[17]The model abstracts from mortgage termination through refinancing for computational reasons, which is a common simplification of the related theoretical literature cited in the introduction. Otherwise the mortgage balance becomes a separate state variable. This is unlikely to be a major limitation in the context of default because refinancing is only feasible when the borrower has positive equity in the house or substantial other liquid assets. Thus refinancing does not directly compete with the default decision in a situation of negative equity and low liquid wealth.

In case the house is sold at the current real price P_t, the homeowner needs to repay the current outstanding nominal mortgage balance M_t and can pocket the rest. The budget constraint of a seller reads as

$$A_{t+1} = (1+r)\left(A_t + Y_t - R + P_t - \frac{M_t}{\Pi_t} - C_t\right). \tag{10}$$

Here R is the real rent for a property of the same size. It is assumed that an agent who terminates the mortgage through prepayment or default needs to rent an equivalent house for the rest of life, which is a common assumption in the literature.[18] The resulting parsimonious specification simplifies the computational solution of the model considerably. However the assumption also captures the economically important fact that in the real world a defaulting borrower is closed out of the mortgage market for an extended period of the time and experiences a strong fall in his credit rating. This is one of the costs of defaulting from the borrower's point of view. In the absence of such costs a rational borrower would find it optimal to default already at very small levels of negative equity independently of his liquidity position, which would lead to clearly counterfactual predictions.

Real rents are assumed to be proportional to the initial house price and then constant over time as

$$R = \alpha P_0. \tag{11}$$

This specification involves both a highly realistic feature of rents and an approximation. The realistic feature is that during the period of study real rents remained almost constant, while real house prices first increased and then decreased enormously. The specification implies that after origination the rent-price ratio decreases when real house prices increase. Such a negative relationship between the rent-price ratio and real house prices exists in the data provided by Davis, Lehnert, and Martin (2008) not only during the recent period, but at least since 1975. In this paper I take these observations as given and specify the exogenous variables of the model accordingly. Explaining this pattern

[18]Thus a change of housing status from owning to renting is irreversible. The assumption also rules out downsizing of the house after a default which could play a role in the default decision of borrowers in the real world.

is an important area for future research. However a fully realistic specification would also require to make α cohort-specific. Instead I use an approximation for computational reasons such that α is constant across cohorts and calibrated to a suitable average.

In contrast, if the agent decides to default on the mortgage by "walking away"[19] or is already a renter the budget constraint is given by

$$A_{t+1} = (1 + r)(A_t + Y_t - R - C_t). \tag{12}$$

It is assumed that for reasons not explicitly modeled here the household faces a borrowing constraint for unsecured credit given by

$$A_{t+1} \geq 0. \tag{13}$$

Together with the budget constraints above this implies that the amount of resources available for consumption in a period depend on current wealth and the house tenure choice. By modelling borrowing constraints the model builds on the buffer-stock saving framework of Zeldes (1989), Deaton (1991) and Carroll (1997).

Remaining wealth at the end of the contract for a homeowner is given by $W_{T+1} = A_{T+1} + Y_{T+1} + P_{T+1}$ and for a renter by $W_{T+1} = A_{T+1} + Y_{T+1}$.

4.4 Labor Income Process

The household's real net labor income Y_t is subject to idiosyncratic unemployment shocks and exogenously given by

$$Y_t = \begin{cases} (1-\tau)Y_0 & \text{if employed} \\ \rho(1-\tau)Y_0 & \text{if unemployed} \end{cases} \tag{14}$$

where Y_0 is initial real gross income, τ is the tax rate and ρ is the net replacement rate of unemployment insurance. Over time employment status evolves according to a Markov transition process with the two states "employed" and "unemployed" and constant job

[19] The specification assumes a non-recourse loan which is a common assumption for the U.S. mortgage market even though formally there are recourse laws in some states. However the empirical study of Ghent and Kudlyak (2011) finds that recourse only deters borrowers from defaulting who own relatively high valued properties (above $200,000 in real 2005 terms). Since my data set contains borrowers with on average much lower house values the neglect of recourse does not seem to be a major concern.

separation and job finding probabilities. Employed agents lose their job with probability s and stay employed with probability $(1-s)$. Unemployed agents find a job with probability f and stay unemployed with probability $(1-f)$. I focus on income fluctuations due to unemployment risk here because unemployment involves a severe fall in labor income from one month to another. This makes it a very plausible cause for short run liquidity problems. This also allows to relate the model closely to the double-trigger hypothesis and the empirical evidence that default is correlated with state unemployment rates.[20]

4.5 House Price Process

Real house prices are exogenous and evolve over time as specified in section 3.4 and equation (5). It is assumed that homeowners view the aggregate component g_t^{agg} of house price appreciation to be stochastic and distributed according to an i.i.d. normal distribution with mean μ and variance σ^2. This process for the aggregate house price component is only used for forming agents' expectations. In the simulation the realizations of g_t^{agg} are those observed in the data. For the individual component agents know that g_t^{ind} is distributed normally with mean zero and time-varying variances that depend on the parameters κ and λ as specified in section 3.4. In order to reduce the computational burden when computing policy functions the parameters μ, σ, κ and λ are not varied across the nine census divisions. Instead they are set equal to national averages, cf. section 5.2 on the calibration. But the house price realizations in the simulation of the model are generated from the division specific data and distributions.

4.6 Initial Conditions

The homeowner solves the dynamic stochastic optimization problem conditional on initial asset holdings A_0, initial employment status, an initial loan-to-value ratio $LTV = \frac{M_0}{P_0}$ and a debt to (gross) income ratio $DTI = \frac{m}{Y_0}$, which refers to the ratio of the monthly

[20] My formulation abstracts from deterministic changes to labor income like a life-cycle profile and keeps the labor income of employed and unemployed agents constant over time. One reason for this is again the lack of demographic information on the borrowers in my data set. In any case these borrowers belong to the lower half of the income distribution and people in lower income classes tend to have relatively flat income profiles. Nevertheless if income during unemployment rises over time then this prolongs the period until buffer stock savings are exhausted and default occurs.

mortgage payment to gross income. I assume that borrowers were employed when they got their loan. With respect to initial assets A_0, I use the computed policy functions to set initial assets equal to the buffer-stock desired by a borrower in period 1 who is employed and faces a house value equal to P_0. Thus I shut down possible effects from borrowers first converging to their desired buffer-stock and being more vulnerable to income shocks during the time immediately after origination. The initial house price P_0 is normalized to 100. LTV and DTI then uniquely determine M_0 and Y_0.

4.7 Computation, Simulation and Test

The model is solved computationally for the optimal policy functions. The borrower's finite horizon optimization problem is characterized by four state variables (liquid wealth $X_t = A_t + Y_t$, employment status, house price P_t and time t) and two choice variables (consumption C_t and the mortgage termination choice). Note that for a fixed-rate mortgage the mortgage balance M_t evolves deterministically over time and is thus captured by the state variable t. The solution proceeds backwards in time and iterates on the value function (the value functions are provided in online appendix C.1). The continuous state and control variables are discretized and the utility maximization problem in each period is solved by grid search. Expected values of future variables are computed by Gaussian Quadrature. Between grid points the value function is evaluated using cubic interpolation.

Given the policy functions the model is simulated for subsequent cohorts of loans originated each month between January 2002 and December 2008. For each cohort I draw $25,000$ individual histories of house prices as explained in section 3.4 and employment histories from the Markov process of section 4.4.

The general idea of the performed computational exercise is the same as in the reduced form section. I use only the default data from the 2002 cohort (and other data sources) to determine model parameters as explained in section 5. The test of the model then consists again in informally comparing the out-of-sample model predictions on default rates of the 2003 to 2008 cohorts to the actual observations.

5 Parametrization

The structural model is parameterized in two steps. First the mortgage contract, house price expectations, rents, labor income, interest and inflation rates are calibrated to data on the respective variables, i.e. to data other than default rates. The preference parameters are divided into a set that is predetermined and another that is estimated such that the model fits the cumulative default rates of the 2002 loan cohort. All parameter values are summarized in table 2 below.

5.1 Mortgage Contract Characteristics

This paper restricts attention to 30-years ($T = 360$ months) fixed-rate mortgages. I use average characteristics at origination of the loans in my data set to determine the loan-to-value ratio, mortgage rate and debt-to-income ratio. The average initial loan-to-value ratio of these loans is 98.2%, so I set $LTV = 98.2\%$. The nominal mortgage rate r^m is set to 6.4% per annum which is the average mortgage rate for newly originated loans in my data set. The debt-to-income ratio DTI is set to 40% as in the data.

5.2 House Price Expectations

As explained before, when computing policy functions the parameters μ, σ, κ and λ are not varied across the nine census divisions. The monthly house price index from the FHFA at the national level between 1991 and 2010 deflated by the Consumer Price Index (CPI) is used to estimate the parameters μ and σ of the aggregate component. I find that at a monthly frequency $\mu = 0.065\%$ and $\sigma = 0.55\%$. These values imply expected yearly aggregate real house price growth of about 0.8% and a yearly standard deviation of 1.9%. This means that agents in the model have expectations on real aggregate house price growth that on average were correct in the years 1991 to 2010 as far as the mean and standard deviation are concerned.

The parameters κ and λ are determined as a simple average of the ones estimated by the FHFA for each of the nine census divisions. This gives $\kappa = 0.00187$ and $\lambda = -4.51E-6$ and implies that the individual house price growth shock g_{it}^{ind} in the first month after house purchase is expected to have a standard deviation around 2.5%.

5.3 Income Process

The average tax rate τ is set to 16% and the net replacement rate of unemployment insurance ρ to 62%. This is based on the OECD Tax-Benefit calculator[21] for the United States. Specifically, the average loan amount, mortgage rate and debt-to-income ratio are used to determine the average gross income of the borrowers in the data set. Based on gross income the calculator reports the net income in work and out of work which then determine the average tax and net replacement rates. These calculations take taxes, social security contributions, in-work and unemployment benefits into account. Precise numbers especially for the tax rate also depend on the demographics of the household. I have used the average values for a married couple with one earner and no children.

Data from the Bureau of Labor Statistics on the national unemployment rate and median unemployment duration are used to compute time-series of monthly job finding and separation probabilities. This is done using steady state relationships. In a steady state the median duration of unemployment d and the unemployment rate u should satisfy $(1-f)^d = 0.5$ and $u = \frac{s}{s+f}$. These two equations are then solved for the time-series of f_t and s_t implied by the time-series of u_t and d_t.[22] I then set $s = 1.8\%$ and $f = 31\%$ which are the average values of the computed monthly finding and separation probabilities during 1990 to 2010. These values imply a steady state unemployment rate around 5.5%. I keep these values of s and f constant in the simulation of the model. However using the observed changing values when simulating the model yields very similar results (without recomputing policy functions).

5.4 Other Prices

Nominal interest rates for 1-year Treasuries and changes to the Consumer Price Index (CPI) are used to compute real interest rates and inflation rates. Based on this data

[21] http://www.oecd.org/social/soc/benefitsandwagestax-benefitcalculator.htm

[22] As a check on this procedure I predict the unemployment rate from the dynamic equation of unemployment $u_{t+1} = u_t + s_t(1-u_t) - f_t u_t$ using the computed time series of finding and separation probabilities as inputs. It turns out that this gives an excellent fit to the path of the actual unemployment rate.

between 1990 and 2010 the real interest rate r is set equal to 1.4% per year. The inflation rate π is set to 2.4% annually which is the average value during the simulation period. The initial rent-price ratio parameter α is set equal to 4.0% on a yearly basis which is the average rent-price ratio between 2002 and 2008 in the data provided by Davis, Lehnert, and Martin (2008).

Table 2: Model Parameters

Mortgage contract	Contract length in months	T	360
	Mortgage rate (yearly)	r^m	6.4%
	Initial loan-to-value ratio	LTV	98.2%
	Initial debt-to-income ratio	DTI	40%
House price process	Mean of aggregate component	μ	0.065%
	Std. dev. of aggregate component	σ	0.55%
	Linear coefficient in indiv. variance	κ	0.00187
	Quadratic coefficient in indiv. variance	λ	-4.51E-6
Income process	Job separation probability	s	1.8%
	Job finding probability	f	31%
	Tax rate	τ	16%
	Net replacement rate of UI	ρ	62%
Other prices	Real interest rate (yearly)	r	1.4%
	Inflation rate (yearly)	π	2.4%
	Rent-price ratio (yearly)	α	4.0%
Preferences	CRRA coefficient	γ	5
	Discount factor (yearly)	β	0.9
	Utility benefit of owning	θ	0.28

5.5 Preferences

In order to reduce the computational burden and due to identification concerns, I first choose reasonable values for β and γ and then estimate only θ. For the intertemporal elasticity of substitution, which is the inverse of γ, Guvenen (2006) reviews empirical estimates ranging from around 1 to 0.1, which implies values of γ ranging from 1 to 10. Furthermore he argues that conflicting estimates can be reconciled if the rich have a high and the poor have a low elasticity. I choose $\gamma = 5$, which is in the middle of this range. Following Guvenen's reasoning, one could also argue for higher values because borrowers in my data set belong to the lower half of the income distribution. For β I choose a value of 0.9 at an annual frequency which may be a bit on the low side. But adapting Guvenen's argument to β, the reason is that I am analyzing borrowers who were only able to make a very small down-payment. This could be due to the fact that they are very impatient. The agents who are net savers could then have a higher discount factor. Online appendix C.2 contains a sensitivity analysis with respect to β and γ.[23]

Given values of β and γ, the parameter θ representing the direct utility benefit from owning the house is estimated by the simulated method of moments. Again the parameter is chosen such that cumulative default rates simulated from the model match those observed for the 2002 cohort, and the remaining data is used to test the ability of the estimated model to predict out of sample. This yields an estimate for θ of 0.28. Relative to a situation of $\theta = 0$ a homeowner at the initial conditions values this direct utility flow of $\theta = 0.28$ about as much as a permanent increase in income of 1.5%. Thus the estimated direct utility benefit is of a relatively modest magnitude here.[24]

6 Results

This section first explains the mechanism generating default in the model. Then the main results how well the model fits the rise in default rates across loan cohorts are presented. Finally, I explain some insights into the modelling of double-trigger behavior and an empirically accurate house price sensitivity.

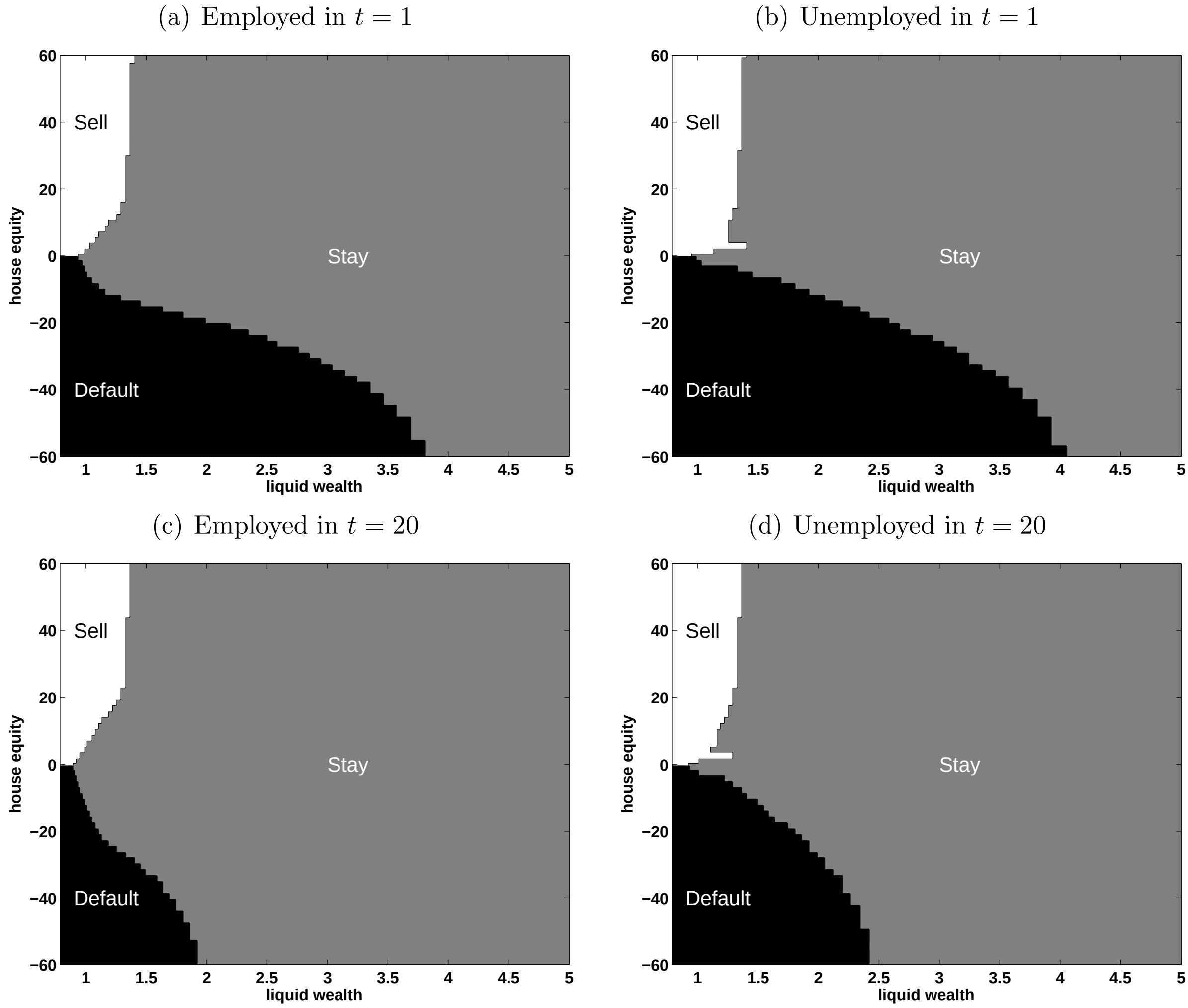
[23]Results from the sensitivity analysis can be summarized as follows. The model relies on a sufficiently high value of γ and low value of β to generate double-trigger behavior. The reason is that a low willingness to substitute intertemporally and a high impatience to consume today worsen the liquidity problem caused by unemployment. As a result being employed and being unemployed are sufficiently different states which is required for double-trigger behavior. If this is not the case then a sizeable portion of employed agents default in all cohorts which brings the model too close to a frictionless option model and the overshooting problems witnessed already in section 3.

[24]The estimated direct utility benefit parameter θ may also capture deviations of house price expectations from empirically observed long-run house price growth rates to which the model is calibrated. For instance if expected real house price growth μ is increased by one percentage point on an annual basis then θ is instead estimated at a lower value of about 0.26. This reflects the general importance of house price expectations for this literature. The empirical performance of the model with such modestly overoptimistic house price expectations is very similar to the benchmark calibration. However a model with very overoptimistic house price expectations and accordingly a direct utility benefit θ close to zero does not perform well empirically because in such a model a strong fall in house prices will also induce employed and not liquidity constraint households to default, cf. the detailed explanation in section 6.3.

6.1 The Default Mechanism

The repayment policy function of a borrower in the model is presented in figure 5 as a function of house equity, liquid wealth, employment status and time. Several features are noteworthy. First, negative equity is a necessary condition for default. Instead, with positive equity selling is strictly preferred to defaulting because the borrower is the residual claimant of the house value after the mortgage balance has been repaid.

Figure 5: Repayment Policy Function



Notes: Repayment choice as a function of the state variables liquid wealth, house equity, employment status and time. Blue region: Default. Green region: Sell. Red region: Stay.

Second, negative equity is not sufficient for default. There are many combinations of state variables where a borrower with negative equity prefers to stay in the house and service the mortgage. In a negative equity situation the basic trade-off of the borrower is the following (postponing the role of the borrowing constraint until the next paragraph).

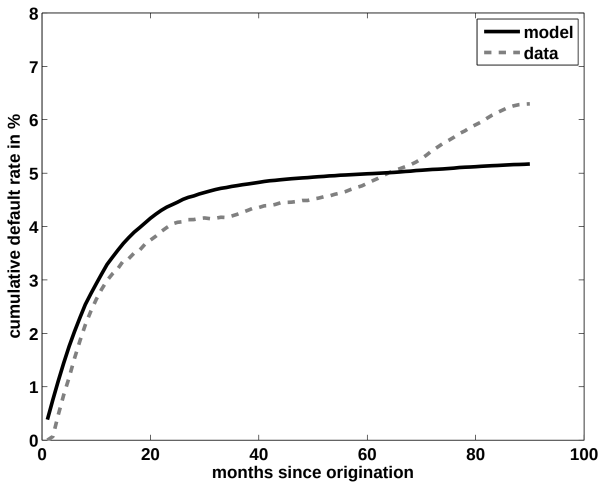
The cost of staying in the house is that the borrower needs to make the mortgage payment, which is higher than the rent for an equivalent property. The benefit of staying is that the borrower receives the utility benefit of owning a house and keeps the option to default, sell or stay later. Specifically, there are possible future states of the world with positive equity. But the probability of reaching these states depends on the current house price. This establishes a default threshold level of the house price. Of course, when making this decision the rational borrower will also need to discount these future gains and take risk aversion into account.

Third, the level of negative equity at which the borrower exercises the default option depends on non-housing state variables: liquid wealth and employment status. Specifically, a borrower who is unemployed and/or has low liquid wealth will default at lower levels of negative equity. There are two reasons for terminating the mortgage in these states. One is that current borrowing constraints may bind and the borrower terminates the mortgage to increase current consumption. The other reason is that in these states it becomes very likely that borrowing constraints bind in the future and the agent is forced to terminate the mortgage then. But an anticipated future mortgage default creates an incentive to default already today to save the difference between the mortgage payment and the rent in the meantime. This also explains why unemployment, which is persistent, shifts the default frontier to the right.

Fourth, over time the default region shrinks. This is mainly due to the effect of inflation that diminishes the real difference between the effective mortgage payments and rents. This has two implications. First, a liquidity constrained borrower cannot increase current consumption much by a mortgage default. Second, staying in the home eventually dominates renting in all states because the real value of the mortgage payment falls below the real rent.

In order to better understand default behavior over the life-cycle of a loan, figure 6 presents the cumulative default rate for loans originated in 2002. This is the cohort for which I have the longest time dimension and on which the model is estimated. Accordingly, the dynamics of default over the life-cycle of this cohort are captured relatively well by the model.

Figure 6: Cumulative Default Rates of 2002 Cohort: Model vs. Data



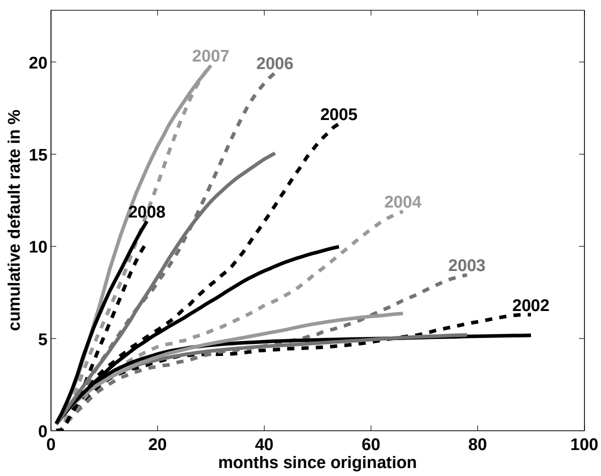
Though this cohort faces growing average house prices during the immediate time after origination as seen in figure 1(b), some individuals experience falling house prices and negative equity as a consequence of individual house price shocks. Households with negative equity default when prolonged stretches of unemployment have exhausted their buffer stock savings, cf. the default region of the state space in figure 5. In fact more than 99% of all borrowers in this cohort who default are unemployed when they default. This number is similar in the other loan cohorts and never falls below 93%. Thus the presented model does indeed micro-found the double-trigger hypothesis, cf. the discussion in section 6.3 below.

Eventually, the cumulative default rate levels off due to two reasons. First, borrowers who are still active have amortized their mortgages sufficiently such that most have positive equity. Second, due to the mortgage tilt effect the difference between the real mortgage payment and real rents shrinks over time such that a default becomes less appealing.

6.2 The Rise in Cumulative Default Rates

The next step is to compare the default behavior of different cohorts during the time period of the U.S. mortgage crisis. Figure 7 and table 3 compare model predictions and empirical observations on cumulative default rates for cohorts of loans originated each year between 2002 and 2008. The model can explain the broad pattern in the data. The more adverse house price paths of later cohorts cause more borrowers to have negative equity. In the model the borrowers with negative equity who also experience liquidity problems due to unemployment default on their mortgage. This means the model attributes the rise in cumulative default rates across cohorts to the different aggregate house price paths witnessed in figure 1(b).

Figure 7: Cumulative Default Rates of all Cohorts: Model (solid lines) vs. Data (dashed lines)



During roughly the first two years after loan origination the model accurately predicts the observed increase of default rates across loan cohorts. In contrast for the later period after loan origination the increase in default rates across cohorts is too weak in the model relative to the data. In the model this is due to a strong effect of inflation, the mortgage tilt effect. This effect diminishes the difference between real mortgage payments and rents over time. The model is sensitive to this difference. As a consequence default rates do not react strongly enough to falling house prices in periods long after loan origination.

Indeed the model gives a much better fit to the data when it is calibrated to a lower inflation rate, cf. online appendix C.4. One possible interpretation is that in the real world borrowers do not fully understand and underestimate the mortgage tilt effect relative to the rational agent in the model. Another fact is that during 2008 to 2010 the inflation rate

Table 3: Cumulative Default Rates (in %) of all Cohorts at certain Points since Origination: Model vs. Data

Cohort		Months since Origination							
		6	18	30	42	54	66	78	90
2002	Data	1.5	3.6	4.2	4.4	4.6	5.1	5.8	6.3
	Model	2.0	4.0	4.6	4.9	5.0	5.1	5.2	5.2
2003	Data	1.3	3.4	4.2	4.7	5.6	7.1	8.4	
	Model	1.9	3.7	4.3	4.6	4.8	5.0	5.1	
2004	Data	1.7	4.3	5.4	7.0	9.5	11.9		
	Model	1.9	3.8	4.7	5.4	6.0	6.4		
2005	Data	1.9	5.1	7.9	12.2	16.6			
	Model	2.0	4.9	7.0	8.8	9.9			
2006	Data	2.1	7.3	13.4	19.4				
	Model	2.5	7.5	12.5	15.2				
2007	Data	3.2	11.8	19.8					
	Model	3.9	14.4	20.1					
2008	Data	2.3	10.2						
	Model	4.0	11.3						

was on average only about 1.4%, which is much lower than the calibrated value of 2.4%. This could also explain the discrepancy between the model and the data particularly if during the crisis borrowers expected inflation to be low for an extended period of time in the future.

Another plausible explanation is that other life events like marital break-up or other income or expenditure shocks that were excluded from the model could be responsible for default in later periods. The structural model only analyzes whether and how unemployment shocks can act as the trigger event and finds that they could definitely play an important role especially during the early months after origination. But assessing the role of other life events and a decomposition of actual default rates into the different causes within the double-trigger paradigm is an interesting area for future research. However with respect to divorce I have also analyzed one alternative specification that included this risk in the model and calibrated it to observed divorce rates. When repeating the analysis the fit to the data was very similar to the one observed for the benchmark model presumably because observed divorce rates are relatively small.

6.3 A Lesson for Building Empirically Accurate Micro-Founded Models

The above results show how one can broadly explain the rise of default rates during the mortgage crisis within a structural double-trigger model. Obviously an important element in this model are the unemployment shocks and a liquidity constraint, which are also included in a mortgage context in a few other papers cited in the introduction. However during the investigation it became clear that these assumptions are as such not sufficient to yield an empirically accurate model. This insight is important for future work and not contained in the prior theoretical literature. The problem is that a strong fall in house prices may also induce employed and not currently liquidity constrained home owners to default. This will then lead to a too strong increase in default rates in the model, which I document in detail in the online appendix C.3. Accordingly, there needs to be a reason that prevents employed agents from defaulting after a strong fall of house prices. In the presented model the direct utility benefit from living in the bought house plays this role. This feature is a simple adjustment to the model, but enormously important for truly micro-founding double-trigger behavior. Accordingly, this is the first paper that achieves this aim. This also ensures an accurate sensitivity of default rates to changes in aggregate house prices, which is of great relevance for the use of such structural models for any policy or macroeconomic risk analysis.

7 Analysis of Two Crisis Mitigation Policies

During mortgage crises governments frequently conduct mitigation policies. Presumably the reason is that governments are concerned about a destabilization of the financial system due to the massive losses that mortgage lenders incur from mortgage default in such situations. This section applies the presented structural model for policy analysis in this context. For simplicity I study a situation where the government decides to neutralize all losses of lenders by a suitable bailout policy and ask the question: Should the government bail out lenders or subsidize homeowners for not defaulting? Thus I only analyze the relative choice between these two options and not whether stabilization policies should be conducted at all. I also do not attempt to provide a full welfare analysis and instead focus on which of these policies has a lower cost for taxpayers.

The two analyzed policy measures do not directly resemble specific policies enacted during the recent crisis. However they are arguably related. In fact actual stabilization policy has encompassed both types of measures. The Troubled Asset Relief Program (TARP) may for instance be viewed as a bailout to lenders because it allowed the government to buy distressed mortgage-backed securities, and to provide guarantees and loans at favorable conditions to banks. In contrast the Home Affordable Modification Program (HAMP) helped borrowers to receive loan modifications that lowered their current mortgage payments because the government shared the costs this created for lenders. Thus this program may be viewed as a subsidy to homeowners to prevent default. The assumption that the government wants to neutralize all losses of lenders is of course stark and may seem unrealistic. But here this is done on purpose to facilitate the analysis. The reason is that this allows to compare the costs of the two policies in a situation where both achieve exactly the same goal. In reality governments could of course scale down these policies in a suitable way.

In case lenders are bailed out the government needs to cover the negative equity of defaulters, i.e. by how much the outstanding mortgage balance exceeds the value of the collateral. In contrast, the government could also give subsidies to homeowners who would otherwise default such that they continue to service the mortgage. An alternative interpretation of such a subsidy policy is a temporary mortgage payment reduction for the borrower. This policy might well be cheaper because homeowners are willing to accept some negative equity and thus bear some of the losses on the house value unless they face severe liquidity problems. The subsidies then only have to overcome the temporary liquidity shortage to neutralize the losses for lenders. However it is also possible that subsidizing homeowners simply delays default to a later period such that the subsidy policy ends up being more expensive in the long run. These opposing effects make a quantitative analysis desirable.

The two policies are compared by calculating the average cost per borrower who would default in absence of an intervention. For the bailout of lenders this simply amounts to the average negative equity of a defaulter which can readily be computed during the simulation. For the subsidy policy I calculate for each potential defaulter the minimum subsidy amount required to make the borrower stay in the house. When doing this the standard policy functions are used. This means borrowers will consume out of the subsidy, but further negative incentive effects are ruled out. The total sum of all subsidies to a cohort is divided by the number of defaulters without any intervention to make it comparable to the other bailout policy. The required real payment streams of both policies are compared by calculating present discounted values using the real interest rate r. Of course these calculations can only be as accurate as the model captures actual default behavior. Thus it is an important advantage that the model is broadly consistent with empirical evidence. There is also an argument to be made to focus more on the earlier cohorts that are observed for more time periods in order to accurately account for the delayed default effect of the subsidy.

Table 4 presents the results of this analysis for the different cohorts. Bailing out lenders implies average real present discounted costs between 4.5% and 9.8% of the initial house price per borrower who defaults. In contrast subsidizing homeowners on average only costs between 0.6% and 1.0% of the initial house price in real present discounted value terms. Depending on the cohort bailing out lenders is thus between 7.1 and 9.9 times more expensive than subsidizing homeowners. These are large differences.

Table 4: Costs of Different Mitigation Policies

Cohort	2002	2003	2004	2005	2006	2007	2008
Bailout to Lenders	4.5	4.6	5.3	6.7	8.3	9.8	7.7
Subsidy to Borrowers	0.6	0.6	0.7	0.7	0.9	1.0	0.9
Ratio Bailout / Subsidy	7.1	7.4	8.1	9.0	9.5	9.9	9.0

Notes: Rows 1 and 2 present the average real discounted cost of the respective policy per borrower who would default without an intervention expressed in percent of the initial house price. Row 3 reports the ratio between row 1 and row 2.

A couple of caveats apply to this policy analysis. First, these are partial equilibrium results. But it seems that general equilibrium effects of subsidizing homeowners would also be more favorable because keeping borrowers in their houses avoids downward pressure on house prices due to foreclosure sales. Second, the subsidy would also help lenders to avoid further administrative costs related to foreclosures and housing sales. Both of these points further strengthen the case for the subsidy.

However there are also reasons to believe that the costs of the subsidy might be underestimated in these calculations or at least that a real world implementation of this policy would need to pay attention to further details. One is that there may be practical problems and high informational requirements associated with implementing such an individually targeted minimum subsidy to homeowners. Other concerns are related to moral hazard issues. The subsidy could for example make unemployed borrowers more reluctant to accept new job offers and prolong their unemployment spells, cf. Mulligan (2012) for arguments on adverse labor market effects from mortgage modification. However this problem could potentially be addressed by making the subsidy policy conditional on the borrower exerting a reasonable job search effort and accepting job offers he receives. In the long-run both policies may also have negative incentive effects on the screening efforts of lenders and may lead to more risky loans. Thus these calculations are probably most accurate for a situation where the government surprises private agents with such policies, which are then implemented temporarily during a crisis and only applied to old and not new loans.

Several of the raised points would merit further investigation in future work. Nevertheless the magnitude of the differences between policies suggest that there is indeed considerable potential to improve on simply bailing out lenders in order to reduce the cost of mitigating a mortgage crisis for taxpayers.

8 Conclusions

This paper has included the economic structure of different default theories explicitly in the analysis and presented simulations of these models for the observed path of aggregate house prices and a realistic microeconomic distribution. Theoretical predictions were then compared to data on default rates on prime fixed-rate mortgages to assess the explanatory power of the theories during the U.S. mortgage crisis. This comparison revealed that the frictionless default theory is too sensitive to the mean shifts in the house price distribution observed in recent years. In contrast, the double-trigger hypothesis attributing default to the joint occurrence of negative equity and a life event is consistent with the dynamics of default rates during the U.S. mortgage crisis.

Based on this finding a structural dynamic stochastic model with liquidity constraints, unemployment shocks and a direct utility benefit of owning the bought house was presented to provide micro-foundations for the double-trigger hypothesis. In this model the liquidity problems associated with unemployment act as a trigger event for default in negative equity situations. The direct utility flow from living in the bought house prevents employed not liquidity constrained borrowers from defaulting after a strong fall in house prices. Both features are important for micro-founding double-trigger behavior. The model is broadly consistent with the data and explains most of the rise in mortgage default rates as a consequence of aggregate house price dynamics. This accurate sensitivity of default rates to changes in aggregate house prices is a key requirement to make such models useful for policy or macroeconomic risk analysis.

The structural model was used for a first pass to formally analyze two important ways to conduct mitigation policies in a mortgage crisis, namely a bailout to lenders or a subsidy to homeowners to prevent default. These abstract policy options are broadly related to actual policies such as in turn the Troubled Asset Relief Program (TARP) and the Home Affordable Modification Program (HAMP). The analysis shows that if the government desires to neutralize losses for lenders then subsidizing homeowners is about $7 - 10$ times cheaper than bailing out lenders when liquidity problems are a key determinant of mortgage default. These results suggest a large potential to reduce the cost of mitigating a mortgage crisis for taxpayers. In future research extended versions of the model could also be used to analyze a range of related policy measures meant to prevent or mitigate mortgage crises. An example is how the design of unemployment insurance may help to prevent mortgage default. In such analyses it would also be useful to pay more attention to incentive and information problems associated with these policies.

The paper provides evidence that the observed aggregate house price dynamics play a very important role for the rise in mortgage default during the U.S. mortgage crisis. Together with the presented evidence on stable loan characteristics this cautions against attributing too much of the increase in default just to lax lending standards. Though the extreme movements of house prices were a rare historical event, the reaction of borrowers can be explained quantitatively by the double-trigger theory of mortgage default. This finding may also help to draw lessons from the recent crisis for the prevention of future mortgage crises.

References

Adelino, M., K. Gerardi, and P. S. Willen (2013). Why don't Lenders renegotiate more Home Mortgages? Redefaults, Self-Cures and Securitization. *Journal of Monetary Economics 60*(7), 835–853.

Amromin, G. and A. L. Paulson (2009). Comparing Patterns of Default among Prime and Subprime Mortgages. *Federal Reserve Bank of Chicago Economic Perspectives Q2*, 18–37.

Bajari, P., S. Chu, and M. Park (2010). An Empirical Model of Subprime Mortgage Default from 2000 to 2007. *Working Paper*.

Bhutta, N., J. Dokko, and H. Shan (2010). The Depth of Negative Equity and Mortgage Default Decisions. *Working Paper*.

Calhoun, C. A. (1996). *OFHEO House Price Indexes: HPI Technical Description*. Washington, D.C.: Office of Federal Housing Enterprise Oversight.

Campbell, J. Y. and J. F. Cocco (2003). Household Risk Management and Optimal Mortgage Choice. *Quarterly Journal of Economics 118*, 1449–1494.

Campbell, J. Y. and J. F. Cocco (2015). A Model of Mortgage Default. *Journal of Finance 70*(4), 1495–1554.

Carroll, C. D. (1997). Buffer-Stock Saving and the Life-Cycle/Permanent Income Hypothesis. *Quarterly Journal of Economics 112*(1), 1–55.

Case, K. E., R. J. Shiller, and A. K. Thompson (2012). What have They been thinking? Homebuyer Behavior in Hot and Cold Markets. *Brookings Papers on Economic Activity Fall 2012*, 265–298.

Chatterjee, S. and B. Eyigungor (2015). A Quantitative Analysis of the U.S. Housing and Mortgage Markets and the Foreclosure Crisis. *Review of Economic Dynamics 18*(2), 165–184.

Corbae, D. and E. Quintin (2015). Leverage and the Foreclosure Crisis. *Journal of Political Economy 123*(1), 1–65.

Corradin, S. (2014). Household Leverage. *Journal of Money, Credit and Banking 46*(6), 567–613.

Davis, M. A., A. Lehnert, and R. F. Martin (2008). The Rent-Price Ratio for the Aggregate Stock of Owner-Occupied Housing. *Review of Income and Wealth 54*(2), 279–284.

Deaton, A. (1991). Saving and Liquidity Constraints. *Econometrica 59*(5), 1221–1248.

Demyanyk, Y. and O. Van Hemert (2011). Understanding the Subprime Mortgage Crisis. *Review of Financial Studies 24*(6), 1848–1880.

Deng, Y., J. M. Quigley, and R. Van Order (2000). Mortgage Terminations, Heterogeneity and the Exercise of Mortgage Options. *Econometrica 68*(2), 275–307.

Duffie, D. and K. J. Singleton (1993). Simulated Moments Estimation of Markov Models of Asset Prices. *Econometrica 61*(4), 929–952.

Elul, R., N. S. Souleles, S. Chomsisengphet, D. Glennon, and R. Hunt (2010). What 'Triggers' Mortgage Default? *American Economic Review Papers and Proceedings 100*(2), 490–494.

Ferreira, F. and J. Gyourko (2015). A New Look at the U.S. Foreclosure Crisis: Panel Data Evidence of Prime and Subprime Borrowers from 1997 to 2012. *Working Paper*.

Foote, C. F., K. S. Gerardi, L. Goette, and P. S. Willen (2008). Subprime Facts: What (We Think) We Know about the Subprime Crisis and What We Don't. *Federal Reserve Bank of Boston Public Policy Discussion Paper No. 08-2*.

Foote, C. F., K. S. Gerardi, L. Goette, and P. S. Willen (2009). Reducing Foreclosures: No Easy Answers. *NBER Macroeconomics Annual 24*, 89–138.

Foote, C. F., K. S. Gerardi, and P. S. Willen (2008). Negative Equity and Foreclosure: Theory and Evidence. *Journal of Urban Economics 64*(2), 234–245.

Fuster, A. and P. S. Willen (2013). Payment Size, Negative Equity, and Mortgage Default. *Working Paper*.

Garriga, C. and D. E. Schlagenhauf (2009). Home Equity, Foreclosures, and Bail-outs. *working paper*.

Gerardi, K. S., K. F. Herkenhoff, L. E. Ohanian, and P. S. Willen (2013). Unemployment, Negative Equity and Strategic Default. *Working Paper*.

Gerardi, K. S., A. Lehnert, S. M. Sherlund, and P. S. Willen (2008). Making Sense of the Subprime Crisis. *Brookings Papers on Economic Activity* (Fall), 69–159.

Gerardi, K. S., A. H. Shapiro, and P. S. Willen (2007). Subprime Outcomes: Risky Mortgages, Homeownership Experiences, and Foreclosures. *Federal Reserve Bank of Boston Working Paper No. 07-15*.

Ghent, A. C. and M. Kudlyak (2011). Recourse and Residential Mortgage Default: Theory and Evidence from U.S. States. *Review of Financial Studies 24*(9), 3139–3186.

Guiso, L., P. Sapienza, and L. Zingales (2013). The Determinants of Attitudes towards Strategic Default on Mortgages. *Journal of Finance 68*(4), 1473–1515.

Guvenen, F. (2006). Reconciling Conflicting Evidence on the Elasticity of Intertemporal Substitution: A Macroeconomic Perspective. *Journal of Monetary Economics 53*(7), 1451–1472.

Gyourko, J. and J. Tracy (2014). Reconciling Theory and Empirics on the Role of Unemployment in Mortgage Default. *Journal of Urban Economics 80*, 87–96.

Hatchondo, J. C., L. Martinez, and J. M. Sánchez (2015). Mortgage Defaults. *Journal of Monetary Economics 76*, 173–190.

Herkenhoff, K. F. and L. E. Ohanian (2013). Foreclosure Delay and U.S. Unemployment. *Working Paper*.

Jagtiani, J. and W. W. Lang (2011). Strategic Default on First and Second Lien Mortgages During the Financial Crisis. *Journal of Fixed Income 20*(4), 7–23.

Jeske, K., D. Krueger, and K. Mitman (2013). Housing, Mortgage Bailout Guarantees and the Macro Economy. *Journal of Monetary Economics 60*(8), 917–935.

Joint Center for Housing Studies at Harvard University (2008). *The State of the Nation's Housing.*

Kau, J. B. and D. C. Keenan (1995). An Overview of the Option-Theoretic Pricing of Mortgages. *Journal of Housing Research 6*(2), 217–244.

Kau, J. B., D. C. Keenan, and T. Kim (1994). Default Probabilities for Mortgages. *Journal of Urban Economics 35*, 278–296.

Kau, J. B., D. C. Keenan, W. J. Muller, and J. F. Epperson (1992). A Generalized Valuation Model for Fixed-Rate Residential Mortgages. *Journal of Money, Credit and Banking 24*(3), 279–299.

Kau, J. B., D. C. Keenan, W. J. Muller, and J. F. Epperson (1995). The Valuation at Origination of Fixed-Rate Mortgages with Default and Prepayment. *Journal of Real Estate Finance and Economics 11*(1), 5–36.

Mayer, C., K. Pence, and S. M. Sherlund (2009). The Rise in Mortgage Defaults. *Journal of Economic Perspectives 23*(1), 27–50.

McFadden, D. (1989). A Method of Simulated Moments for Estimation of Discrete Response Models Without Numerical Integration. *Econometrica 57*(5), 995–1026.

Mian, A. and A. Sufi (2009). The Consequences of Mortgage Credit Expansion: Evidence from the U.S. Mortgage Default Crisis. *Quarterly Journal of Economics 124*(4), 1449–1496.

Mulligan, C. B. (2012). *The Redistribution Recession: How Labor Market Distortions Contracted the Economy.* Oxford University Press.

Pakes, A. and D. Pollard (1989). Simulation and the Asymptotics of Optimization Estimators. *Econometrica 57*(5), 1027–1057.

Palmer, C. (2013). Why did so many Subprime Borrowers Default during the Crisis: Loose Credit or Plummeting Prices? *Working Paper*.

Quercia, R. G. and M. A. Stegman (1992). Residential Mortgage Default: A Review of the Literature. *Journal of Housing Research 3*(2), 341–379.

Stanton, R. and N. Wallace (2011). The Bear's Lair: Index Credit Default Swaps and the Subprime Mortgage Crisis. *Review of Financial Studies 24*(11), 3250–3280.

Vandell, K. D. (1995). How Ruthless is Mortgage Default: A Review and Synthesis of the Evidence. *Journal of Housing Research 6*(2), 245–264.

Zeldes, S. (1989). Optimal Consumption with Stochastic Income: Deviations from Certainty Equivalence. *Quarterly Journal of Economics 104*(2), 275–298.

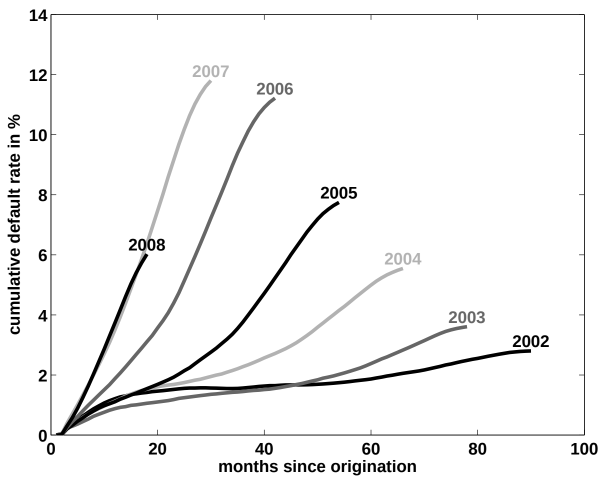
Appendices (Only For Online Publication)

A Appendix to Data and Empirical Facts

The main analysis of the paper is focussed on loans with an initial loan-to-value (LTV) ratio above 95% for the substantive reasons explained in section 2.1. This section shows that the general empirical patterns concerning the rise in default rates and relatively stable loan characteristics at origination across cohorts extend to other loans with lower LTVs as well. In other words the full sample exhibits broadly the same dynamics across cohorts than the sample of highly leveraged borrowers analysed in the paper, though there are of course various level differences.

Figure A.1 presents the observed cumulative default rates for different loan cohorts for the full sample of loans with all possible initial LTVs. Of course the data still refers to prime, fixed-rate, 30-years mortgages as in the main text. Though the level of default rates is in general somewhat lower than for the sample of highly leveraged borrowers, the rise in default rates follows a very similar pattern.

Figure A.1: Cumulative Default Rates in the Full Sample



Evidence on average loan characteristics at origination for different cohorts for the full sample are presented in table A.1. The observed pattern is similar to the one for highly leveraged borrowers in the main text. Average FICO credit scores are almost constant across cohorts. The average loan-to-value ratio changes a bit more across cohorts than for the above 95% LTV sample. But surprisingly in the full sample many of the later cohorts even have a lower average LTV than the first cohort. The dynamic pattern of mortgage rates and debt-to-income (DTI) ratios is very similar to the sample in the main text. Again changes in average mortgage rates are not closely correlated with the increase in default rates, but average DTIs at origination increase somewhat over time.

Table A.1: Average Loan Characteristics at Origination by Loan Cohort for the Full Sample

Cohort	2002	2003	2004	2005	2006	2007	2008	All
LTV in %	77.9	75.1	75.7	73.8	74.8	76.4	78.4	76.0
FICO score	714	714	712	716	712	710	716	714
Mortg. rate in %	6.7	5.9	6.0	6.0	6.6	6.5	6.2	6.2
DTI in %	34	37	36	37	38	39	38	37

These empirical facts show that the dynamics of default rates and loan characteristics across cohorts are very similar in the full and the more restricted sample. This is evidence against the considered sample being somehow special and suggests that several of the drawn conclusions may well extend to prime, fixed-rate, 30-years mortgages more generally. Furthermore, in section B.4 below I show explicitly that under plausible assumptions on second mortgages the main conclusions of the reduced-form exercise generalize to loans with an initial LTV of the first mortgage between 75% and 84%.

At least for the prime mortgage market the facts documented in this section are prima facie evidence against explanations that emphasize the role of a deterioration in lending standards or increased borrower leverage for the rise in default rates like Corbae and Quintin (2015). For the full sample I have also conducted an additional analysis of compositional changes with respect to initial LTVs. For this purpose I grouped all loans with respect to their initial LTV into 10 bins with a width of 10 percentage points (the two exceptions are the first bin with all LTVs between $1-14\%$ and the last bin with all LTVs above 95%). Using the observed default rates of each of these bins for each cohort and the actual LTV composition of each cohort I then computed two counterfactual aggregate default rates for all cohorts, i.e. the equivalent to figure A.1. In the first counterfactual scenario I keep the default rates of each LTV bin fixed at their observed level of the 2002 cohort and only vary the LTV composition across cohorts according to the actual data. In the second counterfactual I only vary the default rates of each bin across cohorts as observed in the actual data, but counterfactually keep the LTV composition fixed as it is observed for the 2002 cohort. This decomposition analysis shows that for all practical purposes the aggregate default dynamics across cohorts witnessed in figure A.1 are entirely driven by increases in default profiles for each initial LTV bin and not by changes in the composition of initial LTVs. The same conclusion applies if one uses a cohort other than the 2002 one for fixing one of the two margins in the counterfactual calculations. These results reinforce the conclusion in the main paper that the fall in house prices and not compositional effects are key for understanding the rise in default rates at least for prime fixed-rate mortgages during the crisis.

B Appendix to Reduced Form Models

B.1 Estimation Procedure

The model parameters are estimated by a simulated method of moments procedure. Let θ stand in for the parameter to be estimated in the respective model. The idea of the estimation is to choose θ such that the cumulative default rates for the 2002 cohort simulated from the model match as well as possible those observed in the data. Collect the variables d_{it} in one vector $D_i = [d_{i1}, \dots, d_{iT}]'$ for each individual. The mean of this vector $\overline{D} = \frac{1}{N}\sum_{i=1}^{N} D_i$ represents the empirically observed cumulative default rate. The expected value of D_i is $E[D_i] = D(\theta)$ and denote the expected value evaluated by simulation of S individuals from the model by $\widetilde{D}(\theta)$. The deviation of the model from the data is then given by $G(\theta) = \overline{D} - \widetilde{D}(\theta)$. The simulated method of moment estimator of θ minimizes $G(\theta)'WG(\theta)$ where W is a weighting matrix. I weight all moments equally by using an identity matrix as the weighting matrix. θ is then estimated by minimizing a least squares criterion function given by

$$\sum_{t=1}^{T} \left(\overline{d_t} - \widetilde{d}_t(\theta)\right)^2 \tag{15}$$

where $\overline{d_t}$ and $\widetilde{d}_t(\theta)$ are the t-th element in the vectors $\overline{D}$ and $\widetilde{D}(\theta)$, respectively. Here $\widetilde{d}_t(\theta)$ is evaluated using a frequency simulator such that $\widetilde{d}_t(\theta) = \frac{1}{S}\sum_{j=1}^{S}\widetilde{d_{jt}}(\theta)$ and $\widetilde{d_{jt}}(\theta)$ represents the outcome for period t of applying the decision rules to the drawn history j of the underlying shocks. The minimization problem is solved by a grid search algorithm.

B.2 Robustness Checks

This section reports a battery of robustness checks that were performed to scrutinize the reduced form results. I find that the results are robust across all the modifications considered here. For brevity I do not report the graphs corresponding to figure 3 for all these checks, but these are available upon request.

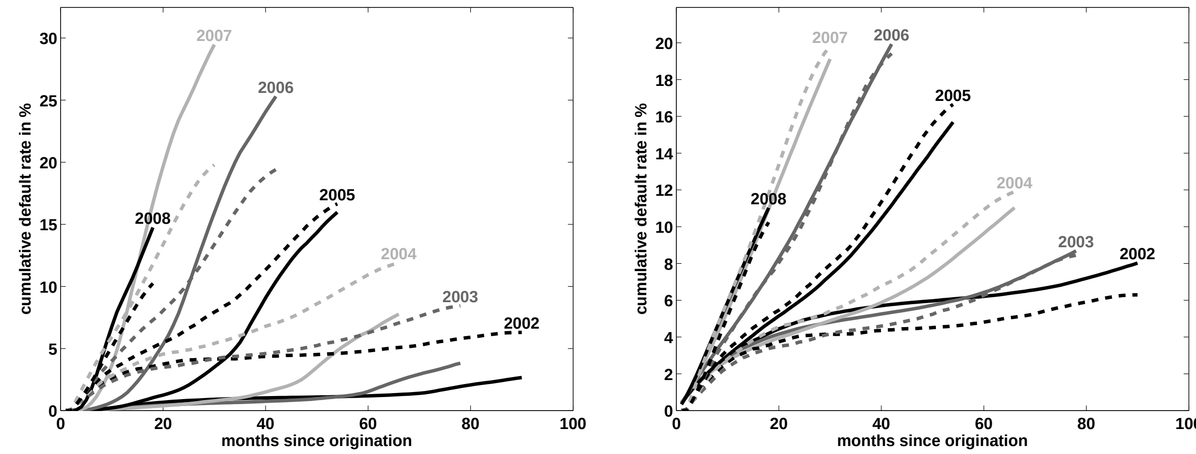
Instead of estimating the models on the 2002 cohort with low default rates, I also estimate them on the 2008 cohort with very high default rates. This does not affect the good fit of the shock model. But now the threshold model greatly undershoots the default rates of early cohorts and also still overshoots the 2006 and 2007 cohort. Thus the comparison across models is unaffected. In fact I have also used the 2003 and 2005 cohorts to estimate the models and always found the same results across the two models.

Another robustness check replaces the out-of-sample test with an in-sample test. Here I estimate the two models on all cohorts and then examine the fit within that sample. This exercise is informative on the best possible fit both models can give to the data. These results are thus worth to report in more detail. The estimated parameters are then -15.9% for ϕ and 1.36% for ψ and the results are shown in figure B.1. The threshold model still has considerable problems to match the data even under these most favorable circumstances. It generally undershoots earlier cohorts and the early months after origination for all cohorts and at the same time still overshoots the late months of the 2006 and 2007 cohorts. In contrast, the shock model gives an excellent fit to the data. The conclusions across models are essentially unchanged.

I also examine the role of the variation in mortgage rates and the distribution of loan-to-value ratios across cohorts in three alternative specifications. In the first specification, I keep the within cohort LTV distribution fixed across cohorts according to the average

Figure B.1: In-Sample Fit of the Two Models

(a) Threshold Model (b) Shock Model



Notes: Solid lines: model. Dashed lines: data.

frequency. The second specification abstracts from within cohort heterogeneity such that everyone has the same LTV according to the respective within cohort average. The third specification is the same as the second except that the LTV and mortgage rate are not varied across cohorts. All these changes have very modest effects on both models and leave the conclusions across models unaffected. This implies that the double-trigger model attributes the rise in default rates to the variation in aggregate house prices and not the changes in contract characteristics across cohorts. It also suggests that abstracting from this heterogeneity across cohorts in the structural model is not too restrictive.

In section 3.4 it was assumed that the individual house price shocks are normally distributed. The major argument supporting this choice is that by the central limit theorem the mean of individual shocks converges asymptotically to a normal distribution anyway. But since the analysis also covers periods where t is still small, I perform an additional check here. Instead of using a normal distribution for the individual shocks I specify them as being uniformly distributed on the interval $[-b_t, b_t]$. The parameter b_t is then chosen such that the variance of the uniformly distributed shock in period t in the respective census division is identical to the one used in the standard framework. I find that the results are almost identical.

Another potential concern is that the simplicity of the presented reduced-form models with only one constant parameter somehow biased the results against the frictionless option model. There is also no strong reason why the default threshold parameter ϕ and default shock probability ψ should be constant over the course of a loan. It turns out that the results are robust to changing this assumption. As a check I have performed a scenario where the respective default parameter depends fully on the month since origination t. The constant parameters in the model are then replaced with ϕ_t and ψ_t that are allowed to differ each period from $t = 1, \ldots, T$ when fitting the models to the 2002 cohort. Under these circumstances both models almost perfectly match the 2002 cohort. The cumulative default rates simulated for the other cohorts then inherit the non-smoothness of the first differences of the cumulative default rate of the 2002 cohort. But subject to that qualification the conclusions on the out-of-sample fit remain essentially unchanged. The threshold model still greatly overshoots the later cohorts. The shock model generates default rates comparable in magnitude to the benchmark.

B.3 Using an alternative definition of default

In this section I use a different definition of default. Instead of using a 60 or more days definition of default as in the main text, I now consider a loan to be in default once it is at least 120 days past due.[25] This is a more demanding definition and by all accounts being 120 days past due is considered as a very serious delinquency. This change of definition affects the levels of the data on cumulative default rates which is used to estimate and test the models. Obviously the level of default rates is lower now, however the broad dynamics across cohorts are similar to the ones analyzed in the main text.

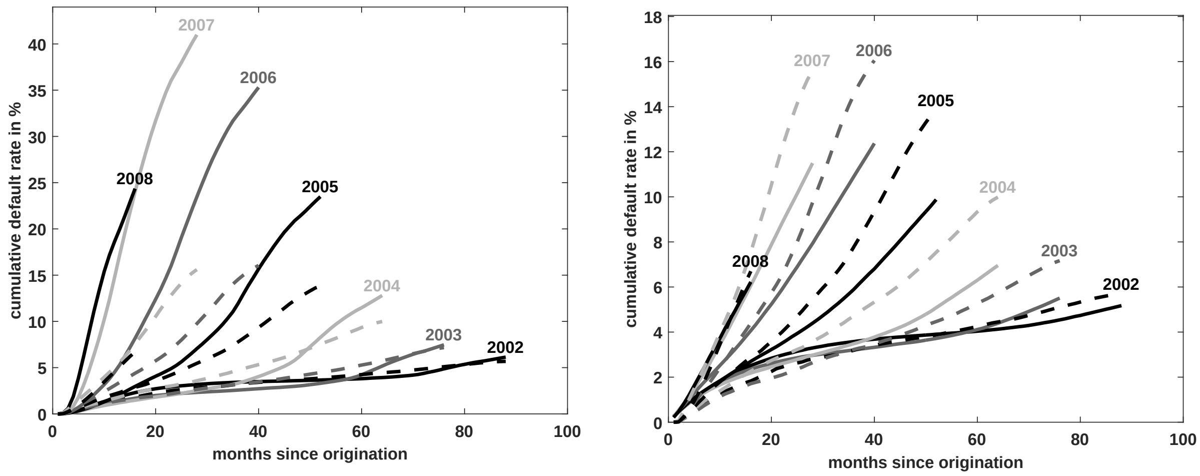
Again I estimate both reduced form models on the 2002 cohort and use the remaining cohorts to test the estimated models. For the threshold model this yields an estimate of ϕ of -11.9% and for the shock model ψ is estimated as 0.83%. The results are reported in figure B.2. These are qualitatively very similar to the ones of the main text and the conclusions across models are unchanged. Though one can debate what is the appropriate definition of default, I conclude from this exercise that this issue is not key for my results.

Furthermore I have also investigated the effect of using a definition of default that

[25] Now I backdate the period of default by 3 months to capture the time when the first payment has been missed.

Figure B.2: Using an alternative definition of default (120+ days)

(a) Threshold Model (b) Shock Model



Notes: Solid lines: model. Dashed lines: data.

requires a loan to be in foreclosure. This also generates similar results (which are available upon request) and does not resolve the empirical problems of a frictionless option-model documented in section 3.

B.4 Extension to lower Loan-to-Value Ratios

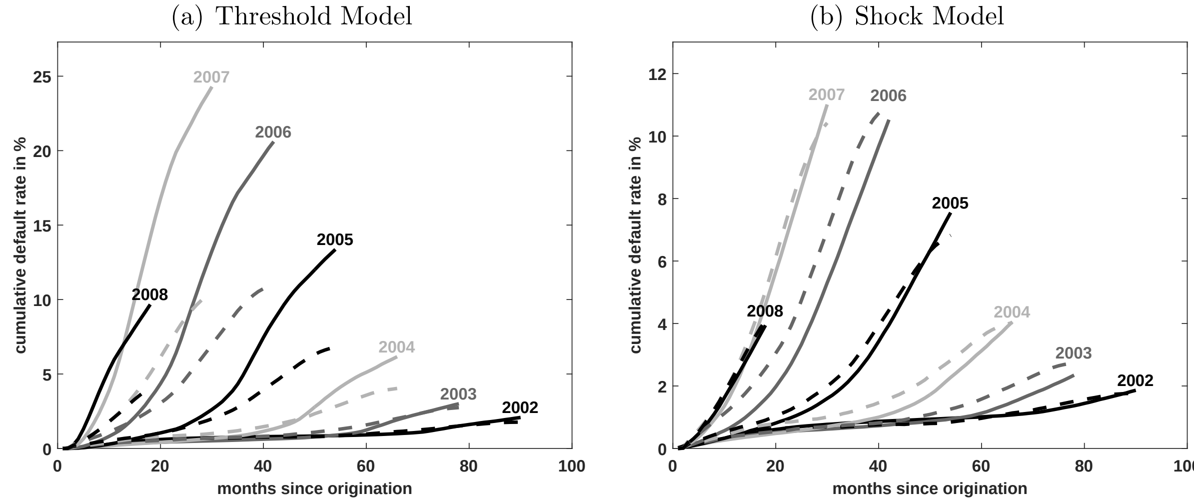
The paper is focussed on loans with a LTV above 95% because these borrowers should be least likely to have a second mortgage on their home, cf. the discussion in section 2.1. The question arises whether the results of the paper also generalize to loans with a lower LTV. This section provides some evidence on this by repeating the reduced-form analysis of section 3 for loans with a LTV of the first mortgage between 75% and 84%. Due to the discussed data problems this section is necessarily somewhat tentative. Nevertheless, some very interesting results emerge.

First I take the data for the loans with a LTV of the first mortgage between 75% and 84% at face value and assume that no one has a second mortgage. Accordingly the LTV varies within cohorts in steps of one percentage point between 75% and 84%. Changes to the distribution of loans over this support across cohorts observed in the mortgage data are again taken into account. The mortgage rate is again kept constant within a cohort and set equal to the respective cohort average. When estimating the models on the 2002 cohort I find that neither of the two models can capture this data well. Even for the most extreme parameter values of $\phi = 0$ and $\psi = 1$, both models undershoot the cumulative default rate of the 2002 cohort substantially for at least the first 60 months after origination. The reason is that the equity buffer generated by the down-payment is substantial for these borrowers. Because the 2002 cohort faced strongly increasing average house prices immediately after origination, too few borrowers in the simulation experience negative equity compared to observed default rates. It is important that both models fail if we take this data at face value. One can draw two possible conclusions from these results. Either we need a completely new theory of default for these loans or it is crucial to take second mortgages into account. I present evidence on the second explanation next.

Elul, Souleles, Chomsisengphet, Glennon, and Hunt (2010) report that 26% of all borrowers have a second mortgage and this adds on average 15% to the combined LTV. But they neither report a break-down of these statistics by the LTV of the first mortgage nor when borrowers take out the second mortgage. Faced with this situation I model a very simple form of intra-cohort heterogeneity taking these estimates of the frequency and size of second mortgages into account. I assume that 74% of borrowers have only one mortgage with a distribution of LTVs as in the mortgage data. But 26% of borrowers in each cohort independently of the LTV of the first mortgage also have a second mortgage adding 15% to the combined LTV. This implies that the support of the LTV distribution is expanded and now also includes values between 90% and 99%. It is assumed that borrowers got the second mortgage at the same time as the first one and pay the same mortgage rate on both. Admittedly, these are very crude assumptions. This exercise can only provide preliminary evidence until better data is available and should be regarded with considerable caution.

For this setup the reduced-form models are estimated again on the 2002 cohort. This yields estimates of $\phi = -7.8\%$ and $\psi = 2.25\%$. The estimated models are again tested on their ability to predict out-of-sample. Figure B.3 presents the results for all cohorts. The threshold model overshoots the data again. In contrast, the shock model provides an excellent fit to the data. Thus the double-trigger theory also provides a better explanation for this data under the maintained assumptions on second mortgages. Due to the discussed data problems I would personally put a lower weight on these results compared to the benchmark results. But these results are at least suggestive that the main conclusions on the relative merit of the two theories may well extend to loans with a lower LTV.

Figure B.3: Results for borrowers with a first mortgage LTV of $75-84\%$ taking second mortgages into account



Notes: Solid lines: model. Dashed lines: data.

C Appendix to Structural Model

C.1 Value Functions

The state variables of the optimization problem for an owner are liquid wealth $X_t = A_t + Y_t$, employment status L_t, house price P_t and time t. The choice variables are consumption C_t and the mortgage termination choice. The value function of an owner $V^o(.)$ can then be written as

$$V^o(X_t, L_t, P_t, t) = \max\left\{V^s(X_t, L_t, P_t, t), V^r(X_t + P_t - \frac{M_t}{\Pi_t}, L_t, t), V^r(X_t, L_t, t)\right\}$$

which reflects the optimal choice between staying in the house with value $V^s(X_t, L_t, P_t, t)$, selling with value $V^r(X_t + P_t - \frac{M_t}{\Pi_t}, L_t, t)$ and defaulting with value $V^r(X_t, L_t, t)$. Selling and defaulting involve a permanent transition to the rental market. In case of staying the value $V^s(X_t, L_t, P_t, t)$ is given by

$$\begin{aligned}
V^s(X_t, L_t, P_t, t) &= \max_{C_t} \left\{ \frac{C_t^{1-\gamma}}{1-\gamma} + \theta + \beta \mathrm{E_t}\left[V^o(X_{t+1}, L_{t+1}, P_{t+1}, t+1) \right] \right\} \\
\text{s.t.} \quad X_{t+1} &= (1+r)\left(X_t - \frac{m}{\Pi_t} + \tau r^m \frac{M_t}{\Pi_t} - C_t \right) + Y_{t+1} \\
C_t &\leq X_t - \frac{m}{\Pi_t} + \tau r^m \frac{M_t}{\Pi_t}.
\end{aligned}$$

The value function of a renter $V^r(X_t, L_t, t)$ is given by

$$\begin{aligned}
V^r(X_t, L_t, t) &= \max_{C_t} \left\{ \frac{C_t^{1-\gamma}}{1-\gamma} + \beta \mathrm{E_t}\left[V^r(X_{t+1}, L_{t+1}, t+1) \right] \right\} \\
\text{s.t.} \quad X_{t+1} &= (1+r)\left(X_t - R - C_t \right) + Y_{t+1} \\
C_t &\leq X_t - R.
\end{aligned}$$

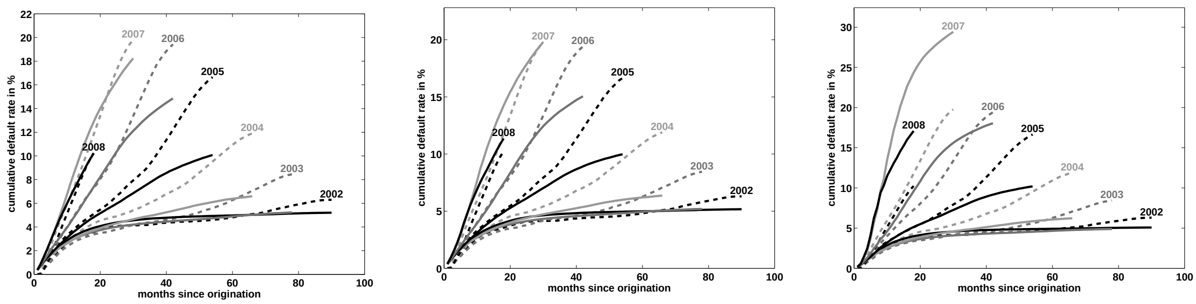
C.2 Dependence on Preference Parameters

This section explores how the model depends on the predetermined preference parameters. Specifically, I compute results for alternative parameter values for β and γ in order to get an idea how the model behaves in different parts of the parameter space. The benchmark preference parameter values are $\beta = 0.9$ and $\gamma = 5$.

First I consider alternative values of β equal to 0.85 and 0.95. For these value of β the parameter θ is then reestimated in order to fit the 2002 cohort. This yields values of θ of 0.39 and 0.16 respectively. The results of these experiments are compared to the benchmark results in figure C.1.

Figure C.1: Sensitivity to Preference Parameter β

(a) $\beta = 0.85$ (b) $\beta = 0.9$ (c) $\beta = 0.95$



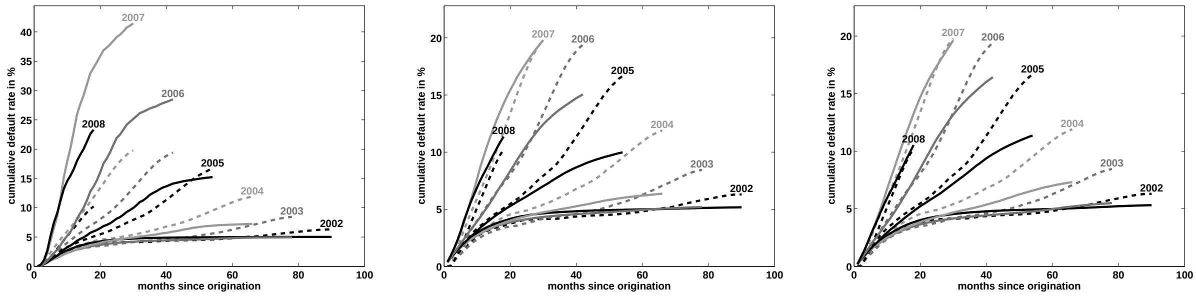
Notes: Solid lines: model. Dashed lines: data.

Next I consider alternative values of γ equal to 2 and 8. θ is then estimated as 0.06 and

0.64 respectively. Figure C.2 compares these alternative calibrations to the benchmark.

Figure C.2: Sensitivity to Preference Parameter γ

(a) $\gamma = 2$ (b) $\gamma = 5$ (c) $\gamma = 8$



Notes: Solid lines: model. Dashed lines: data.

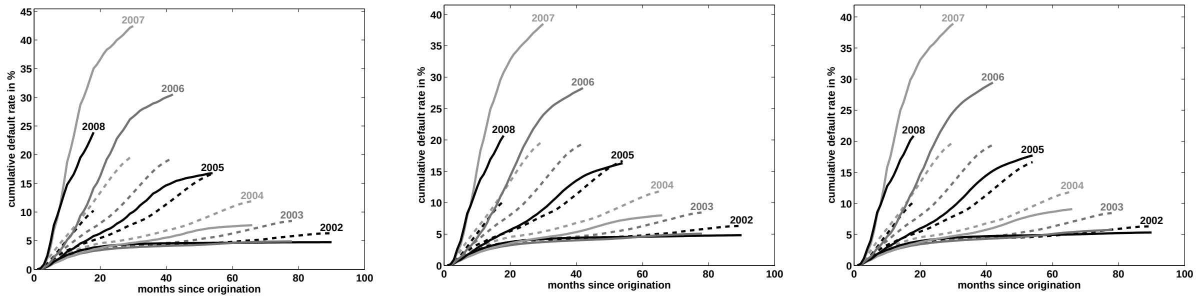
These results show that the model works as well or better than in the benchmark calibration for higher values of γ and/or lower values of β. These parameter changes make the agent less willing to substitute intertemporally and/or more impatient to consume today. This worsens the liquidity problem caused by unemployment. The model can only feature double-trigger behavior when being employed and being unemployed are sufficiently different. In contrast, for lower values of γ and higher values of β temporary income reductions can more easily be smoothed out. The model then implies that a sizeable portion of employed agents default in all cohorts. This brings the model too close to a frictionless option model and the model then inherits all the problems of such a specification witnessed already in section 3.

C.3 Details on the Role of a Direct Utility Benefit

In this section I show that versions of the model which abstract from a direct utility benefit of living in the bought house are excessively sensitive to changes in aggregate house prices. As an illustration I conduct simulations of the model where the utility flow parameter θ is set to 0. I then adjust the discount factor β to again match the default rates of the 2002 cohort for $\theta = 0$. Figure C.3 presents these results for the benchmark value of the CRRA coefficient $\gamma = 5$, but also for values of γ of 2 and 8. Each of those experiments is associated with a different value of β. These results confirm the key role of including a direct utility benefit of living in the bought house in order to prevent employed and not currently liquidity constrained agents from defaulting when house prices fall strongly. Without this feature the model cannot accurately capture the sensitivity of default rates to aggregate house prices.

Figure C.3: Performance of the model without a direct utility benefit of living in the bought house ($\theta = 0$) for different values of γ

(a) $\gamma = 2$, $\beta = 0.947$ (b) $\gamma = 5$, $\beta = 0.997$ (c) $\gamma = 8$, $\beta = 1.042$



Notes: Solid lines: model. Dashed lines: data.

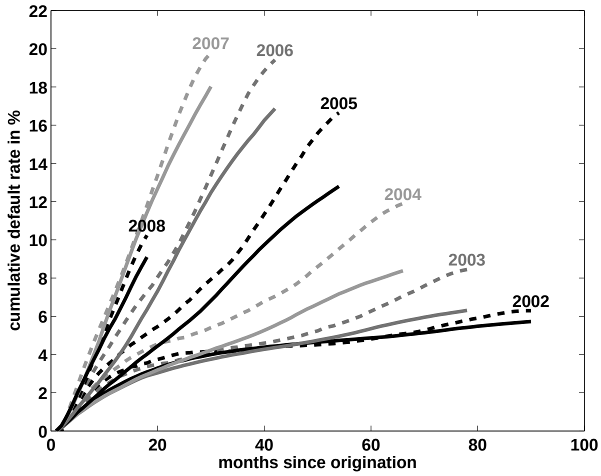
C.4 Role of Inflation

In this section I show that the mortgage tilt effect caused by inflation plays an important role for the performance of the model in later periods after origination. In the benchmark calibration the inflation rate is set to 2.4%, which is the average value during the simulation period (2002-2010). In order to document the sensitivity of default rates to the inflation rate I investigate an alternative calibration where π is set ad-hoc to 1%. This alternative calibration is only meant to be an illustration how default decisions are affected when borrowers either expect the inflation rate to be a bit lower (1.4% lower) during the simulation period and the future than it was during these years, or in some other way underestimate the mortgage tilt effect. Unfortunately I have no data on how large such a deviation of borrower expectations from observed inflation may have been in reality, which implies that I can only look at an ad-hoc scenario. All other parameters are unchanged, but θ is reestimated at a value of 0.44 to fit the 2002 cohort.

Figure C.4 presents the results of this exercise. The fit of the model improves in the later period after origination relative to the benchmark results. This shows that the strength of the mortgage tilt effect is responsible for the problems of the model in the main text to capture default decisions in later periods after origination.

Using this alternative calibration I have also repeated the policy analysis from section

Figure C.4: Performance of the Model for a Lower Inflation Rate



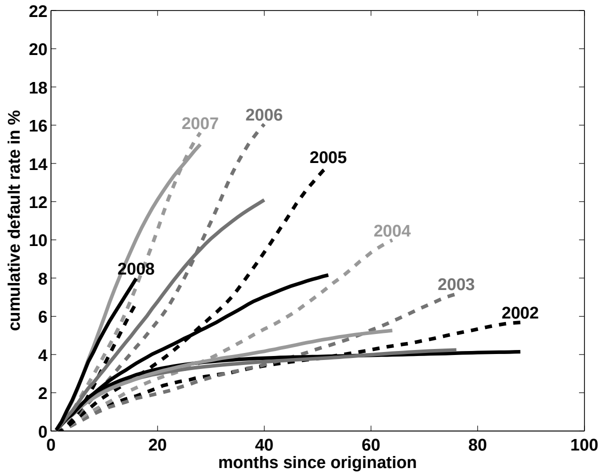
Notes: Solid lines: model. Dashed lines: data.

7. This allows to check how the policy results change in a model that captures the data even better than the benchmark (but admittedly makes an ad-hoc assumption on the inflation rate). The absolute costs of both policies tend to increase a bit relative to the benchmark and the relative cost of the bailout to lenders also increases. Bailing out lenders is then between 8.6 and 11.2 times more expensive than subsidizing homeowners using this alternative calibration. Thus the conclusions across policies are robust or even strengthened relative to the benchmark calibration.

C.5 Using an alternative definition of default

This section reports the results for the structural model when the 120 days definition instead of the 60 days definition is used to measure default empirically as in section B.3. All other procedures are as in the main text. The estimate of θ is then 0.32. Figure C.5 reports the results for all cohorts. The fit of the model to this data is qualitatively similar to the one of the main text that uses a 60 days default definition. The results of the policy analysis are also essentially unchanged and bailing out lenders is found to be between 8.7 and 11.2 times more expensive than subsidizing homeowners. This analysis shows that the main results of the structural model are robust to using an alternative reasonable definition of default.

Figure C.5: Model Results for the 120+ Days Default Definition



Notes: Solid lines: model. Dashed lines: data.

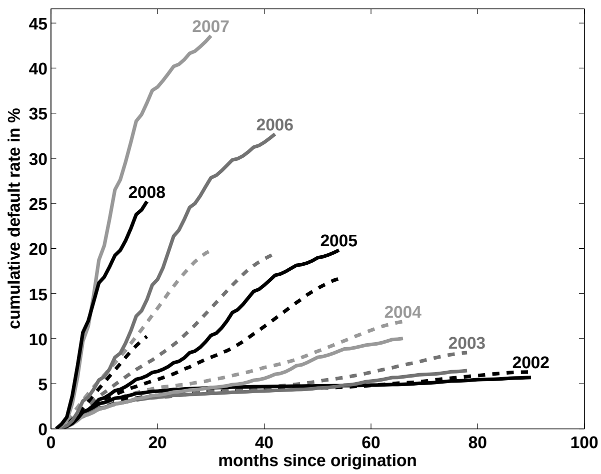
C.6 A Structural Single-Trigger Model

This section investigates a structural single-trigger model in the spirit of the frictionless option theoretic literature. The aim here is to confirm that the conclusions on the relative merit of the two theories drawn in the reduced form section 3 also carry over to a comparison of structural models of these theories.

For this purpose it is convenient that the structural double trigger model of sections 4 and 5 in fact nests a single-trigger model if one suitably modifies some model parameters. The single-trigger paradigm relies on the assumption that borrowers have access to perfect and complete financial markets such that they can perfectly insure against income fluctuations. Accordingly, mortgage default decisions are unaffected by income fluctuations and liquidity problems. The structural model nests such a case when the following changes to model parameters are made. The net replacement rate in case of unemployment ρ is set to 1 such that income is identical in the employed and unemployed state. Given the actual replacement rate of unemployment insurance and the unemployment rate implied by the job separation and finding rates from the benchmark calibration this insurance costs about 2% of net income when employed. Accordingly employed net income $(1-\tau)Y_0$ is lowered by 2% such that it is net of insurance fees, which is achieved by resetting the debt to income ratio to a slightly higher value of $DTI = 0.4085$. These modifications are equivalent to the borrower buying insurance against the part of income risk not covered by the public unemployment insurance system. Naturally in the resulting model default is no longer driven by employment status because income does not vary across employment states. The second modification is to delete the borrowing constraint of equation (13) from the model such that it features a perfect credit market for unsecured credit. In practice this is achieved by changing equation (13) to $A_{t+1} \geq -\xi$ and then setting ξ to a large number instead of zero as in the benchmark calibration. Specifically, I set ξ equal to five times annual net income and confirm that during the simulation the optimal asset path chosen by borrowers remains far away from this constraint. The other features, parameters and initial conditions of the model remain the same as in the benchmark model and calibration. Thus the structural single trigger model is put as much as possible on an equal footing with the structural double trigger model. The direct utility benefit parameter θ is then estimated at a value of -0.035 to match the cumulative default rate of the 2002 cohort, which this model specification also matches very well.

Figure C.6 presents the resulting fit of the structural single-trigger model to all cohorts. The model strongly overpredicts the rise in default rates. Its performance is broadly comparable to the one of the reduced-form single-trigger model. When comparing the structural single and double-trigger models to each other one again finds that the double-trigger model explains the data much better than the single trigger model. The impression from visually inspecting figure C.6 and figure 7 of the main text is confirmed by simple measures of goodness of fit of the model predictions to the data. For instance the root mean squared error (mean absolute error) of the out-of-sample forecast is 3.5 (2.8) times higher for the structural single-trigger than for the structural double-trigger model. Thus this exercise using structural models confirms the reduced form results on the relative merit of the two theories.

Figure C.6: Performance of a Structural Single-Trigger Model



Notes: Solid lines: model. Dashed lines: data.