# Demographic Change, Human Capital and Welfare*

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**Abstract**

Projected demographic changes in the U.S. will reduce the share of the working-age population. Analyses based on standard OLG models predict that these changes will increase the capital-labor ratio. Hence, rates of return to capital decrease and wages increase, which has adverse welfare consequences for current cohorts who will be retired when the rate of return is low. This paper argues that adding endogenous human capital accumulation to the standard model dampens these forces. We find that this adjustment channel is quantitatively important. The standard model with exogenous human capital predicts welfare losses up to 12.5% (5.6%) of lifetime consumption, when contribution (replacement) rates to the pension system are kept constant. These numbers reduce to approximately 8.7% (4.4%) when human capital can endogenously adjust.

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*Keywords: population aging; human capital; rate of return; distribution of welfare*

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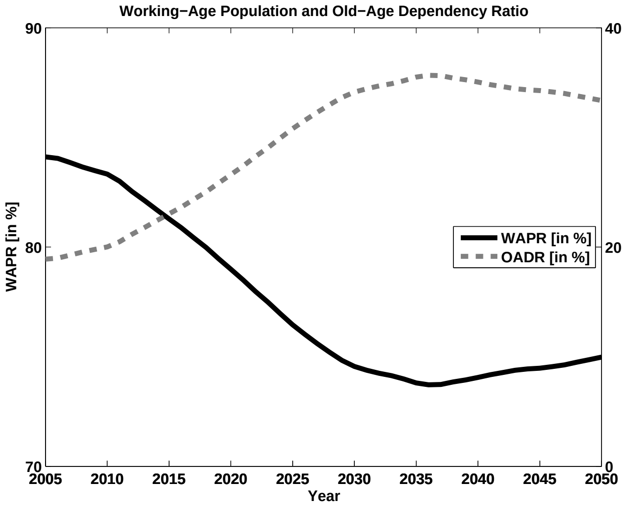
# 1 Introduction

As in all major industrialized countries, the population of the United States is aging over time. This process is driven by increasing life expectancy and a decline in birth rates from the peak levels of the baby boom. Consequently, the fraction of the working-age population will decrease, and the fraction of elderly people will increase. Figure 1 presents two summary measures of these demographic changes: the working-age population ratio is predicted to decrease from 84% in 2005 to 75% in 2050, while the old-age dependency ratio will increase from 19% in 2005 to 34% in 2050. These projected changes in the population structure will have important macroeconomic effects on the balance between physical capital and labor. Specifically, labor is expected to be scarce relative to physical capital, with an ensuing decline in real returns on physical capital and increases in gross wages. These relative price changes have adverse welfare effects, especially for individuals close to retirement because they receive a lower return on their assets accumulated for retirement and cannot profit from increased wages.

This paper argues that a strong incentive to invest in human capital emanates from the combined effects of increasing life expectancy and changes in relative prices, particularly if social security systems are reformed such that contribution rates remain constant. At general equilibrium, such endogenous human-capital adjustments substantially mitigate the effects of demographic change on macroeconomic aggregates and individual welfare.

Figure 1: Working-Age and Old-Age Dependency Ratio




*Notes:* Working-age population ratio (WAPR, left scale): ratio of population of age $16-64$ to total adult population of age $16-90$. Old-age dependency ratio (OADR, right scale): ratio of population of age $65-90$ to working-age population.
*Source:* Own calculations based on Human Mortality Database (2008).

The key contribution of our paper is to show that the human-capital adjustment mechanism is quantitatively important. We add endogenous human-capital accumulation to an otherwise standard large-scale OLG model in the spirit of Auerbach and Kotlikoff (1987). The central focus of our analysis is then to work out the differences between our model, with endogenous human capital adjustments and endogenous labor supply, and the "standard" models in the literature, with fixed (exogenous) productivity profiles.

We find that the decrease of the return to physical capital induced by demographic change in a model with endogenous human capital is only one-third of that predicted in the standard model. Welfare consequences from increasing wages, declines in rates of return, changes to pension contributions and benefits induced by demographic change are substantial. When human capital cannot adjust, some of the agents alive in 2005 will experience welfare losses up to 12.5% (5.6%) of lifetime consumption with constant pension contribution (replacement) rates. However, importantly, we find that these maximum losses are only 8.7% (4.4%) of lifetime consumption when the human capital adjustment mechanism is taken into account. Ignoring this adjustment channel thus leads to quantitatively important biases of the welfare assessment of demographic change.

Our work relates to a vast number of papers that have analyzed the economic consequences of population aging and possible adjustment mechanisms. Important examples in closed economies with a focus on social-security adjustments include Huang et al. (1997), De Nardi et al. (1999) and, with respect to migration, Storesletten (2000). In open economies, Börsch-Supan et al. (2006), Attanasio et al. (2007) and Krüger and Ludwig (2007), among others, investigate the role of international capital flows during a demographic transition. We add to this literature by highlighting an additional mechanism through which households can respond to demographic change.

Our paper is closely related to the theoretical work on longevity, human capital, taxation and growth[1] and to Fougère and Mérette (1999) and Sadahiro and Shimasawa (2002), who also quantitatively investigate demographic change in large-scale OLG models with individual human-capital decisions. In contrast to their work,

[1] See, for example, de la Croix and Licandro (1999), Boucekkine et al. (2002), Kalemli-Ozcan et al. (2000) Echevarria and Iza (2006), Heijdra and Romp (2008), Ludwig and Vogel (2009) and Lee and Mason (2010). Our paper is also related to the literature emphasizing the role of endogenous human-capital accumulation for the analysis of changes to the tax or social-security system, as in Lord (1989), Trostel (1993), Perroni (1995), Dupor et al. (1996) and Lau and Poutvaara (2006), among others.

we focus our analysis on relative price changes during a demographic transition and therefore consider an exogenous growth specification.[2] We also extend their analysis along various dimensions. We use realistic demographic projections instead of stylized scenarios. More importantly, our model contains a labor supply-human capital formation-leisure trade-off. It can thus capture effects from changes in individual labor supply, i.e., human capital utilization, on the return of human-capital investments. As has already been stressed by Becker (1967) and Ben-Porath (1967), it is important to model human-capital and labor supply-decisions jointly in a life-cycle framework. Along this line, a key feature of our quantitative investigation is to employ a Ben-Porath (1967) human-capital model and calibrate it to replicate realistic life-cycle wage profiles.[3] Furthermore, we place particular emphasis on the welfare consequences of an aging population for households living through the demographic transition.

The paper is organized as follows. In Section 2, we present our quantitative model. Section 3 describes the calibration strategy and our computational solution method. Our results are presented in Section 4. Finally, Section 5 concludes the paper. A separate online appendix[4] contains additional results, robustness checks, a description of our demographic model and technical details.

[2]Whether the trend growth rate endogenously fluctuates during the demographic transition or is held constant is of minor importance for the questions we are interested in. This is shown in our earlier unpublished working paper. The results are available upon request.

[3]The Ben-Porath (1967) model of human capital accumulation is one of the workhorses in labor economics used to understand such issues as educational attainment, on-the-job training, and wage growth over the life cycle, among others. See Browning, Hansen, and Heckman (1999) for a review. Extended versions of the model have been applied to study the significant changes to the U.S. wage distribution and inequality observed since the early 1970s by Heckman, Lochner, and Taber (1998) and Guvenen and Kuruscu (2009).

[4]The online appendix is available at www.wiso.uni-koeln.de/aspsamp/cmr/alexludwig/downloads/HKApp.pdf.

# 2 The Model

We employ a large-scale OLG model à la Auerbach and Kotlikoff (1987) with endogenous labor supply and endogenous human-capital formation. The population structure is exogenously determined by time- varying demographic processes for fertility and mortality, the main driving forces of our model.[5] In a perfectly competitive environment, firms produce with standard constant returns to scale production function. We assume that the U.S. is a closed economy.[6] Agents contribute a share of their wages to the pension system, and retirees receive a share of their average indexed past yearly earnings as pensions. Technological progress is exogenous.

## 2.1 Timing, Demographics and Notation

Time is discrete, and one period corresponds to one calendar year $t$. Each year, a new generation is born. Birth in this paper refers to the first time households make their own decisions and is set to the age of 16 (model age $j = 0$). Agents retire at an exogenously given age of 65 (model age $jr = 49$). Agents live at most until age 90 (model age $j = J = 74$). At a given point in time $t$, individuals of age $j$ survive to age $j + 1$ with probability $\varphi_{t,j}$, where $\varphi_{t,J} = 0$. The number of agents of age $j$ at time $t$ is denoted by $N_{t,j}$, and $N_t = \sum_{j=0}^{J} N_{t,j}$ is the total population in $t$.

[5]We do not model endogenous life expectancy, fertility or endogenous migration and assume that all exogenous migration is completed before agents begin making economically relevant decisions (cf. online Appendix C). Thus, we also abstract from potential feedback effects of social-security policies on fertility, as studied by Ehrlich and Kim (2007).

[6]For our question, the assumption of a closed economy is a valid approximation. As documented in Krüger and Ludwig (2007), demographically induced changes in the return to physical capital and wages from the U.S. perspective do not differ much between closed- and open-economy scenarios. The reason is that demographic processes are correlated across countries and, in terms of speed of the aging processes, the U.S. is somewhere in the middle with respect to all OECD countries.

## 2.2 Households

Each household comprises one representative agent who makes decisions regarding consumption and saving, labor supply and human-capital investment. The household maximizes lifetime utility at the beginning of economic life ($j = 0$) in period $t$,

$$\max \sum_{j=0}^{J} \beta^j \pi_{t,j} \frac{1}{1-\sigma} \{c_{t+j,j}^{\phi} (1 - \ell_{t+j,j} - e_{t+j,j})^{1-\phi}\}^{1-\sigma}, \quad \sigma > 0, \ \phi \in (0,1), \tag{1}$$

where the per-period utility function is a function of individual consumption $c$, labor supply $\ell$ and the time invested in formation of human capital, $e$. The agent is endowed with one unit of time, thus, $1 - \ell - e$ is leisure time. $\beta$ is the pure time-discount factor, $\phi$ determines the weight of consumption in utility, and $\sigma$ is the inverse of the inter-temporal elasticity of substitution with respect to the Cobb-Douglas aggregate of consumption and leisure time. $\pi_{t,j}$ denotes the (unconditional) probability to survive until age $j$, $\pi_{t,j} = \prod_{i=0}^{j-1} \varphi_{t+i,i}$, for $j > 0$ and $\pi_{t,0} = 1$.

Agents earn labor income (pension income when retired) as well as interest payments on their savings and receive accidental bequests. When working, they pay a fraction $\tau_t$ from their gross wages to the social-security system. The net wage income in period $t$ of an agent of age $j$ is given by $w_{t,j}^n = \ell_{t,j} h_{t,j} w_t (1 - \tau_t)$, where $w_t$ is the gross wage per unit of supplied human capital at time $t$. There are no annuity markets, and households leave accidental bequests. These are collected by the government and redistributed in a lump-sum fashion to all households. Accordingly, the dynamic budget constraint is given by

$$a_{t+1,j+1} = \begin{cases} (a_{t,j} + tr_t)(1 + r_t) + w^n_{t,j} - c_{t,j} & \text{if } j < jr \\ \\ (a_{t,j} + tr_t)(1 + r_t) + p_{t,j} - c_{t,j} & \text{if } j \geq jr, \end{cases} \tag{2}$$

where $a_{t,j}$ denotes assets, $tr_t$ are transfers from accidental bequests, $r_t$ is the real interest rate, the rate of return to physical capital, and $p_{t,j}$ is pension income. Initial household assets are zero ($a_{t,0} = 0$), and the transversality condition is $a_{t,J+1} = 0$.

## 2.3 Formation of Human Capital

The key element of our model is the endogenous formation of human capital. Households enter economic life with a predetermined and cohort invariant level of human capital $h_{t,0} = h_0$. Afterwards, they can invest a fraction of their time into acquiring additional human capital. We adopt a version of the Ben-Porath (1967) human-capital technology[7] given by

$$h_{t+1,j+1} = h_{t,j}(1 - \delta^h) + \xi(h_{t,j}e_{t,j})^{\psi} \quad \psi \in (0, 1), \ \ \xi > 0, \ \ \delta^h \geq 0, \tag{3}$$

where $\xi$ is a scaling factor, the average learning ability, $\psi$ determines the curvature of human-capital technology, $\delta^h$ is the depreciation rate of human capital, and $e_{t,j}$ is time invested in human-capital formation.

The costs of investing in human capital in this model are only the opportunity costs of foregone wage income and leisure. We understand the process of accumulating human capital to be a mixture of knowledge acquired by formal schooling and

[7]This functional form is widely used in the human-capital literature, cf. Browning, Hansen, and Heckman (1999) for a review.

on-the- job training programs after schooling is complete. Human capital can be accumulated until retirement age, but an agent's optimally chosen time investment converges to zero some time before retirement.

## 2.4 The Pension System

The pension system is a simple balanced-budget, pay-as-you-go system that resembles key features of the U.S. system. Workers contribute a fraction $\tau_t$ of their gross wages, and pensioners receive a fraction $\rho_t$ of their average indexed past yearly earnings.[8] The level of pensions in each period is given by $p_{t,j} = \rho_t w_{t+jr-j} \bar{h}_{t+jr-j} \frac{s_{t,j}}{jr-1}$, where $w_{t+jr-j} \bar{h}_{t+jr-j} \frac{s_{t,j}}{jr-1}$ are average indexed past yearly earnings (AIYE)[9], $w_{t+jr-j} \bar{h}_{t+jr-j}$ are average earnings of all workers in the period when a retiree of current age $j$ reaches retirement age $jr$, and $\bar{h}_t$ is defined as $\bar{h}_t = \frac{\sum_{j=0}^{jr-1} \ell_{t,j} h_{t,j} N_{t,j}}{\sum_{j=0}^{jr-1} N_{t,j}}$. We refer to $\bar{h}_t$ as the average (hours weighted) human-capital stock. The sum up to age $j$ of past individual earnings of an agent relative to average economy-wide earnings in the respective year is given by $s_{t,j} = \sum_{i=0}^{j} \frac{\ell_{t-j+i,i} h_{t-j+i,i}}{\bar{h}_{t-j+i}}$. This links pensions to individuals' past earnings. Using the above formula for $p_{t,j}$, the budget constraint of the pension system is given by

$$\tau_t w_t \sum_{j=1}^{jr-1} \ell_{t,j} h_{t,j} N_{t,j} = \rho_t \sum_{j=jr}^{J} N_{t,j} w_{t+jr-j} \bar{h}_{t+jr-j} \frac{s_{t,j}}{jr-1} \quad \forall t. \tag{4}$$

[8]The U.S. system applies an additional bend-point formula to pensions, which results in intra-generational redistribution. However, in our model, without intra-cohort heterogeneity, we do not take this feature of the actual system into account. For a description of the current U.S. system, see Diamond and Gruber (1999) and Geanakoplos and Zeldes (2009).

[9]Our concept of AIYE is an approximation to the "average indexed monthly earnings" (AIME) in the current U.S. system where only the 35 years of working life with the highest individual earnings relative to average earnings are counted for the calculation of AIME. We ignore this feature for computational reasons and count all years of working life.

Below, we consider two opposite scenarios of parametric adjustment of the pension system to demographic change. In our first scenario ("const. $\tau$"), we hold the contribution rate constant, $\tau_t = \bar{\tau}$, and endogenously adjust the replacement rate to balance the budget of the pension system. In the other extreme scenario ("const. $\rho$"), we hold the replacement rate constant, $\rho_t = \bar{\rho}$, and endogenously adjust the contribution rate.

### 2.5 Firms and Equilibrium

Firms operate in a perfectly competitive environment and produce one homogenous good, according to the Cobb-Douglas production function

$$Y_t = K_t^{\alpha}(A_t L_t)^{1-\alpha}, \tag{5}$$

where $\alpha$ denotes the share of capital used in production. $K_t, L_t$ and $A_t$ are the stocks of physical capital, effective labor and the level of technology, respectively. Output can be either consumed or used as an investment good. We assume that labor inputs and human capital of different agents are perfect substitutes, and effective labor input $L_t$ is accordingly given by $L_t = \sum_{j=0}^{jr-1} \ell_{t,j} h_{t,j} N_{t,j}$. Factors of production are paid their marginal products, i.e., $w_t = (1-\alpha)\frac{Y_t}{L_t}$ and $r_t = \alpha \frac{Y_t}{K_t} - \delta_t$, where $w_t$ is the gross wage per unit of efficient labor, $r_t$ is the interest rate, and $\delta_t$ denotes the depreciation rate of physical capital. Total factor productivity, $A_t$, is growing at the exogenous rate of $g_t^A$: $A_{t+1} = A_t(1 + g_t^A)$.

The definition of equilibrium is standard and can be found in our online Appendix A.

## 2.6 Thought Experiments

We take as an exogenous driving process a time-varying demographic structure. Computations begin in year 1750 ($t = 0$), assuming an artificial initial steady state[10]. We then compute the model equilibrium from 1750 to 2500 ($t = T = 751$) when the new steady state is assumed and reached[11] and report simulation results for the main projection period of interest, from 2005 ($t = 256$) to 2050 ($t = 306$). We use data during our calibration period, $1960 - 2005$ (from $t_0 = 211$ to $t_1 = 256$), to determine several structural model parameters (cf. section 3).

Our main objective is to compare the time paths of aggregate variables and welfare across two model variants for two social-security scenarios. Our first model variant is one in which households adjust their human capital, and our second variant is one in which human capital is held constant across cohorts. Therefore, our strategy is to first solve for transitional dynamics using the model described above. Next, we use the endogenously obtained profile of time invested in human-capital formation to compute an average time investment and associated human-capital profile, which is then fed into our alternative model in which agents are restricted with respect to their time-investment choice. We do so separately for the two opposite social-security scenarios described in subsection 2.4. The average time investment is computed as $\bar{e}_j = \frac{1}{t_1 - t_0 + 1} \sum_{t=t_0}^{t_1} e_{t,j}$ for our calibration period ($t_0 = 211$ and $t_1 = 256$). In the

[10]The artificial initial steady state and long phase-in period are only used to generate suitable starting values for our main projection period. Bar and Leukhina (2010) provide an explicit model of the demographic transition and economic development that began in 17th Century England.

[11]In fact, changes in variables that are constant in steady state are already numerically irrelevant circa the year 2400.

alternative model, we then add the constraint $e_{t,j} = \bar{e}_j$. The human-capital profile is then obtained from (3) by iterating forward on age.[12]

# 3 Calibration and Computation

To calibrate the model, we choose model parameters such that simulated moments match selected moments in NIPA data and the endogenous wage profiles match the empirically observed wage profile in the U.S. during the calibration period 1960 – 2005.[13] The calibrated parameters are summarized in Table 1.

Table 1: Model Parameters

| | | | |
|---|---|---|---|
| Preferences | $\sigma$ | Inverse of Inter-Temporal Elasticity of Substitution | 2.00 |
| | $\beta$ | Pure Time Discount Factor | 0.993 |
| | $\phi$ | Weight of Consumption | 0.401 |
| Human Capital | $\xi$ | Scaling Factor | 0.16 |
| | $\psi$ | Curvature Parameter | 0.65 |
| | $\delta^h$ | Depreciation Rate of Human Capital | 0.8% |
| | $h_0$ | Initial Human Capital Endowment | 1.00 |
| Production | $\alpha$ | Share of Physical Capital in Production | 0.33 |
| | $\bar{\delta}$ | Depreciation Rate of Physical Capital | 3.8% |
| | $\bar{g}^A$ | Exogenous Growth Rate | 1.8% |

[12] By imposing the restriction of identical time-investment profiles for all cohorts (instead of, e.g., imposing the restriction only on cohorts born after 2005), we shut down direct effects from changing mortality on human capital and indirect anticipation effects of changing returns. This alternative model is a "standard" model of endogenous labor supply and an exogenously given age-specific productivity profile—as used in numerous studies on the consequences of demographic change—with the only exception being that the time endowment is age-specific. By setting the time endowment to $1 - \bar{e}_j$ rather than 1, we avoid re-calibration across model variants. For details, see below.

[13] We perform this moment matching in the endogenous human-capital model and the constant contribution-rate scenario. We do not recalibrate model parameters across social-security scenarios or for the alternative human-capital model because simulated moments do not differ much. Furthermore, we are interested in how our welfare conclusions are affected by imposing various constraints on the model—either through our social-security scenarios or by restricting human-capital formation—and any parametric change in this comparison would confound our welfare analysis.

## 3.1 Demographics

Actual population data from $1950 - 2005$ are collected from the Human Mortality Database (2008). Our demographic projections beyond 2004 are based on a population model that is described in detail in the online Appendix C.[14] Prior to 1950, we keep the population structure constant, as in 1950.

## 3.2 Household Behavior

The parameter $\sigma$, the inverse of the inter-temporal elasticity of substitution, is set to 2.[15] The time-discount factor $\beta$ is calibrated to match the empirically observed capital-output ratio of 2.8 which requires $\beta = 0.993$. The weight of consumption in the utility function, $\phi$, is calibrated such that households spend one-third of their time working, on average, which requires $\phi = 0.401$.

## 3.3 Individual Productivity Profiles

We choose values for the parameters of the human-capital production function such that average simulated wage profiles resulting from endogenous human-capital formation replicate empirically observed wage profiles. Data for age-specific productivity are collected from Huggett et al. (2010)[16]. We first normalize $h_0 = 1$ and then deter-

[14]The key assumptions of this model are as follows: First, the total fertility rate is constant at 2005 levels of 2.0185, until 2100, when fertility is adjusted slightly to keep the number of newborns constant for the remainder of the simulation period. Second, life expectancy monotonically increases from a current (2004) average life expectancy at birth of 77.06 years to 88.42 years in 2100, when it is held constant. Third, total migration is constant at the average migration for $1950-2005$ for the remainder of the simulation period. These assumptions imply that a stationary population is reached in about 2200.

[15]In the online appendix, Section B.4, a sensitivity analysis shows that our main quantitative results are robust when we change the predetermined parameter $\sigma$ to 1 or 3, respectively.

[16]We thank Mark Huggett for sending us the data.

mine the values of the structural parameters $\{\xi, \psi, \delta^h\}$ by indirect inference methods (Smith 1993; Gourieroux et al. 1993). To this end, we run separate regressions of the data and simulated wage profiles on a third-order polynomial in age, given by

$$\log w_j = \eta_0 + \eta_1 j + \eta_2 j^2 + \eta_3 j^3 + \epsilon_j. \tag{6}$$

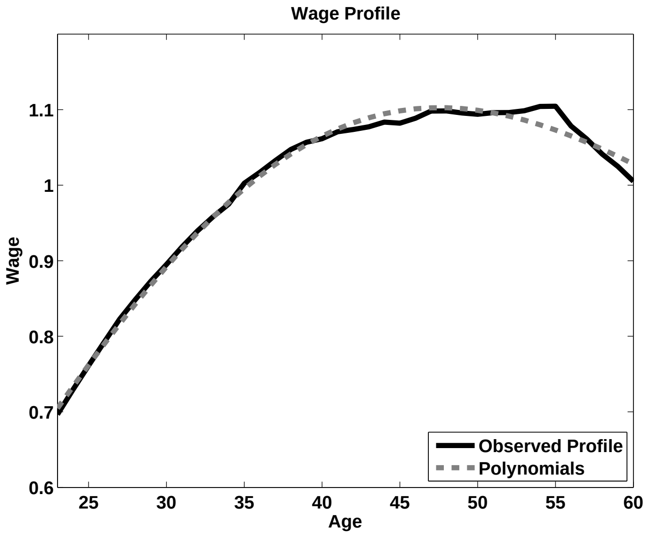
Here, $w_j$ is the age-specific productivity, and $\epsilon_j$ is a residual. Denote the coefficient vector determining the slope of the polynomial estimated from the actual wage data by $\overrightarrow{\eta} = [\eta_1, \eta_2, \eta_3]'$ and the one estimated from simulated human capital profiles during $1960-2004$ by $\overrightarrow{\hat{\eta}} = [\hat{\eta}_1, \hat{\eta}_2, \hat{\eta}_3]'$. The latter coefficient vector is a function of the structural model parameters $\{\xi, \psi, \delta^h\}$. Finally, the values of our structural model parameters are determined by minimizing the distance $\|\overrightarrow{\eta} - \overrightarrow{\hat{\eta}}\|$. See subsection 3.6 for further details.

Figure 2 presents the empirically observed productivity profile and the estimated polynomials. Our coefficients[17] and the shape of the wage profile are in line with others reported in the literature, especially with those obtained by Hansen (1993) and Altig et al. (2001). The estimate of $\delta^h = 0.008$ is reasonable (Arrazola and de Hevia (2004), Browning, Hansen, and Heckman (1999)), and the estimate of $\psi = 0.65$ is exactly in the middle of the range reported in Browning, Hansen, and Heckman (1999).[18]

[17]The coefficient estimates from the data are $\eta_0$: -1.6262, $\eta_1$: 0.1054, $\eta_2$: -0.0017 and $\eta_3$: 7.83e-06. We do not display the polynomial profile estimated from simulated data in Figure 2 because it perfectly tracks the polynomial obtained from the data.

[18]In a sensitivity analysis, we have shown that the estimate of the average time-investment productivity, $\xi = 0.16$, depends on the predetermined value of $h_0$, whereas the other parameters are rather insensitive to this choice. We have also found that parameterizations with a different value for $h_0$ yield the same results for the effects of demographic change on aggregate variables and welfare.

Figure 2: Wage Profiles




*Notes:* Observed profile: average life-cycle wage profiles collected from Huggett, Ventura, and Yaron (2010). Polynomials: predicted wage profile based on estimated polynomial coefficients of (6). Both profiles were normalized by their respective means.

## 3.4 Production

We calibrate the capital share in production, $\alpha$, to match the income share of labor in the data, which requires that $\alpha = 0.33$. We estimate a series of TFP and actual depreciation using NIPA data. We HP-filter these data series and then feed them into the model for the period 1950 to 2005. Thereafter, both parameters, $g$ and $\delta$, are held constant at their respective means. The average growth rate of total factor productivity, $\bar{g}^A$, is calibrated to match the growth rate of the Solow residual in the data. Accordingly, $\bar{g}^A = 0.018$. Finally, we calibrate $\bar{\delta}$ (and thereby scale the exogenous time path of depreciation, $\delta_t$) such that our simulated data match an average investment-output ratio of 20%, which requires $\bar{\delta} = 0.038$.

## 3.5 The Pension System

In our first social-security scenario ("const. $\tau$"), we fix contribution rates and adjust replacement rates of the pension system. We calculate contribution rates from NIPA data for $1960 - 2004$ and freeze the contribution rate at the year-2004 level for all following years. When simulating the alternative social-security scenario with constant replacement rates ("const. $\rho$"), we feed the equilibrium replacement rate obtained in the "const. $\tau$" scenario into the model and hold it constant at the 2004 level for all remaining years. Then the contribution rate endogenously adjusts to balance the budget of the social-security system.

## 3.6 Computational Method

For a given set of structural model parameters, the solution of the model is determined by outer- and inner-loop iterations. On the aggregate level (outer loop), the model is solved by guessing initial time paths of four variables: the capital intensity, the ratio of bequests to wages, the replacement rate (or contribution rate) of the pension system and average human capital for all periods from $t = 0$ until $T$. On the individual level (inner loop), we begin each iteration by guessing the terminal values for consumption and human capital. We then proceed by backward induction and iterate over these terminal values until the inner-loop iterations converge. In each outer loop, disaggregated variables are aggregated each period. We then update aggregate variables until convergence, using the Gauss-Seidel-Quasi-Newton algorithm developed in Ludwig (2007).

To calibrate the model in the "const. $\tau$" scenario, we consider additional "outer outer" loops to determine structural model parameters by minimizing the distance between the simulated average values and their respective calibration targets for the calibration period $1960 - 2004$. To summarize the description above, the parameter values determined in this way are $\beta$, $\phi$, $\delta$, $\xi$, $\psi$ and $\delta^h$.

# 4 Results

Before using our model to investigate the effects of future demographic change, we show how well it can replicate observed individual life-cycle profiles of the past. Next, we turn to the analysis of the transitional dynamics for the period 2005 to 2050, whereby we focus especially on the developments of major macroeconomic variables and the welfare effects of demographic change.

## 4.1 Backfitting

We first examine consumption profiles. We recognize that our model fails to replicate the empirically observed cross-sectional consumption profile in the 1990 Consumer Expenditure Survey[19], cf. Figure 3(a). The increase of consumption over the life cycle is too steep, and the peak is too late compared to the data. Because the decrease of consumption after the peak is solely caused by falling survival rates in a model without idiosyncratic risk, we cannot expect to match this dimension of the

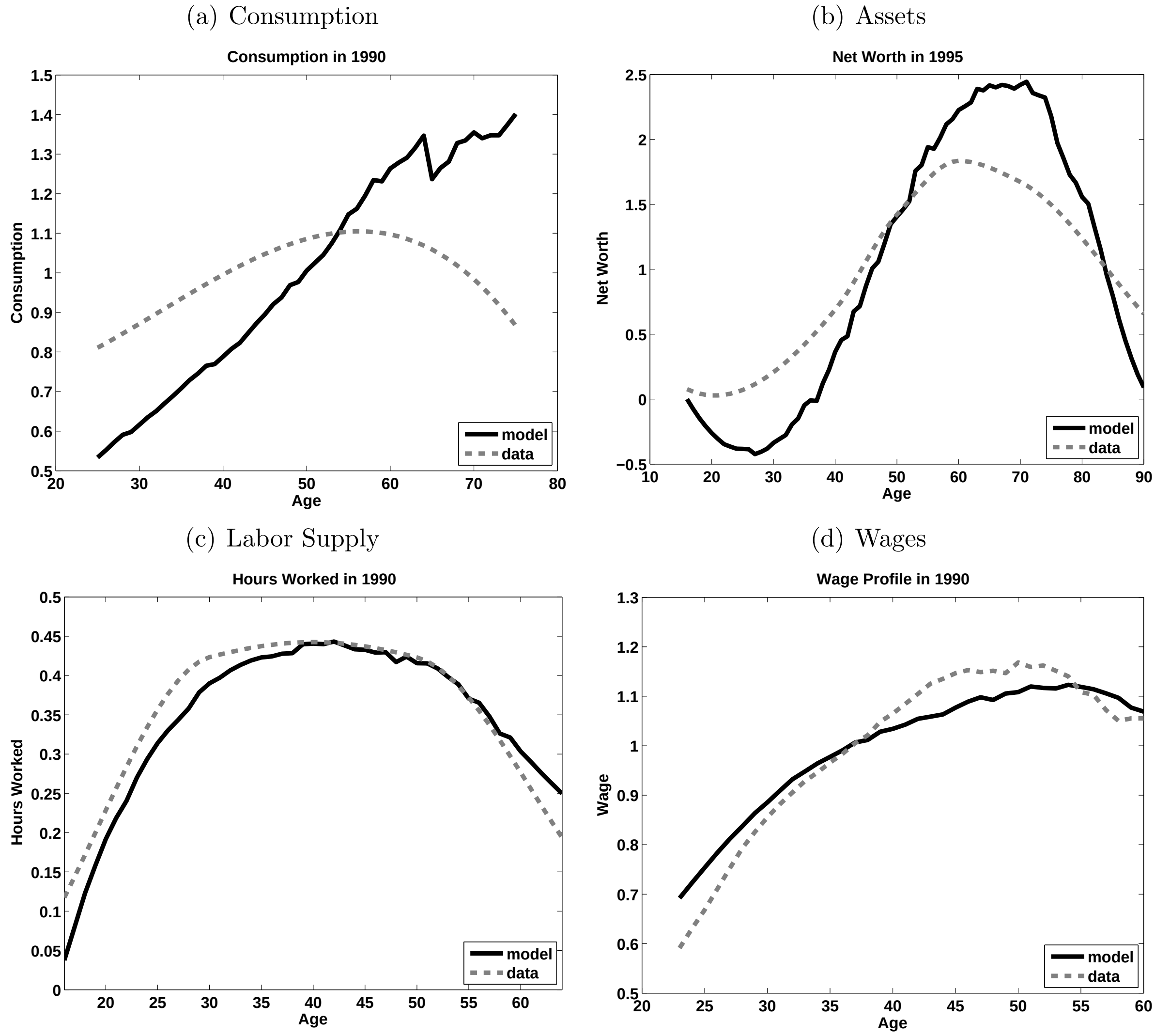
[19]The empirical profile is based on the observations on non-durable consumption for 1990 in the data set of Aguiar and Hurst (2009). We equalize the data using the traditional OECD scale that attributes weights of $(1.0, 0.7, 0.5)$ to the first adult, further adults (above age 16) and children, respectively. We then estimate a third-order polynomial in age on the adult-equivalent consumption data and show the predicted profile in the figure.

data (cf. Hansen and İmrohoroğlu (2008), Fernández-Villaverde and Krüger (2007)). As shown in Ludwig et al. (2007), in a model without human-capital adjustments, omitting idiosyncratic risk has only a negligible effect on welfare calculations. This is because welfare calculations are based on differences in consumption profiles, and the exact shape of the consumption profile is therefore less important. However, verifying the robustness of this finding in a model with endogenous human capital such as ours requires the introduction of idiosyncratic risk. We leave this extension for future research, mainly for technical reasons.[20]

We next examine asset profiles. Figure 3(b) shows household net worth data from the Survey of Consumer Finances for a cross-section in 1995, obtained from Bucks et al. (2006), and the corresponding cross-sectional asset profile in the model. Our model matches the broad pattern in the data. Observed discrepancies are threefold: First, as borrowing constraints are absent from our model, initial assets are negative, whereas they are positive in the data. Second, the run-up of wealth until retirement age is stronger in our model than it is in the data. Third, decumulation of assets is stronger as well. This last fact is due to the fact that our model neither has health risks, as in De Nardi et al. (2009), nor explicit bequest motives, cf., e.g., Attanasio (1999).

[20]Introducing idiosyncratic risk into our large model with two continuous-state variables would render computation of the transition path practically infeasible. However, to address the sensitivity of our welfare results with respect to the consumption profile, we have performed an additional sensitivity check, whereby we introduce lump-sum transfers that redistribute resources from aged individuals to young individuals within a household, such that the present value of lifetime resources is unaffected. This increases savings at younger ages. Our calibration then offsets this increase in savings via a lower $\beta$. This yields a flatter consumption profile. The total effect of lump-sum transfers on the consumption profile, therefore, mimics the effects of precautionary savings. This sensitivity analysis shows that our findings continue to hold in a model that achieves a better fit of the consumption profile and is otherwise as close as possible to our benchmark model. The results are available upon request.

Figure 3: Cross-Sectional Profiles




*Notes:* Model and data profiles for consumption, assets, labor supply and wages. All profiles are cross-sectional profiles for 1990, except for the asset profile, which is for 1995. Consumption, asset and wage profiles are normalized by their respective means. Hours data are normalized by 76 total hours per week.

*Data Sources:* Based on CEX consumption data collected from Aguiar and Hurst (2009), SCF net worth data obtained from Bucks et al. (2006), hours worked data from McGrattan and Rogerson (2004) and PSID wage data.

Our model does a good job of matching the cross-sectional hours profile observed in 1990 Census data collected from McGrattan and Rogerson (2004); see figure 3(c).[21]

[21] The hours data are normalized, with total hours per week equal to 76. This might appear to be a low number for total available hours, but such a magnitude is needed to make the McGrattan and Rogerson (2004) hours

Given our preference specification, the inverse u-shape of hours worked translates into a u-shaped pattern of Frisch labor-supply elasticities over the life-cycle. This implicitly captures higher elasticities at the extensive margin at the beginning and end of the life-cycle, cf., e.g., Rogerson and Wallenius (2009). Using a Frisch elasticity concept with constant (variable) time invested in human-capital formation, we find that agents of age 30-50 have an average elasticity of 0.8 (1.3). A more detailed discussion of these concepts can be found in our online Appendix B.2. The hour-weighted average Frisch elasticity across all ages, a "macro" elasticity, is approximately 1.1 (1.9). These numbers are higher than the standard microeconomic estimates reported in the literature, which are typically approximately 0.5. See, e.g., Domeij and Flodén (2006). However, these standard estimates are based on prime-age, full-time employed, male workers. In contrast, Browning, Hansen, and Heckman (1999) report that the scarce empirical estimates for females are much higher than for males. Our model is a unisex model and should, accordingly, represent both sexes. Furthermore, Imai and Keane (2004) argue that standard estimates are downward-biased by not considering endogenous human- capital accumulation explicitly and thereby not correctly accounting for the true opportunity cost of time.[22] We therefore regard our value of the Frisch elasticity as very reasonable. In Section B.4 of the online

data broadly consistent with the common belief that agents spend about one-third of their time working and the standard practice of macroeconomists to calibrate their models (which we have followed). The McGrattan and Rogerson (2004) data only contain average hours for certain age bins, e.g., average hours for persons of age 15-24. We associate the average hours with the age mean of that age bin, e.g., associate the value for ages 15-24 with age 20 and then use cubic interpolation to construct the empirical hours profile for all other ages. A similar procedure is used to construct the empirical asset profile.

[22] Imai and Keane (2004) make this argument in the context of a learning-by-doing model, but similar biases might be present in our model. We are unaware of any attempt to estimate the Frisch elasticity with varying time invested empirically in a framework such as ours, which would require inclusion of the marginal utility of human capital in the set of conditioning variables.

appendix, a sensitivity analysis further shows that our main quantitative results are robust to using a higher value of $\sigma$, which implies a lower Frisch elasticity.

Finally, Figure 3(d) shows the cross-sectional wage profile observed in the PSID data in 1990. Our model matches the broad pattern observed in the data.[23]

## 4.2 Transitional Dynamics

We divide our analysis of transitional dynamics into two parts. First, we analyze the behavior of several important aggregate variables from 2005 to 2050. Second, we investigate the welfare consequences of demographic change for generations already alive in 2005 and for households born in the future. Throughout, we demonstrate how the design of the social-security system affects our results.

### 4.2.1 Aggregate Variables

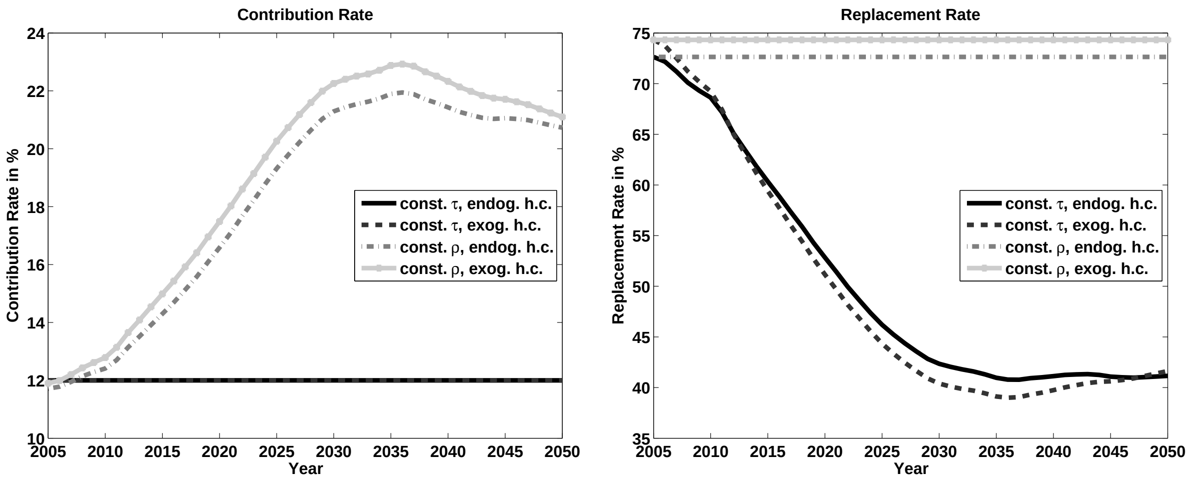
The evolution of policy variables in the two social-security scenarios is presented in Figure 4. In the "const. $\tau$" scenario, pensions become less generous over time, which is represented by a decrease in the replacement rate, from approximately 73% (74%) in 2005 to 41% (42%) in 2050 for the endogenous (exogenous) human-capital model. In contrast, in the "const. $\rho$" scenario, the generosity of the pension system remains at the 2005 level, implying that contribution rates have to increase from approximately 12% in 2005 to 21% in 2050 in both human-capital models.[24]

[23] The wage data were selected using the same criteria as in Huggett, Ventura, and Yaron (2010). To smooth the data, we show a centered average of five subsequent PSID samples. In Section B.1 of the online appendix, we present the fit of our model to cross-sectional data on wages and hours in the years 1970, 1980, 1990 and 2000. The model profiles are broadly consistent with the data at all those points in time.

[24] As explained in Section 2.4, our model of the pension system abstracts from the fact that in the United States, only the 35 years of working life with the highest individual earnings relative to average earnings are counted for

Figure 4: Evolution of Policy Variables

(a) Contribution Rate (b) Replacement Rate




*Notes:* Pension system contribution and replacement rates for the two social-security scenarios. "const. $\tau$": constant contribution-rate scenario. "const. $\rho$": constant replacement-rate scenario. "endog. h.c.": endogenous human-capital model. "exog. h.c.": exogenous human-capital model.

Figure 5 reports the dynamics of four major macroeconomic variables for the two model variants—with endogenous and exogenous human capital—in the "const. $\tau$" social-security scenario, and Figure 6 does so in the "const. $\rho$" scenario.

In Figures 5(a) and 6(a), we show the evolution of the rate of return to physical capital for the different models.[25] In the "standard" models with endogenous labor supply only, the rate of return decreases from an initial level of approximately 8.1% in 2005 to 7.1% in the "const. $\tau$" scenario and to 7.7% in the "const. $\rho$" scenario in 2050.[26] This magnitude is in line with results reported elsewhere in the literature, cf.,

the calculation of average indexed past earnings. This leads us to overstate the replacement rate but does not directly affect the level of pensions. Furthermore, we assume a balanced budget, whereas the U.S. system runs a social-security trust fund that collects excess paid-in contributions. This biases upward the replacement rate and the level of pensions.

[25]There are two reasons for the small level differences in 2005 across the various scenarios. First, our calibration targets are averages for the period $1960 - 2004$. Second, as already discussed in Section 3, we do not recalibrate across scenarios. Such level differences in initial values can be observed in all of the following figures.

[26]The high initial level of the rate of return is caused by the previous baby boom, which increased the labor

e.g., Börsch-Supan et al. (2006) and Krüger and Ludwig (2007), whereas Attanasio et al. (2007) find slightly larger effects. On the contrary, in the two models with endogenous human-capital adjustment, the rate of return is expected to fall by only 0.4 (0.2) percentage points in the "const. $\tau$" ("const. $\rho$") scenario. This difference in the decrease of the rate of return until 2050 between the exogenous and endogenous human-capital models is large, at a factor of about 2.5.

In Figures 5(b) and 6(b), we depict the evolution of average hours worked by all working-age individuals. Average hours worked increase both for the endogenous and exogenous human-capital models. Observe that there are level differences between the two model variants. This is mainly caused by differences in time invested in human-capital formation.

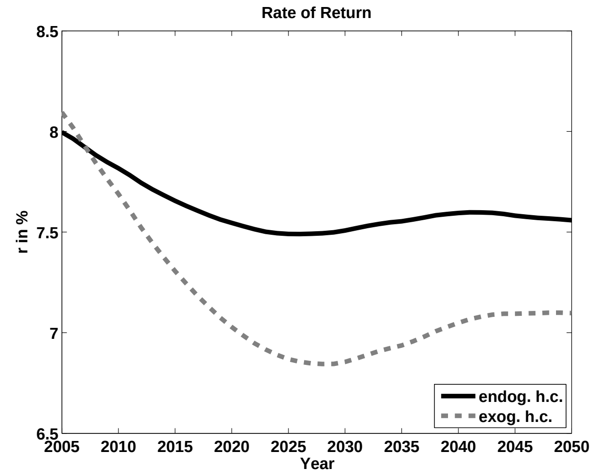
Figures 5(c) and 6(c) show that time invested in human-capital formation increases when agents are allowed to adjust their human capital. The reasons for this adjustment are that both higher wage growth and a lower rate of return to physical capital strengthen the incentive to accumulate human capital. Specifically, with endogenous human capital in the "const. $\tau$" ("const. $\rho$") scenario, average human capital per working hour increases by approximately 17% (11%) until 2050.

Finally, we focus on the evolution of the growth rate of GDP per capita, as shown in Figures 5(d) and 6(d). When the U.S. aging process peaks in 2025 (cf. figure 1), the growth rate of per-capita GDP falls in all scenarios to its lowest level. The drop is least pronounced for the endogenous human capital model with a fixed contribution

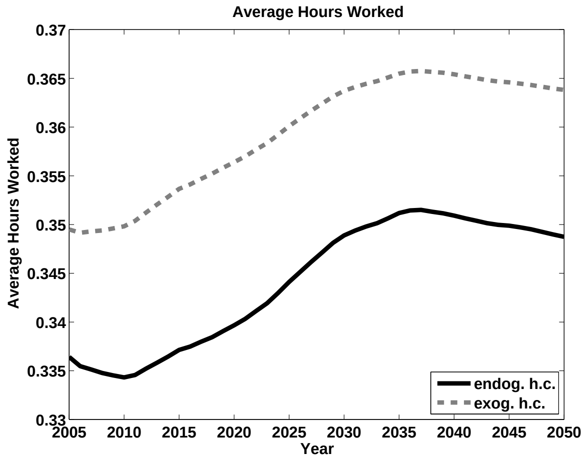
force and hence decreased capital intensity.

Figure 5: Aggregate Variables for Constant Contribution-Rate Scenario

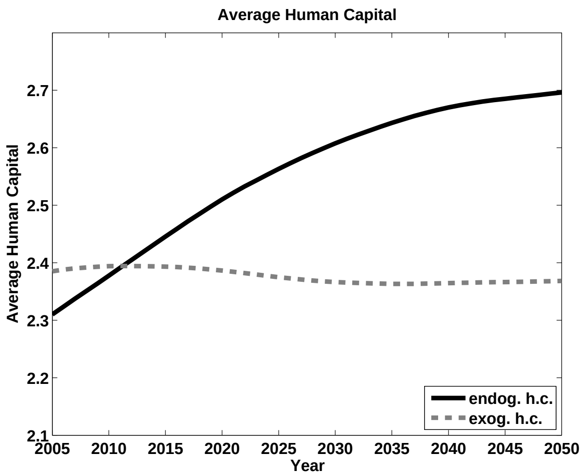
(a) Rate of Return to Physical Capital



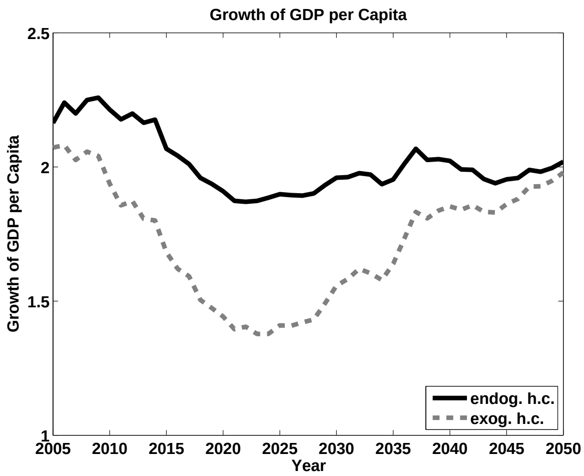

(b) Average Hours Worked




(c) Average Human Capital




(d) Growth of GDP per Capita in %



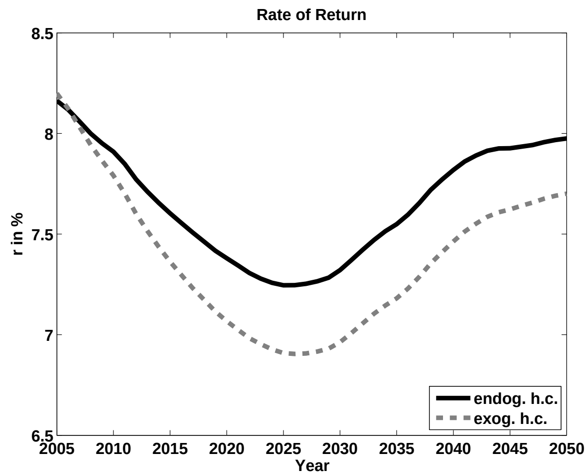

*Notes:* Rate of return to physical capital, average hours worked of the working-age population, average human capital per working hour and growth of GDP per capita in the constant contribution-rate social-security scenario for two model variants. "endog. h.c.": endogenous human-capital model. "exog. h.c.": exogenous human capital model.

rate. There, the growth rate gradually declines from 2.2% in 2005 to 1.9% in 2025.[27] Comparing the two "const. $\tau$" scenarios, it can be observed that not adjusting the human-capital profile entails a large drop in the growth rate. The maximum difference circa 2025 is 0.4 percentage points. Although the difference across human-

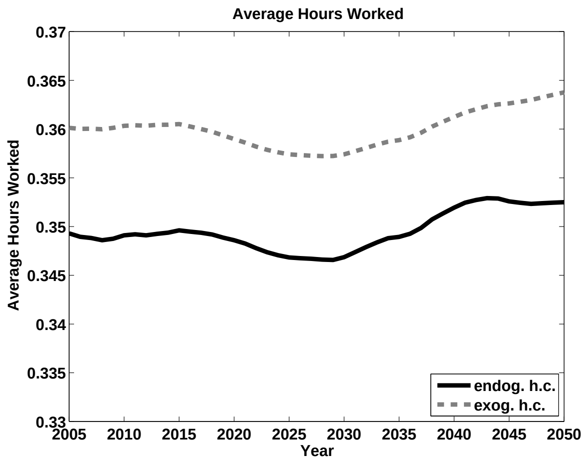
[27]The high initial growth rate is a consequence of the past baby boom, cf. footnote 26.

Figure 6: Aggregate Variables for Constant Replacement-Rate Scenario

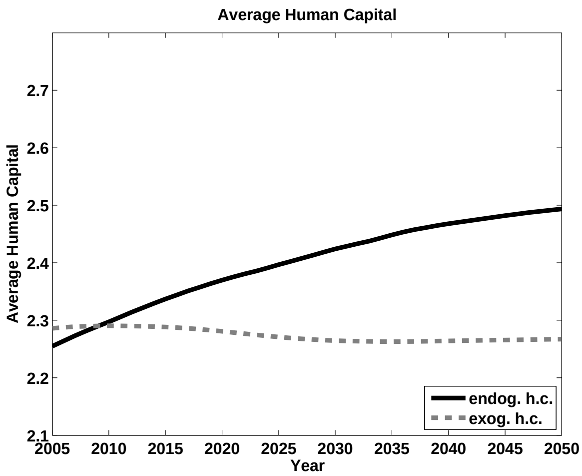
(a) Rate of Return to Physical Capital



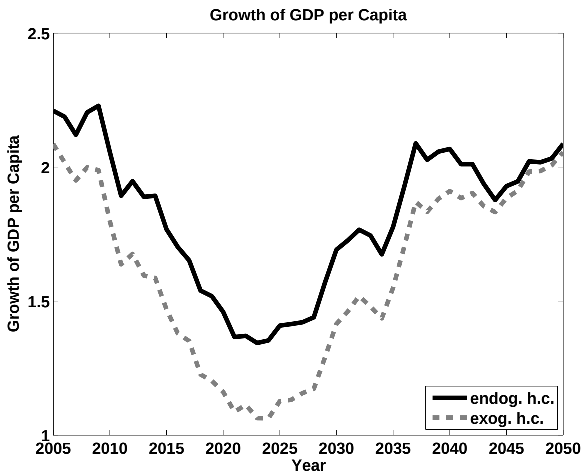

(b) Average Hours Worked




(c) Average Human Capital




(d) Growth of GDP per Capita in %




*Notes:* Rate of return to physical capital, average hours worked of the working-age population, average human capital per working hour and growth of GDP per capita in the constant replacement-rate social-security scenario for two model variants. "endog. h.c.": endogenous human-capital model. "exog. h.c.": exogenous human-capital model.

capital models is only 0.2 percentage points in the case that the replacement rate is held constant ("const. $\rho$" scenarios), the same conclusion applies. The aging process induces relative price changes, such that agents increase their time invested in human-capital formation and thereby cushion the negative effects of demographic change on growth.[28]

#### 4.2.2 Welfare Effects

In our model, a household's welfare is affected by two consequences of demographic change. First, her lifetime utility changes because her own survival probabilities increase. Second, households face a path of declining interest rates, increasing gross wages and decreasing replacement rates (increasing contribution rates) relative to the situation without a demographic transition.

We want to isolate the welfare consequences of the second effect. To this end, we compare for an agent born at time $t$ and of current age $j$ her lifetime utility when she faces equilibrium factor prices, transfers and contribution (replacement) rates, as documented in the previous section, with her lifetime utility when she instead faces prices, transfers and contribution (replacement) rates that are held constant at their 2005 value. For both of these scenarios, we fix the households' individual survival probabilities at their 2005 values.[29] Following Attanasio et al. (2007) and Krüger and Ludwig (2007), we then compute the consumption-equivalent variation $g_{t,j}$, i.e., the percentage increase in consumption that needs to be given to an agent with characteristics $t, j$ at each date in her remaining lifetime at fixed prices to make her as well off as she would be in the situation with changing prices.[30] Positive

[28] In our online Appendix B.3.1, we show that the cumulative effect of the growth rate differences between the endogenous and exogenous human-capital models on the level of GDP per capita is large. With human-capital adjustments, the detrended level of GDP per capita will increase by approximately 14% (10%) more until 2050 in the "const. $\tau$" ("const. $\rho$") scenario than without these adjustments.

[29] Of course, they fully retain their age dependency. We show in the online Appendix B.3.2 that varying the survival probabilities according to the underlying demographic projections leaves our conclusions on welfare in the comparison across the two models essentially unchanged.

[30] With our assumptions on preferences, $g_{t,j}$ can be calculated as $g_{t,j} = \left(\frac{\bar{V}_{t,j}}{\bar{V}_j^{2005}}\right)^{\frac{1}{\phi(1-\sigma)}} - 1$, where $\bar{V}_{t,j}$ denotes

numbers of $g_{t,j}$ thus indicate that households obtain welfare gains from the general-equilibrium effects of demographic change, and negative numbers indicate welfare losses.

**Welfare of Generations Alive in 2005**

Of particular interest is how the welfare of all generations already alive in 2005 will be affected by demographic change. This analysis allows for an inter-generational welfare comparison of the consequences of demographic change in terms of well-being that would not be possible using aggregate statistics such as per-capita GDP. Newborns and young generations benefit from increasing wages as well as decreasing returns, if they borrow to finance their human-capital formation. However, older—and thus asset-rich—generations are expected to lose lifetime utility. First, they benefit less from increasing wages because they do not significantly adjust their human capital and because their remaining working period is short. Second, falling returns diminish their capital income. Third, retirement income decreases in our scenario with constant contribution rates.

The results shown in Figure 7 can be summarized as follows: First, the welfare of newborn agents is essentially unchanged in the "const. $\tau$" scenarios, whereas in the "const. $\rho$" scenario, newborns experience welfare losses of roughly 4.4% (5.0%) in the endogenous (exogenous) human-capital model. As explained in our online Appendix B.3.2, the fact that these welfare changes are almost identical in the two human-capital models is due to a complex interaction between the value of human-capital

lifetime utility at changing prices and $\bar{V}_j^{2005}$ at fixed 2005 prices.

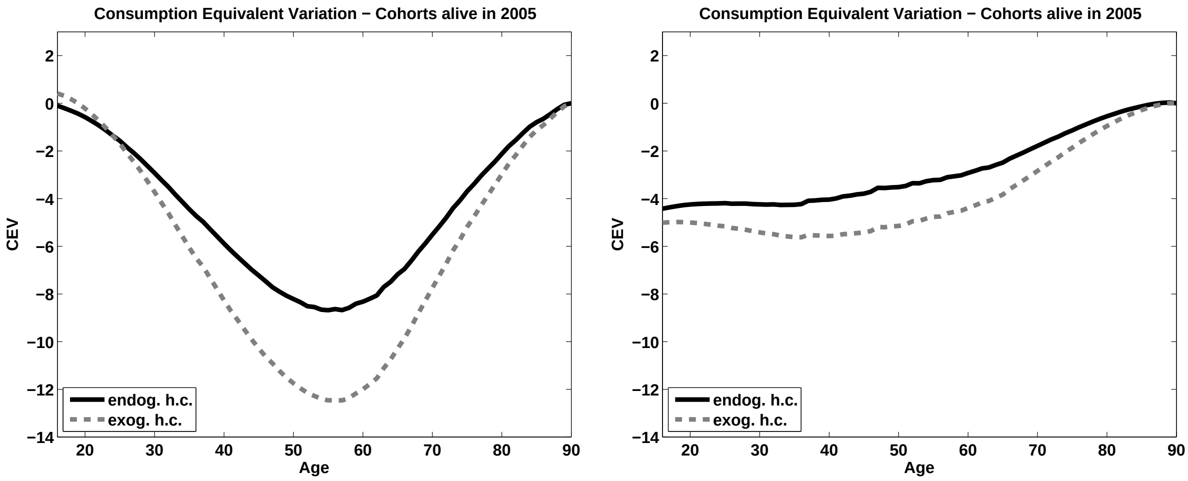
adjustments, which is positive, and differential general-equilibrium effects, which partially offset this interaction. Second, middle-aged agents incur the highest losses in the "const. $\tau$" scenarios: the maximum loss of agents is much larger compared to a scenario with fixed replacement rates. Clearly, constant replacement rates decrease net wages of the young but keep pensions more generous. This decreases lifetime utility of the young but narrows the loss of utility of the old (compared to a situation with falling replacement rates). The redistribution through the pension system shifts the balance somewhat in favor of the old. This also explains why the maximum of the losses occurs at a much higher age in the "const. $\tau$" scenario in which agents close to retirement lose interest income *and* receive lower pensions. Third, independent of future pension policy, agents lose relatively less in the endogenous human-capital model. Younger agents can adjust their human capital in response to higher wages, whereas older (asset-rich) households benefit from a smaller drop in the interest rate (cf. Figures 5(a) and 6(a)) and higher pension payments.[31]

Table 2 finally provides numbers on the maximum welfare loss displayed in Figure 7 as a summary statistic. In the exogenous human-capital model, the maximum welfare loss is approximately 12.5% (5.6%) of lifetime consumption in the "const. $\tau$" ("const. $\rho$") scenario, while it is only 8.7% (4.4%) of lifetime consumption in the endogenous human-capital model. This exemplifies that ignoring the adjustment channel through human-capital formation leads to quantitatively important biases of the welfare assessments of demographic change.

[31] In the online Appendix B.3.2, we decompose the welfare differences between the endogenous and exogenous human-capital models for the "const. $\tau$" scenario into effects stemming from differential changes in factor prices and the relative rise in social-security benefits, which is caused by additional human-capital formation.

Figure 7: Consumption Equivalent Variation of Agents Alive in 2005

(a) Constant Contribution-Rate Scenario (b) Constant Replacement-Rate Scenario




*Notes:* Consumption equivalent variation (CEV) in the two social-security scenarios.

Table 2: Maximum Utility Loss for Generations alive in 2005

| | Human Capital | |
|---|---|---|
| | Endogenous | Exogenous |
| Const. $\tau$ ($\tau_t = \bar{\tau}$) | -8.7% | -12.5% |
| Const. $\rho$ ($\rho_t = \bar{\rho}$) | -4.4% | -5.6% |

**Welfare of Future Generations**

We next examine the welfare consequences for all future newborns. Agents born into a "const $\tau$"-world experience welfare gains of up to 1.1% and losses of up to 1.7% of lifetime consumption, depending on whether they are born before or after 2005. However, welfare losses for future generations may be quite large, despite the human-capital channel, if the social-security system is not reformed ("const $\rho$"). These losses are between 5.2% and 10.7% of lifetime consumption with exogenous human capital and not much lower with endogenous human-capital adjustments.[32]

# 5 Conclusion

This paper finds that increased investments in human capital may substantially mitigate the macroeconomic impact of demographic change, with profound implications for individual welfare. As labor will be relatively scarce and capital will be relatively abundant in an aging society, interest rates will fall. As we emphasize, these effects will be much smaller once we account for changes in human-capital formation. For the U.S., our simulations predict that if contribution rates (replacement rates) are held constant, then the rate of return will fall by only 0.5 (0.9) percentage points until 2025 with endogenous human capital, compared to 1.2 (1.3) percentage points in the standard model with a fixed human-capital profile.

We also document that the increase in wages, declines in rates of return and changes to pension contributions and benefits induced by demographic change have

[32]See graphs in the online Appendix B.3.2 for more details.

substantial welfare consequences. When human capital cannot adjust, some of the agents alive in 2005 will experience welfare losses of up to 12.5% (5.6%) of lifetime consumption with constant contribution (replacement) rates. However, importantly, we find that these maximum losses are only 8.7% (4.4%) of lifetime consumption when the human-capital adjustment mechanism is taken into account.

However, we have operated in a frictionless environment, where all endogenous human-capital adjustments are driven by relative price changes. If, instead, human-capital formation is affected by market imperfections, such as borrowing constraints, then these automatic adjustments will be inhibited. In this case, appropriate education and training policies in aging societies are an important topic for future research and the policy agenda.

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