Central Bank Asset Purchases as a Corrective Policy[1]

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Abstract

We show that central bank asset purchases can serve as a superior corrective policy under financial frictions. We develop an analytically tractable incomplete markets model with idiosyncratic risk and limited commitment, where conventional monetary policies are neutral. Asset purchases at above-market prices affect the allocation by driving a wedge between relevant returns for borrowers and lenders. Monetary policy can thereby restore constrained efficiency by correcting for pecuniary externalities under financial constraints. Contrary to equivalent Pigouvian policies, asset purchases neither require type-specific nor lump-sum taxes. We further show that countercyclical asset purchases can enhance social welfare under aggregate shocks.

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1 Introduction

The European Central Bank (ECB) and US Federal Reserve (Fed), which have traditionally traded money in exchange for (short-term) treasuries, have included large-scale purchases of private debt securities in secondary markets into their set of policy instruments. According to empirical evidence, interventions in secondary markets of the Fed and the ECB during and after the recent financial crisis have altered assets prices and portfolio decisions.[3] In theory, asset purchases have been shown to be beneficial under stressed financial markets.[4] Yet, it is an open question whether they can play a useful role in non-crisis times, when financial market participants are also constrained, though to a lesser extent. The usefulness of asset purchases is disputable, since they might reduce efficiency by distorting prices. Contrary to this view, we prove that central bank asset purchases can serve as a corrective policy that address pecuniary externalities induced by financial constraints (see Davila and Korinek, 2018).[5] We further show that conventional monetary policies cannot serve for this purpose, whereas asset purchases can in principle be mimicked by Pigouvian subsidies. Yet, the latter rely on type-specific and non-distortionary taxes, which are not required for welfare-enhancing asset purchases.

Credit market interventions have distributional consequences, i.e. different effects on holders and issuers of debt. Subsidizing debt or lending directly to agents benefits borrowers, whereas savers/lenders tend to loose from lower interest rates. In contrast, distributional effects of asset purchases are ambiguous: When a central bank offers a favorable price for debt in secondary markets, *direct* effects of this intervention are beneficial for savers/lenders. After receiving liquid funds in exchange for less liquid assets, these agents can further invest the proceeds, such that market prices and other participants in financial markets are also affected. In tranquil times, when neither asset liquidation nor liquidity hoarding are urgent, these *indirect* effects of asset purchases can be crucial.

[3] Krishnamurthy and Vissing-Jorgensen (2011) and Krishnamurthy et al. (2017) provide evidence on price effects via different channels of asset purchase programmes conducted by the Fed and the ECB.

[4] See, e.g., Curdia and Woodford (2011) or Del Negro et al. (2017).

[5] Inefficiency is induced by a financial constraint that leads to a positive feedback between asset demand, prices, and borrowing capacities, like in Lorenzoni (2008), Stein (2012), or Bianchi and Mendoza (2018). Our paper complements this literature on prudential policies by focussing on gains of ex-post corrective policies conducted by the central bank.

Concretely, borrowers might gain from the pass-through of price effects on borrowing costs; the latter being aimed and observed for recent asset purchases programmes.[6]

We examine central bank purchases of private debt securities in an environment where financial markets are not unusually stressed. Instead, we acknowledge that financial markets are regularly distorted by frictions, i.e. by financial constraints that bind for a subset of agents (see Kiyotaki and Moore, 1997), inducing deviations from first best and leading to inefficiencies due to pecuniary externalities (see Davila and Korinek, 2018). The model features a positive feedback between collateral demand, prices, and the borrowing capacity, closely relating to the main mechanism in studies on prudential policies.[7] First, we prove that central bank asset purchases can enhance ex-ante social welfare by addressing these pecuniary externalities, which is not possible by conventional monetary policies. Specifically, lenders can be incentivized to increase their supply of funds by offering an above-market price for debt purchases. This raises the lenders' effective real return on debt, while the real interest rate for borrowers falls in equilibrium.[8] The wedge between the effective real returns for borrowers and lenders can be used to correct for pecuniary externalities. Second, we show that asset purchases can enhance welfare even further by alleviating the borrowing constraint via their impact on the relative price of collateral. Third, we find that asset purchases can mitigate financial amplification of aggregate shocks. Notably, the total effects of asset purchases can in principle be replicated by a combination of Pigouvian subsidies on debt and collateral, which require type-specific and non-distortionary taxes. The latter are not required for asset purchases, since the central bank can set the price of money and money can costlessly be produced. Asset purchases are thus a superior corrective policy under limited availability of fiscal instruments.

[6] ECB purchases of ABS in 2014 were expected to "facilitate credit conditions" (ECB press release, 2nd Oct. 2014). Hancock and Passmore (2011) report that Fed's MBS purchases in 2008 do not only affect MBS yields, but also reduced mortgage rates.

[7] Lorenzoni (2008), Bianchi (2011), Stein (2012), Bianchi and Mendoza (2018), and Schmitt-Grohe and Uribe (2018) focus on ex-ante corrective policies under financial constraints that bind only in crises states. In constrast, we consider regularly binding financial constraints (see Kiyotaki and Moore, 1997) and examine ex-post policies that are introduced when borrowers are constrained. As also shown by Bianchi (2016) and Jeanne and Korinek (2016), subsidizing borrowers is then beneficial.

[8] This is consistent with empirical evidence on recent asset purchase programmes. Specifically, the loan rate falls by a reduction in the (il-)liquidity premium, which accords to empirical evidence on price effects of US Federal Reserve asset purchases (see Gagnon et al., 2011).

We develop an analytically tractable incomplete market model with limited commitment to repay debt. The model is sufficiently stylized to isolate the main mechanism, while it includes relevant elements of monetary policy implementation. It is not intended to provide a description of a policy measure implemented in a specific episode, but it is rather aimed to identify efficiency gains from applying an established policy instrument. To assess the effects of asset purchases with central bank money, we specify a monetary economy. We respect the special role of central bank money for the settlement of transactions and assume, for convenience, that it serves as the unique means of payment for non-durable consumption goods (see Lucas and Stokey, 1987). We abstract from financial intermediation, endogenous production, and price rigidities. This facilitates isolation of asset purchase effects, since conventional monetary policies are then neutral with regard to the equilibrium allocation. Agents differ by their valuation of non-durables, giving rise to borrowing/lending in terms of money. We consider limited commitment, implying that borrowing is constrained by collateral (as in Kiyotaki and Moore, 1997). The central bank supplies money only against eligible assets, which basically consist of short-term treasury securities. In addition, we consider central bank purchases of private debt securities in secondary markets,[9] which can be non-neutral by affecting borrowers and lenders in different ways. When the monetary policy rate, i.e. price of money in terms of eligible assets, is set below the marginal rate of intertemporal substitution, eligible assets are scarce and money supply is effectively rationed.[10] Then, Wallace's (1981) irrelevance result for open market operations does not apply. Lenders participate in asset purchases programs when the central bank offers an above-market price. They supply the proceeds, at least partially, to borrowers, leading to a lower real loan rate than in the laissez faire equilibrium (which prevails under conventional monetary policies).

To facilitate aggregation and to enable the derivation of analytical results, we apply linear-quadratic preferences and derive an equilibrium representation with a representa-

[9] This can alternatively be interpreted as a central bank collateral framework where private debt securities are eligible. In contrast to related studies on unconventional policies (see Curdia and Woodford, 2011, or Gertler and Karadi, 2011), the central bank does not directly trade with borrowers.

[10] Under money rationing, the central bank can simultaneously control the price and the amount of money, and can thereby implement welfare dominating allocations compared to policy regimes that satiate money demand (see also Schabert, 2015).

tive lender and a representative borrower. Then, we establish the existence of constrained inefficiencies due to pecuniary externalities in this framework, which correspond to the "collateral externalities" in Davila and Korinek (2018) or in Bianchi and Mendoza (2018). Notably, the pecuniary externalities affect the representative agents only via a consolidated collateral and liquidity constraint, such that "distributive externalities" (see Davila and Korinek, 2018) are not relevant. We assume that lump-sum taxes are not available, whereas the public sector is able to transfer funds in a lump-sum way. As a consequence, the implementation of the first best allocation is not feasible.[11]

To identify gains of financial market interventions, we consider a well-established reference case (see, e.g., Bianchi and Mendoza, 2018). Specifically, we examine the problem of a social planer who maximizes social welfare, while choosing debt on the behalf of private agents subject to the equilibrium conditions under laissez faire. The resulting constrained efficient allocation can in principle be implemented by a Pigouvian subsidy on debt, which however requires type-specific lump-sum taxes. We show that a central bank can exactly replicate the Pigouvian debt subsidy by a suitably sized asset purchase programme at an above-market price. Moreover, we show that asset purchases can further implement allocations that welfare-dominate the constrained efficient allocation. The reason is that asset purchases tend to increase the price of collateral ("credit goods") relative to non-durables ("cash goods"), which alleviates the borrowing constraint in terms of non-durables. This effect stems from the increased lenders' willingness to pay for housing relative to non-durables when they receive a higher effective return on lending (i.e. on relinquishing non-durables). In principle, this can alternatively be generated by a non-monetary instrument, in particular, by a Pigouvian subsidy on housing that is implemented with type-specific and non-distortionary taxes.[12]

To provide numerical examples for the previous results as well as short-run effects

[11] Precisely, a positive initial stock of public sector liabilities is required for a feasible consumption sequence, such that the present value of public sector earnings has to ensure public sector solvency. Without (lump-sum) tax revenues central bank interest earnings have to be positive. This rules out zero interest rates and complete self-ensurance of agents with money, which would be required for the implementation of the first best allocation.

[12] This is shown in Appendix G, where we relate the monetary instruments under asset purchases to fiscal instruments in a corresponding real (non-monetary) economy.

of asset purchases, we use a CRRA utility function. While the latter facilitates the calibration of the model, we rely on pooled end-of-period funds within households (as in Lucas and Stokey, 1987, or Woodford, 2016) to define a competitive equilibrium with a representative borrower and a representative lender. For this equilibrium, which differs from the previous one solely by the agents' marginal utilities, we confirm the results derived before. Neglecting aggregate risk, we find that asset purchases can increase social welfare (measured in consumption equivalents) by up to 1% compared to the laissez faire case, while the largest contribution (about 0.9%) stems from restoring constrained efficiency. Notably, the representative borrower gains and the representative lender looses in the long-run equilibrium, indicating that the indirect (price) effects of asset purchases dominate the direct effects. We further introduce aggregate risk in form of a stochastic aggregate income, i.e., aggregate endowment with non-durable goods, and examine state-contingent policies under commitment.[13] We find that asset purchases should be conducted in a countercyclical way, such that borrowing is particularly stimulated in low income states. The reason is that borrowers suffer not only from a reduction in income, but also from a decline in the price of collateral (i.e. housing). Countercyclical asset purchases then stimulate (dampen) borrowing and thus borrowers' consumption in situations where the borrowing capacity is reduced (enhanced). Hence, asset purchases serve as an ex-post corrective policy that mitigates financial amplification and can thereby support prudential policies; an analysis of this interaction being left for future research.

Related literature Our analysis relates to studies on unconventional monetary policies under price rigidities. Curdia and Woodford (2011) and Gertler and Karadi (2011) show that direct central bank lending to ultimate borrowers can be beneficial if financial market frictions are sufficiently severe. Chen et al. (2012) find that changing the composition of treasury debt as under US Federal Reserve large scale asset purchase programmes during the financial crisis had moderate GDP growth and inflation effects. Del Negro et al. (2017) examine government purchases of equity in response to an adverse shock to resaleability and show that the introduction of this type of policy after 2008 have

[13] For simplicity, we thereby abstract from the issue of time inconsistency, which has been examined by Bianchi and Mendoza (2018) in the context of financial market interventions.

prevented a repeat of the Great Depression. Woodford (2016) extends Stein's (2012) fire sale model to assess the impact of central bank purchases of long-term treasuries on financial stability when crises are exogenously triggered. Our paper further relates to Araújo et al. (2015), who show that asset purchases exert ambiguous welfare effects under endogenous collateral constraints. In contrast to our paper, they do not examine pecuniary externalities, consider "no special uses of currency", and assume that all central bank trades occur at market prices. The specification of central bank operations in our paper relates to Schabert (2015), who examines welfare gains from money rationing in a New Keynesian model without idiosyncratic shocks and with frictionless financial markets. Finally, our analysis relates to Bianchi's (2016) and Jeanne and Korinek's (2016) studies on real ex-post policies, like bailouts or borrowing subsidies,[14] in non-monetary economies where pecuniary externalities are also induced by financial constraints. Using a single agent (small open economy) model, Benigno et al. (2016) examine welfare-improvements of various tax instruments for ex-post and ex-ante interventions. None of these nor other studies consider the difference between monetary and fiscal interventions for corrections of pecuniary externalities.

In Section 2, we present the model. Section 3 provides analytical results on welfare-enhancing financial market interventions. In Section 4, we present numerical examples and analyze state-contingent asset purchases under aggregate risk. Section 5 concludes.

2 The model

In this Section, we develop a simple incomplete markets model with idiosyncratic preference shocks and limited commitment. Acknowledging core properties of central bank money, we consider its specific role for transactions and that it can be costlessly produced. To suitably take account of central bank asset purchase programmes, we specify regular treasury open market operations and additional consider secondary market interventions of the central bank. We disregard endogenous production and price rigidities, such that conventional monetary policy measures are neutral.

[14] Borrowing subsidies are also examine by Correia et al. (2016) in a model with frictional intermediation and costly enforcement. They show that credit subsidies are desirable and – in contrast to our analysis – superior to monetary policy measures.

2.1 Overview and timing of events

The economy consists of households, a central bank, and a government. To respect the special role of central bank money, we assume that it is essential because of its unique role for the settlement of transactions. For simplicity, we assume that money serves as the exclusive means of payment for non-durable goods. Money is supplied by the central bank only in exchange for eligible assets, which basically consist of short-term treasuries. Debt contracts are only available in nominal terms. Due to limited enforcement of debt contracts, households can borrow money among each other only against collateral, i.e., durable goods. The central bank sets the price of money in terms of eligible assets and decides on the amount of money supplied via treasury open market operations. In addition, it can supply money via purchases of collateralized loans in secondary markets. When the central bank trades assets at an above-market price, eligible assets are scarce and money supply is rationed. In this case, asset purchases can be non-neutral. The government issues short-term debt and receives seigniorage. Lump-sum taxes are not available, though, lump-sum transfers. The timing of events in each period is as follows:

i.) Households enter a period with holdings of money, government bonds and durable goods, and receive an exogenously determined endowment of non-durable goods.

ii.) Open market operations are conducted, where the central bank sells or purchases assets outright or supplies money under repurchase agreements (repos) against treasury securities at the policy rate.

iii.) Idiosyncratic preference shocks are realized and durable goods are traded. Households with a high realization of the preference shock are willing to consume more non-durables than households with a low realization, such that the former tend to borrow money from the latter against collateral. Collateralized loans might be purchased by the central bank and the proceeds are available to extend loan supply.

iv.) Non-durable goods are traded and repos are settled.

v.) The asset market opens, where agents repay collateralized loans, the government issues bonds, and the central bank reinvests earnings from maturing bonds.

2.2 Private sector

There are infinitely many and infinitely lived households i of measure one, which are characterized by identical initial stocks of wealth. Their utility increases with consumption $c_{i,t}$ of a non-durable good and holdings of a durable good, i.e. housing $h_{i,t}$; the supply of the latter being normalized to one. Households i receives an endowment of non-durable goods in each period $y_{i,t}$, where $y_{i,t} = y_t$ and y_t denotes aggregate endowment that is exogenously determined with mean one. Households can differ with regard to their marginal valuation of consumption of the non-durable good due to preference shocks $\epsilon_i > 0$, which are i.i.d. across households and time. The instantaneous utility function $u_{i,t}$ is

$$u_{i,t} = u(\epsilon_i, c_{i,t}, h_{i,t}), \tag{1}$$

where $h_{i,t}$ denotes the end-of-period stock of housing. We assume that $u_{i,t}$ is strictly increasing, concave, separable in consumption of non-durables and housing, and satisfies the usual Inada conditions. The idiosyncratic shock ϵ_i exhibits two possible realizations, $\epsilon_i \in \{\epsilon_l, \epsilon_b\}$, with mean one, equal probabilities $\pi_\epsilon = 0.5$, and $\epsilon_l < \epsilon_b$. Households rely on money for purchases of non-durable goods, whereas we treat housing as a "credit good" (see Lucas and Stokey, 1987). They hold money $M^H_{i,t-1}$ at the beginning of each period and they can acquire additional money $I_{i,t}$ from the central bank, for which they rely on eligible assets. Specifically, households can get money $I_{i,t}$ from the central bank in open market operations, where money is supplied in exchange for treasury securities discounted with the policy rate R^m_t in either outright transactions or repos. Hence, acquisition of money is constrained by holdings of government bonds $B_{i,t-1}$,

$$0 \leq I_{i,t} \leq \kappa^B_t B_{i,t-1}/R^m_t, \tag{2}$$

where the central bank sets $\kappa^B_t \in [0, 1]$, which can be interpreted as an allotment rate.[15] In contrast to purchases of private debt, purchases of public debt can affect the allocation only via an increase in the supply of money, while associated effects on treasury rates will be irrelevant for the equilibrium allocation.

[15] Notably, the non-negativity in (2) does not rule out deflationary paths.

When household i draws the realization ϵ_b (ϵ_l), which materializes after treasury open market transactions are conducted, it is willing to consume more (less) than households who draw ϵ_l (ϵ_b). Hence, ϵ_b-type households tend to borrow an additional amount of money from ϵ_l-type households. We assume that borrowing and lending among households only takes place in form of short-term nominal debt at the price $1/R_t^L$. Like Jermann and Quadrini (2012) and Woodford (2016), we assume that loan contracts are signed at the beginning of the period and repaid at the end of each period. The assumption of intraperiod debt greatly simplifies the analysis, while it can be shown that main results also hold for interperiod debt (see Appendix G).

The crucial element of the model is a financial constraint, which can be microfounded as follows by limited commitment and the possibility of debt renegotiation: We assume that borrowers can threaten to repudiate the debt contract and that lenders protect themselves by collateralizing borrowers' housing. Following a repudiation, they can seize a fraction z of the borrower's housing and can sell it at the current price Q_t in the housing market. We consider the case where borrowers have all the bargaining power and are able to negotiate the loan down to the liquidation value zQ_t of their housing (see Hart and Moore, 1994). Lenders take this possibility into account, such that the debt repayment does not exceed the value of the seizable collateral. Hence, debt $-L_{i,t} > 0$ of a borrower i with housing h_i is constrained by

$$-L_{i,t} \leq zQ_t h_{i,t}, \tag{3}$$

where P_t denotes the aggregate price level, Q_t the housing price, and $z \in (0,1)$ the exogenous liquidation share of collateral. For simplicity, we assume that bonds held after open market operations $(1-\kappa_t^B)B_{i,t-1}$ cannot be used as collateral. Allowing for this or for intraperiod bond trades, would just quantitatively affect the results.

As the main object of our analysis, we consider the possibility that the central bank purchases collateralized loans in addition to treasuries: After the preference shocks are realized and while loans are supplied, the central bank offers money in exchange for a

fraction $\kappa_t \in [0,1]$ of loans at the price $1/R_t^m$:

$$0 \leq I_{i,t}^L \leq \kappa_t L_{i,t}/R_t^m. \tag{4}$$

By purchasing loans, the central bank can thus influence lenders' valuation of loans. Notably, a loan price that is set differently from the open market price of bonds ($1/R_t^m$) would not provide an additional degree of freedom, since it just matters in combination with the independent instrument κ_t. Asset purchases ($\kappa_t > 0$) further raise the amount of loanable funds, such that lenders can increase their supply of loans. Loan purchases are conducted in form of repos, where loans are repurchased by lenders before they mature (such that lenders earn the interest on loans). After loans are issued and asset purchases are conducted, the market for non-durables opens. Money is assumed to serve as the means of payment for non-durable goods, for which household i can use money holdings $M_{i,t-1}^H$ as well as new injections $I_{i,t}$ and $I_{i,t}^L$ plus/minus loans:

$$P_t c_{i,t} \leq I_{i,t} + I_{i,t}^L + M_{i,t-1}^H - L_{i,t}/R_t^L. \tag{5}$$

It should be noted that the previous constraints (2)-(5) are affected by various prices, which are taken as given by private agents. Precisely, they do not internalize that their behavior affects the real price of housing $q_t = Q_t/P_t$ (see 3) and of loans $1/R_t^L$ (see 5). These pecuniary externalities matter for the combined liquidity and borrowing constraints, and they relate to the "collateral externalities" in Davila and Korinek (2018). After consumption goods are traded, repurchase agreements are settled, i.e. agents buy back loans and treasuries under repos from the central bank, and transfers are paid. Then, the asset market opens, where households repay intraperiod loans, invest in treasuries, and might trade assets among each other. Thus, the budget constraint of household i is

$$\begin{aligned} &M_{i,t-1}^H + B_{i,t-1} + L_{i,t}\left(1 - 1/R_t^L\right) + P_t y_t + P_t \tau_t \\ &\geq M_{i,t}^H + (B_{i,t}/R_t) + \left(I_{i,t} + I_{i,t}^L\right)(R_t^m - 1) + P_t c_{i,t} + P_t q_t \left(h_{i,t} - h_{i,t-1}\right), \end{aligned} \tag{6}$$

where $1/R_t$ denotes the price of treasuries in period t and $\tau_t \geq 0$ a lump-sum transfer. Maximizing $E\sum_{t=0}^{\infty} \beta^t u_{i,t}$, where the discount factor satisfies $\beta \in (0,1)$, subject to (1)-(6), $M_{i,t-1}^H > 0$, $B_{i,t-1} > 0$, and taking prices as given, leads to the following first order

conditions for non-durables, holdings of treasuries and money, and additional money from treasury open market operations $\forall i \in \{b, l\}$:

$$u'(\epsilon_i, c_{i,t}) = \lambda_{i,t} + \psi_{i,t}, \tag{7}$$

$$\lambda_{i,t} = \beta R_t E_t \left[\left(\lambda_{i,t+1} + \kappa^B_{t+1} \eta_{i,t+1} \right) / \pi_{t+1} \right], \tag{8}$$

$$\lambda_{i,t} = \beta E_t \left[\left(\lambda_{i,t+1} + \psi_{i,t+1} \right) / \pi_{t+1} \right], \tag{9}$$

$$\overline{E}_t \psi_{i,t} = \left(R^m_t - 1 \right) \overline{E}_t \lambda_{i,t} + \overline{E}_t R^m_t \eta_{i,t}, \tag{10}$$

where $\pi_t = P_t / P_{t-1}$ denotes the inflation rate and $\overline{E}_t$ the expectations at the beginning of period t before individual shocks are drawn. Further, $\lambda_{i,t} \geq 0$ is the multiplier on the asset market constraint (6), $\eta_{i,t} \geq 0$ the multiplier on the money supply constraint (2), and $\psi_{i,t} \geq 0$ the multiplier on the cash-in-advance constraint (5), where all constraints are expressed in real terms. Condition (10) reflects that idiosyncratic shocks are not revealed before treasury open market operations are initiated. The following type-specific first order conditions for loans and housing have to be satisfied, for <u>borrowers</u>

$$\lambda_{i,t} \left(1 - 1/R^L_t \right) - \left(\psi_{i,t} / R^L_t \right) + \zeta_{i,t} = 0, \tag{11}$$

$$u'(h_{i,t}) + \zeta_{i,t} z q_t + \beta E_t q_{t+1} \lambda_{i,t+1} - q_t \lambda_{i,t} = 0, \tag{12}$$

and for <u>lenders</u>, where we additionally consider the first order condition for money acquired from loan purchases $I^L_{l,t}$,

$$\lambda_{i,t} \left(1 - 1/R^L_t \right) - \left(\psi_{i,t} / R^L_t \right) + \mu_{i,t} \kappa_t = 0, \tag{13}$$

$$u'(h_{i,t}) + \beta E_t q_{t+1} \lambda_{i,t+1} - q_t \lambda_{i,t} = 0, \tag{14}$$

$$-\lambda_{i,t} \left(1 - 1/R^m_t \right) + \left(\psi_{i,t} / R^m_t \right) - \mu_{i,t} = 0. \tag{15}$$

Condition (15) describes lenders' willingness to sell loans to the central bank. The conditions (11) and (13) further show that the multiplier on the cash-in-advance constraint (5) is positive if the loan rate R^L_t exceeds one, as the latter measures the price of cash goods. Further, the associated complementary slackness conditions,[16] as well as (2)-(5),

[16] The complementary slackness conditions are $\eta_{i,t}[\kappa^B_t b_{i,t-1}(\pi_t R^m_t)^{-1} - i_{i,t}] = 0$, $\zeta_{i,t}\left[z q_t h_{i,t} + l_{i,t}\right] = 0$, $\mu_{i,t}\left[\kappa_t l_{i,t} / R^m_t - i^L_{i,t}\right] = 0$, and $\psi_{i,t}\left[i_{i,t} + i^L_{i,t} + m^H_{i,t-1} - \left(l_{i,t}/R^L_t\right) - c_{i,t}\right] = 0$, where real variables are

(6) as an equality, and the associated transversality conditions hold.

Combining (7) and (9) to $\frac{\psi_{i,t}}{u'(\epsilon_i,c_{i,t})} = 1 - \beta\frac{E_t[u'(\epsilon_i,c_{i,t+1})/\pi_{t+1}]}{u'(\epsilon_i,c_{i,t})}$ shows that the liquidity constraint (5) binds if the nominal marginal rate of intertemporal substitution $\frac{u'(\epsilon_i,c_{i,t})}{\beta E_t(u'(\epsilon_i,c_{i,t+1})/\pi_{t+1})}$ exceeds one. Then, the money supply constraint (4) is binding, $\mu_{i,t} > 0$, implying that lenders are willing to refinance loans at the central bank to the maximum amount. This is the case when $R_t^m < R_t^L$, i.e. when the central bank offers a price for loans $1/R_t^m$ that exceeds the market price $1/R_t^L$. This can be seen from combining (15) with (7), (9), and (18) to

$$\frac{\mu_{i,t}}{u'(\epsilon_i, c_{i,t})} = \frac{1}{1-\kappa}\left(\frac{1}{R_t^m} - \frac{1}{R_t^L}\right). \tag{16}$$

If, however, the policy rate equals the loan rate, $R_t^m = R_t^L$, lenders have no incentive to refinance loans and (4) becomes slack (see 16). The conditions for loan demand (11) and loan supply (15) reveal that the credit market allocation can be affected by the borrowing constraint (for $\zeta_{i,t} > 0$) as well as by loan purchases (for $\mu_{i,t} > 0$). The borrowers' loan demand condition (11) can – by using (7), (9), and (15) – be rewritten as

$$\frac{1}{R_t^L} = \beta\frac{E_t(u'(\epsilon_i, c_{i,t+1})/\pi_{t+1})}{u'(\epsilon_i, c_{i,t})} + \frac{\zeta_{i,t}}{u'(\epsilon_i, c_{i,t})}. \tag{17}$$

Hence, a positive multiplier $\zeta_{i,t}$ tends to raise the RHS of (17), implying a relative increase in current marginal utility of consumption, which can be mitigated by a lower loan rate. Put differently, under a binding borrowing constraint (3) the borrowers' nominal marginal rate of intertemporal substitution exceeds the loan rate R_t^L. Further, the lenders' loan supply condition (13) can – by using (7) and (9) – be written as $\frac{1}{R_t^L} = \kappa_t \cdot \frac{1}{R_t^m} + (1-\kappa_t) \cdot \beta\frac{E_t[u'(\epsilon_i,c_{i,t+1})/\pi_{t+1}]}{u'(\epsilon_i,c_{i,t})}$, which shows that R_t^L depends on the lender's nominal marginal rate of intertemporal substitution as well as on the policy rate R_t^m, if the central bank purchases loans, $\kappa_t > 0$.[17] The loan supply condition can be rewritten as

$$\frac{1}{R_t^L} = \left[\frac{1-\kappa_t}{1-\kappa_t R_t^L/R_t^m}\right] \cdot \beta\frac{E_t(u'(\epsilon_i, c_{i,t+1})/\pi_{t+1})}{u'(\epsilon_i, c_{i,t})}, \tag{18}$$

given by $b_{i,t} = B_{i,t}/P_t$, $l_{i,t} = L_{i,t}/P_t$, $m_{i,t}^H = M_{i,t}^H/P_t$, $i_{i,t} = I_{i,t}/P_t$, $i_{i,t}^L = I_{i,t}^L/P_t$.

[17] Hence, a higher share of purchased loans κ_t for a given policy rate $R_t^m < R_t^L$, or a lower policy rate R_t^m for a given share of purchased loans, $\kappa_t > 0$, tend to reduce the loan rate, while the loan rate approaches the policy rate, $R_t^L \to R_t^m$, for $\kappa_t \to 1$.

which shows that the term in the square brackets drives a policy induced wedge between the effective real returns for borrowers and lenders. Further note that (7), (9), and (10) imply $\frac{\overline{E}_t \eta_{i,t}}{\overline{E}_t u'(\epsilon_i, c_{i,t})} = \frac{1}{R_t^m} - \beta \frac{\overline{E}_t[u'(\epsilon_i, c_{i,t+1})/\pi_{t+1}]}{\overline{E}_t u'(\epsilon_i, c_{i,t})}$, where the term $\frac{\beta \overline{E}_t[u'(\epsilon_i, c_{i,t+1})/\pi_{t+1}]}{\overline{E}_t u'(\epsilon_i, c_{i,t})}$ cannot be larger than the inverse of the loan rate $1/R_t^L$ (see 17 and 18). Thus, a policy rate satisfying $R_t^m < R_t^L$ ensures that money is scarce, such that the liquidity constraint (5) is binding, and that agents liquidate bonds as far as possible, such that the money supply constraint (2) is binding as well as (4). Then, access to money supply will effectively be constrained by the available amount of eligible assets, i.e. bonds and loans, and asset purchases can be non-neutral.

2.3 Public sector

The government issues one-period nominal bonds at the price $1/R_t$ and pays lump-sum transfers $\tau_t \geq 0$ equally to all agents. For our main analysis, we assume that lump-sum taxes ($\tau_t < 0$) are not available. For comparative purposes, we will consider Pigouvian subsidies as alternative corrective policies (in Section 3.2), which rely on type-specific taxes/subsidies and non-distortionary transfers/taxes. As described above, short-term government bonds serve as eligible assets for central bank operations. Hence, sufficiently large holdings of treasuries can in principle support self-insurance against illiquidity risk (see also Woodford, 1990). If interest rates were equal to zero, money holdings would be costless and the first best allocation could be implemented. Yet, non-availability of lump-sum taxes renders implementation of first best impossible (see 2.4). Without loss of generality, we therefore specify the supply of short-term treasuries in an ad-hoc way. Specifically, their total stock B_t^T grows at a rate $\Gamma > 0$,

$$B_t^T = \Gamma B_{t-1}^T, \tag{19}$$

given $B_{-1}^T > 0$.[18] The government further receives seigniorage revenues τ_t^m from the central bank, such that its budget constraint reads $(B_t^T/R_t) + P_t \tau_t^m = B_{t-1}^T + P_t \tau_t$.

The central bank supplies money in open market operations either outright or tem-

[18] Note that the growth rate Γ might affect the long-run inflation rate if the money supply constraint (2) is binding. As shown in Appendix D, the central bank can nonetheless implement a desired inflation target (different from Γ) by suited long-run adjustments of its money supply instruments.

porarily via repos against treasuries, M_t^H and M_t^R, where $I_t = M_t^H - M_{t-1}^H + M_t^R$. It can further increase the supply of money by purchasing loans from lenders, I_t^L, i.e. it supplies money under repos against loans. At the beginning of each period, its holdings of treasuries and the stock of outstanding money are given by B_{t-1}^c and M_{t-1}^H. It then receives treasuries and loans in exchange for money. Before the asset market opens, where the central bank rolls over maturing assets, repos in terms of treasuries and collateralized loans are settled. Hence, its budget constraint reads $(B_t^c/R_t) - B_{t-1}^c + P_t\tau_t^m = R_t^m\left(M_t^H - M_{t-1}^H\right) + \left(R_t^m - 1\right)\left(I_t^L + M_t^R\right)$. It earns interest from money supply as well as asset holdings and it transfers $P_t\tau_t^m$ units of bonds to the government,

$$P_t\tau_t^m = \left(1 - 1/R_t\right)B_t^c + R_t^m\left(M_t^H - M_{t-1}^H\right) + \left(R_t^m - 1\right)\left(I_t^L + M_t^R\right). \tag{20}$$

Central bank asset holdings therefore evolve according to $B_t^c - B_{t-1}^c = M_t^H - M_{t-1}^H$. Further assuming that initial values satisfy $B_{-1}^c = M_{-1}^H$, gives the central bank balance sheet $B_t^c = M_t^H$. The central bank has four instruments at its disposal. It sets the policy rate R_t^m and can decide how much money to supply against treasuries, for which it can adjust $\kappa_t^B \in (0, 1]$. The central bank can further decide whether it supplies money in exchange for treasuries either outright or temporarily via repos. Specifically, it controls the ratio of treasury repos to outright purchases $\Omega_t > 0 : M_t^R = \Omega_t M_t^H$, where a sufficiently large value for Ω_t ensures that injections are always positive, $I_{i,t} > 0$. Finally, it decides on the fraction of loans $\kappa_t \in [0, 1]$ that it is willing to exchange for money under repos.

2.4 Equilibrium properties

In equilibrium, agents' optimal plans are satisfied and prices adjust such that all markets clear: $0 = \sum_i l_{i,t}$, $h = \sum_i h_{i,t}$, $y = \sum_i c_{i,t}$, $m_t^H = \sum_i m_{i,t}^H$, $m_t^R = \sum_i m_{i,t}^R$, $b_t = \sum_i b_{i,t}$, and $b_t^T = b_t^c + b_t$, where $l_{i,t} = L_{i,t}/P_t$, $m_{i,t}^H = M_{i,t}^H/P_t$, $m_t^R = M_t^R/P_t$, $b_{i,t} = B_{i,t}/P_t$, $b_t = B_t/P_t$, $b_t^c = B_t^c/P_t$, and $b_t^T = B_t^T/P_t$. As a reference case, we consider the first best allocation, which maximizes welfare of ex-ante identical agents

$$E\sum_{t=0}^{\infty}\beta^t\sum_i u_{i,t}, \tag{21}$$

s.t. $h = \sum_i h_{i,t}$, and $y = \sum_i c_{i,t}$. Applying the law of large numbers and indexing all agents drawing ϵ_l (ϵ_b) in period t with l (b), we can summarize first best as a set of sequences $\{c^*_{b,t}, c^*_{l,t}, h^*_{b,t}, h^*_{l,t}\}_{t=0}^{\infty}$ satisfying $h^*_{b,t} + h^*_{l,t} = h$, $c^*_{l,t} + c^*_{b,t} = y$,

$$u_c(\epsilon_b, c^*_{b,t}) = u_c(\epsilon_l, c^*_{l,t}), \text{ and } h^*_{b,t} = h^*_{l,t}. \tag{22}$$

Under the first best allocation, the marginal utilities of non-durables and of the end-of-period stock of housing are identical for borrowers and lenders (see 22). This will not be the case in a competitive equilibrium where the borrowing constraint (3) is binding. Only if the equilibrium lending rate R^L_t is equal to one and the supply of eligible assets is sufficiently large, such that money is abundantly available, agents would be able to self-ensure against liquidity risk (see Woodford, 1990). This is however not feasible due to the non-availability of lump-sum taxes. Given that initial public liabilities held by private agents are strictly positive, the public sector relies on a positive present value of revenues to satisfy intertemporal solvency. Without lump-sum taxes, positive interest earnings of the central bank are thus required, ruling out that interest rates are permanently equal to zero. To see this, use (20) to get the public sector budget constraint $B_{t-1} + R^m_t M^H_{t-1} = (R^m_t - 1)\left(I^L_t + M^R_t\right) - P_t\tau_t + (B_t/R_t) + R^m_t M^H_t$ and iterate it forward from $t = 0$ onwards

$$\begin{aligned} 0 &< \left(B_{-1} + R^m_0 M^H_{-1}\right) P_0^{-1} \\ &= E_0 \sum_{t=0}^{\infty} \left(\prod_{m=1}^{t} \frac{\pi_m}{R_{m-1}} \right) \left\{ (R^m_t - 1)\left(i^L_t + \Omega_t m^H_t\right) - \tau_t + \left(R^m_t - R^m_{t+1}/R_t\right) m^H_t \right\}, \end{aligned} \tag{23}$$

where we used that solvency requires $\lim_{t\to\infty} E_0(\Pi^t_{m=1}\frac{\pi_m}{R_{m-1}})\left((b_t/R_t) + R^m_t m^H_t\right) = 0$ and $m^R_t = \Omega_t m^H_t$. First best requires consumption to be unconstrained, such that the multiplier on the liquidity constraint (5) has to equal zero, $\psi_{i,t} = 0$. Then, (9) simplifies to $\lambda_{i,t} = \beta E_t\left[\lambda_{i,t+1}/\pi_{t+1}\right]$ and (8) implies $R_t = 1$ and $\eta_{i,t} = 0$, while (10) can only be satisfied if $R^m_t = 1$. In this case, the term in the curly brackets on the RHS of (23) would reduce to $-\tau_t \leq 0$, whereas the LHS of (23) has to be strictly positive to ensure non-zero initial consumption of non-durables (see 2 and 5 for $t = 0$).[19]

Proposition 1 *The first best allocation cannot be implemented.*

[19] Given Inada conditions, non-durable consumption has to be strictly positive in equilibrium.

In contrast, (23) can be satisfied for a non-negative transfer sequence $\{\tau_t \geq 0\}_{t=0}^{\infty}$ if $R_t^m > 1$, given that $i_t^L \geq 0$, (5) requires $m_t^H > 0$, and (8)-(10) imply $R_t^m - R_{t+1}^m/R_t \geq 0$ $\forall t \geq 0$. To enable government solvency, we introduce the following assumption.

Assumption 1 *Initial public sector liabilities $B_{-1} >$ and $M_{-1}^H > 0$ are sufficiently small, such that (23) is satisfied in any competitive equilibrium with $R_t^m > 1$.*

A definition of a competitive equilibrium under Assumption 1 is given in Appendix A. For asset purchases to be non-neutral, money has to be supplied at a favorable price, i.e. the policy rate has to be set below the lender's marginal rate of intertemporal substitution, implying $R_t^m < R_t^L$. Then, money supply is rationed by the available amount of assets eligible for central bank operations. If, however, the policy rate equals nominal marginal rate of intertemporal substitution, $R_t^m = R_t^L$, or eligible assets are unboundedly available, money supply is non-rationed and asset purchases are neutral (see 16).

3 Analytical results

In this Section, we examine pecuniary externalities and policy effects in an analytical way. In the first part of this Section, we impose two further assumptions to facilitate aggregation. We then characterize the competitive equilibrium in terms of a representative borrower and a representative lender, and we show that conventional monetary policies are neutral. In the second part of this Section, we show how asset purchases can enhance welfare by addressing pecuniary externalities and by easing borrowing conditions. Based on the laissez faire case, we identify a constrained efficient allocation and we show that it can in principle be implemented by a Pigouvian subsidy on borrowing. Then, we show that it can also be implemented by asset purchases. Moreover, asset purchases can implement allocations that welfare-dominate the constrained efficient allocation. In Appendix G, we demonstrate for a real (non-monetary) economy that asset purchases can in principle be mimicked by a combination of Pigouvian subsidies on debt and collateral. The latter rely on taxes that are non-distortionary and type-specific in our economy with heterogenous agents (see also Davila and Korinek, 2018). These types of taxes are however assumed not to be available in our model and are not required for welfare-enhancing central bank asset purchases.

3.1 Aggregation and conventional monetary policy

Here, we examine the case of a conventional monetary policy: The central bank sets the policy rate equal to the loan rate, $R_t^m = R_t^L$, such that both money supply constraints (2) and (4) are not binding ($\eta_{i,t} = \mu_{i,t} = 0$). Even when the central bank were willing to buy loans, lenders would then not gain from selling loans and prices would not be affected. Thus, asset purchases are then irrelevant (see 16). We apply two further assumptions that allow deriving the main results in an analytical way. Firstly, we assume that preferences are given by a linear-quadratic form, which enables aggregation over individual choices of all agents who draw ϵ_l, since all conditions are linear in the agents' choice variables.[20]

Assumption 2 *Instantaneous utility of households satisfies*

$$u(\epsilon_i, c_{i,t}, h_{i,t}) = \epsilon_i(\delta c_{i,t} - (1/2)c_{i,t}^2) + (\gamma h_{i,t} - (1/2)h_{i,t}^2), \tag{24}$$

where $\partial u/\partial c_{i,t} = u'(\epsilon_i, c_{i,t}) > 0$ *and* $\partial u/\partial h_{i,t} = u'(h_{i,t}) > 0$.

Depending on their history of idiosyncratic shocks and thus individual net wealth positions, the borrowing constraint (3) is binding for some borrowers. Their share increases with a larger difference in the agents' valuation of consumption and a lower liquidation value of collateral. To further facilitate aggregation over borrowers, we, secondly, assume that the associated multiplier is strictly positive for all agents drawing ϵ_b, $\zeta_{b,i,t} > 0$, which can be guaranteed by a sufficiently large difference in agents' valuation of consumption relative to the liquidation value of collateral, $(\epsilon_b - \epsilon_l)/z$.

Assumption 3 *The ratio* $(\epsilon_b - \epsilon_l)/z$ *is sufficiently large such that the borrowing constraint (3) is binding for all agents drawing* ϵ_b.

Now, let $x_{l,t} = \sum_{l,i} x_{l,i,t}$ ($x_{b,t} = \sum_{b,i} x_{b,i,t}$) be the value of any generic variable $x_{l,t}$ ($x_{b,t}$) aggregated over all agents drawing ϵ_l (ϵ_b) in period t. Using Assumptions 1-3 and the law of large numbers, we can aggregate over all lenders (agents drawing ϵ_l) and borrowers (agents drawing ϵ_l) and characterize a competitive equilibrium for $1 < R_t^m$ in terms of a representative borrower and a representative lender.

[20] The main results on policy interventions are confirmed for alternative (CRRA) preferences in Section 4. Aggregation will then be enabled by pooling funds within households at the end of each period.

Proposition 2 *A competitive equilibrium under a conventional monetary policy can be characterized as a set of sequences* $\{c_{b,t},\, c_{l,t},\, h_{b,t},\, q_t,\, \pi_t\}_{t=0}^{\infty}$ *satisfying*

$$\epsilon_l(\delta - c_{l,t}) = \beta 0.5 E_t \left[(\epsilon_l(\delta - c_{l,t+1}) + \epsilon_b(\delta - c_{b,t+1})) \left\{ R_t^L/\pi_{t+1} \right\} \right], \tag{25}$$

$$\left\{ R_t^L/q_t \right\} (2h_{b,t} - h)/z = \epsilon_b(\delta - c_{b,t}) - \beta 0.5 E_t[(\epsilon_b(\delta - c_{b,t+1}) + \epsilon_l(\delta - c_{l,t+1})) \left\{ R_t^L/\pi_{t+1} \right\}], \tag{26}$$

$$\epsilon_l(\delta - c_{l,t}) \left\{ q_t/R_t^L \right\} = \gamma - (h - h_{b,t}) + \beta E_t[\epsilon_l(\delta - c_{l,t+1}) \left\{ q_{t+1}/R_{t+1}^L \right\}], \tag{27}$$

$$c_{b,t} - c_{l,t} = z h_{b,t} 2 \left\{ q_t/R_t^L \right\}, \tag{28}$$

$$y_t = c_{b,t} + c_{l,t}, \tag{29}$$

and $R_t^L = R_t^m > 1$*, for* $\{y_t\}_{t=0}^{\infty}$ *and a sequence* $\{R_t^m \geq 1\}_{t=0}^{\infty}$ *set by the central bank.*

Proof. See Appendix B. ■

Notably, cash holdings and loans rather than individual net wealth positions are relevant for agents' consumption and housing choices. Thus, the combined cash-in-advance and collateral constraint will be crucial for consumption and end-of-period housing decisions of the representative borrower and the representative lender (see 28). A central element is therefore the relative price of collateral in terms of the cash good, q/R^L. Further note that the multiplier on the borrowing constraint satisfies

$$\zeta_{b,t} = [\epsilon_b(\delta - c_{b,t}) - \epsilon_l(\delta - c_{l,t})]/R_t^L = (2h_{b,t} - h)/(zq_t) > 0, \tag{30}$$

indicating that both, the housing and the consumption choice (that would ideally satisfy $h_b = h_l$ and $\epsilon_l(\delta - c_{b,t}) = \epsilon_l(\delta - c_{l,t})$, see 22), are distorted by a binding borrowing constraint ($\zeta_{b,t} > 0$). On the one hand, the marginal utility of consumption is then larger for borrowers than for lenders, $\epsilon_b(\delta - c_{b,t}) > \epsilon_l(\delta - c_{l,t})$. On the other hand, borrowers' housing exceeds lenders' housing, $h_{b,t} > h/2$, as the former is characterized by a relatively higher valuation of housing due to its ability to serve as collateral.

Proposition 2 reveals that the nominal interest rate and thus the policy rate matter jointly with either the housing price or the inflation rate. Precisely, the conditions (25)-(29) impose restrictions on the allocation, $c_{b,t}, c_{l,t}$, and $h_{b,t}$, the relative price of housing q_t/R_t^L, and the real interest rate R_t^L/π_{t+1} (see curly brackets in 25-28), but not separately on q_t, π_t, and R_t^L. Thus, a conventional monetary policy measure, i.e. a change in the policy rate $R_t^m = R_t^L$, leaves relative prices and the allocation unaffected, while they

affect inflation and the housing price.[21]

Corollary 1 *Under a conventional monetary policy regime, changes in the monetary policy rate do neither affect relative prices nor the equilibrium allocation, while the housing price and the inflation rate increase with the nominal interest rate.*

The reason for the neutrality summarized in Corollary 1 is that conventional monetary policies can only affect equilibrium prices that are equally relevant for both agents, while the aggregate supply of durable and non-durable goods is exogenously determined. Given that conventional monetary policy is neutral, we summarize this case as laissez faire.

3.2 Pecuniary externalities and constrained efficiency

For the remainder of this section, we focus on long-run equilibria. Variables without time-indices denote long-run values. We aim at examining efficiency gains that can be reaped by a social planer, who takes into account how prices that are relevant for financial constraints are affected by agents' decisions. Condition (27) implies the price of housing relative to non-durables q/R_L to be negatively related to lenders' housing $h - h_b$ and positively related to lenders' consumption c_l,

$$\frac{q}{R^L} = \frac{\gamma - (h - h_b)}{(1-\beta)\epsilon_l(\delta - c_l)}. \tag{31}$$

An increase in the relative price q/R^L tends to raise the difference between consumption of borrowers and lenders (see 28), since it increases borrowers' consumption possibilities by raising the price of collateral in terms of consumption. Yet, the impact of the demand for housing and consumption on the relative price q/R^L is not internalized by individuals, giving rise to inefficiencies induced by pecuniary externalities. Notably, the latter correspond to Davila and Korinek's (2018) "collateral externalities". In contrast, "distributional externalities" are not relevant for the consumption and housing choices of the representative borrower and the representative lender, due to the irrelevance of individual net wealth positions under Assumptions 1-3.

For example, borrowers do not internalize that an increase in their housing (thus a decrease in lenders' housing) tends to increase the relative price q/R^L as lenders' willingness

[21] The latter effect is due to the liquidity constraint and the well-known inflation tax on cash goods, implying that higher interest rates reduce the demand for non-durables and raise housing demand.

to pay for housing increases. Hence, there exists an uninternalized positive feedback loop between collateral demand, prices, and borrowing, as in Bianchi (2011), Stein (2012), or Bianchi and Mendoza (2018). Notably, these studies focus on prudential financial regulation and restrict their attention on financial constraints that are only binding in crisis states. They show that ex-ante interventions should reduce debt to mitigate adverse deleveraging effects. Considering regularly binding constraints (like Kiyotaki and Moore, 1997), we instead examine ex-post interventions that are introduced while borrowers are constrained. In these cases, policies should support borrowing, as for example shown by Bianchi (2016) and Jeanne and Korinek (2016) for bailouts and borrowing subsidies.

We consider a well-established benchmark case for constrained (long-run) efficiency. As in Bianchi and Mendoza (2018), the social planer chooses a welfare-maximizing allocation subject to the laissez faire equilibrium conditions (see Proposition 2), where it chooses debt on the behalf of private agents (such that 25 and 26 are irrelevant).

Definition 1 *The constrained efficient long-run allocation solves the problem*

$$\max_{c_l, c_b, h_b} \{u(\epsilon_b, c_b, h_b) + u(\epsilon_l, c_l, h - h_b)\}/(1-\beta), \tag{32}$$
$$s.t. \quad y = c_l + c_b,\ c_b - c_l \leq 2zh_b \cdot \left(q/R^L\right),\ \textit{and (31).}$$

For comparative purposes, we consider a widely-applied policy to correct for pecuniary externalities induced by the financial constraints, namely, a Pigouvian debt tax/subsidy. In contrast to single agent (open economy) models (e.g., Benigno et al., 2016, Bianchi and Mendoza, 2018), this type of corrective policy has to be type-specific in a model with heterogeneous agents, which has also shown by Davila and Korinek (2018). Specifically, suppose there exists a tax/subsidy on debt $\tau^L \gtreqless 0$ and a lump-sum transfer/tax for borrowers $\tau^R_{b,t} = \tau^L l_t / R^L_t$. Then, the borrower's effective real interest rate $r^\tau_{b,t}$ is

$$r^\tau_{b,t} = \frac{R^L_t/\pi_{t+1}}{1-\tau^L}, \tag{33}$$

and the borrower's loan price net of taxes is $(1-\tau^L)/R^L_t$. Notably, the rate τ^L alters the budget constraint (6) and the cash-in-advance constraint of borrowers (5), which has to be the case for the compensating lump-sum transfer/tax as well.[22] Then, condition (28)

[22] Concretely, the representative borrower's cash-in-advance constraint then changes to $P_t c_{b,t} \leq I_{b,t} +$

remains unchanged, while the marginal costs of borrowing are altered by τ^L entering (26). In the long-run equilibrium under the Pigouvian tax/subsidy, the borrowing condition (26) reads $(1-\tau^L)\epsilon_b(\delta - c_b) = \beta 0.5[(\epsilon_b(\delta - c_b) + \epsilon_l(\delta - c_l))\left\{R^L/\pi\right\} + \left\{R^L/q\right\}(2h_b - h)/z$. Using condition (31) to substitute out q/R^L in the latter and in (28), and (25) to substitute out the real interest rate, yields

$$(1-\tau^L)\epsilon_b(\delta - c_b) - \epsilon_l(\delta - c_l) = (2h_b - h)\frac{(1-\beta)\epsilon_l(\delta - c_l)}{z(\gamma - h + h_b)}, \tag{34}$$

which determines the associated tax rate τ^L for a particular allocation. The following proposition describes the implementation of the constrained efficient allocation.

Proposition 3 *Suppose that type-specific and non-distortionary taxes are available. Then, the constrained efficient long-run allocation can be implemented by a Pigouvian subsidy on borrowing ($\tau^L < 0$) if but not only if $z/(1-\beta) \geq 1$. Compared to laissez-faire ($\tau^L = 0$), the Pigouvian subsidy raises (reduces) borrowers' (lenders') durables as well as non-durables and raises the real interest rate R^L/π.*

Proof. See Appendix C. ■

Proposition 3 implies that a corrective policy that stimulates borrowing can enhance social welfare if $z/(1-\beta) \geq 1$. This condition is sufficient to ensure that the relative price of collateral increases with borrowers' expenditures, which corresponds to condition 1 in Davila and Korinek's (2018). An increase in borrowing, which is induced by a Pigouvian subsidy, tends to increase borrowers' consumption and has to be supported by a larger stock of borrowers' housing. As implied by (31) and $c_l = y - c_b$, the price of housing in terms of consumption, q/R^L, increases with the latter, $\partial(q/R^L)/\partial h_b > 0$, and decreases with the former, $\partial(q/R^L)/\partial c_b < 0$. The impact of housing demand on the relative price q/R^L dominates under the following condition (see proof of Proposition 3)

$$\frac{(1-\tau^L)u_{c_b} - u_{c_l}}{u_{c_b} - u_{c_l}} = \frac{1 + \left[\partial(q/R^L)/\partial h_b\right] \cdot h_b/(q/R^L)}{1 + z h_b \cdot \left[\partial(q/R^L)/\partial c_l\right]} > 1, \tag{35}$$

where $\partial(q/R^L)/\partial c_l = u_{h_l}(-u_{c_l c_l}/u_{cl}^2)(1-\beta)^{-1} > 0$ and $\partial(q/R^L)/\partial h_b = (-u_{h_l h_l}/u_{c_l}) \cdot (1-\beta)^{-1} > 0$. Given that housing is a durable good, permanent adjustments in housing demand are associated with relatively large price changes, i.e. amplified by the multiplier

$M_{b,t-1}^H - L_{b,t}[(1-\tau^L)/R_t^L] - P_t\tau_{b,t}^R$, while lenders are not taxed/subsidized.

$1/(1-\beta)$, such that condition (35) is ensured by $z/(1-\beta) \geq 1$ with $z \in (0,1)$. This condition is likely to be satisfied by reasonable values for the parameters β and z (see Section 4.1). Then, borrowing is (constrained) inefficiently low under laissez faire, given that the private agents do not internalize the favorable effects of increased housing demand on the relative price q/R^L. The social planer can then correct for this pecuniary externality by a Pigouvian subsidy, which causes agents to borrow more and leads to an increase in borrowers' consumption and housing (see Proposition 3). Notably, the subsidy tends to reduce borrowing costs r_b^τ (see 33), while it raises the real interest rate R^L/π, causing an increase in loan supply and a decline in lenders' consumption and housing.

3.3 Welfare-enhancing asset purchases

We now turn to central bank purchases of collateralized loans, $\kappa_t > 0$ (see 4). Firstly, we will show that an asset purchase policy can be equivalent to a Pigouvian subsidy on debt that implements the constrained efficient allocation, by driving a wedge between the relevant real returns for borrowers and lenders (see 18). Thus, asset purchases can enhance social welfare by addressing pecuniary externalities, which is not possible under a conventional monetary policy regime (see Corollary 1). Secondly, we will show that asset purchases can even implement allocations that welfare-dominate the constrained efficient allocation by increasing the relative price of collateral.[23]

The central bank has to offer an above-market price for loan purchases to affect relevant prices and the equilibrium allocation. Thus, it has to set the policy rate below the market loan rate $R_t^m < R_t^L$, implying that money supply is effectively rationed by holdings of eligible collateral. Concretely, the money supply constraints in terms of treasuries (2) and collateralized loans (4) are binding (see 2.2),

$$i_{i,t} = \kappa_t^B 0.5 b_{t-1}/(\pi_t R_t^m) \text{ for } i \in \{l,b\} \quad \text{and} \quad i_{l,t}^L = \kappa_t l_t / R_t^m. \tag{36}$$

If, $R_t^m < R_t^L$, purchases of loans drive a wedge between the borrowers' and the lenders' effective real interest (loan) rate, such that the pecuniary externalities discussed in Section 3.2 can be addressed like with the Pigouvian debt subsidy. Concretely, under an asset

[23] In Appendix G, we show that this effect can in principle also be generated by a housing subsidy.

purchase regime, the effective real return for a lender $r_{l,t}^{ap}$ is distorted by the wedge $\frac{1-\kappa_t}{1-\kappa_t R_t^L/R_t^m} \geq 1$ (see 18), which increases with the fraction of purchased loans κ_t and with the price discount R_t^L/R_t^m. In contrast, the borrowers' real interest rate $r_{b,t}^{ap}$ is not directly affected by the policy instruments:

$$r_{b,t}^{ap} = \frac{R_t^L}{\pi_{t+1}} \text{ and } r_{l,t}^{ap} = \frac{R_t^L}{\pi_{t+1}} \cdot \frac{1-\kappa_t}{1-\kappa_t R_t^L/R_t^m}. \tag{37}$$

Moreover, by purchasing eligible assets the central bank increases the amount of loanable funds. Using (36) to rewrite the liquidity constraints (5) and taking differences yields

$$c_{b,t} - c_{l,t} = z h_{b,t} \left[2 - \kappa_t R_t^L/R_t^m\right] \left\{q_t/R_t^L\right\}, \tag{38}$$

where we substituted out loans with the binding borrowing constraint (78). Comparing (28) with (38), suggests that asset purchases rather reduce than raise the difference between borrowers' and lenders' consumption relative to borrowers' housing. This relates to the direct effect of loan purchases, endowing lenders with additional funds. Yet, there is an opposed indirect (price) effect, as the higher effective return for lenders increases their willingness to pay for housing relative to non-durables. Hence, to identify the impact on the equilibrium allocation, changes in the relative price q_t/R_t^L are decisive.

Applying Assumptions 1-3 and the same aggregation procedure as in Section 3.1, using (19), $B_t^c = M_t^H$, and $B_t^T = B_t^c + B_t$, we can characterize an equilibrium under money rationing, $R_t^m \in (1, R_t^L)$, in terms of a representative borrower and a representative lender.

Proposition 4 *A competitive equilibrium under money rationing can be characterized as a set of sequences* $\{c_{b,t}, c_{l,t}, h_{b,t}, m_t^H, b_t, q_t, R_t^L, \pi_t\}_{t=0}^{\infty}$ *satisfying (26), (29), (38),*

$$\begin{aligned}
\epsilon_l(\delta - c_{l,t}) &= \beta 0.5 E_t[(\epsilon_b(\delta - c_{b,t+1}) + \epsilon_l(\delta - c_{l,t+1}))(R_t^L/\pi_{t+1})] \frac{1-\kappa}{1-\kappa R_t^L/R_t^m}, && (39)\\
q_t \beta E_t &\left[0.5(\epsilon_l(\delta - c_{l,t+1}) + \epsilon_b(\delta - c_{b,t+1}))/\pi_{t+1}\right] && (40)\\
&= \gamma - (h - h_{b,t}) + \beta^2 E_t q_{t+1} \left[0.5(\epsilon_l(\delta - c_{l,t+2}) + \epsilon_b(\delta - c_{b,t+2}))/\pi_{t+2}\right], \\
c_{b,t} &= 0.5(1 + \Omega_t) m_t^H + z q_t h_{b,t}/R_t^L && (41)\\
(1 + \Omega_t) m_t^H &= \kappa_t^B b_{t-1} \pi_t^{-1}/R_t^m + m_{t-1}^H \pi_t^{-1}, && (42)\\
b_t + m_t^H &= \Gamma \left(b_{t-1} + m_{t-1}^H\right)/\pi_t, && (43)
\end{aligned}$$

and the transversality conditions, for $\{y_t\}_{t=0}^{\infty}$ *and sequences* $\{0 \leq \kappa_t < R_t^m/R_t^L,\ \kappa_t^B > 0,\ \Omega_t > 0,\ 1 < R_t^m < R_t^L\}_{t=0}^{\infty}$*, given* $P_{-1} > 0$*,* $M_{-1}^H > 0$*, and* $B_{-1} > 0$*.*

Evidently, there are more instruments available for the central bank when it supplies money in a rationed way compared to a conventional monetary policy regime. In fact, the fraction of eligible bonds κ_t^B and the repo share Ω_t can be adjusted to support a particular competitive equilibrium, such that (41)-(43) are always satisfied. Given that only these three conditions impose restrictions on the sequences $\{b_t,\, m_t^H,\, \kappa_t^B,\, \Omega_t\}_{t=0}^{\infty}$, there is one degree of freedom left for the policy instrument κ_t^B or Ω_t. Notably, the long-run inflation rate π in principle depends on the growth rate of treasuries Γ (see 43). Yet, the central bank can implement a desired inflation rate by suited adjustments of its instruments κ_t^B and Ω_t, as shown in Appendix D. By setting κ_t^B (or Ω_t) accordingly, the central bank can actually use inflation as a choice variable, while (41)-(43) can be ensured to be satisfied by suited values for Ω_t (or κ_t^B). Under money rationing, the central bank therefore has three instruments at its disposal, namely, the inflation rate π_t, the policy rate R_t^m, and the share of purchased loans κ_t.

Implementation of the constrained efficient allocation A competitive equilibrium under money rationing, as given in Proposition 4, can further be summarized by a set of sequences $\{c_{b,t},\, c_{l,t},\, h_{b,t},\, q_t,\, R_t^L\}_{t=0}^{\infty}$ satisfying (26), (29), (38)-(40) for a monetary policy setting $\{\kappa_t,\, R_t^m,\, \pi_t\}_{t=0}^{\infty}$. Notably, inflation only affects (26) and (39), where it appears jointly with the loan rate to measure the real interest rate R_t^L/π_{t+1}. For convenience, we introduce the price discount s_t, as the ratio between the loan rate and the policy rate

$$s_t = R_t^L/R_t^m \geq 1,$$

which can be used as a choice variable (instead of R^m). Substituting out q_t and R_t^L/π_{t+1} with (39) and (40) in (26) and (38), a long-run competitive equilibrium under money rationing can be reduced to a set $\{c_b, c_l, h_b\}$ satisfying $y = c_l + c_b$,

$$\left[\frac{1-\kappa}{1-\kappa s}\right] u_{c_b} = u_{c_l}\left(1 + (1-\beta)\left(u_{h_l} - u_{h_b}\right)/(z u_{h_l})\right), \tag{44}$$

$$\frac{c_b - c_l}{h_b} = \frac{z}{1-\beta}\frac{u_{h_l}}{u_{c_l}} \cdot \left[\frac{(1-\kappa)(2-\kappa s)}{(1-\kappa s)}\right], \tag{45}$$

(where $u_{c_b} = \epsilon_b(\delta - c_b)$, $u_{c_l} = \epsilon_l(\delta - c_l)$, $u_{h_l} = \gamma - (h - h_b)$, and $u_{h_b} = \gamma - h_b$) given the policy instruments $\kappa \in (0, 1/s)$ and $s = R^L/R^m > 1$. For a given allocation $\{c_b, c_l, h_b\}$

and policy $\{\kappa, s\}$, the borrowers' real rate r_b^{ap} is determined by (25).[24] Notably, the instruments, κ and s, affect the RHS of (45) not only by altering the supply of money (see 38), but also via the relative price q/R^L. The reason is that asset purchases raise the lenders' return on loans and, thus, their willingness to pay for housing relative to non-durables, which can be seen from combining (39) and (40):

$$\frac{q}{R^L} = \frac{u_{h_l}}{(1-\beta)u_{c_l}} \cdot \frac{1-\kappa}{1-\kappa s}. \tag{46}$$

We can now easily show that the constrained efficient allocation can be implemented by a suited asset purchase policy $\{\kappa, s\}$: The corresponding conditions under the Pigouvian subsidy with $\widetilde{\tau}^L < 0$ that implements the constrained efficient allocation are $y = c_l + c_b$,

$$\left[1-\widetilde{\tau}^L\right] u_{c_b} = u_{c_l}\left(1 + (1-\beta)\left(u_{h_l} - u_{h_b}\right)/(z u_{h_l})\right), \tag{47}$$

$$\frac{c_b - c_l}{h_b} = \frac{z}{1-\beta}\frac{u_{h_l}}{u_{c_l}} \cdot [2], \tag{48}$$

where (47) stems from (34), and (48) from combining (28) and (31). The comparison of the terms in square brackets in (44)-(45) with the corresponding terms in (47)-(48) immediately reveals that the implementation of the constrained efficient allocation under the Pigouvian debt subsidy requires both monetary policy instruments, s and κ, to simultaneously satisfy $\frac{1-\kappa}{1-\kappa s} = 1 - \widetilde{\tau}^L$ and $\frac{(1-\kappa)(2-\kappa s)}{(1-\kappa s)} = 2$, and therefore

$$\kappa = -\widetilde{\tau}^L > 0 \text{ and } s = 2/(1-\widetilde{\tau}^L). \tag{49}$$

Thus, the central bank can replicate the Pigouvian subsidy by purchasing loans up to a fraction κ that equals the subsidy rate and by offering a price $1/R^m = (1/R^L) \cdot 2/(1+\kappa)$ (see 49), which is feasible for $\kappa \in (0, 1/s)$ and $s > 1$.

Proposition 5 *Suppose that money supply is rationed and $z \geq 1-\beta$. Then, the constrained efficient long-run allocation can be implemented by the central bank via asset purchases.*

As shown above, the central bank can implement the constrained efficient allocation under the Pigouvian subsidy by purchasing loans, such that its instruments κ and s

[24] Inflation, which induces changes in the nominal rate R^L, can still be chosen by the central bank.

satisfy (49).[25] As in the case of the Pigouvian subsidy $\widetilde{\tau}^L < 0$, asset purchases tend to reduce the equilibrium real loan rate R^L/π, which is the relevant rate for borrowers (see 37), compared to laissez faire (see also Section 4.2). At the same time the lender's effective real rate r_l^{ap} increases, such that the representative borrower (lender) consumes more (less) than under laissez faire (see Proposition 3).

Further enhancement of social welfare An asset purchase policy can, however, not only affect the real interest rates of borrowers and lenders, r_b^{ap} and r_l^{ap}, in different ways (like the Pigouvian debt subsidy), but can further relax borrowing conditions. Nonetheless, first best cannot be implemented with an asset purchase policy,[26] because an increase in borrowing has to be associated with a higher value of collateral. However, an asset purchase policy can implement allocations that welfare-dominate the constrained efficient allocation. Importantly, the two instruments $\{\kappa, s\}$ do not affect the private sector behavior in identical ways (see 44 and 45). A higher price discount s (either induced by a lower policy rate R^m or a higher inflation rate π) increases the amount of money supplied per loan, whereas a higher κ increases the fraction of purchased loans. To enhance social welfare compared to the constrained efficient allocation, the central bank can act as follows: On the one hand, it can ease the constraint imposed on the consumption differential relative to borrowers' housing $(c_b - c_l)/h_b$ (see 45) compared to the Pigouvian debt subsidy (48) by setting $\{\kappa, s\}$ to satisfy

$$\frac{(1-\kappa)(2-\kappa s)}{(1-\kappa s)} > 2 \tag{50}$$

$\Leftrightarrow s > 2/(1+\kappa)$. Given that the borrowing constraint is effectively relaxed under (50), the consumption differential relative to borrowers' housing (which is inefficiently small), can be increased compared to the constrained efficient allocation. On the other hand, any change in the instruments also affects the wedge $\frac{1-\kappa}{1-\kappa s}$ between the effective real returns for lenders and borrowers (see 26 and 39). Hence, the central bank can use two distinct

[25] In fact, the Pigouvian subsidy directly alters the effective borrowers' real interest rate r_b^τ (see 33), whereas asset purchases directly change the effective lenders' real interest rate r_l^{ap} (see 37).

[26] To see this, recall that under first best $u_{c_b} = u_{c_l}$ holds (see 22). Condition (44) would in this case imply $\frac{1-\kappa}{1-\kappa s} > 1$ to equal $1 + \frac{1-\beta}{z}\frac{u_{h_l} - u_{h_b}}{u_{h_l}}$ and thus $u_{h_l} > u_{h_b} \Rightarrow h_b > h_l$, which violates the second requirement for a first best allocation (see 22).

channels for its two instrument $\{\kappa, s\}$. Specifically, it can relax borrowing conditions by ensuring (50) and simultaneously steer relative prices in an efficient way (considering 46), which includes addressing the pecuniary externality associated with the borrowing constraint. The term $\frac{(1-\kappa)(2-\kappa s)}{1-\kappa s}$ that alters the tightness of the borrowing constraint (45) is monotonically increasing in s, but not in κ. Hence, an optimal choice of both instruments would be associated with an infinitely large value for s, while κ has to be adjusted to avoid adverse effects of price distortions (see 44).[27] We therefore examine the problem for a given value for the price discount s and assess the optimal fraction of purchased loans κ. By treating one instrument (s) as given, the optimal policy problem is then analogous to the problem in (32). Concretely, to manipulate relative prices in an efficient way, the choice for κ for a given s has to satisfy

$$\frac{\frac{1-\kappa}{1-\kappa s} \cdot u_{c_b} - u_{c_l}}{u_{c_b} - u_{c_l}} = \frac{1 + h_b \left(-u_{h_l h_l}\right) / u_{h_l}}{1 + \frac{1}{1-\beta} h_b u_{h_l} \left(-u_{c_l c_l}\right) u_{c_l}^{-2} \cdot \frac{z}{2} \frac{(1-\kappa)(2-\kappa s)}{1-\kappa s}}, \tag{51}$$

which corresponds to the condition for the optimal borrowing subsidy (35). Hence, an asset purchase policy $\kappa \in (0, 1/s)$ and $s > 1$ that satisfies (50) and (51) implements an allocation that welfare-dominates the constrained efficient allocation.

Proposition 6 *Suppose that money supply is rationed. Then, the central bank can implement long-run allocations via asset purchases that welfare-dominate the constrained efficient long-run allocation.*

When the central bank raises the term $\frac{(1-\kappa)(2-\kappa s)}{(1-\kappa s)}$ (above 2) via a larger price discount s than implied by (49), it relaxes the borrowing constraint (see 45). To avoid distorting relative price (see 46), the wedge $\frac{1-\kappa}{1-\kappa s}$ then has to take a lower value (see 51). Social welfare can thus be enhanced compared to the constrained efficient allocation, by increasing s and reducing κ compared to (49). Notably, this welfare improvement could equivalently be induced by non-monetary instruments that influences the price q/R^L. In Appendix G, we set up a real (non-monetary) economy and show that the total effects of asset purchases can in principle also be generated by a combination of Pigouvian subsidies on debt and housing, for which type-specific and non-distortionary taxes are necessary. In contrast, asset purchases do not require the latter fiscal instruments.

[27] This is confirmed below in the numerical analysis (see Section 4.2).

4 Numerical results

In this section, we provide numerical examples illustrating the analytical results as well as results for the short-run. To facilitate the calibration of the model, we introduce a more standard (CRRA) utility function. Applying such a utility function, however, implies that we cannot easily aggregate over individual households as in Section 3. To simplify the analysis of financial market interventions, we abstract from implications of an endogenous distribution of agents' net wealth. For this, we assume that funds are pooled within households at the end of each period, such that household members are identical at the beginning of each period before they split up into borrowers and lenders. An equilibrium in terms of a representative borrower and a representative lender then only differs from the previous version by non-linear – instead of linear – marginal utilities. We further introduce aggregate risk via a random aggregate endowment and demonstrate that asset purchases should be conducted in a countercyclical way.

4.1 A version with CRRA preferences

We consider infinitely many households of measure one, which consist of infinitely many members i. In each period, ex-ante identical household members draw the idiosyncratic preference shock, which induces some members to borrow and others to lend. Like Lucas and Stokey (1987) or Woodford (2016), we assume that at the end of each period (after loans are repaid) household members obtain equal shares of total household wealth, such that they are again equally endowed before new preference shocks are drawn in the next period. Thus, aggregation is facilitated by a redistribution of wealth within each household (rather than by linearity of agents' behavioral relations). We assume that period utility of household member i is given by a separable CRRA utility function

$$u^{CRRA}(\epsilon_i, c_{i,t}, h_{i,t}) = \epsilon_i \frac{c_{i,t}^{1-\sigma} - 1}{1-\sigma} + \gamma \frac{h_{i,t}^{1-\sigma} - 1}{1-\sigma}, \text{ where } \gamma, \sigma > 0, \tag{52}$$

and $i \in \{b, l\}$, such that Assumption 2 (and thus 24) does not apply. We further allow for aggregate risk in form of a random process for y_t, which will be used in Section 4.3.

We assume that log aggregate endowment with non-durables follows an AR1 process

$$\log y_t = \rho \log y_{t-1} + \varepsilon_{y,t}. \tag{53}$$

Otherwise, the model presented in Section 2 is unchanged, such that the competitive equilibrium in terms of a representative borrower and a representative lender is identical to those given in Propositions 2 and 4, except for marginal utilities being non-linear in this version (see Definition 3 in Appendix E). As in the case of linear-quadratic preferences, it can be shown that a Pigouvian subsidy on debt, $\tau^L < 0$ implements a constrained efficient allocation (see Appendix F). The latter can also be implemented via asset purchases, given that Proposition 5 apparently holds for both types of preferences (see 44-48).

Calibration To solve the model numerically, we have to assign values for the elasticity of intertemporal substitution σ, the discount factor β, the utility weight for housing γ, the liquidation value of collateral z, the degree of household heterogeneity $\Delta\epsilon = \epsilon_b - \epsilon_l$, the autocorrelation coefficient ρ of the AR1 process, and the standard deviation of the innovations σ_ε. We further have to assign values to the growth rate of treasuries Γ and for the repo share Ω. Given that both are not relevant for the equilibrium allocation (see Appendix D), we apply the values $\Gamma = \pi$ and $\Omega = 1$, for convenience. We interpret a model period as one year and calibrate the model consistent with postwar US data. We estimate the process (53) using (linearly detrended) annual US data for real gdp per capita (for 1947-2008), leading to $\rho = 0.752$ and $\sigma_\varepsilon = 0.0216$. The inverse of the elasticity of intertemporal substitution σ is set equal to 2, which is a typical value applied in business cycle studies. The constant liquidation value of collateral z is set equal to 0.55, which is similar to values applied in related studies (see Iacoviello, 2005).

For the remaining three parameters, β, γ, and $\Delta\epsilon$, we apply values that allow to match three targets for the reference case without financial market interventions. Notably, the data samples are not aligned due to limited availability. The first target is the mean share of installment loans to income of 21% (1998-2004, Survey of Consumer Finances), which correspond to the specification in our model, where loans are demanded for consumption rather than for housing. The second target is the mean yield on MBS of 6.6% for pre-2009 US data, taken from Hancock and Passmore (2011), which corresponds to the rate on

collateralized loans $R^L - 1$. The third target is the cross sectional standard deviation of real log consumption of 0.64 (see De Giorgi and Gambetti, 2012). While it is not possible to exactly match all three targets, our choice $\beta = 0.8$, $\gamma = 0.002$, and $\Delta\epsilon = 0.76$ yields to a reasonable match given by an interest rate on collateralized debt 1.06, a loan to income share of 0.2, and a standard deviation of real log consumption of 0.6.

4.2 Welfare gains of asset purchases without aggregate risk

We first consider the case without aggregate risk ($\sigma_\varepsilon = 0$) and compute the equilibrium allocation and associated prices for different policy regimes. As a reference case, we consider a laissez-faire regime, i.e. where monetary policy is conducted in a conventional way (and thus neutral, see Corollary 1) and no corrective policy is applied. Figure 1 shows how asset purchases, which require monetary policy instruments to satisfy $\kappa \in (0, 1/s)$ and $s > 1$, affect relative prices and the equilibrium allocation. The effects are computed for a range of values $\kappa \in (0, 0.75)$ and $s \in (1, 1.2)$, while inflation is fixed at a relatively high value that guarantees a positive equilibrium loan rate ($\pi - 1 = 7\%$). All variables are expressed in terms of percentage deviations from their corresponding laissez faire values.

Higher values for the share of purchased loans κ as well as a larger price discount s increase the effects of the central bank intervention. As revealed in the first row of Figure 1, higher values for κ and s reduce the real rate for borrowers r_b, while they tend to raise the real rate for lender r_l. Notably, the latter is not the case for combinations of a low s and a high κ, where lenders receive a large amount of money at a small discount, such that they tend to consume more and their real rate r_l is below the laissez faire case (< 0). Overall, the price effects of the policy interventions can be substantial, in particular for the relative housing price q/R^L, which can be more than twice as large as in the laissez faire case for a combination of high κ and high s. Consistently, borrowers' consumption of non-durables also tend to increase in these cases, except for low values of s and κ (see above). The effects on borrowers' housing reveal that a larger share of purchased loans κ tend to raise collateral demand, which is not generally the case for a larger discount s, where the central bank supplies more money per loan unit.

The last two rows of Figure 1 present the welfare effects of asset purchases. For

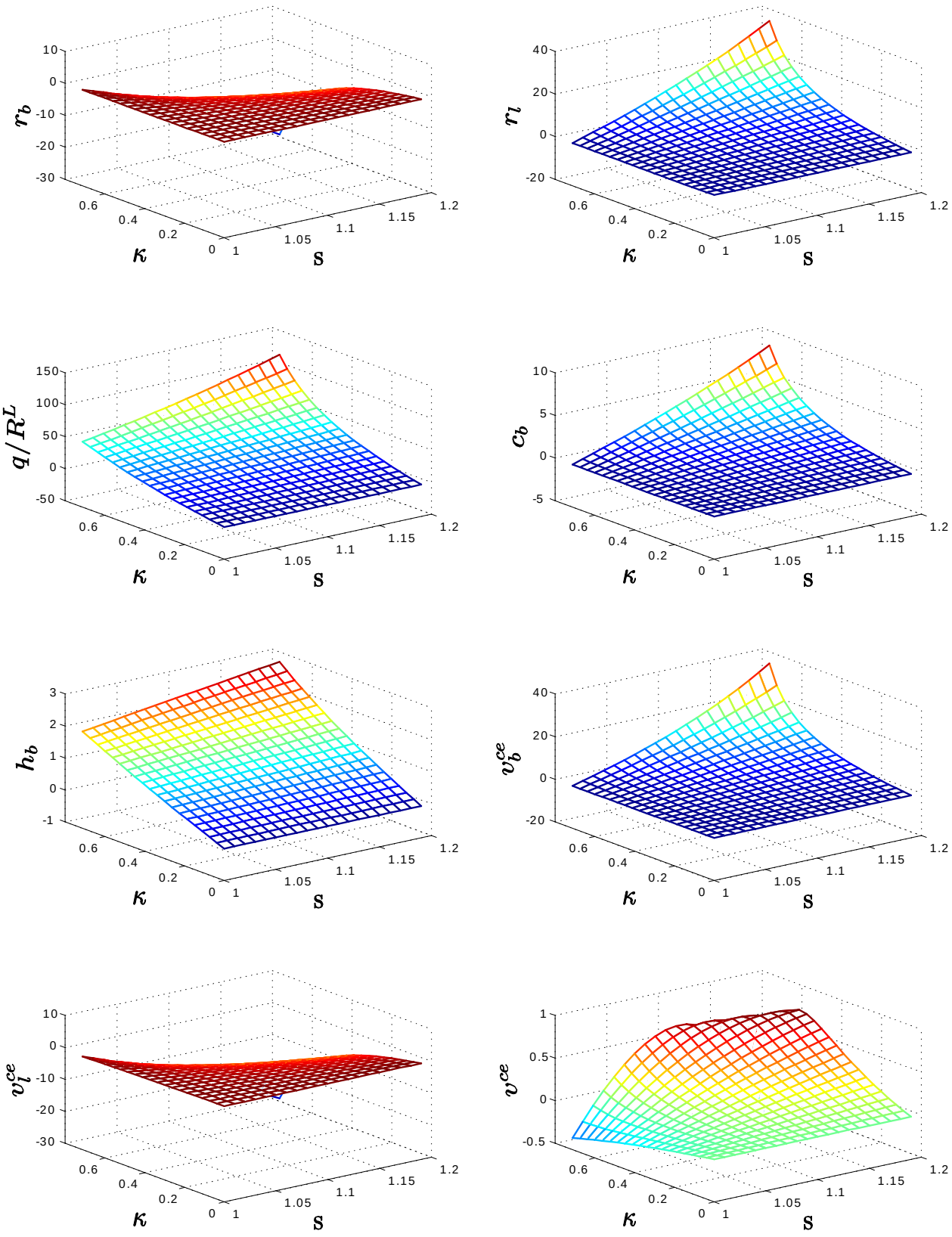

Figure 1: Effects of κ and s for $\pi = 1.07$ (in % deviations from laissez faire values)

this, we compute the utility values for a representative borrower and a representative lender, $u(\epsilon_b, c_b, h_b)$ and $u(\epsilon_l, c_l, h_l)$, and present the permanent consumption equivalents for borrowers' and lenders' welfare, $v_b^{ce} = [(1-\sigma)\, u(\epsilon_b, c_b, h_b)]^{1/(1-\sigma)}$ and $v_l^{ce} = [(1-\sigma)\, u(\epsilon_l, c_l, h_l)]^{1/(1-\sigma)}$, as well as for ex-ante social welfare, $v^{ce} = [(1-\sigma)\, 0.5\{u(\epsilon_l, c_l, h_l) + u(\epsilon_b, c_b, h_b)\}]^{1/(1-\sigma)}$. Apparently, welfare of lenders falls with larger values for κ and s, whereas welfare of borrowers tends to increase. Notably, the welfare gain for borrowers increases by more than 30% compared to the laissez faire case. Yet, the impact on ex-ante social welfare is much smaller, since the two welfare components mostly offset each other. For small values of s, the distortive effects of asset purchases dominate and social welfare decreases with κ compared to the laissez faire case. For larger values of s, social welfare tends to increase in κ and s, while changes in social welfare are non-monotonic as the loss of lenders strongly increases for combinations of large κ and s values. In total, the welfare gain does not exceed 1% of social welfare under laissez faire.

To assess how monetary policy instruments under asset purchases should be combined in an efficient way (see also Section 3.3), we vary the price discount $s > 1$ and compute the optimal fraction of purchased loans κ according to the efficiency condition (51) as well as to (44), (45), and $y = c_l + c$, where we now apply the utility function (52). Figure 2 shows the effects of a change in s for the benchmark value for the liquidation share of collateral, $z = 0.55$ (black solid line) and a lower value $z = 0.45$ (red dashed line), while we mark the values of the constrained efficient long-run allocation with (blue) circles ($\kappa = -\widetilde{\tau}^L = 0.31$ for $z = 0.55$ and $\kappa = -\widetilde{\tau}^L = 0.38$ for $z = 0.45$). All values are now given in absolute terms, except for social welfare v^{ce}, which is again given in terms of percentage deviations from the corresponding laissez faire value.

As shown in the first line of Figure 2, a higher price discount s is accompanied by a lower share of purchased loans κ, in accordance with the efficiency condition (51), and by a higher value for the term $\frac{(1-\kappa)(2-\kappa s)}{1-\kappa s}$, implying a relaxation of the borrowing constraint. This is reflected by an increase in the consumption differential relative to borrowers' housing. The lower value of κ in fact reduces the wedge $\frac{1-\kappa}{1-\kappa s}$ to correct for distortionary effects on market prices and to address the pecuniary externality. Then, the relative price of housing q/R^L, which is larger than in the laissez faire case, decreases

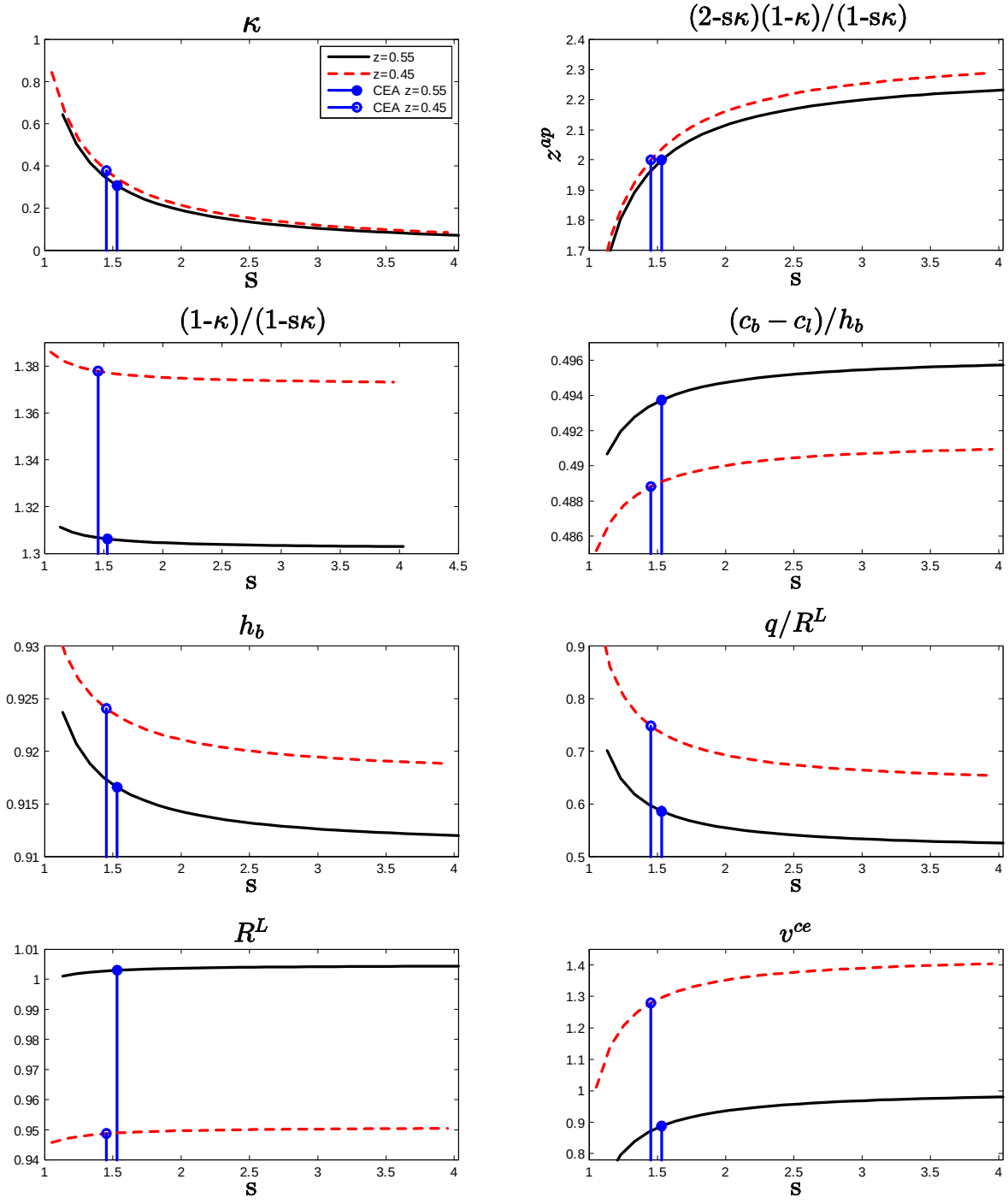

Figure 2: Equilibrium objects under asset purchases and the constrained efficient allocation (CEA) for variations in s [Note: All values are given in absolute terms, except for social welfare, which is in % deviations from laissez faire values.]

with s (see 46). These effects are accompanied by a decrease in borrowers' housing and thus an increase in lenders' housing induced by the enhanced lenders' willingness to save. Overall, an increase in s and the associated reduction in κ enhance ex-ante social welfare compared the constrained efficient allocation by relaxing the collateral requirement (see also Proposition 6). Concretely, the welfare gain under the constrained efficient allocation of about 0.9% (compared to laissez faire) can be increased up to 1%, with diminishing gains for larger values for s.[28] For a more severe collateral constraint, (see red dashed line for $z = 0.45$), the effects of changes in s are analogous, while, intuitively, the absolute welfare gain from policy interventions is larger (about 1.4%).

4.3 Aggregate risk and state contingent asset purchases

We now introduce aggregate risk, by considering a positive standard deviation σ_ε for aggregate endowment (see 53), and examine short-run effects of state contingent asset purchases. Due to aggregate shocks, welfare losses stemming from financial frictions can be amplified in the short-run, i.e. when the economy deviates from a stationary equilibrium due to $\varepsilon_t \neq 0$. To understand the welfare-enhancing role of state contingent interventions, consider first the laissez faire case. When the economy is hit by an adverse income shock, $\varepsilon_t < 0$, consumption of durables and non-durables of all agents decreases. As the lenders' demand for housing shifts downward (see 31), the housing price and thereby the collateral value fall, which tends to tighten agents' borrowing capacity. In such a situation, a corrective policy that tends to increase the collateral price and stimulates borrowing can enhance welfare, which can be achieved by asset purchases at above-market prices.

To identify welfare-enhancing state contingent asset purchases, we set-up the problem of a social planer under commitment, where we disregard the issue of time-inconsistency and restrict our attention to time-invariant processes of the solution to the policy plan (see Appendix H).[29] As discussed in Section 4.2, an optimal choice for the fraction of purchased loans κ_t and the price discount s_t would lead to an infinitely large value for the

[28] Notably, these welfare gains closely relate to the values found by Benigno et al. (2016).

[29] An analysis of time consistent corrective policies in an environment with an occasionally binding financial constraint is provided by Bianchi and Mendoza (2018)

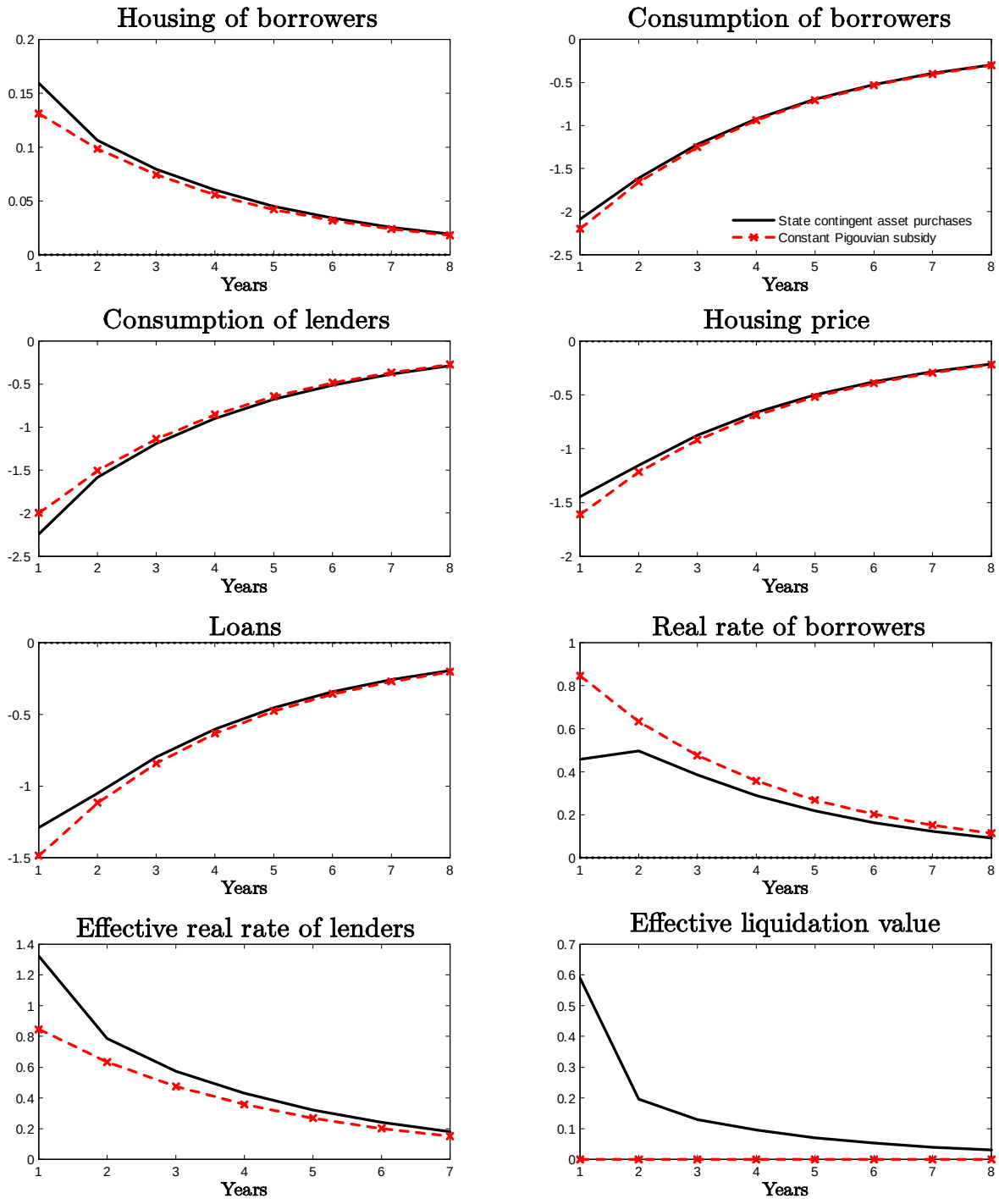

Figure 3: Responses to a minus one st.dev. aggregate income shock (in % deviations from a non-stochastic mean)

latter, while the welfare gains beyond the constrained efficient allocation are limited (see Figure 2). We therefore focus on asset purchases that implement the potentially time-varying constrained efficient allocation and show how this policy responds to aggregate shocks. Precisely, we set the means κ and s to implement the long-run constrained efficient allocation (as in 49), while we set s_t in a state contingent way keeping κ_t constant at κ. To avoid the policy plan under commitment to exhibit a unit root, we introduce a fixed depreciation of housing to the fraction δ_h and an equally sized newly constructed housing supply, to ensure a fixed supply h over time (see Appendix H).[30] This assumption has neither an impact on the mechanism nor on the main results. The model is solved applying a second order perturbation method.[31]

Figure 3 presents impulse responses to a negative income shock by one standard deviation, which hits all agents equally. The black solid line shows the responses under the state contingent asset purchase policy, while the red dashed line with crosses shows the responses for the time invariant policy (with identical long-run equilibrium allocations). Apparently, the adverse shock reduces consumption of borrowers and lenders. Compared to the time-invariant policy, state contingent interventions reduce the increase in the borrowers' real rate $r_{b,t}^{ap}$ and amplifies the increase in the lenders' real rate $r_{l,t}^{ap}$. While the differences in the responses of housing and consumption under both regimes are relatively small, differences in the interest rates are much more pronounced. State contingent asset purchases mitigate adverse income shock effects on borrowers' consumption and housing by stimulating borrowing via a relaxation of the borrowing constraint, i.e. asset purchases raise the effective liquidation value of collateral $\widetilde{z}_t = \frac{z}{2}\frac{(1-\kappa)(2-\kappa s)}{(1-\kappa s)}$ (see last panel of Figure 3). Hence, state contingent asset purchases should be countercyclical in the sense that they stimulates (dampens) borrowing in downturns (upturns).

We finally assess how an asset purchase policy should respond to an exogenous worsening of financial conditions and examine an unexpected fall in the liquidation value of collateral (or loan-to-value ratio) z_t, which can be interpreted as a situation with stressed financial markets.[32] The state contingent asset purchase policy mitigates the adverse

[30] For the numerical analysis we set δ_h to 1%, for convenience.

[31] We use dynare's implementation of a second order perturbation.

[32] Concretely, we assume that z_t is generated by $\log z_t = \rho \log z_{t-1} + (1-\rho)\log z_{t-1} + \varepsilon_{z,t}$, where $\varepsilon_{z,t}$

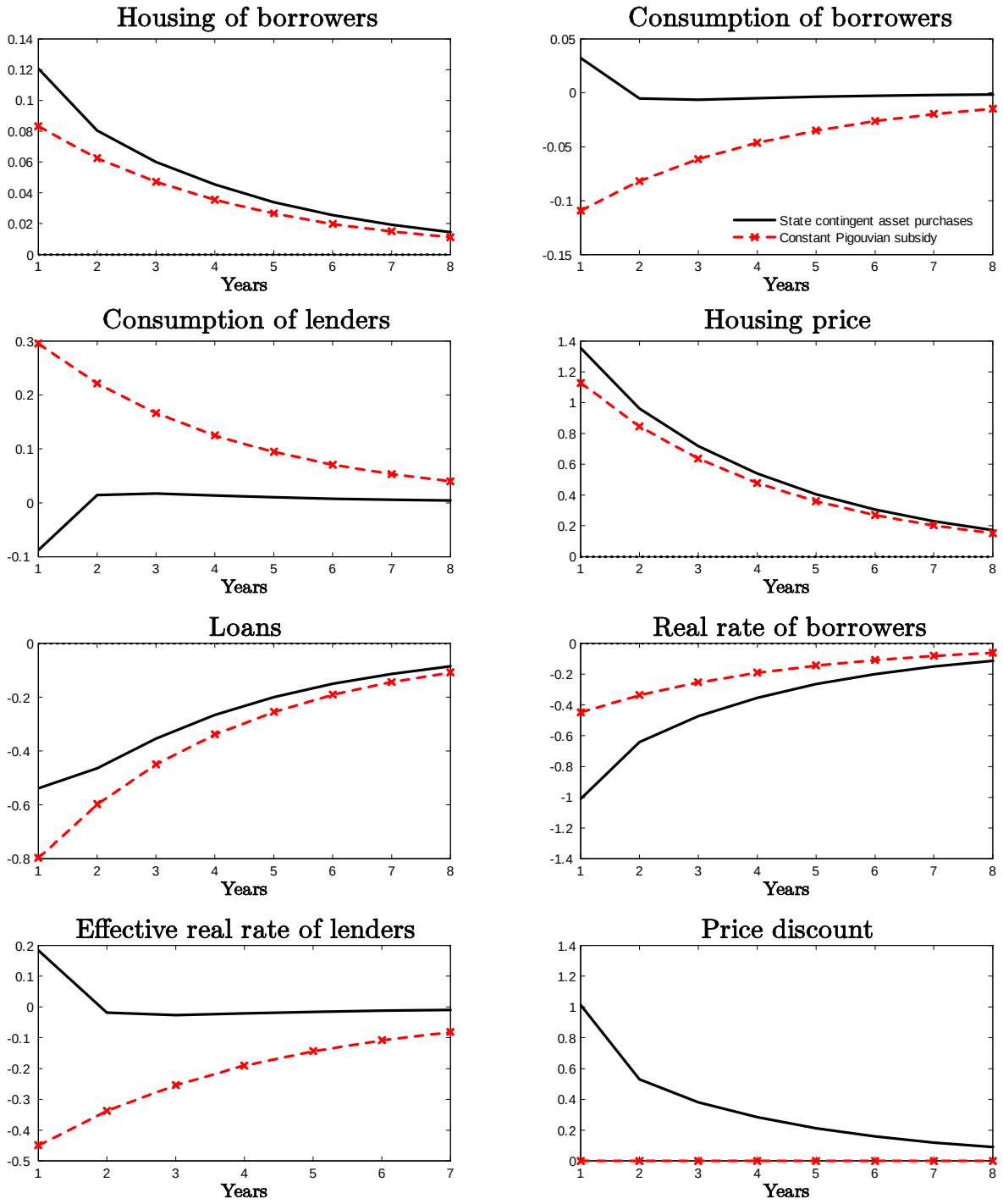

Figure 4: Responses to a minus one st.dev. liquidation value shock (in % deviations from a non-stochastic mean)

effects of the liquidation value shock on loans and on borrowers' consumption (see Figure 4). By raising the price discount s_t, which tends to lower (raise) the borrowers' (lenders') real interest rate, the central bank reduces the fall in loans, which is associated with an increase in borrowers' housing and thus in the price of housing. While borrowers' consumption decreases under a time-invariant policy, state contingent asset purchases can stabilize borrowers' consumption subsequent to the z_t-shock. Overall, responses to both types of shocks imply that asset purchases should stimulate borrowing in adverse states and, symmetrically, mitigate the build-up of debt in favorable states. Thereby, a state contingent asset purchases reduce the shocks amplification due to a positive feedback loop between collateral demand, prices, and borrowing.

5 Conclusion

Is there a useful role for central bank asset purchases in non-crisis times? This paper shows that central bank asset purchases in secondary markets can enhance social welfare as a corrective policy that mitigates financial amplification. The central bank can incentivize lenders to enhance the supply of funds by purchasing debt at an above-market price. This causes the borrower's real interest rate to fall relative to the effective real interest of lenders, which allows addressing pecuniary externalities induced by a collateral constraint. The effects of asset purchases can in principle also be generated by a combination of Pigouvian subsidies on debt and housing. In contrast to the latter, asset purchases do not rely on the availability of type-specific and non-distortionary sum taxes. This superiority of asset purchases is based on specific properties of central bank money: It can costlessly be produced and exhibits a unique role for the settlement of transactions.

We further show that state-contingent asset purchases should be conducted in a countercyclical way to reduce amplification of aggregate shocks via financial frictions. Asset purchases can thereby contribute to financial stability, in addition to macroprudential financial regulation. In this paper, we do not analyze (ex-post) asset purchases together with ex-ante regulation to isolate novel effects of unconventional monetary policy, leaving a joint analysis for future research.

is i.i.d. with mean zero and standard deviation σ_ε.

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Appendix

A Competitive equilibrium

Definition 2 *With Assumption 1, a competitive equilibrium is a set of sequences* $\{c_{i,t},\ h_{i,t},\ l_{i,t},\ i_{i,t},\ i^L_{i,t},\ \zeta_{i,t},\ \lambda_{i,t},\ m^H_{i,t},\ b_{i,t},\ b^T_t,\ \pi_t = P_t/P_{t-1},\ R^L_t,\ q_t, \tau_t\}^\infty_{t=0}$ *satisfying* $\forall i \in [0,1]$

$$
\begin{aligned}
& m^H_{i,t-1}\pi_t^{-1} + b_{i,t-1}\pi_t^{-1} + l_{i,t}\left(1 - 1/R^L_t\right) + y_t + \tau_t && (54)\\
=\ & m^H_{i,t} + (b_{i,t}/R_t) + \left(i_{i,t} + i^L_{i,t}\right)\left(R^m_t - 1\right) + c_{i,t} + q_t\left(h_{i,t} - h_{i,t-1}\right), \\
\lambda_{i,t} =\ & \beta E_t[u'(\epsilon_i, c_{i,t+1})/\pi_{t+1}], && (55)\\
\frac{1}{R^L_t} =\ & \beta \frac{E_t\left[u'(\epsilon_i, c_{i,t+1})/\pi_{t+1}\right]}{u'(\epsilon_i, c_{i,t})} + \frac{\zeta_{i,t}}{u'(\epsilon_i, c_{i,t})} && (56)\\
& \text{or } \frac{1}{R^L_t} = \beta \frac{E_t\left[u'(\epsilon_i, c_{i,t+1})/\pi_{t+1}\right]}{u'(\epsilon_i, c_{i,t})} \cdot \frac{1-\kappa_t}{1 - \kappa_t R^L_t/R^m_t}, \\
c_{i,t} =\ & i_{i,t} + i^L_{i,t} + m^H_{i,t-1}\pi_t^{-1} - l_{i,t}/R^L_t \ \text{ if } \psi_{i,t} > 0, && (57)\\
& \text{or } c_{i,t} \le i_{i,t} + i^L_{i,t} + m^H_{i,t-1}\pi_t^{-1} - l_{i,t}/R^L_t \ \text{ if } \psi_{i,t} = 0, \\
R^m_t i_{i,t} =\ & \kappa^B_t b_{i,t-1}\pi_t^{-1} \quad \text{if } \eta_{i,t} > 0, && (58)\\
& \text{or } R^m_t i_{i,t} < \kappa^B_t b_{i,t-1}\pi_t^{-1} \quad \text{if } \eta_{i,t} = 0, \\
0 \le R^m_t i^L_{i,t} =\ & \kappa_t l_{i,t} \quad \text{if } \mu_{i,t} > 0 && (59)\\
& \text{or } 0 \le R^m_t i^L_{i,t} \le \kappa_t l_{i,t} \quad \text{if } \mu_{i,t} = 0, \\
-l_{i,t} =\ & z q_t h_{i,t} \quad \text{if } \zeta_{i,t} > 0, && (60)\\
& \text{or } -l_{i,t} \le z_t q_t h_{i,t} \quad \text{if } \zeta_{i,t} = 0, \\
q_t \lambda_{i,t} =\ & u_{h,i,t} + \zeta_{i,t} z q_t + \beta E_t q_{t+1}\lambda_{i,t+1}, && (61)\\
i_t =\ & (1+\Omega_t) m^H_t - m^H_{t-1}\pi_t^{-1}, && (62)\\
b^T_t =\ & b_t + m^H_t, && (63)\\
b^T_t =\ & \Gamma b^T_{t-1}/\pi_t, && (64)\\
0 =\ & \Sigma_i l_{i,t}, && (65)\\
h =\ & \Sigma_i h_{i,t}, && (66)\\
b_{t-1}\pi_t^{-1} =\ & \left(R^m_t - 1\right)\left(i^L_t + \Omega_t m^H_t\right) - P_t\tau_t + (b_t/R_t) + R^m_t(m^H_t - m^H_{t-1}\pi_t^{-1}), && (67)
\end{aligned}
$$

$b_{t-1} = \sum_i b_{i,t-1}$, $i_t = \sum_i i_{i,t}$, $i^L_t = \sum_i i^L_{i,t}$, *and* $m^H_{t-1} = \sum_i m^H_{i,t-1}$, *where the multipliers* $\psi_{i,t}$, $\mu_{i,t}$, *and* $\eta_{i,t}$ *satisfy*

$$
\begin{aligned}
\psi_{i,t} &= u'(\epsilon_i, c_{i,t}) - \beta E_t\left[u'(\epsilon_i, c_{i,t+1})/\pi_{t+1}\right] \ge 0, \\
\mu_{i,t} &= [(1/R^m_t) - (1/R^L_t)]u'(\epsilon_i, c_{i,t})/(1-\kappa) \ge 0, \\
\sum_i \eta_{i,t} &= \sum_i [u'(\epsilon_i, c_{i,t})/R^m_t] - \beta E_t \sum_i \left[u'(\epsilon_i, c_{i,t+1})/\pi_{t+1}\right] \ge 0,
\end{aligned}
$$

and the transversality conditions, a monetary policy setting $\{R^m_t \ge 1,\ \kappa^B_t > 0,\ \kappa_t \in [0,1],\ \Omega_t > 0\}^\infty_{t=0}$, *given* $\Gamma > 0$, $\{y_t\}^\infty_{t=0}$, *and initial values* $P_{-1} > 0$, $h_{i,-1} > 0$, $M^H_{i,-1} > 0$, $B_{i,-1} > 0$, *and* $B^T_{-1} > 0$.

B Proof of Proposition 2

Consider an equilibrium as described in Definition 2 and let $x_{l,t} = \sum_{l,i} x_{l,i,t}$ for $x_{l,t} \in \{c_{l,t}, h_{l,t}, l_{l,t}, \lambda_{l,t}, i_{l,t}, i^L_{l,t}\}$ and $x_{b,t} = \sum_{b,i} x_{b,i,t}$ for $x_{b,t} \in \{c_{b,t}, h_{b,t}, l_{b,t}, \lambda_{b,t}, i_{b,t}\}$. Using Assumption 2, $R^m_t > 1$, and the law of large numbers, the set of conditions that describe the behavior of lenders (agents drawing ϵ_l) can be aggregated to

$$\begin{aligned} & 0.5m^H_{t-1}\pi_t^{-1} + 0.5b_{t-1}\pi_t^{-1} + l_{l,t}\left(1 - 1/R^L_t\right) + 0.5y_t + 0.5\tau_t && (68)\\ =& m^H_{l,t} + (b_{l,t}/R_t) + \left(0.5\left(i_{l,t} + i_{b,t}\right) + i^L_{l,t}\right)\left(R^m_t - 1\right) + c_{l,t} + q_t\left(h_{l,t} - 0.5h\right) \\ \lambda_{l,t} =& \epsilon_l(\delta - c_{l,t})/R^L_t, && (69)\\ q_t\lambda_{l,t} =& \gamma - h_{l,t} + \beta E_t q_{t+1} 0.5\left(\lambda_{l,t+1} + \lambda_{b,t+1}\right), && (70)\\ \frac{\epsilon_l(\delta - c_{l,t})}{R^L_t} =& \beta E_t\left[\frac{0.5(\epsilon_l(\delta - c_{l,t+1}) + \epsilon_b(\delta - c_{b,t+1}))}{\pi_{t+1}}\right], && (71)\\ c_{l,t} =& 0.5\left(i_{l,t} + i_{b,t}\right) + 0.5m^H_{t-1}\pi_t^{-1} - l_{l,t}/R^L_t, && (72) \end{aligned}$$

where $m^H_{l,t-1} = \Sigma_{l,i} m^H_{l,i,t-1} = 0.5m^H_{t-1}$, $b_{l,t-1} = \sum_{l,i} b_{l,i,t-1} = 0.5b_{t-1}$, and $h_{l,t-1} = \sum_{l,i} h_{l,i,t-1} = 0.5h$, and (72) uses that treasury open market operations are conducted before idiosyncratic shocks are drawn. Using Assumptions 2 and 3, $R^m_t > 1$, and the law of large numbers, the conditions of borrowers (agents drawing ϵ_b) can be aggregated to

$$\begin{aligned} & 0.5m^H_{t-1}\pi_t^{-1} + 0.5b_{t-1}\pi_t^{-1} + l_{b,t}\left(1 - 1/R^L_t\right) + 0.5y_t + 0.5\tau_t && (73)\\ =& m^H_{b,t} + (b_{b,t}/R_t) + 0.5\left(i_{l,t} + i_{b,t}\right)\left(R^m_t - 1\right) + c_{b,t} + q_t\left(h_{b,t} - 0.5h\right), \\ \lambda_{b,t} =& \beta E_t\left[0.5(\epsilon_l(\delta - c_{l,t+1}) + \epsilon_b(\delta - c_{b,t+1}))/\pi_{t+1}\right], && (74)\\ q_t\lambda_{b,t} =& \gamma - h_{b,t} + \zeta_{b,t} z q_t + \beta E_t q_{t+1} 0.5\left(\lambda_{l,t+1} + \lambda_{b,t+1}\right), && (75)\\ \frac{\epsilon_b(\delta - c_{b,t})}{R^L_t} =& \beta E_t\left[\frac{0.5(\epsilon_l(\delta - c_{l,t+1}) + \epsilon_b(\delta - c_{b,t+1}))}{\pi_{t+1}}\right] + \zeta_{b,t}, && (76)\\ c_{b,t} =& 0.5\left(i_{l,t} + i_{b,t}\right) + 0.5m^H_{t-1}\pi_t^{-1} - l_{b,t}/R^L_t, && (77)\\ -l_{b,t} =& z q_t h_{b,t}, && (78) \end{aligned}$$

where $m^H_{b,t-1} = \Sigma_{b,i} m^H_{b,i,t-1} = 0.5m^H_{t-1}$, $b_{b,t-1} = \sum_{b,i} b_{b,i,t-1} = 0.5b_{t-1}$, and $h_{b,t-1} = \sum_{b,i} h_{b,i,t-1} = 0.5h$. Using Assumption 1, the consolidated public sector budget constraint, $h = h_{l,t} + h_{b,t}$, $l_t = l_{l,t} = -l_{b,t}$, and that (69), (71), and (74) imply $\lambda_t = \lambda_{b,t} = \lambda_{l,t}$, and substituting out $\zeta_{b,t}$, λ_t, and l_t, conditions (68)-(78) can be reduced to (25)-(29). ■

C Proof of Proposition 3

The social planer problem (32) can be written as

$$\max_{c_b,c_l,h_b}\min_{\chi_1,\chi_2}(1-\beta)^{-1}\left\{[u(\epsilon_b,c_b,h_b)+u(\epsilon_l,c_l,h-h_b)]\right. \\ \left.+\chi_1\left[y-c_b-c_l\right]+\chi_2\left[2zh_b\cdot(q/R^L)-c_b+c_l\right]\right\}, \tag{79}$$

where $(q/R^L)=\frac{1}{1-\beta}u_{h_l}/u_{c_l}$. The first order conditions are $u_{c_b}=\chi_1+\chi_2$,

$$u_{c_l}=\chi_1-\chi_2\left(1+2zh_b\cdot\left[\partial(q/R^L)/\partial c_l\right]\right),$$
$$u_{h_l}-u_{h_b}=\chi_2\left(2z\cdot\left(q/R^L\right)+2zh_b\cdot\left[\partial(q/R^L)/\partial h_b\right]\right),$$

where $\partial(q/R^L)/\partial c_l=\frac{u_{h_l}}{1-\beta}(-u_{c_lc_l})/u_{cl}^2>0$ and $\partial(q/R^L)/\partial h_b=\frac{1}{1-\beta}(-u_{h_lh_l})/u_{c_l}>0$. Substituting out the multipliers χ_1 and χ_2 as well as (q/R^L) with $(q/R^L)=\frac{1}{1-\beta}u_{h_l}/u_{c_l}$, we get the following condition for the constrained efficient allocation

$$\frac{u_{c_b}-u_{c_l}}{u_{h_l}-u_{h_b}}\frac{z}{1-\beta}\frac{u_{h_l}}{u_{c_l}}=\frac{1+zh_b\cdot\left[\partial(q/R^L)/\partial c_l\right]}{1+\left[\partial(q/R^L)/\partial h_b\right]\cdot h_b/(q/R^L)}. \tag{80}$$

To disclose the implications for the associated tax/subsidy rate, we compare (80) with the competitive equilibrium condition (34), which can be rewritten as

$$\frac{(1-\tau^L)u_{c_b}-u_{c_l}}{u_{h_l}-u_{h_b}}\frac{z}{1-\beta}\frac{u_{h_l}}{u_{c_l}}=1. \tag{81}$$

Apparently, the LHS of (81) differs from the LHS of (80) only by the tax rate τ^L, while the RHSs differ due to the derivatives of the relative price (q/R^L). For $z(q/R^L)\cdot\left[\partial(q/R^L)/\partial c_l\right]<\left[\partial(q/R^L)/\partial h_b\right]\Leftrightarrow$

$$z(q/R^L)\frac{-u_{c_lc_l}}{u_{c_l}}<\frac{-u_{h_lh_l}}{u_{h_l}}, \tag{82}$$

the RHS of (80) is smaller than one, implying a subsidy $\tau^L<0$. Inserting the marginal utilities of (24), $u_{c_l}=\epsilon_l(\delta-c_l)$, $u_{c_lc_l}=-\epsilon_l$, $u_{h_l}=\gamma-(h-h_b)$, $u_{h_lh_l}=-1$, and using the constraints $c_b-c_l=2zh_b(q/R^L)$ and $y=c_l+c_b$, the inequality (82) reads $\left((1-\beta)/z\right)\cdot\epsilon_l\left((c_b-c_l)/(2h_b)\right)^2<1$. Using that (30) implies $h_b\geq 0.5h$, $\epsilon_l<1$, and that $y=h=1$, we can conclude that $(1-\beta)/z\leq 1$ is a sufficient condition for (82) and thus for a borrowing subsidy, $\tau^L<0$, to implement the constrained efficient allocation.

To identify the impact on the allocation of durable and non-durable goods, we apply the competitive equilibrium conditions (29), (81), and $c_b - c_l = 2zh_b\frac{\gamma-(h-h_b)}{(1-\beta)\epsilon_l(\delta-c_l)}$, and substitute out c_l with $c_l = y - c_b$ to get $F(\boldsymbol{\tau}^L, h_b, c_b) = 0$ and $G(h_b, c_b) = 0$, where

$$F(\boldsymbol{\tau}^L, h_b, c_b) = \frac{(1-\boldsymbol{\tau}^L)\cdot\epsilon_b(\delta-c_b)-\epsilon_l(\delta-y+c_b)}{(2h_b-h)(1-\beta)\epsilon_l(\delta-y+c_b)(1/z)} - \frac{1}{\gamma-h+h_b},$$
$$G(h_b, c_b) = 2zh_b\frac{\gamma-(h-h_b)}{(1-\beta)\epsilon_l(\delta-y+c_b)} - 2c_b + y.$$

The partial derivatives of $G(h_b, c_b)$, where G_x abbreviates $\partial G/\partial x$, are given by

$$G_{h_b} = 2z\frac{2h_b-h+\gamma}{\epsilon_l(\delta-y+c_b)(1-\beta)} > 0, \quad G_{c_b} = -2\left(\frac{z}{\epsilon_l}\frac{h_b(\gamma-h+h_b)}{(1-\beta)(c_b-y+\delta)^2}+1\right) < 0,$$

implying $\partial h_b/\partial c_b = -G_{c_b}/G_{h_b} > 0$. The partial derivatives of $F(\boldsymbol{\tau}^L, h_b, c_b)$ are given by

$$F_{\boldsymbol{\tau}^L} = -\frac{\epsilon_b(\delta-c_b)}{\epsilon_l(1-\beta)(2h_b-h)(\delta-y+c_b)(1/z)} < 0,\ F_{h_b} = -\frac{2\gamma-h}{(2h_b-h)(\gamma-h+h_b)^2} < 0,$$
$$F_{c_b} = -\frac{2\epsilon_b(\delta-y/2)\left(1-\tau^L\right)}{\epsilon_l(\delta-y+c_b)^2(1-\beta)(2h_b-h)(1/z)} < 0.$$

Thus, consumption of the representative borrower decreases with the tax rate, since $\partial c_b/\partial\tau^L = -(G_{h_b}F_{\tau^L})/(F_{c_b}G_{h_b} - F_{h_b}G_{c_b}) < 0$. Hence, introducing a subsidy $\tau^L < 0$ increases consumption and housing of the representative borrower (by $\partial h_b/\partial c_b > 0$). Given that consumption (housing) of lenders decreases for a given aggregate supply (stock of housing), the lenders' consumption Euler equation (25), which can be written as $1 = \beta 0.5[1+\epsilon_b(\delta-c_b)/(\epsilon_l(\delta-c_l))]\left(R^L/\pi\right)$, further implies that the real interest rate R^L/π increases with the subsidy. ■

D Monetary policy instruments under asset purchases

Suppose that government bonds are supplied at a rate that is not identical to the inflation target, $\Gamma \neq \pi^*$. Then, the real value of the total stock of bonds $b_t^T = b_t + m_t^H$ might grow or shrink in a long-run equilibrium at a constant rate Γ/π (see 43). The money demand condition (41) then requires for constant steady state values c_b, R^L, h_b, q, and z, that the term $\widetilde{m}_t = (1+\Omega_t)m_t^H$ is also constant in the long-run. Rewriting (42), and (43) as $\kappa_t^B b_t = R_t^m\pi_t[\widetilde{m}_t - \widetilde{m}_{t-1}(1+\Omega_{t-1})^{-1}\pi_t^{-1}]$ and $[b_t + \widetilde{m}_t/(1+\Omega_t)] = \Gamma\left[b_{t-1} + \widetilde{m}_{t-1}/(1+\Omega_{t-1})\right]/\pi_t$,

and substituting out b_t and b_{t-1} , gives

$$\left[\frac{\pi_t R_t^m}{\kappa_t^B}\left(\widetilde{m}_t - \frac{\widetilde{m}_{t-1}\pi_t^{-1}}{1+\Omega_{t-1}}\right) + \frac{\widetilde{m}_{t-1}}{1+\Omega_{t-1}}\right] \tag{83}$$
$$= \frac{\Gamma}{\pi_{t-1}}\left[\frac{\pi_{t-1}R_{t-1}^m}{\kappa_{t-1}^B}\left(\widetilde{m}_{t-1} - \frac{\widetilde{m}_{t-2}\pi_{t-1}^{-1}}{1+\Omega_{t-2}}\right) + \frac{\widetilde{m}_{t-2}}{1+\Omega_{t-2}}\right].$$

Taking the limit $t \to \infty$ of both sides of (83), we can use that for a constant long-run inflation rate π and a constant policy rate R^m a steady state is characterized by a constant value for $\widetilde{m}_t$. The term in the square brackets in (83), i.e. the total supply of short-term treasuries, grows/shrinks with the constant rate Γ/π. When the growth rate of bonds exceeds the inflation rate, $\Gamma > \pi$, this can be guaranteed by a permanently shrinking value for κ_t^B. Thus, the central bank can let κ_t^B shrink at the rate $\pi/\Gamma < 1$ and can let the share of money supplied outright, $1/\Omega_t = M_t^H/M_t^R$, go to zero in the long-run, i.e. it can set κ_t^B and $1/\Omega_t$ according to $\lim_{t\to\infty} \kappa_t^B/\kappa_{t-1}^B = \pi/\Gamma$ and $\lim_{t\to\infty} 1/\Omega_t = 0$ if $\Gamma > \pi$.

For $\Gamma < \pi$, the term in the square bracket in (83) permanently shrinks, which can not be supported by a growing value κ_t^B without violating the restriction $\kappa_t^B \leq 1$. In this case, the central bank can keep κ_t^B constant and can let the share Ω_t of money supplied under repos grow in a long-run equilibrium. For $\Gamma < \pi$, it can thus set Ω_t in the long-run according to $\lim_{t\to\infty} (1+\Omega_t)/(1+\Omega_{t-1}) = \pi/\Gamma > 1$.

E A CRRA version with representative agents

Definition 3 *With Assumption 1, a competitive equilibrium of the economy with preferences satisfying (52) and wealth redistribution within households is of a set of sequences* $\{c_{b,t},\, c_{l,t},\, \pi_t,\, R_t^L,\, h_{b,t},\, q_t,\, b_t,\, b_t^T,\, m_t^H\}_{t=0}^{\infty}$ *satisfying*

$$\epsilon_b c_{b,t}^{-\sigma}/R_t^L \tag{84}$$
$$= \beta E_t[0.5(\epsilon_b c_{b,t+1}^{-\sigma} + \epsilon_l c_{l,t+1}^{-\sigma})/\pi_{t+1}] + \gamma((h-h_{b,t})^{-\sigma} - h_{b,t}^{-\sigma})/[q_t z],$$
$$\epsilon_l c_{l,t}^{-\sigma}/R_t^L = \beta E_t[0.5(\epsilon_b c_{b,t+1}^{-\sigma} + \epsilon_l c_{l,t+1}^{-\sigma})/\pi_{t+1}]\frac{1-\kappa_t}{1-\kappa_t R_t^L/R_t^m}, \tag{85}$$
$$q_t \epsilon_l c_{l,t}^{-\sigma}\frac{1/R_t^L - \kappa_t/R_t^m}{1-\kappa_t} = \gamma\left(h-h_{b,t}\right)^{-\sigma} + \beta E_t\left[q_{t+1}\epsilon_l c_{l,t+1}^{-\sigma}\frac{1/R_{t+1}^L - \kappa_{t+1}/R_{t+1}^m}{1-\kappa_{t+1}}\right], \tag{86}$$
$$c_{b,t} - c_{l,t} \leq z q_t h_{b,t}\left[(2/R_t^L) - (\kappa_t/R_t^m)\right], \tag{87}$$
$$c_{l,t} + c_{b,t} = y_t, \tag{88}$$
$$0.5(1+\Omega_t)m_t^H \geq c_{b,t} + z_t q_t h_{b,t}/R_t^L, \tag{89}$$
$$\kappa_t^B b_{t-1}\pi_t^{-1}/R_t^m \geq (1+\Omega_t)m_t^H - m_{t-1}^H\pi_t^{-1}, \tag{90}$$

$$b_t^T = \Gamma b_{t-1}^T/\pi_t, \tag{91}$$
$$b_t^T = b_t + m_t^H, \tag{92}$$

the transversality conditions, a monetary policy setting $\{1 < R_t^m,\ \kappa_t \in [0,1],\ \kappa_t^B > 0,\ \Omega_t > 0\}_{t=0}^{\infty}$*, given* $\{y_t\}_{t=0}^{\infty}$*,* $\Gamma > 0$*,* $P_{-1} > 0$*,* $M_{-1}^H > 0$*,* $B_{-1} > 0$*, and* $B_{-1}^T > 0$*.*

The first best allocation apparently satisfies $\epsilon_b c_{b,t}^{-\sigma} = \epsilon_l c_{l,t}^{-\sigma}$ and $h_{b,t} = h_{l,t} = 2h$. Under binding borrowing, liquidity, and money supply constraints, a long-run competitive equilibrium consists of a set $\{c_l,\ c_b,\ R^L,\ h_b,\ q\}$ satisfying

$$1/R^L = \beta\left(c_l^{\sigma}/\epsilon_l\right) 0.5(\epsilon_b c_b^{-\sigma} + \epsilon_l c_l^{-\sigma})\pi^{-1}\frac{1-\kappa}{1-\kappa R^L/R^m}, \tag{93}$$
$$(1-\tau^L)\epsilon_b c_b^{-\sigma} = R^L\beta 0.5(\epsilon_b c_b^{-\sigma} + \epsilon_l c_l^{-\sigma})\pi^{-1} + R^L\left(\gamma/qz\right)\left((h-h_b)^{-\sigma} - h_b^{-\sigma}\right), \tag{94}$$
$$\gamma(h-h_b)^{-\sigma} = q\left(1-\beta\right)\epsilon_l c_l^{-\sigma}\frac{1/R^L - \kappa/R^m}{1-\kappa}, \tag{95}$$
$$c_b - c_l = zqh_b[(2/R^L) - (\kappa/R^m)], \tag{96}$$
$$y = c_l + c_b, \tag{97}$$

for a monetary policy setting $\{1 < R^m < R^L,\ \kappa \in [0,1),\ \pi > \beta\}$. Once the set $\{c_l,\ c_b,\ R^L,\ h_b,\ q\}$ is determined, the values m^H and b are given by $m^H = \left(c_b - zqh_b/R^L\right)\frac{1}{0.5(1+\Omega)}$ and $b = \frac{R^m\pi}{\kappa^B}\left(1+\Omega-\pi^{-1}\right)m^H$ given κ^B and Ω.

F Pigouvian subsidy under CRRA preferences

In this Appendix, we consider an economy under CRRA preferences and pooling of wealth within households as summarized in Definition 3. We show that a constrained efficient allocation can again be implemented by a Pigouvian debt subsidy, as already shown for the case of linear-quadratic preferences (see Proposition 3). Here, a long-run equilibrium consists of the set $\{c_l,\ c_b,\ R^L,\ h_b,\ q\}$ satisfying (97)

$$(1-\tau^L)\epsilon_b c_b^{-\sigma} = R^L\beta 0.5(\epsilon_b c_b^{-\sigma} + \epsilon_l c_l^{-\sigma})\pi^{-1} + R^L\left(\gamma/qz\right)\left((h-h_b)^{-\sigma} - h_b^{-\sigma}\right), \tag{98}$$
$$\epsilon_l c_l^{-\sigma}/R^L = \beta 0.5(\epsilon_b c_b^{-\sigma} + \epsilon_l c_l^{-\sigma})/\pi, \tag{99}$$
$$c_b - c_l \leq zqh_b 2/R^L, \tag{100}$$
$$\gamma(h-h_b)^{-\sigma} = q\beta(1-\beta)0.5(\epsilon_b c_b^{-\sigma} + \epsilon_l c_l^{-\sigma})/\pi, \tag{101}$$

given $\{\tau^L, \pi\}$. Substituting out the housing price q with (101) in (100), leads to

$$0 \leq zh_b 2\frac{\gamma(h-h_{b,t})^{-\sigma_h}}{(1-\beta)\epsilon_l c_l^{-\sigma}} - c_b + c_l, \tag{102}$$

where we further used (99) to substitute out the real rate R^L/π.

Proposition 7 *Suppose that preferences satisfy (52), that wealth is redistributed within households, and that type-specific and non-distortionary taxes are available. Then, the constrained efficient long-run allocation can be implemented by a Pigouvian subsidy on borrowing, if but not only if* $\epsilon_b/\epsilon_l \leq 3^\sigma$.

Proof. The problem of a social planer, who aims at maximizing social welfare (21) while deciding on private sector borrowing, can again be summarized as (79). Likewise, a constrained efficient allocation can be implemented by a Pigouvian borrowing subsidy if (82) is satisfied. Applying the partial derivatives of the CRRA utility function (52), $u_{c_l} = \epsilon_l c_l^{-\sigma}$, $u_{c_l c_l} = -\sigma u_{cl}/c_l$, $u_{h_l} = \gamma(h-h_b)^{-\sigma}$, $u_{h_l h_l} = -\sigma u_{hl}(h-h_b)^{-1}$, we can rewrite (82) as $((c_b/c_l)-1)(h/h_b-1)-2<0$. Since the ratio c_b/c_l is smaller under a binding borrowing constraint than under first best, $(c_b^*/c_l^*) = (\epsilon_b/\epsilon_l)^{1/\sigma}$ and $h/2 \leq h_b \Leftrightarrow (h/h_b)-1 \leq 1$ holds, we can conclude that $((c_b/c_l)-1)(h/h_b-1)-2 < ((\epsilon_b/\epsilon_l)^{1/\sigma}-1)(h/h_b-1)-2 \leq (\epsilon_b/\epsilon_l)^{1/\sigma}-3$. Hence, the constrained efficient allocation can be implemented by a Pigouvian borrowing, if but not only if the preference shock satisfies $\epsilon_b/\epsilon_l \leq 3^\sigma$. ■

G Corrective policies in a non-monetary economy

In this Appendix, we develop a corresponding real model and examine corrective fiscal instruments that relate to asset purchases. In contrast to the main model, we disregard money and bonds, and consider interperiod, i.e., one-period, real loans, instead of intraperiod nominal loans. We consider a continuum of households with a continuum of members. The budget constraint for a household member drawing ϵ_b is

$$\left(1+r_{t-1}^L\right) l_{b,t-1} + 0.5y_t \geq (1-\tau_t^L) l_{b,t} + c_{b,t} + q_t\left(h_{b,t}-h_{b,t-1}\right) + \tau_{b,t}^R, \tag{103}$$

and $\left(1+r_{t-1}^L\right) l_{l,t-1} + 0.5y_t \geq l_{l,t} + c_{l,t} + q_t\left(h_{l,t}-h_{l,t-1}\right)$ for a household member drawing ϵ_l, where r_{t-1}^L is the real loan rate. As in Section 4.2, household members share financial wealth and durables at the end of each period. Notably, $\tau_t^L > 0$ (< 0) is a tax (subsidy) on borrowing ($l_{i,t} < 0$) and $\tau_{b,t}^R$ is a borrower-specific lump-sum tax/transfer, where

implementation of a Pigouvian tax/subsidy on debt requires $\tau^R_{b,t} = \tau^L_t l_{b,t}$. The collateral constraint corresponds to the nominal version (4)

$$-l_{b,t} \leq z q_t h_{b,t}. \tag{104}$$

The collateral value is measured before maturity, which can be rationalized by the borrowers' ability to renegotiate the debt contract before debt matures (see, e.g., Jeanne and Korinek, 2016). Maximizing lifetime utility subject to (103) and (104) leads to

$$\lambda_{b,t} = u_{c_b,t},\ \lambda_{l,t} = u_{c_l,t}, (1-\tau^L_{b,t})\lambda_{b,t} = \beta 0.5\left(1+r^L_t\right) E_t\left(\lambda_{b,t+1} + \lambda_{l,t+1}\right) + \zeta_{b,t},$$
$$\lambda_{l,t} = \beta 0.5\left(1+r^L_t\right) E_t\left[0.5\left(\lambda_{b,t+1} + \lambda_{l,t+1}\right)\right],\ q_t\lambda_{l,t} = u_{h_l,t} + \beta E_t q_{t+1} 0.5\left(\lambda_{b,t+1} + \lambda_{l,t+1}\right),$$
$$q_t\lambda_{b,t} = u_{h_b,t} + \zeta_{b,t} z q_t + \beta E_t q_{t+1} 0.5\left(\lambda_{b,t+1} + \lambda_{l,t+1}\right).$$

Combining the binding budget constraints to $-2l_{b,t} = (c_{b,t} - c_{l,t}) + q_t\left(h_{b,t} - h_{l,t}\right)$, and substituting out multipliers and loans with the binding constraint (104), we can define a competitive equilibrium as a set of sequences $\{c_{b,t},\, c_{l,t},\, r^L_t,\, h_{b,t},\, q_t\}_{t=0}^{\infty}$ satisfying

$$u_{c_l,t} = \beta E_t\left[0.5(u_{c_l,t+1} + u_{c_b,t+1})\left(1+r^L_t\right)\right], \tag{105}$$
$$(1-\tau^L_t)u_{c_b,t} = u_{c_l,t} + [(u_{h_l,t} - u_{h_b,t})q_t^{-1} + (u_{c_b,t} - u_{c_l,t})]/z, \tag{106}$$
$$u_{c_l,t}q_t = u_{h_l,t} + \beta E_t[0.5(u_{c_l,t+1} + u_{c_b,t+1})q_{t+1}], \tag{107}$$
$$c_{b,t} - c_{l,t} = q_t\left(2\widehat{z}h_{b,t} + h\right), \tag{108}$$

and $y_t = c_{b,t} + c_{l,t}$, given $\{\tau^L_t\}_{t=0}^{\infty}$, where we assume that $\widehat{z} = z - 1/2 > 1$. Notably, the constraint on the consumption differential (108) closely relates to the corresponding condition in the monetary economy (see 28). The conditions (105) and (106), which determine the real interest rate and the tax/subsidy rate for a given allocation, do not constitute binding constraints for a social planer who chooses a welfare maximizing allocation subject to (107), (108), and $y = c_b + c_l$. Considering long-run equilibria, the problem of the social planer can be summarized as

$$\max_{c_b,c_l,h_b} \min_{\chi_1,\chi_2} (1-\beta)^{-1}\ \{[u(\epsilon_b, c_b, h_b) + u(\epsilon_l, c_l, h - h_b)] \tag{109}$$
$$+ \chi_1\left[y - c_b - c_l\right] + \chi_2[(2\widehat{z}h_b + h)\cdot q - c_b + c_l]\},$$

where $q = u_{h_l}/\Xi$ and $\Xi = u_{c_l} - \beta 0.5(u_{c_l} + u_{c_b}) > 0$. The first order conditions are

$$u_{c_b} = \chi_1 + \chi_2 \left(1 - (2\widehat{z}h_b + h) \cdot [\partial q/\partial c_b]\right), \quad u_{c_l} = \chi_1 - \chi_2 \left(1 + (2\widehat{z}h_b + h) \cdot [\partial q/\partial c_l]\right),$$
$$u_{h_l} - u_{h_b} = \chi_2 \left(2\widehat{z} \cdot q + (2\widehat{z}h_b + h) \cdot [\partial q/\partial h_b]\right),$$

and can – by substituting out the multipliers χ_1 and χ_2 – be combined to

$$\frac{u_{c_b} - u_{c_l}}{u_{h_l} - u_{h_b}} \frac{u_{h_l}}{\Xi} \widehat{z} = \frac{2 + (2\widehat{z}h_b + h) \cdot ((\partial q/\partial c_l) - (\partial q/\partial c_b))}{2 + (2\widehat{z}h_b + h) \cdot (\partial q/\partial h_b)/(q\widehat{z})}. \tag{110}$$

Combining (105) and (106), gives the competitive equilibrium condition $(1 - \tau_t^L)u_{c_b,t} = u_{c_l,t} + [(u_{h_l,t} - u_{h_b,t})q_t^{-1} + (u_{c_b,t} - u_{c_l,t})]/z$, which simplifies in the long-run to

$$\left(\frac{(1 - \tau^L)u_{c_b} - u_{c_l}}{(u_{h_l} - u_{h_b})} z - \frac{u_{c_b} - u_{c_l}}{u_{h_l} - u_{h_b}}\right) \frac{u_{h_l}}{\Xi} = 1. \tag{111}$$

It can easily be seen that the RHS of (110) is smaller than one if

$$(z - 1/2)q \left[(\partial q/\partial c_l) - (\partial q/\partial c_b)\right] < \partial q/\partial h_b, \tag{112}$$

which implies for the LHSs of (110) and (111): $\frac{u_{c_b} - u_{c_l}}{u_{h_l} - u_{h_b}} \frac{u_{h_l}}{\Xi} \widehat{z} < \frac{(1-\tau^L)zu_{c_b} - zu_{c_l} - (u_{c_b} - u_{c_l})}{u_{h_l} - u_{h_b}} \frac{u_{h_l}}{\Xi} \Rightarrow$ $0 < \frac{1}{z2}(1 - u_{c_l}/u_{c_b}) < -\tau^L$.

Hence, the implementation of a constrained efficient allocation requires a debt subsidy if (112) holds, which can by using the derivatives $\partial q/\partial c_b = u_{h_l}\beta 0.5 u_{c_b c_b}\Xi^{-2} < 0$, $\partial q/\partial c_l = -u_{h_l}u_{c_l c_l}(1 + \beta 0.5)\Xi^{-2} > 0$, and $\partial q/\partial h_b = -u_{h_l h_l}/\Xi > 0$, be rewritten as

$$(z - 1/2)q \frac{-u_{c_l c_l} + \beta 0.5 (u_{c_l c_l} - u_{c_b c_b})}{u_{c_l} - \beta 0.5(u_{c_l} + u_{c_b})} < \frac{-u_{h_l h_l}}{u_{h_l}}. \tag{113}$$

Comparing (112) and (113) with the corresponding expressions for the monetary economy, i.e. $z(q/R^L) \cdot \left[\partial(q/R^L)/\partial c_l\right] < \left[\partial(q/R^L)/\partial h_b\right]$ and $z(q/R^L)\frac{-u_{c_l c_l}}{u_{c_l}} < \frac{-u_{h_l h_l}}{u_{h_l}}$ (see 82), shows that the conditions for the constrained efficient allocations are qualitatively unchanged. Specifically, a subsidy on borrowing, which is neutralized by a borrower-specific lump-sum tax, can implement the constrained efficient allocation. For this, changes in housing demand have to dominate changes in consumption demand with regard to their impact on the (relative) price of housing, corresponding to the case considered in Proposition 5.

To see how total effects of central bank asset purchases can in principle be replicated by

fiscal instruments, we additionally introduce a Pigouvian tax/subsidy τ_t^h on end-of-period housing. Notably, to neutralize the effects of the housing tax/subsidy on households' budgets, lump-sum taxes/transfers have to be type-specific (see also Davila and Korinek, 2018). Hence, the budget constraints are then given by

$$\left(1+r_{t-1}^L\right) l_{b,t-1} + y_t \geq (1-\tau_t^L) l_{b,t} + c_{b,t} + q_t \left(\left(1+\tau_t^h\right) h_{b,t} - h_{b,t-1}\right) + \tau_{b,t}^R + \tau_{b,t}^h,$$

and $\left(1+r_{t-1}^L\right) l_{l,t-1} + y_t \geq l_{l,t} + c_{b,t} + q_t \left(\left(1+\tau_t^h\right) h_{l,t} - h_{l,t-1}\right) + \tau_{l,t}^h$, where $\tau_{b,t}^h = -q_t \tau_t^h h_{b,t}$ and $\tau_{l,t}^h = -q_t \tau_t^h h_{l,t}$. The conditions (106) and (107) then change to $(1-\tau_t^L) u_{c_b,t} = u_{c_l,t} + [(u_{h_l,t} - u_{h_b,t}) q_t^{-1} + \left(1+\tau_t^h\right)(u_{c_b,t} - u_{c_l,t})]/z$ and $\left(1+\tau_t^h\right) q_t u_{c_l,t} = u_{h_l,t} + \beta E_t q_{t+1} 0.5 (u_{c_l,t+1} + u_{c_b,t+1})$. Without aggregate risk, the housing price then satisfies $q = u_{h_l}/[(1+\tau^h) u_{c_l} - \beta 0.5 (u_{c_l} + u_c)]$ and the consumption difference is given by

$$c_b - c_l = \frac{u_{h_l}}{(1+\tau^h) u_{c_l} - \beta 0.5 (u_{c_l} + u_c)} \cdot (\widehat{z} h_b + h).$$

A comparison with (45) shows that a subsidy on housing $\tau^h < 0$, which is neutralized by type-specific lump-sum taxes, can in principle replicate the additional effect of asset purchases (while it also alters the subsidy rate on debt). By raising the price of collateral the borrowing constraint can be relaxed, which can enhance efficiency compared to the allocation where only the Pigouvian subsidy on debt is available. Notably, the housing subsidy can in principle also be combined with a type-specific tax on non-durables.

H Constrained efficiency under aggregate risk

Suppose that housing depreciates every period at the rate δ_h, while new housing is constructed at the same rate, such that total supply again equals h. For an individual agent, the investment decision in housing under asset purchases is then described by

$$q_t \epsilon_l c_{l,t}^{-\sigma} \frac{1/R_t^L - \kappa_t/R_t^m}{1-\kappa_t} = \gamma (h - h_{b,t})^{-\sigma} + \beta E_t (1-\delta_h) q_{t+1} \epsilon_l c_{l,t+1}^{-\sigma} \frac{1/R_{t+1}^L - \kappa_{t+1}/R_{t+1}^m}{1-\kappa_{t+1}}, \tag{114}$$

instead of (86). For the analysis of a welfare-enhancing asset purchase policy, we apply the conditions (84), (85), (87), (88), and (114) and define

$$\widetilde{z}_t = \frac{z}{2} \frac{(2-\kappa_t s_t)(1-\kappa_t)}{1-\kappa_t s_t}, \qquad x_t = \frac{q_t}{R_t^L} \frac{1-\kappa_t s_t}{1-\kappa_t}, \tag{115}$$

where $s_t = R_t^L/R_t^m$. Further combining (84) and (85) to

$$\epsilon_b c_{b,t}^{-\sigma}\left(q_t/R_t^L\right) = x_t\epsilon_l c_{l,t}^{-\sigma} + \gamma((h-h_{b,t})^{-\sigma} - h_{b,t}^{-\sigma})/z, \tag{116}$$

and recalling that the policy maker has two instruments at his disposal to adjust the two terms $\frac{(2-\kappa_t s_t)(1-\kappa_t)}{1-\kappa_t s_t}$ and $\frac{1-\kappa_t s_t}{1-\kappa_t}$, the constraints for an optimal choice of the set of sequences $\{c_{b,t}, c_{l,t}, h_{b,t}, x_t, \widetilde{z}_t\}_{t=0}^{\infty}$ are given by (88)

$$c_{b,t} - c_{l,t} \leq \widetilde{z}_t h_{b,t} 2x_t, \tag{117}$$

$$\epsilon_l c_{l,t}^{-\sigma} x_t = \gamma(h-h_{b,t})^{-\sigma_h} + \beta E_t \epsilon_l c_{l,t+1}^{-\sigma}(1-\delta_h)x_{t+1}, \tag{118}$$

while q_t/R_t^L is determined by (116). Consider a given sequence $\{\widetilde{z}_t\}_{t=0}^{\infty}$. The constrained efficient allocation solves the following problem of a planer who maximizes ex-ante social welfare under commitment, while choosing debt on the behalf of private agents:

$$\max_{\{c_{b,t}, c_{l,t}, h_{b,t}, x_t\}_{t=0}^{\infty}} E\sum_{t=0}^{\infty}\beta^t[u(\epsilon_b, c_{b,t}, h_{b,t}) + u(\epsilon_l, c_{l,t}, h-h_{b,t})] \text{ s.t. (88), (117), and (118).}$$

Neglecting the conditions for period $t=0$, the solution has to satisfy the following first order conditions $0 = \epsilon_b c_{b,t}^{-\sigma} - \mu_t - \lambda_t$, $0 = \epsilon_l c_{l,t}^{-\sigma} + \mu_t - \lambda_t - \sigma\psi_t\epsilon_l c_{l,t}^{-\sigma-1}x_t + \sigma\psi_{t-1}(1-\delta_h)\epsilon_l c_{l,t}^{-\sigma-1}x_t$, $0 = \gamma\left(h_{b,t}^{-\sigma_h} - ((h-h_{b,t})^{-\sigma_h}\right) + \mu_t\widetilde{z}_t 2x_t - \psi_t\sigma_h\gamma(h-h_{b,t})^{-\sigma_h-1}$, and $0 = \mu_t\widetilde{z}_t h_{b,t}2 + \psi_t\epsilon_l c_{l,t}^{-\sigma} - \psi_{t-1}(1-\delta_h)\epsilon_l c_{l,t}^{-\sigma}$, where λ_t, μ_t, and ψ_t denote the multiplier on the constraints (88), (117), and (118), respectively. The last condition would imply a unit root in the multiplier ψ_t under a binding borrowing constraint, $\mu_t > 0$, if there were no depreciation ($\delta_h = 0$). Eliminating λ_t and μ_t with the first two conditions, leads to

$$\begin{aligned} 0 =& \gamma\left(h_{b,t}^{-\sigma_h} - ((h-h_{b,t})^{-\sigma_h}\right) + \left(\epsilon_b c_{b,t}^{-\sigma} - \epsilon_l c_{l,t}^{-\sigma}\right)\widetilde{z}_t x_t \\ &+ \left(\psi_t - \psi_{t-1}(1-\delta_h)\right)\sigma\epsilon_l c_{l,t}^{-\sigma-1}x_t\widetilde{z}_t x_t - \psi_t\sigma_h\gamma(h-h_{b,t})^{-\sigma_h-1}\ , \end{aligned} \tag{119}$$

$$0 = \left(\epsilon_b c_{b,t}^{-\sigma} - \epsilon_l c_{l,t}^{-\sigma}\right)\widetilde{z}_t h_{b,t} + \left(\psi_t - \psi_{t-1}(1-\delta_h)\right)\left(\sigma\epsilon_l c_{l,t}^{-\sigma-1}x_t\widetilde{z}_t h_{b,t} + \epsilon_l c_{l,t}^{-\sigma}\right). \tag{120}$$

The optimal plan given $\{\kappa_t\}_{t=0}^{\infty}$ is a set of sequences $\{h_{b,t}, c_{b,t}, c_{l,t}, x_t, s_t, \psi_t, \widetilde{z}_t, q_t/R_t^L\}_{t=0}^{\infty}$ satisfying (88), (115)-(120). Alternatively, κ_t can be endogenized given $\{s_t\}_{t=0}^{\infty}$.