Debt Non-Neutrality, Policy Interactions, and Macroeconomic Stability¹

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Abstract
We study the consequences of non-neutrality of government debt for macroeconomic stabilization policy in a sticky price model. Ricardian equivalence fails because public debt has a negative impact on its marginal rate of return and thus on private savings, which is induced by assuming transaction services of government bonds. In equilibrium, the dynamics of inflation have to be consistent with a stationary real debt sequence. Under aggressive monetary policy regimes, macroeconomic fluctuations tend to be stabilized if nominal budget deficits are low. In particular, a smooth evolution of debt limits inflation expectations under cost-push shocks, such that inflation variances can be reduced. Under a balanced budget policy, the central bank’s output gap - inflation volatility trade-off is improved relative to an environment where debt is neutral.

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1 Introduction

In the New Keynesian sticky price model temporary price stickiness entails monetary non-neutrality, and the nominal interest rate policies affect both inflation and output fluctuations. Policy makers in this model would like to eliminate fluctuations in both variables but simultaneous achievement of these goals is not possible if there are ‘cost-push’ impulses, i.e. stochastic disturbances that are not caused by variations in output demand. The central bank then faces a trade-off over the choice of output gap or inflation volatility.

In many analyses of the resulting macroeconomic stabilization problem (e.g. Clarida et al., 1999) fiscal policy is irrelevant for output and inflation determination since government debt is typically neutral and Ricardian equivalence (Barro, 1974) holds. However, the irrelevance of government debt is a strong assumption that does not seem to be supported by empirical evidence. The present paper, in contrast, studies the consequences of non-neutrality of public debt for the macroeconomic stabilization problem. Specifically, we investigate in how far fiscal policy can contribute to macroeconomic stabilization for a given monetary policy. Thus, we aim at discovering ways in which the non-neutrality of debt can be used to improve the trade-off faced by the central bank.

To this end, we present a sticky-price model where Ricardian equivalence fails because government bonds provide liquidity services that facilitate goods market transactions. This assumption has for example been used by Bansal and Coleman (1996), Lahiri and Vegh (2003), and by Canzoneri and Diba (2005) in different contexts. Liquidity services of government bonds can be justified through the observation that (short-run) government bonds can in general be more easily transformed into money than other assets, or through their potential role as collateral for many types of transactions, which is typically priced by a liquidity premium. The total rate of return on bonds thus consists of the real interest rate and a liquidity service component. Importantly, this establishes a positive link between government debt and aggregate goods demand. While an increase in real debt tends to lower its total rate of return via the liquidity service component, which induces agents to intertemporally substitute consumption to the present, it is in general associated with a higher real interest rate in an arbitrage-free equilibrium, consistent with a large body of empirical evidence.

The model is used to analyze the stabilization performance of simple fiscal and mon-

\footnote{For example, Elmendorf and Mankiw (1999) in their review of the earlier empirical literature conclude that most, though not all, studies find some relation between government debt and variables like output and real interest rates that point to a failure of Ricardian equivalence. The most recent available empirical results seem to converge on a set of compatible findings. Gale and Orszag (2004), Engel and Hubbard (2005), and Laubach (2008) all report a positive empirical relation between the level of government debt or deficits and the real interest rate in the U.S., a finding which Ardagna et al. (2004) corroborate for a panel of OECD countries.}

\footnote{See the references in footnote 1.}
etary policy rules. Monetary policy sets the short-run nominal interest rate in response to inflation, and fiscal policy specifies a fraction $1 - \kappa$ of nominal expenditures to be covered by budget deficits which lead to nominal bond issuance, while the rest is paid for by lump-sum taxes.

We find the following results. First, a low share of deficit finance is beneficial for inflation stabilization for any given monetary policy. To see why, note that cost push shocks lead to a rise in inflation, and a decline in output. Higher inflation further reduces the real value of public debt (given that policy ensures a unique and saddle path stable steady state). Since this reinforces the output contraction, it contributes to stabilizing inflation which turns out to have a lower variance if fiscal policy relies less on deficits (higher $\kappa$). Interestingly, if monetary policy is strongly anti-inflationary through a high reaction coefficient of the interest rate with respect to inflation, also the output variance can decrease for a lower deficit share. As a consequence, the well-known policy trade-off under cost-push shocks can disappear, in particular both inflation and output fluctuations can be minimized through a balanced budget fiscal policy.

Second, under a balanced budget regime both the variances of inflation and the output gap can be lower in the present model than in the corresponding model with neutral debt. The reason that debt non-neutrality can be beneficial for stabilization policy is that it limits admissible fluctuations of inflation by the requirement that the equilibrium sequence of real public debt must be stationary. Thus, inflationary expectations are contained in equilibrium, and more so when nominal debt is kept on a smoothly evolving path as under a policy of nominal budget balance.

Notably, the condition for saddle path stability in this model deviates from the Taylor principle usually found in New Keynesian models (e.g. Woodford, 2003a), in that it involves both monetary and fiscal policy rule parameters. The crucial restriction for stability is that policy must ensure that higher inflation reduces the real value of debt; in the model, this is automatically ensured for any monetary policy rule parameter (including an interest rate peg) if the government budget is balanced.

Models with non-neutral debt have been proposed before in the literature, with various sources for Ricardian non-equivalence. Overlapping generations models (Leith and Wren-Lewis, 2000, based on Blanchard, 1985) postulate that debt has direct wealth effects due to limited intergenerational altruism. Binding borrowing constraints are another reason why Ricardian equivalence breaks down. This has been discussed in a stylized model by Woodford (1990), or more recently in a heterogeneous agent context by Heathcote (2005), who also analyzes the effects of shocks to distortionary income tax rates. Distortionary taxation is possibly the most often discussed case of Ricardian non-equivalence, in which debt is non-neutral because the timing of taxes implies incentives to intertemporally shift factor supply. This has been discussed in real models by e.g. McGrattan (1994) and Schmitt-Grohe and Uribe (1997), and in sticky price models by Benigno and Woodford.
The present paper differs from these approaches in that it postulates a direct link between debt and aggregate demand which derives from the latter providing transaction services, such that non-neutrality arises even under absence of borrowing constraints, with infinite planning horizons (or perfect intergenerational altruism), and even if, as we assume, adjustments to the government budget are financed by lump-sum taxes. In our model, the stabilizing effect of keeping the nominal budget close to balance arises due to contained inflationary expectations in the presence of cost-push shocks. If, on the other hand, the government would use a distortionary tax rate to balance its budget, this would tend to exacerbate fluctuations, since the need to raise income tax rates in the wake of an adverse shock would (from the adverse effect on labor supply and thus wage costs) increase both the output drop and the inflation surge following a cost-push disturbance.

In the present model, we abstract from additional distortions through income taxation in order to isolate the debt-demand channel that we introduce in the most transparent way.

The rest of this paper is organized as follows. Section 2 presents the model and Section 3 the main results. Section 4 concludes.

2 The model

Throughout the paper, nominal variables are denoted by upper-case letters, while real variables are denoted by lower-case letters. A bar over a variable denotes a constant steady state value, and a caret operator marks a logarithmic deviation from steady state, \( \hat{z}_t = \log(z_t/z) \) for any variable \( z_t \).

Private sector There is a continuum of households indexed with \( j \in [0, 1] \). Households have identical asset endowments and identical preferences. Household \( j \) maximizes the expected sum of a discounted stream of instantaneous utilities \( u \) :

\[
E_0 \sum_{t=0}^{\infty} \beta^t u(c_{jt}, l_{jt}),
\]

where \( E_0 \) is the expectation operator conditional on the time 0 information set, and \( \beta \in (0, 1) \) is the subjective discount factor. The instantaneous utility \( u \) is assumed to be increasing in consumption \( c_{jt} \), decreasing in working time \( l_{jt} \), strictly concave, twice continuously differentiable, and to satisfy the usual Inada conditions. It is further restricted to be additively separable, \( u(c_{jt}, l_{jt}) = v(c_{jt}) - \mu(l_{jt}) \).

At the beginning of period \( t \) household \( j \) is endowed with holdings of money \( M_{jt-1} \) and government bonds \( B_{jt-1} \), which are carried over from the previous period. Purchases of the consumption good are assumed to be associated with real transaction costs. While it is commonly assumed that only money provides transaction services, here also holdings of government bonds reduce transaction costs. We view this assumption as reasonable, since government bonds can in general easily be transformed into money and serve as collateral.
for many types of transactions. We assume that the goods market opens before the asset market, such that households rely on the beginning-of-period holdings of government liabilities to reduce transaction costs.\(^3\)

The transaction cost function \(h(c_{jt}, M_{jt-1}/P_t, B_{jt-1}/P_t)\) is non-negative, increasing in \(c\), strictly decreasing in \(M_{jt-1}/P_t\) and in \(B_{jt-1}/P_t\), twice continuously differentiable in all arguments, and satisfies \(h_{cc} \geq 0\), \(h_{mm} > 0\), \(h_{bb} > 0\), \(\lim_{m \rightarrow 0} h_m = -\infty\), \(\lim_{b \rightarrow 0} h_b = -\infty\), and \(h_{cm} = h_{cb} (= h_{mb}) = 0\); the latter assumption implies that the transaction cost function is separable in all arguments (like in Lahiri and Vegh, 2003). We further assume that transaction costs are private costs that are paid to a particular sector whose only function is to rebate its receipts immediately to the household sector through lump-sum transfers. Both assumptions are made to isolate the effect of debt on the household’s consumption/saving decision and to facilitate comparisons with studies on cashless economies (see Woodford, 2003a). In particular, they ensure that money and bonds do not exert wealth effects, but only substitution effects.

In order to introduce supply side disturbances, we assume that households monopolistically supply differentiated labor services. Differentiated labor services \(l_{jt}\) are transformed into aggregate labor input \(l_t\), which can be employed for the production of the final good. The transformation is conducted via the aggregator \(l_t^{1-\vartheta_t} = \int_0^1 l_{jt}^{1-\vartheta_t} dj_t\). The elasticity of substitution between differentiated labor services \(\vartheta_t > 1\) varies exogenously over time. Cost minimization then leads to the following demand for differentiated labor services \(l_{jt}\),

\[
l_{jt} = (w_{jt}/w_t)^{-\vartheta_t} l_t, \quad \text{with} \quad w_t^{1-\vartheta_t} = \int_0^1 w_{jt}^{1-\vartheta_t} dj_t,
\]

where \(w_{jt}\) and \(w_t\) are the individual and the aggregate real wage rate, respectively. Household \(j\) faces a lump-sum tax \(P_t\tau_t\) (where \(P\) is the aggregate price level), and receives labor income \(P_t w_{jt} l_{jt}\) and dividends \(D_{jt}\) from monopolistically competitive firms and from financial intermediaries. After the goods market is closed, the financial market opens where households can either invest in government bonds \(B_{jt}\) at the price \(1/R_t\), or in money \(M_{jt}\). Household \(j\)’s flow budget constraint reads

\[
M_{jt} + B_{jt}/R_t + P_t c_{jt} + P_t h(c_{jt}, M_{jt-1}/P_t, B_{jt-1}/P_t) \leq P_t w_{jt} l_{jt} + B_{jt-1} + M_{jt-1} - P_t \tau_t + P_t \tau_t^* + D_{jt},
\]

where \(\tau_t^*\) is a lump-sum rebate of seignorage revenue received from the central bank.\(^4\) The household maximizes (1) subject to (2), (3), and non-negativity constraints on money and bonds for given initial values \(M_{j(-1)} = M_{-1} > 0\), and \(B_{j(-1)} = B_{-1} > 0\). The first order

\(^3\)The partial derivative of \(h\) with respect to the real value of beginning-of-period \(t\) money (bond) holdings \(M_{jt-1}/P_t, (B_{jt-1}/P_t)\) is denoted by \(h_{m_t}(h_b)\).

\(^4\)This assumption is purely for convenience, in order to remove this item from the government budget identity.
conditions for the household’s problem are given by

\begin{equation}
\lambda_{jt}(1 + h_e(c_{jt})) = \nu'(c_{jt}),
\end{equation}

(4)

\begin{equation}
\xi_t^{-1} w_j \lambda_{jt} = \mu'(l_{jt}),
\end{equation}

(5)

\begin{equation}
\beta E_t \left\{ \lambda_{jt+1} \frac{1}{\lambda_{jt}} \left[ (1 - h(b_{jt+1}) \frac{1}{\pi_{t+1}}) \frac{R_t}{\pi_{t+1}} \right] \right\} = 1/\beta,
\end{equation}

(6)

and \( \beta E_t \left[ \lambda_{jt+1} \pi_{t+1}^{-1} (1 - h_m(m_{jt+1}^{-1})) \right] = \lambda_{jt} \), where \( \lambda_{jt} \) is the Lagrange multiplier on the budget constraint, \( \pi_t = P_t/P_{t-1} \) is the gross inflation rate, \( m_{jt} \equiv M_{jt}/P_t \) and \( b_{jt} \equiv B_{jt}/P_t \) are real cash and government bond holdings, respectively, and \( \xi_t \equiv \vartheta_t/(\vartheta_t - 1) \) denotes the wage mark-up, which is assumed to follow an exogenous stochastic process (see below). Further, the transversality conditions \( \lim_{t \to \infty} E_0 \beta^t \lambda_{jt} b_{jt} = 0 \) and \( \lim_{t \to \infty} E_0 \beta^t \lambda_{jt} m_{jt} = 0 \) hold.

Equations (4) and (5) are first order conditions for consumption and labor supply. The central model element can be seen in equation (6), which is the first order condition for bond holdings. Here, the growth rate of the shadow price of wealth \( \lambda_{jt} \) is related to the expected total rate of return on government bonds (given in the square brackets), consisting of the real interest rate \( R_t/\pi_{t+1} \), and the marginal benefit from transaction services \( -h(b_{jt+1}) \pi_{t+1}^{-1} \). By the assumption \( h(b) > 0 \), the latter is decreasing in the stock of real bonds. Thus, a higher stock of bonds reduces their total rate of return. This either causes an intertemporal reallocation of consumption, since it requires the growth rate \( \lambda_{jt+1}/\lambda_{jt} \) to rise, or raises the real interest rate \( R_t/\pi_{t+1} \).

The production sector is standard: aggregate output \( y_t \) is defined as \( y_t^\epsilon = \int_0^1 y_t^\epsilon d\phi \), \( \epsilon > 1 \), where \( y_t^\epsilon \) is the amount produced by firm \( i \) denoting one out of a unit continuum of monopolistically competitive intermediate producers facing the demand constraint \( y_t = (P_{it}/P_t)^{-\epsilon} y_t \), with \( P_{it}^{1-\epsilon} = \int_0^1 P_{it}^{1-\epsilon} d\phi \) (\( P_{it} \) and \( P_t \) being the price of good \( i \) and the aggregate price level) and technology \( y_t = a_t l_{it} \), where \( l_t = \int_0^1 l_t d\phi \) and \( a_t \) denotes an exogenous productivity level. Labor demand satisfies: \( mc_{it} = w_t \), where \( mc \) is real marginal costs. Nominal price stickiness à la Calvo (1983) - Yun (1996) forces a measure \( \phi \in [0,1) \) of firms to adjust their previous period’s prices according to the simple rule \( P_t = \pi P_{t-1} \), where \( \pi \) denotes the average inflation rate, while the measure \( 1 - \phi \) chooses new prices \( \tilde{P}_{it} \) as the solution to \( \max_{P_{it}} E_t \sum_{s=0}^{\infty} \phi^s q_{it+t+s}(\pi^s \tilde{P}_{it+t+s} - P_{it+t+s} mc_{it+t+s}) \), s.t. \( y_{it+t+s} = (\pi^s \tilde{P}_{it+t+s})^{-1} P_{it+t+s} y_{it+s} \), where \( q_{it+t+s} \) is the appropriate discount factor. The first order condition for the optimal price setting of re-optimizing producers is

\begin{equation}
\tilde{P}_{it} = \frac{\epsilon}{\epsilon - 1} \frac{E_t \sum_{s=0}^{\infty} \phi^s q_{it+t+s} P_{it+t+s}^{1-\epsilon}(mc_{it+t+s})}{E_t \sum_{s=0}^{\infty} \phi^s q_{it+t+s} P_{it+t+s}(mc_{it+t+s})},
\end{equation}

(7)

where we used \( mc_{it} = mc_t \). Further, \( P_t \) satisfies \( P_t^{-\epsilon} = \phi (\pi P_t)^{-1-\epsilon} + (1 - \phi) \tilde{P}_t^{-\epsilon} \) and aggregate output \( y_t = (P_t^\epsilon/P_t)^{1-\epsilon} \), where \( (P_t^\epsilon)^{-\epsilon} = \int_0^1 P_t^{1-\epsilon} d\phi \) and thus \( (P_t^\epsilon)^{-\epsilon} = \phi (\pi P_t)^{-1-\epsilon} + (1 - \phi) \tilde{P}_t^{-\epsilon} \).
Public sector  The public sector consists of the fiscal authority and the central bank. The central bank generates revenues from money creation $P_t\tau^c_t$ that are directly rebated to households for simplicity. We assume that real index debt is not available, and that the fiscal authority issues nominally risk-less one-period bonds $B_t \geq 0$ at the price $1/R_t$ paying $B_t$ units of currency in period $t+1$. The government collects lump-sum taxes $\tau_t$ from households,

$$B_{t-1} = B_t/R_t + P_t\tau_t.$$  

(8)

Government expenditures on goods are normalized to zero, such that the services on outstanding debt are the only flow that needs to be financed, either by issuing new debt or by raising taxes. We postulate a simple fiscal policy rule that sets tax receipts as a linear reaction function in response to government outlays, with reaction coefficient $\kappa \in (0,1]$. This allows us to characterize the government’s choice of a nominal budget deficit by a single parameter, $\kappa$, and to focus on short-run deficit dynamics.\(^5\) The fiscal rule has the advantage that it can be related to restrictions on public deficits, like in the European stability and growth pact, or in some U.S. states; in particular, it includes the case of a balanced budget rule. Specifically, the fiscal rule is

$$P_t\tau_t = \kappa \frac{i_t B_{t-1}}{1+i_t}, \quad \kappa \in (0,1],$$  

(9)

with $i_t = R_t - 1$, where the feedback coefficient $\kappa$ denotes the fraction of expenditures financed by tax revenues, as opposed to deficits (we refer to $1-\kappa$ as the share of deficit finance). Inserting this in (8) shows

$$B_t - B_{t-1} = (1-\kappa)i_t B_{t-1}.$$  

(10)

Thus, in the case $\kappa = 1$ the budget balances in every period, such that nominal government bonds are constant over time, $B_{t-1} = B_t$. The assumption $\kappa > 0$ ensures that government debt grows on average with a rate which is strictly smaller than the interest rate $E_0 B_t/B_{t-1} < R_t$, implying $\lim_{t \to \infty} B_t \Pi_{v=1}^{t-1} R_v^{-1} = 0$. Hence, for any non-explosive sequence of real money and inflation, this policy guarantees government solvency $\lim_{s \to -\infty}(b_{t+s} R_t^{-1} \Pi_{v=1}^{t+s} m_{t+s}) \Pi_{v=1}^{t+v}/R_{t-1+v} = 0$.

The central bank transfers seigniorage to the households, $P_t\tau^c_t = M_t - M_{t-1}$, and controls the nominal interest rate $R_t$ on government bonds as a standard Taylor rule.

$$R_t = R(\pi_t) = \kappa_R \pi_t^{\rho_\pi}, \quad \rho_\pi > 0, \quad R_t \geq 1,$$  

(11)

We assume the support of all shocks to be small enough such that the central bank can choose $\kappa_R$ to ensure that $R_t \geq 1$ holds for all $t$.

\(^5\)In contrast, the literature on long-run sustainability of public finances, e.g. Bohn (1998), typically relates primary surpluses to levels of debt, rather than deficits.
Rational expectations equilibrium All households are symmetric such that we drop the individual index \( j \) to denote aggregate variables. The stochastic discount factor in the firms’ maximization problem \( \tilde{q}_{t+1} \) satisfies \( \tilde{q}_{t+1} = \beta^s (\lambda_{t+1} \tilde{P}_{t+1}^{-1} / \lambda_{t} \tilde{P}_t^{-1}) \). Since money enters the transaction cost function in a separable way, it is irrelevant for the analysis of the equilibrium behavior of the remaining variables and will therefore be neglected in what follows, and the private nature of transaction costs ensures \( c_t = y_t \).

Definition 1 A rational expectations equilibrium is a set of sequences \( \{y_t, l_t, \pi_t, P_t^*, \tilde{P}_t, m_{ct}, w_t, b_t, R_t\}_{t=0}^\infty \) satisfying the firms’ first order conditions \( mc_t = w_t \), (7) with \( \tilde{P}_t = \tilde{P}_t \), and \( P_t^{\lambda^*-} = \phi (\pi_{t-1})^{1-\epsilon} + (1 - \phi) \tilde{P}_t^{1-\epsilon} \), the households’ first order conditions
\[
\beta E_t \left[ \nu'(y_{t+1}) (1 + h_{ct,t+1}) \right]_{t+1} \pi_{t+1}^{-1} (1 - h_b (b_t \pi_{t+1}^{-1})) \] \( R_t = \left[ \nu'(y_t) (1 + h_{ct,t}) \right]_{t}^{-1} \), (12)
and \( \pi_t = P_t / P_{t-1} \), the aggregate resource constraint \( y_t = (P_t^*/P_t) a_t l_t \), where \( (P_t^*)^{-\epsilon} = \phi (\pi_{t-1})^{1-\epsilon} + (1 - \phi) \tilde{P}_t^{1-\epsilon} \), and the transversality condition \( \lim_{t \to \infty} \beta E_0 \frac{\nu'(a_0)}{1 + \beta (\rho)} b_t = 0 \), for fiscal and monetary policy satisfying \( b_t = (1 + (1 - \kappa) \epsilon) b_t^{\pi_t^{-1}} \) and (11), and given sequences of \( \{\xi_t\}_{t=0}^\infty \) and \( \{a_t\}_{t=0}^\infty \), and initial values \( P_{-1} > 0 \), \( P_{-1} > 0 \), and \( b_{-1} \equiv B_{-1}/P_{-1} > 0 \).

As implied by definition 1, the equilibrium sequence of public debt cannot separately be determined from the equilibrium sequences of the other variables, which leads to the failure of Ricardian equivalence. This property is due to the assumption that the level of debt affects its total rate of return \( h_b < 0 \). Below, we will compare the results coming from this model with one in which government debt is neutral.

A cautionary note is in order here: in the model with \( h_b = 0 \), the equilibrium would look very different. In this case, different sequences of public debt would leave the behavior of households and firms unchanged (another way of stating that Ricardian equivalence would hold). Thus, when \( h_b = 0 \) a rational expectations equilibrium would be defined \( \{y_t, l_t, \pi_t, P_t^*, \tilde{P}_t, m_{ct}, w_t, R_t\}_{t=0}^\infty \), excluding public debt and thus be independent of the fiscal policy regime (10). Hence, if \( h_b = 0 \), any sequence of real public debt would be consistent with a particular rational expectations equilibrium. When we turn to the analysis of locally stable equilibria, this difference becomes crucial: the debt sequence then has to be stationary in a fundamental equilibrium only in the model with \( h_b < 0 \).

Steady state A deterministic steady state \( (a_t = 1 \text{ and } \xi_t = \xi) \) of the model is characterized by constant values for output, inflation, and government bonds. Due to the assumption that transaction costs are private and separable, the first order conditions on consumption and labor, and the aggregate resource constraint uniquely determine steady state output by \( \mu'(\pi) \nu'(\pi)^{-1} (1 + h_c(\pi)) = (\theta - 1)(\epsilon - 1)/\varphi(\epsilon) \). A steady state further requires \( b_t = \bar{b} \) and \( \pi_t = \bar{\pi} \) (see 10 and 12). The fiscal and monetary policy specification
leads to the restriction
\[ \pi = 1 + (1 - \kappa)(\kappa R \pi^\rho - 1), \]  
(13)
on the steady state inflation rate. Whether condition (13) has a unique or multiple solutions for the steady state inflation rate depends on both policy parameters. The equilibrium condition for bond holdings (12) can be used to uniquely determine the steady state level of government bonds for a given steady state inflation rate,
\[ h_b(\bar{b}/\pi) = 1 - \pi / [\beta \kappa R \pi^\rho]. \]  
(14)
The steady state inflation rate and, thus, the steady state level of government bonds, is determined by (13). As policy satisfies \( \kappa \in (0,1) \) and \( \overline{R} = \kappa R \pi^\rho \geq 1 \), we know that \( G(\pi) \equiv (1 + (1 - \kappa)(\kappa R \pi^\rho - 1)) - \pi \) is strictly positive for \( \pi \to 0 \). Hence, \( G(\pi) = 0 \) has a unique solution if \( G'(\pi) < 0 \iff \rho_\pi < \left[ (\kappa R \pi^\rho - 1)(1 - \kappa) \right]^{-1} \). Note that (14) and \( h_b < 0 \) imply \( \kappa R \pi^\rho - 1 < 1/\beta \). Thus, we assume that the central bank chooses a sufficiently small value for \( \kappa_R \). Then, there exists a unique steady state inflation rate if
\[ \rho_\pi < \beta / (1 - \kappa). \]  
(15)
If (15) is satisfied, the model further exhibits a unique steady state level of government bonds. It may be noted that the existence of a steady state relies on the two effects of inflation on public debt. On the one hand, the real value of nominal debt decreases with inflation. On the other hand, higher inflation induces the central bank to raise the nominal interest rate such that the fiscal authority might issue new debt to finance additional interest rate payments. If \( G'(\pi) < 0 \), then there exists an inflation rate where both effects exactly offset each other, such that real public debt is constant.

Further note that condition (15) is sufficient for saddle point stability of the steady state such that indeterminacy (i.e. the steady state being a sink) and instability are ruled out. This result implies that – contrary to much of the New Keynesian literature – monetary policy activism in the sense \( \rho_\pi > 1 \) is not necessary for local determinacy in our model, due to the stabilizing influence of government debt; indeed, a nominal interest rate peg \( (\rho_\pi = 0) \) or a balanced budget \( (\kappa = 1) \) would be sufficient. Intuitively, the condition ensures that outside the steady state an increase in inflation will decrease real debt, which raises transaction costs and reduces aggregate goods demand, thus preventing inflation from rising further.\(^6\)

3 Results

This section consists of two parts. In the first, we show how public debt contributes to forecasts of macroeconomic aggregates and thereby affects the inflation variance. In

\(^6\)Further details on the steady state and local equilibrium determinacy are available in an additional appendix available at www.wiso.uni-dortmund.de/\textasciitilde mak-ansc/index.html.
the second and main part of this section, we (numerically) examine how the stance of monetary and fiscal policy affects the policy trade-off under cost push shocks, highlighting the stabilizing role of low deficits.

The model is log-linearized at the steady state and reduced to a set of equilibrium conditions in \( \hat{\pi}_t, \hat{b}_t, \hat{R}_t, \) and \( \hat{x}_t \), where \( x_t \) denotes the output gap, which is defined as deviation of actual output from its level \( y_t^* \) that would be realized if prices were perfectly flexible and cost push shocks were absent. In log-linearized form, the output gap satisfies 
\[
\hat{x}_t = \hat{y}_t - y_t^*,
\]
where \( \hat{y}_t = \{(1 + \vartheta) / (\vartheta + \sigma)\} \hat{a}_t \) with \( \sigma = -\frac{\varphi(1 - \varphi)}{1 + \varphi} > 0 \) and \( \vartheta = \frac{\varphi(1 - \varphi)}{1 + \varphi} > 0 \). Thus, a rational expectations equilibrium of the linearized model is a set of sequences \( \{\hat{x}_t, \hat{\pi}_t, \hat{b}_t, \hat{R}_t\}_{t=0}^\infty \) satisfying
\[
\begin{align*}
\sigma \hat{x}_t &= \sigma E_t \hat{x}_{t+1} - \hat{R}_t + (1 - \Psi) E_t \hat{\pi}_{t+1} + \Psi \hat{b}_t, \\
\hat{\pi}_t &= \beta E_t \hat{\pi}_{t+1} + \omega \hat{x}_t + \varphi_t, \\
\hat{b}_t &= \hat{b}_{t-1} + \eta \hat{R}_t - \hat{\pi}_t, \quad \eta \in [0, 1) \text{ and } \partial \eta / \partial \kappa < 0, \\
\hat{R}_t &= \rho \hat{\pi}_t,
\end{align*}
\tag{16}
\tag{17}
\tag{18}
\tag{19}
\]

Together with an exogenous stochastic process for the cost-push shock \( \varphi_t = (1 - \phi)(1 - \beta \phi)\phi^{-1} \xi_t \), with \( \xi_t = \rho_c \xi_{t-1} + \varepsilon_{ct} \) and \( \rho_c \in [0, 1) \), where \( \varepsilon_{ct} \) is i.i.d. with zero mean, \( \alpha_t = \pi \) for all \( t \), \( \omega = (\sigma + \vartheta)(1 - \phi)(1 - \beta \phi) / \phi \), \( \eta = \frac{1 - \kappa}{1 + (1 - \kappa)(1 - \vartheta)} \), and \( \Psi = \frac{\varphi(1 - \varphi)}{1 + \varphi} \).

Equation (16) specifies the evolution of real aggregate demand as a function of the nominal interest rate and inflation. If debt were neutral, consumption growth would only depend on the real interest rate; crucially, this is different here as real debt \( \hat{b}_t \) enters the demand equation. Equation (18) is the log-linearized flow budget constraint of the public sector under the assumed fiscal policy rule. Note that the composite parameter \( \eta \) is strictly decreasing in \( \kappa \).

### 3.1 Public debt and inflation expectations under flexible prices

We now turn to the impact of public debt non-neutrality on the stabilization of macroeconomic fluctuations. The focus here is on the modification that debt non-neutrality and fiscal policy entail concerning stabilization policy under cost-push shocks. When debt is non-neutral, the interaction between fiscal and monetary policies affects the variances of the output gap and inflation in our model, whereas only monetary policy is responsible for macroeconomic fluctuations when debt is neutral. The distinguishing feature of debt non-neutrality is that the evolution of public debt has to follow a convergent sequence, which imposes a restriction on feasible equilibrium sequences of inflation and output (-gap).

To disclose this mechanism, we first apply a simplified version of the model with flexible prices, \( \phi = 0 \), and an interest rate peg, \( \rho_\pi = 0 \). The equilibrium conditions for the log-linearized version of the model are in this case given by (16), (18), (19), and \( \hat{x}_t = -\left(\sigma + \frac{\varphi(1 - \varphi)}{1 + \varphi}\right)^{-1} \xi_t \). Comparing both versions, it turns out that inflation is less volatile
under debt non-neutrality when the cost-push shock is not too strongly autocorrelated.

**Proposition 1 (Variances under \( \phi = 0 \))** Suppose that prices are flexible and that the central bank pegs the nominal interest rate, \( \rho_\pi = 0 \). Then, the inflation variance is smaller under debt non-neutrality (\( \Psi > 0 \)) than under debt neutrality (\( \Psi = 0 \)) if \( \rho_c \leq 1/2 \).

**Proof.** See appendix.

The reason for this result is that in equilibrium the sequence of inflation has to be consistent with a stationary debt sequence. Consider, for example, a positive cost-push shock. This leads to an immediate decline in output and a rise in the expected total real rate of return from government bonds. Under debt neutrality, this is solely brought about by a rise in (expected) inflation. Under debt non-neutrality, the rate of return is jointly determined by inflation and real debt. Inflation then first increases and leads to lower real debt, thereby raising the real return. Once the cost-push shock has died out, inflation is below steady state, inducing the real value of public debt to converge back to its steady state value. Thus, debt non-neutrality induces inflation to depend on the real value of beginning-of-period debt, which is responsible for a reduction of the inflation variance if the autocorrelation of cost-push shocks is sufficiently small, e.g. \( \rho_c \leq 0.5 \) (since otherwise, the recovery of real debt and of inflation is delayed such that its variances can be higher than in the case where debt is neutral). In the next section, we show that this mechanism also holds in the sticky price version of the model under more general specifications for interest rate policy and also for more persistent shocks.

### 3.2 Policy interactions and macroeconomic stabilization

We now return to the sticky-price case and investigate the role of government debt for macroeconomic fluctuations by means of calculating variances for empirically plausible parameter values. The impact of different (fiscal and monetary) policy parameters on macroeconomic volatility is assessed by comparing the unconditional variances of the output gap and inflation relative to the unconditional variance of their source, i.e., the cost-push shock process \( \varphi_t \).

#### 3.2.1 Inflation and output gap variances

For convenience, we present the results in graphical form applying a set of deep parameters in accordance with values often found in the literature.\(^7\) In particular, we set preference parameters equal to \( \sigma = \vartheta = 2 \) and \( \beta = 0.99 \), the average (quarterly) gross nominal interest rate to \( \bar{R} = 1.01 \), the autocorrelation of cost-push shocks to \( \rho_c = 0.9 \), and the fraction of non-optimally price adjusting firms to \( \phi = 0.7 \), as is standard in the New Keynesian literature. We set the transaction cost elasticity equal to \( \Psi = 0.05 \) for the benchmark specification.

\(^7\)A closed form derivation of the solution can be found in the additional appendix mentioned in footnote 6.
Figure 1 displays the relative variance of inflation $\frac{\text{var}_\pi}{\text{var}_\varphi}$, and figure 2 the relative variance of the output gap $\frac{\text{var}_x}{\text{var}_\varphi}$, each for various values of the fiscal feedback parameter $\kappa$ and the inflation elasticity $\rho_\pi$ of the nominal interest rate. Evidently, interest rate policy faces the usual trade-off when the model is driven by cost-push shocks, in that higher values of $\rho_\pi$ lower the variance of inflation, but increase the variance of the output gap. What is new here is the influence of the fiscal policy parameter $\kappa$: a higher value of $\kappa$, i.e. a lower share of deficit financing, is generally associated with a lower inflation variance, while it has an ambiguous (but generally small) influence on the output gap variance. The lowest inflation volatility is achieved with a balanced budget policy.

Before turning to explanations, it is useful to compare the performance of different stabilization policies to the case where debt is neutral. Therefore, figure 3 shows the relative inflation and output gap variances for selected monetary and varying fiscal policy parameters in comparison to the latter case, which is labelled DN (for debt neutral). Recall that the DN model can be summarized by (19), $\sigma \tilde{x}_t = \sigma E_t \tilde{x}_{t+1} - \rho_\pi \tilde{R}_t + E_t \tilde{\pi}_{t+1}$, and $\tilde{\pi}_t = \beta E_t \tilde{\pi}_{t+1} + \omega \tilde{x}_t + \tilde{\varphi}_t$ (which accords to the prototype New Keynesian model). The relative output gap and inflation variances are displayed for $\rho_\pi = 1.5$ by the solid horizontal lines in figure 3.

Figure 3 further displays relative variances of the model with non-neutral debt for three different values of the monetary policy feedback parameter, $\rho_\pi \in \{1.25, 1.5, 1.75\}$

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8 In this case, $\Psi = 0$, steady state inflation is determined by $\pi = \pi_0 \beta$ and is, thus, independent of fiscal policy.
Figure 2: Relative output gap variance

(only points where the parameter combination entails saddle path stability are shown). Not surprisingly, higher $\rho_\pi$ values reduce the inflation variance and raise the output gap variance; this effect is already well known from the DN case. What is new here is seen by comparing the unmarked solid lines in figure 3 with the lines marked with squares. These show that for a given monetary policy stance – in this case for the example value $\rho_\pi = 1.5$ – the variances of both the output gap and inflation are lower for $\Psi > 0$ (lines marked with squares) than in the DN case (solid lines without markers) if the $\kappa$ is sufficiently high. Thus, a smooth debt sequence (through a high $\kappa$ value) appears to stabilize inflation and output gap fluctuations. The reason is that if a cost-push shock hits the economy, the output gap declines while inflation rises. Looking at (18) and (19), the inflation increase reduces the real value of public debt (despite the positive partial effect from a higher real interest rate) if $\eta \rho_\pi - 1 < 0$ is fulfilled, for which the equilibrium uniqueness condition (15) is sufficient. The debt reduction exerts – via a higher marginal rate of return – a negative impact on consumption, which tends to exacerbate the output contraction and to mitigate the rise in inflation caused by cost-push shocks. In equilibrium, the dampening effect on (future) inflation is strong enough to limit the real interest rate increase so much that, in the end, the output gap variance can be even lower than in the DN case.

The central mechanism here is that if government debt is relevant for the determination of the equilibrium values of inflation and the output gap, the equilibrium response of inflation is constrained by the requirement that real debt must return to its steady state value subsequent to a shock (see section 3.1). With a sufficiently aggressive monetary policy (high $\rho_\pi$), this implies that during the adjustment process future real rates of
interest must be lower than in steady state, which also reduces the impact of the shock on consumption and thus can mitigate the output gap volatility.

Figure 3 further shows that the inflation variance is always declining with higher $\kappa$, while its impact on the output gap variance is ambiguous. In fact, the inflation variance reaches a minimum in the balanced budget case $\kappa = 1$, where the negative influence of inflation on the real value of debt is strongest. The effects on the output gap variance are ambiguous, since debt reduction on the one hand reduces output partially, but the resulting inflation decrease makes room for lower real interest rates. Given an aggressive monetary policy ($\rho_\pi = 1.75$), however, there is no trade-off involved in fiscal policy: both the output gap and inflation variance decrease in $\kappa$ and are minimized by a balanced budget policy ($\kappa = 1$).\footnote{That a policy of budget balance is helpful for stabilizing output and inflation might not be true if the budget were balanced through adjustments of a distortionary income tax rate. Indeed, such a specification would be prone to indeterminacy in the first place (see Schmitt-Grohe and Uribe, 1997), and the inflation increasing and output reducing effects of cost-push shocks would be aggravated if labor taxes were raised to keep budget balance.}

### 3.2.2 Policy trade-offs under a balanced budget regime

The above analysis suggests that the policy trade-off between stabilizing inflation vs. the output-gap can in principle be alleviated by an appropriately chosen fiscal-monetary policy mix. To clarify this property, we take a closer look at the policy trade-off in this section. The left hand panel in figure 4 presents the relation between relative volatilities. Each symbol identifies a simulation run for a given interest rate policy parameter $\rho_\pi$.\footnote{That a policy of budget balance is helpful for stabilizing output and inflation might not be true if the budget were balanced through adjustments of a distortionary income tax rate. Indeed, such a specification would be prone to indeterminacy in the first place (see Schmitt-Grohe and Uribe, 1997), and the inflation increasing and output reducing effects of cost-push shocks would be aggravated if labor taxes were raised to keep budget balance.}
where $\rho_\pi$ was varied in 0.1 steps in the interval between 1.1 and 2.5, while $\kappa = 1$ is maintained throughout. The panel illustrates the trade-off between the inflation variance and the output gap variance faced by monetary policy. Obviously, this trade-off still exists in the case of debt non-neutrality: the line traced out by the stars is decreasing in this presentation, drawn for $\Psi = 0.05$, as it is for the debt neutral case, the latter signifying the debt neutral case. However, the line traced out by the stars is always below of the one traced out by the circles. This finding shows that the trade-off can be mitigated under debt non-neutrality.

The consequences for monetary policy performance are illustrated in the right hand panel of figure 4. There it is assumed that the central bank is engaged in what has been called ‘flexible inflation targeting’ (e.g. Svensson, 1999), and minimizes the loss (to the policy maker) entailed by inflation and output gap fluctuations given by the objective

$$E_0 \sum_{t=0}^{\infty} \beta^t \left( \hat{\pi}_t^2 + \lambda \tilde{\pi}_t^2 \right),$$

where $\lambda \geq 0$ is the relative weight on output gap fluctuations. We use this policy objective to compare the performance of different interest rate reaction coefficients $\rho_\pi$ in our model under a balanced budget regime with the debt neutral case. Given that the weight $\lambda$ is usually chosen to be small in the literature, ranging for example between 0.05 (Woodford, 2003b, Giannoni and Woodford, 2004) and 0.25 (Walsh, 2003, McCallum and Nelson, 2004), we use $\lambda = 0.1$ for figure 4. Such a loss function can also be derived by applying a second order approximation of household objective (1) at an undistorted steady state (see Woodford, 2003a), which in our model requires a balanced budget policy.

The right hand panel of figure 4 shows that the policy maker’s loss under debt non-neutrality is always smaller than in the DN case. Thus, the central bank’s cost of pursuing inflation stabilization are lower when fiscal policy is supportive in the sense of keeping the evolution of nominal debt on a smooth path. However, for a less stabilized debt sequence ($\kappa < 1$), the trade-off can even be worsened, and losses can be higher under non-neutral debt and active monetary policy.

4 Conclusion

This paper has explored the consequences of a specific type of public debt non-neutrality in a sticky-price business cycle model and examined the interaction of fiscal and monetary policy. Government debt matters for aggregate demand through its negative effects on its total rate of return, which is induced by considering transactions services of bonds. A rise in public debt thereby exerts an expansionary intertemporal substitution effect on consumption and thus a tendency for rising inflation. There is fiscal-monetary policy interaction, in that the central bank’s interest rate reaction to changes in inflation influences
interest rate services of public debt. The share of deficit finance in turn feeds back on output and inflation in equilibrium.

It is shown that effects of public debt on consumption growth can lead to results which substantially depart from those known from models with neutral debt. Non-neutrality of debt constrains the equilibrium inflation sequence to be consistent with a stationary real debt sequence. As a consequence, debt non-neutrality affects the ability of public policy to stabilize the inflation and output volatility arising from cost-push shocks. The well-known trade-off that these impart on monetary policy is existent here as well. However, the trade-off is improved under debt non-neutrality if the government keeps its budget close to balance, because a smooth evolution of the stock of debt favorably affects private expectations about future output and inflation. As a consequence, macroeconomic fluctuations will be smaller.

Appendix: Proof of proposition 1

Under an interest rate peg and flexible prices, the model with $\Psi > 0$ can be reduced to the following conditions in inflation and real public debt:

$-\gamma (1 - \rho) \hat{\xi}_t = (1 - \Psi) E_t \hat{\pi}_{t+1} + \Psi \hat{b}_t$

and

$\hat{b}_t = \hat{b}_{t-1} - \hat{\pi}_t$, where $\gamma = \sigma (\sigma + \varphi / \mu) ^{-1} > 0$. Applying the method of undetermined coefficients for a generic solution form featuring real public debt as a state variable $\hat{b}_t = \delta_b \hat{b}_{t-1} + \delta_{bc} \hat{c}_t$ and $\hat{\pi}_t = \delta_{\pi b} \hat{b}_{t-1} + \delta_{\pi c} \hat{c}_t$, leads to the following fundamental solution $\delta_b = 0$, $\delta_{\pi b} = 1$, $\delta_{bc} = -\delta_{\pi c}$ and $\delta_{\pi c} = \gamma (1 - \rho_c) / (1 - \rho_c + \Psi \rho_c)$.

The (unconditional) inflation variance satisfies $\text{var}(\hat{\pi}_t) = \text{var}(\hat{b}_{t-1}) + \delta_{\pi c}^2 \text{var}(\hat{\xi}_t) +$
$2\delta_{\pi e} \text{cov}(\tilde{b}_{t-1}, \tilde{\xi}_t)$, where $\text{cov}(\tilde{b}_{t-1}, \tilde{\xi}_t) = \delta_{\pi e} \rho_c \text{var}_\xi$ and covariance stationarity implies $\text{var}(\tilde{b}_{t-1}) = \text{var}(\tilde{b}_t) = \delta_{\pi e}^2 \text{var}_\xi$. Thus, the inflation variance is given by $\text{var}(\tilde{\pi}_t) = \left( \delta_{\pi e}^2 + \delta_{\pi e}^2 + 2\delta_{\pi e} \delta_{\pi e} \rho_c \right) \text{var}_\xi = 2\delta_{\pi e}^2 (1 - \rho_c) \text{var}_\xi$. When debt is neutral ($\Psi = 0$) the minimum state variable solution for inflation reads $\tilde{\pi}_t = \delta_{\pi e} \tilde{\xi}_t$ where $\delta_{\pi e} = -\gamma (1 - \rho_c) / \rho_c$, and its variance is $\text{var}_\pi = \delta_{\pi e}^2 \text{var}_\xi$. Hence, inflation is more volatile in the latter case if $2\delta_{\pi e}^2 (1 - \rho_c) < \left( (1 - \rho_c) \rho_c^{-1} \gamma \right)^2 \Leftrightarrow \sqrt{2(1 - \rho_c) \rho_c} < 1 - \rho_c + \Psi \rho_c$, which is more likely to hold for larger values for $\Psi$. For $\Psi \to 0$, this condition simplifies to $\left(1 - \rho_c^2\right) (2 \rho_c - 1) < 0 \Leftrightarrow \rho_c < 1/2$. 

References


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