Markovian Households*

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March 16, 2016

Abstract

This paper studies the consumption-savings problem of two-person households whose individual members cannot commit to future actions and might not cooperate. The interaction between household members is modeled as a stationary Markov-perfect game with the household's asset position as the single endogenous state variable. Intuitive first-order conditions are derived that show when lack of cooperation distorts household decision making relative to the case of full cooperation. A model version with idiosyncratic labor income risk then is used to explore the implications of lack of cooperation for precautionary savings, intra-household risk sharing and welfare.

Keywords: Two-Person Households, Markov-Perfect Equilibrium, Household Savings, Intra-Household Allocation

JEL Classification: C72, D13, D91

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*Preliminary version. Comments are welcome. I thank Christian Bredemeier, Dominik Sachs and Andreas Schabert for helpful comments and suggestions. Financial support from the German Research Foundation (DFG Priority Programme 1578) is gratefully acknowledged.

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1 Introduction

During the last three decades, a large empirical and theoretical literature has evolved that stresses the importance of analyzing household decision making not just at the aggregate household level but also at the intra-household level.[1] Within this literature, several model environments have been developed that allow to study how the interaction between different household members shapes the allocation of household resources. Broadly, these models can be divided into two categories: cooperative and non-cooperative models. In both categories, the household decision process has mostly been studied in static environments. While there a few recent papers that consider cooperative households in a dynamic context, dynamic non-cooperative models of the household are still the exception.[2]

This paper studies a model that allows to understand how cooperation and lack thereof affects the decision making of a two-person household in a dynamic environment with endogenous labor supply and uncertainty. More specifically, the model studies the consumption-savings problem of a household whose individual members cannot commit to future actions and might not cooperate. First, I characterize the basic properties of household decision making with and without cooperation. Then, I use a calibrated model version with idiosyncratic income risk to explore the implications of lack of cooperation for precautionary savings, intra-household risk sharing and welfare.

In the model, both household members consume a private consumption good and supply labor, subject to a single joint household budget constraint. The household has access to a risk-less one-period bond that allows the transfer of resources across periods. The individual household members are infinitely-lived and exhibit spousal altruism, i.e. they care about the utility that their spouse derives from its own consumption and leisure. The household members may differ with respect to the wage rate that they earn on the labor market, their utility function and their degree of spousal altruism. The interaction between the household members is modeled as a Markov-perfect game (see Maskin and Tirole, 2001) with the household's asset position as the single endogenous state variable. In each period, the equilibrium choices of the household members are hence characterized by policy functions that only depend on the joint asset position. Since the Markov-perfect equilibrium concept is a refinement of the subgame-perfect equilibrium concept, it guarantees that the decisions of the

[1] See Chiappori and Donni (2011), Chiappori and Mazzocco (2014) and Chiappori and Meghir (2015) for recent surveys of this literature.

[2] Examples of dynamic models that use a cooperative framework are Mazzocco (2007, 2008), Ligon (2011), Ortigueira and Siassi (2013). Examples of non-cooperative household models that study dynamic decision problems are Browning (2000), Konrad and Lommerud (2000), Hertzberg (2012) and Eckstein and Lifshitz (2015). This paper is particularly related to Hertzberg (2012). For details, see the literature review at the end of this section.

household members are time consistent.[3] While the household members take the allocation of future consumption and working time as given, they fully understand that these actions will depend on future household asset holdings which they can affect via their current consumption and labor supply decisions.

I consider two cases for the household problem. In the first case, the household members agree on a joint household objective function that reflects their relative degree of altruism. Based on this objective, the two individuals cooperate and jointly decide about the intra-household allocation of consumption and working time, as well as savings. In the second case, the household members do not cooperate. Instead, the allocation of household resources is determined in a non-cooperative game played by the two individuals. More specifically, the household members choose their consumption and labor supply to maximize their own objective function, taking as given the decisions of the spouse.

For both cases, the equilibrium can be characterized by a set of intuitive optimality conditions. While the optimal labor supply conditions are standard and do not differ for the cooperative and non-cooperative household, the allocation of consumption within the household and over time is chosen differently. In the cooperative case, these decisions are associated with a sharing rule and a standard Euler equation. For a given level of savings, the sharing rule states that the ratio of the household members' marginal utilities of consumption is equal to the inverse of the ratio of their respective weights in the joint household objective. As in the collective model of the household proposed by Chiappori (1988, 1992), this sharing rule guarantees the efficient distribution of income within the household. The non-cooperative solution to the household problem does not involve an explicit sharing rule but an additional Euler equation instead. There are therefore two Euler equations in this case, one for each of the two household members. Relative to the Euler equation associated with the cooperative household, these two Euler equations feature an additional term which can be interpreted as a wedge that distorts the consumption-savings trade-off. More specifically, this wedge reflects the household members' inability to control the decisions of their spouse as well as their lack of commitment and disagreement about the valuation of spousal consumption and labor supply.

Under the assumption that household members are homogeneous, i.e. they have the same preferences with respect to their own consumption and leisure, face the same wage rate and exhibit the same degree of altruism, I show that the cooperative and the non-cooperative solutions to the house-

[3] By requiring household members to only condition their decisions on the minimal payoff-relevant state, equilibria that involve reputational considerations based on trigger strategies with complex history-dependence are ruled out by construction.

hold problem coincide when the household members are perfectly altruistic, i.e. when they place the same weight on their own utility and spousal utility. When the household members are imperfectly altruistic and place a lower weight on spousal utility than on their own, the non-cooperative household exhibits an undersaving (or overborrowing) bias relative to the cooperative case which is captured by the wedge in the Euler equation. For a household member, saving increases the amount of resources available to the household in the next period. However, these additional resources will encourage the spouse to consume more and work less in the next period, leaving less resources for the saving household member to consume. When imperfectly altruistic, the household member thus has an incentive to save less and consume more in the current period.[4] Using numerical examples, I show that the savings distortion remains when household members are heterogeneous and investigate how outcomes vary with the exact type of heterogeneity. In particular, I show that non-participation of one household member in the labor market affects the allocation of consumption within a non-cooperative household relative to a cooperative one. More specifically, the household member that does not participate in the labor force obtains more resources compared to the cooperative case.

Given that lack of cooperation leads to undersaving when imperfectly altruistic household members do not cooperate, it is interesting to ask how costly the savings distortion is in terms of welfare. To answer this question in a quantitatively meaningful way, I study a version of the household problem with incomplete financial markets and costly borrowing in which the two household members are both subject to idiosyncratic productivity shocks. When financial markets only provide partial insurance against idiosyncratic labor income risk, a role for precautionary savings emerges. In addition, when the household members' idiosyncratic income risks are not perfectly positively correlated, spousal labor supply adjustments are a useful instrument to share risks within the household and reduce the impact of adverse shocks on household consumption (see e.g. Ortigueira and Siassi, 2013). The main finding is that the savings distortion can substantially reduce precautionary savings of non-cooperative households relative to cooperative ones when the individual members are imperfectly altruistic. As a result, non-cooperative households rely more on labor supply adjustments to smooth consumption in response to bad shocks, making intra-household risk-sharing more important for these types of households relative to cooperative ones. However, non-cooperative households not only experience more volatile labor supply but also more volatile consumption compared to cooperative households since they have lower buffer stock savings. A welfare exercise reveals that the welfare costs of lack

[4] Such an intertemporal distortion has previously been shown by Hertzberg (2012) in a related model framework. A detailed discussion about the relation to this paper can be found at the end of this section.

of cooperation are sizable for even modest deviations from perfect spousal altruism.

Related Literature This paper is related to the large and growing literature on multi-person households (see e.g. Chiappori and Donni, 2011). As mentioned earlier, despite its size, there are relatively few papers that study such households in a dynamic context (see Chiappori and Mazzocco, 2014, for details). Two exceptions that use an intertemporal version of the collective household model to study consumption-savings problems with endogenous labor supply as in this paper are Mazzocco (2008), who estimates the preferences of the household members, and Ortigueira and Siassi (2013), who study the importance of intra-household risk sharing in a general equilibrium incomplete markets model.

Examples of non-cooperative two-person households in a dynamic context are Browning (2000), Konrad and Lommerud (2000), Doepke and Tertilt (2014) and Eckstein and Lifshitz (2015).[5] Browning (2000) studies a non-cooperative savings problem in a two-period model with two household members where the husband might not live as long as the wife. Konrad and Lommerud (2000) use an intertemporal model to study investment in human capital. In their two-stage model, the within-household allocation is determined non-cooperatively at the first stage and via Nash-bargaining at the second stage, with the non-cooperative outcome as the threat point. Doepke and Tertilt (2014) build a non-cooperative household model with human capital investment to study the impact of transfers to females on growth. As in this paper, Eckstein and Lifshitz (2015) also study cooperative and non-cooperative solutions to the dynamic decision problem of a two-person household with endogenous labor supply but abstract from savings.

Another paper that develops a non-cooperative dynamic model is Hertzberg (2012), which is closely related to this paper. The author also studies the consumption-savings problem of a non-cooperative household with two imperfectly altruistic members that lack commitment and share their wealth, relating it to the commitment case. As in this paper, Hertzberg (2012) shows that imperfect spousal altruism leads to an undersaving bias when household members do not cooperate. However, there are crucial differences between our studies. Whereas the model studied by Hertzberg (2012) is a deterministic one with a finite time horizon and exogenous labor income, the one studied in this paper features an infinite time horizon, endogenous labor supply and uncertainty. These differences have several important implications. First, to analyze his model, Hertzberg (2012) studies the subgame-

[5]Examples of studies that look at non-cooperative decision making in a static context are Lundberg and Pollak (1993), Konrad and Lommerud (1995), Chen and Woolley (2001), Lechene and Preston (2011).

perfect equilibrium of the non-cooperative game between the household members, while I study the interaction as a stationary Markov-perfect game. More specifically, Hertzberg (2012) derives closed form solutions for the model via backward induction that prove the existence of a savings distortion. To derive these solutions, the author relies on specific functional forms for the utility functions of the household members. By contrast, I analyze the equilibrium of the non-cooperative game via time-invariant first-order conditions. In particular, I derive generalized Euler equations for the household members (see e.g. Krusell et al., 2002), which demonstrate the presence of savings wedges that distort intertemporal decision making. These Euler equations admit a very intuitive economic interpretation and allow me to illustrate the determinants of the savings distortion. In addition, I do not have to assume specific functional forms to show that non-cooperative households save less than cooperative ones when the household members are imperfectly altruistic. Methodologically, our two studies can thus be seen as complementary. Second, the inclusion of endogenous labor supply allows me to study how within-household heterogeneity with respect to wages or labor-force participation affects savings and the intra-household allocation of resources. In particular, I show that lack of cooperation does not only lead to intertemporal distortions but can also result in intratemporal distortions compared to a cooperative setting. The third important difference between our studies is that I perform a welfare analysis of the costs of lack of cooperation in the absence of commitment by using a calibrated model with uncertainty, whereas Hertzberg (2012) uses numerical examples to illustrate the welfare properties of his deterministic life-cycle model. These welfare comparisons can be viewed as complementary since Hertzberg (2012) highlights a life-cycle savings motive for the household, whereas I consider a precautionary savings motive. Compared to Hertzberg's (2012) analysis, my setting with idiosyncratic income risk also allows me to study how imperfect spousal altruism affects intra-household risk sharing and precautionary savings.

The remainder of this paper is organized as follows. In Section 2, I present the model and discuss the implications of lack of cooperation for household decision making. Section 3 studies a version of the non-cooperative household problem with Stackelberg leadership. Section 4 introduces labor income risk into the model of Section 2 and investigates how non-cooperative interaction between the household members affects precautionary savings, intra-household risk sharing and welfare. Section 5 concludes.

2 Model

The model considers the dynamic decision problem of a two-person household. Time is discrete, starts in $t = 0$ and goes on forever. A household consists of two infinitely-lived individuals, one male (M) and one female (F), and will be interpreted as a married couple. Throughout the paper, I will not model household dissolution and assume that the individuals form a couple at $t = 0$ and never break up. The objective of household member $i \in \{F, M\}$ is given by

$$\sum_{t=0}^{\infty} \beta^t U_i(c_{it}, c_{\sim it}, n_{it}, n_{\sim it}),$$

where

$$U_i(c_{it}, c_{\sim it}, n_{it}, n_{\sim it}) = u_i(c_{it}, n_{it}) + \theta_i u_{\sim i}(c_{\sim it}, n_{\sim it}).$$

The period objective function U_i is the sum of two parts.[6] The first part, u_i, is the utility that household member i derives from its own consumption c_i and labor supply n_i. This function satisfies $u_{c,i}, -u_{n,i} > 0$ and $u_{cc,i}, u_{nn,i} < 0$, where $u_{x,i}$ ($u_{xx,i}$) denotes the first (second) derivative of u_i with respect to the argument $x \in \{c, n\}$. The second part, $\theta_i u_{\sim i}$, reflects household member i's altruism towards household member $\sim i \equiv \{F, M\} \setminus i$, where the parameter $\theta_i > 0$ measures the degree of altruism.[7] For $\theta_i = 1$, i is perfectly altruistic and places the same weight on its "private" utility u_i and the utility $u_{\sim i}$ that its spouse derives from consuming $c_{\sim i}$ and working $n_{\sim i}$. For $\theta_i < 1$ ($\theta_i > 1$), individual i places a lower (higher) weight on the utility that its spouse $\sim i$ derives from its consumption and leisure than on its own private utility u_i. The two household members might differ from each other with respect to their wage rate $w_i > 0$, utility function u_i and altruism θ_i.[8] The wage rates w_i are exogenous and constant, i.e. the model is a partial equilibrium one.[9] The household does not face any uncertainty about the future. Section 4 will relax this assumption and introduce uncertainty about the household members' future labor productivities.

[6]Mazzocco (2008) uses the same preferences in the context of an intertemporal collective household model.

[7]In the literature, individual preferences that involve this type of altruism are sometimes also referred to as "caring preferences" (see e.g. Chiappori, 1992, or Chen and Woolley, 2001).

[8]I abstract from heterogeneity in discount factors since this case has already received a lot of attention in the literature (see e.g. Hertzberg, 2013, Jackson and Yariv, 2014, Schaner, 2015, and references therein) and is well understood. In particular, it is known that heterogeneous time preferences lead to a time-inconsistency problem even when a group of individuals decides collectively, i.e. under cooperation. In this paper, I want to highlight a time-inconsistency problem that arises due to other factors and explore how it depends on household cooperation.

[9]Of course, one can interpret the model as a general equilibrium one, assuming a small open economy setting, competitive labor markets and a firm sector that employs a production technology with constant returns to scale and labor as the only input.

The household faces the joint period budget constraint

$$w_F n_{Ft} + w_M n_{Mt} + a_t (1 + r_t) = c_{Ft} + c_{Mt} + a_{t+1}. \tag{1}$$

It has access to a one-period bond a_{t+1} that yields a net return of r_{t+1} in the subsequent period. A negative asset position means that the household is in debt. The assumption of a joint household budget constraint is commonly used in the literature (see e.g. Mazzocco, 2008, and Ortigueira and Siassi, 2013). It is also an empirically plausible assumption. As noted by Hertzberg (2012), for 2002, the General Social Survey documents that the majority of married couples in the United States (53.35%) share their financial resources (see Smith et al., 2011).

I assume that the interest rate $r_{t+1} = r(a_{t+1})$ can vary with the savings position:

$$\frac{\partial r(a_{t+1})}{\partial a_{t+1}} \leq 0.$$

In particular, $r(a_{t+1})$ may decrease with a_{t+1} for negative asset values values, i.e. the interest rate increases with the size of the credit. The relationship between the interest rate and the asset position is exogenously imposed and might reflect e.g. the lender's marginal costs of monitoring and enforcing a loan. When making their decisions, the household members internalize that the interest rate changes with savings a_{t+1}. For the remainder, it will be convenient to define the marginal (net) interest rate $R(a_{t+1}) \equiv r(a_{t+1}) + a_{t+1} \times (\partial r(a_{t+1}) / \partial a_{t+1})$ associated with savings a_{t+1}.

The debt-elastic interest rate schedule is introduced for two reasons. First, since I analyze a partial equilibrium model with exogenous prices, an interior and unique solution for the steady state asset level is not generally assured.[10] The debt-elasticity of the interest rate $r(a_{t+1})$ provides a mechanism that induces an interior and unique steady state even for given prices.[11] Second, for analytical and computational tractability, it is not feasible to introduce a standard debt limit $a_{t+1} \geq \underline{a}$ into the model. More specifically, I will need differentiability of the equilibrium objects for the analysis which does not necessarily hold when a debt limit is imposed. As will be shown in greater detail below, the debt-elastic interest rate will have very similar implications for the household savings decision as an ad hoc debt limit.

The interaction between the household members is modeled as a dynamic game. I assume that

[10] This will even be true when $r_{t+1} = \bar{r}$ and $\beta = 1/(1+\bar{r})$, where $\bar{r}$ is a positive constant.

[11] Schmitt-Grohé and Uribe (2003) also consider a debt-elastic interest rate to induce stationarity in a model of a small open economy with a constant world interest rate.

the household members act in a time consistent way and are not able to commit to actions beyond the current period. More specifically, I restrict attention to stationary Markov-perfect equilibria (see Maskin and Tirole, 2001). The strategies of the household members are thus assumed to be Markov, i.e. they are only conditioned on the minimal payoff-relevant state (see Maskin and Tirole, 2001), which is the joint asset position a_t in this case. The Markov-perfect equilibrium (MPE) is a refinement of the subgame-perfect (Nash) equilibrium and therefore ensures that the equilibrium strategies are time consistent. It also rules out the possibility of reputational considerations that are based on the use of trigger strategies. As in Krusell et al. (2002), I restrict attention to stationary equilibria with differentiable policy and value functions. This restriction is made to facilitate the analysis by allowing the derivation of first-order conditions that intuitively highlight the main forces of the model. I will first look at a household whose members optimize under full cooperation, and then consider the case of non-cooperative household members.

2.1 Household Problem under Cooperation

Under cooperation, the household members agree on an objective function for the household and jointly decide about current consumption and labor supply of both household members as well as savings without commitment. The literature on multi-person households typically refers to this case as the "collective model of the household" (see e.g. Chiappori, 1988, 1992). In the cooperative case, the household objective is given as

$$\sum_{t=0}^{\infty} \beta^t U\left(c_{Ft}, c_{Mt}, n_{Ft}, n_{Mt}\right),$$

with period objective function

$$U\left(c_{Ft}, c_{Mt}, n_{Ft}, n_{Mt}\right) = U_F\left(c_{Ft}, c_{Mt}, n_{Ft}, n_{Mt}\right) + \mu U_M\left(c_{Ft}, c_{Mt}, n_{Ft}, n_{Mt}\right),$$

where $\mu > 0$ is a constant relative Pareto weight. As in Browning et al. (2006), the period objective can be rewritten as

$$U\left(c_{Ft}, c_{Mt}, n_{Ft}, n_{Mt}\right) = \left(1 + \mu \theta_M\right) u_F(c_{Ft}, n_{Ft}) + \left(\mu + \theta_F\right) u_M(c_{Mt}, n_{Mt}).$$

When the household members agree on how to evaluate intra-household allocations, the household decision problem is equivalent to that of a utilitarian planner who assigns constant weights $1+\mu\theta_M$ and $\mu+\theta_F$ to the utility functions u_F and u_M and chooses consumption and labor supply of the household members without commitment. In the remainder, I will assume that $\mu = 1$, i.e. the weights placed on u_F and u_M only reflect the relative altruism of the household members. If e.g. $\theta_M > \theta_F$ holds, the household objective assigns a higher weight to u_F compared to u_M.

Given the focus on MPE, the household problem is formulated recursively.[12] In each period, the household solves

$$\max_{c_F,c_M,n_F,n_M} \sum\nolimits_{i\in\{F,M\}} (1+\theta_{\sim i})\, u_i(c_i,n_i) + \beta\mathcal{V}(a'(a,c_F,c_M,n_F,n_M)),$$

where savings a' are given by the period budget constraint

$$a' = w_F n_F + w_M n_M + a\,(1+r(a)) - c_F - c_M.$$

The continuation value $\mathcal{V}$ satisfies

$$\mathcal{V}(a) = \sum\nolimits_{i\in\{F,M\}} (1+\theta_{\sim i})\, u_i(\mathcal{C}_i(a),\mathcal{N}_i(a)) + \beta\mathcal{V}(\mathcal{A}(a)),$$

where $\mathcal{C}_F\,(a)$, $\mathcal{C}_M\,(a)$, $\mathcal{N}_F\,(a)$, $\mathcal{N}_M\,(a)$ and $\mathcal{A}\,(a)$ are the policy functions that determine consumption, labor supply and savings in the subsequent period.[13] These functions only depend on the beginning-of-period asset position. Since the household members cannot commit to future actions, they take these policies as given. However, they recognize that they can affect the future allocation via the asset position a'. In a stationary equilibrium, $c_i = \mathcal{C}_i\,(a)$ and $n_i = \mathcal{N}_i\,(a)$, $i \in \{F,M\}$, as well as $a' = \mathcal{A}\,(a)$ hold. The policy functions that govern future decisions thus coincide with the policy functions that determine the optimal decisions in the current period. The MPE for the household problem under cooperation is formally defined as follows[14]:

Definition 1. *A stationary Markov-perfect equilibrium for the household problem under cooperation is given by a set of functions* $\{\mathcal{A},\mathcal{C}_F,\mathcal{C}_M,\mathcal{N}_F,\mathcal{N}_M,\mathcal{V}\}$ *such that for all a,*

[12] The time indices are hence dropped and a prime is used to denote variables of the next period.

[13] Note that the continuation value $\mathcal{V}$ of the household can only be written as a single value for the whole household since the individual members share the same discount factor.

[14] The definition of the MPE is formulated by using notation similar to that used in Niemann et al. (2013) for a public policy problem.

$$(i)\ \ \{\mathcal{X}(a)\}_{\mathcal{X}\in\{\mathcal{C}_F,\mathcal{C}_M,\mathcal{N}_F,\mathcal{N}_M\}} = \underset{c_F,c_M,n_F,n_M}{\arg\max} \left\{ \begin{array}{l} \sum_{i\in\{F,M\}}(1+\theta_{\sim i})\,u_i(c_i,n_i) \\ +\beta\mathcal{V}(a'(a,c_F,c_M,n_F,n_M)) \end{array} \right\},$$

$$(ii)\ \ \mathcal{V}(a) = \sum_{i\in\{F,M\}}(1+\theta_{\sim i})\,u_i(\mathcal{C}_i(a),\mathcal{N}_i(a)) + \beta\mathcal{V}(\mathcal{A}(a)),$$

$$(iii)\ \ a'(a,c_F,c_M,n_F,n_M) = w_F n_F + w_M n_M + a\,(1+r(a)) - c_F - c_M,$$

$$(iv)\ \ \mathcal{A}(a) = a'(a,\mathcal{C}_F(a),\mathcal{C}_M(a),\mathcal{N}_F(a),\mathcal{N}_M(a)).$$

The equilibrium for the household decision problem under full cooperation between the two household members can be characterized by the conditions stated in the following proposition:

Proposition 1. *The Markov-perfect equilibrium for the household decision problem under cooperation satisfies the conditions*

$$\frac{u_{c,F}(\mathcal{C}_F(a),\mathcal{N}_F(a))}{u_{c,M}(\mathcal{C}_M(a),\mathcal{N}_M(a))} = \frac{1+\theta_F}{1+\theta_M}, \tag{2}$$

$$-u_{n,i}(\mathcal{C}_i(a),\mathcal{N}_i(a)) = u_{c,i}(\mathcal{C}_i(a),\mathcal{N}_i(a))w_i, i\in\{F,M\}, \tag{3}$$

$$u_{c,F}(\mathcal{C}_F(a),\mathcal{N}_F(a)) = \beta u_{c,F}(\mathcal{C}_F(\mathcal{A}(a)),\mathcal{N}_F(\mathcal{A}(a)))\,(1+R(\mathcal{A}(a))), \tag{4}$$

$$\mathcal{A}(a)+\mathcal{C}_F(a)+\mathcal{C}_M(a) = w_F\mathcal{N}_F(a)+w_M\mathcal{N}_M(a)+a\,(1+r(a)), \tag{5}$$

for all a.

Proof. See Appendix A.1.1.

Condition (2) states that the cooperative solution to the household problem allocates consumption and working time of the household members such that the ratio of the household members' marginal utility of consumption $u_{c,F}(c_F,n_F)/u_{c,M}(c_M,n_M)$ equals the inverse of the ratio of the respective utility weights $(1+\theta_F)/(1+\theta_M)$. In the literature, this condition is often referred to as a "sharing rule" (see e.g. Chiappori, 1992). If $\theta_i > \theta_{\sim i}$, the optimal intra-household allocation implies that the marginal utility of consumption is higher for i relative to $\sim i$. Condition (3) is a standard labor supply condition which requires that the marginal rate of substitution between consumption and labor supply of household member i equals the respective wage rate w_i. Condition (4) is the Euler equation for the household that governs how resources are allocated intertemporally. In principle, the set of equilibrium conditions also involves an Euler equation for household member M. However, this condition is redundant here since the sharing rule (2) implies that the growth rate of the marginal utility of consumption is equalized across household members. Condition (5) is simply the household budget constraint evaluated at the equilibrium policies. Given consumption and labor supply of the household members, this budget constraint determines equilibrium savings $a' = \mathcal{A}(a)$. Note that the exact values for the degree of altruism of the household members do not matter for the household allocation if $\theta_F = \theta_M$.

The equilibrium conditions (2)-(5) are standard in the literature that uses the collective household model in a dynamic context (see e.g. Mazzocco, 2008, or Ortigueira and Siassi, 2013). One exception is the term $R(a') = r(a') + a' \times (\partial r(a')/\partial a')$ that appears in the Euler equation (4), measuring the (assumed) marginal effect of a' on borrowing costs.[15] However, the debt-elastic interest rate has a very similar effect on the saving behavior of the household as an ad hoc borrowing constraint $a' \geq \underline{a}$ that prevents the household from reducing its asset position below the level $\underline{a}$. Define the "risk-free rate" $\bar{r} = R(0) = r(0)$. Just like the debt-elastic interest rate, a binding borrowing constraint $a' = \underline{a}$ would also imply that $u_{c,i}(c_i, n_i) \geq \beta u_{c,i}(c_i', n_i')(1 + \bar{r})$ holds (see Ortigueira and Siassi, 2013). In both cases, the household will not borrow beyond a certain level, either because of being outright rationed or due to an interest rate schedule that increases with debt.

When the household members agree on a joint household objective function and collectively make their decisions based on it, the implemented outcome does not depend on whether the household can commit to future actions or not, i.e. the optimal cooperative household allocation under commitment is time consistent (see Appendix A.2). Since the optimal household allocation under commitment is Pareto optimal with respective to the joint household objective function, the cooperative solution without commitment is Pareto optimal as well, making the cooperative household solution a useful benchmark for the non-cooperative case.

2.2 Household Problem without Cooperation

Now assume that the individual household members do not cooperate. The interaction between the household members is modeled as a non-cooperative simultaneous-move game.[16] In each period, the household members F and M simultaneously choose consumption c_i and labor supply n_i, taking as given the decisions of their spouse $c_{\sim i}$ and $n_{\sim i}$. In addition, they also take as given the policy functions that determine the allocation of household resources in the next period $\mathcal{X}(a')$, $\mathcal{X} \in \{\mathcal{A}, \mathcal{C}_F, \mathcal{N}_F, \mathcal{C}_M, \mathcal{N}_M\}$, given savings a'.

[15]To economize on notation, I will occasionally exploit stationarity and write a' instead of $\mathcal{A}(a)$.

[16]Section 3 relaxes the simultaneous-move assumption and considers the case of Stackelberg leadership.

From the perspective of household member i, the joint household budget constraint is

$$a' = w_i n_i + w_{\sim i} n_{\sim i} + a(1 + r(a)) - c_i - c_{\sim i}. \tag{6}$$

Given the consumption and labor supply decisions of both household members, savings a' are determined residually to satisfy the budget constraint. The household members however internalize the impact of their own decisions on household savings. The decision problem of household member $i \in \{F, M\}$ is given by

$$\max_{c_i, n_i} u_i(c_i, n_i) + \theta_i u_{\sim i}(c_{\sim i}, n_{\sim i}) + \beta \mathcal{V}_i(a'(a, c_i, c_{\sim i}, n_i, n_{\sim i})), \tag{7}$$

where spousal consumption $c_{\sim i}$ and labor supply $n_{\sim i}$ are taken as given, and savings a' are given by the budget constraint (6).

The continuation value $\mathcal{V}_i$ is defined recursively as

$$\mathcal{V}_i(a) = u_i(\mathcal{C}_i(a), \mathcal{N}_i(a)) + \theta_i u_{\sim i}(\mathcal{C}_{\sim i}(a), \mathcal{N}_{\sim i}(a)) + \beta \mathcal{V}_i(\mathcal{A}(a)). \tag{8}$$

The definition of the MPE for the non-cooperative household problem is as follows:

Definition 2. *A stationary Markov-perfect equilibrium for the household problem without cooperation is given by a set of functions $\{\mathcal{A}, \mathcal{C}_F, \mathcal{C}_M, \mathcal{N}_F, \mathcal{N}_M, \mathcal{V}_F, \mathcal{V}_M\}$ such that for all a,*

(i) $\{\mathcal{X}(a)\}_{\mathcal{X} \in \{\mathcal{C}_i, \mathcal{N}_i\}} = \arg\max_{c_i, n_i} \left\{ \begin{array}{c} u_i(c_i, n_i) + \theta_i u_{\sim i}(\mathcal{C}_{\sim i}(a), \mathcal{N}_{\sim i}(a)) \\ + \beta \mathcal{V}_i(a'(a, c_i, \mathcal{C}_{\sim i}(a), n_i, \mathcal{N}_{\sim i}(a))) \end{array} \right\}, i \in \{F, M\},$

(ii) $\mathcal{V}_i(a) = u_i(\mathcal{C}_i(a), \mathcal{N}_i(a)) + \theta_i u_{\sim i}(\mathcal{C}_{\sim i}(a), \mathcal{N}_{\sim i}(a)) + \beta \mathcal{V}_i(\mathcal{A}(a)), i \in \{F, M\},$

(iii) $a'(a, c_F, c_M, n_F, n_M) = w_F n_F + w_M n_M + a(1 + r(a)) - c_F - c_M,$

(iv) $\mathcal{A}(a) = a'(a, \mathcal{C}_F(a), \mathcal{C}_M(a), \mathcal{N}_F(a), \mathcal{N}_M(a)).$

The key difference relative to Definition 1 is that the two household members now independently make their consumption and working decisions based on their own objective, with the joint household budget constraint providing the link between the individual household members' actions. Condition (i) requires that the strategies chosen by the household members in equilibrium are indeed optimal responses to each other, i.e. the policy functions form a Nash equilibrium. The equilibrium conditions for the non-cooperative household problem are given by the following proposition:

Proposition 2. *The Markov-perfect equilibrium for the household decision problem without cooperation satisfies the conditions (3),(5), and*

$$u_{c,i}(\mathcal{C}_i(a),\mathcal{N}_i(a)) = \beta u_{c,i}(\mathcal{C}_i(\mathcal{A}(a)),\mathcal{N}_i(\mathcal{A}(a))) \tag{9}$$

$$\times (1+R(\mathcal{A}(a))+\Gamma_i(\mathcal{A}(a))),\ i \in \{F,M\}, \tag{10}$$

with

$$\Gamma_i(a) = \left(\theta_i \frac{u_{c,\sim i}(\mathcal{C}_{\sim i}(a),\mathcal{N}_{\sim i}(a))}{u_{c,i}(\mathcal{C}_i(a),\mathcal{N}_i(a))} - 1\right)\left[\frac{\partial \mathcal{C}_{\sim i}(a)}{\partial a} - w_{\sim i}\frac{\partial \mathcal{N}_{\sim i}(a)}{\partial a}\right], \tag{11}$$

for all a.

Proof. See Appendix A.1.2.

The optimal labor supply conditions are the same for the non-cooperative and the cooperative household problem. The important difference is that the ratio $u_{c,F}(c_F,n_F)/u_{c,M}(c_M,n_M)$ now is not pinned down by a sharing rule (see condition (2)) but by two generalized Euler equations instead. In contrast to the cooperative case, two intertemporal optimality conditions thus matter for the equilibrium outcome. Following Krusell et al. (2002), the Euler equations can be called generalized due to the presence of the function Γ_i which involves derivatives of the policy functions $\mathcal{C}_{\sim i}$ and $\mathcal{N}_{\sim i}$ with respect to the asset position. The function Γ_i can be interpreted as a wedge that distorts the optimal consumption-savings decision of household member i relative to the cooperative case.

This wedge admits an intuitive interpretation. To see this, it is helpful to multiply Γ_i with $u_{c,i}$, which yields

$$(\theta_i u_{c,\sim i}(\mathcal{C}_{\sim i}(a),\mathcal{N}_{\sim i}(a)) - u_{c,i}(\mathcal{C}_i(a),\mathcal{N}_i(a))) \times \left[\frac{\partial \mathcal{C}_{\sim i}(a)}{\partial a} - w_{\sim i}\frac{\partial \mathcal{N}_{\sim i}(a)}{\partial a}\right].$$

The expression in squared brackets measures the change in household resources that a marginal increase in wealth a induces by changing the consumption and working decisions of household member $\sim i$. The expression in round brackets measures the marginal valuation of this change in resources from the perspective of household member i. Suppose that $\partial \mathcal{C}_{\sim i}(a)/\partial a > 0$ and $\partial \mathcal{N}_{\sim i}(a)/\partial a = 0$, i.e. there is no wealth effect on labor supply and a marginal increase in assets a only leads to an increase in consumption of household member $\sim i$, leaving its labor supply decision unchanged. The first term in round brackets, $\theta_i u_{c,\sim i}$, is the marginal increase in utility that individual i derives from the increase in spousal consumption due to its altruism. The second term, $-u_{c,i}$, is the marginal decrease in utility that household member i experiences because a higher value of $c_{\sim i}$ reduces the amount of resources that i can spend on its own consumption. The term in round brackets thus measures the net marginal change in utility of household member i associated with a change in available resources given by the term in squared brackets. If the two terms in round brackets sum up to zero, the wedge Γ_i disappears from the Euler equation. If the marginal valuation does not equal zero, the wedge Γ_i will deviate from zero as well and distort the consumption-savings trade-off relative to the cooperative case.

When the household members are heterogeneous, it is difficult to make a clear statement about how the wedges Γ_F and Γ_M affect the behavior of the household in detail. Therefore, the next section will consider homogeneous household members, i.e. household members which share the same attributes $x_i = x_{\sim i} = x$ for $x \in \{\theta, u, w\}$, which implies that $\mathcal{X}_i(a) = \mathcal{X}_{\sim i}(a) = \mathcal{X}(a)$ for $\mathcal{X} \in \{\mathcal{C}, \mathcal{N}\}$ in a symmetric equilibrium. Under this assumption, one can isolate the role of lack of cooperation for household decision making. The case of heterogeneous household members will further be examined in Section 2.5.

2.3 Homogeneous Household Members

When the household members share the same attributes, the equilibrium for the cooperative and the non-cooperative household problem can be characterized by the following lemma:

Lemma 1. *If household members are homogeneous, i.e. $x_i = x_{\sim i} = x$ for $x \in \{\theta, u, w\}$, the Markov-perfect equilibrium for the household decision problem satisfies the conditions*

$$u_c(\mathcal{C}(a), \mathcal{N}(a))w = -u_n(\mathcal{C}(a), \mathcal{N}(a)), \tag{12}$$

$$\begin{aligned} u_c(\mathcal{C}(a), \mathcal{N}(a)) &= \beta u_c\left(\mathcal{C}(\mathcal{A}(a)), \mathcal{N}(\mathcal{A}(a))\right) \\ &\quad \times \left(1 + R(\mathcal{A}(a)) + \Gamma(\mathcal{A}(a))\right), \end{aligned} \tag{13}$$

with

$$\Gamma(a) = \begin{cases} (\theta - 1)\left[\frac{\partial \mathcal{C}(a)}{\partial a} - w\frac{\partial \mathcal{N}(a)}{\partial a}\right], & \text{without cooperation,} \\ 0, & \text{with cooperation,} \end{cases} \tag{14}$$

as well as

$$\mathcal{A}(a) + 2\mathcal{C}(a) = 2w\mathcal{N}(a) + a\left(1 + r(a)\right), \tag{15}$$

for all a.

Proof. Without differences in altruism, wage rates and utility functions $\mathcal{X}_F = \mathcal{X}_M = \mathcal{X}$ holds for

$\mathcal{X} \in \{\mathcal{C}, \mathcal{N}\}$, which implies that the first-order conditions for F and M coincide. As a result, only one of the two optimal labor supply conditions and (for the non-cooperative household solution) one Euler equation is needed to pin down the equilibrium outcome. For the cooperative household solution, the sharing rule (2) becomes redundant as it is always satisfied for homogeneous household members.

As summarized by the following corollary, perfect altruism and the absence of within-household heterogeneity imply that the equilibrium outcome does not depend on the household members' ability to cooperate.

Corollary 1. *If household members are homogeneous and perfectly altruistic ($\theta = 1$), the outcome of the MPE under cooperation coincides with the outcome of the MPE without cooperation.*

Proof. If $\theta = 1$, $\Gamma(a) = 0$ holds regardless of whether household members cooperate or not. The equilibrium conditions (12)-(15) then are identical for the household decision problems with and without cooperation.

In contrast to the cooperative solution to the household problem, the exact value of the degree of altruism θ matters for the non-cooperative solution if $\theta \neq 1$. In particular, θ governs the magnitude (and the sign) of the savings distortion given by the wedge Γ_i. To understand the savings distortion, suppose that $\partial\mathcal{C}(a)/\partial a > 0$ as well as $\partial\mathcal{N}(a)/\partial a \leq 0$ hold.[17] With $u_c > 0$, it then follows from (14) that $\Gamma(a) \lesseqqgtr 0$, if $\theta \lesseqqgtr 1$. If individual household members cannot commit to future actions and do not cooperate, imperfect altruism ($\theta < 1$) hence leads a two-person household to save less (or borrow more) relative to the case of perfect altruism ($\theta = 1$) and thus relative to the cooperative case (see Corollary 1). The presence of imperfect altruism effectively lowers the marginal return from saving for a household member. While an additional unit of resources transfered into the next period yields the marginal return $1 + R(a')$, it will also encourage the spouse to consume more ($\partial\mathcal{C}(a')/\partial a' > 0$) and work less ($\partial\mathcal{N}(a')/\partial a' \leq 0$), which reduces the resources available for the saving household member to consume in the subsequent period. If the household member is perfectly altruistic ($\theta = 1$), this change in future spousal consumption and labor supply is valued like a change in its own consumption and working time. As a result $\Gamma = 0$ holds and the Euler equation has the same shape as in the cooperative case. If the household member is imperfectly altruistic ($\theta < 1$), the increase in future spousal consumption and labor supply is valued less than the decline in utility due to the decrease in available resources, leading to a negative wedge $\Gamma < 0$. In the current period by contrast, the

[17] For standard utility functions that are used in the literature, these properties usually hold.

household member can directly allocate financial resources towards its own consumption. As a result, it has an incentive to reduce savings and allocate more wealth into the present, where it can directly consume the resources itself.[18]

The intuition behind the savings distortion of the non-cooperative household is similar to that in political economy models of public debt (see Persson and Svensson, 1989, and Alesina and Tabellini, 1990), where turnover risk and disagreement between political parties result in a deficit bias. In these models, the incumbent party has an incentive to front-load public spending since it might not be in charge of allocating resources in future periods and disagrees with the way these resources are spent by its successor. Aguiar and Amador (2011) and Chatterjee and Eyigungor (2016) demonstrate that there is in fact a direct link between the behavior of a government in such a political economy model and the behavior of a benevolent policy maker who exhibits quasi-geometric discounting (see Laibson, 1997, or Krusell et al., 2002).[19] Similarly, as first demonstrated by Hertzberg (2012) for a finite-horizon model without labor supply, one can show a direct link between a two-person household which consists of non-cooperative, imperfectly altruistic individuals that discount geometrically and a representative household which discounts in a quasi-geometric fashion.

2.4 Imperfect Spousal Altruism and Quasi-Geometric Discounting

Consider the decision problem of a quasi-geometric household who consumes, works and saves (see e.g. Laibson, 1997, or Krusell et al., 2002).[20] In a given period $t \geq 0$, this household values the stream of current and future consumption and working time $\{c_{t+s}, n_{t+s}\}_{s=0}^{\infty}$ according to

$$u(c_t, n_t) + \delta \sum_{s=1}^{\infty} \beta^s u(c_{t+s}, n_{t+s}),$$

where $u(\cdot)$ is the same period utility function used before, β a standard (long-run) discount factor and $\delta > 0$ the household's short-run discount factor. For $\delta = 1$, the household has standard time-consistent preferences and therefore does not have an incentive to deviate from a plan made in the past about future actions. If the short-run discount factor deviates from one, the household exhibits preference

[18] Another way to think about the undersaving (or overborrowing) bias is pointed out by Hertzberg (2012, p. 13) who proves the existence of such a bias for a finite-horizon setting with exogenous income (see Proposition 1 in his paper). When the household members share their financial wealth, savings are a public good and imperfect altruism leads the members to contribute less to the provision of this good than with perfect spousal altruism.

[19] See also Cao and Werning (2016) for a related discussion in a continuous-time setting.

[20] In the literature, quasi-geometric discounting sometimes is also referred to as quasi-hyperbolic discounting. However, Krusell et al. (2002) point out that, mathematically speaking, quasi-geometric is the more appropriate term to describe this type of discounting.

reversals over time, making such commitments to future actions time inconsistent. More specifically, for $\delta < 1$, the household has an incentive to delay costly actions into future periods. In the context of the consumption-savings problem in this paper, the household would like to delay (costly) saving today and commit to save more in the future. However, in the next period $t+1$, the household will again be tempted to delay saving, making a commitment to save more in the future time inconsistent. For $\delta > 1$, the household's temptation goes into the opposite direction, leading to an "oversaving bias". As pointed out by Laibson (1997), the decision problem of such a quasi-geometric household can be modeled as a dynamic game between multiple successive selves of the household.

To analyze this game, I follow Krusell et al. (2002) and study the stationary MPE that is the infinite-horizon limit of the MPE of the finite-horizon household problem.[21] To make the problem of the representative household comparable to that of the two-person household in size, I rescale consumption and labor supply in the household budget constraint by a factor of two.[22] In recursive notation, the decision problem then is given as

$$\max_{c,n} u(c,n) + \delta\beta\mathcal{V}(a'(a,c,n)),$$

where savings a' are given via the period budget constraint

$$a' = 2wn + a(1+r(a)) - 2c,$$

and the continuation value $\mathcal{V}$ satisfies

$$\mathcal{V}(a) = u(\mathcal{C}(a),\mathcal{N}(a)) + \beta\mathcal{V}(\mathcal{A}(a)).$$

The MPE for the decision problem of the quasi-geometric household is formally defined as follows:

Definition 3. *A stationary Markov-perfect equilibrium for the decision problem of the quasi-geometric household is given by a set of functions $\{\mathcal{A},\mathcal{C},\mathcal{N},\mathcal{V}\}$ such that for all a,*

(i) $\{\mathcal{X}(a)\}_{\mathcal{X}\in\{\mathcal{C},\mathcal{N}\}} = \arg\max_{c,n} \{u(c,n) + \delta\beta\mathcal{V}(a'(a,c,n))\},$

[21] In addition, I again assume differentiability of the policy and value functions. Bernheim et al. (2015) focus on subgame-perfect equilibria in general and study under which conditions quasi-geometric households can overcome their commitment problems by using different (non-Markov) strategies.

[22] Alternatively, one could also follow Hertzberg (2012) and rewrite the equilibrium conditions of the two-person household in terms of aggregate household consumption to obtain the direct link between the non-cooperative and the quasi-geometric household. However, this would require one to place the additional restriction of homogeneity on the household utility function. Details are available upon request.

(ii) $\mathcal{V}(a) = u(\mathcal{C}(a), \mathcal{N}(a)) + \beta \mathcal{V}(\mathcal{A}(a))$,

(iii) $a'(a,c,n) = 2wn + a(1 + r(a)) - 2c$,

(iv) $\mathcal{A}(a) = a'(a, \mathcal{C}(a), \mathcal{N}(a))$.

As in Hertzberg (2012), define $\Delta \equiv 1/(1+\theta_i) - \theta_{\sim i}/(1+\theta_{\sim i})$. This statistic measures the difference between the relative weight that i puts on its own private utility and the relative weight that $\sim i$ puts on i's utility. With homogenous household members, Δ reduces to $(1-\theta)/(1+\theta)$. The next proposition, which is a version of Proposition 2 in Hertzberg (2012), shows a direct link between the quasi-geometric representative household and the non-cooperative two-person household:

Proposition 3. *If $\delta = 1/(1+\Delta)$, the Markov-perfect equilibrium for the decision problem of the representative quasi-geometric household can be characterized by the same conditions given by Lemma 1 for the non-cooperative household.*

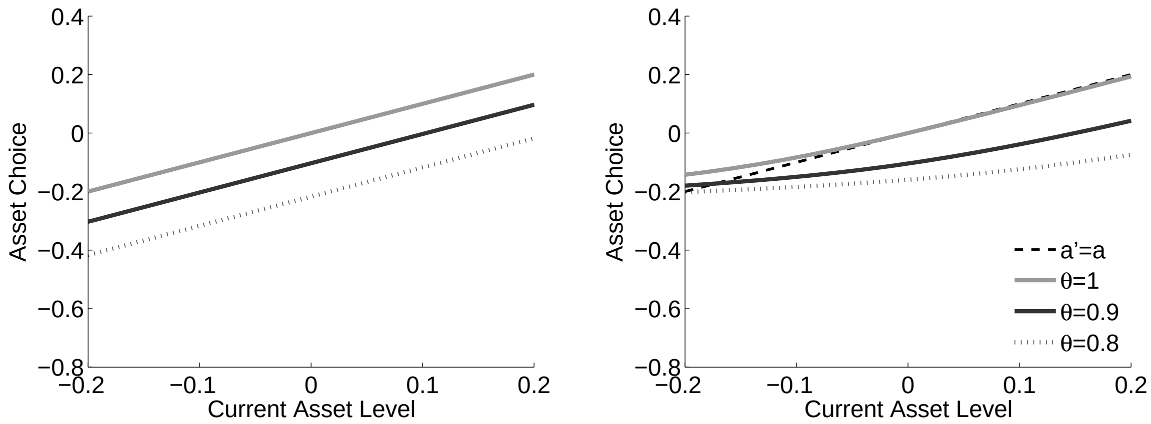
Proof. See Appendix A.1.3.

Since $1/(1+\Delta) = (1+\theta)/2$, Proposition 3 provides a direct link between the short-run discount factor δ and the degree of altruism θ, demonstrating that imperfect spousal altruism leads a non-cooperative two-person household to effectively discount in a quasi-geometric fashion even though its members exhibit standard geometric discounting. Hertzberg (2012) has previously established this link for a finite-horizon model without endogenous labor supply. In contrast to this paper, his derivation of this result relies on a specific functional form for the utility function, a finite time-horizon and exogenous household income.

An important implication of Proposition 3 is that the behavior of the non-cooperative two-person household exhibits the basic properties highlighted for (infinitely-lived) quasi-geometric households in the literature (see e.g. Krusell et al., 2002, Chatterjee and Eyigungor, 2016). In particular, when household members are imperfectly altruistic ($\theta < 1$) and the interest rate is constant by assumption, i.e. $r(a)$ is always debt-inelastic ($\partial r(a)/\partial a = 0$), there is no interior steady state a^* with $\mathcal{A}(a^*) = a^*$ unless the (constant) interest rate is given by $r = 1/\beta - 1 - \Gamma(a^*)$. If however, $\beta(1+r) = 1$ and $\theta < 1$ hold, the wedge $\Gamma(a)$ is negative and the non-cooperative household keeps on accumulating debt (see Figure 1a), leading consumption and/or leisure to sharply decline in the long run.[23] As shown by Figure 1b, the debt-elastic interest rate introduces a mechanism that discourages the persistent accumulation of debt and induces an interior steady state a^*.

[23] For the case $\theta > 1$, the opposite effect would lead to persistent accumulation of savings.

Figure 1: Savings policy function



(a) Constant interest rate ($\partial r/\partial a = 0$) (b) Debt-elastic interest rate ($\partial r/\partial a \leq 0$)

2.5 The Role of Within-Household Heterogeneity

In Section 2.3, I have demonstrated that in the absence of within-household heterogeneity, lack of cooperation distorts household decision making when individual household members are imperfectly altruistic. I will now examine how heterogeneity between household members affects the behavior of the non-cooperative household.

2.5.1 Model Specification

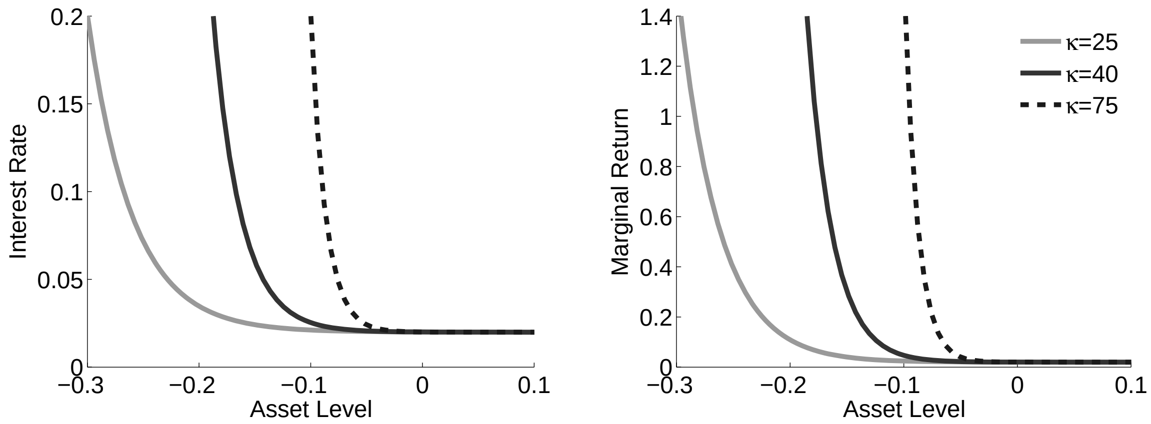
To study the impact of within-household heterogeneity on non-cooperative household decision making, I use numerical examples. More specifically, I choose a baseline calibration for the model under the assumption that household members are homogeneous and then perform comparative statics with respect to a specific attribute x_i, with $x \in \{\theta, w\}$, leaving the other attributes unchanged. In particular, I will consider mean-preserving spreads for one specific attribute $x \in \{\theta, w\}$, keeping the average value constant across household members: $x_F = (1-\varepsilon)x$ and $x_M = (1+\varepsilon)x$, $\varepsilon \in [0,1)$. I do not consider preference heterogeneity for the numerical exercises, i.e. $u_i = u_{\sim i} = u$.

For the utility function, I use the specification

$$u(c,n) = \alpha \frac{c^{1-\gamma}-1}{1-\gamma} + (1-\alpha)\frac{(1-n)^{1-\eta}-1}{1-\eta}, \alpha \in (0,1), \gamma, \eta > 0.$$

The elasticity parameters are normalized to $\gamma \to 1$ and $\eta \to 1$. A consumption share of $\alpha = 0.3$ is chosen to match a steady-state working time of one third for the cooperative household. The average

Figure 2: Interest rate schedule



(a) $r(a)$ (b) $r(a)+a\times(\partial r/\partial a)$

wage rate w is normalized to one, i.e. $w_F = w_M = 1$ in the baseline scenario without heterogeneity.

Following Schmitt-Grohé and Uribe (2003), the interest rate schedule is specified as

$$r(a) = \bar{r} + \psi\left(\exp\left(-\kappa a\right) - 1\right).$$

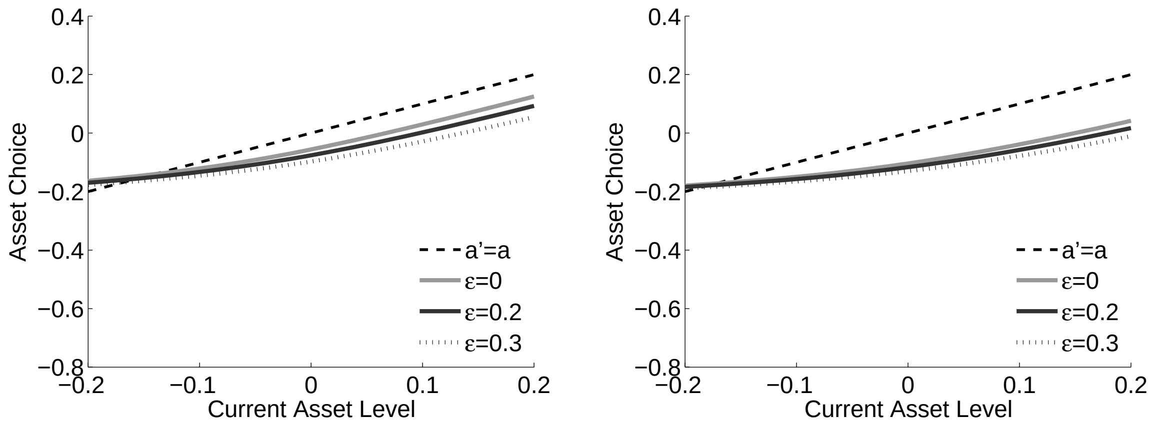
The parameters ψ and κ are set to 0.0001 and 25, whereas $\bar{r}$ is set to 0.02. The interest rate schedule $r(a)$ is shown in Figure 2 for different κ-values. If the household saves, the interest rate is virtually invariant to the asset position and increasing for negative asset values. The chosen parametrization for $r(a)$ ensures that the interest rate smoothly moves away from $\bar{r}$ for negative asset values. This property keeps the policy functions differentiable and makes the use of standard computational methods feasible.[24] The parameter κ governs the steepness of the interest rate schedule. For $\psi = 0$, the interest rate is always constant ($r(a) = \bar{r}$) and thus independent of the asset position. The discount factor β is set to $1/(1+\bar{r})$. The interest rate schedule $r(a)$ thus implies a steady-state asset value of zero for the cooperative household solution.

2.5.2 Degree of Altruism

For the non-cooperative case, Figure 3 depicts the savings policy function for different θ- and ε-values. The overall picture is that an increase in the spread ε tends to increase the household's incentive to borrow, leading to a lower (negative) steady-state asset value. For his finite-horizon setting without endogenous labor supply, Hertzberg (2012) also considers heterogeneity in spousal

[24] Appendix A.4 provides details on the numerical solution algorithm used in this paper.

Figure 3: Savings policy function and differences in the degree of altruism θ_i



(a) $\theta = 0.95$

(b) $\theta = 0.9$

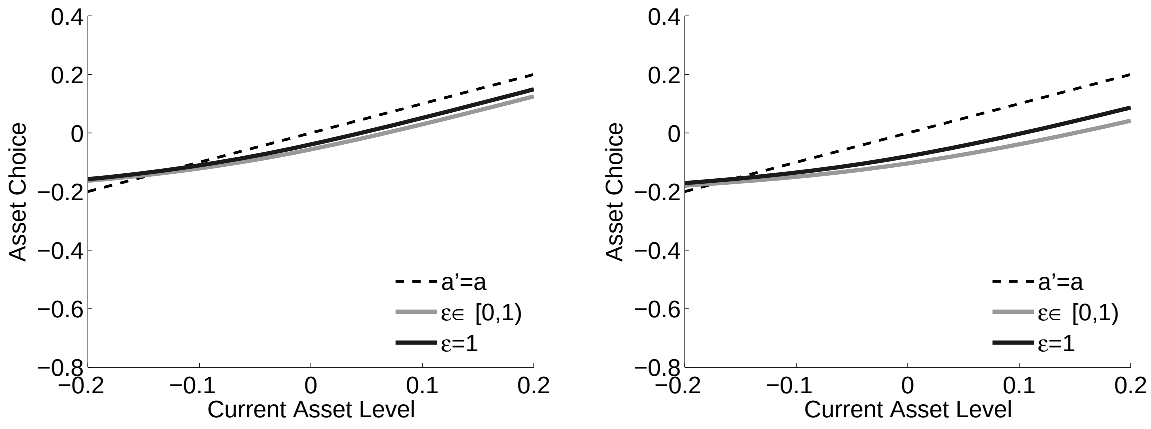
altruism and points out that it is however not the average degree of altruism θ that governs the savings distortion but Δ. Hertzberg's (2012) finding implies that the same aggregate savings distortion can be generated by a different combination of θ_F and θ_M as long as $\Delta = 1/(1+\theta_F) - \theta_M/(1+\theta_M)$ remains unchanged. For my model, I can confirm this result numerically. Note that Δ increases with the spread ε.

While the aggregate savings distortion can be generated by different combinations of relative spousal altruism, the intra-household allocation of consumption and labor supply will however vary and always reflect relative spousal altruism. As in the cooperative case, the ratio of the marginal utility of consumption $u_{c,F}$ and $u_{c,M}$ is also given by the ratio $(1+\theta_F)/(1+\theta_M)$ in the non-cooperative case, which is consistent with Hertzberg's (2012) findings.

2.5.3 Wage Rate

It is well documented that female workers tend to receive a lower wage rate in the labor market than male workers. This phenomenon is also referred to as the "gender pay gap" in the literature (see e.g. Blau and Kahn, 2000). Does this pay gap matter in the context of the household problem studied in this paper? It turns out that it does not if $\varepsilon \in [0,1)$. Again, I consider mean-preserving spreads ε for the household members, such that $w_F = 1-\varepsilon$ and $w_M = 1+\varepsilon$ since the average wage rate has been normalized to one. While different spreads ε imply different expressions for Γ_i, the effect of these differences cancels out across household members, leaving the savings policy of the household and the ratio $u_{c,F}/u_{c,M}$ unaffected. Labor supply of both household members is however adjusted to the

Figure 4: Savings policy function and differences in the wage rate w_i



(a) $\theta = 0.95$ (b) $\theta = 0.9$

changed intra-household wage distribution.

For $\varepsilon = 1$, household savings are affected relative to $\varepsilon \in [0,1)$. In this case, the labor supply condition does not hold with equality since the female household member finds it optimal not to participate in the labor market, i.e. $n_F = 0$. This non-participation leads to an asymmetry in the Euler equations for F and M that does not cancel out since $\partial \mathcal{N}_F(a)/\partial a = 0$, such that there is no wealth effect on female labor supply in Γ_M. As a result, the overborrowing bias is relaxed for M and savings are higher compared to $\varepsilon \neq 1$ (see Figure 4).

An important implication of F's non-participation in the labor market is that it affects the within-household distribution of household consumption relative to the cooperative case. In particular, $u_{c,F}/u_{c,M}$ now declines, i.e. F consumes more than M. This finding is important since it shows that lack of cooperation can lead to intertemporal as well as intratemporal deviations from the cooperative case if household members are imperfectly altruistic.

3 Stackelberg Leadership

So far, the non-cooperative formulation of the household problem has maintained the assumption that both household members choose their strategies simultaneously. Similar to Hertzberg (2012), I will now discuss the implications of relaxing this assumption by modeling the interaction between the household members as a Stackelberg game. In particular, I assume that household member M

is the Stackelberg leader and that household member F is the Stackelberg follower.[25] Leadership is assumed to be permanent, i.e. M will always be the leader and F always remain the follower.[26]

The Stackelberg follower F solves the decision problem

$$\max_{c_F,n_F} u_F(c_F,n_F)+\theta_F u_M(c_M,n_M)+\beta\mathcal{V}_F(a'(a,c_F,c_M,n_F,n_M).$$

As before, household member F takes the decisions of the spouse c_M and n_M as given. However, now it does so because these variables have already been determined at the time it acts and not because the household members act simultaneously.

The decision problem of the Stackelberg leader M is given by

$$\max_{c_M,n_M}\left\{\begin{array}{c} u_M(c_M,n_M)+\theta_M u_F(\hat{\mathcal{C}}_F(a,c_M,n_M),\hat{\mathcal{N}}_F(a,c_M,n_M)) \\ +\beta\mathcal{V}_M(a'(a,\hat{\mathcal{C}}_F(a,c_M,n_M),c_M,\hat{\mathcal{N}}_F(a,c_M,n_M),n_M)) \end{array}\right\}.$$

For both household members, the continuation values $\mathcal{V}_i$ satisfy again the recursion (8) and savings $a'(\cdot)$ are again given by (6).

Importantly, the Stackelberg leader M now internalizes that his choices c_M and n_M affect the decisions of the Stackelberg follower F, $c_F=\hat{\mathcal{C}}_F(a,c_M,n_M)$ and $n_F=\hat{\mathcal{N}}_F(a,c_M,n_M)$, even in the current period and thereby also have an additional (indirect) effect on future assets a'. For the Stackelberg follower F, the tuple (a,c_M,n_M) is the intra-period state that it conditions its strategies on. Since in equilibrium $c_M=\mathcal{C}_M(a)$ and $n_M=\mathcal{N}_M(a)$ hold, the equilibrium policy functions for c_F and n_F will however only depend on the asset position a, as formalized by the following definition:

Definition 4. *A stationary Markov-perfect equilibrium for the non-cooperative household problem with Stackelberg leadership is given by a set of functions* $\{\mathcal{A},\mathcal{C}_F,\mathcal{C}_M,\hat{\mathcal{C}}_F,\mathcal{N}_F,\mathcal{N}_M,\hat{\mathcal{N}}_F,\mathcal{V}_F,\mathcal{V}_M\}$ *such that for all a,*

(i) $$\{\mathcal{X}(a)\}_{\mathcal{X}\in\{\mathcal{C}_M,\mathcal{N}_M\}}=\arg\max_{c_M,n_M}\left\{\begin{array}{c} u_M(c_M,n_M)+\theta_M u_F(\hat{\mathcal{C}}_F(a,c_M,n_M),\hat{\mathcal{N}}_F(a,c_M,n_M)) \\ +\beta\mathcal{V}_M(a'(a,\hat{\mathcal{C}}_F(a,c_M,n_M),c_M,\hat{\mathcal{N}}_F(a,c_M,n_M),n_M)) \end{array}\right\},$$

[25]For a dynamic model of a two-person household with endogenous labor supply but without savings, Eckstein and Lifshitz (2015) also consider a non-cooperative solution with Stackelberg leadership, with the female as the Stackelberg follower. They refer to this setting as the "classical" household type and contrast it with the "modern" type which involves a simultaneous-move game played by the spouses.

[26]One could assume that the Stackelberg leader is determined at the beginning of the period in a random way or that leadership alternates between periods. Since the household problem with Stackelberg leadership is already quite involved without such additional assumptions, I only consider the case of permanent leadership.

(ii) $\left\{\hat{\mathcal{X}}(a,c_F,n_F)\right\}_{\mathcal{X}\in\{\mathcal{C}_F,\mathcal{N}_F\}} = \underset{c_F,n_F}{\arg\max}\left\{\begin{array}{c} u_F(c_F,n_F)+\theta_F u_M(c_M,n_M) \\ +\beta\mathcal{V}_F(a'(a,c_F,c_M,n_F,n_M)\end{array}\right\},$

(iii) $\mathcal{V}_i(a) = u_i(\mathcal{C}_i(a),\mathcal{N}_i(a))+\theta_i u_{\sim i}(\mathcal{C}_{\sim i}(a),\mathcal{N}_{\sim i}(a))+\beta\mathcal{V}_i(\mathcal{A}(a)),\ i\in\{F,M\},$

(iv) $a'(a,c_F,c_M,n_F,n_M) = w_F n_F + w_M n_M + a(1+r(a)) - c_F - c_M,$

(v) $\mathcal{X}(a) = \hat{\mathcal{X}}(a,\mathcal{C}_M(a),\mathcal{N}_M(a)),\ \mathcal{X}\in\{\mathcal{C}_F,\mathcal{N}_F\},$

(vi) $\mathcal{A}(a) = a'(a,\mathcal{C}_F(a),\mathcal{C}_M(a),\mathcal{N}_F(a),\mathcal{N}_M(a)).$

The equilibrium for the non-cooperative Markov-perfect game with Stackelberg leadership involves the same optimal labor supply conditions (3) and the same generalized Euler equation for the Stackelberg follower (9) as in the simultaneous-move case.[27] For the Stackelberg follower, the consumption-savings trade-off thus is not affected by the sequential-move assumption.

The important difference is the shape of the Euler equation for the Stackelberg leader,

$$\begin{aligned} &\frac{u_{c,M}(\mathcal{C}_M(a),\mathcal{N}_M(a))+\theta_M\Lambda(a)}{\Theta(a)} \qquad (16)\\ &= \beta\left[\begin{array}{l} \frac{u_{c,M}(\mathcal{C}_M(\mathcal{A}(a)),\mathcal{N}_M(\mathcal{A}(a)))+\theta_M\Lambda(\mathcal{A}(a))}{\Theta(\mathcal{A}(a))}(1+R(\mathcal{A}(a))) \\ +\left(\begin{array}{c} u_{c,M}(\mathcal{C}_M(\mathcal{A}(a)),\mathcal{N}_M(\mathcal{A}(a))) \\ -\frac{u_{c,M}(\mathcal{C}_M(\mathcal{A}(a)),\mathcal{N}_M(\mathcal{A}(a)))+\theta_M\Lambda(\mathcal{A}(a))}{\Theta(\mathcal{A}(a))}\end{array}\right)\left[\frac{\partial\mathcal{C}_M(a')}{\partial a'}-\frac{\partial\mathcal{N}_M(a')}{\partial a'}w_M\right] \\ +\left(\begin{array}{c} \theta_M u_{c,F}(\mathcal{C}_F(\mathcal{A}(a)),\mathcal{N}_F(\mathcal{A}(a))) \\ -\frac{u_{c,M}(\mathcal{C}_M(\mathcal{A}(a)),\mathcal{N}_M(\mathcal{A}(a)))+\theta_M\Lambda(\mathcal{A}(a))}{\Theta(\mathcal{A}(a))}\end{array}\right)\left[\frac{\partial\mathcal{C}_F(a')}{\partial a'}-\frac{\partial\mathcal{N}_F(a')}{\partial a'}w_F\right]\end{array}\right], \end{aligned}$$

where

$$\begin{aligned} \Lambda(a) &= u_{c,F}(\mathcal{C}_F(a),\mathcal{N}_F(a))\left[\frac{\partial\hat{\mathcal{C}}_F(a,c_M,n_M)}{\partial c_M}-\frac{\partial\hat{\mathcal{N}}_F(a,c_M,n_M)}{\partial c_M}w_F\right], \\ \Theta(a) &= 1+\frac{\partial\hat{\mathcal{C}}_F(a,\mathcal{C}_M(a),\mathcal{N}_M(a))}{\partial c_M}-w_F\frac{\partial\hat{\mathcal{N}}_F(a,\mathcal{C}_M(a),\mathcal{N}_M(a))}{\partial c_M}. \end{aligned}$$

This GEE differs from the one in the simultaneous-move case (9) due to the presence of the derivatives $\partial\hat{\mathcal{C}}_F/\partial c_M$ and $\partial\hat{\mathcal{N}}_F/\partial c_M$, which appear in the functions $\Theta(a)$ and $\Lambda(a)$. These additional terms capture the Stackelberg leader's ability to affect the decision of the Stackelberg follower within the same period.[28] Condition (16) can be seen as a version of the GEE (9) that is adjusted for

[27] The derivation of the optimality conditions can be found in Appendix A.3.

[28] With Stackelberg leadership, $\partial\hat{\mathcal{X}}_F(a,c_M,n_M)/\partial n_M = -\left(\partial\hat{\mathcal{X}}_F(a,c_M,n_M)/\partial c_M\right)w_M$ holds for $\mathcal{X}\in\{\mathcal{C},\mathcal{N}\}$ (see Appendix A.3), i.e. there is a direct link between the effect of c_M and n_M on F's actions.

the leader's intra-period effect of his actions given by $\Theta(a)$ and $\Lambda(a)$. To illustrate this, suppose that the Stackelberg leader cannot affect F's consumption and working decisions in the same period, i.e. $\partial\hat{\mathcal{X}}_F(a,c_M,n_M)/\partial c_M = 0$ for $\mathcal{X} \in \{\mathcal{C},\mathcal{N}\}$. In this case, $\Lambda(a) = 0$ and $\Theta(a) = 1$ hold and the Euler equation reduces to (9). When $\partial\hat{\mathcal{X}}_F(a,c_M,n_M)/\partial c_M \neq 0$ for $\mathcal{X} = \mathcal{C}$ or $\mathcal{X} = \mathcal{N}$, the term $(u_{c,M}(c_M,n_M) + \theta_M\Lambda(a))/\Theta(a)$ replaces $u_{c,M}(c_M,n_M)$ as M's marginal value of wealth, leaving the consumption-savings trade-off otherwise essentially unchanged relative to the simultaneous-move case. The main message of this section thus is that the savings distortion associated with the non-cooperative household problem is not an artifact of the simultaneous-move assumption, which is consistent with Hertzberg's (2012) findings.

4 Household Problem with Labor Income Risk

In the deterministic model environment studied so far, the household savings motive was driven by the household's impatience, as given by the discount factor β, relative to $1/(1+\bar{r})$. To allow for a realistic evaluation of the quantitative and welfare implications of lack of cooperation, this section studies a calibrated version of the model presented in Section 2 with idiosyncratic labor income risk and incomplete financial markets. Financial markets only provide partial insurance against income fluctuations since the household can only trade a non-state contingent one-period bond a'. In addition, the debt-elastic interest rate schedule $r(a')$ makes it costly to borrow against future labor income in response to adverse productivity shocks. As a result, there now is a role for precautionary savings such that household savings are not only governed by the household's impatience. In contrast to most incomplete markets models (see Heathcote et al., 2009), the household does not face income risk at the "aggregate" household level.[29] Instead, the two household members are both subject to idiosyncratic, i.e. member-specific, labor productivity shocks that are not perfectly positively correlated. In this case, as highlighted e.g. by Attanasio et al. (2005), adjustment of spousal labor supply offers an additional insurance channel that allows the household to smooth consumption in response to idiosyncratic shocks.

With labor income risk, the household budget constraint is given by

$$a' = e_F w_F n_F + e_M w_M n_M + a(1+r(a)) - c_F - c_M, \tag{17}$$

[29] A notable exception within the incomplete markets literature are Ortigueira and Siassi (2013) who study the importance of intra-household risk sharing in a general equilibrium incomplete markets model with endogenous labor supply.

where e_i is the random labor productivity of household member $i \in \{F,M\}$. When $e_i w_i$ is defined as the effective wage rate of household member i, changes in productivity e_i can be interpreted as shocks to the wage rate. The household members are assumed to have perfect information about the productivity value of their spouse $e_{\sim i}$. Labor productivities e_i follow first-order Markov processes with discrete support $e_i \in \{e_1, ..., e_I\}$. I assume that the productivities might be correlated across household members. It will however be important that the shocks are not perfectly positively correlated. In such a case, the productivity shocks would have the same impact as a single "aggregate shock" at the household level and intra-household risk sharing via spousal labor supply would not be possible.

When the household members cooperate, the decision problem under uncertainty is given by

$$\max_{c_F, c_M, n_F, n_M} \sum\nolimits_{i \in \{F,M\}} (1+\theta_{\sim i})\, u_i(c_i, n_i) + \beta \mathbb{E}_{e'|e} \left[\mathcal{V}(a'(a, e, c_F, c_M, n_F, n_M), e') \right],$$

where savings are given by the period budget constraint (17), $e = (e_F, e_M)$ summarizes the exogeneous household productivity state and $\mathbb{E}_{e'|e}[\cdot]$ denotes the conditional expectation operator. The value and policy functions now depend on asset holdings a as well as on the labor productivities of both household members e. The continuation value $\mathcal{V}$ satisfies

$$\mathcal{V}(a,e) = \sum\nolimits_{i \in \{F,M\}} (1+\theta_{\sim i})\, u_i(\mathcal{C}_i(a,e), \mathcal{N}_i(a,e)) + \beta \mathbb{E}_{e'|e} \left[\mathcal{V}(\mathcal{A}(a,e), e') \right].$$

Without cooperation, household member $i \in \{F,M\}$ takes spousal consumption $c_{\sim i}$ and labor supply $n_{\sim i}$ as given and solves

$$\max_{c_i, n_i} u_i(c_i, n_i) + \theta_i u_{\sim i}(c_{\sim i}, n_{\sim i}) + \beta \mathbb{E}_{e'|e} \left[\mathcal{V}_i(a'(a, e, c_i, c_{\sim i}, n_i, n_{\sim i}), e') \right],$$

where savings are given by

$$a' = e_i w_i n_i + e_{\sim i} w_{\sim i} n_{\sim i} + a\,(1 + r(a)) - c_i - c_{\sim i},$$

and $\mathcal{V}_i$ satisfies

$$\mathcal{V}_i(a,e) = u_i(\mathcal{C}_i(a,e), \mathcal{N}_i(a,e)) + \theta_i u_{\sim i}(\mathcal{C}_{\sim i}(a,e), \mathcal{N}_{\sim i}(a,e)) + \beta \mathbb{E}_{e'|e} \left[\mathcal{V}_i(\mathcal{A}(a,e), e') \right].$$

The definitions of the MPE for the household problems under uncertainty are straightforward extensions of Definition 1 and Definition 2, and are therefore omitted here. With productivity shocks, the conditions that describe the MPE outcome for the cooperative and the non-cooperative household problem are also very similar to the ones in the model of Section 2. The only changes are that: (i) the wage rate w_i is replaced by the effective wage rate $e_i w_i$, (ii) the minimal payoff-relevant state now includes the individual productivity values summarized by e in addition to the asset position a, and (iii) the household members have to form expectations with respect to future productivities e', conditional on current productivities e.

In the remainder of this section, I will assume that the household members share the same attributes x_i, with $x \in \{\theta, u, w\}$, to isolate the impact of income risk on household behavior. In addition, the productivities e_F and e_M will follow the same Markov process. Importantly, the realizations of the idiosyncratic productivity shocks will not be perfectly positively correlated across household members. As a result, transitory differences between the household members will occur, generating the possibility of intra-household risk sharing.

4.1 Model Calibration

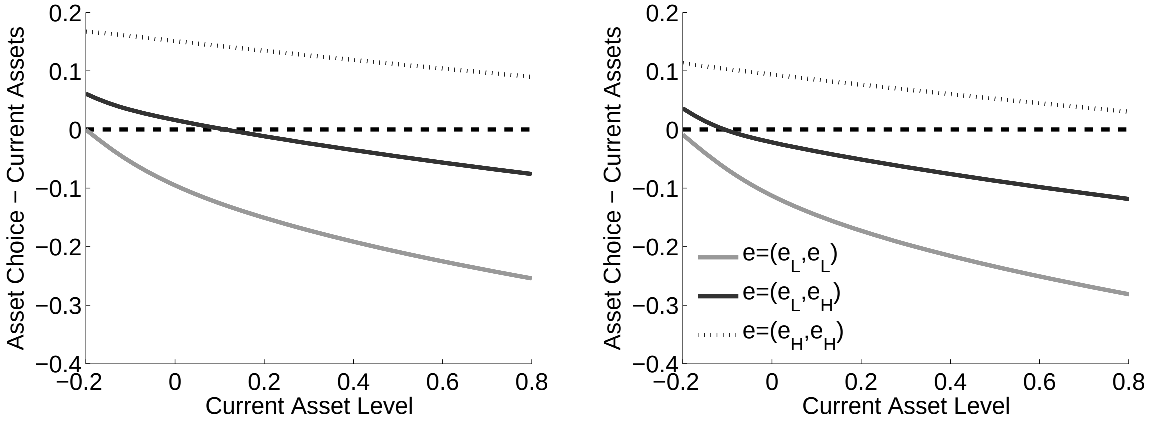
The model with labor income risk is specified as follows. For the utility function $u(\cdot)$, I follow Ortigueira and Siassi (2013), setting the parameter γ to a standard value of 2 and η to a value of 3. The consumption share α now is set to 0.4 to match an average working time of 36% (see Floden and Lindé, 2001), which implies a Frisch elasticity of roughly 60%.[30] For the interest rate schedule $r(a)$, I keep the parameter values from Section 2.5. A model period corresponds to one year. Labor productivity follows the log-normal AR(1)-process

$$e_{it} = e_{it-1}^{\rho} \exp\left(\sigma \varepsilon_{it}\right), \varepsilon_{it} \overset{i.i.d.}{\sim} \mathcal{N}(0,1).$$

The productivity parameters ρ and σ are specified as in Floden and Lindé (2001). Using the Panel Study of Income Dynamics data set, they estimate the values $(\rho, \sigma) = (0.9136, 0.2064)$ based on wage data for the United States. I approximate the productivity process as a two-state Markov chain via the Rouwenhorst method as described in Kopecky and Suen (2010), assuming that labor productivity shocks are uncorrelated across household members. Labor productivity e_i can thus take on one of two

[30]For the chosen utility function, the Frisch elasticity of labor supply is $\frac{1}{\eta}\frac{1-n^*}{n^*}$, where n^* is the average working time of the (symmetric) household members (see Ortigueira and Siassi, 2013).

Figure 5: Savings policy function with labor income risk ($\theta = 0.9$)



(a) With cooperation

(b) Without cooperation

values: $e_i \in \{e_L, e_H\}$, with $e_L < e_H$. Following Domeij and Floden (2006), the discount factor β is set to 0.95. This value satisfies the condition $\beta(1+\bar{r}) < 1$, which is standard in the incomplete markets literature to prevent households from accumulating assets without bound.[31]

4.2 Precautionary Savings and Imperfect Altruism

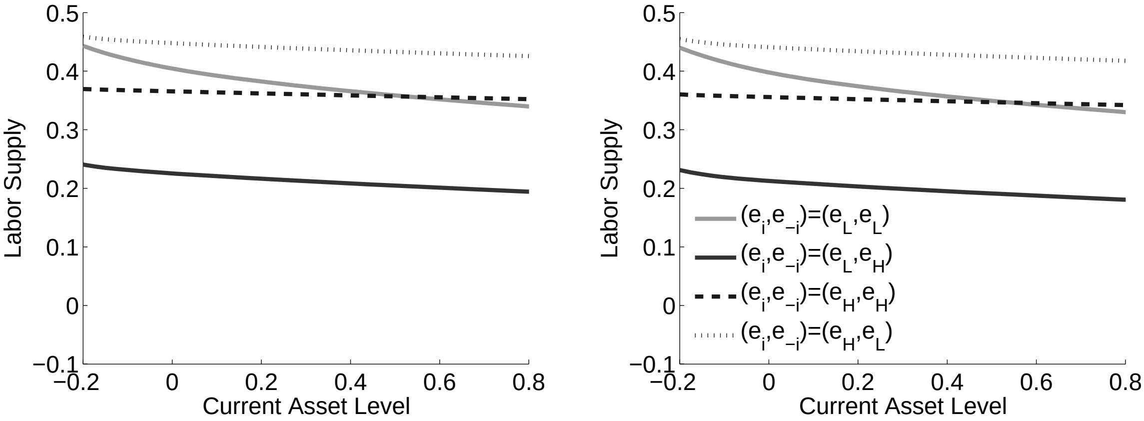
First, I assess how imperfect altruism affects precautionary savings. Figure 5 depicts the savings policy function for a cooperative household (left panel) and a non-cooperative one (right panel), assuming that the degree of altruism is $\theta = 0.9$ in both cases.[32] Since the household members are symmetric, the productivity states $e = (e_L, e_H)$ and $e = (e_H, e_L)$ lead to the same savings policy function. As probably expected given the results of Section 2, the non-cooperative household saves less in all possible states.

Table 1 lists the average assets of the non-cooperative household relative to its annual household labor income for different degrees of altruism. These values are calculated based on 75 million simulated model periods. When the household members cooperate or do not cooperate but are perfectly altruistic ($\theta = 1$), the average asset position is 36.52% of annual total household income. For the non-cooperative household, savings are already notably lower (29.26%) when the members only exhibit small deviations from perfect altruism ($\theta = 0.98$). The undersaving bias induced by imperfect

[31] When labor supply is exogenous and $\beta(1+\bar{r}) = 1$ holds, the precautionary savings motive leads to the accumulation of infinitely high asset holdings in the long run (see e.g. Ljungqvist and Sargent, 2004). By contrast, Marcet et al. (2007) show that when labor supply is endogenous, household savings will be (possibly large but) finite even for $\beta(1+\bar{r}) = 1$.

[32] When the household members share the same permanent attributes, the exact value of θ is irrelevant for the behavior of the cooperative household.

Figure 6: Policy function for labor supply of household member i ($\theta = 0.9$)



(a) With cooperation

(b) Without cooperation

spousal altruism and lack of cooperation thus tends to dominate the precautionary saving motive. When the degree of altruism is further reduced to $\theta = 0.90$, the household holds substantially less assets relative to the cooperative case (4.14%).

4.3 Intra-Household Risk Sharing

When household member i is hit by a low productivity shock, its spouse can work more to increase household earnings and thereby reduce the impact of i's bad labor market outcome on household consumption. This risk-sharing aspect of spousal labor supply can be seen in Figure 6 which displays the labor supply policy function for a household member i. The interesting two cases are the states $e = (e_L, e_H)$ and $e = (e_H, e_L)$. Compared to the good state $e = (e_H, e_H)$, a bad shock to the labor productivity of household member $\sim i$ leads to an increase in i's working time, partially recovering $\sim i$'s income loss. If household member i is in a low productivity state, a bad shock to $\sim i$'s productivity leads to a much weaker response of i's labor supply. As shown by Figure 5, in this case, the household relies more on its savings (or borrowing) to smooth consumption. Since lack of cooperation increases the household's willingness to borrow (or dissave) in response to adverse shocks, its members need to work less for a given asset position and productivity state.

However, one should not infer from Figures 6 that the non-cooperative household smooths consumption in a more effective way compared to a cooperative one. To avoid large adjustments in consumption and leisure, the cooperative household accumulates large asset holdings that are used as a buffer stock in bad times. By contrast, the non-cooperative household tends to have lower as-

	$\theta = 1$	$\theta = 0.98$	$\theta = 0.96$	$\theta = 0.94$	$\theta = 0.92$	$\theta = 0.90$
Average assets	36.52	29.26	22.35	15.84	9.75	4.14
Welfare measure	0	0.32	0.64	0.95	1.25	1.54

Table 1: Average asset holdings of the non-cooperative household (in % of total household income) and welfare measure ζ (in %) for different degrees of spousal altruism θ

set holdings which then require larger changes in consumption and labor supply in response to bad shocks, increasing the volatility of consumption and labor supply.

4.4 The Welfare Cost of Lack of Cooperation

Given the impact of lack of cooperation on the behavior of a household, it is interesting to ask how much non-cooperative decision making matters in terms of welfare. To address this question, I calculate the welfare-equivalent permanent change in consumption ζ that the members of a non-cooperative household need to experience to achieve the same expected life-time utility as in the cooperative case. Formally, I solve for the ζ-value that satisfies

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t U\left(c^C_{Ft}, c^C_{Mt}, n^C_{Ft}, n^C_{Mt}\right)\right] = \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t U\left((1+\zeta)\,c^N_{Ft}, (1+\zeta)\,c^N_{Mt}, n^N_{Ft}, n^N_{Mt}\right)\right],$$

with

$$U\left(c_F, c_M, n_F, n_M\right) = (1+\theta_M)\,u_F(c_F, n_F) + (1+\theta_F)\,u_M(c_M, n_M),$$

where the sequences of consumption and labor supply for the members of a household with ($j = C$) and without cooperation ($j = N$) are denoted as $\{c^j_{Ft}, c^j_{Mt}, n^j_{Ft}, n^j_{Mt}\}^{\infty}_{t=0}$. Remember that $\theta_F = \theta_M = \theta$ holds. I calculate the unconditional expectation of life-time utility for the two household types by computing the sum of realized discounted utilities for 2500 periods and taking the average value over 50,000 samples. For each sample, the first 1000 observations are not used to reduce the impact of initial conditions.

The results are displayed in Table 1. The welfare cost of lack of cooperation ζ decreases monotonically with the degree of spousal altruism θ for $\theta < 1$. For small deviations from perfect altruism ($\theta = 0.98$), the calculated ζ-value is already notable (0.32%). For values lower than $\theta = 0.94$, ζ exceeds one percent, going up to 1.54% for $\theta = 0.9$. Lack of cooperation therefore entails sizable welfare losses for the household members for even modest deviations from perfect spousal altruism.

5 Conclusion

This paper has studied the consumption-savings problem of two-person households whose individual members cannot commit to future actions and might not cooperate. The interaction between the individual household members was modeled as a Markov-perfect game. Intuitive first-order conditions were derived that illustrate how lack of cooperation leads to a savings distortion relative to the case of full cooperation. More specifically, a non-cooperative household tends to save less (borrow more) than a cooperative one when its members exhibit imperfect spousal altruism. A calibrated model version with incomplete markets and idiosyncratic labor income risk was used to quantify the implications of lack of cooperation for precautionary savings and welfare. Even modest deviations from perfect altruism were shown to induce a decline in precautionary savings for the non-cooperative household, leading to substantial welfare losses relative to the cooperative case.

References

ALESINA, A. AND G. TABELLINI (1990): "A Positive Theory of Fiscal Deficits and Government Debt," *Review of Economic Studies*, 57(3), 403-414.

ATTANASIO, O., H. LOW, AND V. SANCHEZ-MARCOS (2005): "Female Labor Supply as Insurance Against Idiosyncratic Risk," *Journal of the European Economic Association*, 3(2-3), 755-764.

BERNHEIM, B.D., D. RAY, AND S. YELTEKIN (2015): "Poverty and Self-Control," *Econometrica*, 83(5), 1877-1911.

BLAU, F.D. AND L.M. KAHN (2000): "Gender Differences in Pay," *Journal of Economic Perspectives*, 14(4), 75-99.

BROWNING, W. (2000): "The Saving Behaviour of a Two-Person Household," *Scandinavian Journal of Economics*, 102(2), 235-251.

BROWNING, W., P.-A. CHIAPPORI, V. LECHENE (2006): "Collective and Unitary Models: A Clarification ," *Review of Economics of the Household*, 4(1), 5-14.

CAO, D. AND I. WERNING (2016): "Dynamic Savings Choices with Wealth-Dependent Time Inconsistency," mimeo.

CHATTERJEE, S., D. CORBAE, M. NAKAJIMA, AND J. RIOS-RULL (2007): "A Quantitative Theory of Unsecured Consumer Credit with Risk of Default," *Econometrica*, 75(6), 1525-89.

CHATTERJEE, S. AND B. EYIGUNGOR (2016): "Continuous Markov Equilibria with Quasi-Geometric Discounting," *Journal of Economic Theory*, 163(3), 467-494.

CHEN, Z. AND F. WOOLLEY (2001): "A Cournot-Nash Model of Family Decision Making," *Economic Journal* , 111, 722-748.

CHIAPPORI, P.-A. (1988): "Rational Household Labor Supply," *Econometrica*, 56(1), 63-89.

CHIAPPORI, P.-A. (1992): "Collective Labor Supply and Welfare," *Journal of Political Economy*, 100(3), 437-467.

CHIAPPORI, P.-A. AND O. DONNI (2011): "Nonunitary Models of Household Behavior: A Survey of the Literature," in *Household Economic Behaviors*, ed. by Molina, J.A., Springer, New York, 1-40.

CHIAPPORI, P.-A. AND M. MAZZOCCO (2014): ”Static and Intertemporal Household Decisions,” mimeo.

CHIAPPORI, P.-A. AND C. MEGHIR (2015): ”Intrahousehold Inequality,” in *Handbook of Income Distribution*, Vol. 2, ed. by Atkinson, A.B. and F. Bourguignon, North-Holland, Amsterdam, 1369-1418.

DOEPKE, M. AND M. TERTILT (2014): ”Does Female Empowerment Promote Economic Development?” mimeo.

DOMEIJ, D. AND M. FLODEN (2006): ”The Labor-Supply Elasticity and Borrowing Constraints: Why Estimates Are Biased,” *Review of Economic Dynamics* , 9(2), 242-262.

ECKSTEIN, Z. AND O. LIFSHITZ (2015): ”Household Interaction and the Labor Supply of Married Women,” *International Economic Review*, 56(2), 427-455.

FLODEN, M. AND J. LINDÉ (2001): ”Idiosyncratic Risk in the United States and Sweden: Is There a Role for Government Insurance?,” *Review of Economic Dynamics* , 4(2), 406-437.

HEATHCOTE, J., K. STORESLETTEN, AND G. VIOLANTE (2009): ”Quantitative Macroeconomics with Heterogeneous Households,” *Annual Review of Economics*, 1(1), 319-354.

HERTZBERG, A. (2012): ”Exponential Individuals, Hyperbolic Households,” mimeo.

HERTZBERG, A. (2013): ”Heterogeneous Time Preferences within the Household,” mimeo.

JACKSON, M.O. AND L. YARIV (2014): ”Present Bias and Collective Dynamic Choice in the Lab,” *American Economic Review*, 104(12), 4184-4204.

JUDD, K. (1998): *Numerical Methods in Economics*, MIT Press, Cambridge, MA.

KONRAD, K.A. AND K.E. LOMMERUD (1995): ”Family Policy with Non-Cooperative Families,” *Scandinavian Journal of Economics*, 97(4), 581-601.

KONRAD, K.A. AND K.E. LOMMERUD (2000): ”The Bargaining Family Revisited,” *Canadian Journal of Economics*, 33(2), 471-487.

KOPECKY, K. AND R. SUEN (2010): ”Finite State Markov-Chain Approximations to Highly Persistent Processes,” *Review of Economic Dynamics*, 13(3), 701-714.

KRUSELL, P. AND SMITH, A.A. (2003): ”Consumption–Savings Decisions with Quasi–Geometric Discounting,” *Econometrica*, 71(1), 365-375.

KRUSELL, P., B. KURUSCU, AND T. SMITH (2002): ”Equilibrium Welfare and Government Policy with Quasi-Geometric Discounting,” *Journal of Economic Theory*, 105(1), 42-72.

LAIBSON, D. (1997): ”Golden Eggs and Hyperbolic Discounting,” *Quarterly Journal of Economics*, 112(2), 443-478.

LECHENE, V. AND I. PRESTON (2011): ”Noncooperative Household Demand,” *Journal of Economic Theory*, 146(2), 504-527.

LIGON (2011): ”Dynamic Bargaining in Households (with an Application to Bangladesh),” mimeo.

LJUNGQVIST, L. AND T.J. SARGENT (2004): *Recursive Macroeconomic Theory*, 2nd Edition, MIT Press, Cambridge, MA.

LUNDBERG, S. AND R.A. POLLAK (1993): ”Separate Speres Bargaining and the Marriage Market,” *Journal of Political Economy*, 101(6), 988-1010.

MARCET, A., F. OBIOLS-HOMS, AND P. WEIL (2007): ”Incomplete Markets, Labor Supply and Capital Accumulation,” *Journal of Monetary Economics*, 54(8), 2621-2635.

MASKIN, E. AND J. TIROLE (2001): ”Markov Perfect Equilibrium: I. Observable Actions,” *Journal of Economic Theory*, 100(2), 191-219.

MAZZOCCO, M. (2007): ”Household Intertemporal Behaviour: A Collective Characterization and a Test of Commitment.,” *Review of Economic Studies*, 74(3): 857-895.

MAZZOCCO, M. (2008): ”Individual Rather Than Household Euler Equations: Identification and Estimation of Individual Preferences Using Household Data,” mimeo.

NIEMANN, S., P. PICHLER, AND G. SORGER (2013): ”Central Bank Independence And The Monetary Instrument Problem,” *International Economic Review*, 54(8), 1031-1055.

ORTIGUEIRA, S. AND N. SIASSI (2013): ”How Important is Intra-Household Risk Sharing for Savings and Labor Supply?” *Journal of Monetary Economics*, 60(6), 650-666.

PERSSON, T. AND L.E.O. SVENSSON (1989): ”Why a Stubborn Conservative Would Run a Deficit: Policy with Time-Inconsistent Preferences,” *Quarterly Journal of Economics*, 104(2), 325-345.

SCHANER, S. (2015): "Do Opposites Detract? Intrahousehold Preference Heterogeneity and Inefficient Strategic Savings," *American Economic Journal: Applied Economics*, 7(2), 135-74.

SMITH, T.W., P.V. MARSDEN, AND M. HOUT (2011): *General Social Survey, 1972-2010: Cumulative Codebook*, The Roper Center for Public Opinion Research, University of Connecticut.

SCHMITT-GROHÉ, S. AND M. URIBE (2003): "Closing Small Open Economy Models," *Journal of International Economics*, 61(1), 163-185.

A Appendix

A.1 Proofs

This section contains the proofs of the propositions in the main text.

A.1.1 Proof of Proposition 1.

Under full cooperation of the household members, the first-order conditions for consumption c_i and labor supply n_i are

$$\begin{aligned}(1+\theta_{\sim i})\,u_{c,i}(c_i,n_i)+\beta\frac{\partial\mathcal{V}(a')}{\partial a'}\frac{\partial a'}{\partial c_i} &= 0,\\ (1+\theta_{\sim i})\,u_{n,i}(c_i,n_i)+\beta\frac{\partial\mathcal{V}(a')}{\partial a'}\frac{\partial a'}{\partial n_i} &= 0,\end{aligned}$$

with $i\in\{F,M\}$. Differentiating the budget constraint with respect to consumption c_i and labor supply n_i yields

$$\frac{\partial a'}{\partial c_i} = -1, \tag{18}$$

$$\frac{\partial a'}{\partial n_i} = w_i. \tag{19}$$

Using these derivatives, the first-order conditions can be written as

$$(1+\theta_{\sim i})\,u_{c,i}(c_i,n_i) = \beta\frac{\partial\mathcal{V}(a')}{\partial a'}, \tag{20}$$

$$-(1+\theta_{\sim i})\,u_{n,i}(c_i,n_i) = \beta\frac{\partial\mathcal{V}(a')}{\partial a'}w_i, \tag{21}$$

or combined as

$$-u_{n,i}(c_i,n_i) = u_{c,i}(c_i,n_i)w_i. \tag{22}$$

By combining the first-order conditions for c_F and c_M, one can derive the sharing rule

$$u_{c,F}(c_F,n_F) = \frac{1+\theta_F}{1+\theta_M}u_{c,M}(c_M,n_M). \tag{23}$$

Using

$$\mathcal{V}(a) = \textstyle\sum_{i\in\{F,M\}}(1+\theta_{\sim i})\,u_i(\mathcal{C}_i(a),\mathcal{N}_i(a))+\beta\mathcal{V}(\mathcal{A}(a)),$$

and

$$\mathcal{A}(a) = w_F \mathcal{N}_F(a) + w_M \mathcal{N}_M(a) + a(1 + r(a)) - \mathcal{C}_F(a) - \mathcal{C}_M(a),$$

the derivatives of $\mathcal{V}(a)$ and $\mathcal{A}(a)$ are given by

$$\begin{aligned} \frac{\partial \mathcal{V}(a)}{\partial a} &= \sum\nolimits_{i \in \{F,M\}} \left[\begin{array}{c} (1+\theta_{\sim i})\, u_{c,i}(\mathcal{C}_i(a), \mathcal{N}_i(a)) \frac{\partial \mathcal{C}_i(a)}{\partial a} \\ +(1+\theta_{\sim i})\, u_{n,i}(\mathcal{C}_i(a), \mathcal{N}_i(a)) \frac{\partial \mathcal{N}_i(a)}{\partial a} \end{array} \right] \\ &\quad + \beta \frac{\partial \mathcal{V}(\mathcal{A}(a))}{\partial a'} \frac{\partial \mathcal{A}(a)}{\partial a}, \end{aligned} \tag{24}$$

and

$$\frac{\partial \mathcal{A}(a)}{\partial a} = \sum\nolimits_{i \in \{F,M\}} \left[w_i \frac{\partial \mathcal{N}_i(a)}{\partial a} - \frac{\partial \mathcal{C}_i(a)}{\partial a} \right] + 1 + R(a). \tag{25}$$

After combining (24) with (20) for $i = F$ and (25),

$$\begin{aligned} \frac{\partial \mathcal{V}(a)}{\partial a} &= \sum\nolimits_{i \in \{F,M\}} \left[\begin{array}{c} (1+\theta_{\sim i})\, u_{c,i}(\mathcal{C}_i(a), \mathcal{N}_i(a)) \frac{\partial \mathcal{C}_i(a)}{\partial a} \\ +(1+\theta_{\sim i})\, u_{n,i}(\mathcal{C}_i(a), \mathcal{N}_i(a)) \frac{\partial \mathcal{N}_i(a)}{\partial a} \end{array} \right] \\ &\quad + (1+\theta_M)\, u_{c,F}(\mathcal{C}_F(a), \mathcal{N}_F(a)) \left[\sum\nolimits_{i \in \{F,M\}} \left[w_i \frac{\partial \mathcal{N}_i(a)}{\partial a} - \frac{\partial \mathcal{C}_i(a)}{\partial a} \right] + 1 + R(a) \right], \end{aligned}$$

and using (22) to replace $u_{n,i}(c_i, n_i)$ with $-u_{c,i}(c_i, n_i) w_i$ for $i \in \{F, M\}$, one obtains

$$\begin{aligned} \frac{\partial \mathcal{V}(a)}{\partial a} &= \sum\nolimits_{i \in \{F,M\}} \left[(1+\theta_{\sim i})\, u_{c,i}(\mathcal{C}_i(a), \mathcal{N}_i(a)) \left[\frac{\partial \mathcal{C}_i(a)}{\partial a} - w_i \frac{\partial \mathcal{N}_i(a)}{\partial a} \right] \right] \\ &\quad + (1+\theta_M)\, u_{c,F}(\mathcal{C}_F(a), \mathcal{N}_F(a)) \left[\begin{array}{c} \sum_{i \in \{F,M\}} \left[w_i \frac{\partial \mathcal{N}_i(a)}{\partial a} - \frac{\partial \mathcal{C}_i(a)}{\partial a} \right] \\ +1 + R(a) \end{array} \right], \end{aligned}$$

This expression can further be rewritten by using (23) to rewrite

$$(1+\theta_M)\, u_{c,F}(\mathcal{C}_F(a), \mathcal{N}_F(a)) \sum\nolimits_{i \in \{F,M\}} \left[w_i \frac{\partial \mathcal{N}_i(a)}{\partial a} - \frac{\partial \mathcal{C}_i(a)}{\partial a} \right],$$

as

$$\sum\nolimits_{i \in \{F,M\}} \left[(1+\theta_{\sim i})\, u_{c,i}(\mathcal{C}_i(a), \mathcal{N}_i(a)) \left[w_i \frac{\partial \mathcal{N}_i(a)}{\partial a} - \frac{\partial \mathcal{C}_i(a)}{\partial a} \right] \right],$$

which yields

$$\begin{aligned}
\frac{\partial \mathcal{V}(a)}{\partial a} &= \sum\nolimits_{i\in\{F,M\}}\left[(1+\theta_{\sim i})\,u_{c,i}(\mathcal{C}_i(a),\mathcal{N}_i(a))\left[\frac{\partial \mathcal{C}_i(a)}{\partial a}-w_i\frac{\partial \mathcal{N}_i(a)}{\partial a}\right]\right]\\
&\quad+\sum\nolimits_{i\in\{F,M\}}\left[(1+\theta_{\sim i})\,u_{c,i}(\mathcal{C}_i(a),\mathcal{N}_i(a))\left[w_i\frac{\partial \mathcal{N}_i(a)}{\partial a}-\frac{\partial \mathcal{C}_i(a)}{\partial a}\right]\right]\\
&\quad+(1+\theta_M)\,u_{c,F}(\mathcal{C}_F(a),\mathcal{N}_F(a))\,(1+R(a)),
\end{aligned}$$

or

$$\frac{\partial \mathcal{V}(a)}{\partial a}=(1+\theta_M)\,u_{c,F}(\mathcal{C}_F(a),\mathcal{N}_F(a))(1+R(a)).$$

Updating this expression one period ahead and plugging it into (20) for $i=F$ yields the Euler equation for the cooperative household

$$u_{c,F}(c_F,n_F)=\beta u_{c,F}(\mathcal{C}_F(a'),\mathcal{N}_F(a'))(1+R(a')).$$

A.1.2 Proof of Proposition 2.

Household member $i\in\{F,M\}$ solves the decision problem (7), taking as given current spousal decisions $c_{\sim i}$ and $n_{\sim i}$ as well as next period's intra-household allocation given by $\mathcal{X}_i(a')$ and $\mathcal{X}_{\sim i}(a')$, with $\mathcal{X}\in\{\mathcal{C},\mathcal{N}\}$. The optimal consumption and labor supply decisions of the household member satisfy

$$\begin{aligned}
u_{c,i}(c_i,n_i)+\beta\frac{\partial \mathcal{V}_i(a')}{\partial a'}\frac{\partial a'}{\partial c_i} &= 0,\\
u_{n,i}(c_i,n_i)+\beta\frac{\partial \mathcal{V}_i(a')}{\partial a'}\frac{\partial a'}{\partial n_i} &= 0.
\end{aligned}$$

Using the derivatives (18) and (19), the first-order conditions can be written as

$$u_{c,i}(c_i,n_i) = \beta\frac{\partial \mathcal{V}_i(a')}{\partial a'}, \tag{26}$$

$$-u_{n,i}(c_i,n_i) = \beta\frac{\partial \mathcal{V}_i(a')}{\partial a'}w_i, \tag{27}$$

or combined as

$$-u_{n,i}(c_i,n_i)=u_{c,i}(c_i,n_i)w_i. \tag{28}$$

Using (8) and

$$\mathcal{A}(a)=w_i\mathcal{N}_i(a)+w_{\sim i}\mathcal{N}_{\sim i}(a)+a\,(1+r(a))-\mathcal{C}_i(a)-\mathcal{C}_{\sim i}(a),$$

the derivatives of $\mathcal{V}_i(a)$ and $\mathcal{A}(a)$ with respect to a can be calculated:

$$\begin{aligned}
\frac{\partial \mathcal{V}_i(a)}{\partial a} &= u_{c,i}(\mathcal{C}_i(a),\mathcal{N}_i(a))\frac{\partial \mathcal{C}_i(a)}{\partial a} + u_{n,i}(\mathcal{C}_i(a),\mathcal{N}_i(a))\frac{\partial \mathcal{N}_i(a)}{\partial a} \\
&+\theta_i u_{c,\sim i}(\mathcal{C}_{\sim i}(a),\mathcal{N}_{\sim i}(a))\frac{\partial \mathcal{C}_{\sim i}(a)}{\partial a} + \theta_i u_{n,\sim i}(\mathcal{C}_{\sim i}(a),\mathcal{N}_{\sim i}(a))\frac{\partial \mathcal{N}_{\sim i}(a)}{\partial a} \\
&+\beta \frac{\partial \mathcal{V}_i(\mathcal{A}(a))}{\partial a'}\frac{\partial \mathcal{A}(a)}{\partial a},
\end{aligned} \tag{29}$$

and

$$\frac{\partial \mathcal{A}(a)}{\partial a} = w_i\frac{\partial \mathcal{N}_i(a)}{\partial a} + w_{\sim i}\frac{\partial \mathcal{N}_{\sim i}(a)}{\partial a} - \frac{\partial \mathcal{C}_i(a)}{\partial a} - \frac{\partial \mathcal{C}_{\sim i}(a)}{\partial a} + 1 + R(a). \tag{30}$$

Combined, the conditions (26), (29) and (30) yield

$$\begin{aligned}
\frac{\partial \mathcal{V}_i(a)}{\partial a} &= u_{c,i}(\mathcal{C}_i(a),\mathcal{N}_i(a))\frac{\partial \mathcal{C}_i(a)}{\partial a} + u_{n,i}(\mathcal{C}_i(a),\mathcal{N}_i(a))\frac{\partial \mathcal{N}_i(a)}{\partial a} \\
&+\theta_i u_{c,\sim i}(\mathcal{C}_{\sim i}(a),\mathcal{N}_{\sim i}(a))\frac{\partial \mathcal{C}_{\sim i}(a)}{\partial a} + \theta_i u_{n,\sim i}(\mathcal{C}_{\sim i}(a),\mathcal{N}_{\sim i}(a))\frac{\partial \mathcal{N}_{\sim i}(a)}{\partial a} \\
&+u_{c,i}(\mathcal{C}_i(a),\mathcal{N}_i(a))\frac{\partial \mathcal{A}(a)}{\partial a} \\
&= u_{c,i}(\mathcal{C}_i(a),\mathcal{N}_i(a))\frac{\partial \mathcal{C}_i(a)}{\partial a} + u_{n,i}(\mathcal{C}_i(a),\mathcal{N}_i(a))\frac{\partial \mathcal{N}_i(a)}{\partial a} \\
&+\theta_i u_{c,\sim i}(\mathcal{C}_{\sim i}(a),\mathcal{N}_{\sim i}(a))\frac{\partial \mathcal{C}_{\sim i}(a)}{\partial a} + \theta_i u_{n,\sim i}(\mathcal{C}_{\sim i}(a),\mathcal{N}_{\sim i}(a))\frac{\partial \mathcal{N}_{\sim i}(a)}{\partial a} \\
&+u_{c,i}(\mathcal{C}_i(a),\mathcal{N}_i(a))\left(w_i\frac{\partial \mathcal{N}_i(a)}{\partial a} + w_{\sim i}\frac{\partial \mathcal{N}_{\sim i}(a)}{\partial a} - \frac{\partial \mathcal{C}_i(a)}{\partial a} - \frac{\partial \mathcal{C}_{\sim i}(a)}{\partial a} + 1 + R(a)\right),
\end{aligned}$$

which can be rewritten as

$$\begin{aligned}
\frac{\partial \mathcal{V}_i(a)}{\partial a} &= u_{c,i}(\mathcal{C}_i(a),\mathcal{N}_i(a))\left[\frac{\partial \mathcal{C}_i(a)}{\partial a} - w_i\frac{\partial \mathcal{N}_i(a)}{\partial a}\right] \\
&+\theta_i u_{c,\sim i}(\mathcal{C}_{\sim i}(a),\mathcal{N}_{\sim i}(a))\left[\frac{\partial \mathcal{C}_{\sim i}(a)}{\partial a} - w_{\sim i}\frac{\partial \mathcal{N}_{\sim i}(a)}{\partial a}\right] \\
&+u_{c,i}(\mathcal{C}_i(a),\mathcal{N}_i(a))\left[w_i\frac{\partial \mathcal{N}_i(a)}{\partial a} - \frac{\partial \mathcal{C}_i(a)}{\partial a}\right] \\
&+u_{c,i}(\mathcal{C}_i(a),\mathcal{N}_i(a))\left[w_{\sim i}\frac{\partial \mathcal{N}_{\sim i}(a)}{\partial a} - \frac{\partial \mathcal{C}_{\sim i}(a)}{\partial a}\right] \\
&+u_{c,i}(\mathcal{C}_i(a),\mathcal{N}_i(a))(1+R(a)),
\end{aligned}$$

by using the labor supply condition (28) to replace $u_{n,i}(c_i,n_i)$ with $-u_{c,i}(c_i,n_i)w_i$ and $u_{n,\sim i}(c_{\sim i},n_{\sim i})$ with $-u_{c,\sim i}(c_{\sim i},n_{\sim i})w_{\sim i}$. After collecting terms, this expression can be reduced to

$$\begin{aligned}\frac{\partial \mathcal{V}_i(a)}{\partial a} &= u_{c,i}(\mathcal{C}_i(a),\mathcal{N}_i(a))(1+R(a)) \\ &\quad +[\theta_i u_{c,\sim i}(\mathcal{C}_{\sim i}(a),\mathcal{N}_{\sim i}(a))-u_{c,i}(\mathcal{C}_i(a),\mathcal{N}_i(a))] \\ &\quad \times\left[\frac{\partial \mathcal{C}_{\sim i}(a)}{\partial a}-w_{\sim i}\frac{\partial \mathcal{N}_{\sim i}(a)}{\partial a}\right].\end{aligned} \tag{31}$$

After updating (31) one period ahead and combining it with (26), one obtains the Euler equation for household member i:

$$\begin{aligned}u_{c,i}(c_i,n_i) &= \beta u_{c,i}(\mathcal{C}_i(a'),\mathcal{N}_i(a'))(1+R\left(a'\right)) \\ &\quad +\beta\left[\theta_i u_{c,\sim i}(\mathcal{C}_{\sim i}(a'),\mathcal{N}_{\sim i}(a'))-u_{c,i}(\mathcal{C}_i(a'),\mathcal{N}_i(a'))\right] \\ &\quad \times\left[\frac{\partial \mathcal{C}_{\sim i}(a')}{\partial a'}-w_{\sim i}\frac{\partial \mathcal{N}_{\sim i}(a')}{\partial a'}\right].\end{aligned}$$

A.1.3 Proof of Proposition 3.

The first-order conditions for the decision problem of the quasi-geometric household are given by

$$u_c(c,n) = 2\delta\beta\frac{\partial \mathcal{V}(a')}{\partial a'}, \tag{32}$$

$$-u_n(c,n) = 2\delta\beta\frac{\partial \mathcal{V}(a')}{\partial a'}w, \tag{33}$$

where I used that $\partial a'/\partial c=-1$ and $\partial a'/\partial n=w$. By combining these two conditions, one obtains the standard labor supply condition

$$-u_n(c,n)=u_c(c,n)w. \tag{34}$$

Differentiating $\mathcal{V}(a)$ with respect to a yields

$$\frac{\partial \mathcal{V}(a)}{\partial a}=u_c(\mathcal{C}(a),\mathcal{N}(a))\frac{\partial \mathcal{C}(a)}{\partial a}+u_n(\mathcal{C}(a),\mathcal{N}(a))\frac{\partial \mathcal{N}(a)}{\partial a}+\beta\frac{\partial \mathcal{V}(\mathcal{A}(a))}{\partial a'}\frac{\partial \mathcal{A}(a)}{\partial a},$$

and differentiating $\mathcal{A}(a)$ with respect to a gives

$$\frac{\partial \mathcal{A}(a)}{\partial a}=2w\frac{\partial \mathcal{N}(a)}{\partial a}-2\frac{\partial \mathcal{C}(a)}{\partial a}+1+R\left(a\right).$$

Together with condition (32), the two expressions above can be combined to

$$\begin{aligned}\frac{\partial \mathcal{V}(a)}{\partial a} &= u_c(\mathcal{C}(a),\mathcal{N}(a))\frac{\partial \mathcal{C}(a)}{\partial a} + u_n(\mathcal{C}(a),\mathcal{N}(a))\frac{\partial \mathcal{N}(a)}{\partial a} \\ &\quad + \frac{1}{2\delta}u_c(\mathcal{C}(a),\mathcal{N}(a))\frac{\partial \mathcal{A}(a)}{\partial a} \\ &= u_c(\mathcal{C}(a),\mathcal{N}(a))\frac{\partial \mathcal{C}(a)}{\partial a} + u_n(\mathcal{C}(a),\mathcal{N}(a))\frac{\partial \mathcal{N}(a)}{\partial a} \\ &\quad + \frac{1}{2\delta}u_c(\mathcal{C}(a),\mathcal{N}(a))\left(2w\frac{\partial \mathcal{N}(a)}{\partial a} - 2\frac{\partial \mathcal{C}(a)}{\partial a} + 1 + R(a)\right),\end{aligned}$$

which can be written as

$$\begin{aligned}\frac{\partial \mathcal{V}(a)}{\partial a} &= u_c(\mathcal{C}(a),\mathcal{N}(a))\left[\frac{\partial \mathcal{C}(a)}{\partial a} - w\frac{\partial \mathcal{N}(a)}{\partial a}\right] \\ &\quad + \frac{1}{\delta}u_c(\mathcal{C}(a),\mathcal{N}(a))\left[w\frac{\partial \mathcal{N}(a)}{\partial a} - \frac{\partial \mathcal{C}(a)}{\partial a}\right] \\ &\quad + \frac{1}{2\delta}u_c(\mathcal{C}(a),\mathcal{N}(a))(1+R(a)) \\ &= u_c(\mathcal{C}(a),\mathcal{N}(a))\left(1-\frac{1}{\delta}\right)\left[\frac{\partial \mathcal{C}(a)}{\partial a} - w\frac{\partial \mathcal{N}(a)}{\partial a}\right] \\ &\quad + \frac{1}{2\delta}u_c(\mathcal{C}(a),\mathcal{N}(a))(1+R(a)),\end{aligned}$$

by using condition (34). Updating this condition one period ahead and combining it with condition (32) yields the Euler equation for the quasi-geometric household:

$$u_c(c,n) = \beta u_c(\mathcal{C}(a'),\mathcal{N}(a'))\left(1+R\left(a'\right)+\Gamma(a')\right),$$

with

$$\Gamma(a) = 2(\delta-1)\left[\frac{\partial \mathcal{C}(a)}{\partial a} - w\frac{\partial \mathcal{N}(a)}{\partial a}\right].$$

With $\delta = (\theta+1)/2$, this wedge equals the one in the non-cooperative two-person case.

A.2 Household Problem under Commitment

This section derives the optimality conditions for the cooperative household decision problem under commitment. In this case, the cooperative household chooses current and future consumption, labor supply and savings $\{c_{Ft}, n_{Ft}, c_{Mt}, n_{Mt}, a_{t+1}\}_{t=0}^{\infty}$ to maximize the joint objective function $\sum_{t=0}^{\infty}\beta^t U(c_{it}, c_{\sim it}, n_{it}, n_{\sim it})$ subject to a sequence of budget constraints (1) for all periods $t \geq 0$. The first-order conditions for this problem are

$$
\begin{aligned}
(1+\theta_{\sim i})\,u_{c,i}(c_{it},n_{it}) &= \lambda_t, i\in\{F,M\}\,,\\
-\,(1+\theta_{\sim i})\,u_{n,i}(c_{it},n_{it}) &= \lambda_t w_i, i\in\{F,M\}\,,\\
\lambda_t &= \beta\lambda_{t+1}(1+R(a_{t+1})),
\end{aligned}
$$

with λ_t denoting the Lagrange multiplier associated with the household budget constraint of period t. After using $\lambda_{t+s} = (1+\theta_{\sim i})\,u_{c,i}(c_{it+s},n_{it+s})$ to eliminate the multipliers λ_t and λ_{t+1}, these conditions reduce to

$$
\begin{aligned}
\frac{u_{c,F}(c_{Ft},n_{Ft})}{u_{c,M}(c_{Mt},n_{Mt})} &= \frac{1+\theta_F}{1+\theta_M},\\
-u_{n,i}(c_{it},n_{it}) &= u_{c,i}(c_{it},n_{it})w_i, i\in\{F,M\}\,,\\
u_{c,F}(c_{Ft},n_{Ft}) &= \beta u_{c,F}(c_{Ft+1},n_{Ft+1})(1+R(a_{t+1})),
\end{aligned}
$$

which are the same conditions as listed by Proposition 1 for the case without commitment when written using sequential notation.[33] Under cooperation, the household hence does not have an incentive to deviate from the optimal plan under commitment when re-optimizing from period to period, i.e. the optimal plan under commitment is time consistent.

The cooperative solution to the household problem under commitment is only time consistent since the household members have standard time-consistent preferences and there is (by assumption) no disagreement about how to evaluate consumption and labor supply within and across periods. It is therefore crucial that the household members share the same discount factor because heterogeneous time preferences would introduce disagreement into the household problem (see e.g. Jackson and Yariv, 2014). In addition, for the time-consistency result, it is also important that the interest rate charged in a given period does not depend on the actions of the household in the subsequent period. If, for instance, the household could default on a loan ($a < 0$) and the lender would charge an actuarially fair interest rate that reflects the risk of default as a function of the size of the credit as in Chatterjee et al. (2007), a time-inconsistency problem would arise. Only a household who can commit to a plan for all future actions would internalize the adverse effect that a default has on the interest rate in the previous period, leading to a different outcome relative to the case without commitment.

[33] The household budget constraint is the same as well.

A.3 Derivations for the Stackelberg Leadership Case

I will first derive the first-order conditions for the Stackelberg follower F and then the ones for the Stackelberg leader M.

A.3.1 Stackelberg Follower

The first-order conditions for c_F and n_F are

$$\begin{aligned} u_{c,F}(c_F, n_F) &= \beta \frac{\partial \mathcal{V}_F(a')}{\partial a'}, \\ -u_{n,F}(c_F, n_F) &= \beta \frac{\partial \mathcal{V}_F(a')}{\partial a'} w_F, \end{aligned}$$

or combined

$$-u_{n,F}(c_F, n_F) = u_{c,F}(c_F, n_F) w_{F.}$$

Above, I used the conditions (18) and (19). The first-order conditions for F are the same as in the simultaneous-move case. Differentiating the continuation value $\mathcal{V}_i(a)$, $i \in \{F, M\}$, and the savings policy $\mathcal{A}(a)$ with respect to assets a also yields the same expressions as in the simultaneous-move case. As a result, the generalized Euler equation for F is the same as in the model version of Section 2.

A.3.2 Stackelberg Leader

The first-order conditions for c_M and n_M are

$$\begin{aligned} 0 &= u_{c,M}(c_M, n_M) + \theta_M \left[\begin{array}{c} u_{c,F}(\hat{\mathcal{C}}_F(a, c_M, n_M), \hat{\mathcal{N}}_F(a, c_M, n_M)) \frac{\partial \hat{\mathcal{C}}_F(a, c_M, n_M)}{\partial c_M} \\ + u_{n,F}(\hat{\mathcal{C}}_F(a, c_M, n_M), \hat{\mathcal{N}}_F(a, c_M, n_M)) \frac{\partial \hat{\mathcal{N}}_F(a, c_M, n_M)}{\partial c_M} \end{array} \right] \\ &\quad + \beta \frac{\partial \mathcal{V}_M(a')}{\partial a'} \left[\frac{\partial a'}{\partial c_M} + \frac{\partial a'}{\partial c_F} \frac{\partial \hat{\mathcal{C}}_F(a, c_M, n_M)}{\partial c_M} + \frac{\partial a'}{\partial n_F} \frac{\partial \hat{\mathcal{N}}_F(a, c_M, n_M)}{\partial c_M} \right], \\ 0 &= u_{n,M}(c_M, n_M) + \theta_M \left[\begin{array}{c} u_{c,F}(\hat{\mathcal{C}}_F(a, c_M, n_M), \hat{\mathcal{N}}_F(a, c_M, n_M)) \frac{\partial \hat{\mathcal{C}}_F(a, c_M, n_M)}{\partial n_M} \\ + u_{n,F}(\hat{\mathcal{C}}_F(a, c_M, n_M), \hat{\mathcal{N}}_F(a, c_M, n_M)) \frac{\partial \hat{\mathcal{N}}_F(a, c_M, n_M)}{\partial n_M} \end{array} \right] \\ &\quad + \beta \frac{\partial \mathcal{V}_M(a')}{\partial a'} \left[\frac{\partial a'}{\partial n_M} + \frac{\partial a'}{\partial c_F} \frac{\partial \hat{\mathcal{C}}_F(a, c_M, n_M)}{\partial n_M} + \frac{\partial a'}{\partial n_F} \frac{\partial \hat{\mathcal{N}}_F(a, c_M, n_M)}{\partial n_M} \right]. \end{aligned}$$

Since u_M and u_F enter F's objective function additively, c_M and n_M only affect the decisions of F via the budget constraint. As a result, the following relationship holds

$$\frac{\partial \hat{\mathcal{X}}_F(a,c_M,n_M)}{\partial n_M} = -\frac{\partial \hat{\mathcal{X}}_F(a,c_M,n_M)}{\partial c_M} w_M, \tag{35}$$

for $\mathcal{X} \in \{\mathcal{C},\mathcal{N}\}$. By using this condition as well as the conditions (18) and (19), the first-order conditions for the Stackelberg leader can be written as

$$\begin{aligned} 0 &= u_{c,M}(c_M,n_M) \\ &\quad +\theta_M \begin{bmatrix} u_{c,F}(\hat{\mathcal{C}}_F(a,c_M,n_M),\hat{\mathcal{N}}_F(a,c_M,n_M))\frac{\partial \hat{\mathcal{C}}_F(a,c_M,n_M)}{\partial c_M} \\ +u_{n,F}(\hat{\mathcal{C}}_F(a,c_M,n_M),\hat{\mathcal{N}}_F(a,c_M,n_M))\frac{\partial \hat{\mathcal{N}}_F(a,c_M,n_M)}{\partial c_M} \end{bmatrix} \\ &\quad +\beta\frac{\partial \mathcal{V}_M(a')}{\partial a'}\left[-1-\frac{\partial \hat{\mathcal{C}}_F(a,c_M,n_M)}{\partial c_M}+w_F\frac{\partial \hat{\mathcal{N}}_F(a,c_M,n_M)}{\partial c_M}\right], \end{aligned} \tag{36}$$

and

$$\begin{aligned} 0 &= u_{n,M}(c_M,n_M) \\ &\quad -\theta_M \begin{bmatrix} u_{c,F}(\hat{\mathcal{C}}_F(a,c_M,n_M),\hat{\mathcal{N}}_F(a,c_M,n_M))\frac{\partial \hat{\mathcal{C}}_F(a,c_M,n_M)}{\partial c_M}w_M \\ +u_{n,F}(\hat{\mathcal{C}}_F(a,c_M,n_M),\hat{\mathcal{N}}_F(a,c_M,n_M))\frac{\partial \hat{\mathcal{N}}_F(a,c_M,n_M)}{\partial c_M}w_M \end{bmatrix} \\ &\quad +\beta\frac{\partial \mathcal{V}_M(a')}{\partial a'}\left[w_M+w_M\frac{\partial \hat{\mathcal{C}}_F(a,c_M,n_M)}{\partial c_M}-w_M w_F\frac{\partial \hat{\mathcal{N}}_F(a,c_M,n_M)}{\partial c_M}\right], \end{aligned}$$

which can further bew written as

$$\begin{aligned} 0 &= u_{n,M}(c_M,n_M) \\ &\quad -w_M \begin{bmatrix} \theta_M \begin{bmatrix} u_{c,F}(\hat{\mathcal{C}}_F(a,c_M,n_M),\hat{\mathcal{N}}_F(a,c_M,n_M))\frac{\partial \hat{\mathcal{C}}_F(a,c_M,n_M)}{\partial c_M} \\ +u_{n,F}(\hat{\mathcal{C}}_F(a,c_M,n_M),\hat{\mathcal{N}}_F(a,c_M,n_M))\frac{\partial \hat{\mathcal{N}}_F(a,c_M,n_M)}{\partial c_M} \end{bmatrix} \\ +\beta\frac{\partial \mathcal{V}_M(a')}{\partial a'}\left(-1-\frac{\partial \hat{\mathcal{C}}_F(a,c_M,n_M)}{\partial c_M}+w_F\frac{\partial \hat{\mathcal{N}}_F(a,c_M,n_M)}{\partial c_M}\right) \end{bmatrix}. \end{aligned} \tag{37}$$

Combined, (36) and (37) yield the standard labor supply condition

$$-u_{n,M}(c_M,n_M) = u_{c,M}(c_M,n_M)w_M.$$

Condition (36) can further be rewritten as

$$\begin{aligned}
0 &= u_{c,M}(c_M, n_M) \\
&\quad +\theta_M u_{c,F}(\hat{\mathcal{C}}_F(a,c_M,n_M), \hat{\mathcal{N}}_F(a,c_M,n_M)) \\
&\quad \times \left[\frac{\partial \hat{\mathcal{C}}_F(a,c_M,n_M)}{\partial c_M} - \frac{\partial \hat{\mathcal{N}}_F(a,c_M,n_M)}{\partial c_M} w_F\right] \\
&\quad +\beta \frac{\partial \mathcal{V}_M(a')}{\partial a'}\left[-1 - \frac{\partial \hat{\mathcal{C}}_F(a,c_M,n_M)}{\partial c_M} + w_F \frac{\partial \hat{\mathcal{N}}_F(a,c_M,n_M)}{\partial c_M}\right],
\end{aligned}$$

by using the labor supply condition $-u_{n,F}(c_F, n_F) = u_{c,F}(c_F, n_F) w_F$ to replace $u_{n,F}$. This condition can then be used to eliminate $\partial \mathcal{V}_M(a')/\partial a'$ in (29), leading to

$$\begin{aligned}
\frac{\partial \mathcal{V}_M(a)}{\partial a} &= u_{c,M}(\mathcal{C}_M(a), \mathcal{N}_M(a)) \frac{\partial \mathcal{C}_M(a)}{\partial a} + u_{n,M}(\mathcal{C}_M(a), \mathcal{N}_M(a)) \frac{\partial \mathcal{N}_M(a)}{\partial a} \\
&\quad +\theta_M u_{c,F}(\mathcal{C}_F(a), \mathcal{N}_F(a)) \frac{\partial \mathcal{C}_F(a)}{\partial a} + \theta_M u_{n,F}(\mathcal{C}_F(a), \mathcal{N}_F(a)) \frac{\partial \mathcal{N}_F(a)}{\partial a} \\
&\quad + \frac{\partial \mathcal{A}(a)}{\partial a} \frac{u_{c,M}(c_M, n_M) + \theta_M \Lambda(a)}{\Theta(a)} \\
&= u_{c,M}(\mathcal{C}_M(a), \mathcal{N}_M(a)) \left[\frac{\partial \mathcal{C}_M(a)}{\partial a} - \frac{\partial \mathcal{N}_M(a)}{\partial a} w_M\right] \\
&\quad +\theta_M u_{c,F}(\mathcal{C}_F(a), \mathcal{N}_F(a)) \left[\frac{\partial \mathcal{C}_F(a)}{\partial a} - \frac{\partial \mathcal{N}_F(a)}{\partial a} w_F\right] \\
&\quad + \frac{\partial \mathcal{A}(a)}{\partial a} \frac{u_{c,M}(c_M, n_M) + \theta_M \Lambda(a)}{\Theta(a)},
\end{aligned} \tag{38}$$

with

$$\begin{aligned}
\Theta(a) &= 1 + \frac{\partial \hat{\mathcal{C}}_F(a,c_M,n_M)}{\partial c_M} - w_F \frac{\partial \hat{\mathcal{N}}_F(a,c_M,n_M)}{\partial c_M}, \\
\Lambda(a) &= \theta_M u_{c,F}(\hat{\mathcal{C}}_F(a,c_M,n_M), \hat{\mathcal{N}}_F(a,c_M,n_M)) \\
&\quad \times \left[\frac{\partial \hat{\mathcal{C}}_F(a,c_M,n_M)}{\partial c_M} - \frac{\partial \hat{\mathcal{N}}_F(a,c_M,n_M)}{\partial c_M} w_F\right].
\end{aligned}$$

The second equality above uses the labor supply conditions for F and M to replace $u_{n,F}$ and $u_{n,M}$. Updating (38) one period ahead and inserting it into (36) yields

$$\begin{aligned}
&\frac{u_{c,M}(c_M, n_M) + \theta_M \Lambda(a)}{\Theta(a)} \\
&= \beta \left[\begin{array}{c}
u_{c,M}(\mathcal{C}_M(a'), \mathcal{N}_M(a')) \left[\frac{\partial \mathcal{C}_M(a')}{\partial a'} - \frac{\partial \mathcal{N}_M(a')}{\partial a'} w_M\right] \\
+\theta_M u_{c,F}(\mathcal{C}_F(a'), \mathcal{N}_F(a')) \left[\frac{\partial \mathcal{C}_F(a')}{\partial a'} - \frac{\partial \mathcal{N}_F(a')}{\partial a'} w_F\right] \\
+ \frac{\partial \mathcal{A}(a')}{\partial a'} \frac{u_{c,M}(c_M, n_M) + \theta_M \Lambda(a')}{\Theta(a')}
\end{array}\right].
\end{aligned}$$

After eliminating $\mathcal{A}(a')$ via (30), one finally obtains the generalized Euler equation for the Stackelberg leader:

$$\begin{aligned} &\frac{u_{c,M}(c_M,n_M)+\theta_M\Lambda(a)}{\Theta(a)} \\ = \quad &\beta\left[\begin{array}{c} \frac{u_{c,M}(\mathcal{C}_M(a'),\mathcal{N}_M(a'))+\theta_M\Lambda(a')}{\Theta(a')}(1+R(a')) \\ +\left(\begin{array}{c} u_{c,M}(\mathcal{C}_M(a'),\mathcal{N}_M(a')) \\ -\frac{u_{c,M}(\mathcal{C}_M(a'),\mathcal{N}_M(a'))+\theta_M\Lambda(a')}{\Theta(a')}\end{array}\right)\left[\frac{\partial\mathcal{C}_M(a')}{\partial a'}-\frac{\partial\mathcal{N}_M(a')}{\partial a'}w_M\right] \\ +\left(\begin{array}{c} \theta_M u_{c,F}(\mathcal{C}_F(a'),\mathcal{N}_F(a')) \\ -\frac{u_{c,M}(\mathcal{C}_M(a'),\mathcal{N}_M(a'))+\theta_M\Lambda(a')}{\Theta(a')}\end{array}\right)\left[\frac{\partial\mathcal{C}_F(a')}{\partial a'}-\frac{\partial\mathcal{N}_F(a')}{\partial a'}w_F\right]\end{array}\right]\end{aligned}$$

A.4 Numerical Solution

The model from Section 2 is solved via a time-iteration algorithm (see Judd, 1998).[34] It involves the following six steps:

1. Construct a discrete grid for household asset holdings $[\underline{a},\overline{a}]$.
2. Choose initial values for the policy functions $\mathcal{X}_{start}(a)$, $\mathcal{X}\in\{\mathcal{A},\mathcal{C}_F,\mathcal{C}_M,\mathcal{N}_F,\mathcal{N}_M\}$, at all grid points $a\in[\underline{a},\overline{a}]$.
3. Set $\mathcal{X}_{next}(a)=\mathcal{X}_{start}(a)$, $\mathcal{X}\in\{\mathcal{A},\mathcal{C}_F,\mathcal{C}_M,\mathcal{N}_F,\mathcal{N}_M\}$, for all $a\in[\underline{a},\overline{a}]$ and choose an error tolerance ε.
4. For each grid point $a\in[\underline{a},\overline{a}]$, compute the policies $\mathcal{X}_{new}(a)$, $\mathcal{X}\in\{\mathcal{A},\mathcal{C}_F,\mathcal{C}_M,\mathcal{N}_F,\mathcal{N}_M\}$, that satisfy the equilibrium conditions, given next period's policy functions $\mathcal{X}_{next}(a)$.
5. If $|\mathcal{X}_{new}(a)-\mathcal{X}_{next}(a)|<\varepsilon$, $\mathcal{X}\in\{\mathcal{A},\mathcal{C}_F,\mathcal{C}_M,\mathcal{N}_F,\mathcal{N}_M\}$, for all $a\in[\underline{a},\overline{a}]$, go to step 6, else set $\mathcal{X}_{next}(a)=\mathcal{X}_{new}(a)$ and repeat step 4.
6. Take $\mathcal{X}_{new}(a)$, $\mathcal{X}\in\{\mathcal{A},\mathcal{C}_F,\mathcal{C}_M,\mathcal{N}_F,\mathcal{N}_M\}$, as approximations of the respective equilibrium objects $\mathcal{X}(a)$ in the infinite-horizon model.

The policies in step 4 are computed via a non-linear equation solver. Given the policy functions $\mathcal{X}_{next}$ that determine the allocation in the subsequent period, the equation solver finds values for current savings, consumption and labor supply that satisfy the equilibrium conditions at all grid points. In the

[34] It is straightforward to modify the algorithm to solve the model with labor income risk discussed in Section 4.

cooperative case, these conditions are the ones listed by Proposition 1. In the non-cooperative case, Proposition 2 summarizes the relevant equilibrium conditions. To approximate the policy functions, I use Chebyshev polynomials. The use of Chebyshev polynomials preserves the differentiability of the policy functions which is crucial here since the equilibrium conditions in the non-cooperative case involve derivatives of future policy functions with respect to savings.

While equilibrium multiplicity is a potential problem when looking at MPE for dynamic games with infinitely-lived agents (see Krusell and Smith, 2003), I do not find evidence for multiple (differentiable) equilibria after varying the initial guesses used for the policy functions in the second step of the numerical algorithm. The algorithm always converged to the same policy functions regardless of the initial guesses.