

Discretionary Monetary and Fiscal Policy with Endogenous Sovereign Default*

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Abstract

How does the option to default on debt payments affect the conduct of public policy? To answer this question, this paper studies optimal monetary and fiscal policy without commitment for a model with nominal public debt and strategic sovereign default. When the government can default on its debt, public policy changes in the short and the long run relative to a setting without default option. The risk of default increases the volatility of interest rates, impeding the government's ability to smooth tax distortions across states. It also limits public debt accumulation which reduces the government's incentive to use surprise inflation on average. For the United States, the welfare consequences of the default option are found to be positive but of negligible size.

Keywords: Optimal Monetary and Fiscal Policy, Lack of Commitment, Public Debt, Long-Term Bonds, Sovereign Default, Markov-Perfect Equilibrium

JEL Classification: E31, E63, H63

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1 Introduction

While sovereign default was viewed as an emerging markets phenomenon for a long time, the recent European debt crisis has illustrated its ongoing relevance for developed economies (see e.g. Lane, 2012). Even the federal government of the United States, whose debt instruments have usually been treated as risk-less by market participants and economists alike, now faces increased concerns about the sustainability of its debt, as highlighted, for instance, by its credit-rating downgrade in 2011. Events like the debt-ceiling crisis of 2013 or comments made by then-presidential nominee Donald J. Trump about his potential willingness to consider debt restructuring as a policy option for the federal government further fueled such concerns.¹ Thinking about how the possibility of sovereign default can affect policy-making in developed economies has thus become more than just an interesting thought experiment.

The contribution of this paper is to study the consequences of allowing a policy maker not only to use standard instruments of monetary and fiscal policy but also to choose outright sovereign default. To do so, the paper studies optimal monetary and fiscal policy without commitment for a representative agent cash-credit economy that is subject to productivity shocks.² In the model, a benevolent government finances exogenous expenditures by setting a labor income tax rate, choosing the money growth rate, issuing nominal long-term bonds and deciding on whether to repay its outstanding debt or not. The default decision is modeled as a binary choice (see Eaton and Gersovitz, 1981). Following the quantitative sovereign default literature (see e.g. Hamann, 2004; Aguiar and Gopinath, 2006; Arellano, 2008), a default is costly because it leads to a deadweight loss of resources that takes the form of a reduction in aggregate productivity and triggers a debt restructuring process that involves a temporary exclusion of the government from financial markets.

As is common in the literature on optimal monetary and fiscal policy, I consider a closed economy. This paper thus contributes to the study of domestic debt default which, despite being a historically recurring phenomenon with severe economic consequences, has not received a lot of attention in the sovereign default literature (see Reinhart and Rogoff, 2011). In a closed economy, a default does not redistribute resources from foreign lenders to domestic citizens. The government may still choose not to repay its debt to relax its budget constraint and reduce distortionary taxes. The model is calibrated to

¹See e.g. <https://www.washingtonpost.com/news/wonk/wp/2013/10/04/absolutely-everything-you-need-to-know-about-the-debt-ceiling/> and <https://www.nytimes.com/2016/05/07/us/politics/donald-trumps-idea-to-cut-national-debt-get-creditors-to-accept-less.html> for details.

²As is common in the optimal policy literature (see Chari and Kehoe, 1999), I assume that there is only one benevolent policy maker, referred to as the government, who is in charge of both, monetary and fiscal policy. Niemann (2011), Niemann et al. (2013a) and Martin (2015) study time-consistent public policy without sovereign default in models where a central bank and a fiscal authority interact. See Roettger (2016) for a model with independent monetary and fiscal authorities that allows for sovereign default and political frictions.

the US economy, assuming that the resource costs of default are sufficiently high to rule out equilibrium default. Reducing these costs then allows to study how the risk of default affects public policy.

I study the Markov-perfect equilibrium of the public policy problem (see [Klein et al., 2008](#)). The government's decisions hence only depend on the payoff-relevant state of the economy which consists of aggregate productivity, the beginning-of-period public debt position and whether the government is in financial autarky or not. Since the government optimizes sequentially, it cannot commit to future policies and does not internalize that its current decisions affect household expectations in previous periods. However, the government is aware that expected future policy will depend on its borrowing decision because it will affect the incentive to reduce the real debt burden via default or inflation in the next period. The option to default thus matters for the government's response to adverse shocks by allowing it to adjust the real debt burden as well as by affecting the cost of borrowing and thus the attractiveness of debt as a shock absorber.

Compared to the otherwise identical economy without default option (or equivalently an economy with prohibitively high costs of default) the availability of sovereign default results in lower average inflation. Since the gains of inflation decline when a default takes place, it is lower when default is chosen instead of repayment. However, this direct effect of default on average inflation is of negligible size. Instead, the key mechanism that leads inflation to be lower when the default option is available is an indirect one. The attractiveness and hence the probability of default increases with public debt and decreases with aggregate productivity. With default risk, the bond price become more debt elastic in recessions and the marginal revenue from debt issuance accordingly decreases faster. Consequently, the government borrows less which reduces its incentive to use inflation to adjust real debt payments. The increased sensitivity of the bond price to productivity shocks also impedes the government's ability to smooth tax distortions across states. Relative to an economy without default option, tax and inflation rates are thus more volatile, amplifying the impact of productivity shocks on the economy.

From a welfare perspective, it is not obvious whether it is desirable to endow the government with the option to default when it cannot commit to future actions. As discussed above, the risk of default affects public policy in the short and the long run. With productivity shocks, the government would like to smooth tax distortions by running a budget deficit (surplus) during bad (good) times, following the logic of [Barro \(1979\)](#). Default risk makes debt issuance more expensive in recessions which leads to welfare losses due to more volatile public policy. The long-run implications of sovereign default might however lead to welfare gains that outweigh these costs. As in [Martin \(2009\)](#) and [Diaz-Gimenez et al. \(2008\)](#), the government chooses positive average debt positions because of its lack of commitment and

a monetary friction. By increasing the cost of borrowing in recessions, risk of default renders public debt accumulation less attractive, reducing average debt and - as a result - inflation. A welfare exercise reveals that having the option to default results in positive but negligible welfare gains. For the United States, lack of commitment to debt service might hence not be particularly important from a welfare perspective.

Related Literature This paper is related to the literature on optimal Markov-perfect monetary and fiscal policy with nominal government debt. [Martin \(2009, 2011, 2013\)](#) extensively studies the short-and long-run properties of public debt and inflation when the government lacks commitment. In particular, he shows that a monetary economy with discretionary policy and nominal public debt can generate positive public debt positions of plausible size. For a similar model environment, [Diaz-Gimenez et al. \(2008\)](#) show how public policy and welfare depend on whether debt is indexed to inflation or not. Among other things, they find that without commitment welfare can be lower when debt is indexed. In a model with nominal rigidities, [Niemann et al. \(2013b\)](#) study how the presence of lack of commitment and nominal government debt affect the persistence of inflation. Despite highlighting the role of lack of commitment for public policy, these studies maintain the assumption that there is no commitment problem related to debt repayment and thus abstract from sovereign default. Furthermore, at odds with the data, these papers assume that the government only issues one-period bonds. By contrast, I allow for perpetuities as in [Hatchondo and Martinez \(2009\)](#) and [Chatterjee and Eyigungor \(2012\)](#) which allows to match the average debt structure.³

This work is also related to recent papers that study domestic debt default. In a model with incomplete markets and idiosyncratic income risk, [D'Erasmus and Mendoza \(2013\)](#) show that a sovereign default can occur in equilibrium as an optimal distributive policy. [Pouzo and Presno \(2016\)](#) extend the incomplete markets model of [Aiyagari et al. \(2002\)](#) by considering a policy maker who cannot commit to debt payments. [Sosa-Padilla \(2014\)](#) studies Markov-perfect fiscal policy in a model where a sovereign default triggers a banking crisis. [Niemann and Pichler \(Forthcoming\)](#) study optimal fiscal policy without commitment for a deterministic closed economy where government bonds are valued for their liquidity services, which gives rise to endogenous output costs of default. All of these papers feature real economies and hence do not discuss monetary policy. They also only consider one-period debt, making this paper, to the best of my knowledge, the first one to study a quantitative model of domestic sovereign default with long-term debt.

³[Leeper et al. \(2016\)](#) and [Matveev \(2016\)](#) study time-consistent monetary and fiscal policy with long-term bonds but focus on cash-less New Keynesian models without uncertainty. They also abstract from sovereign default.

This paper also relates to the quantitative sovereign default literature that studies how risk of default affects business cycles in emerging economies.⁴ With this literature, it shares the assumption of the government's lack of commitment and the way sovereign default is modeled. Within this literature, the studies that are closest to this paper are [Cuadra et al. \(2010\)](#), [Nuño and Thomas \(2016\)](#) and [Du and Schreger \(2016\)](#). [Cuadra et al. \(2010\)](#) study a production economy with endogenous fiscal policy but abstract from monetary policy and - as is common in the sovereign default literature - look at a small open economy that trades real bonds with foreign investors. [Nuño and Thomas \(2016\)](#) consider a small open endowment economy with nominal defaultable debt and a benevolent government that chooses monetary policy under discretion. The authors find that the economy tends to be better off when the government issues foreign currency debt or joins a monetary union since this eliminates its inflation bias. [Du and Schreger \(2016\)](#) study a model of a small open economy where the government borrows in local currency from foreign investors, enabling it to reduce the real debt burden by using inflation. As in this paper, the authors allow for nominal long-term bonds. Since domestic entrepreneurs have liabilities denominated in foreign currency but earn revenues in local currency, inflation hurts firm balance sheets by depreciating the local currency.⁵ In contrast to these papers, the closed-economy model studied in this paper does not rely on the assumption that the government is impatient relative to its creditors to generate empirically plausible debt levels.

In independent and contemporaneous work, [Sunder-Plassmann \(2017\)](#) also studies time-consistent public policy for a monetary economy with sovereign default. However, there are several differences between our studies. Similar to [Diaz-Gimenez et al. \(2008\)](#), the focus of her paper is on comparing the properties of a model economy with nominal government debt with those of an otherwise identical model economy with indexed government debt. By contrast, I focus on how the ability to default changes the conduct of monetary and fiscal policy, using a model that can replicate short- and long-run properties of the US economy as the baseline scenario. Another difference between our two studies is that her model only considers one-period debt, whereas I allow for long-term bonds.

Finally and most importantly, in contrast to [Sunder-Plassmann \(2017\)](#), my setting features an endogenous debt recovery rate, which is crucial for a number of reasons. By allowing for a positive and endogenous haircut, the model can account for the empirical observations that default events rarely lead to haircuts of 100% and that debt recovery rates vary with the size of public debt ([Cruces and Trebesch, 2013](#)). An endogenous haircut also matters from a theoretical perspective. For the government, default

⁴A recent summary of this literature can be found in [Aguilar and Amador \(2014\)](#).

⁵[Na et al. \(2015\)](#) also develop a quantitative sovereign default model where the government can depreciate the local currency but consider external debt that is denominated in foreign currency.

and inflation are imperfect substitutes since they can both reduce the real debt burden if outstanding debt is denominated in local currency. The degree of substitution between the two policy options depends on how flexibly they can be used to adjust debt payments. Allowing for partial default is crucial to capture this policy dimension. On the one hand, Default events typically involve reductions of debt payments that are larger and more sudden compared to what an inflationary monetary policy could accomplish. On the other hand, while a government can arguably affect the size of a haircut, a default is usually followed by a potentially lengthy debt restructuring process that cannot be entirely controlled by the government and ultimately determines the debt recovery rate. The debt restructuring process in this paper, which is modeled following [Hatchondo et al. \(2016\)](#), is able to capture the trade-off between the potentially larger adjustment of debt payments that a default can accomplish relative to inflation and the associated uncertainty about the ultimate size and timing of debt repayment.

Recently, [Aguiar et al. \(2013\)](#) have also developed a model to jointly study inflation and sovereign default when a government cannot commit to future policy. However, their analysis differs from mine in several ways. First, their model features a small open endowment economy that is not subject to fundamental shocks and borrows from abroad. Second, the authors assume that the government experiences an ad-hoc utility cost of inflation. Third, in the spirit of [Cole and Kehoe \(2000\)](#), they exclusively focus on self-fulfilling debt crises.

Layout The rest of the paper is organized as follows. Section 2 presents the model that is analyzed quantitatively in Section 3. The welfare implications of sovereign default are discussed in Section 4. Section 5 concludes.

2 Model

The model extends a standard cash-credit economy (see [Lucas and Stokey, 1983](#)) by introducing long-term government bonds (see [Hatchondo and Martinez, 2009](#); [Chatterjee and Eyigungor, 2012](#)), strategic sovereign default (see [Eaton and Gersovitz, 1981](#); [Arellano, 2008](#)) and endogenous debt recovery (see [Hatchondo et al., 2016](#)).

Time is discrete, starts in $t = 0$ and goes on forever. The economy is populated by a unit mass continuum of homogeneous infinitely-lived households and a benevolent government. Taking government policies and prices as given, the households optimize in a competitive fashion. They supply labor n_t to produce the marketable good y_t , using a linear technology to be specified below. In addition, they choose consumption of a cash good c_{1t} and a credit good c_{2t} , and decide on money (\tilde{m}_{t+1}) and nominal

government bond (\tilde{b}_{t+1}) holdings. The unit price of a government bond is denoted as q_t . While all assets are nominal and thus subject to inflation risk, only government bonds are subject to default risk. A role for money is introduced by tying cash-good consumption c_{1t} to beginning-of-period money holdings via a cash-in-advance constraint (see [Lucas and Stokey, 1983](#); [Svensson, 1985](#))

$$\tilde{m}_t \geq \tilde{p}_t c_{1t},$$

with \tilde{p}_t denoting the price of consumption in terms of \tilde{m}_t .

To finance exogenous government spending g and outstanding nominal debt payments $\delta \tilde{B}_t$, the government chooses from a set of policies that includes the money growth rate μ_t , a linear labor income tax rate τ_t , the binary default decision $d_t \in \{0, 1\}$, and issuance of nominal non-state contingent long-maturity bonds \tilde{I}_t . Following [Hatchondo and Martinez \(2009\)](#) and [Chatterjee and Eyigungor \(2012\)](#), government bonds are modeled as perpetuities that promise to pay an infinite stream of coupon payments that decline geometrically over time, where the coupon parameter $\delta \in (0, 1]$ governs the average maturity of debt $1/\delta$ and the size of coupon payments. More specifically, a bond issued in period t promises to pay the nominal cash flow $\tilde{p}_t \delta (1 - \delta)^{k-1}$ in periods $t + k$, for $k \geq 1$.⁶ The memory-less nature of these perpetuities implies that the law of motion for the stock of nominal government debt can be recursively written as $\tilde{B}_{t+1} = (1 - \delta) \tilde{B}_t + \tilde{I}_t$.⁷ A default on outstanding public debt occurs when $d_t = 1$ is chosen, while the government fully repays its obligations for $d_t = 0$. In the default case, the government is excluded from financial markets until debt repayment to bond holders is settled (see [Hatchondo et al., 2016](#)).

The government's credit status is given by the indicator variable $h_t \in \{0, 1\}$. If $h_t = 0$, the government has access to the bond market, whereas it is in financial autarky for $h_t = 1$. Given the credit status at the end of the previous period h_{t-1} , the law of motion for h_t is

$$h_t = [\zeta_t(1 - e_t) + 1 - \zeta_t] h_{t-1} + d_t(1 - h_{t-1}).$$

If the government enters period t with a good credit status ($h_{t-1} = 0$) and defaults ($d_t = 1$), its credit status switches to $h_t = 1$. Conditional on having left the previous period $t - 1$ in autarky, with probability θ , in period t the government receives the offer to repay the fraction $\omega \in [0, 1]$ of its outstanding debt and

⁶Du and Schreger (2016) consider similar nominal perpetuities in a model with sovereign default and risk-neutral foreign investors.

⁷For $\delta = 1$, the perpetuity bond reduces to a standard one-period bond.

immediately leave autarky in return (see [Hatchondo et al., 2016](#)).⁸ The acceptance decision is denoted as $e_t \in \{0, 1\}$, where $e_t = 1$ means that the offer is accepted. As in [Hatchondo et al. \(2016\)](#), even if the offer to repay the reduced debt burden is declined, the debt position is nevertheless reduced to ωB_t .⁹ For the model formulation it will be useful to define the indicator variable $\zeta_t \in \{0, 1\}$, which equals one if the government receives a repayment offer and zero if not. If the government does not accept an offer, i.e. $e_t = 0$, it remains in autarky ($h_t = 1$) and might receive a new offer in the next period, again with probability θ . Conditional on not being in autarky, the government will have access to the bond market until it chooses to default.

2.1 Private Sector

Households have preferences given by

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_{1t}, c_{2t}, n_t) \right],$$

with discount factor $\beta \in (0, 1)$ and period utility function $u : \mathbb{R}_+^3 \rightarrow \mathbb{R}$. The utility function is twice continuously differentiable and satisfies $u_1, u_2, -u_n > 0$ and $u_{11}, u_{22}, u_{nn} < 0$ with u_x (u_{xx}) denoting the first (second) derivative of $u(\cdot)$ with respect to $x \in \{c_1, c_2, n\}$. Households have initial assets (b_0, m_0) and take as given prices $\{\tilde{p}_t, q_t\}_{t=0}^{\infty}$ and government policies $\{d_t, e_t, \mu_t, \tau_t, \tilde{B}_{t+1}\}_{t=0}^{\infty}$. The aggregate money stock evolves according to $\tilde{M}_{t+1} = (1 + \mu_t)\tilde{M}_t$. Households also take as given the government's credit status $\{h_t\}_{t=0}^{\infty}$. The labor productivity $\{a_t\}_{t=0}^{\infty}$ of the households is subject to random shocks and follows a stationary first-order Markov process with continuous support $\mathbb{A} \subseteq \mathbb{R}_+$ and transition function $f_a(a_{t+1}|a_t)$.¹⁰

Households maximize their expected lifetime utility subject to their period budget constraint and the cash-in-advance constraint,

$$\frac{\tilde{m}_t}{\tilde{p}_t} \geq c_{1t}.$$

⁸[Pouzo and Presno \(2016\)](#) endogenize the debt recovery rate and the duration of financial exclusion in a closed economy environment without monetary policy. Examples of sovereign default models that endogenize the recovery rate by modeling debt renegotiation between a small open economy and foreign investors are [Yue \(2010\)](#) and [Bai and Zhang \(2012\)](#). See [Niemann and Pichler \(Forthcoming\)](#) for a model of a deterministic production economy with financial frictions that allows for an endogenous recovery rate by letting the government directly choose the haircut on sovereign debt.

⁹This assumption reduces the notation needed for the model formulation because the acceptance decision e_t can in this case be characterized by the same policy functions as the default decision d_t .

¹⁰The focus on productivity shocks allows me to study how the possibility of sovereign default affects the business cycle properties of a monetary economy.

Given the debt restructuring process outlined above, the household period budget constraint is given as

$$\begin{aligned}
c_{1t} + c_{2t} + \frac{\tilde{m}_{t+1}}{\tilde{p}_t} + q_t \frac{\tilde{b}_{t+1}}{\tilde{p}_t} &\leq (1 - \tau_t) \psi(a_t, h_t) n_t + \frac{\tilde{m}_t}{\tilde{p}_t} \\
&+ \mathbf{1}_{\{h_{t-1}=0 \wedge d_t=0\}} \left[(\delta + (1 - \delta) q_t) \frac{\tilde{b}_t}{\tilde{p}_t} \right] \\
&+ \mathbf{1}_{\{(h_{t-1}=0 \wedge d_t=1) \vee (h_{t-1}=1 \wedge \zeta_t=0)\}} \left[q_t \frac{\tilde{b}_t}{\tilde{p}_t} \right] \\
&+ \mathbf{1}_{\{h_{t-1}=1 \wedge \zeta_t=1 \wedge e_t=0\}} \left[q_t \frac{\omega \tilde{b}_t}{\tilde{p}_t} \right] \\
&+ \mathbf{1}_{\{h_{t-1}=1 \wedge \zeta_t=1 \wedge e_t=1\}} \left[(\delta + (1 - \delta) q_t) \frac{\omega \tilde{b}_t}{\tilde{p}_t} \right],
\end{aligned}$$

where $\mathbf{1}_{\{\cdot\}}$ denotes the indicator function, which equals one if the statement in curly brackets is true and zero otherwise. The indicator functions allow to express the size of debt payments received by the household from the governments as well as the value of its beginning-of-period bond holdings \tilde{b}_t conditional on the government's credit status in the previous period h_{t-1} , whether a repayment offer has been made (ζ_t) and accepted (e_t), and the government's repayment decision d_t .

Households use their labor supply n_t to produce a marketable good according to the linear technology $y_t = \psi(a_t, h_t) n_t$. They take as given their effective labor productivity $\psi: \mathbb{R}_+ \times \{0, 1\} \rightarrow \mathbb{R}_+$ which depends on random productivity a_t and the government's credit status h_t (see Cuadra et al., 2010). Effective productivity $\psi(\cdot)$ increases with exogenous productivity ($\partial \psi(a_t, h_t) / \partial a_t \geq 0$) and is negatively affected if the government has a bad credit status ($\psi(a_t, 0) \geq \psi(a_t, 1)$).¹¹

2.2 Public Sector

Conditional on the government's credit status, the government budget constraint is

$$g - \tau_t \psi(a_t, h_t) n_t = \begin{cases} \frac{\tilde{M}_{t+1} + q_t \tilde{B}_{t+1}}{\tilde{p}_t} - \frac{\tilde{M}_t + (\delta + (1 - \delta) q_t) \tilde{B}_t}{\tilde{p}_t}, & \text{if } h_t = 0 \\ \frac{\tilde{M}_{t+1} - \tilde{M}_t}{\tilde{p}_t}, & \text{if } h_t = 1 \end{cases}$$

In the default (and autarky) case, the government has to finance public spending g with income tax revenues $\tau_t \psi(a_t, 1) n_t$ and seigniorage $\tau_t^m \equiv (\tilde{M}_{t+1} - \tilde{M}_t) / \tilde{p}_t$. When the government repays its debt, it additionally has to make debt payments but can access the bond market and issue new debt.

Following the quantitative sovereign default literature (see e.g. Arellano, 2008; Cuadra et al., 2010),

¹¹It is straightforward to modify the model to include a representative firm that is owned by households and produces the homogeneous good y_t , using labor supplied by households at a real wage w_t . Due to linearity of the production function, the wage rate will equal effective productivity $\psi(a_t, h_t)$ and profits will be zero, such that the behavior of the economy will not change with such a firm sector.

a sovereign default entails two types of costs for the economy. First, the government is excluded from the bond market in the default period and remains in autarky until it accepts an offer to repay its debt.¹² Second, the economy experiences a direct resource loss governed by $\psi(\cdot)$. As in [Cuadra et al. \(2010\)](#) and [Pouzo and Presno \(2016\)](#), these costs capture in reduced form productivity losses that occur in periods of default (and financial autarky). Despite being arguably ad hoc, such a specification allows me not to take a stand on how exactly a sovereign default is propagated through the economy. While there is evidence for domestic output costs, there is still no consensus on which mechanism is the most relevant one (see [Panizza et al., 2009](#)). In addition, two recent papers show that models with endogenous default costs that arise due to private credit disruptions ([Mendoza and Yue, 2012](#)) or banking crises ([Sosa-Padilla, 2014](#)) deliver similar qualitative and quantitative results as those with exogenous default costs. Furthermore, with exogenous resource costs of default, I can analyze the impact of the default option on public policy in a transparent and flexible way as it allows me to directly control the attractiveness of default via the size of the resource costs.

2.3 Private Sector Equilibrium

The first-order conditions for the household problem are

$$-\frac{u_n(t)}{u_2(t)} = (1 - \tau_t)\psi(a_t, h_t), \quad (1)$$

$$u_2(t) = \beta \mathbb{E}_t \left[u_1(t+1) \frac{\tilde{p}_t}{\tilde{p}_{t+1}} \right], \quad (2)$$

$$u_2(t)q_t = \beta \mathbb{E}_t \left[((1 - d_{t+1})(\delta + (1 - \delta)q_{t+1}) + d_{t+1}q_{t+1}) u_2(t+1) \frac{\tilde{p}_t}{\tilde{p}_{t+1}} \right], \quad (3)$$

and

$$u_2(t)q_t = \beta \mathbb{E}_t \left[\left\{ \begin{array}{l} \zeta_{t+1} \omega(e_{t+1}(\delta + (1 - \delta)q_{t+1}) + (1 - e_{t+1})q_{t+1}) \\ + (1 - \zeta_{t+1})q_{t+1} \end{array} \right\} u_2(t+1) \frac{\tilde{p}_t}{\tilde{p}_{t+1}} \right]. \quad (4)$$

In addition, the following complementary slackness conditions need to be satisfied as well:

$$\lambda_t = u_1(t) - u_2(t) \geq 0, \tilde{m}_t / \tilde{p}_t - c_{1t} \geq 0, \lambda_t (\tilde{m}_t / \tilde{p}_t - c_{1t}) = 0,$$

with λ_t denoting the Kuhn-Tucker multiplier on the cash-in-advance constraint.¹³

¹²Note that households can still trade the distressed government bonds among each other when the government is in financial autarky.

¹³In a household optimum, the household budget constraint holds with equality.

Intuitively, the cash-in-advance constraint is binding whenever the marginal utility of cash-good consumption exceeds the marginal utility of credit-good consumption. The inequality

$$u_1(t) - u_2(t) \geq 0, \quad (5)$$

needs to hold in equilibrium to satisfy $\lambda_t \geq 0$. Equation (1) characterizes the optimal household labor supply decision which is distorted for non-zero tax rates $\tau_t \neq 0$. The conditions (2)-(4) are the Euler equations for money holdings as well as investment in government bonds, conditional on h_t . If the government is in financial autarky, only the secondary market for public debt is operative and (4) is the relevant Euler equation for government bonds, whereas condition (3) is the Euler equation for regular times. Since all assets are nominal, they need to compensate households for expected (gross) inflation $\tilde{p}_{t+1}/\tilde{p}_t$. Government bonds furthermore reflect default and bond price risk as well.

As in [Martin \(2009\)](#), I normalize nominal variables by the beginning-of-period aggregate money stock \tilde{M}_t , $x_t \equiv \tilde{x}_t/\tilde{M}_t$ for $x \in \{B, b, m, p\}$, which renders the model stationary.¹⁴ It implies that the inflation rate in period t is given as

$$\pi_t \equiv \frac{p_t(1 + \mu_{t-1})}{p_{t-1}} - 1,$$

such that inflation equals money growth in the long run and an increase in the price index p_t directly raises inflation π_t .

After normalizing nominal variables, the Euler equations become

$$u_2(t) = \beta \mathbb{E}_t \left[u_1(t+1) \frac{p_t}{p_{t+1}} \frac{1}{1 + \mu_t} \right], \quad (6)$$

$$u_2(t)q_t = \beta \mathbb{E}_t \left[((1 - d_{t+1})(\delta + (1 - \delta)q_{t+1}) + d_{t+1}q_{t+1})u_2(t+1) \frac{p_t}{p_{t+1}} \frac{1}{1 + \mu_t} \right], \quad (7)$$

and

$$u_2(t)q_t = \beta \mathbb{E}_t \left[\left\{ \begin{array}{l} \zeta_{t+1} \omega(e_{t+1}(\delta + (1 - \delta)q_{t+1}) + (1 - e_{t+1})q_{t+1}) \\ + (1 - \zeta_{t+1})q_{t+1} \end{array} \right\} u_2(t+1) \frac{p_t}{p_{t+1}} \frac{1}{1 + \mu_t} \right]. \quad (8)$$

¹⁴Note that, by construction, the normalized aggregate money stock is constant and equal to one.

For the economy, the goods and asset market clearing conditions are as follows:

$$\begin{aligned}\psi(a_t, h_t)n_t &= c_{1t} + c_{2t} + g, \\ b_{t+1} &= B_{t+1}, \\ m_{t+1} &= 1.\end{aligned}$$

If real balances are high enough, households equalize marginal utility across cash and credit goods, i.e. condition (5) holds with equality. If not, households are cash constrained and the allocation of consumption is distorted. As in [Martin \(2009\)](#), in a monetary equilibrium, i.e. an equilibrium in which money is valued,

$$c_{1t} = 1/p_t,$$

needs to hold. Note that this still allows for an unconstrained consumption allocation if the cash-in-advance constraint is just binding, i.e. when $\lambda_t = 0$ and $u_1(t) = u_2(t)$ hold simultaneously.

2.4 Public Policy Problem

In this section, I formulate the public policy problem. The government is benevolent and sets its policy instruments to maximize the expected life-time utility of the households, anticipating the response of the private sector to its policies. However, it cannot commit itself to a state-contingent (Ramsey) policy plan for all current and future policies but optimizes from period to period instead. To analyze the decision problem of the government, I restrict attention to stationary Markov-perfect equilibria (see [Klein et al., 2008](#)). In a Markov-perfect equilibrium, the optimal decisions of the government in any period will be characterized by time-invariant functions that only depend on the minimal payoff-relevant state of the economy in that respective period. In the model, this state consists of the beginning-of-period debt-to-money ratio B_t , labor productivity a_t and the government's credit status h_t . By requiring the government to only condition its decisions on the current payoff-relevant aggregate state, the Markov-perfect equilibrium concept rules out the possibility that the government is able to keep promises made in the past. This is because at the start of a period, the government does not care about the past and only considers its payoff in current and future periods.¹⁵ By construction, the government thus is ensured to act in a time-consistent way.

The Markov-perfect policy problem will be formulated recursively. In the remainder, I will thus adopt

¹⁵The focus on Markov-perfect strategies also rules out the possibility of reputational considerations based on complex trigger strategies as in [Chari and Kehoe \(1990, 1993\)](#).

the notation of dynamic programming. Time indices are hence dropped and a prime is used to denote next period's variables. Given the aggregate state at the start of a period, the government takes as given the policy function $\mathcal{D}(B', a')$ that determines next period's default decision as well as the policy functions $\mathcal{X}^r(B', a')$ and $\mathcal{X}^d(B', a')$, with $\mathcal{X} \in \{C_2, \mathcal{N}, \mathcal{P}, \mathcal{Q}\}$, that determine consumption, labor supply, the price index and the bond price in the next period for the case of repayment (r) and default (d).¹⁶ Expectations of these variables enter the household optimality conditions (6) and (7) and thus matter for the allocation in the current period.^{17,18} Despite lacking the ability to commit to future policies, the government fully recognizes today that it affects (expected) future policies via its choice of B' , which in turn have an effect on the behavior of the private sector in the current period. In a stationary Markov-perfect equilibrium, the policy functions that govern future decisions then coincide with the policy functions that determine current public policy for all states.

As in Klein et al. (2008), one can interpret the formulation of the public policy problem as a Markov-perfect game played between successive governments. Following this interpretation, in each period, a different government is in charge of choosing public policy. Each government then chooses its optimal strategies, taking as given the optimal responses of the government in the next period.

In every period, the government anticipates how the private sector responds to its actions as given by the private sector equilibrium conditions.¹⁹ Applying the normalisation of nominal variables used earlier, the government budget constraint can be written as

$$g - \tau_t \psi(a_t, h_t) n_t = \begin{cases} (1 + \mu_t) \frac{1 + q_t B_{t+1}}{p_t} - \frac{1 + (\delta + (1 - \delta) q_t) B_t}{p_t}, & \text{if } h_t = 0, \\ \frac{\mu_t}{p_t}, & \text{if } h_t = 1, \end{cases}$$

where $\tilde{M}_{t+1} = (1 + \mu_t) \tilde{M}_t$ is used as well. Using the household optimality conditions (1), (6)-(7), the binding cash-in-advance constraint and the aggregate resource constraint, the government budget constraint

¹⁶Remember that cash-good consumption c_1 is directly linked to the price index p via the cash-in-advance constraint.

¹⁷While these functions also enter the Euler equation for distressed government bonds, (8), in expectation, there is no direct feedback between this bond price and the behavior of the government in periods of default/autarky.

¹⁸Households do not have a strategic impact on future government policies but form rational expectations about them based on the policy functions listed above.

¹⁹The government thus plays a Stackelberg game against the (passive) private sector in every period.

can be further rewritten as

$$\begin{aligned}
& \beta \mathbb{E}_{a'|a} \begin{bmatrix} u'_1 \frac{1-\mathcal{D}(B',a')}{\mathcal{P}^r(B',a')} \\ + u'_1 \frac{\mathcal{D}(B',a')}{\mathcal{P}^d(B',a')} \end{bmatrix} + \beta \mathbb{E}_{a'|a} \begin{bmatrix} u'_2 \frac{1-\mathcal{D}(B',a')}{\mathcal{P}^r(B',a')} (\delta + (1-\delta) \mathcal{Q}^r(B',a')) \\ + u'_2 \frac{\mathcal{D}(B',a')}{\mathcal{P}^d(B',a')} \mathcal{Q}^d(B',a') \end{bmatrix} B' \\
& - (u_2/p) \left(\delta + (1-\delta) \frac{\mathbb{E}_{a'|a} \left[u'_2 \frac{1-\mathcal{D}(B',a')}{\mathcal{P}^r(B',a')} (\delta + (1-\delta) \mathcal{Q}^r(B',a')) + u'_2 \frac{\mathcal{D}(B',a')}{\mathcal{P}^d(B',a')} \mathcal{Q}^d(B',a') \right]}{\mathbb{E}_{a'|a} \left[u'_1 \frac{1-\mathcal{D}(B',a')}{\mathcal{P}^r(B',a')} + u'_1 \frac{\mathcal{D}(B',a')}{\mathcal{P}^d(B',a')} \right]} \right) B \\
& + u_n n + u_2 c_2 = 0,
\end{aligned} \tag{9}$$

for the repayment case and as

$$\begin{aligned}
& \beta \mathbb{E}_{a'|a} \left[\theta \times \left\{ u'_1 \frac{\mathcal{D}(\omega B',a')}{\mathcal{P}^d(\omega B',a')} + u'_1 \frac{1-\mathcal{D}(\omega B',a')}{\mathcal{P}^r(\omega B',a')} \right\} + (1-\theta) \times u'_1 \frac{1}{\mathcal{P}^d(B',a')} \right] \\
& + u_n n + u_2 c_2 = 0,
\end{aligned} \tag{10}$$

for the default (and autarky) case. This constraint can be seen as the period implementability constraint for the government.²⁰ Note that $e' = 1 - \mathcal{D}(\omega B', a')$ was used for the derivation of the constraint in the default case. Declining an offer to repay can hence be thought of as defaulting on it (see [Hatchondo et al., 2016](#)).

In addition to the implementability constraint, the government also has to respect the following two private sector equilibrium conditions:

$$0 = \psi(a, h)n - 1/p - c_2 - g, \tag{11}$$

$$0 \leq u_1 - u_2. \tag{12}$$

The household budget constraint is satisfied by Walras' Law, given the government budget constraint, the binding cash-in-advance constraint and the market clearing conditions.

Although the government cannot borrow in periods of autarky, it can still affect the end-of-period debt position B' . To see this, recall that B' is the end-of-period debt-to-money ratio \tilde{B}'/\tilde{M}' . While the numerator of this ratio (the nominal debt value) is fixed to \tilde{B} due to financial autarky, the denominator (the end-of-period money stock) might change and is equal to $(1+\mu)\tilde{M}$.²¹ With definition $B = \tilde{B}/\tilde{M}$, it then follows that in periods of default (and autarky)

$$B' = \frac{B}{1+\mu},$$

²⁰The derivation of the implementability constraint can be found in Appendix A.1.

²¹Applying the same normalisation of nominal variables used in this paper, [Niemann et al. \(2013b\)](#) study a model without default where a fiscal authority chooses \tilde{B}'/\tilde{B} and a monetary authority sets \tilde{M}'/\tilde{M} .

holds, which can be rewritten as

$$0 = B' - B \times \left(\beta \mathbb{E}_{a'|a} \left[\left\{ \begin{array}{c} \theta \times \left\{ u'_1 \frac{\mathcal{D}(\omega B', a')}{\mathcal{P}^d(\omega B', a')} + u'_1 \frac{1 - \mathcal{D}(\omega B', a')}{\mathcal{P}^r(\omega B', a')} \right\} \\ + (1 - \theta) \times \frac{u'_1}{\mathcal{P}^d(B', a')} \end{array} \right\} \times \frac{p}{u_2} \right] \right)^{-1}, \quad (13)$$

by eliminating the money growth rate via condition (6) and rearranging terms.

Let $\mathbb{B} \equiv [\underline{B}, \bar{B}]$ be the set of feasible aggregate debt values, with $-\infty < \underline{B} \leq 0$ and $0 < \bar{B} < \infty$. Conditional on having a good credit standing ($h = 0$), the decision problem of the government solves the following functional equation:

$$\mathcal{V}(B, a) = \max_{d \in \{0, 1\}} \left\{ (1 - d) \mathcal{V}^r(B, a) + d \mathcal{V}^d(B, a) \right\}, \quad (14)$$

with the value of repayment given as

$$\mathcal{V}^r(B, a) = \max_{c_2, n, p, B' \in \mathbb{B}} u(1/p, c_2, n) + \beta \mathbb{E}_{a'|a} [\mathcal{V}(B', a')] \text{ s.t. (9), (11), (12),}$$

and the value of default (and autarky) as

$$\mathcal{V}^d(B, a) = \max_{c_2, n, p, B' \in \mathbb{B}} u(1/p, c_2, n) + \beta \mathbb{E}_{a'|a} \left[\theta \mathcal{V}(\omega B', a') + (1 - \theta) \mathcal{V}^d(B', a') \right] \text{ s.t. (10) - (13).}$$

As is standard in the sovereign default literature, the government is assumed to honor its obligations whenever it is indifferent between default and repayment. If the government is in financial autarky, it solves the same problem as in the default case. When in autarky, the government will have the offer to regain access to financial markets in the subsequent period with probability θ . With probability $1 - \theta$, it will not receive an offer and remain in financial autarky.

2.5 Equilibrium

The Markov-perfect equilibrium is defined as follows:

Definition 1 A stationary Markov-perfect equilibrium consists of two sets of functions $\{\mathcal{D}, \mathcal{B}^r, \mathcal{C}_2^r, \mathcal{N}^r, \mathcal{P}^r, \mathcal{Q}^r, \mathcal{V}, \mathcal{V}^r\} : \mathbb{B} \times \mathbb{A} \rightarrow \{0, 1\} \times \mathbb{B} \times \mathbb{R}_+^4 \times \mathbb{R}^2$ and $\{\mathcal{B}^d, \mathcal{C}_2^d, \mathcal{N}^d, \mathcal{P}^d, \mathcal{Q}^d, \mathcal{V}^d\} : \mathbb{B} \times \mathbb{A} \rightarrow \mathbb{B} \times \mathbb{R}_+^4 \times \mathbb{R}$, such that for all $(B, a) \in \mathbb{B} \times \mathbb{A}$:

$$\mathcal{D}(B, a) = \arg \max_{d \in \{0, 1\}} \left\{ (1 - d) \mathcal{V}^r(B, a) + d \mathcal{V}^d(B, a) \right\},$$

$$\begin{aligned} \{\mathcal{X}^r(B, a)\}_{\mathcal{X} \in \{C_2, \mathcal{N}, \mathcal{P}, \mathcal{B}\}} &= \arg \max_{c_2, n, p, B' \in \mathbb{B}} u(1/p, c_2, n) + \beta \mathbb{E}_{a'|a} [\mathcal{V}(B', a')] \\ &\text{s.t. (9), (11), (12),} \end{aligned}$$

$$\begin{aligned} \{\mathcal{X}^d(B, a)\}_{\mathcal{X} \in \{C_2, \mathcal{N}, \mathcal{P}, \mathcal{B}\}} &= \arg \max_{c_2, n, p, B' \in \mathbb{B}} u(1/p, c_2, n) + \beta \mathbb{E}_{a'|a} \left[\begin{array}{l} \theta \mathcal{V}(\omega B', a') \\ + (1 - \theta) \mathcal{V}^d(B', a') \end{array} \right] \\ &\text{s.t. (10) - (13),} \end{aligned}$$

as well as

$$\mathcal{V}(B, a) = (1 - \mathcal{D}(B, a)) \times \mathcal{V}^r(B, a) + \mathcal{D}(B, a) \times \mathcal{V}^d(B, a),$$

$$\mathcal{V}^r(B, a) = u(\mathcal{P}^r(B, a)^{-1}, C_2^r(B, a), \mathcal{N}^r(B, a)) + \beta \mathbb{E}_{a'|a} [\mathcal{V}(\mathcal{B}^r(B, a), a')],$$

$$\mathcal{V}^d(B, a) = u(\mathcal{P}^d(B, a)^{-1}, C_2^d(B, a), \mathcal{N}^d(B, a)) + \beta \mathbb{E}_{a'|a} \left[\begin{array}{l} \theta \mathcal{V}(\omega \mathcal{B}^d(B, a), a') \\ + (1 - \theta) \mathcal{V}^d(\mathcal{B}^d(B, a), a') \end{array} \right],$$

$$\mathcal{Q}^r(B, a) = \frac{\mathbb{E}_{a'|a} \left[\begin{array}{l} u'_2 \frac{1 - \mathcal{D}(\mathcal{B}^r(B, a), a')}{\mathcal{P}^r(\mathcal{B}^r(B, a), a')} (\delta + (1 - \delta) \mathcal{Q}^r(\mathcal{B}^r(B, a), a')) \\ + u'_2 \frac{\mathcal{D}(\mathcal{B}^r(B, a), a')}{\mathcal{P}^d(\mathcal{B}^r(B, a), a')} \mathcal{Q}^d(\mathcal{B}^r(B, a), a') \end{array} \right]}{\mathbb{E}_{a'|a} \left[u'_1 \frac{1 - \mathcal{D}(\mathcal{B}^r(B, a), a')}{\mathcal{P}^r(\mathcal{B}^r(B, a), a')} + u'_1 \frac{\mathcal{D}(\mathcal{B}^r(B, a), a')}{\mathcal{P}^d(\mathcal{B}^r(B, a), a')} \right]},$$

and

$$\mathcal{Q}^d(B, a) = \frac{\mathbb{E}_{a'|a} \left[\begin{array}{l} \theta \times \omega \left\{ \begin{array}{l} + u'_2 \frac{1 - \mathcal{D}(\omega \mathcal{B}^d(B, a), a')}{\mathcal{P}^r(\omega \mathcal{B}^d(B, a), a')} (\delta + (1 - \delta) \mathcal{Q}^r(\omega \mathcal{B}^d(B, a), a')) \\ + u'_2 \frac{\mathcal{D}(\omega \mathcal{B}^d(B, a), a')}{\mathcal{P}^d(\omega \mathcal{B}^d(B, a), a')} \mathcal{Q}^d(\omega \mathcal{B}^d(B, a), a') \end{array} \right\} \\ + (1 - \theta) \times u'_2 \frac{1}{\mathcal{P}^d(\mathcal{B}^d(B, a), a')} \mathcal{Q}^d(\mathcal{B}^d(B, a), a') \end{array} \right]}{\mathbb{E}_{a'|a} \left[\begin{array}{l} \theta \times \left\{ u'_1 \frac{\mathcal{D}(\omega \mathcal{B}^d(B, a), a')}{\mathcal{P}^d(\omega \mathcal{B}^d(B, a), a')} + u'_1 \frac{1 - \mathcal{D}(\omega \mathcal{B}^d(B, a), a')}{\mathcal{P}^r(\omega \mathcal{B}^d(B, a), a')} \right\} \\ + (1 - \theta) \times u'_1 \frac{1}{\mathcal{P}^d(\mathcal{B}^d(B, a), a')} \end{array} \right]}.$$

The last two conditions are functional equations for the equilibrium bond prices $\mathcal{Q}^r(\cdot)$ and $\mathcal{Q}^d(\cdot)$. They are derived by combining the money demand condition (6) with the bond demand conditions (7) and (8), respectively.

The equilibrium definition highlights the stationarity of the policy problem as the functions that solve the decision problem of the government in a given period coincide with the policy functions that govern the optimal decisions of the government in future periods.

3 Quantitative Analysis

In this section, the role of sovereign default for public policy is investigated. Because the model cannot be solved analytically due to the discrete default option, numerical methods are applied. Appendix A.2 contains details regarding the numerical computation of the equilibrium. The next section presents the model specification. Simulation results are presented and discussed in Section 3.2.

3.1 Model Specification

To explore the model properties by computational means, functional forms and parameters need to be chosen.

Functional Forms Productivity follows a log-normal AR(1)-process,

$$a_t = a_{t-1}^{\rho} \exp(\sigma \varepsilon_t), \quad \varepsilon_t \stackrel{i.i.d.}{\sim} N(0, 1).$$

The household utility function is specified as

$$u(c_1, c_2, n) = \gamma_1 \frac{c_1^{1-\sigma_1} - 1}{1-\sigma_1} + \gamma_2 \frac{c_2^{1-\sigma_2} - 1}{1-\sigma_2} + (1 - \gamma_1 - \gamma_2) \frac{(1-n)^{1-\sigma_n} - 1}{1-\sigma_n},$$

with $\gamma_1, \gamma_2, \sigma_i > 0, i \in \{1, 2, n\}$ and $\gamma_1 + \gamma_2 < 1$.²²

The resource costs of default are specified as in Cuadra et al. (2010):

$$\psi(a, d) = a - d \times \max\{0, a - \tilde{a}\}.$$

If a default takes place, effective productivity equals \tilde{a} when a exceeds \tilde{a} while there are no costs of default when productivity a is below the threshold \tilde{a} . This default cost specification implies that a default is more costly in booms than in recessions.²³ In the quantitative sovereign default literature, it is well known that this feature is crucial for default to mostly take place in bad states and hence for counter-cyclical sovereign risk to emerge (see e.g. Aguiar and Amador, 2014). This property is consistent with empirical evidence (see Tomz and Wright, 2007) and also present in models with endogenous costs of default (see Mendoza and Yue, 2012, Sosa-Padilla, 2014).²⁴

²²For $\sigma_i = 1, i \in \{1, 2, n\}$, household utility is logarithmic for the respective variable.

²³The model results do not change if a convex cost specification as in Chatterjee and Eyigungor (2012) is adopted.

²⁴Allowing for default costs that enter the the aggregate resource constraint (or the government budget constraint) in a lump-sum way does not change the results of this paper as long as these losses are also relatively higher in good than in bad states, preserving countercyclical default incentives.

Parameter	Description	Value
β	Discount factor	0.9600
δ	Debt maturity parameter	0.2500
g	Government spending	0.0546
γ_1	Cash-good weight	0.0261
γ_2	Credit-good weight	0.1407
ρ	Persistence of productivity	0.7210
σ_1	Cash-good curvature	1.7940
σ_2	Credit-good curvature	1.8060
σ_n	Leisure curvature	3.0000
σ	Std. dev. productivity shock	0.0269
ω	Offer rate	0.6300

Table 1: Parameter values for baseline calibration without default

Parameters A model period corresponds to one year. The selected model parameters are listed in Table 1. As in [Martin \(2009, 2013\)](#), they are chosen to replicate certain short- or long-run properties of the US economy for the time period 1962-2006. Following [Martin \(2013\)](#), the productivity parameters are set to match the autocorrelation and standard deviation of US log real GDP, resulting in the values $(\rho, \sigma) = (0.721, 0.0269)$. Targeting an empirically plausible average debt maturity of four years, the model parameter δ is set to 0.25. The discount factor β is set to 0.96 and the expenditure parameter g to 0.0546 to match an annual real risk-free rate of 4% and an average public spending-to-GDP ratio of 18%, respectively. Parameters γ_1 and γ_2 are chosen to target a cash-credit good ratio of 0.37 and an average working time of 0.3 (see [Martin, 2009](#)), respectively. For the leisure elasticity parameter σ_n , I choose a rather standard value of 3.

As discussed in detail by [Diaz-Gimenez et al. \(2008\)](#) and [Martin \(2009, 2011, 2013\)](#), the size and sign of the long-run debt position crucially depend on how the revenues that the government receives from money issuance (first term on the LHS of (9)) change with B' . More specifically, a non-zero long-run debt position requires that these money revenues increase when more debt is issued, counteracting the simultaneous decline in marginal revenues from debt issuance caused by a decline of the bond price, which reflects an increase in expected inflation that investors want to be compensated for.²⁵ The chosen utility function implies that the parameter σ_1 , which governs the elasticity of cash-good consumption, is crucial for the behavior of money revenues and hence the long-run debt position. Importantly, the government only has an incentive to accumulate positive debt for $\sigma_1 > 1$ (see [Diaz-Gimenez et al., 2008; Martin, 2009, 2011, 2013](#), for details). I choose a value of $\sigma_1 = 1.794$ to match an average annual debt-to-

²⁵Looking at Markov-perfect public policy in a real economy setting with endogenous government spending and without default, [Debortoli and Nunes \(2013\)](#) show - for analytical and quantitative examples - that long-run debt only deviates from zero for a small range of parameter values. Similar results are found by [Krusell et al. \(2005\)](#) for a related model with exogenous government spending.

GDP ratio of 30.08%. Targeting the US average annual inflation rate of 4.40%, the cash-good parameter σ_2 is set to 1.806. The incentive to default and hence the probability thereof critically depend on \tilde{a} . For the baseline calibration, I choose a value for \tilde{a} which is low enough such that default never arises in equilibrium and high enough to ensure a well-defined competitive equilibrium in the default case. This benchmark economy yields the same results as a model without default option and will be referred to as "baseline economy". The model with default option will be referred to as "default economy". For the quantitative analysis, I will consider different values for \tilde{a} in order to understand how the incentive to default affects public policy.

The offer parameter ω is set to 0.63 as in [Hatchondo et al. \(2016\)](#), which generates an empirically plausible average haircut between 37% and 40% for the simulated model versions with equilibrium default (see [Cruces and Trebesch, 2013](#)). The probability of receiving an offer θ is set to 0.5. The results are not sensitive to the exact value used for θ , assuming the default cost parameter \tilde{a} is adjusted to keep the average default probability unchanged.

3.2 Results

Table 2 presents the averages of statistics calculated for 2500 simulated economies with 2500 periods each. The first 500 observations of each sample are discarded to eliminate the role of initial conditions. Output is given in logs and real terms, debt-to-GDP in terms of end-of-period debt divided by nominal GDP.

Average debt and inflation are lower for the model versions with default option and both increasing with the resource cost of default. The possibility of default reduces average inflation through a direct and an indirect effect. When the government chooses to default, there is no incentive to use inflation to reduce the real debt burden anymore since there is no debt service in periods of default and financial autarky. As a result, inflation is lower in such periods on average compared to periods of repayment. The role of this direct effect is however limited by the frequency of default and does not contribute much to the average inflation rate. The indirect effect of default on inflation is related to how the risk of default affects the government's borrowing behavior. As can be seen in panel a) of Figure 1, the probability of default $\mathbb{E}_{a'|a}[\mathcal{D}(B', a')]$ increases with borrowing and is higher in low productivity states, reflecting the government's incentive to default in bad times.

As in sovereign default models with risk-neutral investors (see [Arellano, 2008](#)), the model allows to express the bond price as a function of an arbitrary (and hence potentially off-equilibrium) end-of-period

	Baseline	$\tilde{a} = 0.945$	$\tilde{a} = 0.955$	$\tilde{a} = 0.965$	$\tilde{a} = 0.975$
<u>Mean</u>					
Default probability	-	0.0020	0.0061	0.0129	0.0252
Debt-to-GDP	0.3071	0.3001	0.2826	0.2449	0.2114
Tax rate	0.1788	0.1793	0.1805	0.1830	0.1856
Inflation rate	0.0442	0.0411	0.0336	0.0177	0.0022
Haircut	-	0.3700	0.3753	0.3861	0.3930
Years in autarky after a default	-	2.0024	2.0461	2.1285	2.1972
Nominal yield	0.0877	0.0865	0.0827	0.0726	0.0681
<u>Standard deviation</u>					
Output	0.0274	0.0276	0.0277	0.0278	0.0279
Tax rate	0.0045	0.0054	0.0065	0.0074	0.0084
Inflation rate	0.0285	0.0293	0.0310	0.0327	0.0345
Nominal yield	0.0118	0.0151	0.0212	0.0290	0.0394
<u>Correlation with output</u>					
Debt-to-GDP	-0.4366	-0.3686	-0.1940	-0.0134	0.1722
Tax rate	-0.9896	-0.9793	-0.9711	-0.9746	-0.9779
Inflation rate	-0.3121	-0.2549	-0.1703	-0.0836	-0.0002
Nominal yield	-0.6687	-0.5933	-0.5153	-0.4971	-0.5243

Table 2: Selected model statistics

debt position $B' \in \mathbb{B}$ and current productivity a ,²⁶

$$q(B', a) = \frac{\mathbb{E}_{a'|a} \left[u'_2 \frac{1 - \mathcal{D}(B', a')}{\mathcal{P}^r(B', a')} (\delta + (1 - \delta) \mathcal{Q}^r(B', a')) + u'_2 \frac{\mathcal{D}(B', a')}{\mathcal{P}^d(B', a')} \mathcal{Q}^d(B', a') \right]}{\mathbb{E}_{a'|a} \left[u'_1 \frac{1 - \mathcal{D}(B', a')}{\mathcal{P}^r(B', a')} + u'_1 \frac{\mathcal{D}(B', a')}{\mathcal{P}^d(B', a')} \right]}.$$

This bond price schedule is depicted in panel b) of Figure 1.²⁷ The presence of default risk strongly reduces the bond price in low productivity states and raises the cost of debt issuance in recessions compared to the baseline economy. This mechanism discourages the government from issuing as much debt as in an economy without default and thereby restricts the build up of public debt positions that would make higher inflation more attractive. When the cost of default is reduced, its attractiveness and hence its probability increase for a given debt position, which makes the bond price schedule become steeper in recessions for higher \tilde{a} -values, amplifying the mechanism just outlined and making average debt and inflation decline with \tilde{a} . Less average debt also implies that the tax base of the income tax increases relative to that of inflation. Hence, the benefit of raising inflation is lower, leading to a higher average labor tax rate in the baseline economy.²⁸ While the accumulation of debt crucially depends on the government's ability to collect seigniorage (see Section 3.1), the average seigniorage-to-GDP ratio is rather

²⁶See also Martin (2009) or Niemann et al. (2013a).

²⁷The equilibrium bond price then satisfies $\mathcal{Q}^r(B, a) = q(B^r(B, a), a)$.

²⁸According to Martin (2013), the US tax revenue-to-GDP ratio was 18.2% for the time period 1962-2006, which is close to the respective value predicted by the baseline model (17.88%).

small and of empirically plausible size (0.98%).²⁹

While there is a clear negative relationship between \tilde{a} and average debt as well as inflation, the effect that lowering the resource cost of default has on the average default frequency is not clear ex ante. Although a higher value for \tilde{a} increases the incentive to default for a given debt position, it also lowers the average debt position, which in turn reduces the incentive to default on average. However, for the simulated economies, the first effect dominates the latter, resulting in a negative relationship between the cost of default and the default frequency.

The default option also affects the cyclical behavior of the economy via its effect on borrowing conditions. Its impact on the government's borrowing behavior can be seen by looking at the cyclicity of public debt. While borrowing is countercyclical for the baseline model, it becomes less countercyclical as \tilde{a} goes up and even procyclical for $\tilde{a} = 0.975$, which is associated with a default probability typically only found in emerging economies. Since productivity is persistent, a negative shock to productivity raises the risk of default as the incentive to default is more likely to be strong in the subsequent period. The high debt elasticity of the bond price in low-productivity states forces the government to issue less debt in order to avoid an even larger decline of the bond price. As a result, the government has to resort to larger adjustments of inflation and taxes to finance debt payments and government spending.³⁰ By contrast, in the no-default economy, borrowing conditions do not deteriorate very much in response to a negative productivity shock, allowing the government to effectively smooth tax distortions across states, which translates into a lower degree of macroeconomic volatility. By increasing the cost of default, the government's behavior thus moves closer to that of an emerging economy.

To study the bond pricing consequences of the default option, it is helpful to look at the distribution of the nominal yield i_t of a bond, which is visualized in Figure 2 for selected model versions. Following [Du and Schreger \(2016\)](#), i_t is defined as the internal rate of return that satisfies

$$q_t = \sum_{s=1}^{\infty} \frac{CF_{t+s}}{(1+i_t)^s},$$

where CF_{t+s} denotes the promised payment in period $t+s$. Due to the perpetuity structure of the bond, the nominal yield is simply given as $i_t = \delta/q_t - \delta$ (see [Du and Schreger, 2016](#)).

While the yield distribution is bell-shaped and single-peaked for the baseline model economy, the

²⁹Using the same definition of seigniorage as in the model, [Aisen and Veiga \(2008\)](#) calculate that average seigniorage is 0.3% of GDP for the United States.

³⁰This mechanism is related to the one studied by [Cuadra et al. \(2010\)](#) in a model of a small open economy with real one-period government debt. The authors show that countercyclical default risk can rationalize the procyclical consumption taxation observed in emerging economies.

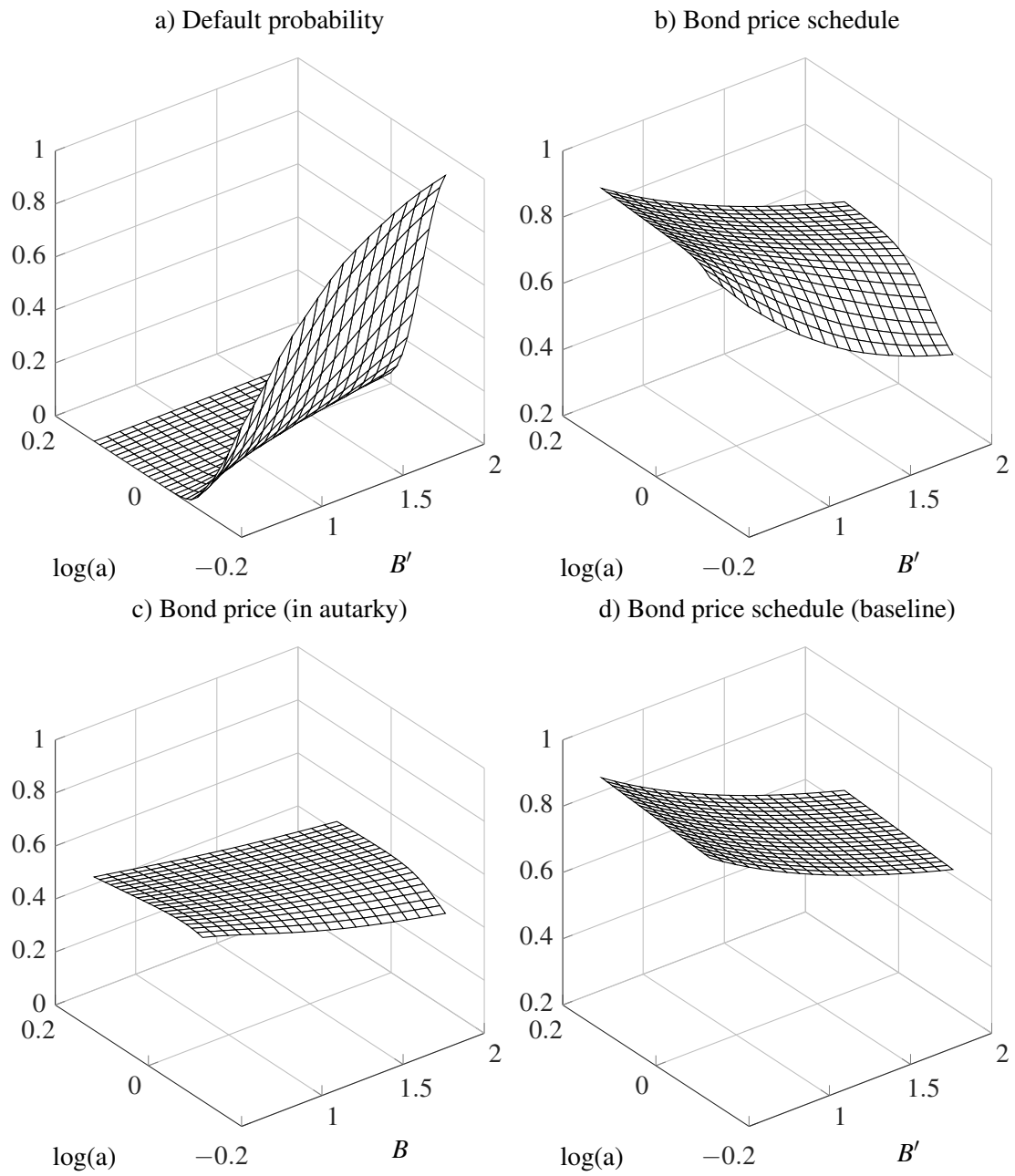


Figure 1: Default probability $\mathbb{E}_{a'|a}[\mathcal{D}(B', a')]$, bond price schedule $q(B', a)$ and bond price $Q^d(B, a)$ for the model economy with $\tilde{a} = 0.965$, as well as the bond price schedule for the baseline model

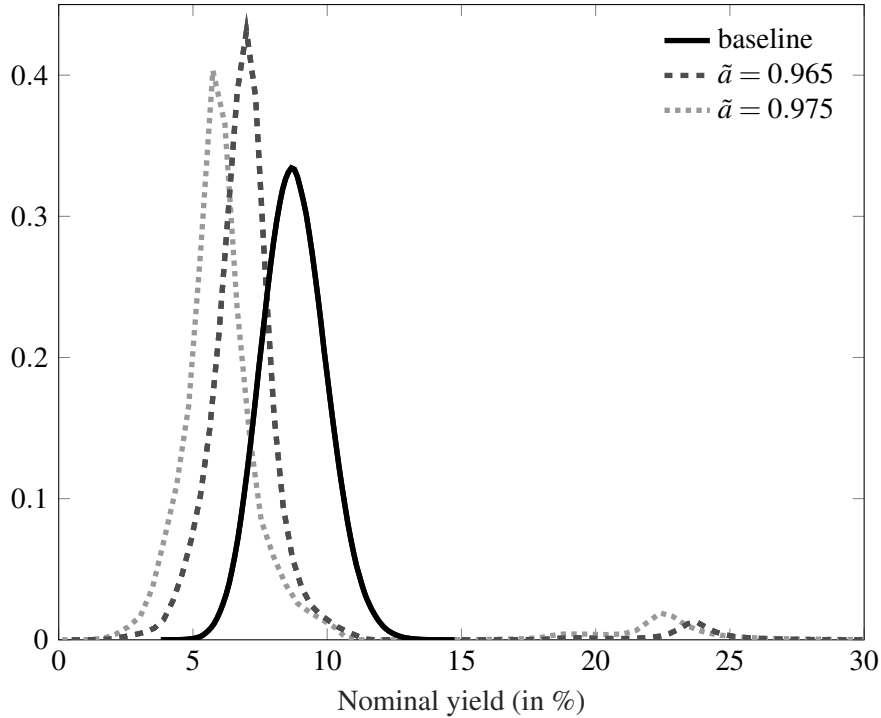


Figure 2: Nominal yield distribution for selected model versions

default economies exhibit a two-peaked distribution, reflecting the differences between the bond price in the repayment (left part) and the autarky (right part) case. Despite the higher average default frequency, the average nominal yield is lower in economies with equilibrium default and decreasing with $\tilde{\alpha}$ since these types of economies experience less inflation on average (see Table 2). The cost of default does not only affect decision making in periods of repayment but also has a direct effect on the outcome of the debt scheduling process. Since a higher value for $\tilde{\alpha}$ implies that staying in financial autarky becomes less costly, it makes waiting for a better settlement offer more attractive for the government. As a result, the higher default cost parameter $\tilde{\alpha}$ is, i.e. the lower the resource costs of default are, the higher the average haircut and the average spell in financial autarky become (see Table 2). The average bond price in autarky is however increasing in $\tilde{\alpha}$ due to the associated reduction in average inflation.

4 The Welfare Implications of Sovereign Default

This section discusses the welfare implications of sovereign default. With commitment, the option to default will not decrease welfare since the government would otherwise refrain from using it.³¹ Without commitment, this is not necessarily the case anymore.³² The previous section has shown that the default

³¹For a real small open economy with incomplete markets and costly sovereign default, Adam and Grill (2017) show that welfare can be increased when the Ramsey planner can commit to a state-contingent default plan.

³²The same is true in the context of consumer default where there exists a trade-off when introducing the option to file bankruptcy. On the one hand, indebted consumers receive the ability to make debt payments state contingent. On the other

	$\tilde{a} = 0.945$	$\tilde{a} = 0.955$	$\tilde{a} = 0.965$	$\tilde{a} = 0.975$
Welfare gain Δ (in %)	0.0013	0.0075	0.0279	0.0387

Table 3: Welfare comparison

option has implications for public policy in the short and the long run. On the one hand, by increasing the sensitivity of the bond price with respect to debt and productivity, countercyclical risk of default entails short-run costs because the government loses some of its ability to smooth tax distortions across states. On the other hand, default risk might lead to welfare gains due to its impact on long-run debt. The model features a long-run borrowing motive that stems from the presence of two frictions, lack of commitment and a liquidity constraint (see e.g. [Diaz-Gimenez et al., 2008](#), or [Martin, 2009](#)). By limiting public debt accumulation via more sensitive bond prices, the default option reduces average inflation and the misallocation of consumption compared to the no-default setting.

To evaluate whether the addition of the default option to the set of policy instruments is welfare enhancing, welfare measure Δ is calculated. It measures the percentage increase in credit-good consumption that households in the baseline economy without default need to be given in each period to achieve the same expected lifetime utility as in an economy with default:

$$\mathbb{E}_0 \left[\sum_{t=0}^T \beta^t u(c_{1t}^{\tilde{a}}, c_{2t}^{\tilde{a}}, n_t^{\tilde{a}}) \right] = \mathbb{E}_0 \left[\sum_{t=0}^T \beta^t u(c_{1t}^{\text{baseline}}, c_{2t}^{\text{baseline}}(1 + \Delta), n_t^{\text{baseline}}) \right].$$

Consumption and labor supply in an economy with resource cost parameter \tilde{a} are denoted as $x_t^{\tilde{a}}$, $x \in \{c_1, c_2, n\}$, whereas the respective variables for the baseline economy without default are denoted as x_t^{baseline} . Expected lifetime utility is calculated for both types of economies by averaging realized lifetime utility of 2500 samples with simulated time series of effective length $T = 2000$ each.

The calculated values for welfare measure Δ are presented in [Table 3](#). For the baseline economy, credit-good consumption needs to be increased regardless of which default economy it is compared to, i.e. the default option is welfare-enhancing. The computed Δ -values are however very small, ranging from only 0.0013% to 0.0387% of annual credit-good consumption. Since these welfare gains are of negligible size, one could argue that, from a welfare perspective, the model predicts that lack of commitment to repayment is not particularly important for the case of the United States.³³

hand, this flexibility comes at the cost of higher borrowing costs that compensate lenders for the increased risk of default (see e.g. [Livshits et al., 2007](#)).

³³Since monetary policy moves towards the Friedman Rule as \tilde{a} is increased, it might be that welfare gains decline at some point or even become negative. Unfortunately, the numerical algorithm becomes unstable if the average default frequency exceeds empirically plausible values well above 3%.

5 Conclusion

To understand the implications of the option to default on debt payments for public policy, this paper has studied optimal monetary and fiscal policy without commitment for a cash-credit economy with nominal debt and endogenous government default. While a default allows the government to reduce inflation and distortionary labor taxation by relaxing its budget constraint, the default option mainly induces lower rates of inflation by constraining debt issuance via endogenous default risk premia. This mechanism reduces the average debt position and as a result the government's incentive to use surprise inflation in the long-run. When the default option is available, taxes and inflation become more volatile because the government's ability to smooth tax distortions across states is reduced by the presence of default risk. For the case of the United States, the paper finds that the consequences of the option to default for welfare are positive but of negligible size.³⁴

³⁴In a previous version of this paper, titled "Monetary and Fiscal Policy with Sovereign Default", I calibrated a model version without long-term bonds and positive debt recovery to the case of Mexico and considered the opposite of the policy experiment performed in this paper: Matching a historically plausible default frequency, I used the model to study the implications of a counterfactual elimination of the default option on policy outcomes. The qualitative implications of that model were the same as in the model version calibrated to the United States.

A Appendix

A.1 Derivation of the Implementability Constraint

I will only derive the implementability constraint for the repayment case. The constraint for the default case is derived similarly. First, take the household optimality conditions (1),(6)-(7) and rewrite them (in recursive notation) as

$$\begin{aligned}\tau &= 1 + \frac{u_n}{u_2} \frac{1}{\psi(a,0)}, \\ \frac{1+\mu}{p} &= \beta \mathbb{E}_{a'|a} \left[\frac{u'_1}{u_2} \frac{1}{p'} \right], \\ \frac{(1+\mu)q}{p} &= \beta \mathbb{E}_{a'|a} \left[\left((1-d')(\delta + (1-\delta)q') + d'q' \right) \frac{u'_2}{u_2} \frac{1}{p'} \right].\end{aligned}$$

After using these expressions to eliminate the terms on the LHS of these equations in the government budget constraint

$$g - \tau \psi(a,0)n + \frac{1 + (\delta + (1-\delta)q)B}{p} = (1+\mu) \frac{1+qB'}{p},$$

one obtains

$$\begin{aligned}& g - \left(1 + \frac{u_n}{u_2} \frac{1}{\psi(a,0)} \right) \psi(a,0)n + 1/p + \left(\delta + (1-\delta) \frac{\mathbb{E}_{a'|a} \left[\left((1-d')(\delta + (1-\delta)q') + d'q' \right) \frac{u'_2}{u_2} \frac{1}{p'} \right]}{\mathbb{E}_{a'|a} \left[\frac{u'_1}{u_2} \frac{1}{p'} \right]} \right) (B/p) \\ &= \beta \mathbb{E}_{a'|a} \left[\frac{u'_1}{u_2} \frac{1}{p'} \right] + \beta \mathbb{E}_{a'|a} \left[\left((1-d')(\delta + (1-\delta)q') + d'q' \right) \frac{u'_2}{u_2} \frac{1}{p'} \right] B',\end{aligned}$$

or

$$\begin{aligned}& g - \psi(a,0)n - \frac{u_n}{u_2} n + 1/p + \left(\delta + (1-\delta) \frac{\mathbb{E}_{a'|a} \left[\left((1-d')(\delta + (1-\delta)q') + d'q' \right) \frac{u'_2}{u_2} \frac{1}{p'} \right]}{\mathbb{E}_{a'|a} \left[\frac{u'_1}{u_2} \frac{1}{p'} \right]} \right) (B/p) \\ &= \beta \mathbb{E}_{a'|a} \left[\frac{u'_1}{u_2} \frac{1}{p'} \right] + \beta \mathbb{E}_{a'|a} \left[\left((1-d')(\delta + (1-\delta)q') + d'q' \right) \frac{u'_2}{u_2} \frac{1}{p'} \right] B'.\end{aligned}$$

Now, eliminate $\psi(a,0)n$ via the resource constraint $\psi(a,0)n = 1/p + c_2 + g$,

$$\begin{aligned}& g - (1/p + c_2 + g) - \frac{u_n}{u_2} n + 1/p \\ &+ \left(\delta + (1-\delta) \frac{\mathbb{E}_{a'|a} \left[\left((1-d')(\delta + (1-\delta)q') + d'q' \right) \frac{u'_2}{u_2} \frac{1}{p'} \right]}{\mathbb{E}_{a'|a} \left[\frac{u'_1}{u_2} \frac{1}{p'} \right]} \right) (B/p) \\ &= \beta \mathbb{E}_{a'|a} \left[\frac{u'_1}{u_2} \frac{1}{p'} \right] + \beta \mathbb{E}_{a'|a} \left[\left((1-d')(\delta + (1-\delta)q') + d'q' \right) \frac{u'_2}{u_2} \frac{1}{p'} \right] B'.\end{aligned}$$

After multiplying both sides of the equation with u_2 and using the policy functions to replace next period's variables, one arrives at the implementability constraint (9).

A.2 Numerical Solution

The task of the numerical solution algorithm is to find the policy, bond price and value functions $\mathcal{X}^j(B, a)$, $\mathcal{X} \in \{\mathcal{B}, \mathcal{C}_2, \mathcal{N}, \mathcal{P}, \mathcal{Q}, \mathcal{V}\}$, $j \in \{r, d\}$.³⁵ Following [Hatchondo et al. \(2010\)](#), I use value function iteration and approximate these functions on discrete grids for debt and productivity, employing Chebyshev interpolation to allow for off-grid values of B and linear interpolation for a -values that are not on the grid.

The solution algorithm involves the following steps:

1. Construct discrete grids for debt $[\underline{B}, \bar{B}]$ and productivity $[\underline{a}, \bar{a}]$.
2. Choose initial values for the policy and value functions $\mathcal{X}_{start}^j(B, a)$, for $\mathcal{X} \in \{\mathcal{B}, \mathcal{C}_2, \mathcal{N}, \mathcal{P}, \mathcal{Q}, \mathcal{V}\}$ and $j \in \{r, d\}$, at all grid point combinations.
3. Set $\mathcal{X}_{next}^j = \mathcal{X}_{start}^j$, $j \in \{r, d\}$ and fix an error tolerance ε .
4. For each discrete grid point combination $(B, a) \in [\underline{B}, \bar{B}] \times [\underline{a}, \bar{a}]$, find the optimal policies $\mathcal{X}_{new}^j(B, a)$, $\mathcal{X} \in \{\mathcal{B}, \mathcal{C}_2, \mathcal{N}, \mathcal{P}\}$, and the associated bond prices $\mathcal{Q}_{new}^j(B, a)$ and values $\mathcal{V}_{new}^j(B, a)$, for $j \in \{r, d\}$.
5. If $|\mathcal{X}_{new}^j(B, a) - \mathcal{X}_{next}^j(B, a)| < \varepsilon$, for $\mathcal{X} \in \{\mathcal{B}, \mathcal{C}_2, \mathcal{N}, \mathcal{P}, \mathcal{Q}, \mathcal{V}\}$ and $j \in \{r, d\}$, at all grid point combinations, go to step 6, else set $\mathcal{X}_{next}^j = \mathcal{X}_{new}^j$, $j \in \{r, d\}$ and repeat step 4.
6. Use $\mathcal{X}_{new}^j(\cdot)$, $j \in \{r, d\}$, as approximations of the respective equilibrium objects in the infinite-horizon economy.

For the debt grid, the individual points are constructed by using Chebyshev nodes. Since the asymmetric default cost specification leads to a kink at $a = \bar{a}$ for $\mathcal{X}^d(B, a)$, $\mathcal{X} \in \{\mathcal{C}_2, \mathcal{P}, \mathcal{Q}, \mathcal{V}\}$, I follow [Hatchondo et al. \(2010\)](#) and partition the productivity grid into two equally spaced parts to account for this discontinuity when interpolating along the productivity dimension.

As is known in the literature (see e.g. [Krusell and Smith, 2003](#); [Martin, 2009](#)), there might be multiple Markov-perfect equilibria in models with infinitely-lived agents. In particular, there could be equilibria with discontinuous policy functions which do not arise in the infinite-horizon limit of a finite-horizon

³⁵The policy functions for the default and acceptance decisions can be calculated based on the value functions $\mathcal{V}^r(\cdot)$ and $\mathcal{V}^d(\cdot)$.

model version. To avoid such equilibria, I follow [Hatchondo et al. \(2010\)](#) and solve for the infinite-horizon limit of a finite-horizon model version.³⁶ In practice, this means that I compute the value and policy functions for the final period problem where no borrowing takes place and use these objects as initial values \mathcal{X}_{start}^j , $j \in \{r, d\}$, for step 2.

For a given state $(B, a) \in [\underline{B}, \bar{B}] \times [\underline{a}, \bar{a}]$, the objective function of the government is the sum of two parts, the period utility function $u(1/p, c_2, n)$ and (in the repayment case) the continuation value $\beta \mathbb{E}_{a'|a} [\mathcal{V}_{next}(B', a')]$, with $\mathcal{V}_{next}(B, a) = \max \{ \mathcal{V}_{next}^r(B, a), \mathcal{V}_{next}^d(B, a) \}$. The optimal policies for step 4 are then computed as follows. I use a sub-routine that calculates the optimal static policies c_2 , n , and p for given debt and productivity values $(B, a) \in [\underline{B}, \bar{B}] \times [\underline{a}, \bar{a}]$ and an arbitrary, i.e. possibly off-grid, borrowing value \hat{B}' . More specifically, these static policies are computed by using a sequential quadratic programming algorithm (see e.g. [Nocedal and Wright, 1999](#), for details). Using the static policy sub-routine, (c_2, n, p) and thus period utility $u(1/p, c_2, n)$ can be expressed as functions of (B, a, \hat{B}') . As a result, given $(B, a) \in [\underline{B}, \bar{B}] \times [\underline{a}, \bar{a}]$, the government objective for the repayment case can be expressed as a function of \hat{B}' as well: $u(1/p, c_2, n) + \beta \mathbb{E}_{a'|a} [\mathcal{V}_{next}(\hat{B}', a')]$.³⁷

For each discrete grid point combination $(B, a) \in [\underline{B}, \bar{B}] \times [\underline{a}, \bar{a}]$, the optimal debt policy $\mathcal{B}_{new}^j(B, a)$, $j \in \{r, d\}$, then is computed via a global non-linear optimizer, calling the static policy routine to calculate the objective function for each candidate debt value \hat{B}' . The optimal policies $\mathcal{X}^j(B, a)$, $\mathcal{X} \in \{C_2, \mathcal{N}, \mathcal{P}\}$ then are computed by using the static policy routine for the optimal borrowing value $\mathcal{B}_{new}^j(B, a)$. The algorithm iterates on the policy, bond and value functions until the maximum absolute difference between value, bond and policy functions obtained in two subsequent iterations is below $\varepsilon = 10^{-5}$ for each combination $(B, a) \in [\underline{B}, \bar{B}] \times [\underline{a}, \bar{a}]$.

To approximate expected values in an accurate way, one needs to account for the default threshold. This can be seen by looking at the expected option value of default:

$$\mathbb{E}_{a'|a} [\mathcal{V}_{next}(B', a')] = \int_0^{\hat{a}(B')} \mathcal{V}_{next}^d(B', a') f_a(a'|a) da' + \int_{\hat{a}(B')}^{\infty} \mathcal{V}_{next}^r(B', a') f_a(a'|a) da'.$$

As in [Hatchondo et al., 2010](#), Gauss-Legendre quadrature nodes and weights are used to approximate the integrals above. The default threshold $\hat{a}(B)$ satisfies $\mathcal{V}_{next}^r(B, \hat{a}(B)) - \mathcal{V}_{next}^d(B, \hat{a}(B)) = 0$ and is computed via bisection method.

³⁶[Martin \(2009\)](#) also solves for the infinite-horizon limit. As pointed out by him, using a [Svensson \(1985\)](#)-type beginning-of-period cash-in-advance constraint in a finite-horizon model requires a terminal money value for a monetary equilibrium to exist. Otherwise, households will not be willing to invest in money in the final period and by backward induction not in any of the previous periods. The impact of the final-period value of money vanishes over time and does not affect the final results.

³⁷The same logic applies to the default (and autarky) case.

In quantitative models of sovereign default with long-term debt a positive debt recovery rate can incentivize the government to maximally increase its debt in periods prior to default (see [Chatterjee and Eyigungor, 2015](#), or [Hatchondo et al., 2016](#), for details). Following [Chatterjee and Eyigungor \(2015\)](#), I rule out such counterfactual borrowing behavior by imposing the restriction that the probability of default is not permitted to exceed the upper bound $\iota \in [0, 1]$ ³⁸ whenever the government issues positive amounts of debt. For all model versions, I set $\iota = 0.75$, which, as in [Chatterjee and Eyigungor \(2015\)](#), results in a constraint that is loose enough to not bind for the model simulations.

³⁸[Chatterjee and Eyigungor \(2015\)](#) rationalize such a restriction as an underwriting standard.

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