

# Interest Rate Spreads and Forward Guidance

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## Abstract

We provide evidence that liquidity premia on assets that are more relevant for private agents' intertemporal choices than near-money assets increase in response to expansionary forward guidance announcements. We introduce a structural specification of liquidity premia based on assets' differential pledgeability in a basic New Keynesian model to replicate this finding. This model predicts that output and inflation effects of forward guidance do not increase with the length of the guidance period and are substantially smaller than if liquidity premia were neglected. This indicates that there are no puzzling forward guidance effects when endogenous liquidity premia are taken into account.

**JEL classification:** E32, E42, E52

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# 1 Introduction

Ever since the financial crisis of 2007-2009 and monetary policy rates close to zero, forward guidance – the communication of central banks about the likely future course of their policy stance – has gained considerable importance for the conduct of monetary policy by major central banks. Based on the New Keynesian paradigm, communicating low future rates should substantially stimulate aggregate demand today and may even break deflationary spirals at zero interest rates (see, e.g., Eggertsson and Woodford 2003). Empirical studies, however, suggest that New Keynesian models tend to overstate the effects of forward guidance announcements,<sup>1</sup> which has led Del Negro et al. (2015) to claim that there exists a "forward guidance puzzle". Several theoretical studies have already addressed this issue and have shown that various perturbations of the basic New Keynesian model can reduce this puzzle (see below).<sup>2</sup> This paper contributes to this literature by focussing on an empirical observation that has – up to now – been unnoticed in the context of the macroeconomic effects of forward guidance: Liquidity premia unambiguously increase after expansionary monetary policy announcements, implying that interest rates that are relevant for private sector saving and investment decisions fall by less compared to the monetary policy rate and to rates of return on near-money assets. In this paper, we provide direct evidence on this pattern and rationalize it by introducing an endogenous liquidity premium into a basic New Keynesian model. This extended model further predicts much weaker macroeconomic effects of forward guidance compared to the case without a liquidity premium, indicating that there are no puzzling forward guidance effects once one acknowledges the endogenous response of interest rate spreads to monetary policy announcements.

Our analysis is motivated by the empirical observation that responses to forward guidance announcements vary substantially for interest rates on different assets (see Campbell et al. 2012 or Del Negro et al. 2017). Campbell et al. (2012), for example, have estimated the response of interest rates to changes in the anticipated future paths of the monetary policy instrument. Applying the method of Gürkaynak et al. (2005) to extract surprise components in the announcements of FOMC meetings,<sup>3</sup> they find that interest rates on corporate bonds react less strongly than those on government bonds. Since highly rated corporate bonds and government bonds mainly differ by liquidity,

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<sup>1</sup>See, for instance, Gertler and Karadi (2015), Campbell et al. (2012), or Del Negro et al. (2015).

<sup>2</sup>Examples are McKay et al. (2016) or Del Negro et al. (2015), Farhi and Werning (2017), Angeletos and Lian (2018), and Gabaix (2018).

<sup>3</sup>This method has widely been used to analyze the effects of monetary policy and forward guidance on financial markets and has for instance, also been applied by Swanson (2017) and Gertler and Karadi (2015).

as for example argued by Krishnamurthy and Vissing-Jorgensen (2012), we take these findings as indicative for forward guidance effects on liquidity premia. In the first part of this paper, we corroborate this idea by extending the analysis of Campbell et al. (2012) to various interest spreads that have been suggested by the literature to be mainly affected by liquidity premia and a common liquidity factor as used by Del Negro et al. (2017). We apply the method of Gürkaynak et al. (2005) for the time period from 1990 and 2016<sup>4</sup> and find that forward guidance announcements affect interest rates on near-money assets and less liquid assets in different ways and, in particular, that an announcement of reductions in the current or future monetary policy rate substantially raises interest rate spreads, which are applied as measures for liquidity premia by Krishnamurthy and Vissing-Jorgensen (2012) and Nagel (2016), and, most importantly, the common liquidity factor. These effects of monetary policy announcements are suggestive for a mitigation of forward guidance effects, given that private sector savings and investment decisions are rather based on interest rates on less liquid assets than on interest rates on near-money assets. In the second part of the paper, we aim at assessing the macroeconomic predictions of a basic New Keynesian model that can replicate the liquidity premium response to monetary policy announcements.

There exist several specifications that generate liquidity premia of near-money assets and that have been used in macroeconomic studies. The most widely applied approach assumes that government bonds raise agents' utility directly, similar to the money-in-the-utility function specification as a short-cut for modelling liquidity services of money. Generally, liquidity services of bonds stem from their ability to serve as a substitute for money, as for example found by Nagel (2016). Hence, an increase in real money tends to decrease the marginal gains from liquidity services provided by bonds; these marginal gains are decisive for endogenous changes in the liquidity premium. Concretely, such a specification predicts that the spread between the interest rate on a risk-free nominal bond which provides no liquidity service and the interest rate on government bonds tends to decrease – rather than to increase – when real money increases due to an expansionary monetary policy.<sup>5</sup> Likewise, Campbell et al.'s (2017) (government)

<sup>4</sup>Our results are qualitatively unchanged when we consider a sample ending in 2008, i.e., a sample excluding the recent zero lower bound (ZLB) episode.

<sup>5</sup>Nagel (2016) assumes that real bonds and real money contribute to current utility by CES aggregate, and provides evidence for an imperfect substitutability between them. He finds a *positive unconditional* correlation between the federal funds rate and liquidity premia, in particular, measured by the spread between the interest rate on generalized collateral (GC) repos and the treasury bill rate, whereas we provide evidence for a *negative conditional* correlation between the same spreads and policy-induced innovations in the current and the future policy rate. Notably, both findings are consistent from the perspective of the model developed in this paper when the unconditional correlation is not mainly driven by monetary policy shocks, but is dominated by the responses to other (e.g., demand) shocks, which

bonds-in-the-utility-function specification implies that the marginal utility gain from government bond holdings increases with their interest rate,  $R$ , such that a liquidity premium tends to decrease in response to an expansionary monetary policy.<sup>6</sup>

While this approach has its merits (e.g., flexibility and simplicity), it can hardly be squared with the empirical observation that liquidity premia increase in response to expansionary monetary policy. For this reason, we apply a more structural approach to specify liquidity premia, which allows replicating the empirical evidence. We acknowledge that assets differ with regard to their pledgeability in financial transactions, as in Schabert (2015) and Williamson (2016). Specifically, we consider that the central bank supplies money only against eligible assets at a discount rate (or, repo rate)  $R^m$ , which is the inverse of the amount of money supplied against one unit of eligible assets and serves as the monetary policy rate.<sup>7</sup> Since government bonds provide access to money through their eligibility in open-market operations, they are an (imperfect) substitute to money and their interest rate  $R$  closely follows the monetary policy rate. In contrast, other assets such as corporate bonds have to pay higher rates of return in order to compensate for their ineligibility in open-market operations. Due to their rate-of-return dominance, these less liquid assets serve as agents' preferred store of wealth, such that the interest rates on these assets accord to agents' marginal rate of intertemporal substitution.

In this model, the liquidity value of bonds differs from the liquidity value of money whenever the policy rate exceeds zero, i.e. when the central bank supplies money less than one-for-one for bonds. The liquidity value of money increases, as usual, with agents' willingness to pay for money or, put differently, it decreases with their willingness to postpone transactions, for which money is essential, from today to tomorrow. This marginal rate of intertemporal substitution is not directly controlled by the central bank and evolves endogenously. An expansionary monetary policy, i.e., a reduction in the policy rate, which is tantamount to the central bank supplying more money per eligible asset in open market operations, tends to raise the liquidity value of a government bond. Since the expansionary policy increases contemporaneous compared to future transactions, the marginal rate of intertemporal substitution tends to decrease as well, though to a smaller extent than the policy rate. As a consequence, the liquidity value of bonds and thus the interest rate spread between corporate and government bonds, i.e.,

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are commonly considered sources of business cycle fluctuations (see Figure 10 in Appendix G).

<sup>6</sup>Specifically, Campbell et al. (2017) assume that real government bonds measured at their issuance price  $1/R$  contribute to current utility in a concave way. For their analysis, they assume a zero supply of bonds, such that changes in the liquidity premium are exogenously determined by "liquidity preference shocks".

<sup>7</sup>Empirically, the difference between the treasury repo rate and the federal funds rate is negligible (less than 1 b.p. on average) compared to other spreads in this paper (see Figure 6 in Appendix D).

the liquidity premium, increases.

We show that the introduction of the liquidity premium in a basic New Keynesian model allows to reproduce qualitatively and quantitatively the observed spread responses to forward guidance and predicts a substantially weaker current GDP response to these announcements than a basic New Keynesian model without a liquidity premium. A central feature for the transmission of monetary policy announcements in the latter model is that interest rates that are relevant for private agents' intertemporal choices move one-to-one with the monetary policy rate. In contrast, the monetary policy rate and the marginal rate of intertemporal substitution are endogenously separated in our model with a liquidity premium.<sup>8</sup> Due to this separation, a reduction in the current policy rate lowers the price of money and thus raises real activity rather by stimulating transactions for which money is essential than by inducing agents to frontload consumption due to policy-enforced changes in the marginal rate of intertemporal substitution. While both models can generate similar output and inflation effects to conventional monetary policy shocks, i.e. unexpected changes in the current policy rate, the predictions of the two models substantially differ with regard to the effects that announcements of future policy rate changes have.

Consider, as a thought experiment, an isolated reduction of the policy rate in a future period  $T$ . As this raises the amount of money provided per bond, aggregate demand is stimulated in that particular future period. Due to the stimulating effect of monetary policy, inflation rises in period  $T$ , which causes forward-looking price setters to raise prices already before. Given that this surge in inflation is not associated with an accommodative policy, the real value of money and bonds is deflated, such that aggregate demand tends to *fall* prior to period  $T$ . In contrast, a New Keynesian model without a liquidity premium predicts that aggregate demand *increases* in all preceding periods, since consumption falls with the sum of all future marginal rates of intertemporal substitution, which – by assumption – are identical to the future policy rates. Notably, a typical forward guidance announcement, where the central bank commits to reduce the policy rate from today onwards until a period  $T$ , increases current real activity in our model with the liquidity premium, since the expansionary effects of the current policy-rate reduction dominate the adverse effect of anticipated future inflation. Overall, forward guidance, i.e., a reduction of the current policy rate accompanied with an announcement to keep future policy rates low, leads to a rise in the liquidity premium and increases of output and inflation that substantially differ from the predictions of a basic New Keynesian model without a liquidity premium.

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<sup>8</sup>This separation in fact accords to the evidence provided by Canzoneri et al. (2007).

While we derive main model properties in an analytical way, we further show that our model can quantitatively replicate the response of the liquidity premium to a forward guidance announcement as found in our econometric analysis. Yet, the main purpose of the quantitative analysis is to compare the model’s quantitative effects on output and inflation with the predictions of a model version without a liquidity premium. We consider two experiments where the central bank reduces the policy rate by 25 basis points and announces to keep it at this level for another year or another two years, respectively. The announcement triggers output to increase by about 0.1 percent relative to its steady state value from the time of the announcement until the policy rate is raised back to normal. Compared to the prediction of our model with a liquidity premium, we find the immediate output and inflation effects of the one-year forward guidance in a model version without the liquidity premium, which corresponds to a standard New Keynesian model, to be about 12 times larger. Moreover, the length of the guidance period hardly affects the impact output response in the model with the endogenous liquidity premium, which again clearly differs from the prediction of a basic New Keynesian model, where the size of the initial output response increases with the length of the guidance period (see also McKay et al., 2016).

Our strategy to consider a special role of government bonds for the analysis of forward guidance effects relates to Campbell et al. (2016) and Michaillat and Saez (2018), who both assume that government bonds enter the utility function. While Campbell et al. (2016) find that the spread “on its own does not explain the absence of very large effects of forward guidance”, Michaillat and Saez (2018) show that the forward guidance puzzle vanishes if the marginal utility of bonds is sufficiently large (leading to a well-behaved steady state under a zero nominal interest rate).<sup>9</sup> Both studies restrict their attention to the case where the supply of government bonds equals zero, such that the resulting interest rate spread is exogenous. The main difference of our paper to these two studies – as well as to other studies cited below – is that our analysis is motivated and based on direct empirical evidence on forward guidance effects, which our model can replicate by accounting for the liquidity value of government bonds in a structural way. Notably, our specification of the liquidity premium also improves the empirical performance of macroeconomic models in other respects. Concretely, our modelling strategy has proved to be helpful in solving puzzles related to uncovered interest rate parity and the effects of fiscal policy, see Linnemann and Schabert (2015) and Bredemeier et al. (2017), and it

<sup>9</sup>Diba and Loisel (2017) augment a New Keynesian model by assuming that the central bank simultaneously controls the interest rate on reserves as well as the supply of reserves, and obtain local determinacy properties that imply muted effects of forward guidance. Like in their model and in Michaillat and Saez (2018), our model also predicts equilibrium determinacy under an interest rate peg.

explains Canzoneri et al.'s (2007) puzzling finding that the spread between the marginal rate of intertemporal substitution and the policy rate is negatively related to the level of the policy rate. However, the model predicts that assets differ in an extreme way with regard to their ability to provide liquidity services. A first class of assets that includes money and government bonds are either directly accepted as means of payment or can be exchanged at the central bank for that purpose. The other class of assets that includes for example corporate bonds are completely illiquid and cannot be used for transaction purposes before maturity. A comprehensive specification would include asset classes with incomplete liquidity, for example securities that can be liquidated at worse conditions than government bonds. Given that our specification suffices for the purposes of this paper, we leave such extensions for future research.

While our paper provides direct empirical evidence on the effects of forward guidance on interest rate spreads and evaluates the relevance of these effects for macroeconomic outcomes, a growing number of studies has suggested extensions of and alternatives to the standard New Keynesian model that bring about more muted output responses to forward guidance. Del Negro et al. (2015) address the excess response to policy announcements in the New Keynesian model by introducing a perpetual youth structure, which leads to a higher discounting of future events and thereby reduces current responses. Campbell et al. (2016) differentiate between Delphic and Odyssean forward guidance and find that the predictions of their medium scale model, in which government bond holdings provide direct utility, do not reflect the forward guidance puzzle. McKay et al. (2016, 2017) show that the effects of forward guidance are more limited in a model with heterogeneous agents that face the risk of hitting a borrowing constraint. A further set of papers by Carlstrom et al. (2015), Chung et al. (2015), and Kiley (2016) demonstrate that the effects are dampened when firms are subject to sticky information instead of a direct sticky price friction, as this confines the forward-lookingness of the Phillips curve. Relatedly, Wiederholt (2015) shows that forward guidance has limited effects in a model where households have dispersed inflation expectations. Farhi and Werning (2017) show that the interaction of bounded rationality and incomplete insurance markets reduces the predicted output effects of the New Keynesian model substantially. Angeletos and Lian (2018) relax the assumption that news such as forward guidance announcements are common knowledge, which leads to an attenuation of their effects. Gabaix (2018) departs from full rationality and introduces myopia to the New Keynesian model in the form of an incomplete understanding of future disturbances which intuitively mutes their effects. Caballero and Farhi (2017) construct a model where the economy is pushed to the zero lower bound because of a shortage of safe assets. Forward

guidance does not foster recovery, but leads to higher risk premia in their setting.

The remainder of the paper is structured as follows. Section 2 provides empirical evidence on the response of liquidity premia to monetary policy announcements. Section 3 presents the model. We derive analytical results on forward guidance effects for a simplified version and present impulse responses obtained numerically for the full model in Section 4. Section 5 concludes.

## **2 Empirical Effects of Forward Guidance on Liquidity Premia**

In this section, we document empirically that liquidity premia on near-money assets tend to rise in response to forward guidance announcements that financial markets consider to be accommodative. We explain how we measure the value of liquidity services of near-money assets by various interest rate spreads in Section 2.1. In Section 2.2, we provide an analysis of these interest rate spreads at all FOMC meeting dates between 1990 and 2016. Notably, the results do not change for the sample 1990-2008, where we disregard periods with unconventional monetary policies (see Table 3 in Appendix B). We use the approach of Gürkaynak et al. (2005), which separates the effects of unanticipated forward guidance announcements from those of simultaneously announced changes in other monetary policy instruments, such as the current federal funds rate. We apply this approach to identify the response of liquidity premia to monetary policy announcements.

### **2.1 Measurement of Liquidity Premia**

We use various market-based measures for the value of liquidity services of near-money assets by calculating interest rate spreads between assets that differ by the degree of liquidity in financial markets, but feature similar characteristics in terms of safety and maturity. In this way, we rule out that movements in the spreads are due to differences in credit risk or term premia. As the measure for highly liquid near-money assets, we use US Treasuries at various maturities. Those can be seen as close substitutes for money as Treasuries are allowed to serve as collateral for obtaining liquidity from the Fed.

We use the following spreads relative to Treasuries as measures of liquidity premia. According to Krishnamurthy and Vissing-Jorgensen (2012), the spread between highly rated corporate bonds and Treasuries is primarily driven by liquidity. We therefore use the spreads between highly rated commercial papers and corporate bonds with maturities of 3 months and 3, 5, and 10 years on the one hand and Treasuries of the same maturities on the other hand. As some credit risk may remain even in very highly rated corporate

bonds, we also follow Krishnamurthy and Vissing-Jorgensen (2012) in using spreads between relatively illiquid certificates of deposit (CD), which are very safe due to coverage by the Federal Deposit Insurance Corporation (FDIC), and Treasury bills at maturities of 3 and 6 months. Finally, we use the spread between the rate on 3-month general collateral repurchase agreements (GC repos, hereafter) and the 3-month T-bill rate, suggested by Nagel (2016) as a particularly clean measure of the value of liquidity, since GC repos are entirely illiquid before maturity but in other aspects virtually identical to T-bills. We end up with eight different spreads, for which we collect daily data with observations ranging from January 1990 to September 2016. A detailed description of the data set and the construction of the spreads is given in Appendix A.<sup>10</sup>

We acknowledge that these spreads may contain non-liquidity-related components, for instance due to differences in credit risk or additional safety attributes of Treasuries as discussed by Krishnamurthy and Vissing-Jorgensen (2012). We therefore follow Del Negro et al. (2017) and construct a factor model with all spreads to extract their common component over time, which can be interpreted as a purified liquidity premium. This further yields the advantage of having one single summary measure for the value of liquidity. We calculate the liquidity factor for a sample from 1990-01-02 to 2016-09-16 using principle component analysis. To account for missing values in our data, we employ the method of Stock and Watson (2002) that relies on an expectation maximization algorithm.<sup>11</sup> To give the resulting factor  $f_t$  a quantitative interpretation as a measure of the liquidity premium, we use that  $f_t$  is related to the liquidity premium  $LP_t$  by

$$LP_t = a + bf_t, \tag{1}$$

where  $a$  and  $b$  are unknown parameters, see Del Negro et al. (2017). We apply the assumptions proposed by Del Negro et al. (2017) to recover  $a$  and  $b$ . First, we assume that the average value of the liquidity premium before the outbreak of the financial crisis in July, 2007 equals 46 basis points. This number is the estimate for the liquidity value of Treasuries by Krishnamurthy and Vissing-Jorgensen (2012) for a sample from 1926 to 2008. Second, Del Negro et al. (2017) argue that the asset in their sample with the highest spread to Treasuries at the peak of the financial crisis (a BBB rated bond whose

<sup>10</sup>In Appendix D, Figure 4 shows the time series of the liquidity premium  $LP$  in equation (1) and Figure 5 provides time series plots of all spreads along with a linear projection on the common factor and a constant. Summary statistics on all spreads and the liquidity premium derived from the factor model are given in Table 4 in Appendix D.

<sup>11</sup>As a robustness check for our treatment of missing values, we also calculated the common factor for the maximum balanced sample of our data, which ranges from 1997-01-02 to 2013-06-28. We find that the common factor is very similar to the one estimated on the entire sample.

credit risk is hedged by a credit default swap) was essentially illiquid. The average size of this spread of 342 basis points in the last quarter of 2008 therefore gives a value for the liquidity premium at this time. Using these two assumptions, we can construct a daily time series for the liquidity premium in equation (1) that we plot in Figure 4 in the Appendix. Figure 5 in the Appendix provides time series plots of all individual liquidity spreads along with a linear projection of the common factor and a constant on each spread. They show that the common liquidity factor captures a large part of the variation for the majority of the series.

## 2.2 Regression Analysis

We now analyze the effect of forward guidance on the valuation of liquidity in financial markets using the approach of Kuttner (2001) and Gürkaynak et al. (2005). This method takes into account the following points. First, forward guidance announcements are usually given simultaneously with announcements about the federal funds rate or – at least in the years following the financial crisis – simultaneously with other monetary policy measures. Second, since financial markets are forward looking, only unanticipated components of the policy changes should matter for market interest rates and spreads. Anticipated policy actions should already be priced into the markets *ex ante*, therefore leading to only limited reactions after publication. Ignoring this may wrongly suggest that a policy had no effect. Related to this issue, a by words accommodative policy announcement can actually have negative effects on markets when the press release was interpreted as bad news for the economy. Finally, the central bank can affect markets by refraining from taking action in a situation, where a policy adjustment was expected – i.e., also reactions on the non-appearance of a forward guidance announcement can be informative for the effects of forward guidance if such an announcement had been expected by market participants.

Following Gürkaynak et al. (2005), we extract the surprise component of forward guidance announcements from changes in federal funds and Eurodollar futures rates around FOMC meetings. We consider all 237 FOMC meetings between January 1990 and December 2016. After constructing such monetary surprise measures, we extract their first two principle components and rotate them in a way maintaining orthogonality and achieving that the second factor has no effect on the current federal funds rate. This transformations allows a structural interpretation of the two factors. Following the terminology of Gürkaynak et al. (2005), we denote the first one as the “target factor”, which measures the unanticipated change in the current federal funds rate, and the second one as the “path factor”, which measures the unanticipated change of

expectations about the path of the federal funds rate over the next 12 months.<sup>12</sup> To allow for an interpretation in basis points, we normalize the two factors as in Campbell et al. (2012), such that an increase of 0.01 in the target factor corresponds to a surprise change of 1 basis point in the federal funds rate target and that an increase of 0.01 in the path factor corresponds to a surprise change of 1 basis point in the 12-months-ahead Eurodollar futures rate. Hence, a change in the target factor by one unit is to be interpreted as a change in expected future short-term interest rates over the next 12 months, where the 12-months ahead Euro-dollar future rate changes by 100 basis points while the current (spot) federal funds rate is unchanged.

We estimate the effect of the target and the path factor on the change of the various liquidity spreads and the underlying assets with the regression model

$$\Delta y_t = \beta_0 + \beta_1 \tilde{F}_{1,t} + \beta_2 \tilde{F}_{2,t} + \beta_3 qe_t + e_t, \quad (2)$$

where  $\Delta y_t$  is the one-day change of a liquidity spread or asset return around the FOMC meeting at time  $t \in T$ ,  $\beta_0$  is a constant,  $\beta_1$  and  $\beta_2$  are the coefficients on the target factor,  $\tilde{F}_1$ , and the path factor,  $\tilde{F}_2$ , respectively, and  $e_t$  is an error term.  $\beta_3$  is the coefficient on the dummy variable  $qe_t$ , which takes a value of 1 at FOMC meetings with important decisions regarding quantitative easing.<sup>13</sup> This variable ensures that our results are not driven by these events, which were shown, e.g., by Krishnamurthy and Vissing-Jorgensen (2011), to have affected financial markets considerably.

Results on the response of the liquidity measures to the surprise changes in monetary policy are given in Table 1. The first row shows the effect of a change in the current federal funds rate, as measured by the target factor, while the second row shows the effect of a change in forward guidance, as measured by the path factor. We start by presenting results for the liquidity premium from our factor model (1). We find that the premium reacts strongly on both, changes in the current and the expected path of the federal funds rate. A 1% reduction of the current federal funds rate target, as measured by the target factor, increases the valuation of liquidity by 0.41%, while the liquidity premium rises by 0.28% today in response to a 1% reduction of the expected federal funds rate in 12 months, as measured by the path factor. Accordingly, markets

<sup>12</sup>Details on the construction of the two factors can be found in Appendix E. Swanson (2017) also uses the approach by Gürkaynak et al. (2005), but estimates three factors, giving the third one the interpretation to capture changes in asset purchase programmes. We also address the separate effect of quantitative easing policies in our analysis, though in a different way (see below).

<sup>13</sup>The variable  $qe_t$  takes a value of 1 at the following 6 dates. 2009-03-18: Announcement of QE1. 2010-11-03: Announcement of QE2. 2011-09-21: Announcement of "Operation Twist" 2012-09-13: Announcement of QE3. 2012-12-12: Announcement of additional long-term Treasury purchases. 2013-12-18: Begin to taper asset purchases.

**Table 1:** Response of Liquidity Premia to Changes in Monetary Policy

	Liquidity Premium $LP$	Commercial Paper / Corporate Bond spread					
		3M	3Y	5Y	10Y(A)	10Y(B)	
Current Federal Funds Rate $\tilde{F}_1$	-0.41*** (0.13)	-0.30*** (0.11)	0.15* (0.088)	0.043 (0.055)	-0.053 (0.056)	-0.032 (0.042)	
Expected Future Federal Funds Rates $\tilde{F}_2$	-0.28*** (0.059)	-0.11* (0.068)	-0.13*** (0.048)	-0.17*** (0.035)	-0.30*** (0.037)	-0.31*** (0.035)	
$R^2$	0.29	0.10	0.09	0.19	0.46	0.51	
Number of Observations $T$	237	122	165	165	237	237	
		GC spread		CD spread			
		3M	3M	3M	6M		
Current Federal Funds Rate $\tilde{F}_1$		-0.37** (0.15)	-0.26* (0.15)	-0.35* (0.18)			
Expected Future Federal Funds Rates $\tilde{F}_2$		-0.16*** (0.060)	-0.011 (0.084)	-0.11 (0.12)			
$R^2$		0.55	0.21	0.41			
Number of Observations $T$		213	212	212	212		

*Notes:* Responses of liquidity spreads to changes in monetary policy at FOMC meetings between January 1990 and September 2016. Constant and QE-dummy included in all regressions. Heteroskedasticity-robust (White) standard errors in parentheses. Asterisks mark significance at 10% (\*), 5% (\*\*), 1% (\*\*\*). Maturity measured in months (M) or years (Y). Corporate Bond 10Y(A) and (B): long-term bonds with AAA and BAA rating, respectively. CD: Certificate of Deposit; GC: General Collateral Repo. Spreads calculated relative to Treasuries of same maturity.

value the liquidity property of near-money assets higher in response to both types of expansionary monetary policy. This novel finding constitutes the main result of our empirical analysis. Regressions of the individual spreads that serve as measures for a liquidity premium provide additional supportive evidence. Coefficients on the target and path factor have a negative sign in almost all cases. Intuitively, the coefficients as well as the significance of forward guidance changes become stronger for longer maturities, whereas the effect of the current federal funds rate on liquidity spreads is particularly pronounced for shorter maturities.

We further provide regression results on the reaction of the asset returns, underlying the spreads, which are given in Table 2 in Appendix B. In line with the observations made in Table 1, both Treasuries and the interest rates on relatively illiquid assets tend to increase with the (expected) federal funds rate. The increasing liquidity premium in response to expansionary monetary policy is, accordingly, driven by a relatively stronger reaction of the return of Treasuries. These results also confirm that the effect of forward guidance increases with the maturity of the assets, while the effect of changes in the current federal funds rate become smaller with longer maturities.

As a robustness check, Table 3 in Appendix B repeats the analysis for a sample that excludes the recent zero lower bound episode (sample end in December 2008). Overall, the results from this exercise are very similar and indicate that our main findings are not affected by the recent ZLB episode.

### 3 The Model

In this section, we present a New Keynesian model with an endogenous liquidity premium for the analysis of forward guidance. To endogenize the liquidity premium, we consider high powered money, i.e., reserves, being supplied by the central bank via open market operations only against eligible securities (as in Schabert, 2015). Our model distinguishes between several assets in order to account for rates of return, which respond differently to forward guidance shocks in the data. Decisively, assets differ with respect to liquidity, i.e., to their ability to serve as substitutes for central bank money. The price of central bank money equals the monetary policy rate and is set by the central bank. The interest rate on eligible assets (i.e., Treasury bills) is closely related to the policy rate, as they are close substitutes to central bank money, whereas interest rates on non-eligible assets differ by a liquidity premium. Given that the latter assets (rather than money or Treasury bills) serve as agents' store of value, their real interest rates reflect private agents' intertemporal consumption and investment choices. To isolate the main mechanism, we abstract from modelling any financial market friction, such that

the model features only a single non-standard element in form of the liquidity premium. For transparency, we further neglect model features that are relevant for an empirically more plausible specification, in particular, for inside money. In Appendix F, we show that the introduction of a banking sector does not change the results.

In each period, the timing of events in the economy, which consists of households, intermediate goods producing firms, retailers, and the public sector unfolds as follows: At the beginning of each period, aggregate shocks materialize. Then, agents can acquire reserves from the central bank via open market operations. Subsequently, the labor market opens, goods are produced, and the goods market opens, where money is used as a means of payment. At the end of each period, the asset market opens. Throughout the paper, upper case letters denote nominal variables and lower case letters real variables.

### 3.1 Private sector

There is a continuum of infinitely lived households indexed with  $i \in [0, 1]$  with identical wealth endowments and preferences. Though, they will behave in an identical way, we do use the index  $i$  at the beginning to describe individual choices. They maximize the expected sum of a discounted stream of instantaneous utilities  $u_t$ ,

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_{i,t}, \tilde{c}_{i,t}, n_t), \quad (3)$$

where  $u(c_{i,t}, \tilde{c}_{i,t}, n_{i,t}) = [(c_{i,t} - 1)^{1-\sigma} + \gamma(\tilde{c}_{i,t} - 1)^{1-\sigma}] (1 - \sigma)^{-1} - \theta n_{i,t}^{1+\sigma_n} / (1 + \sigma_n)$  with  $\sigma \geq 1$ , and  $\sigma_n, \theta, \gamma \geq 0$ ,  $c_{i,t}$  denotes consumption of cash goods,  $\tilde{c}_{i,t}$  denotes consumption of credit goods,  $n_{i,t}$  working time,  $E_0$  the expectation operator conditional on the time 0 information set, and  $\beta \in (0, 1)$  is the subjective discount factor. Households can buy short-term government bonds  $B_{i,t}$  and risk-free debt  $L_{i,t}$  issued by private agents (i.e., households and firms). They can further hold money  $M_{i,t}$  and receive additional money  $I_{i,t}$  from the central bank. The budget constraint of the household reads

$$\begin{aligned} (B_{i,t}/R_t) + (L_{i,t}/R_t^L) + M_{i,t} + I_{i,t} (R_t^m - 1) + P_t c_{i,t} + P_t \tilde{c}_{i,t} + P_t \tau_t \\ \leq M_{i,t-1} + B_{i,t-1} + L_{i,t-1} + P_t w_t n_{i,t} + P_t \varphi_t, \end{aligned} \quad (4)$$

where  $P_t$  denotes the goods price level,  $1/R_t$  the price of government bonds,  $1/R_t^L$  the price of privately issued debt,  $(R_t^m - 1)$  is the price of newly received money,  $w_t$  the real wage rate,  $\tau_t$  a lump-sum tax, and  $\varphi_t$  profits from retailers. We assume that households rely on money  $M_{i,t}$  for purchases of cash goods. Thus, households demand for money is induced by the following constraint, which resembles a standard cash-in-advance con-

straint,

$$P_t c_{i,t} \leq M_{i,t-1} + I_{i,t}. \quad (5)$$

Here, we abstract from modelling banks and inside money creation (see Appendix F for the inclusion of a banking sector), and assume that households directly trade with the central bank. The central bank supplies money via open market operations either outright or temporarily under repurchase agreements. In both cases, Treasury bills serve as collateral for central bank money, while the price of reserves in open market operations in terms of Treasuries (the repo rate) equals  $R_t^m$ . Specifically, reserves are supplied by the central bank only in exchange for Treasuries  $\Delta B_{i,t}^C$ , while the price of money is the repo rate  $R_t^m$ :

$$I_{i,t} = \Delta B_{i,t}^C / R_t^m \quad \text{and} \quad \Delta B_{i,t}^C \leq B_{i,t-1}. \quad (6)$$

Hence, (6) describes a central bank money supply constraint, which shows that reserves  $I_{i,t}$  can be acquired in exchange for the discounted value of Treasury bills carried over from the previous period  $B_{i,t-1}/R_t^m$ . Notably, individual households can trade treasuries and money among each other, while they can, obviously, not change the total stock of money and government bonds. Maximizing the objective (3) subject to the budget constraint (4), the goods market constraint (5), the money supply constraint (6), for given initial values leads to the following first-order conditions for working time, consumption of credit and cash goods, real government bonds  $b_{i,t}$ , privately issued real debt, real injections  $i_{i,t}$ , and real money holdings  $m_{i,t}$ :  $-u_{n,i,t} = w_t \lambda_{i,t}$ ,  $u_{\tilde{c},i,t} = \lambda_{i,t}$ ,

$$u_{c,i,t} = \lambda_{i,t} + \psi_{i,t}, \quad (7)$$

$$\beta E_t [(\lambda_{i,t+1} + \eta_{i,t+1}) \pi_{t+1}^{-1}] = \lambda_{i,t} / R_t, \quad (8)$$

$$\beta E_t [\lambda_{i,t+1} \pi_{t+1}^{-1}] = \lambda_{i,t} / R_t^L, \quad (9)$$

$$(R_t^m - 1) \lambda_{i,t} + R_t^m \eta_{i,t} = \psi_{i,t}, \quad (10)$$

$$\beta E_t [(\lambda_{i,t+1} + \psi_{i,t+1}) \pi_{t+1}^{-1}] = \lambda_{i,t}, \quad (11)$$

where  $u_{n,t} = \partial u_t / \partial n_t$ ,  $u_{\tilde{c},t} = \partial u_t / \partial \tilde{c}_t$ , and  $u_{c,t} = \partial u_t / \partial c_t$  denote marginal (dis-)utilities, and  $\lambda_{i,t}$ ,  $\psi_{i,t}$ , and  $\eta_{i,t}$  denote the multipliers on the real versions of the budget constraint (4), the goods market constraint (5), and the money supply constraint (6), rearranged to  $i_{i,t} R_t^m \leq b_{i,t-1} / \pi_t$ . Finally, the complementary slackness conditions are  $0 \leq m_{i,t-1} \pi_t^{-1} + i_{i,t} - c_{i,t}$ ,  $\psi_t \geq 0$ ,  $\psi_t (m_{i,t-1} \pi_t^{-1} + i_{i,t} - c_{i,t}) = 0$  and  $0 \leq b_{i,t-1} \pi_t^{-1} - R_t^m i_{i,t}$ ,  $\eta_{i,t} \geq 0$ ,  $\eta_{i,t} (b_{i,t-1} \pi_t^{-1} - R_t^m i_{i,t}) = 0$ , as well as (4) with equality and associated transversality conditions hold.

Substituting out  $\lambda_{i,t}$  in (11) with (7), shows that the multiplier on the cash-in-advance

constraint, which measures the liquidity value of money, satisfies

$$\psi_{i,t}/u_{c,i,t} = 1 - 1/R_{i,t}^{IS} \quad \text{with} \quad R_{i,t}^{IS} \equiv u_{c,i,t}/\beta E_t(u_{c,i,t+1}/\pi_{t+1}), \quad (12)$$

where  $\psi_{i,t}/u_{c,i,t}$  measures agents' marginal willingness to spend for money and  $1/R_{i,t}^{IS}$  is the inverse of the nominal marginal rate of intertemporal substitution in terms of the cash good. Accordingly, the loan rate equals the nominal marginal rate of intertemporal substitution in terms of the credit good  $\tilde{R}_t^{IS} \equiv u_{\tilde{c},i,t}/\beta E_t(u_{\tilde{c},i,t+1}/\pi_{t+1})$ , see (9). Given that we relate the loan rate to empirically observed interest rates mentioned in Section 2, we use the notation  $R_t^L$  (instead of  $\tilde{R}_t^{IS}$ ), for convenience. Notably,  $R_{i,t}^{IS}$  only equals the policy rate (like in a basic New Keynesian model) if the money supply constraint (6) is not binding,  $\eta_{i,t} = 0$ . Then, condition (10) can – by using (7) and (11) – be written as  $\psi_{i,t}/u_{c,i,t} = (R_t^m - 1)\beta E_t(u_{c,i,t+1}/\pi_{t+1})/u_{c,i,t}$ , implying  $R_t^m = R_{i,t}^{IS}$ . When the central bank sets the policy rate at a lower value, agents receive a positive rent when they acquire money in open market operations. Then, they will demand money, until the money supply constraint (6) is binding. This can be seen from substituting out  $\lambda_{i,t}$  in (10) with (7), to get a measure for the real liquidity value of government bonds

$$\eta_{i,t}/u_{c,i,t} = (\psi_{i,t}/u_{c,i,t}) - (1 - 1/R_{i,t}^m), \quad (13)$$

or by using (12),  $\eta_{i,t}/u_{c,i,t} = (1/R_t^m) - (1/R_{i,t}^{IS})$ . Hence, the liquidity value of bonds in real terms is smaller than the liquidity value of money as long as its relative price in open market operations does not equal one,  $R_t^m > 1$ . Notably, liquidity is positively valued by households if  $R_t^{IS} > 1$ , such that the demand for money is well defined, even when the policy rate is at the zero lower bound,  $R_t^m = 1$ . Further note that the interest rate of non-eligible debt  $R_t^L$  tends to be larger than the treasury rate  $R_t$  (see 8 and 9), if the money supply constraint is binding  $\eta_{i,t} > 0$ . Then, bonds have a positive liquidity value and there is a positive liquidity premium  $R_t^L > R_t$ , consistent with empirical evidence.

Further, there are intermediate goods producing firms, which sell their goods to monopolistically competitive retailers that are subject to a Calvo-type sticky price friction. The retailers sell a differentiated good to bundlers, who assemble final goods using a Dixit-Stiglitz technology. The intermediate goods producing firms are identical, perfectly competitive, owned by the households, and produce an intermediate good  $y_t^m$  with labor  $n_t$  according to the production function  $y_t^m = n_t^\alpha$ , with the labor elasticity of production  $\alpha$ . Firms can also issue and hold risk-free debt  $L_t^f$ . The problem of a representative firm can then be summarized as  $\max E_t \sum_{k=0}^{\infty} p_{t,t+k} \varrho_{t+k}$ , where  $p_{t,t+k} = \beta^k \lambda_{t+k}/\lambda_t$  and  $\varrho_t$  denotes real dividends  $\varrho_t = (P_t^m/P_t)n_t^\alpha - w_t n_t - l_{t-1}^f \pi_t^{-1} + l_t^f/R_t^L$ . The first-order conditions

for debt and labor demand are then given by  $1 = R_t^L E_t[p_{t,t+1}\pi_{t+1}^{-1}]$  and  $w_t = P_t^m/P_t\alpha n_t^{\alpha-1}$ . Monopolistically competitive retailers, indexed with  $k \in [0, 1]$  buy intermediate goods  $y_t^m$  at the price  $P_t^m$  to relabel them to a good  $y_{k,t}$ . The latter are sold at a price  $P_{k,t}$  to perfectly competitive bundlers. Only a random fraction  $1 - \phi$  of the retailers is able to reset their price  $P_{k,t}$  in an optimizing way each period, while the remaining retailers of mass  $\phi$  adjust the price with steady-state inflation  $\pi$ ,  $P_{k,t} = P_{k,t-1} \cdot \pi$ . The problem of a price adjusting retailer reads  $\max_{\tilde{P}_{k,t}} E_t \sum_{s=0}^{\infty} \phi^s \beta^s \phi_{t,t+s} ((\prod_{k=1}^s \tilde{P}_{k,t}/P_{t+s}) - mc_{t+s}) y_{k,t+s}$ , where marginal costs are  $mc_t = P_t^m/P_t$ . The first-order condition can be written as  $\tilde{Z}_t = \frac{\varepsilon}{\varepsilon-1} Z_t^1/Z_t^2$ , where  $\tilde{Z}_t = \tilde{P}_t/P_t$ ,  $Z_t^1 = \xi_t c_t^{-\sigma} y_t mc_t + \phi \beta E_t (\pi_t/\pi)^\varepsilon Z_{t+1}^1$  and  $Z_t^2 = \xi_t c_t^{-\sigma} y_t + \phi \beta E_t (\pi_{t+1}/\pi)^{\varepsilon-1} Z_{t+1}^2$ .

The perfectly competitive bundlers combine the various  $y_{k,t}$  to the final consumption good  $y_t$  using the technology  $y_t^{\frac{\varepsilon-1}{\varepsilon}} = \int_0^1 y_{k,t}^{\frac{\varepsilon-1}{\varepsilon}} dk$ , where  $\varepsilon > 1$  is the elasticity of substitution between the different varieties. The cost minimizing demand for each good is given by  $y_{k,t} = (P_{k,t}/P_t)^{-\varepsilon} y_t$ . The bundlers sell the final good  $y_t$  to the households at the price  $P_t$ , which can be written as the consumer price index (CPI)  $P_t^{1-\varepsilon} = \int_0^1 P_{k,t}^{1-\varepsilon} dk$ . The price index satisfies  $1 = (1 - \phi) \tilde{Z}_t^{1-\varepsilon} + \phi (\pi_t/\pi)^{\varepsilon-1}$ . In a symmetric equilibrium,  $y_t^m = \int_0^1 y_{k,t} dk$  and  $y_t = a_t n_t^\alpha / s_t$  will hold, where  $s_t = \int_0^1 (P_{k,t}/P_t)^{-\varepsilon} dk$  is an index of price dispersion that evolves according to  $s_t = (1 - \phi) \tilde{Z}_t^{-\varepsilon} + \phi s_{t-1} (\pi_t/\pi)^\varepsilon$  for a given  $s_{-1}$ .

## 3.2 Public Sector

The government issues one-period bonds  $B_t^T$  and obtains potential profits of the central bank  $P_t \tau_t^m$ . Revenues beyond those used to repay debt from last period are transferred to the households in a lump-sum fashion,  $P_t \tau_t$ . The government budget constraint is then given by  $(B_t^T/R_t) + P_t \tau_t^m = B_{t-1}^T + P_t \tau_t$ . Given that one period equals one quarter in our setting, this debt corresponds to 3-month Treasury bills. Government debt is held by banks in the amount of  $B_t$  and by the central bank in the amount of  $B_t^C$ , such that  $B_t^T = B_t + B_t^C$ . We assume that the supply of Treasury bills is exogenously determined by a constant growth rate  $\Gamma$

$$B_t^T = \Gamma B_{t-1}^T, \quad (14)$$

where  $\Gamma > \beta$ . Equation (14) describes the supply of the single money market instrument that the central bank declares eligible, which can be augmented without affecting the main model properties. In particular, we abstract from explicitly modelling long-term government debt, for convenience, and compute implied long-term interest rates (see Section 4.2).

The central bank supplies money in exchange for Treasury bills either outright,  $M_t$ , or under repos  $M_t^R$ . At the beginning of each period, the central bank's stock of Treasuries equals  $B_{t-1}^C$  and the stock of outstanding money equals  $M_{t-1}$ . It then receives an amount  $\Delta B_t^C$  of Treasuries in exchange for newly supplied money  $I_t = M_t - M_{t-1} + M_t^R$ . After repurchase agreements are settled, its holdings of Treasuries and the amount of outstanding money are reduced by  $B_t^R$  and by  $M_t^R$ , respectively. Before the asset market opens, where the central bank can reinvest its payoffs from maturing assets  $B_t^C$ , it holds an amount equal to  $B_{t-1}^C + \Delta B_t^C - B_t^R$ . Its budget constraint is thus given by  $(B_t^C/R_t) + P_t\tau_t^m = \Delta B_t^C + B_{t-1}^C - B_t^R + M_t - M_{t-1} - (I_t - M_t^R)$ , which after substituting out  $I_t$ ,  $B_t^R$ , and  $\Delta B_t^C$  using  $\Delta B_t^C = R_t^m I_t$ , can be rewritten as  $(B_t^C/R_t) - B_{t-1}^C = R_t^m (M_t - M_{t-1}) + (R_t^m - 1) M_t^R - P_t\tau_t^m$ . Following central bank practice, we assume that interest earnings are transferred to the government,  $P_t\tau_t^m = B_t^C (1 - 1/R_t) + (R_t^m - 1) (M_t - M_{t-1} + M_t^R)$ , such that central bank holdings of Treasuries evolve according to  $B_t^C - B_{t-1}^C = M_t - M_{t-1}$ . Restricting the initial values to  $B_{-1}^C = M_{-1}$  leads to the central bank balance sheet

$$B_t^C = M_t. \quad (15)$$

Regarding the implementation of monetary policy, we assume that the central bank sets the policy rate  $R_t^m$  following a Taylor-type feedback rule, while respecting the ZLB:

$$R_t^m = \max \left\{ 1; (R_{t-1}^m)^{\rho_R} [R_m (\pi_t/\pi)^{\rho_\pi} (y_t/\tilde{y}_t)^{\rho_y}]^{1-\rho_R} \exp \left( \varepsilon_t^m \cdot \prod_{k=1}^K \varepsilon_{t,t-k}^m \right) \right\}, \quad (16)$$

where  $\tilde{y}_t$  is the efficient level of output,  $\rho_\pi \geq 0$ ,  $\rho_y \geq 0$ ,  $0 \leq \rho_R < 1$ ,  $R^m \geq 1$ , and  $\varepsilon_t^m$  denotes a contemporaneous monetary policy shock. Following Laséen and Svensson (2011),  $\prod \varepsilon_{t,t-k}^m$  describes a series of anticipated policy shocks, which materialize in period  $t$ , but were announced in period  $t - k$ , that are used to model forward guidance.

The target inflation rate  $\pi$  is controlled by the central bank and will be assumed to equal the growth rate of Treasuries  $\Gamma$ , which is in line with US data (see 4.2.1). Finally, the central bank fixes the fraction of money supplied under repurchase agreements relative to money supplied outright at  $\Omega \geq 0$ :  $M_t^R = \Omega M_t$ . For the subsequent analysis,  $\Omega$  will be set at a sufficiently large value to ensure that central bank money injections  $I_t$  are non-negative.

### 3.3 Equilibrium Properties

Given that households, firms, and retailers behave in an identical way, we can omit indices. A rational expectations equilibrium is characterized in Definition 1 in Appendix C. The main difference to a basic New Keynesian model is the money supply constraint (6). The model in fact reduces to a New Keynesian model with a conventional cash-in-advance constraint if the money supply constraint (6) is slack, which is summarized in Definition 2 in Appendix C.<sup>14</sup>

Since short-term Treasuries and money are close substitutes, the Treasury bill rate  $R_t$  relates to the expected future policy rate, which can be seen from combining the equilibrium version of (7) with (8), (10), and (11),  $(1/R_t) \cdot E_t \varsigma_{t+1} = E_t[(1/R_{t+1}^m) \cdot \varsigma_{t+1}]$ , where  $\varsigma_{t+1} = u_{c,t+1}/\pi_{t+1}$ . Thus, the Treasury bill rate equals the expected policy rate up to first order,

$$R_t = E_t R_{t+1}^m + \text{h.o.t.}, \quad (17)$$

where h.o.t. represents higher order terms. Notably, the relation (17), which implies households' indifference between holdings of money and treasuries, accords to the empirical evidence provided by Simon (1990). Combining the equilibrium versions of (7), (9), and (11) further shows that the loan rate  $R_t^L$ , which equals the marginal rate of intertemporal substitution of credit goods, relates to the expected marginal rate of intertemporal substitution of cash goods by  $(1/R_t^L) \cdot E_t \varsigma_{t+1} = E_t[(1/R_{t+1}^{IS}) \cdot \varsigma_{t+1}]$ . Hence, the loan rate equals to the expected value of  $R_{t+1}^{IS}$  up to first order,

$$R_t^L = E_t R_{t+1}^{IS} + \text{h.o.t.}, \quad (18)$$

As implied by (17) and (18), the model predicts a positive spread between the loan rate and the Treasury bill rate, in accordance with the data, as long as the central bank sets the policy rate below the marginal rate of intertemporal substitution of cash goods, implying a positive liquidity value of government bonds (see 13).

## 4 The Effect of Forward Guidance in the Model

In this section, we examine the models' predictions regarding the macroeconomic effects of forward guidance. We begin with deriving main model properties in an analytical way in Section 4.1. Subsequently, we study its quantitative predictions numerically in Section 4.2. In the first part, we focus, for analytical clarity, on the real liquidity value

<sup>14</sup>It should be noted that a binding money supply constraint does not imply that monetary policy is less efficient compared to a regime, where money is supplied in an unbounded way, as shown by Schabert (2015).

of government bonds (see 13), which provides the basis for the liquidity premium. Short and long-term versions of the spread between the loan rate  $R_t^L$  and the treasury rate  $R_t$  which correspond to the spreads examined in Section 2, will be examined in the second part of this section for the quantitative analysis.

## 4.1 Analytical Results

As mentioned above, the model features two substantially different versions depending on whether the money supply constraint (6) is binding, which leads to an endogenous liquidity premium, or whether money supply is de facto unconstrained, implying that the policy rate  $R_t^m$  equals the marginal rate of intertemporal substitution  $R_t^{IS}$ . Technically, this means that we assume that the central bank sets the policy rate in the long run either below or equal to  $R^{IS} = \pi/\beta$  (where time indices are omitted to indicate steady state values) and examine the local dynamics in the neighborhood of the particular steady state.<sup>15</sup> In the neighborhood of a steady state, the equilibrium sequences are approximated by the solutions to the linearized equilibrium conditions, where  $\hat{a}_t$  denotes relative deviations of a generic variable  $a_t$  from its steady state value  $a$ :  $\hat{a}_t = \log(a_t/a)$ . To facilitate the derivation of analytical results, we assume that outright money supply is negligible,  $\Omega \rightarrow \infty$ , which reduces the set of endogenous state variables. We further assume for convenience that there are no credit goods  $\gamma = 0$ , and that the central bank targets long-run price stability  $\pi = 1$ , which is supported by the supply of eligible government debt  $\Gamma = 1$ .<sup>16</sup>

**Definition 3** *A rational expectations equilibrium for  $\Omega \rightarrow \infty$ ,  $\Gamma = \pi = \alpha = 1$ , and  $\rho_{R,y} = \gamma = 0$  is a set of convergent sequences  $\{\hat{c}_t, \pi_t, \hat{b}_t, \hat{R}_t^{IS}, \hat{R}_t^m, \hat{R}_t^L, \hat{R}_t\}_{t=0}^\infty$  satisfying*

$$\hat{c}_t = \hat{b}_{t-1} - \hat{\pi}_t - \hat{R}_t^m \text{ if } R_t^m < R_t^{IS}, \quad (19)$$

$$\text{or } \hat{c}_t \leq \hat{b}_{t-1} - \hat{\pi}_t - \hat{R}_t^m \text{ if } R_t^m = R_t^{IS},$$

$$\sigma \hat{c}_t = \sigma E_t \hat{c}_{t+1} - \hat{R}_t^{IS} + E_t \hat{\pi}_{t+1}, \quad (20)$$

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \chi \left[ (\sigma_n + \sigma) \hat{c}_t + \hat{R}_t^{IS} \right], \quad (21)$$

$$\hat{b}_t = \hat{b}_{t-1} - \hat{\pi}_t, \quad (22)$$

$\hat{R}_t = E_t \hat{R}_{t+1}^m$ , and  $\hat{R}_t^L = E_t \hat{R}_{t+1}^{IS}$ , where  $\chi = (1 - \phi)(1 - \beta\phi)/\phi$  for a monetary policy rate

<sup>15</sup>We further assume that shocks are sufficiently small such that the zero lower bound is never binding.

<sup>16</sup>Notably, the latter assumption is not necessary for the implementation of long-run price stability, since the central bank can in principle adjust the share of short-term Treasuries that are eligible for money supply operations to implement the desired inflation target, as shown by Schabert (2015).

satisfying

$$\widehat{R}_t^m = \rho_\pi \widehat{\pi}_t + \widehat{\varepsilon}_t^m + \sum_{k=1}^K \widehat{\varepsilon}_{t,t-k}^m, \quad (23)$$

where  $\rho_\pi > 0$ , for a given  $b_{-1} > 0$ .

Suppose the money supply constraint (6) were not binding, such that the policy rate equals the marginal rate of intertemporal substitution,  $R_t^m = R_t^{IS}$ , and there is no liquidity premium. This would be the case if eligible assets are supplied abundantly or if there are no collateral requirements in open market operations. Given that condition (19) is then slack, the model reduces to a standard New Keynesian model with a cash-in-advance constraint. This latter constraint implies that the policy rate affects the marginal rate of substitution between consumption and working time and therefore enters the aggregate supply constraint (21). In this setting, forward guidance exerts the stark effects that were criticized in the literature (see below and Del Negro et al., 2015), such as large initial output and inflation effects as well as cumulative output responses that are growing in the horizon of forward guidance (see Section 4.2.2).

In the subsequent analysis, we focus on the case where the policy rate is set below the marginal rate of intertemporal substitution, i.e.,  $R_t^m < R_t^{IS}$ , the money supply constraint and, hence, (19) is binding, which implies a liquidity premium. As shown in Appendix D there exist unique locally convergent equilibrium sequences, if but not only if

$$\rho_\pi < [(1 + \beta)\chi^{-1} + 1 - \sigma]/\sigma \quad (24)$$

is satisfied. Condition (24) implies that an active monetary policy ( $\rho_\pi > 1$ ) is not necessary for equilibrium determinacy. Importantly, this model property allows to consider an exogenous path for the monetary policy rate ( $\rho_\pi = 0$ ) without inducing local equilibrium indeterminacy.<sup>17</sup> The reason for the irrelevance of the Taylor principle is that the equilibrium dynamics are similar to the case where the central bank controls the money growth rate, while extreme changes in the policy rate, i.e., violations of (24), should be avoided to ensure that explosive dynamics do not occur. It should further be noted that the sufficient condition (24) is far from being restrictive for a broad range of reasonable parameter values.

A typical forward guidance announcement of the FOMC in the last years stated to keep policy rates at low levels for a specific period of time. To assess the effect of this kind of forward guidance in our model, we consider the following simple experiment: The central bank announces in period  $t$  to reduce the policy rate for the periods  $t$  and

<sup>17</sup>This property relates to the findings of Diba and Loisel (2017) and Michaillat and Saez (2018).

$t + 1$ . A simple way to have full control about policy rates in our model is to set the inflation feedback  $\rho_\pi$  to zero such that the policy rate is only affected by the shocks  $\widehat{\varepsilon}^m$ . Formally, our forward-guidance experiment consists of two components: a shock to the policy rate in  $t$ , i.e.,  $\widehat{\varepsilon}_t^m < 0$ , and a shock in  $t + 1$  that is announced in  $t$  of the same size, i.e.,  $\widehat{\varepsilon}_{t+1,t}^m = \widehat{\varepsilon}_t^m$ . Since we do not consider announcements for periods that lie more than one period in the future, we apply  $K = 1$  for (23). For the linearized model given in Definition 3, we can analytically derive the main effects of this policy experiment, in particular, on the real liquidity value of bonds  $\eta_t/u_{c,t}$  (see 13), that we summarize in the following proposition.<sup>18</sup>

**Proposition 1** *Suppose that  $R^m < \pi/\beta$ ,  $\sigma \in (1, \beta/\chi)$ , and  $\rho_\pi = 0$  which guarantees that (24) is satisfied. The effect of a forward guidance announcement in period  $t$  that reduces the monetary policy rate in  $t$  and  $t + 1$  can be separated into the partial effects of the reduction of the contemporaneous policy rate,  $\widehat{\varepsilon}_t^m < 0$ , and the effects of the announcement of the reduction in the future policy rate,  $\widehat{\varepsilon}_{t+1,t}^m = \widehat{\varepsilon}_t^m < 0$ .*

1. *The reduction of the contemporaneous policy rate,  $\widehat{\varepsilon}_t^m < 0$ , leads to an increase in the liquidity value of government bonds, an increase in inflation, and an increase in consumption and output in the current period,  $t$ .*
2. *The announcement of the reduction in the future policy rate,  $\widehat{\varepsilon}_{t+1,t}^m < 0$ , also leads to an increase in the liquidity value of government bonds, an increase in inflation, but to a decrease in consumption and output in the period of the announcement,  $t$ .*
3. *In total, the forward guidance announcement, with  $\widehat{\varepsilon}_t^m < 0$  and  $\widehat{\varepsilon}_{t+1,t}^m = \widehat{\varepsilon}_t^m$ , leads to an increase in the liquidity value of government bonds and an increase in inflation as well as an increase in consumption and output in the current period,  $t$ .*

**Proof.** See Appendix E. ■

An isolated reduction in the current policy rate,  $\widehat{\varepsilon}_t^m < 0$  (see Part 1 of Proposition 1), stimulates aggregate demand by easing money supply, such that output as well as inflation increases in period  $t$ . Since the central bank supplies more money per unit of an eligible asset, the real liquidity value of government bonds  $\eta_t/u_{c,t}$  increases relative to the real liquidity value of money  $\psi_t/u_{c,t}$  (see 13); the latter being positively affected by the endogenous nominal marginal rate of intertemporal substitution  $R_t^{IS} = u_{c,t}/\beta E_t(u_{c,t+1}/\pi_{t+1})$ , see (12). The increase in current consumption relative to future consumption tends to reduce the nominal marginal rate of intertemporal substitution, while this impact on the real liquidity value of government bonds,

<sup>18</sup>Note that the parameter restriction  $\rho_\pi < \beta\chi^{-1}$  is hardly restrictive, given that in our calibration used in Section,  $\beta\chi^{-1} = 19.72$  which is by far larger than values typically applied for  $\rho_\pi$  of about 1.5.

$\eta_t/u_{c,t} = (1/R_t^m) - (1/R_t^{IS})$ , is dominated by the direct impact of the policy rate reduction for moderate values of the intertemporal elasticity of substitution  $\sigma \in (1, \beta/\chi)$ .

Now consider the isolated reduction in the future policy rate,  $\widehat{\varepsilon}_{t+1}^m < 0$  (see Part 2 of Proposition 1). Its stimulating impact on future output and inflation causes forward-looking price setters also to raise prices in the current period  $t$ . Given that this is not accompanied by an accommodative policy, the real value of money and bonds is deflated in period  $t$ , such that current aggregate demand falls, as can be seen from combining (5) and (6),  $c_t \leq m_{t-1}\pi_t^{-1} + b_{t-1}\pi_t^{-1}/R_t^m$ . Due to the positive consumption growth between  $t$  and  $t+1$  and a higher inflation rate in  $t+1$ , the nominal marginal rate of intertemporal substitution increases, such that the real liquidity value of bonds  $\eta_t/u_{c,t}$  increases even for an unchanged policy rate in period  $t$ .

In total, the forward guidance announcement of keeping the policy rate at a lower level for  $t$  and  $t+1$ , i.e., the joint effect of  $\widehat{\varepsilon}_t^m$  and  $\widehat{\varepsilon}_{t+1}^m$  for  $\widehat{\varepsilon}_t^m = \widehat{\varepsilon}_{t+1}^m$ , apparently leads to an increase in current inflation and in the real liquidity value for bonds, since these effects are also induced by either shock in isolation. For moderate values of the intertemporal elasticity of substitution, it can further be shown that the adverse consumption impact of the future policy rate reduction is dominated by the expansionary impact of the current policy rate reduction, such that current consumption increases as well (see Part 3 of Proposition 1).

The separation of interest rates has important implications for the aggregate effects of forward guidance. While the reduction in the current policy rate stimulates real activity, the additional announcement of a reduction in tomorrow's policy rate dampens this effect. This prediction is in stark contrast to that of a basic New Keynesian model where increased inflation today due to the announcement of low future interest rates unambiguously reduces the relevant real interest rate since the nominal rate is directly controlled by the central bank. This additional reduction in the real interest rate reinforces increases in consumption and is responsible for the response of current output to increase with the distance between the announcement date and the future period until which the policy rate is reduced, see Section 4.2.2.

## 4.2 Numerical Results

In this section, we describe the parameterization of the model and present quantitative effects of forward guidance. Motivated by recent forward guidance announcements of the FOMC that stated to keep policy rates at low levels over a period of a 1 to 3 years, we study the effects of policy rate reductions that last several quarters. We show that our model can replicate liquidity premium responses as found in Section 2, while it

generates moderate output and inflation effects that are substantially smaller than in a model version without the liquidity premium, which corresponds to a conventional New Keynesian model.

### 4.2.1 Parameter values

A period is assumed to be one quarter. For a first set of parameters, we apply values that are standard in the literature on business cycle analysis. The elasticity of substitution between individual varieties of the intermediate goods producing firms  $\epsilon$  is set to 6, which implies a steady state mark-up of 20%, the inverse Frisch elasticity  $\sigma_n$  is set to 2, and the production elasticity  $\alpha$  is set to  $2/3$ . The probability that firms are not able to reset prices in the Calvo model is set to  $\phi = 0.8$ , and the reaction coefficients of the interest rate rule (16) are set to  $\rho_\pi = 1.5$ ,  $\rho_y = 0.05$ , and  $\rho_R = 0.8$ .

A second set of parameters is set to match mean observations in our data set from Section 2 (January 1990 to September 2016). The rate of inflation and the policy rate in steady state are set to the average values of the CPI inflation and the federal funds rate.<sup>19</sup> The corresponding values are  $\pi = 1.0243^{1/4}$  and  $R^m = 1.0304^{1/4}$ . We set the long-run liquidity premium between Treasuries that are eligible for open market operations and the less liquid assets that are non-eligible to 53 basis points, which is the mean value of the common liquidity factor from Section 2.1 between January 1990 and September 2016. This implies  $R^{IS} = \pi/\beta = 1.036^{1/4}$  and with the empirical mean inflation rate also gives  $\beta = 0.9972$ . The growth rate  $\Gamma$  of the T-bills in (14) is set to the long-run inflation rate, which roughly accords to the average T-bill growth rate in the pre crisis sample. The ratio of money supplied under repos  $\Omega$  is set to 1.5, which is based on data about the mean fraction of repos to total reserves of depository institutions in the US between 2003 and 2007.<sup>20</sup> This value further ensures that money injections by the central bank  $I_t$  are, in line with the data, always positive. The utility weight of credit goods  $\gamma$  is set to match the share of non-cash transactions of 86%, taken from Bennett et al. (2014), from which we subtract the average expenditure shares for durables and investment, as both are not specified in the model, leading to a share of cash goods (in non-durable consumption expenditures) of 39%.

Given that the central bank does not set the marginal rate of intertemporal substitution, its dynamics are affected by the elasticity of intertemporal substitution  $1/\sigma$ . For the

<sup>19</sup>We use monthly data from FRED between January 1990 and December 2016 that we aggregate to quarterly values as the basis for the long-run means. For the CPI we take the series [CPIAUCSL] and for the federal funds rate we take the series [FEDFUNDS].

<sup>20</sup>The time period is restricted by data availability and the start of the financial crisis.

numerical analysis, we choose a value of  $\sigma = 1.5$ , which lies between values that are typically applied in the business cycle literature. For this value, we find that the combined empirical response of the common liquidity factor to equal changes in the factors  $\tilde{F}_1$  and  $\tilde{F}_2$  falls in the range of the model-implied responses of the liquidity premia measured by the short-term spread between the loan rate and the treasury rate,  $\hat{R}_t^L - \hat{R}_t$ , and the response of the implied long-term liquidity premium,  $\prod_s^q (\hat{R}_{t+s}^L - \hat{R}_{t+s})^{1/q}$  for a maturity  $q = 4$  that corresponds to the experiment of an announced policy rate reduction by 25 b.p. per quarter from  $t$  to  $t + 4$ .<sup>21</sup> We also considered a higher value of the elasticity of intertemporal substitution as a robustness check, see Figure 7 in Appendix G.

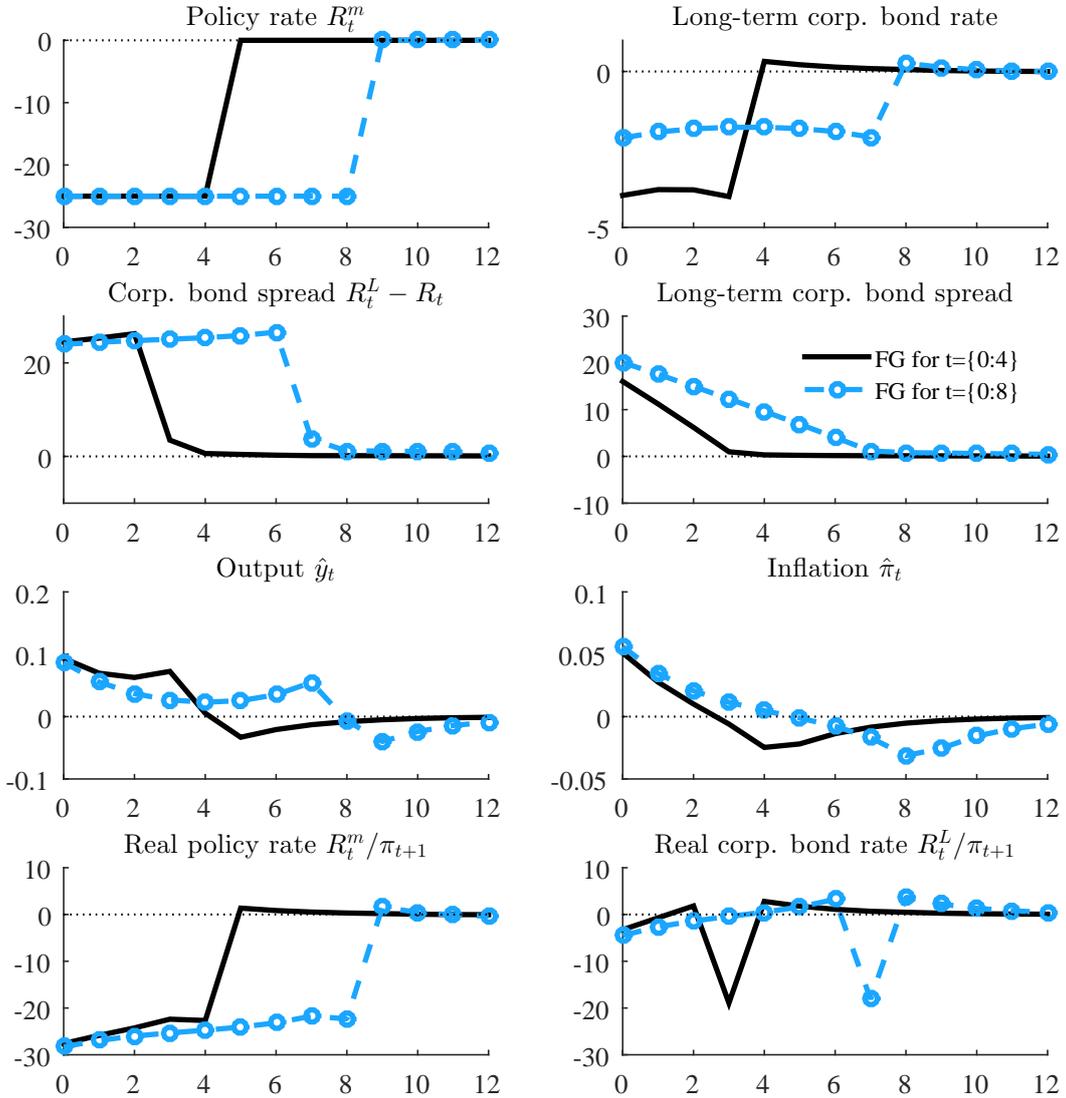
For the policy experiments, we consider paths of the monetary policy rate announced in advance. For this, it is convenient to assume that the contemporaneous shock,  $\varepsilon_t^m$ , and all announced monetary policy innovations,  $\prod \varepsilon_{t-k}^m$ , in (16) are completely transitory white-noise innovations that are identically and independently distributed as  $N(0, \sigma_{m,k}^2)$ . We model forward guidance as a path for the monetary policy rate  $\{R_{T+h}^m\}_{h=1}^H$  in the upcoming  $H$  periods that the central bank announces at the beginning of period  $T + 1$ , before which the economy is assumed to rest in steady state. We then back out a sequence of present and anticipated future monetary policy innovations  $\varepsilon_{T+1}^m = \{\varepsilon_{T+1}^m, \varepsilon_{T+1+k, T+1}^m\}_{k=1}^K$  that yields this desired interest rate path for our model with the liquidity premium and a reference version without a liquidity premium, where local equilibrium determinacy relies on the Taylor principle. The calculation of the shocks is based on a procedure by Laséen and Svensson (2011) and Del Negro et al. (2015) and is described in Section F of the Appendix. Notably, the results found for our model would not be affected if the forward guidance experiment was not a policy rate reduction at the steady state, but an announcement to keep the policy rate at the ZLB for longer than dictated by the feedback rule.

## 4.2.2 Impulse Responses to Forward Guidance

Figure 1 shows impulse responses to different forward guidance scenarios in our model with the endogenous liquidity premium. The two scenarios shown in the figure are announcements of the central bank to reduce the policy rate  $R_t^m$  by 25 annualized basis points for the next 4 (see black solid line) and 8 quarters (see blue dashed line with

<sup>21</sup>In our empirical analysis, the target factor  $\tilde{F}_1$  summarizes a change in money market rates of 1 percentage point for the spot rate and 0.84, 0.28, 0.12, and 0.03 annualized percentage points for the 3-months, 6-months, 9-months, and 12-months forward rates. In turn, the path factor  $\tilde{F}_2$  affects the rates by 0 (spot rate), 0.20, 0.98, 0.98, and 1 (forward rates) annualized percentage points. Accordingly, a joint reduction in both  $\tilde{F}_1$  and  $\tilde{F}_2$  by one unit captures a reduction of money-market rates by 100 annualized basis points, or 25 basis points per quarter, over one year.

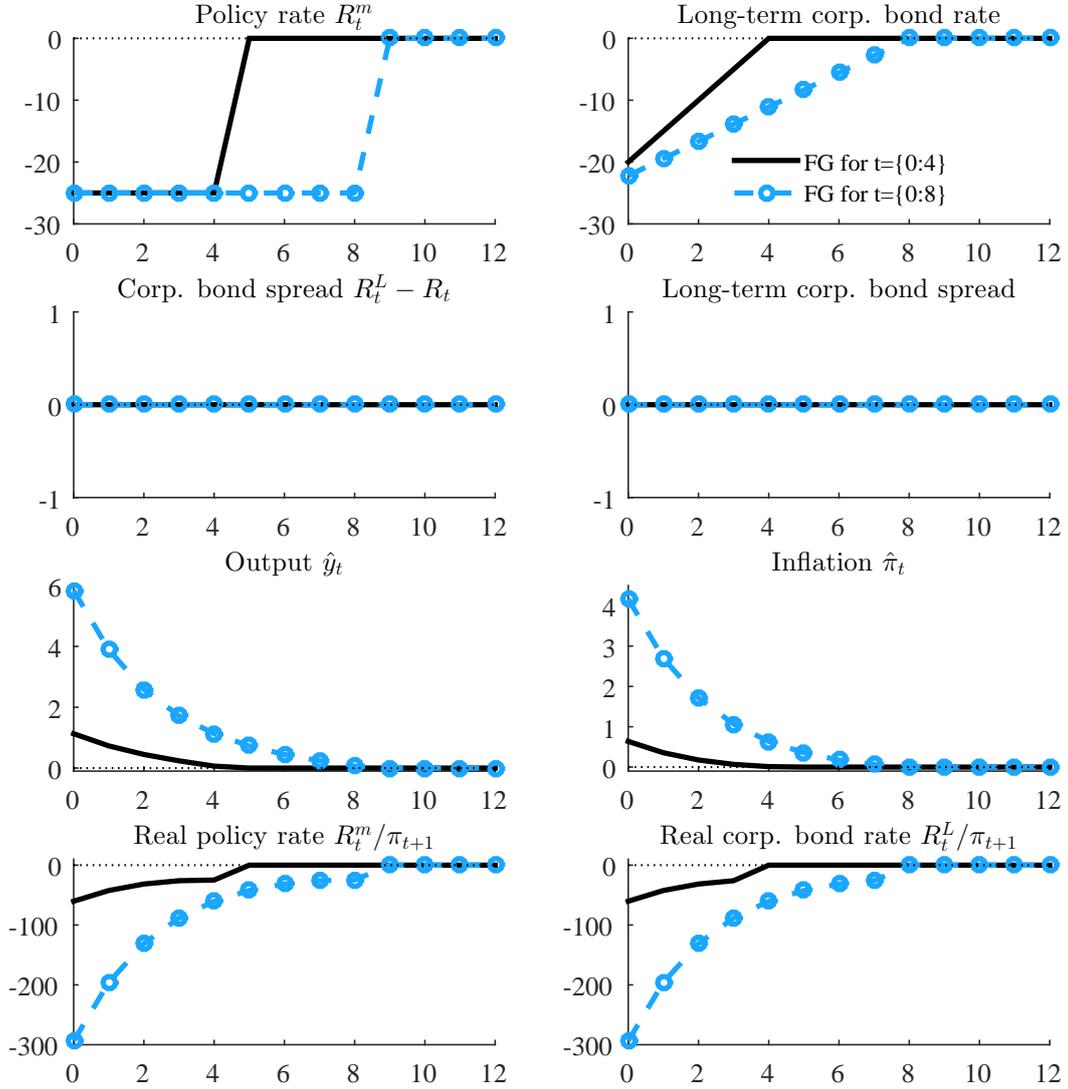
**Figure 1:** Effects of Forward Guidance



*Notes:* Impulse responses to forward guidance about policy rate  $R_t^m$  announced at the beginning of period 0 in model with endogenous liquidity premium. Y-axis: Deviations from steady state in percent ( $\hat{y}_t$ ,  $\hat{\pi}_t$ ) or in basis points (else). X-axis: quarters. Black solid (blue circled) line: Announced policy rate reduction of 25 basis points in quarters 0 to 4 (0 to 8). Long-term corporate bonds rate constructed as  $\prod_s^q (\hat{R}_{t+s}^L)^{1/q}$ , where  $q$  equals the length of the forward guidance period. Long-term treasury rate and long-term spread are constructed accordingly.

circles), respectively. This resembles recent forward guidance experiences, where central banks stated to keep policy rates at low levels over a horizon of about two years. The central bank resets the policy rate to its steady state value after the guidance period until quarter 10. After that, monetary policy is governed by the Taylor rule (16), which then implies values in close proximity of the steady state. The considered path of the nominal interest rate is given in the upper left panel of Figure 1.

**Figure 2:** Forward guidance in a model version without liquidity premium



*Notes:* Impulse responses to forward guidance about policy rate  $R_t^m$  announced at the beginning of period 0 in model without liquidity premium. Y-axis: Deviations from steady state in percent ( $\hat{y}_t$ ,  $\hat{\pi}_t$ ) or in basis points (else). X-axis: quarters. Black solid (blue circled) line: Announced policy rate reduction of 25 basis points in quarters 0 to 4 (0 to 8). Long-term corporate bonds rate constructed as  $\prod_s^q (\hat{R}_{t+s}^L)^{1/q}$ , where  $q$  equals the length of the forward guidance period. Long-term treasury rate and long-term spread are constructed accordingly.

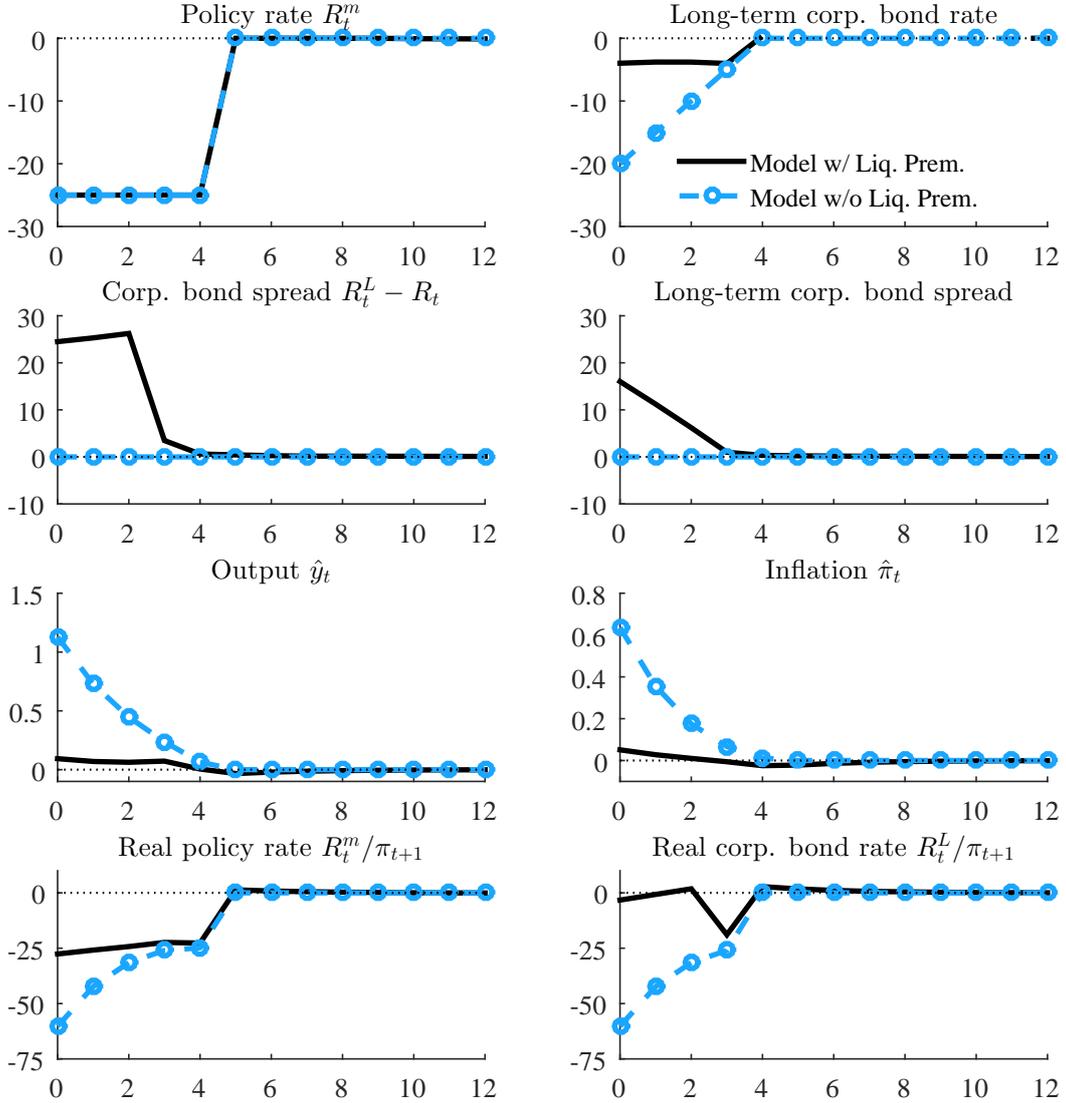
Consistent with the empirical evidence provided by Campbell et al. (2012) and in Table 2 in Appendix B, the implied long-term loan rate  $\prod_s^q (\hat{R}_{t+s}^L)^{1/q}$  for  $q \in \{4, 8\}$  falls on impact, but to a smaller extent than the policy rate. Accordingly, the short-term liquidity premium  $R_t^L - R_t$  as well its long-term counterpart for  $q = 4$  increase on impact, by 24 b.p. and 16 b.p., respectively. Notably, the response of the common liquidity factor, which is based on liquidity premia of various maturities, to the corresponding empirical

experiment (i.e., the combined change of  $\tilde{F}_1$  and  $\tilde{F}_2$  by 25 b.p. per quarter, hence 100 b.p. per annum) equals 17 b.p. in quarterly rates and lies between the responses of the liquidity premia in the model. The interest rate reduction further leads to a moderate increase of current output by about 0.1%, while output remains close to this level until the end of the guidance period. Once the policy rate increases, output experiences a dip before returning to its steady state value. Inflation rises on impact by about 0.05 percentage points and it starts decreasing already before the end of the guidance period. The real policy rate  $R_t^m/\pi_{t+1}$  behaves similar to the nominal rate, where differences reflect the endogenous response of inflation. By contrast, the real loan rate,  $R_t^L/\pi_{t+1}$ , barely moves on impact and only experiences a negative spike before the end of the guidance period (18). Comparing the scenario of forward guidance over 4 quarters with that over 8 quarters reveals that differences in terms of the impact responses of output and the liquidity premium are small while inflation is slightly higher on impact in case of the longer horizon. As can be seen from Figure 2, this observation differs substantially from the prediction of the basic New Keynesian model without an endogenous liquidity premium in which the impact responses of output and inflation increase with the horizon of the forward guidance. The latter has already been shown in other forward guidance studies for a basic New Keynesian model (see e.g. McKay et al., 2016).

Figure 3 compares the effects of forward guidance in the model featuring the endogenous liquidity premium with version of the model without the liquidity premium ( $\eta_t = 0$ , see 13), which corresponds to a conventional New Keynesian model. In both cases, the central bank announces to reduce the policy rate by 25 basis points for the next 4 quarters and to return it to its steady state value afterwards. The results are identical to those of the first scenarios in Figures 1 and 2, respectively, and are repeated in a joint figure here to facilitate comparison. Output and inflation in the model without the liquidity premium increase sharply on impact. Compared to the model with the liquidity premium, the responses on impact are about 12 times higher. When the central bank directly sets the nominal marginal rate of substitution (i.e., when there is no liquidity premium), the real rate falls by more on impact than the nominal rate due to the increase in inflation and, hence, add to the increase of consumption and output. A corresponding announcement in terms of real (instead of nominal) policy rates (see Figure 8 in Appendix G) leads to similar results as in Figure 3.

In Appendix G, we further show results to additional experiments to complement the analysis. Figure 9 shows the effects of an isolated policy rate reduction in  $t = 1$  that is announced in  $t = 0$ . In isolation, also this policy experiment leads to an increase in liquidity premia while the introduction of the endogenous liquidity premium

**Figure 3:** Comparison with a model version without liquidity premium



*Notes:* Impulse responses to policy rate ( $R_t^m$ ) reduction of 25 basis points in quarters 0 to 4, at the beginning of period 0. Y-axis: Deviations from steady state in percent ( $\hat{y}_t$ ,  $\hat{\pi}_t$ ) or in basis points (else). X-axis: quarters. Black line: Baseline model with endogenous liquidity premium. Blue circled line: basic New Keynesian model. Long-term corporate bonds rate constructed as  $\prod_s^q (\hat{R}_{t+s}^L)^{1/q}$ , where  $q$  equals the length of the forward guidance period. Long-term treasury rate and long-term spread are constructed accordingly.

dampens its output and inflation effects considerably compared to the baseline model without a liquidity premium. Figure 10 shows the responses to a commonly used non-monetary demand shock, which is typically found to contribute more to macroeconomic fluctuations than monetary policy shocks. In response to this shock, our model predicts a positive relation between liquidity premia and the monetary policy rate, which accords to evidence provided by Nagel (2016).

Overall, our model can replicate the liquidity premium effects of forward guidance, generates substantially smaller output and inflation effects, and does not predict that the latter effects increase with the announcement horizon.

## 5 Conclusion

We show empirically that liquidity premia tend to rise after forward guidance announcements. We augment the conventional New Keynesian model by an endogenous liquidity premium that separates the monetary policy rate from other interest rates that are more relevant for private-sector transactions, which allows replicating the observed liquidity premium response. We show both analytically and numerically that forward guidance is a much less powerful policy tool in this setting. According to our analysis, no forward guidance puzzle exists when endogenous liquidity premia are taken into account.

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## Appendix

### A Measurement of Liquidity Premia

In this appendix, we describe the data sources and the construction of all interest rate spreads. We collect daily return data and calculate the spreads as the difference in annualized daily returns between Treasuries and an illiquid asset of similar safety and maturity. We use data from FRED (<https://fred.stlouisfed.org>) and from Bloomberg. Original mnemonics in the data source are given in square brackets.

The data for the Treasury rates stem from FRED. We use the 'Treasury Constant Maturity Rates' with the mnemonic [DGS' $xx$ '], where ' $xx$ ' = {3MO, 6MO, 1, 3, 5, 10} refers to the maturity in months (MO) or years (else). We collect daily data from 1990-01-02 to 2016-09-16. Following Krishnamurthy and Vissing-Jorgensen (2012) as well as Del Negro et al. (2017), we construct several spreads between the rates on investment grade rated commercial papers or corporate bonds and Treasuries for different maturities. All series are taken from FRED. As a short-run measure, we use the '3-Month AA/P1 Nonfinancial Commercial Paper Rate' with mnemonic [DCPN3M] and we calculate the spread relative to the series [DGS3MO]. For longer maturities, we employ the following four corporate bond indexes: (1) The 'Bank of America (BofA) Merrill Lynch US Corporate 1-3 Year Effective Yield', mnemonic [BAMLC1A0C13YEY], which is a subset of the 'BofA Merrill Lynch US Corporate Master Index' that includes investment grade rated corporate bonds that were publicly issued in the United States. The series that we use includes all securities with a remaining term to maturity between 1 and 3 years. We calculate the spread as [BAMLC1A0C13YEY] - [DGS3]. (2) The 'BofA Merrill Lynch US Corporate AAA Effective Yield', mnemonic [BAMLC0A1CAAAY], which is a subset of the 'BofA Merrill Lynch US Corporate Master Index' that covers securities with an AAA rating. We calculate the spread as [BAMLC0A1CAAAY] - [DGS5]. (3) 'Moody's Seasoned Aaa Corporate Bond Yield', mnemonic [DAAA], which consists of bonds with an AAA rating and long remaining terms to maturity. We construct the spread relative to the series [DGS10]. (4) 'Moody's Seasoned Baa Corporate Bond Yield', mnemonic [DBAA], which consists of US bonds with an BAA rating and long remaining terms to maturity. We construct the spread relative to the series [DGS10]. The series on commercial papers and the indexes from BofA Merrill Lynch are available to us from 1997-01-02 onwards. We collect data on the indexes by Moody's beginning on 1990-01-02. We collect the series 'Certificate of Deposit: Secondary Market Rate' with maturities of 3 and 6 months from FRED with the mnemonics [DCD90] and [DCD6M]. We calculate the spreads relative to the Treasury series [DGS3MO] and [DGS6MO], respectively. Daily data is available to

us from 1990-01-02 to 2013-06-28. We collect data from Bloomberg with the mnemonic [USRGCGC ICUS Curncy] from 1991-05-21 to 2016-09-16. We follow Nagel (2016) in calculating averages between bid and ask prices. We construct the spread relative to the series [DGS3MO].

## B Additional Empirical Results

For completeness, Table 2 presents the responses to monetary policy shocks of the individual interest rates associated to the interest rate spreads considered in Table 1. Tables 3(a) and 3(b) are the counterparts to Tables 2 and 1 for the sample 1990-2008.

**Table 2:** Response of Asset Returns to Changes in Monetary Policy

	Treasuries					GC
	3M	1Y	3Y	5Y	10Y	3M
Current Federal Funds Rate $\tilde{F}_1$	0.65*** (0.079)	0.62*** (0.065)	0.33*** (0.043)	0.19*** (0.037)	0.028 (0.037)	0.31*** (0.077)
Expected Future Federal Funds Rates $\tilde{F}_2$	0.16*** (0.041)	0.38*** (0.041)	0.69*** (0.058)	0.79*** (0.058)	0.70*** (0.054)	0.0087 (0.038)
$R^2$	0.54	0.76	0.81	0.84	0.77	0.23
no. of obs.	237	237	237	237	237	213
	Commercial Paper / Corporate Bonds					CD
	3M	3Y	5Y	10Y(A)	10Y(B)	3M
$\tilde{F}_1$	0.27** (0.11)	0.38*** (0.11)	0.15** (0.060)	-0.025 (0.040)	-0.0037 (0.031)	0.38*** (0.14)
$\tilde{F}_2$	0.034 (0.069)	0.52*** (0.083)	0.58*** (0.070)	0.39*** (0.038)	0.39*** (0.037)	0.15* (0.077)
$R^2$	0.10	0.43	0.65	0.50	0.50	0.22
no. of obs.	122	165	165	237	237	212

*Notes:* Responses of asset returns at FOMC meetings between January 1990 and September 2016. Constant and QE-Dummy included in all regressions. Heteroskedasticity-robust (White) standard errors in parentheses. Asterisks mark significance at 10% (\*), 5% (\*\*), 1% (\*\*\*). Maturity measured in months (M) or years (Y). Corporate Bond 10Y(A) and (B): long-term bonds with AAA and BAA rating, respectively. CD: Certificate of Deposit; GC: General Collateral Repo.

**Table 3:** Response of Liquidity Premia and Asset Prices to Changes in Monetary Policy in a Sample Ending 2008-12-16

(a) Liquidity Premia

	Liquidity	Commercial Paper / Corporate Bond spread				
	Premium $LP$	3M	3Y	5Y	10Y(A)	10Y(B)
Current Federal Funds Rate $\tilde{F}_1$	-0.41*** (0.13)	-0.28*** (0.10)	0.154* (0.090)	0.031 (0.060)	-0.075 (0.061)	-0.047 (0.045)
Expected Future Federal Funds Rates $\tilde{F}_2$	-0.32*** (0.081)	-0.15 (0.12)	-0.063 (0.076)	-0.14*** (0.047)	-0.29*** (0.042)	-0.30*** (0.034)
$R^2$	0.30	0.09	0.05	0.10	0.43	0.53
no. of obs.	175	73	103	103	175	175

	GC spread		CD spread	
	3M		3M	6M
Current Federal Funds Rate $\tilde{F}_1$	-0.36** (0.15)		-0.27* (0.15)	-0.35* (0.19)
Expected Future Federal Funds Rates $\tilde{F}_2$	-0.20** (0.088)		-0.0014 (0.11)	-0.13 (0.16)
$R^2$	0.21		0.08	0.10
no. of obs.	152		175	175

(b) Asset prices

	Treasury					GC
	3M	1Y	3Y	5Y	10Y	3M
Current Federal Funds Rate $\tilde{F}_1$	0.65*** (0.080)	0.60*** (0.059)	0.32*** (0.040)	0.20*** (0.040)	0.042 (0.041)	0.31*** (0.078)
Expected Future Federal Funds Rates $\tilde{F}_2$	0.18*** (0.054)	0.48*** (0.050)	0.74*** (0.069)	0.77*** (0.067)	0.66*** (0.058)	-0.001 (0.057)
$R^2$	0.55	0.82	0.82	0.84	0.79	0.24
no. of obs.	175	175	175	175	175	152

	Commercial Paper / Corporate Bonds					CD
	3M	3Y	5Y	10Y(A)	10Y(B)	3M
Current Federal Funds Rate $\tilde{F}_1$	0.27*** (0.097)	0.38*** (0.11)	0.15** (0.064)	-0.033 (0.040)	-0.0047 (0.031)	0.37*** (0.14)
Expected Future Federal Funds Rates $\tilde{F}_2$	-0.003 (0.12)	0.62*** (0.13)	0.58*** (0.083)	0.37*** (0.046)	0.36*** (0.045)	0.18* (0.10)
$R^2$	0.11	0.44	0.61	0.53	0.54	0.22
no of obs.	73	103	103	175	175	175

*Notes:* Responses of liquidity spreads or asset returns at FOMC meetings between January 1990 and December 2008. Constant and QE-dummy included in all regressions. Heteroskedasticity-robust (White) standard errors in parentheses. Asterisks mark significance at 10% (\*), 5% (\*\*), 1% (\*\*\*). Maturity measured in months (M) or years (Y). Corporate Bond 10Y(A) and (B): long-term bonds with AAA and BAA rating, respectively. CD: Certificate of Deposit; GC: General Collateral Repo. Spreads calculated relative to Treasuries of same maturity.

## C Definition of Equilibrium

**Definition 1** A rational expectations equilibrium is a set of sequences  $\{c_t, \tilde{c}_t, y_t, n_t, w_t, \lambda_t, m_t^R, m_t, b_t, b_t^T, mc_t, Z_{1,t}, Z_{2,t}, Z_t, s_t, \pi_t, R_t^{IS}\}_{t=0}^\infty$  satisfying

$$c_t = m_t + m_t^R, \text{ if } R_t^{IS} > 1, \text{ or } c_t \leq m_t + m_t^R, \text{ if } R_t^{IS} = 1, \quad (25)$$

$$b_{t-1}/(R_t^m \pi_t) = m_t - m_{t-1} \pi_t^{-1} + m_t^R, \text{ if } R_t^{IS} > R_t^m \text{ or} \quad (26)$$

$$b_{t-1}/(R_t^m \pi_t) \geq m_t - m_{t-1} \pi_t^{-1} + m_t^R, \text{ if } R_t^{IS} = R_t^m,$$

$$m_t^R = \Omega m_t, \quad b_t = b_t^T - m_t, \quad b_t^T = \Gamma b_{t-1}^T / \pi_t, \quad (27)$$

$$\theta n_t^{\sigma_n} = u_{c,t} w_t / R_t^{IS}, \quad 1/R_t^{IS} = \beta E_t[u_{c,t+1}/(u_{c,t} \pi_{t+1})], \quad w_t / (\alpha n_t^{\alpha-1}) = mc_t, \quad (28)$$

$$u_{\tilde{c},t} = \lambda_t, \quad \lambda_t = \beta E_t[u_{c,t+1}/\pi_{t+1}], \quad Z_{1,t} = \lambda_t y_t mc_t + \phi \beta E_t[(\pi_{t+1}/\pi)^\varepsilon Z_{1,t+1}], \quad (29)$$

$$Z_{2,t} = \lambda_t y_t + \phi \beta E_t[(\pi_{t+1}/\pi)^{\varepsilon-1} Z_{2,t+1}], \quad Z_t = \mu Z_{1,t} / Z_{2,t}, \quad (30)$$

$$1 = (1 - \phi) Z_t^{1-\varepsilon} + \phi (\pi_t/\pi)^{\varepsilon-1}, \quad s_t = (1 - \phi) Z_t^{-\varepsilon} + \phi s_{t-1} (\pi_t/\pi)^\varepsilon, \quad (31)$$

$$y_t = n_t^\alpha / s_t, \quad y_t = c_t + \tilde{c}_t, \quad (32)$$

(where  $u_{c,t} = c_t^{-\sigma}$ ,  $u_{\tilde{c},t} = \gamma \tilde{c}_t^{-\sigma}$ , and  $\mu = \varepsilon/(\varepsilon - 1)$ ), the transversality conditions, a monetary policy  $\{R_t^m \geq 1\}_{t=0}^\infty$ ,  $\Omega > 0$ ,  $\pi \geq \beta$ , and a fiscal policy  $\Gamma \geq 1$ , for given initial values  $M_{-1} > 0$ ,  $B_{-1} > 0$ ,  $B_{-1}^T > 0$ , and  $s_{-1} \geq 1$ .

Given a rational expectations equilibrium as summarized in Definition 1, the equilibrium sequences  $\{R_t, R_t^L\}_{t=0}^\infty$  can be determined by  $(1/R_t) \cdot E_t[u_{c,t+1}/\pi_{t+1}] = E_t[(1/R_{t+1}^m) \cdot u_{c,t+1}/\pi_{t+1}]$  and  $(1/R_t^L) \cdot E_t[u_{c,t+1}/\pi_{t+1}] = E_t[(1/R_{t+1}^{IS}) \cdot u_{c,t+1}/\pi_{t+1}]$ . If the money supply constraint (6) is not binding, which is the case if  $R_t^m = R_t^{IS}$  (see 13), the model given in Definition 1 reduces to a standard New Keynesian model with a cash-in-advance constraint, where government liabilities can be determined residually.

**Definition 2** A rational expectations equilibrium under a non-binding money supply constraint (6) is a set of sequences  $\{c_t, \tilde{c}_t, y_t, n_t, w_t, \lambda_t, mc_t, Z_{1,t}, Z_{2,t}, Z_t, s_t, \pi_t, R_t^{IS}\}_{t=0}^\infty$  satisfying  $R_t^{IS} = R_t^m$ , (28)-(32), the transversality conditions, and a monetary policy  $\{R_t^m \geq 1\}_{t=0}^\infty$ ,  $\pi \geq \beta$ , for a given initial value  $s_{-1} \geq 1$ .

## D Appendix: Equilibrium determinacy

For the analysis of local equilibrium determinacy, we disregard shocks which are not relevant for this purpose, for simplicity. The equilibrium conditions (19)-(23) given in Definition 3 for the version with  $R_t^m < R_t^{IS}$  can then be summarized by substituting out  $\hat{R}_t^{IS}$ ,  $\hat{R}_t^m$ , and  $\hat{c}_t$  as follows:  $\delta_1 E_t \hat{\pi}_{t+1} + (\delta_2 + \delta_3) \hat{b}_t = \hat{\pi}_t (1 + \rho_\pi \delta_2)$  and  $\hat{b}_t = \hat{b}_{t-1} - \hat{\pi}_t$ , where  $\delta_1 = (\beta + \chi(1 - \sigma) - \chi \sigma \rho_\pi) \gtrless 0$ ,  $\delta_2 = \chi \sigma_n > 0$ ,  $\delta_3 = \chi \sigma > 0$ , which can in matrix form be rewritten as

$$\begin{pmatrix} \delta_1 & \delta_3 + \delta_2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} E_t \hat{\pi}_{t+1} \\ \hat{b}_t \end{pmatrix} = \begin{pmatrix} 1 + \delta_2 \rho_\pi & 0 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} \hat{\pi}_t \\ \hat{b}_{t-1} \end{pmatrix}.$$

The characteristic polynomial of

$$\mathbf{A} = \begin{pmatrix} \delta_1 & \delta_3 + \delta_2 \\ 0 & 1 \end{pmatrix}^{-1} \begin{pmatrix} 1 + \delta_2 \rho_\pi & 0 \\ -1 & 1 \end{pmatrix} \quad (33)$$

is  $F(X) = X^2 - \frac{\delta_1 + \delta_2 + \delta_3 + \rho_\pi \delta_2 + 1}{\delta_1} X + \frac{\rho_\pi \delta_2 + 1}{\delta_1}$ . Since there is one backward-looking variable and one forward-looking variable,  $F(X)$  has to be characterized by one stable and one unstable root for stability and uniqueness. At  $X = 0$ , the sign of  $F(X)$  is equal to the sign of  $\delta_1$ ,  $F(0) = (\rho_\pi \delta_2 + 1) / \delta_1$ , while  $F(X)$  has the opposite sign at  $X = 1$ :  $F(1) = -\frac{1}{\delta_1} (\delta_2 + \delta_3)$ . First, consider the case where  $\delta_1 = \beta + \chi(1 - \sigma) - \chi\sigma\rho_\pi > 0$ . As  $\sigma \geq 1$  and  $\beta < 1$ ,  $\delta_1$  is then strictly smaller than one. Hence,  $F(1) < 0$  and  $F(0) > 1$  implying that exactly one root is unstable and that the stable root it is strictly positive. Second, consider the case where  $\delta_1 = \beta + \chi(1 - \sigma) - \chi\sigma\rho_\pi < 0 \Leftrightarrow \rho_\pi > \frac{\beta + \chi(1 - \sigma)}{\chi\sigma}$ , such that  $F(1) > 0$  and  $F(0) < 0$ . This implies that there is at least one stable root between zero and one. To establish a condition ensuring that there is exactly one stable root, we further use  $F(-1) = [2(1 + \delta_1) + \delta_3 + (2\rho_\pi + 1)\delta_2] / \delta_1$ . Rewriting the numerator using  $\delta_1 = \beta + \chi(1 - \sigma) - \chi\sigma\rho_\pi$ ,  $\delta_2 = \chi\sigma_n$  and  $\delta_3 = \chi\sigma$ , the condition

$$2(1 + \beta + \chi(1 - \sigma) - \chi\sigma\rho_\pi) + \chi\sigma + (2\rho_\pi + 1)\chi\sigma_n > 0 \quad (34)$$

ensures that  $F(0)$  and  $F(-1)$  have the same sign which implies that there is no stable root between zero and minus one. We now use that (34) holds, if but not only if  $\rho_\pi \leq \frac{1 + \beta}{\chi\sigma} + \frac{1 - \sigma}{\sigma}$ , where the term on the right-hand side strictly exceeds  $\frac{\beta + \chi(1 - \sigma)}{\chi\sigma}$  such that local equilibrium determinacy is ensured by (24).

## E Proof of Proposition 1

For  $\rho_\pi = 0$  and  $R_t^m < R_t^{IS}$ , the equilibrium conditions (19) and (23) simplify to

$$\widehat{c}_t = \widehat{b}_{t-1} - \widehat{\pi}_t - \widehat{R}_t^m, \quad (35)$$

$$\widehat{R}_t^m = \widehat{\varepsilon}_t^m + \widehat{\varepsilon}_{t,t-1}^m, \text{ and } \widehat{R}_{t+1}^m = \widehat{\varepsilon}_{t+1}^m + \widehat{\varepsilon}_{t+1,t}^m. \quad (36)$$

We first use the nominal marginal rate of intertemporal substitution  $\widehat{R}_t^{IS} = \sigma E_t \widehat{c}_{t+1} - \sigma \widehat{c}_t + E_t \widehat{\pi}_{t+1}$  in (21) to obtain  $\widehat{\pi}_t = (\beta + \chi) E_t \widehat{\pi}_{t+1} + \chi\sigma_n \widehat{c}_t + \chi\sigma E_t \widehat{c}_{t+1}$ . Further, combining (22) and (35) gives  $\widehat{c}_t = \widehat{b}_t - \widehat{R}_t^m$  which we use to substitute out consumption from (21). We obtain  $\widehat{\pi}_t = (\beta + \chi) E_t \widehat{\pi}_{t+1} + \chi\sigma_n (\widehat{b}_t - \widehat{R}_t^m) + \chi\sigma E_t (\widehat{b}_{t+1} - \widehat{R}_{t+1}^m)$ , which can be rearranged to  $(1 + \chi(\sigma_n + \sigma)) \widehat{\pi}_t = (\beta + \chi(1 - \sigma)) E_t \widehat{\pi}_{t+1} + \chi(\sigma_n + \sigma) \widehat{b}_{t-1} - \chi\sigma_n \widehat{R}_t^m - \chi\sigma E_t \widehat{R}_{t+1}^m$ . Hence, we can summarize the equilibrium as a set of two equations where the

policy rate summarizes the exogenous states: (22) and

$$(1 + \delta_2 + \delta_3)\widehat{\pi}_t = \delta_1 E_t \widehat{\pi}_{t+1} + (\delta_2 + \delta_3)\widehat{b}_{t-1} - \delta_2 \widehat{R}_t^m - \delta_3 E_t \widehat{R}_{t+1}^m, \quad (37)$$

where  $\delta_1 = \beta + \chi(1 - \sigma)$ ,  $\delta_2 = \chi\sigma_n > 0$ ,  $\delta_3 = \chi\sigma > 0$ . Consider general solution forms

$$\widehat{\pi}_t = \gamma_{\pi b}\widehat{b}_{t-1} + \gamma_{\pi\varepsilon}\widehat{\varepsilon}_{t+1,t}^m + \gamma'_{\pi\varepsilon}\widehat{\varepsilon}_{t,t-1}^m + \gamma_{\pi\varepsilon}^m\widehat{\varepsilon}_t^m, \quad (38)$$

$$\widehat{b}_t = \gamma_b\widehat{b}_{t-1} + \gamma_{b\varepsilon}\widehat{\varepsilon}_{t+1,t}^m + \gamma'_{b\varepsilon}\widehat{\varepsilon}_{t,t-1}^m + \gamma_{b\varepsilon}^m\widehat{\varepsilon}_t^m, \quad (39)$$

$$\widehat{c}_t = \gamma_{cb}\widehat{b}_{t-1} + \gamma_{c\varepsilon}\widehat{\varepsilon}_{t+1,t}^m + \gamma'_{c\varepsilon}\widehat{\varepsilon}_{t,t-1}^m + \gamma_{c\varepsilon}^m\widehat{\varepsilon}_t^m, \quad (40)$$

and (36). Substituting the generic solutions into (22) and collecting terms gives  $(\gamma_{b\varepsilon} + \gamma_{\pi\varepsilon})\widehat{\varepsilon}_{t+1,t}^m + (\gamma'_{b\varepsilon} + \gamma'_{\pi\varepsilon})\widehat{\varepsilon}_{t,t-1}^m = (1 - \gamma_{\pi b} - \gamma_b)\widehat{b}_{t-1} - (\gamma_{\pi\varepsilon}^m + \gamma_{b\varepsilon}^m)\widehat{\varepsilon}_t^m$ . Matching coefficients gives (for  $\widehat{\varepsilon}_{t+1,t}^m \neq 0$ ,  $\widehat{\varepsilon}_{t,t-1}^m \neq 0$ ,  $\widehat{b}_{t-1}^m \neq 0$ ,  $\widehat{\varepsilon}_t^m \neq 0$ , respectively)

$$\gamma_{b\varepsilon} = -\gamma_{\pi\varepsilon}, \quad \gamma'_{b\varepsilon} = -\gamma'_{\pi\varepsilon}, \quad \gamma_{\pi b} = 1 - \gamma_b, \quad \text{and} \quad \gamma_{b\varepsilon}^m = -\gamma_{\pi\varepsilon}^m. \quad (41)$$

Similarly, using (36) and (38)-(40) in (37) and applying  $E_t \widehat{\varepsilon}_{t+2,t+1}^m = E_t \widehat{\varepsilon}_{t+1}^m = 0$  gives

$$\begin{aligned} & (1 + \delta_2 + \delta_3)\gamma_{\pi b}\widehat{b}_{t-1} + (1 + \delta_2 + \delta_3)\gamma_{\pi\varepsilon}\widehat{\varepsilon}_{t+1,t}^m + (1 + \delta_2 + \delta_3)\gamma'_{\pi\varepsilon}\widehat{\varepsilon}_{t,t-1}^m + (1 + \delta_2 + \delta_3)\gamma_{\pi\varepsilon}^m\widehat{\varepsilon}_t^m \\ &= \delta_1\gamma_{\pi b}\gamma_b\widehat{b}_{t-1} + (\delta_2 + \delta_3)\widehat{b}_{t-1} + \delta_1\gamma_{\pi b}\gamma_{b\varepsilon}\widehat{\varepsilon}_{t+1,t}^m + \delta_1\gamma'_{\pi\varepsilon}\widehat{\varepsilon}_{t+1,t}^m - \delta_3\widehat{\varepsilon}_{t+1,t}^m + \delta_1\gamma_{\pi b}\gamma_{b\varepsilon}^m\widehat{\varepsilon}_t^m - \delta_2\widehat{\varepsilon}_t^m \\ & \quad + \delta_1\gamma_{\pi b}\gamma'_{b\varepsilon}\widehat{\varepsilon}_{t,t-1}^m - \delta_2\widehat{\varepsilon}_{t,t-1}^m. \end{aligned}$$

Here, matching coefficients yields the following conditions (for  $\widehat{b}_{t-1}^m \neq 0$ ,  $\widehat{\varepsilon}_{t+1,t}^m \neq 0$ ,  $\widehat{\varepsilon}_{t,t-1}^m \neq 0$ ,  $\widehat{\varepsilon}_t^m \neq 0$ , respectively):

$$(1 + \delta_2 + \delta_3)\gamma_{\pi b} = \delta_1\gamma_{\pi b}\gamma_b + \delta_2 + \delta_3, \quad (42)$$

$$(1 + \delta_2 + \delta_3)\gamma_{\pi\varepsilon} = \delta_1\gamma_{\pi b}\gamma_{b\varepsilon} + \delta_1\gamma'_{\pi\varepsilon} - \delta_3, \quad (43)$$

$$(1 + \delta_2 + \delta_3)\gamma'_{\pi\varepsilon} = \delta_1\gamma_{\pi b}\gamma'_{b\varepsilon} - \delta_2, \quad \text{and} \quad (44)$$

$$(1 + \delta_2 + \delta_3)\gamma_{\pi\varepsilon}^m = \delta_1\gamma_{\pi b}\gamma_{b\varepsilon}^m - \delta_2. \quad (45)$$

Substituting (41) into (42) gives  $0 = (\delta_1\gamma_b - (1 + \delta_2 + \delta_3))(1 - \gamma_b) + \delta_2 + \delta_3$ , where the right-hand side is the characteristic polynomial considered in Appendix D for the special case of  $\rho_\pi = 0$  with  $\gamma_b \in (0, 1)$  under (24). Further, using (41) in (44) and (45), respectively, shows that  $\gamma'_{\pi\varepsilon} = \gamma_{\pi\varepsilon}^m$ . Solving (44) for  $\gamma'_{\pi\varepsilon}$ , using (41), gives

$$\gamma'_{\pi\varepsilon} = \gamma_{\pi\varepsilon}^m = -\delta_2/\delta_4 < 0$$

where  $\delta_4 = 1 + \delta_2 + \delta_3 + \delta_1(1 - \gamma_b)$  and the sign follows from  $\delta_1 > -\delta_3$  and  $\gamma_b \in (0, 1)$  which imply that the denominator is positive. Further,  $\delta_1(1 - \gamma_b) > -\delta_3$  implies that

$\gamma_{\pi\varepsilon}^m, \gamma'_{\pi\varepsilon} \in (-1, 0)$ . Solving (43) for  $\gamma_{\pi\varepsilon}$ , using (41), gives

$$\gamma_{\pi\varepsilon} = (\delta_1 \gamma'_{\pi\varepsilon} - \delta_3) / \delta_4 < 0,$$

where the sign follows from  $\gamma'_{\pi\varepsilon} < 0$  and the positive sign of denominator shown above.

For the responses of consumption, we substitute (36), (38), and (40), into (35) which yields  $\gamma_{cb} \widehat{b}_{t-1} + \gamma_{c\varepsilon} \widehat{\varepsilon}_{t+1,t}^m + \gamma'_{c\varepsilon} \widehat{\varepsilon}_{t,t-1}^m + \gamma_{c\varepsilon}^m \widehat{\varepsilon}_t^m = \widehat{b}_{t-1} - \gamma_{\pi b} \widehat{b}_{t-1} - \gamma_{\pi\varepsilon} \widehat{\varepsilon}_{t+1,t}^m - \gamma'_{\pi\varepsilon} \widehat{\varepsilon}_{t,t-1}^m - \gamma_{\pi\varepsilon}^m \widehat{\varepsilon}_t^m - \widehat{\varepsilon}_{t,t-1}^m - \widehat{\varepsilon}_t^m$ . Matching coefficients gives

$$\gamma_{cb} = 1 - \gamma_{\pi b} = \gamma_b \in (0, 1), \gamma_{c\varepsilon} = -\gamma_{\pi\varepsilon} > 0, \gamma'_{c\varepsilon} = -\gamma'_{\pi\varepsilon} - 1 < 0, \gamma_{c\varepsilon}^m = -\gamma_{\pi\varepsilon}^m - 1 < 0,$$

as well as  $\gamma_{c\varepsilon}^m = \gamma'_{c\varepsilon}$  which follows from the last two conditions along with  $\gamma'_{\pi\varepsilon} = \gamma_{\pi\varepsilon}^m$ .

Finally, we turn to the liquidity value of government bonds  $v_t = \eta_t / u_{c,t}$  (see 13) which is given by  $\widehat{v}_t = \sigma E_t \widehat{c}_{t+1} - \sigma \widehat{c}_t + E_t \widehat{\pi}_{t+1} - \widehat{R}_t^m$ . Substituting in (36), (38), and (40), using the solution coefficients for consumption, gives

$$\begin{aligned} \widehat{v}_t = & ((\sigma - 1) \cdot (\gamma_b - 1) \cdot \gamma_b) \cdot \widehat{b}_{t-1} + ((\sigma - 1) (1 - \gamma_b) \gamma_{\pi\varepsilon} - \sigma (1 + \gamma'_{\pi\varepsilon}) + \gamma'_{\pi\varepsilon}) \cdot \widehat{\varepsilon}_{t+1,t}^m \\ & + \left( (\sigma - 1) \cdot (1 - \gamma_b) \cdot \gamma'_{\pi\varepsilon} - 1 \right) \cdot \widehat{\varepsilon}_{t,t-1}^m + ((\sigma - 1) \cdot (1 - \gamma_b) \cdot \gamma_{\pi\varepsilon}^m - 1) \cdot \widehat{\varepsilon}_t^m. \end{aligned}$$

The signs of the marginal derivatives with respect to the shocks are as follows. First,

$$\begin{aligned} \partial \widehat{v}_t / \partial \widehat{\varepsilon}_{t+1,t}^m &= (\sigma - 1) (1 - \gamma_b) \gamma_{\pi\varepsilon} - \sigma (1 + \gamma'_{\pi\varepsilon}) + \gamma'_{\pi\varepsilon} < 0, \\ \partial \widehat{v}_t / \partial \widehat{\varepsilon}_t^m &= (\sigma - 1) \cdot (1 - \gamma_b) \cdot \gamma_{\pi\varepsilon}^m - 1 < 0, \end{aligned}$$

since  $\sigma \geq 1$ ,  $\gamma_b \in (0, 1)$ ,  $\gamma_{\pi\varepsilon} < 0$ ,  $\gamma'_{\pi\varepsilon} \in (-1, 0)$  and  $\gamma_{\pi\varepsilon}^m < 0$ . For completeness,  $\partial \widehat{v}_t / \partial \widehat{\varepsilon}_{t,t-1}^m = ((\sigma - 1) \cdot (\gamma_b - 1) \cdot \gamma_b) \cdot \gamma_{b\varepsilon} + (\sigma - 1) \cdot (1 - \gamma_b) \cdot \gamma'_{\pi\varepsilon} - 1 < 0$ , since  $\gamma_{b\varepsilon} = -\gamma_{\pi\varepsilon} > 0$ . Hence, the liquidity value of government bonds increases in response to negative realizations of the monetary policy innovations.

The effects of  $\widehat{\varepsilon}_t^m$  and  $\widehat{\varepsilon}_{t,t-1}^m$  in isolation imply that jointly considering  $\widehat{\varepsilon}_t^m = \widehat{\varepsilon}_{t,t-1}^m < 0$  has unambiguous effects on inflation  $\widehat{\pi}_t$  and the liquidity value  $\widehat{v}_t$  since both,  $\widehat{\pi}_t$  and  $\widehat{v}_t$ , increase in both,  $\widehat{\varepsilon}_t^m$  and  $\widehat{\varepsilon}_{t,t-1}^m$ . The two shocks have, however, counteracting effects on current consumption,  $\widehat{c}_t$ . The total effect is

$$\partial \widehat{c}_t / \partial \widehat{\varepsilon}_t^m + \partial \widehat{c}_t / \partial \widehat{\varepsilon}_{t+1,t}^m = -\gamma_{\pi\varepsilon}^m - 1 - \gamma_{\pi\varepsilon}.$$

Using the results for  $\gamma_{\pi\varepsilon}^m$  and  $\gamma_{\pi\varepsilon}$  above, we can express the total effect as  $((\delta_2 + \delta_3 + 1 + \delta_1 (1 - \gamma_b)) (\delta_2 + \delta_3 + \delta_1 \delta_2 / (1 + \delta_2 + \delta_3 + \delta_1 (1 - \gamma_b)))) / \delta_4 - 1$  which is negative if

$$\delta_1 \delta_2 / (1 + \delta_2 + \delta_3 + \delta_1 (1 - \gamma_b)) < 1 + \delta_1 (1 - \gamma_b). \quad (46)$$

If  $\sigma < \beta/\chi$ ,  $\delta_1 > \chi$  and hence positive. It follows that the right-hand side of (46) is larger than one. Concerning, the left-hand-side, we can state

$$\frac{\delta_1\delta_2}{1 + \delta_2 + \delta_3 + \delta_1\gamma_{\pi b}} < \frac{\delta_1\delta_2}{1 + \delta_2 + \delta_3} = \frac{\chi^2\sigma\sigma_n}{(1 + \beta + \chi + \chi\sigma_n)} < \frac{\chi\beta\sigma_n}{(1 + \beta + \chi + \chi\sigma_n)} < 1$$

which uses  $\sigma < \beta/\chi$  and  $\beta < 1$ . Hence, (46) is fulfilled and the total effect of  $\widehat{\varepsilon}_t^m = \widehat{\varepsilon}_{t,t-1}^m < 0$  on  $\widehat{c}_t$  negative if, but not only if,  $\sigma < \beta/\chi$ .

## F Calculation of Anticipated Monetary Policy Shocks

In this appendix, we describe how to calculate the sequence of current and anticipated policy shocks  $\boldsymbol{\varepsilon}_{T+1}^m = \{\varepsilon_{T+1}^m, \{\varepsilon_{T+1+k,T+1}^m\}_{k=1}^K\}$  of length  $H = K + 1$ , which are associated with the announced interest rate path  $\{R_{T+h}^m\}_{h=1}^H$  that we want to study. We solve our model using standard perturbation techniques, yielding policy functions of the type<sup>22</sup>

$$R_T^m - R^m = \gamma_{R_s}(\mathbf{s}_T - \mathbf{s}) + \gamma_{R\varepsilon}\boldsymbol{\varepsilon}_{T+1}^m \quad (47)$$

where  $\gamma_{R_s}$  and  $\gamma_{R\varepsilon}$  are vectors of coefficients that describe how  $R^m$  depends on state variables and shocks, respectively,  $\mathbf{s}_T$  is the vector of state variables, and  $\mathbf{s}$  the vector of their state-state values. The vector of policy functions for state variables  $\mathbf{s}_T$  is

$$\mathbf{s}_{T+1} - \mathbf{s} = \Gamma_{ss} \cdot (\mathbf{s}_T - \mathbf{s}) + \Gamma_{s\varepsilon}\boldsymbol{\varepsilon}_{T+1}^m, \quad (48)$$

with the coefficient matrices  $\Gamma_{ss}$  and  $\Gamma_{s\varepsilon}$ . Using (47) and (48) and assuming that only  $\boldsymbol{\varepsilon}_{T+1}^m$  has non-zero entries, whereas  $\boldsymbol{\varepsilon}_t^m$  for all  $t \neq T + 1$  has zero entries, allows us to write solutions for the policy rate for  $H$  periods ahead that depend on the values of the state variables (in period  $T$ ) and the policy shocks that are announced in  $T + 1$  only:

$$R_{T+1}^m - R^m = \Gamma_1 \cdot (\mathbf{s}_T - \mathbf{s}) + \Gamma_2\boldsymbol{\varepsilon}_{T+1}^m,$$

where  $\mathbf{R}_{T+1}^m = [R_{T+1}^m, R_{T+2}^m, R_{T+3}^m, \dots, R_{T+H}^m]'$ ,  $\Gamma_1 = [\gamma_{R_s}, \gamma_{R_s}\Gamma_{ss}, \gamma_{R_s}\Gamma_{ss}^2, \dots, \gamma_{R_s}\Gamma_{ss}^{H-1}]'$ , and  $\Gamma_2 = [\gamma_{R\varepsilon}, \gamma_{R_s}\Gamma_{s\varepsilon}, \gamma_{R_s}\Gamma_{ss}\Gamma_{s\varepsilon}, \dots, \gamma_{R_s}\Gamma_{ss}^{H-2}\Gamma_{s\varepsilon}]'$ . This constitutes a system of  $H$  linear equations in  $H$  unknown elements of  $\boldsymbol{\varepsilon}_{T+1}^m$  for a given sequence  $\{R_{T+h}^m\}_{h=1}^H$  and a current state  $\mathbf{s}_T$ . The  $H$  shocks can be backed out as

$$\boldsymbol{\varepsilon}_{T+1}^m = \Gamma_2^{-1} \cdot (\mathbf{R}_{T+1}^m - R^m - \Gamma_1 \cdot (\mathbf{s}_T - \mathbf{s})).$$

**Additional material: [click here](#)**

<sup>22</sup>The procedure can obviously be applied to any other endogenous variable also, e.g., the real policy rate.