Boring Banks and Taxes*

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Abstract

How do taxes in the financial sector affect economic outcomes? We analyze a simple general equilibrium model with financial intermediation. We formalize a trade-off between tax policies that burden the owners of banks and tax policies that burden households. We also study the implications of the financial sector’s exemption from value added taxation (VAT). Main results are that an increased taxation of the banks’ profits goes together with a larger financial sector, as measured by the volume of loans and the employment in banking. We also show that the general presumption that the VAT-exemption is beneficial for banks is unjustified.

Keywords: Taxation of the financial sector, Financial activities tax, Value added taxation

JEL Classification: G21, H21, H22

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Thirty-plus years ago, when I was a graduate student in economics, only the least ambitious of my classmates sought careers in the financial world. Even then, investment banks paid more than teaching or public service — but not that much more, and anyway, everyone knew that banking was, well, boring. Paul Krugman, April 2009

1 Introduction

Since the 2007/2008 financial crises policy-makers have repeatedly articulated a need for an increased taxation of the financial sector with the objective that the sector makes a “fair and substantial contribution” to meeting the costs associated with government interventions to repair it”, International Monetary Fund (2010). Various tax policies have since been discussed.2 The European Union seeks to introduce a financial transactions tax, the IMF instead has advocated the use of a financial activities tax, i.e. a tax on all wage and profit incomes that are generated in the financial sector. Another suggestion is to make an end to the financial sector’s exemption from value added taxation (VAT).3 As financial intermediation accounts for a significant fraction of GDP,4 the introduction of a VAT on financial services appears promising in terms of its potential to generate tax revenues.5

It is difficult to relate classical results in the theory of taxation to this policy debate. The workhorse models – such as, for instance, the Ramsey (1927)-model for an analysis of taxes in an economy with many sectors or the Mirrlees (1971)-model for an analysis of non-linear income taxes – do not include a financial sector. As Keen (2011) points out, “one striking aspect of these policy developments and of the wider debate . . . is that they have been almost entirely unguided by the public finance literature on the topic – because there is hardly any.”6 This paper is a first attempt to use standard tools from public finance to analyze in a systematic way some of the tax policies that have been proposed in the debate about an increased taxation of the financial sector. Specifically, we

2 Keen (2011) provides a comprehensive lists of new taxes on financial institutions.
3 In the European Union, the VAT treatment of financial services has been on the policy agenda since the mid-1990s, see e.g. Huizinga (2002), Poddar and English (1997), or de la Feria and Lockwood (2012).
4 For data on financial sector size, see, e.g., Huizinga (2002) or Lockwood (2014). They report values in the range of 3% to 8% of GDP for a range of developed countries. For figures on fiscal exposures and debt levels, see International Montary Fund (2010).
5 Yet, most of the more than 120 countries in the world that have a value added tax fully or partially exempt the financial sector, see Zee (2005). A traditional explanation has been that a taxation of valued added in the financial sector is difficult, as it is hard for authorities to distinguish between risk premia and true profit. Only the latter correspond to value added and thus should be taxed. Various authors have discussed whether it is possible to overcome this difficulty, see the discussion by Huizinga (2002). However, the literature lacks a compelling answer to the the deeper question: Should the financial sector be exempt from VAT?
6 There is, however, a larger body of applied work on the topic. See, e.g., Honohan and Yoder (2010).
incorporate a profitable financial sector into an otherwise very conventional multi-sector model of taxation.

While the financial crisis has reinforced academic interest in excessive risk-taking, moral hazard, and other market failures associated with the modern financial system, we will instead look at boring banks, i.e., a banking sector that does not suffer from market failure. This allows us to separate the analysis of corrective instruments from the efficiency and equity concerns that arise in a model where policy-makers face a trade-off between the well-being of the owners of banks and the well-being of other economic agents. This paper is concerned with the latter and complements the rich and important literature on corrective interventions.\footnote{Corrective taxation is, for instance, discussed in Bianchi and Mendoza (2010), Jeanne and Korinek (2010) and Acharya e.a. (2010).} Specifically, we introduce financial intermediation into a model in which households and firms exchange labor and consumption goods. Financial intermediaries, henceforth simply banks, borrow money from households and lend money to firms. Some firms will go bankrupt and default on their debt so that banks need to charge a risk premium. We assume, however, that, by a law of large numbers, banks can predict how many firms will fail. We also impose an assumption of decreasing returns to scale in the banking sector. This assumption implies that, in a competitive equilibrium, there are positive profits in banking. Hence, bankers in our model can safely follow what is known as the 3-6-3 rule: pay 3 percent interest on deposits, lend money at 6 percent and tee off at the golf course by 3 p.m.\footnote{For a discussion of the “3-6-3 rule of banking”, see Walter (2006).}

We then introduce a government into this environment. The government has an exogenous revenue requirement and various tax instruments at its disposal. We consider taxes on household income and the profits of banks. We also discuss a financial activities tax. Finally, we discuss what a uniform VAT system that includes the financial sector would look like and analyze whether such a system can be replicated by a financial activities tax, as proposed by Keen, Krelove and Norregaard (2010) in the report by the International Monetary Fund (2010).

This, admittedly, very simple setup is useful for a variety of reasons. First, since banks in our model are profitable, policy makers face a choice between a taxation of household income and a taxation of the banks’ profits. We can therefore analyze how equilibrium outcomes change if the households are made better off and the owners of banks are made worse off. Surprisingly, we find that such a tax reform would increase the size of the financial sector if measured by the loan volume or the employment in the banking sector. At the same time, there would be less money in the hands of the banks’ owners.

Second, since there is no aggregate risk, we can easily distinguish between a banks’ true profits and the risk premia that would be required to ensure that a bank breaks even. We can therefore discuss the relation between a tax on profits and a tax on valued added in the financial sector. The typical justification of the financial sector’s VAT exemption
is based on the following hypothesis: If it was possible to distinguish risk premia, which cover costs related to the default of borrower, and true profits, then one could tax the latter, and this would then be equivalent to a tax on value added. We find that this is not the case. If banking is profitable, then a tax on profits will obviously generate positive tax revenue. At the same time, moving to a uniform VAT system that includes the financial sector does not generate any additional tax revenue if those who demand financial services can deduct their VAT payments.

Third, we analyze a financial activities tax (FAT). Such a tax is in fact a combination of a tax on the banks’ profits and a tax on the banks’ wage bill. We show that a tax system that includes such a tax is dominated by a simple tax system that has only a tax on labor income and a tax on the banks’ profits. The simple tax system can generate more tax revenue without making households or bank-owners worse off.

Fourth, our setup is deliberately chosen in such a way that it can be easily related to the classical contributions to the theory of optimal taxation in multi-sector models by Ramsey (1927) and Diamond and Mirrlees (1971). Our contribution is an explicit introduction of financial intermediation into this setup. Moreover, we do it in such a way that the intermediaries can reap economic rents. We therefore view it as a strength of our approach that we stay close to the basic framework by Ramsey (1927) and Diamond and Mirrlees (1971). This makes it possible to relate classical insights from the theory of optimal taxation to the current policy debate about taxes on the financial sector.

Overall, we provide insights for a simple and, well, quite boring banking sector. This reduces the scope of our analysis because the current debate on bank taxes is to a large extent driven by the desire to correct market failures. Indeed, a thorough understanding of taxation in a banking sector including all the frictions that caused so much trouble is needed, eventually. Yet, it is a key first step to figure out the basic mechanics of taxation in the financial sector. Those are best identified in a basis setup like ours. Moreover, there are still banks operating a rather boring business model and – according to commentators – this should be the blueprint for the whole industry.9

Related Literature. There is a controversial literature on the desirability of various taxes on financial sector activities.10 This literature does not treat the price of financial services.

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10Auerbach and Gordon (2002) decompose the price of a consumption good into the cost of producing the good and the transactions cost of acquiring it, e.g., administering a credit card, maintaining ATM machines or transferring funds over time. They find that a value added tax system including these transaction costs is equivalent to a tax on labor income and conclude that financial services should be taxed under an ideal value added tax system. Grubert and Mackie (2000) obtain a different conclusion. Based on a view that financial services “provide the funds used to purchase fully taxable consumption goods” they find that “[as] non-consumption goods, such financial services should not be in the base of a consumption tax.” See Chia and Whalley (1999) for a similar point of view. Boadway and Keen (2003)
services as an endogenously determined object that could respond to taxation. Instead
the focus is on how various tax schemes affect the price-ratios faced by households. It
is thereby implicitly assumed that before-tax fees charged by financial service providers
are exogenously fixed and independent of the tax policy. This approach could possibly
be justified if the banking sector was characterized by constant returns to scale and zero
or normal profits. Empirically, however, in the last decades, enormous economic rents
have been realized in the financial sector. Therefore, a model that is based on a constant
returns to scale assumption does not seem to be the most relevant one.

Like us, Lockwood (2014) analyzes taxes on financial sector activities in a general
equilibrium model that departs from constant returns to scale assumptions. Specifically,
he studies taxes on payment services in a dynamic Ramsey model. These payment services
enable households to save time. Lockwood discusses to what extent this generates a
virtual rent for households and whether it would be desirable to use tax policy to extract
some of this rent.

Overall, the literature on the tax treatment of financial services is rather small. Possibly
this is a reflection of the production efficiency result by Diamond and Mirrlees (1971)
which suggests that intermediate goods or services should remain untaxed. This conclu-
sion, however, rests on the assumption that the intermediation service is produced with
a constant returns to scale technology. That said, in a world in which banking is char-
acterized by constant returns to scale and normal profits, the analysis of Diamond and
Mirrlees (1971) suggests that an ideal value added tax system should include financial
services. In practice, the value added tax is typically collected using an invoice-credit
method. A universal VAT is then equivalent to a sales tax that applies to final consumers
only. If the VAT exempts some businesses, though, the chain of invoice-credits is broken
and not all taxes are passed on to final consumers. Therefore, when a general VAT is
in place, productive efficiency requires the inclusion of financial transactions rather than
their exemption.

Outlook. The remainder is organized as follows. The subsequent section introduces
our model and characterizes competitive equilibrium allocations that arise with a simple
tax system that consists only of a tax on labor income and a tax on the profits in banking.
Section 3 characterizes simple tax systems that are Pareto-efficient. The analysis of a
financial activities tax is in Section 4, and Section 5 looks at an ideal value added tax
system that does not exempt the financial sector. The last section contains concluding
remarks. All proofs and some supplementary material are relegated to the Appendix.

reject this reasoning. They argue that it is a fallacy to believe that goods yielding no direct utility should
automatically go untaxed. They argue, though, that financial services payed for by means of a spread
should go untaxed, while fixed fees should be taxed. Jack (2000) obtains a similar result.

11For a clarification of this point, see Mirrlees (1972).
2 The Model

We develop a model with the following features: There is a household sector that receives labor income and uses this income to buy a final consumption good, henceforth also referred to as food. In addition, there is a production sector that uses labor as an input and produces food. We introduce a need for financial intermediation in the following way: Workers have to be compensated by their employers after having been hired, but before the revenue from selling the firm’s output has been generated. Thus, a firm that wants to hire workers needs a financial intermediary that is willing to provide financing for the wage bill. The intermediary is compensated by a promise that there will be a repayment after the firm’s output has been sold. Households in turn receive a wage payment before the market for the final consumption good opens. We assume, for simplicity, that households cannot store this money but have to deposit it with a financial intermediary.

We will occasionally use the term money in what follows, e.g. firms can pay out wages only if they have a sufficient amount of money available. However, money is a pure accounting unit that is neutral in the sense that a change of relative prices leaves equilibrium allocations unaffected. We could as well formulate the model so that there is an exchange of contingent claims on food rather than an exchange of money. Our choice of terminology has the advantage of being closer to the one that is used in the real-world.

For clarity of exposition, we distinguish two points in time. At $t = 0$ households supply labor and firms hire labor. Moreover, households bring their wage income to a financial intermediary, and firms obtain financing from the financial intermediary. At $t = 1$ households withdraw their deposits and buy food. Some firms fail and go bankrupt. The other ones sell food and use the proceeds to repay their loans. If there are profits in banking, then the bankers spend their profit income also on food.

In the following we describe the optimization problems of a household, a firm and a financial intermediary. To keep the analysis simple, we assume initially that only two tax instruments are available, namely a tax on the banks’ profits and a tax on the households’ labor income. We will later discuss a richer set of tax instruments so as to be able to relate our analysis to current proposals for a reform of the taxation of the financial sector.

**Households.** There is a continuum of measure one of identical households. Households consume a final consumption good, henceforth referred to as food, and supply labor. The utility of the representative household is given by $u(c_h, y_h)$, where $c_h$ denotes the household’s food consumption, and $y_h$ is labor supply. The utility function $u$ is increasing in $c_h$ and decreasing in $y_h$. The household faces the following constraints: At time $t = 0$, it receives a net wage income of $p_w(1 - \tau_w)y_h$, where $p_w$ is the gross wage and $\tau_w$ is the tax on labor income. Until $t = 1$, the net wage income is kept in a deposit account offered by financial intermediaries. Hence,

$$p_w(1 - \tau_w)y_h \geq d_h ,$$

(1)
where \( d_h \) is the volume of deposits acquired by the household. At \( t = 1 \), the household faces the following constraint

\[
p_c c_h \leq p_d d_h, \tag{2}
\]

where the left hand side gives the household’s spending on food, and \( p_c \) is the price of one unit of food. The right hand side gives the amount that can be spent on food: \( p_d \) is the amount of money that is paid out per deposit in \( t = 1 \), thus a household with deposits \( d_h \) can spend an amount \( p_d d_h \) on food and thereby obtain \( \frac{p_c}{p_d} d_h \) units of food.

The household’s problem is to choose \( y_h, d_h \) and \( c_h \) so as to maximize \( u(c_h, y_h) \) subject to the constraints in (1) and (2). From the monotonicity of the household’s preferences it follows that both constraints will hold as equalities which implies that

\[
c_h = \frac{p_d}{p_c} p_w (1 - \tau_w) y_h. \tag{3}
\]

Thus, the household chooses \( c_h \) and \( y_h \) so as to maximize \( u(c_h, y_h) \) subject to (3). Note that the household’s choice of consumption and labor supply depends only on \( q_h := \frac{p_d}{p_c} p_w (1 - \tau_w) \).

We can think of \( q_h \) as an effective net wage for the household: If the household supplies one more unit of labor it can increase its food consumption by \( q_h \). Optimal choices will henceforth be denoted by \( c_h^*(q_h) \) and \( y_h^*(q_h) \), respectively. The household’s indirect utility is denoted by

\[
V(q_h) := u(c_h^*(q_h), y_h^*(q_h)).
\]

Once \( c_h^* \) and \( y_h^* \) are determined, we can solve for the optimal deposit demand by using that both (1) and (2) hold as equalities. This yields

\[
d_h^* \left( \frac{p_c}{p_d}, q_h \right) = \frac{p_c}{p_d} c_h^*(q_h). \tag{4}
\]

**Food producers.** There is a continuum of measure one of identical food producers. Food producers hire labor in \( t = 0 \) to produce food that becomes available in \( t = 1 \). We denote a firm’s labor demand by \( y_f \). Wages have to be paid in \( t = 0 \), i.e. before revenues from sales are realized. To finance the wage bill, a firm has to acquire a loan. Hence it faces the constraint \( l_f \geq p_w y_f \), where \( l_f \) denotes the loan volume. In exchange, food producers promise to repay \( p_l l_f \), where \( p_l \) corresponds to 1 plus the interest rate on loans.

For any one firm, production is successful with probability \( \alpha \). This probability is exogenous from the perspective of food producers. It will be endogenously determined in equilibrium and depend on the volume of credit that is generated by the economy’s banking system.

Successful food producers transform labor into food according to a constant returns to scale technology which yields one unit of food per unit of labor. With probability
1 − α a food producer fails, with the implications that output is zero and that the food producer has to default on its loans. Food producers maximize the expected net cash flow as of \( t = 1 \). Thus, the maximization problem of a food producer is: choose \( y_f \) and \( l_f \) so as to maximize

\[
\alpha \left( p_c y_f - p_l l_f \right) \quad \text{s.t.} \quad l_f \geq p_w y_f .
\] (5)

In the following, we will occasionally appeal to a law of large numbers and interpret \( \alpha \) not only as the probability of success of an individual firm, but also as the fraction of firms that are successful. Industry output is therefore non-random and equal to \( \alpha y_f \).

Optimal behavior of food producers depends on

\[
q_f := \frac{p_c}{p_w} \frac{p_l}{p_t}
\]

and is henceforth denoted by \( l_f^*(q_f) \) and \( y_f^*(q_f) \), respectively. Upon writing a food producer’s objective as

\[
\alpha p_w p_l (q_f - 1) y_f
\]

we can think of \( q_f \) as a measure of the firm’s revenue per unit of food that is sold, and of 1 as being the corresponding measure of marginal costs. Consequently, \( l_f^*(\cdot) \) and \( y_f^*(\cdot) \) become unbounded for \( q_f > 1 \), and are are zero for \( q_f < 1 \). Thus, in a competitive equilibrium with positive food production it has to be the case that \( q_f = 1 \) and that the food producer’s profits are zero. In such an equilibrium, the firms labor demand is indeterminate: any \( y_f \in \mathbb{R}_+ \) is part of a solution to the food producer’s problem. Given a labor demand of \( y_f \), the loan demand follows from the fact that the constraint on cash-flows in \( t = 0 \) binds so that

\[
l_f = p_w y_f .
\]

**Banks.** There is a continuum of measure one of identical banks. In period 0 they obtain funds from households via deposits and grant loans to food producers. We denote by \( d_b \) and \( l_b \), respectively, the volumes of deposits and loans of an individual bank.

A bank transforms loans, i.e. promises to repay by firms, into money that really becomes available in the hands of the bank. We denote by \( \rho(l_b) \) the probability that a single loan is repaid. However, a bank grants many loans. We once more appeal to the law of large numbers and interpret \( \rho(l_b) \) also as the fraction of non-defaulting loans for a bank with loan volume \( l_b \). As we explain in more detail below, we allow for the possibility that the fraction that is repaid depends on the loan volume. Banks maximize profits or equivalently their net cash flow in \( t = 1 \) which is given by

\[
(\rho(l_b) p_l l_b - p_d d_b)(1 - \tau_b) ,
\] (6)

where \( \tau_b \) is a tax on banking profits. In maximizing this expression, a bank faces two constraints. First, banks need to hire workers who handle the deposits and loans. We
assume that an additional loan requires \( \kappa_l \) additional units of labor input and that an additional deposit requires \( \kappa_d \) units. Thus, a bank with \( d_b \) deposits and \( l_b \) loans, also has to have a labor force \( y_b \) that satisfies

\[
y_b \geq \kappa_d d_b + \kappa_l l_b .
\]  

(7)

Second, the money that can be lent to firms in \( t = 0 \) is bounded from above by the money that comes in via deposits. Moreover, some of the money that comes in is used in order to pay the wages of the bank’s workers. Hence,

\[
d_b \geq l_b + p_w y_b .
\]  

(8)

Finally, we assume that the profit income is spent on food so that the bank owners’ food demand is given by

\[
c_b = \frac{(\rho(l_b) p_l l_b - p_d d_b)(1 - \tau_b)}{p_c} .
\]

(9)

To sum up, the problem of a bank is to choose \( d_b, l_b, \) and \( y_b \) in order to maximize the expression in (6) subject to the constraints in (7) and (8). Obviously, the constraints will both bind. We can use this observation to express \( d_b, y_b \) and \( c_b \) as functions of \( l_b \). We can also reformulate the bank’s objective as a function of \( l_b \), only. This yields the following observations: Loans will be chosen so as to maximize

\[
\rho(l_b) l_b - \frac{p_d}{p_l} 1 + p_w \frac{\kappa_l}{\kappa_d} l_b .
\]  

(9)

A bank’s optimal loan supply is a function of its effective costs of generating an additional loan

\[
q_b := \frac{p_d}{p_l} 1 + p_w \frac{\kappa_l}{\kappa_d} .
\]

We denote the bank’s profit-maximizing loan supply henceforth by \( l^*_b(q_b) \). The optimal labor demand is then given by

\[
y^*_b(p_w, q_b) := \frac{\kappa_d + \kappa_l}{1 - p_w \kappa_d} l^*_b(q_b) .
\]  

(10)

The profit-maximizing volume of deposits is given by

\[
d^*_b \left( \frac{p_l}{p_d}, q_b \right) := \frac{p_l}{p_d} q_b l^*_b(q_b) .
\]  

(11)

Finally, the bank’s food demand is given by

\[
c^*_b \left( \frac{p_l}{p_c}, \frac{p_d}{p_c}, q_b \right) = \frac{\rho(l^*_b(q_b)) p_l l^*_b(q_b) - p_d d^*_b(q_b)}{p_c}(1 - \tau_b) .
\]  

(12)
Boring Banking and Very Boring Banking. We analyze the model under two alternative assumptions that are, respectively, referred to as Boring Banking and Very Boring Banking. They differ in the assumptions made on the function \( R(l_b) := \rho(l_b)l_b \). Under Very Boring Banking, financial intermediation exhibits constant returns to scale and the financial sector equilibrium is characterized by a zero-profit condition. Banks cannot bear any tax burden but pass on all taxes to consumers. In this sense, banking is very boring. Under Boring as opposed to Very Boring Banking, banks will earn positive profits in equilibrium. An answer to the question who bears the burden of taxation therefore requires an analytical effort.

Assumption 1 (Very Boring Banking) The function \( R(l_b) = \rho(l_b)l_b \) exhibits constant returns to scale with \( \rho(l_b) = \alpha \), for all \( l_b \).

Under the Assumption of Very Boring Banking, \( \alpha \) is an exogenous parameter equal to the success probability of a food producer. Credit supply of banks is positive only if \( \alpha \geq \frac{p_d}{p_t} \frac{1+p_w \kappa_d}{1-p_w \kappa_d} \) and becomes unbounded if this inequality is strict. Thus, an equilibrium with positive credit supply exists if and only if

\[
\alpha = \frac{p_d}{p_t} \frac{1+p_w \kappa_d}{1-p_w \kappa_d}.
\]

Assumption 2 (Boring Banking)

a) The function \( R(l_b) = \rho(l_b)l_b \) is twice continuously differentiable, increasing, and strictly concave.

b) The functions \( R \) and \( \rho \) satisfy, for all \( l_b \), \( \rho(l_b) \in [0,1] \), \( \rho'(l_b) < 0 \), and \( \left| \frac{R'(l_b)}{l_b R''(l_b)} \right| > 1 \).

Part a) of the assumption implies that banking comes with decreasing returns to scale and positive profits in a competitive equilibrium. The assumption of decreasing returns is meant to capture that the difficulty of monitoring the quality of loans increases with the number of loans. To keep the analysis as simple as possible we refrain, however, from modelling the underlying economic mechanism explicitly.\(^{12}\)

Part b) contains technical assumptions which simplify the solution of the bank’s optimization problem and which help to ensure existence and uniqueness of equilibria.\(^{13}\)

Specifically, under Assumption 2, optimal loan supply \( l_b^*(\cdot) \) is implicitly defined as the solution of the following equation

\[
R'(l_b^*(\cdot)) = \frac{p_d}{p_t} \frac{1+p_w \kappa_d}{1-p_w \kappa_d}.
\]

\(^{12}\)A more microfounded model might look as follows: Due to moral hazard problems, the probability that a food producer is successful depends on the monitoring effort of its bank. With a convex effort cost function, a bank that enlarges the supply of credit chooses to tolerate a worsening of the quality of the average loan and hence a lower success probability.

\(^{13}\)An example of a function \( R \) satisfying Assumption 2 is \( R(l_b) = [\beta_1/(\beta_1 + l_b)]^{\beta_2}l_b \), with \( \beta_1 \in (0,1) \) and \( \beta_2 \in (0,1/2] \).
Moreover, $l'(\cdot)$ is a decreasing function of the cost measure $pl \frac{1 - p_w \kappa_i}{pl - p_w \kappa_d}$. The elasticity of $l'(\cdot)$ with respect to this cost measure is given by

$$\frac{R'(l)}{l R''(l)}$$

An absolute value of this elasticity that exceeds 1 will turn out to be important for the existence of a market clearing price for loans.

**Government consumption and tax revenues.** Government consumption is an exogenously determined number $\gamma$. The government has two means of generating this consumption level. It can spend the tax revenue that is generated in $t = 1$ with a tax on profits on the food market. Alternatively, it can spend the revenue from the taxation of labor income in $t = 0$ on the labor market so as to hire workers that are employed in the public production of food. We assume that the government has access to the same technology as the food production sector: there is a constant returns to scale technology so that one unit of labor can be used to generate $\alpha$ units of food. More formally, the tax revenue from period 0 is given by

$$T(\omega_w, p_w, q_h) := \omega_w p_w y^*(q_h)$$

and is used to hire workers. Hence,

$$y^*(\omega_w, q_h) = \frac{T(\omega_w, p_w, q_h)}{p_w} = \omega_w y^*(q_h)$$

is public labor demand. Tax revenue from period 1 equals

$$T_1(\omega_b, p_l, p_d, q_b) := \omega_b \left( \rho(l^*(q_b)) \frac{p_l}{p_d} l^*(q_b) - p_d d^*(\frac{p_l}{p_d}, q_b) \right),$$

and generates a public demand of food that is given by

$$e^*_g \left( \omega_b, \frac{p_l}{p_c}, \frac{p_d}{p_c}, q_b \right) = \frac{T_1(\omega_b, p_l, p_d, q_b)}{p_c}.$$  

To meet government’s consumption needs it has to be the case that

$$e^*_g \left( \omega_b, \frac{p_l}{p_c}, \frac{p_d}{p_c}, q_b \right) + \alpha y^*(\omega_w, p_w) = \gamma .$$

The set of feasible tax policies consists of the pairs $\tau = (\omega_w, \omega_b)$ for which this government budget constraint holds. As will become clear, there will typically be a whole set of feasible tax policies.
Competitive equilibrium. An allocation is a list \( a = (d, l, y, c) \) in which \( d \) is the volume of deposits, \( l \) is the volume of loans, \( y = (y_b, y_f, y_g, y_h) \) is a vector that contains, respectively, the labor demand by banks, food producers and the government, as well as the household sector’s labor supply. Finally, \( c = (c_b, c_g, c_h) \) gives the food consumption of banks, the government and the household.\(^{14}\)

A price system consists of a vector \( q := (q_b, q_f, q_h) \) which specifies the effective prices, i.e. the prices which govern the economic decisions of banks, food producers, and households. The price system also contains the prices listed in \( p := (p_w, p_c, p_f, p_d) \). It will prove convenient for our characterization of competitive equilibrium allocations to have a separate notation for both price vectors.\(^{15}\)

A competitive equilibrium is a list of prices and allocations so that markets clear and all economic agents behave optimally given the equilibrium prices. A more formal definition that lists all equations that a competitive equilibrium has to satisfy can be found in the Appendix.

The following Proposition, which is proven in the Appendix, provides an equilibrium characterization. It is based on choosing labor as the numeraire good so that \( p_w = 1 \). The Proposition begins with an equation – equation (18) – that characterizes the equilibrium volume of loans. Once this quantity is known, all other equilibrium quantities can be determined. E.g. if we plug the equilibrium volume of loans into equation (19) we obtain the equilibrium volume of deposits. The same approach works with all other endogenous variables. Thus in our equilibrium characterization, everything is a function of the equilibrium volume of loans, with the latter being determined by equation (18).

**Proposition 2.1** Suppose that either Assumption 1 or 2 holds. Let \( p_w = 1 \) and let there be a given tax policy \( \tau \). An allocation \( a = (d, l, y, c) \) and a price system \( (q, p) \) are a competitive equilibrium if and only if they satisfy equations (18) - (30) below: The loan volume \( l \) solves

\[
c^*_h \left( 1 - \tau_w \right) \frac{1 - \kappa_d}{1 + \kappa_l} R'(l) = l R'(l) . \tag{18}
\]

The volume of deposits satisfies

\[
d = \frac{1 + \kappa_l}{1 - \kappa_d} . \tag{19}
\]

Household consumption of food is given by

\[
c_h = c^*_h \left( 1 - \tau_w \right) \frac{1 - \kappa_d}{1 + \kappa_l} R'(l) . \tag{20}
\]

\(^{14}\)We do not include food consumption by food producers because it will be zero for all allocations of interest due to zero profits in food production.

\(^{15}\)In the theory of optimal commodity taxation, in the tradition of Ramsey (1927), it is commonplace to distinguish a vector of consumer prices, typically denoted by \( q \), from a vector of producer prices typically denoted by \( p \).
Labor supply of households is given by
\[ y_h = \frac{1}{1 - \tau_w} \frac{1 + \kappa_l}{1 - \kappa_d R'(l)} c_h^* \left( 1 - \tau_w \right) \frac{1 - \kappa_d}{1 + \kappa_l} R'(l) . \]  
(21)

Labor employed by banks is given by
\[ y_b = \frac{\kappa_d + \kappa_l}{1 - \kappa_d} l . \]  
(22)

Food consumption by banks is given by
\[ c_b = (1 - \tau_b)(\rho(l) - R'(l)) l . \]  
(23)

Government food consumption is given by
\[ c_g = \tau_b(\rho(l) - R'(l)) l . \]  
(24)

Public employment is given by
\[ y_g = \tau_w y_h = \frac{1}{1 - \tau_w} \frac{1 + \kappa_l}{1 - \kappa_d R'(l)} c_h^* \left( 1 - \tau_w \right) \frac{1 - \kappa_d}{1 + \kappa_l} R'(l) . \]  
(25)

Labor hired in food production is given by
\[ y_f = l . \]  
(26)

The success probability of food producers is given by
\[ \alpha = \rho(l) . \]  
(27)

The effective wage for the household is given by
\[ q_h = (1 - \tau_w) \frac{1 - \kappa_d}{1 + \kappa_l} R'(l) . \]  
(28)

The effective cost for banks is given by
\[ q_b = R'(l) . \]  
(29)

Relative prices are such that
\[ \frac{p_l}{p_c} = 1 \quad \text{and} \quad \frac{p_l}{p_d} = \frac{1 + \kappa_l}{1 - \kappa_d R'(l)} . \]  
(30)

The Proposition covers both Boring Banking and Very Boring Banking. If banking is very boring, then there is no profit income that bankers could use to buy food, so that
\[ c_b = (1 - \tau_b)(\rho(l) - R'(l)) l = (1 - \tau_b)(\alpha - \alpha) l = 0 . \]

By contrast, under Boring Banking
\[ c_b = (1 - \tau_b)(\rho(l) - R'(l)) l = (1 - \tau_b)(\alpha - R'(l)) l > 0 . \]
because the average probability that a loan is repaid exceeds the probability for the marginal loan.

The Proposition makes it possible to obtain comparative statics results on how a change of the tax policy affects equilibrium prices and quantities. We will use this intensively in the subsequent analysis. For now, it is noteworthy that, by equation (18), the equilibrium volume of loans is affected by the tax on labor income $\tau_w$, but not by the tax on the banks’ profits $\tau_b$. The tax $\tau_b$ is not distortionary; it only affects the profits in the banking sector, but does not change any individual choices. A change of $\tau_w$, by contrast, affects the equilibrium volume of loans and thereby also all other endogenous variables.

**Existence and uniqueness of competitive equilibria.** To establish the existence of an equilibrium we need to show that there is a value of $l$ that solves equation (18). To establish uniqueness we need to show that this equation has at most one solution.

**Assumption 3** The function $c^*_h$ is a continuous and strictly increasing function of the effective wage rate $q_h$.

According to this assumption, if the effective wage of the household goes up, then his food consumption will increase. This assumption is very weak and is satisfied by all common utility functions. It simply rules out the possibility that the household decreases his labor supply in response to a wage increase so drastically that he can no longer afford his initial consumption level.

Under this assumption the expression on the left-hand-side of equation (18) is a non-increasing function of $l$. Specifically, it is a decreasing function of $l$ under the assumption of Boring Banking and does not depend on $l$ if banking is very boring. The right-hand side of equation (18) is an increasing function of $l$ if banking is boring and if banking is very boring. Thus, there can be at most one solution to equation (18).

For $l = 0$, the consumption level on the left-hand-side of (18) exceeds the right-hand side. Since all relevant functions are assumed to be continuous, a sufficient condition for equilibrium existence therefore is that there is a value of $l$ at which the right-hand-side of (18) exceeds the left-hand-side. We summarize these observations in the following Proposition.

**Proposition 2.2** Suppose that Assumption 3 holds. Also suppose that either Assumption 1 or 2 holds. Finally assume that there exists $l$ so that

$$c^*_h \left( (1 - \tau_w) \frac{1 - \kappa_d}{1 + \kappa_l} R'(l) \right) \leq l R'(l).$$

Then there exists one and only one competitive equilibrium.
3 Pareto-efficient tax systems

A tax system $\tau = (\tau_b, \tau_w)$ is said to be Pareto-efficient if there is no other tax system $\tau' = (\tau'_b, \tau'_w)$ that satisfies the government’s revenue requirement and makes either the households or the bankers better off while making no one worse off.

If banking is very boring and thus yields zero profits, the study of Pareto-efficient tax systems is uninteresting. The only source of revenue is then the income tax. Consequently, there is one and only one Pareto-efficient tax system under which $\tau_w$ is set in such a way that a tax revenue of $\gamma$ is generated. By contrast, if banking is boring, then there are two sources of revenue, the income tax and the tax on the banks’ profits. Consequently, tax policy faces a choice among various Pareto-efficient tax systems.

Before we can turn to a characterization of Pareto-efficient tax systems, we need to clarify how a change in the tax policy affects the equilibrium allocation. We denote by $a^{eq}(\tau_b, \tau_w) = (d^{eq}(\tau_b, \tau_w), l^{eq}(\tau_b, \tau_w), y^{eq}(\tau_b, \tau_w), c^{eq}(\tau_b, \tau_w))$ the equilibrium allocation that is induced by a tax policy $\tau = (\tau_b, \tau_w)$. Analogously, we denote by $q^{eq}(\tau_b, \tau_w)$ and $p^{eq}(\tau_b, \tau_w)$ the corresponding price system. Recall that Proposition 2.1 provides a complete equilibrium characterization. For instance, the equilibrium volume of loans, $l^{eq}(\tau_b, \tau_w)$, is implicitly defined by the equation

$$c^*_h \left( 1 - \tau_w \right) \frac{1 - K_d}{1 + K_l} R'(l^{eq}(\tau_b, \tau_w)) = l^{eq}(\tau_b, \tau_w) R'(l^{eq}(\tau_b, \tau_w)).$$

Using the implicit function theorem, we can now compute the derivatives of $l^{eq}(\tau_b, \tau_w)$ with respect to $\tau_b$ and $\tau_w$. The equilibrium characterization in Proposition 2.1 is such that all equilibrium quantities are expressed as functions $l^{eq}(\tau_b, \tau_w)$. We can thus use the results on

$$\frac{\partial l^{eq}(\tau_b, \tau_w)}{\partial \tau_b} \quad \text{and} \quad \frac{\partial l^{eq}(\tau_b, \tau_w)}{\partial \tau_w}$$

to determine the comparative statics properties of the whole equilibrium allocation. The following proposition results from such an exercise. We omit a proof.

**Proposition 3.1** Suppose that Assumptions 2 and 3 hold.

I. A marginal increase of $\tau_w$ has the following implications: The volume of loans, the supply of food, the number of bankers, household utility and the consumption of bank-owners all go down,

$$\frac{\partial l^{eq}(\tau_b, \tau_w)}{\partial \tau_w} < 0, \quad \frac{\partial y^{eq}(\tau_b, \tau_w)}{\partial \tau_w} < 0, \quad \frac{\partial y^{eq}_b(\tau_b, \tau_w)}{\partial \tau_w} < 0, \quad \frac{\partial V(q^{eq}_h(\tau_b, \tau_w))}{\partial \tau_w} < 0,$$

and

$$\frac{\partial c^{eq}_b(\tau_b, \tau_w)}{\partial \tau_w} < 0.$$
II. A marginal increase of $\tau_b$ has the following implications: The volume of loans, the supply of food, the number of bankers and household utility are unaffected,

$$\frac{\partial l^{eq}(\tau_b, \tau_w)}{\partial \tau_b} = 0, \quad \frac{\partial y_f^{eq}(\tau_b, \tau_w)}{\partial \tau_b} = 0, \quad \frac{\partial y_b^{eq}(\tau_b, \tau_w)}{\partial \tau_b} = 0 \quad \text{and} \quad \frac{\partial V(q_h^{eq}(\tau_b, \tau_w))}{\partial \tau_b} = 0.$$  

The bank-owners’ consumption goes down,

$$\frac{\partial c_b^{eq}(\tau_b, \tau_w)}{\partial \tau_b} < 0.$$  

Proposition 3.1 reflects that the tax on income is distortionary in the sense that it interferes with the choices of banks and households. The tax on profits by contrast is not distortionary. It only affects the fraction of the banks’ profits that go to the treasury. Note that an increase of $\tau_b$ and an increase of $\tau_w$ both reduce the bank’s after-tax-profits.

An implication of Proposition 3.1 is the following: If $\tau^A = (\tau^A_b, \tau^A_w)$ and $\tau^B = (\tau^B_b, \tau^B_w)$ are different Pareto-efficient tax systems with $\tau^A_b > \tau^B_b$, then it has to be the case that $\tau^A_w < \tau^B_w$. If we had $\tau^A_b > \tau^B_b$ and $\tau^A_w \geq \tau^B_w$, then tax system $\tau^B$ would Pareto-dominate tax system $\tau^A$. Proposition 3.1 then also implies that, under the tax system with the lower tax on income and the higher tax on the banks’ profits, households are better off, food production is higher, and more loans are generated. Moreover, if households are better off, then it has to be the case that the bankers are worse-off. If not, the tax system with the low income tax would be Pareto-superior. We summarize these observations in the following Proposition. Again, we omit a formal proof.

**Proposition 3.2** Suppose that Assumptions 2 and 3 hold. Suppose that $\tau^A = (\tau^A_b, \tau^A_w)$ and $\tau^B = (\tau^B_b, \tau^B_w)$ are Pareto-efficient tax systems with $\tau^A_b > \tau^B_b$ and $\tau^A_w < \tau^B_w$. Under tax system $\tau^A$: The volume of loans is larger, there is more food production, there are more bankers, households are better off, but there is less food for the bank-owners:

$$l^{eq}(\tau^A) > l^{eq}(\tau^B), \quad y_f^{eq}(\tau^A) > y_f^{eq}(\tau^B), \quad y_b^{eq}(\tau^A) > y_b^{eq}(\tau^B),$$

and

$$V(q_h^{eq}(\tau^A)) > V(q_h^{eq}(\tau^B)) \quad \text{and} \quad c_b^{eq}(\tau^A) < c_b^{eq}(\tau^B).$$

If we move to a tax system that has a lower income tax and higher taxes on the profits in banking, then the size of the financial sector increases, provided that size is measured by the volume of loans or the employment in the financial sector. The after-tax profits, however, go down. Thus, in the given model, it is possible to increase the tax burden on those who benefit from profits in banking without curtailing real economic activity.

It may appear surprising, at first glance, that the move to a tax system that makes the banks’ owners worse off stimulates economic activity and leads to an enlarged equilibrium loan volume. The reason is that the increase of the tax on the banks’ profits makes it possible to lower the distortionary tax on household income. Thus, households supply more labor, and equilibrium employment goes up, which requires an increase of the loan volume. Otherwise, firms would not be able to pay for the larger workforce.
4 The financial activities tax

A number of specific tax instruments have been proposed in the debates about value added taxation in the financial sector or about the financial sector’s contribution to paying for the cost of the 2008 financial crisis. The proposals include a financial activities tax, i.e. a tax on all labor or profit income generated in the financial sector. A tax on all cash flows generated in the financial sector has also been considered. We will discuss value added taxation more extensively in Section 5 below. Here, we focus on the financial activities tax. Specifically, we will show that, for normative questions, the consideration of simple tax systems – i.e. of tax systems that have only taxes on labor income and bank profits – is without loss of generality. We will show that any tax system that includes a financial activities tax is Pareto-dominated by a simple tax system.

The introduction of a financial activities tax, \( \tau_{fat} \), leaves the optimization problems of households and food producers unaffected. The problem of a bank now looks as follows: Maximize

\[
c_b = \frac{(1 - \tau_{fat})(1 - \tau_b)}{p_c} \left( p(l_b) p_l l_b - p_d d_b \right),
\]

subject to

\[
y_b = \kappa_d d_b + \kappa_l l_b.
\]

and

\[
d_b = l_b + p_w (1 + \tau_{fat}) y_b.
\]

The financial activities tax affects the banks in two ways: It reduces the bank’s after tax profits, for a given behavior. In addition, it affects the profit-maximizing supply of loans which follows from the first order condition

\[
R'(l) = q_b = \frac{p_d \frac{1}{p_l} + p_w (1 + \tau_{fat}) \kappa_l}{1 - p_w (1 + \tau_{fat}) \kappa_d}.
\]

The following Proposition, which is proven in the Appendix, contains the main result of this section: If we get rid of the financial activities tax, we can increase tax revenue without making anybody worse off. A corollary is that getting rid of the financial activities tax makes it possible to lower the tax on income or the tax on banking profits, thereby making everybody better off, without violating the government’s budget constraint. Thus, Pareto-efficient tax systems do not involve a financial activities tax.

**Proposition 4.1** Let \( \tau = (\tau_b, \tau_{fat}, \tau_w) \) be a tax system that includes a financial activities tax. There exists a simple tax system \( \tau^* = (\tau^*_b, \tau^*_w) \) so that

(a) Household utility, after tax profits in banking, and government consumption of food are the same:

\[
V(q_h^e(\tau)) = V(q_h^e(\tau^*)), \quad c_h^e(\tau) = c_h^e(\tau^*), \quad \text{and} \quad c_g^e(\tau) = c_g^e(\tau^*).
\]
(b) Public employment is higher under the simple tax system: \( g_q^e(\tau) < g_q^e(\tau^*) \).

The key steps in the proof are the following: We first show that the requirement that the households are equally well off under both tax systems implies that the equilibrium volume of loans, and the household’s labor supply have to be identical under both tax systems. If we then require, in addition, identical after-tax profits for bankers, we obtain, as an implication, identical revenues from the taxation of the banks’ profits for the government. Finally, from the perspective of banks, a financial activities tax affects the profit-maximizing loan supply in exactly the same way as an increase of the labor input requirements \( \kappa_d \) and \( \kappa_l \). Thus, if banks are supposed to generate the same volume of loans with and without a financial activities tax, they need a larger work force with a financial activities tax. Since labor supply is identical in both situations, the larger employment in banking crowds out public employment. Hence, it has to be the case that total tax revenue in \( t = 0 \) is lower with a financial activities tax.

5 Valued added taxation

The financial sector is exempt from value added taxation (VAT) in most OECD countries. This tax is sizable. Typical tax rates lie between 15 and 20 per cent. This raises a couple of questions: Do banks benefit from the exemption? Does the exemption contribute to a bigger financial sector? Are there other tax instruments that might be able to replicate a VAT on financial services? For instance, Huizinga (2002) suggests that a tax on the net cash flow into the financial sector would be a substitute for a VAT on financial services. Auerbach and Gordon (2002) by contrast, argue that a tax on the incomes that are generated in the financial sector would be the proper substitute.

In the following, we formalize these questions in the context of our model. We extend our analysis by introducing a value added tax on food purchases, denoted by \( \tau_{vat} c \). We also introduce a value added tax on financial services that we denote by \( \tau_{vat} b \). A tax system that exempts the financial sector from value added taxation has \( \tau_{vat} c > 0 \) and \( \tau_{vat} b = 0 \). A hypothetical tax system with an equal treatment of the financial sector and the rest of the economy is characterized by \( \tau_{vat} c = \tau_{vat} b > 0 \). We can then study the impact of the VAT exemption by comparing the competitive equilibria that arise under these two scenarios.

The financial sector engages in financial transactions with the households when acquiring deposits and with the food producers when granting loans. Let us first focus on the interaction between banks and households. We may consider that the tax \( \tau_{vat} b \) drives a wedge between the net price \( p_d \) that is realized by households when selling deposits to banks and the gross price \( p_d (1 + \tau_{vat} b) \) that banks have to pay. However, under a system in which a VAT system includes financial transactions and is collected using an invoice-credit method, the banks would then be able to deduct the VAT payment of \( p_d \tau_{vat} b \) from their tax bill. Thus, the price that is relevant both for households and banks
when entering into deposit contracts is simply \( p_d \), irrespectively of the level of the tax rate \( \tau_{vat}^b \). A similar logic applies to the banks’ interaction with food producers, provided that food producers can deduct their VAT payments. In this case, the relevant price for loan contracts is the net price \( p_l \) both from the perspective of banks and the perspective of food producers.

Real-world VAT systems are such that the final consumers of goods and services cannot deduct their VAT payments. By contrast, those who acquire goods and services as an input for the production of other goods or services can. If we apply this principle to a hypothetical VAT on financial services, then a household who acquires a loan so as to increase current consumption at the expense of future consumption should not be able to tax deduct. A firm that uses a loan to finance an investment should be able to do so. Our model cannot do full justice to this distinction. In particular, since our model is static we cannot get at financial intermediation between households who seek to exchange consumption opportunities over time. We can, however, analyze how the possibility to deduct VAT payments on loan contracts affects equilibrium outcomes. We will therefore distinguish two cases in what follows and show what competitive equilibria look like if food producers can deduct their VAT payments and if they cannot.

**Assumption 4** Food producers pay a VAT-inclusive price \( p_l \left( 1 + \tau_{vat}^b \right) \) for getting a one dollar loan, and can then deduct \( p_l \tau_{vat}^b \) from their tax bill.

**Assumption 5** Food producers pay a VAT-inclusive price \( p_l \left( 1 + \tau_{vat}^c \right) \) for getting a one dollar loan. These VAT-payments cannot be deducted.

In the following, we will first present results under Assumption 4 and then under Assumption 5.

Under Assumption 4, we can establish a neutrality result. The value of \( \tau_{vat}^b \) is inconsequential for the equilibrium allocation. In particular, the equilibrium outcome is the same under a tax system with \( \tau_{vat}^b = 0 \) and a tax system with \( \tau_{vat}^c = \tau_{vat}^b > 0 \). This follows from the possibility to deduct VAT-payments on intermediate goods. If banks have to pay a VAT-inclusive price, \( p_d \left( 1 + \tau_{vat}^c \right) \), for deposits and can, at the same time, deduct the VAT-payment, their behavior is as if there was no VAT to be paid when acquiring deposits and they were just facing the price \( p_d \). Likewise, if food producers have to pay a VAT-inclusive price, \( p_l \left( 1 + \tau_{vat}^c \right) \), for getting a loan and can then deduct the VAT-payment, their behavior is as if there was no VAT to be paid. The only agents who can not deduct their VAT-payments are the households and the bankers who buy food. Thus, only \( \tau_{vat}^c \) is relevant for the equilibrium characterization.

**Proposition 5.1** states this in a formal way. It characterizes the competitive equilibrium for a tax system consisting of an income tax \( \tau_w \), a tax on banking profits \( \tau_b \), a value added tax on food consumption \( \tau_{vat}^c \) and a value added tax on financial transactions \( \tau_{vat}^b \). It follows in a straightforward way from extending the analysis that led to
Proposition 2.1 by incorporating a value added tax on consumption goods and by the above arguments on the irrelevance of \( \tau_{vat}^b \). We therefore omit a formal proof.

**Proposition 5.1** Suppose that Assumptions 3 and 4 hold. Also suppose that either Assumption 1 or 2 holds. Let \( p_w = 1 \) and let there be given tax rates \( \tau_b, \tau_c^{vat}, \tau_b^{vat} \) and \( \tau_w \). Let \( p_c \) be the VAT-inclusive price of food paid by final consumers and let \( p_c(1 - \tau_c^{vat}) \) be the VAT-exclusive price received by food producers. For all \( \tau_b^{vat} \geq 0 \), an allocation \( a = (d, l, y, c) \) and a price system \( (q, p) \) are a competitive equilibrium if and only if they satisfy equations (34) - (40) below: The loan volume \( l \) solves

\[
 c_h^* \left( (1 - \tau_w)(1 - \tau_c^{vat}) \frac{1 - \kappa_d}{1 + \kappa_l} R'(l) \right) = (1 - \tau_c^{vat}) l R'(l). \tag{34}
\]

The volume of deposits satisfies (19). Household consumption of food is given by

\[
 c_h = c_h^* \left( (1 - \tau_w)(1 - \tau_c^{vat}) \frac{1 - \kappa_d}{1 + \kappa_l} R'(l) \right). \tag{35}
\]

Labor supply of households is given by

\[
 y_h = \frac{1}{(1 - \tau_w)(1 - \tau_c^{vat})} \frac{1 + \kappa_l}{1 - \kappa_d} \frac{1}{R'(l)} c_h^* \left( (1 - \tau_w)(1 - \tau_c^{vat}) \frac{1 - \kappa_d}{1 + \kappa_l} R'(l) \right). \tag{36}
\]

Labor employed by banks is given by (22). Food consumption by banks is given by (23). Government food consumption is given by

\[
 c_g = \tau_b \left( \rho(l) - R'(l) \right) l + \tau_c^{vat} \rho(l) l. \tag{37}
\]

Public employment is given by

\[
 y_g = \frac{\tau_w}{(1 - \tau_w)(1 - \tau_c^{vat})} \frac{1 + \kappa_l}{1 - \kappa_d} \frac{1}{R'(l)} c_h^* \left( (1 - \tau_w)(1 - \tau_c^{vat}) \frac{1 - \kappa_d}{1 + \kappa_l} R'(l) \right). \tag{38}
\]

Labor hired in food production is given by (26). The success probability of food producers is given by (27). The effective wage for the household is given by

\[
 q_h = (1 - \tau_w)(1 - \tau_c^{vat}) \frac{1 - \kappa_d}{1 + \kappa_l} R'(l). \tag{39}
\]

The effective cost for banks is given by (29). Relative prices are such that

\[
 \frac{p_l}{p_c} = 1 - \tau_c^{vat} \quad \text{and} \quad \frac{p_l}{p_d} = \frac{1 + \kappa_l}{1 - \kappa_d} \frac{1}{R'(l)}. \tag{40}
\]

The Proposition is noteworthy as a benchmark result: If those who demand financial services can deduct their VAT-payments, then the question whether or not financial services should be subject to value added taxation is uninteresting because such a tax has no impact whatsoever on the equilibrium allocation. Another interesting observation is that, even with an ideal VAT systems, so that \( \tau_b^{vat} = \tau_c^{vat} \) there may be profits in banking while there is no revenue from value added taxation in the financial sector. This
shows that there is no equivalence between a tax on value added in the financial sector, \( \tau_{vat}^{b} \), and a tax on profits, \( \tau_{p} \). The latter does generate tax revenue.

The following Proposition deals with a VAT on financial services under Assumption 5, so that food producers cannot deduct VAT payments on the loans that they acquire from banks. A formal proof requires a straightforward adaptation of the arguments in the proof of Proposition 2.1 and is therefore omitted.

**Proposition 5.2** Suppose that Assumptions 3 and 5 hold. Also suppose that either Assumption 1 or 2 holds. Let \( p_{w} = 1 \) and let there be given tax rates \( \tau_{b}, \tau_{vat}^{c}, \tau_{vat}^{w} \). Let \( p_{c} \) be the VAT-inclusive price of food paid by final consumers and let \( p_{c}(1 - \tau_{vat}^{c}) \) be the VAT-exclusive price received by food producers. An allocation \( a = (d,l,y,c) \) and a price system \((q,p)\) are a competitive equilibrium if and only if they satisfy equations (41) - (47) below: The loan volume \( l \) solves

\[
 c_{h}^{*} \left( \frac{(1 - \tau_{w})(1 - \tau_{vat}^{c})}{1 + \tau_{vat}^{c}} \right) \frac{1 - \kappa_{d}}{1 + \kappa_{l}} R'(l) = \frac{1 - \tau_{vat}^{c}}{1 + \tau_{vat}^{c}} l R'(l) . \tag{41}
\]

The volume of deposits satisfies (19).

Household consumption of food is given by

\[
c_{h} = c_{h}^{*} \left( \frac{(1 - \tau_{w})(1 - \tau_{vat}^{c})}{1 + \tau_{vat}^{c}} \right) \frac{1 - \kappa_{d}}{1 + \kappa_{l}} R'(l) . \tag{42}
\]

Labor supply of households is given by

\[
y_{h} = \frac{1 + \tau_{vat}^{c}}{(1 - \tau_{w})(1 - \tau_{vat}^{c})} \frac{1 + \kappa_{l}}{1 - \kappa_{d}} \frac{1}{R'(l)} c_{h} \left( \frac{(1 - \tau_{w})(1 - \tau_{vat}^{c})}{1 + \tau_{vat}^{c}} \right) \frac{1 - \kappa_{d}}{1 + \kappa_{l}} R'(l) . \tag{43}
\]

Labor employed by banks is given by (22). Food consumption by banks is given by (23). Government food consumption is given by

\[
c_{g} = \tau_{b}(\rho(l) - R'(l)) l + \left( 1 - \frac{1 - \tau_{vat}^{c}}{1 + \tau_{vat}^{c}} \right) \rho(l) l . \tag{44}
\]

Public employment is given by

\[
y_{g} = \frac{\tau_{w}(1 + \tau_{vat}^{c})}{(1 - \tau_{w})(1 - \tau_{vat}^{c})} \frac{1 + \kappa_{l}}{1 - \kappa_{d}} \frac{1}{R'(l)} c_{h} \left( \frac{(1 - \tau_{w})(1 - \tau_{vat}^{c})}{1 + \tau_{vat}^{c}} \right) \frac{1 - \kappa_{d}}{1 + \kappa_{l}} R'(l) . \tag{45}
\]

Labor hired in food production is given by (26). The success probability of food producers is given by (27). The effective wage for the household is given by

\[
 q_{h} = \frac{(1 - \tau_{w})(1 - \tau_{vat}^{c})}{1 + \tau_{vat}^{c}} \frac{1 - \kappa_{d}}{1 + \kappa_{l}} R'(l) . \tag{46}
\]

The effective cost for banks is given by (29). Relative prices are such that

\[
 \frac{p_{l}}{p_{c}} = \frac{1 - \tau_{vat}^{c}}{1 + \tau_{vat}^{c}} \quad \text{and} \quad \frac{p_{l}}{p_{d}} = \frac{1 + \kappa_{l}}{1 - \kappa_{d}} \frac{1}{R'(l)} . \tag{47}
\]
The Proposition 5.2 shows that $\tau_{vat}^v$ is no longer neutral under the assumption that food producers cannot deduct their VAT payments. More specifically, if VAT payments cannot be deducted an increase of $\tau_{vat}^v$ has similar implications as an increase of $\tau_{vat}^c$. What matters for the equilibrium outcome is only the ratio $\frac{1-\tau_{vat}^c}{1+\tau_{vat}^v}$. An increase of $\tau_{vat}^c$ makes this expression smaller, and the same is true for an increase of $\tau_{vat}^v$.

Unless we impose additional assumptions the comparative statics effects of an increase of $\tau_{vat}^v$ or $\tau_{vat}^c$ taxes are difficult to sign. The following Proposition imposes additional assumptions on the household’s utility function to get clear-cut results.

**Proposition 5.3** Suppose that the conditions in Proposition 5.2 hold. Also suppose that the household’s utility function is given by

$$u(c_h, y_h) = c_h - \frac{1}{2}y_h^2.$$  

Then, a marginal increase of $\tau_{vat}^v$ or $\tau_{vat}^c$ has the following implications:

The equilibrium loan volume, employment in banking and food production, food output, household consumption, labor supply, household utility and the bank-owner’s consumption all go down, for all $k \in \{b, c\}$:

$$\frac{\partial l_{eq}^{eq}(\tau)}{\partial \tau_{vat}^v} < 0, \quad \frac{\partial y_{eq}^{eq}(\tau)}{\partial \tau_{vat}^v} < 0, \quad \frac{\partial y_{eq}^{eq}(\tau)}{\partial \tau_{vat}^c} < 0, \quad \frac{\partial (\rho(l_{eq}^{eq}(\tau)) y_{eq}^{eq}(\tau))}{\partial \tau_{vat}^v} < 0,$$

$$\frac{\partial c_{eq}^{eq}(\tau)}{\partial \tau_{vat}^v} < 0, \quad \frac{\partial y_{eq}^{eq}(\tau)}{\partial \tau_{vat}^c} < 0, \quad \frac{\partial V(q_{eq}^{eq}(\tau))}{\partial \tau_{vat}^v} < 0 \quad \text{and} \quad \frac{\partial c_{eq}^{eq}(\tau)}{\partial \tau_{vat}^c} < 0.$$

The average quality of loans goes up: $\frac{\partial (\rho(l_{eq}^{eq}(\tau)))}{\partial \tau_{vat}^k} \geq 0$, for all $k \in \{b, c\}$.

We sketch the key step of the proof: The assumption on the household’s preferences implies that $c_{h}^v(q_{h}) = y_h^2$. Equation (41) which characterizes the equilibrium loan volume can therefore, after some algebra, be written as

$$1 - \tau_{vat}^v \left( 1 - \frac{1}{\tau_{vat}^c} \right)^2 = \frac{l_{eq}^{eq}}{R'(l_{eq}^{eq})}.$$  

Obviously, the right-hand-side is an increasing function of $l_{eq}^{eq}$. Thus, if the left hand-side goes down due to an increase of $\tau_{vat}^v$ or $\tau_{vat}^c$, then $l_{eq}^{eq}$ must go down as well. The other statements in the Proposition then follow in a straightforward way from our equilibrium characterization which expresses all endogenous variables as functions of the equilibrium loan volume.

Propositions 5.1 and 5.2 imply in particular, that there is no equivalence between an hypothetical ideal value added tax system that has $\tau_{vat}^v = \tau_{vat}^c$ and a system with a larger tax on the bank’s profits. If food producers can deduct their VAT on financial services, the change of the tax rate $\tau_{vat}^v$ does not affect equilibrium outcomes, whereas an increase of the tax on the banks’ profits $\tau_{b}$ reduces the banks’ after tax profits and increases the government’s tax revenue, without affecting the households. If VAT payments cannot be
deducted, then an increase of $\tau_{vat}^b$ makes households worse off, and reduces the banks’ profits. An increase of $\tau_b$, by contrast does not affect households, but makes the bank-owners worse off.

We can also relate a financial activities tax (FAT) to a VAT on financial services. As follows from our equilibrium characterization for a tax system that includes a FAT, see Lemma 1 in the Appendix, and from Proposition 5.2 the two are not equivalent. A FAT has two effects, it is equivalent to an increase of the banks’ labor input requirements $\kappa_l$ and $\kappa_d$ and a simultaneous increase of a tax on the bank’s profits. Qualitatively, the implications of an increase of $\tau_{vat}^c$ under Assumption 5 are akin to those of an increase of $\kappa_l$ and $\kappa_d$, which is similar to the first effect of a FAT increase. There is, however, no analogue to the second effect of a FAT increase. So, a FAT is not a perfect substitute for a VAT on financial services.

The observation in Proposition 5.2 that the whole equilibrium allocation depends only on the ratio $\frac{1-\tau_{vat}^c}{1+\tau_{vat}^b}$ implies moreover, that to any VAT system $(\tau_{vat}^c, \tau_{vat}^b)$ with a tax on financial services, there exists an equivalent VAT system $(\bar{\tau}_{vat}^c, \bar{\tau}_{vat}^b)$ with $\bar{\tau}_{vat}^b = 0$ and

$$\bar{\tau}_{vat}^c = 1 - \frac{1 - \tau_{vat}^c}{1 + \tau_{vat}^b}.$$ 

Thus, under Assumption 5, the tax that can be used to replicate an ideal VAT on financial services is the VAT on consumption goods.

6 Concluding Remarks

In this paper, we investigate a simple general equilibrium model with a profitable financial sector. We use this model for an analysis of how various taxes that are under debate since the 2008 financial crises would affect real economic activity, the size of the financial sector, its profits and the well-being of households. The existing literature on this topic neglects the question of how profit margins in banking are determined in equilibrium and how these equilibrium outcomes respond to taxation. Our analysis is a first step towards a more complete analysis of tax incidence in models that include a profitable financial sector.

Our analysis yields the following key insights: First, a higher tax on the profits in banking will enable policy makers to reduce taxes on household income and the consequence will be an increase of economic activity as measured by employment or output. Moreover, this will also come with an increased volume of financial sector activity, as measured by the equilibrium loan volume or the employment in the financial sector. The reason for this effect is that the profit tax is non-distortive as compared to the income tax. It thus allows for greater economic output.

Second, the exemption of the financial sector from value added taxation can not be viewed as being beneficial for bankers. We discuss various approaches towards modeling
an ideal value added tax system that includes the financial sector, and discuss the implications of moving from the current system to the ideal system. This analysis does not support the general presumption that the exemption contributes to an increased size of the financial sector or to excess returns in banking.

Our analysis has left out market failures in banking such as excessive risk-taking or regulatory arbitrage. Including these will be important in future research. This paper, by contrast, studied an environment in which the first welfare theorem holds so that there is no role for corrective taxation. This provides a benchmark which has been missing, since the classical treatments of tax structures in multi-sector models such as Ramsey (1927) and Diamond and Mirrlees (1971) did neither include an intermediate sector that generates economic rents nor an explicit modelling of financial intermediation.

References


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A Appendix

Definition 1 (Competitive Equilibrium) Given a tax system $\tau = (\tau_w, \tau_b)$, an allocation $a = (d, l, y, c)$ and a price system $(q, p)$ are a competitive equilibrium if they satisfy equations (48) - (56) below: There are zero profits in food production,

$$q_f = \frac{p_c}{p_w p_l} = 1 \quad \text{and} \quad l = p_w y_f .$$

(48)

Food producers repay their loans with probability

$$\alpha = \rho(l) .$$

(49)

The labor market clears,

$$y_b + y_f + y_g = y_h .$$

(50)

The food market clears,

$$c_b + c_g + c_h = \alpha y_f .$$

(51)

The “effective” prices $q_h$ and $q_b$ are such that

$$q_h = \frac{p_d p_w (1 - \tau_w)}{p_c} .$$

(52)

and

$$q_b = \frac{p_d}{p_l} \frac{1 + p_w \kappa_l}{1 - p_w \kappa_d} .$$

(53)

Moreover, at the given price system, the households choose $y_h, c_h$ and $d$, i.e.

$$c_h = c_h^*(q_h) , \quad y_h = y_h^*(q_h) \quad \text{and} \quad d = d_h^* \left( \frac{p_c}{p_d}, q_h \right) .$$

(54)

and the banks choose $y_b, c_b, l$ and $d$,

$$c_b = c_b^* \left( \frac{p_l}{p_c}, \frac{p_d}{p_c}, q_b \right) , \quad y_b = y_b^*(p_w, q_b) , \quad l = l_b^*(q_b) \quad \text{and} \quad d = d_b^* \left( \frac{p_l}{p_d}, q_b \right) .$$

(55)

Finally, government consumption of food and labor is consistent with the government’s tax revenue,

$$c_g = c_g^* \left( \tau_b, \frac{p_l}{p_c}, \frac{p_d}{p_c}, q_b \right) \quad \text{and} \quad y_g = y_g^*(\tau_w, p_w) .$$

(56)
A.1 Proof of Proposition 2.1

Part 1. We first show that if an allocation satisfies (48) - (56), then it also satisfies (18) - (30).

The requirement that $\alpha = \rho(l)$ is part of the definition of a competitive equilibrium, see equation (49). Trivially, equation (27) holds as well.

In a competitive equilibrium it has to be the case that $l = l_b^*(q_b)$. From the bank’s profit-maximization problem it follows that $l_b^*(q_b)$ is implicitly defined by the equation $R'(l_b^*(q_b)) = q_b$. Thus, if $l$ is part of a competitive equilibrium allocation then $R'(l) = q_b$, which proves that (29) holds.

By (48), in a competitive equilibrium it has to be the case that $q_f = 1$. Given $p_w = 1$, this implies that $\frac{p_l}{p_c} = 1$. Combining $R'(l) = q_b$ and equation (53) yields $\frac{p_l}{p_d} = \frac{1 + \kappa l}{1 - \kappa d} R'(l)$. Thus (30) holds.

In a competitive equilibrium the prices $q_h$ and $q_b$ satisfy (52) and (53). Using that $p_w = 1$ and that $\frac{p_l}{p_c} = 1$, this implies that

$$q_h = (1 - \tau_w) \frac{1 - \kappa_d}{1 + \kappa_l} q_b .$$

With $R'(l) = q_b$, this yields

$$q_h = (1 - \tau_w) \frac{1 - \kappa_d}{1 + \kappa_l} R'(l) ,$$

which proves that (28) holds.

In a competitive equilibrium it also has to be the case that $c_h = c_h^*(q_h)$, or that

$$c_h = c_h^* \left( (1 - \tau_w) \frac{1 - \kappa_d}{1 + \kappa_l} R'(l) \right) ,$$

which proves that (20) holds.

If $d$ is part of a competitive equilibrium, then by (4) and (54),

$$c_h^* \left( (1 - \tau_w) \frac{1 - \kappa_d}{1 + \kappa_l} R'(l) \right) = c_h = \frac{p_d}{p_l} d . \quad (57)$$

Also, by (10) and (55),

$$d = \frac{p_l}{p_d} q_b \ l = \frac{p_l}{p_d} R'(l) \ l . \quad (58)$$

Equations (57) and (58) imply that

$$c_h^* \left( (1 - \tau_w) \frac{1 - \kappa_d}{1 + \kappa_l} R'(l) \right) = R'(l) \ l , \quad (59)$$

i.e. that equation (18) holds.

In a competitive equilibrium it has to be the case that

$$d = d_b^* \left( \frac{p_l}{p_d}, q_b \right) = \frac{p_l}{p_d} q_b \ l = \frac{1 + \kappa_l}{1 - \kappa_d} \ l .$$
where the first equality follows from (55), the second equality uses (11) and, again, (55), and the third equality uses $p_w = 1$ and (53). This shows that equation (19) holds.

From the household’s budget constraint it follows that in a competitive equilibrium $y_h = \frac{1}{q_h} c_h$. Combining (28) and (20) yields (21). From equations (10) and (55) it follows that (22) holds. To see that (23) holds, note that (12), (55), (30), (19) and (29) imply that

$$\begin{align*}
c_b &= \frac{\rho(l) p_l l - p_d d(1 - \tau_b)}{p_c} \\
&= \left(\frac{\rho(l) l - \frac{p_d}{p} d}{p_c}\right)(1 - \tau_b) \\
&= \left(\frac{\rho(l) l - q_b l}{p_c}\right)(1 - \tau_b) \\
&= \left(\frac{\rho(l) - R'(l)}{l}\right) l (1 - \tau_b).
\end{align*}$$

If combined with (16), (13), (56), the same equations imply that (24) holds. Analogously, equation (25) follows from (56), (3), $p_w = 1$, (30), (13) and (14).

Finally, we need to show that (26) holds. From (50) it follows that

$$y_f = y_h - y_g - y_b.$$

Using (21), (25), and (22) this becomes

$$y_f = \frac{1 + \kappa_l}{1 - \kappa_d} \frac{1}{R'(l)} c_h \frac{\kappa_d + \kappa_l}{1 - \kappa_d} l.$$

Upon using (18) and $c_h = c_h^*(q_h)$ this simplifies to $y_f = l$, which proves that (26) holds.

**Part 2.** We now show that equations (18) - (30) imply that equations (48) - (56) hold.

Trivially, equation (27) implies that equation (49) holds. Equations (30), (26) and $p_w = 1$ imply that (48) holds. Equations (30), (28) and $p_w = 1$ imply that (52) holds. Equations (30), (29) and $p_w = 1$ imply that (53) holds.

Given these price relations, we leave it to the reader to verify that $c_h$, $y_h$, $d$, $l$, $y_b$, and $c_b$, are consistent with optimizing behavior by banks and household, respectively, i.e. that (54) and (55) hold. We also leave it to the reader to verify that government consumption of food and labor is consistent with the behavior of banks and households so that (56) holds.

It remains to be shown that the labor market and the food market clear. To see that the food market clears, note that (20), (18), (23), and (24) imply that

$$c_b + c_g + c_h = \rho(l) l.$$

With $\rho(l) = \alpha$ and $l = y_f$ this becomes

$$c_b + c_g + c_h = \alpha y_f,$$

which proves that (51) holds.

Labor market clearing can be verified as follows: Equations (18), (21) and (25) imply that

$$y_h - y_g = \frac{1 + \kappa_l}{1 - \kappa_d} l.$$

27
From \(y_f = l\) and (22) it follows that
\[
y_h + y_f = \frac{1 + \kappa_l}{1 - \kappa_d} l.
\]
Hence, \(y_h - y_g = y_b + y_f\), which proves that (50) holds. \(\square\)

A.2 Proof of Proposition 4.1

The following Lemma provides the equilibrium characterization if there is a tax system \(\tau = (\tau_b, \tau_{fat}, \tau_w)\) that includes a financial activities tax. It is a straightforward adaptation of Proposition 2.1 so that we omit a proof.

**Lemma 1** Suppose that Assumption 3 holds. Also suppose that either Assumption 1 or 2 holds. Let \(p_w = 1\) and let there be a given tax policy \(\tau = (\tau_b, \tau_{fat}, \tau_w)\) that includes a financial activities tax. An allocation \(a = (d, l, y, c)\) and a price system \((q, p)\) are a competitive equilibrium if and only if they satisfy equations (60) - (72) below: The loan volume \(l\) solves
\[
c_h^* \left( (1 - \tau_w) \frac{1 - (1 + \tau_{fat}) \kappa_d}{1 + (1 + \tau_{fat}) \kappa_l} R'(l) \right) = l R'(l).
\]
(60)

The volume of deposits satisfies
\[
d = \frac{1 + (1 + \tau_{fat}) \kappa_l}{1 - (1 + \tau_{fat}) \kappa_d} l.
\]
(61)

Household consumption of food is given by
\[
c_h = c_h^* \left( (1 - \tau_w) \frac{1 - (1 + \tau_{fat}) \kappa_d}{1 + (1 + \tau_{fat}) \kappa_l} R'(l) \right) .
\]
(62)

Labor supply of households is given by
\[
y_h = \frac{1}{(1 - \tau_w)} \frac{1 + (1 + \tau_{fat}) \kappa_l}{1 - (1 + \tau_{fat}) \kappa_d} \frac{1}{R'(l)} c_h^* \left( (1 - \tau_w) \frac{1 - (1 + \tau_{fat}) \kappa_d}{1 + (1 + \tau_{fat}) \kappa_l} R'(l) \right) .
\]
(63)

Labor employed by banks is given by
\[
y_b = \frac{\kappa_d + \kappa_l}{1 - (1 + \tau_{fat}) \kappa_d} l.
\]
(64)

Food consumption by banks is given by
\[
c_b = (1 - \tau_b)(1 - \tau_{fat})(\rho(l) - R'(l))) l.
\]
(65)

Government food consumption is given by
\[
c_g = (1 - (1 - \tau_b)(1 - \tau_{fat}))(\rho(l) - R'(l))) l.
\]
(66)

Public employment is given by
\[
y_g = \frac{\tau_w}{(1 - \tau_w)} \frac{1 + (1 + \tau_{fat}) \kappa_l}{1 - (1 + \tau_{fat}) \kappa_d} \frac{1}{R'(l)} c_h^* \left( (1 - \tau_w) \frac{1 - (1 + \tau_{fat}) \kappa_d}{1 + (1 + \tau_{fat}) \kappa_l} R'(l) \right) .
\]
(67)
Labor hired in food production is given by
\[ y_f = l. \] (68)

The success probability of food producers is given by
\[ \alpha = \rho(l). \] (69)

The effective wage for the household is given by
\[ q_h = (1 - \tau_w) \frac{1 - (1 + \tau_{fat}) \kappa_d}{1 + (1 + \tau_{fat}) \kappa_l} R'(l). \] (70)

The effective cost for banks is given by
\[ q_b = R'(l). \] (71)

Relative prices are such that
\[ \frac{p_l}{p_c} = 1 \quad \text{and} \quad \frac{p_l}{p_d} = \frac{1 + (1 + \tau_{fat}) \kappa_l}{1 - (1 + \tau_{fat}) \kappa_d} \frac{1}{R'(l)}. \] (72)

We now turn to the proof of Proposition 4.1. Let there be a given tax system $\tau = (\tau_b, \tau_{fat}, \tau_w)$ and the corresponding equilibrium as characterized in Lemma 1. We now construct a simple tax system $\tau_s = (\tau_{sb}, \tau_{sw})$ so that the effective wage for the household $q_h$ and the after-tax-profits of banks are identical under both tax systems. Note that $q_{eq}^h(\tau) = q_{eq}^h(\tau_s)$ is both necessary and sufficient for $V_h(q_{eq}^h(\tau)) = V_h(q_{eq}^h(\tau_s))$. We then study the implications of the simple tax system $\tau_s$ for the government’s tax revenue.

The requirements $q_{eq}^h(\tau) = q_{eq}^h(\tau_s)$ and $c_{eq}^b(\tau) = c_{eq}^b(\tau_s)$ give us two equations in two unknowns, namely the tax rates $\tau_{sb}$ and $\tau_{sw}$.

Step 1. We first note that $q_{eq}^h(\tau) = q_{eq}^h(\tau_s)$ implies that the equilibrium loan volume is the same under tax system $\tau$ and under tax system $\tau_s$, i.e. $l^q(\tau) = l^q(\tau_s)$. To see this, note that Proposition 2.1 and Lemma 1 imply that
\[ c_h^*(q_{eq}^h(\tau)) = l^q(\tau) R'(l^q(\tau)), \]
and
\[ c_b^*(q_{eq}^h(\tau_s)) = l^q(\tau_s) R'(l^q(\tau_s)). \]

Now $q_{eq}^h(\tau) = q_{eq}^h(\tau_s)$ implies that
\[ l^q(\tau) R'(l^q(\tau)) = l^q(\tau_s) R'(l^q(\tau_s)) \]
By Assumptions 1 and 2, the function $f(l) = l R'(l)$ is strictly increasing. Hence, $f(l^q(\tau)) = f(l^q(\tau_s))$ implies $l^q(\tau) = l^q(\tau_s)$. We will simply write $l^q$ in the following.

Step 2. We now show that the government’s food consumption is unaffected if we move from $\tau$ to $\tau_s$. Proposition 2.1 and Lemma 1 imply that
\[ c_b^q(\tau) = (1 - \tau_b)(1 - \tau_{fat})(\rho(l^q) - R'(l^q)) l^q, \]
and
\[ c_b^{eq}(\tau^s) = (1 - \tau^s_b)(\rho(l^{eq}) - R'(l^{eq})) l^{eq}. \]

Hence, the requirement that \( c_b^{eq}(\tau) = c_b^{eq}(\tau^s) \) implies
\[ (1 - \tau_b)(1 - \tau_{fat}) = 1 - \tau^s_b. \] (73)

Proposition 2.1 and Lemma 1 also imply that
\[ c_g^{eq}(\tau) = (1 - (1 - \tau_b)(1 - \tau_{fat}))(\rho(l^{eq}) - R'(l^{eq})) l^{eq}, \]
and
\[ c_g^{eq}(\tau^s) = \tau^s_b(\rho(l^{eq}) - R'(l^{eq})) l^{eq}. \]

These two equations in conjunction with (73) imply that
\[ c_g^{eq}(\tau) = c_g^{eq}(\tau^s). \]

**Step 3.** We finally show that public employment is higher under the simple tax system, i.e. that \( y_g(\tau^s) > y_g(\tau) \). Proposition 2.1, Lemma 1 and \( q_h^{eq}(\tau) = q_h^{eq}(\tau^s) \) imply that
\[ y_h^{eq}(\tau) = y_h^{eq}(\tau^s), \]

i.e. household labor supply is the same under both tax systems. Proposition 2.1, Lemma 1 and \( l^{eq}(\tau) = l^{eq}(\tau^s) \) also imply that
\[ y_f^{eq}(\tau) = y_f^{eq}(\tau^s), \]
and
\[ y_b^{eq}(\tau) > y_f^{eq}(\tau^s), \]

i.e. employment in food production is the same under both tax systems, but employment in banking is higher with a financial activities tax. In a competitive equilibrium it has to be the case that the labor market clears. Thus
\[ y_g^{eq}(\tau) = y_h^{eq}(\tau) - y_b^{eq}(\tau) - y_f^{eq}(\tau), \]
and
\[ y_g^{eq}(\tau^s) = y_h^{eq}(\tau^s) - y_b^{eq}(\tau^s) - y_f^{eq}(\tau^s), \]

Obviously, \( y_h^{eq}(\tau) = y_h^{eq}(\tau^s), y_f^{eq}(\tau) = y_f^{eq}(\tau^s) \), and \( y_b^{eq}(\tau) > y_f^{eq}(\tau^s) \) imply that \( y_g^{eq}(\tau) < y_g^{eq}(\tau^s) \).  \(\square\)